Intermediation with Costly Bilateral Exchange

ROBERT M. TOWNSEND
Carnegie-Mellon University

1. INTRODUCTION

This paper was motivated in part by questions of optimal regulation of financial institutions such as intermediaries. It is believed that an analysis of such questions should take place in a model in which financial institutions emerge endogenously as the choice of agents of the model. Yet there appear to be few models which explain institutions in this way.

The absence of such models is due in part to the acceptance of what are often thought of as necessary conditions for tractable mathematical analysis. In particular, speaking rather loosely, the technologies of production and exchange are usually assumed to be convex. This is true of the standard competitive model as described for example in Arrow–Hahn (1971). It is also true of recent attempts to obtain a theory of more limited exchange as in Foley (1970), Hahn (1971) and Kurz (1974). In these analyses convexity is used in establishing the continuity of choice functions upon which standard fixed point theorems rely. Yet, to repeat, convexity prevents the models from explaining specialization.

There have been several attempts to weaken the convexity assumptions. In Starr (1969) and in Arrow–Hahn (1971) non-convex sets are replaced by their convex hulls, standard existence theorems are applied to the convexified economy, and the excess demands of the resulting social-approximate equilibrium are then shown to have limit zero as the number of agents is increased. The thrust of this work is that the competitive equilibrium is a good approximation for non-convex economies with a large number of agents. This approach has been modified somewhat by Heller (1972) and by Heller and Starr (forthcoming) to allow for various strong non-convexities associated with set-up costs and to explain the use of money.

This paper proposes a simple theory of intermediation and, in contrast to the literature described above, circumvents the difficulties associated with non-convex technologies by a game theoretic approach. A model is described in which exchange is motivated by risk-sharing considerations. But exchange is also resource using; for each bilateral deal there is a fixed cost. This creates the non-convexity.[1] Then, having specified the set of agents, their preferences and the technology available to them, core allocations are characterized. As noted by Shapley and Shubik (1966), core allocations are in some sense socially stable; one might expect to observe institutions and exchange arrangements which support such allocations.[2] Indeed, a non-cooperative game is proposed for the model, and it is shown that the equilibrium allocations of the game are equivalent to core allocations. In this non-cooperative game agents announce strategies under which they are willing to act as intermediaries, buying shares in investment projects and selling shares in the resulting portfolio. In an equilibrium only a subset of agents act as intermediaries, and in this sense the model may be said to explain specialization. These intermediaries emerge endogenously because they are able to economize on the fixed cost of exchange.

The game theoretic approach adopted here also enables one to dispense with the Walrasian auctioneer. Agents of the model propose the exchange arrangements. Moreover the strategies of active intermediaries may include per unit prices with fixed fees or a bid–ask spread with minimum purchase requirements. In this sense the equilibrium of the non-cooperative game may be viewed as a generalization of the notion of a competitive equilibrium which allows a much richer specification of the terms of trade.

This paper proceeds as follows. The structure and technology of the model is described in Section 2. Then markets, feasible allocations, core allocations and intermediaries are defined. Core allocations are characterized in Section 3. In Section 4 a non-cooperative game with intermediation strategies is described, and the equivalence of equilibrium allocations and core allocations is established. Section 5 offers some comments on the strengths and limitations of the model.

2. THE STRUCTURE AND TECHNOLOGY OF THE MODEL

Let I denote the set of agents. It is assumed that I is countable infinite. Each agent $j \in I$ is endowed with a quantity of the unique factor of production of the model, K^j. The endowments of this capital good are identical for all agents and perfectly divisible. Each agent is also endowed with a stochastic technology which transforms the capital good into a distribution of the unique consumption good of the model. Each of these technologies or investment projects displays constant return to scale. Let λ^j denote the output of the consumption good per unit of the capital input y_j in project j. Each λ^j is a random variable which can take on a finite number of values in R^1_+. Also, the $\lambda^j, j \in I$, are assumed to be independent and identically distributed. A state of the economy subsequent to the realization of the λ^j is a complete specification of the value of each of the $\lambda^j, j \in I$. The set of all possible states will be denoted Ω with typical element ω. Let $\mu(\omega)$ denote the probability that state ω will occur.

The preferences of each agent are assumed to conform to the Von Neumann–Morgenstern axioms so that each has objective function

$$V^j(c^j) = \int_{\omega \in \Omega} \mu(\omega) U^j[c^j(\omega)].$$

Here c^j is a random variable with typical element $c^j(\omega) \in R^1_+$, the consumption of agent j in state ω. It is further assumed that the $U^j(\cdot)$ are identical across agents and display constant relative risk aversion. (The superscript j on $U^j(\cdot)$ and $V^j(\cdot)$ will be deleted if no ambiguity results.)

The model is thus constructed in such a way that there are gains to exchange or portfolio diversification. In particular note that agents are risk averse and the λ^i are independent and identically distributed. In contrast, if the λ^i were perfectly correlated, then no risk spreading could be achieved.

Exchange in the model is assumed to be costly. For each bilateral deal a fixed cost of 2α units of the capital good is incurred, α per agent. Note that this cost is independent of the nature of the exchange. Thus if there were three agents and each agent were trading with every other agent as in Figure 1(a), total transactions costs for the three bilateral

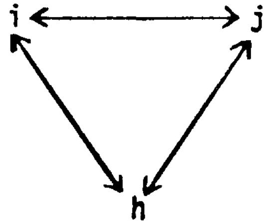

FIGURE 1(a)

Exchange without intermediation.

exchanges would be 6α. Transactions costs could be reduced to 4α if one agent were to act as a go-between or intermediary for the other two, as indicated in Figure 1(b).

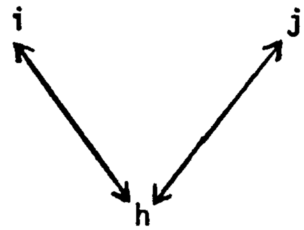

FIGURE 1(*b*)

Exchange with intermediation.

It remains to describe the set of feasible allocations given this transactions technology. Let $C \subseteqq I$ denote a coalition with $\# C$ agents. Let N^j denote the set of agents with whom agent j deals directly. Then a coalition C is said to constitute a *market* if $\# C < \infty$; if for each $j \in C$, $N^j \subseteqq C$; and if there exists no proper subset A of C such that for each $j \in A$, $N^j \subseteqq A$. Thus a market is defined to be the smallest set of agents for which every agent of the set deals with other agents of the set and with no agent outside that set. Let $\eta(M)$ denote the number of bilateral exchanges in a market M. Hence

$$(\# M - 1) \leqq \eta(M) \leqq (\# M)(\# M - 1)/2.$$

Then an allocation $\{c^j, y^j; j \in M\}$ is said to be *feasible for market M* if

$$\Sigma_{j \in M}(K^j - y^j) \geqq (2\alpha)\eta(M)$$

and $\Sigma_{j \in M}\lambda^j(\omega)y^j \geqq \Sigma_{j \in M}c^j(\omega)$, $\omega \in \Omega$. Thus an allocation is said to be feasible for a market if it can be achieved with the resources and technologies of agents of the market, taking into account the resource costs of exchange. An allocation $\{c^j, y^j; j \in C\}$ is said to be *feasible for a coalition C* if there exists a set of markets $\mathscr{A}$ such that $\cup_{M \in \mathscr{A}} M = C$ and the allocation $\{c^j, y^j; j \in M\}$ is feasible for each market $M \in \mathscr{A}$.[3]

The *core* for the economy I is the set of allocations which are feasible for I and which are not blocked by any coalition. An allocation $\{c^j, y^j; j \in I\}$ is said to be *blocked* by a coalition B if there exists an allocation $\{c^j_*, y^j_*; j \in B\}$ which is feasible for B such that $V^j(c^j_*) > V^j(c^j)$ for each agent $j \in B$. Thus an allocation is said to be in the core if there exists no coalition which can do better with its own resources and technology. In a strong sense this definition of the core is standard. However, it may be noted again that core allocations take into consideration the resource costs of exchange. Viewed in another way, this definition takes into consideration the costs of coalition formation.

Finally a formal definition of intermediation is needed. Agent h is said to act as an *intermediary for a market M* if $N^j = h$ for $j \neq h, j \in M$, and $N^h = M - h$.

3. CHARACTERIZATION OF THE CORE: A COOPERATIVE ECONOMY

As has been noted by many authors, the core may be viewed as a cooperative solution to the resource allocation problem. This suggests that the search for core allocations may be facilitated by a characterization of the allocations of a cooperative game. Such an approach is adopted in this section. First a cooperative game is described, and cooperative allocations are analysed. These are shown in Propositions 1–3 to be equivalent to core allocations.

In the cooperative economy specified agents are designated as intermediaries. Each intermediary selects a group of agents for projects in his portfolio. These sets are assumed to be disjoint so that in effect intermediaries are forming markets. Each agent in a market agrees to sell shares in his project to the intermediary for a price of one in terms of the capital good, and the intermediary sells shares in his portfolio for a price of one in terms of the capital good. A share in the output of project j entitles the holder to λ^j units of the consumption good. A share in the portfolio of an intermediary of market M entitles the holder to $\Sigma_{i \in M} \lambda^i / \# M$ units of the consumption good. All transactions costs are shared equally by all agents in a given market. With $(\# M - 1)$ bilateral exchanges in M, these transactions costs will be $(\# M - 1)(2\alpha)$. In these circumstances agents in a market will trade shares with the intermediary on a one-to-one basis up to the limits of their initial endowments, less transactions costs. Each intermediary determines the number of agents in his market, and each will act to maximize.

$$EU\left\{\left[K^j - \frac{(2\alpha)(\# M - 1)}{\# M}\right][\Sigma_{i \in M}\lambda^i / \# M]\right\} \quad \ldots(1)$$

with respect to $\# M$.

Let Γ denote the set of positive integers which maximize (1). For example, if $\{1\} \in \Gamma$, there can be an autarkic allocation in which each agent claims the return only on his own project. The set Γ need not be a singleton; hence each market need not have the same number of agents. Also Γ may be empty; that is, there need not exist a positive integer which maximizes (1). As the size of the market increases, the marginal gain from increased diversification goes to zero for by the law of large numbers $\sum_{i=1}^{n} \lambda^i / n \to E(\lambda^i)$ almost surely as $n \to \infty$; but the marginal per-capita transactions cost also goes to zero as $(2\alpha)(n-1)/n \to 2\alpha$. However, in what follows it will be assumed that $\Gamma \neq \emptyset$, so it is important to examine the generality of this assumption. It is easily shown that if $K^j \leqq 2\alpha$, then $\Gamma \neq \emptyset$; otherwise per-capita transactions costs would equal or exceed endowments in the limit as the number of agents increases. There also exist economies in which $K^j > 2\alpha > 0$ and $\Gamma \neq \emptyset$. For example let $U(W) = W^b$. Suppose that for each $b \in (0, 1)$, $\Gamma = \emptyset$ and consider the subsequence $\{n_v\}$ such that (1) is strictly increasing. By taking the limit of (1) as $v \to \infty$ and then letting $b \to 1$, a contradiction can be obtained.

The equivalence of the core and the allocations of the cooperative economy with $\Gamma \neq \emptyset$ can now be readily established.

Proposition 1. *If $\Gamma \neq \emptyset$, the allocations of the cooperative economy are in the core.*

Proof. Suppose a coalition B were able to block an allocation of the cooperative economy; without loss of generality B may be taken to be a market. Consider exchange, production, and consumption decisions which result in Pareto optimal allocations for B; an allocation is Pareto optimal for B if it is feasible for B and if no agent of B can be made better off without making some agent of B worse off. As $U(\cdot)$ displays constant relative risk aversion, a necessary condition for a consumption allocation to be optimal for B is that $c^i(\omega) = \theta^i \Sigma_{j \in B} c^j(\omega)$ for each $\omega \in \Omega$ and for each $i \in B$ with $0 \leqq \theta^i \leqq 1$, $\Sigma_{i \in B} \theta^i = 1$. Then to determine an optimal production allocation it is sufficient to maximize

$$\int_{\omega \in \Omega} \mu(\omega) U[c^i(\omega)] = \gamma_1 + \gamma_2 \int_{\omega \in \Omega} \mu(\omega) U[\Sigma_{j \in B} c^j(\omega)]$$

for some positive constants γ_1 and γ_2 for any i with $\theta^i \neq 0$. As $U(\cdot)$ is strictly concave, equal amounts should be invested in all projects of B. With total transactions costs minimized at $(2\alpha)(\# B - 1)$,

$$y^j = K^j - \frac{(2\alpha)(\# B - 1)}{\# B}$$

for each $j \in B$. Consider a symmetric allocation of the consumption good over states so that $\theta^j = 1/\# B$ for each $j \in B$. Then

$$c^j(\omega) = \left[K^j - \frac{(2\alpha)(\# B - 1)}{\# B}\right][\Sigma_{i \in B} \lambda^i(\omega) / \# B].$$

By construction this allocation is Pareto optimal for B, and all agents of B have a level of expected utility which is at most equal to the level of the cooperative allocation. Hence, the coalition B could not block the cooperative allocation. Note this establishes that with $\Gamma \neq \varnothing$, the core is non-empty. ||

Proposition 2. *If* $\Gamma = \varnothing$, *there do not exist core allocations.*

Proof. Suppose that there did exist a core allocation. In any market M of the core allocation exchange, production, and consumption decisions must result in an allocation which is Pareto optimal for M; otherwise M could block the core allocation, contradicting the definition of the core. Furthermore, as in a core allocation all agents of M have the same level of expected utility,[4] the allocation of the consumption good must be symmetric so that

$$c^j(\omega) = \left[K^j - \frac{(2\alpha)(\# M - 1)}{\# M}\right][\Sigma_{i \in M}\lambda^i(\omega)/\# M]$$

for each j in M. But as $\Gamma = \varnothing$ there exist some number of agents $n > \# M$ which could form a market with efficient production and exchange and with a symmetric consumption allocation in which all agents are better off than initially. This is the desired contradiction.[5] ||

Proposition 3. *If there exists a core allocation, then* $\Gamma \neq \varnothing$ *and the core allocation* is *an allocation of the cooperative economy.*

Proof. That $\Gamma \neq \varnothing$ follows from the contrapositive of Proposition 2. As in the proof of Proposition 2, in each market M of the core production and exchange must be efficient with a symmetric consumption allocation. As there do not exist blocking coalitions, it follows that $\# M \in \Gamma$. ||

4. INTERMEDIATION STRATEGIES AND NON-COOPERATIVE EQUILIBRIA

It is shown in this section that core allocations are equivalent to the equilibrium allocations of a non-cooperative game. First the strategies of agents are defined, and the behaviour of agents given these strategies is analysed. The notion of a blocking strategy is then made precise, and an equilibrium for the non-cooperative game is defined. The equivalence is established in Propositions 4–5.

Prior to the realization of the state, each agent h adopts a strategy under which he is willing to act as an intermediary for some specified market. This strategy is denoted S^h with components S_0^h, the market proposed by agent h; S_1^h the yield in terms of the consumption good for one share in the portfolio of agent h; S_2^h a price in terms of the capital good at which agent h is willing to buy an unlimited number of shares in any project j of S_0^h; S_3^{hj}, a fixed fee in terms of the capital good for the purchase of shares in the portfolio of agent h by agent j of S_0^h; and S_4^h, a price in terms of the capital good at which agent h is willing to sell an unlimited number of shares in his portfolio to agents j of S_0^h. Also let d^{jh} denote the number of shares purchased by agent j in the portfolio of agent h, and let D^{hj} denote the number of shares in project j sold by j to agent h. Also let $A^{jh} = 1$ if agent j purchases shares in the portfolio of agent h, and let $A^{jh} = 0$ otherwise. (It will also be convenient in what follows to let $d^{jj} = D^{jj} = S_3^{jj} = A^{jj} = 0$.)

Agents must act on the strategies S^h prior to the realization of the state; once the state has occurred, agent make the transfers of the consumption good required to honour claims issued. Given that intermediaries have been selected in some way, all other agents regard the strategies S^h as parameters and maximize expected utility. Hence if agent j is not an active intermediary ($d^{ij} = 0$ for each $i \in I$) agent j acts to maximize

$$\Sigma_{h \cdot j \in S_0^h}[A^{jh}][EU(d^{jh}S_1^h + y^j\lambda^j - D^{hj}\lambda^j)] \qquad \ldots(2)$$

with respect to $\{y^j, D^{hj}, d^{jh}, A^{jh}\}$ subject to the following constraints:

$$[\Sigma_{h:\, j\in S_0^h}(A^{jh})(K^j+D^{hj}S_2^h-d^{jh}S_4^h-S_3^{hj}-\alpha-y^j)] \geqq 0 \qquad \ldots(3)$$

$$c^j(\omega) = \{\Sigma_{h:\, j\in S_0^h}[A^{jh}][d^{jh}S_1^h(\omega)+y^j\lambda^j(\omega)-D^{hj}\lambda^j(\omega)]\} \geqq 0 \text{ for each } \omega\in\Omega \qquad \ldots(4)$$

$$A^{jh} = 1 \Rightarrow A^{ji} = 0 \text{ for each } i \text{ such that } j\in S_0^i, \quad i\neq h. \qquad \ldots(5)$$

Constraint (3) is the budget constraint prior to the realization of the state. Constraint (4) requires that agent j be able to honour claims on shares which he issues on his project. Constraint (5) limits agent j to choosing one intermediary in which to invest.

An intermediary who is active for his proposed market S_0^h will have expected utility

$$EU(\Sigma_{j\in S_0^h}D^{hj}\lambda^j+y^h\lambda^h-\Sigma_{j\in S_0^h}d^{jh}S_1^h). \qquad \ldots(6)$$

It is required that the vector (S^h, y^h) be *feasible* in that S_0^h be finite and the following constraints be satisfied:

$$[K^h+\Sigma_{j\in S_0^h}d^{jh}S_4^h+\Sigma_{j\in S_0^h}S_3^{hj}-\Sigma_{j\in S_0^h}D^{hj}S_2^h-\alpha(\# N^h)-y^h] \geqq 0 \qquad \ldots(7)$$

$$c^h(\omega) = [\Sigma_{j\in S_0^h}D^{hj}\lambda^j(\omega)+y^h\lambda^h(\omega)-\Sigma_{j\in S_0^h}d^{jh}S_1^h(\omega)] \geqq 0 \quad \text{for each } \omega\in\Omega. \qquad \ldots(8)$$

Constraint (7) is the budget constraint, and (8) ensures that claims issued on the strategy S^h can be honoured. If there is any discretion, y^h is chosen to maximize (6) subject to (7) and (8).

In the non-cooperative game agents do not cooperate explicitly. In particular the determination of who is to act as an intermediary and under what terms is not exogenous to the model. Each agent is free to announce any intermediation strategy, and it is assumed that each selects a strategy in such a way as to maximize expected utility. In order to define an equilibrium in such a situation, the following definition of a blocking strategy is needed. A strategy S^b with an input choice y^b for some agent $b\in I$ is said to block a consumption allocation $\{c^j;\, j\in I\}$ if when (2) is maximized for each agent $j\in S_0^b-b$ subject to constraints (3)–(5) with $A^{jb}=1$, the vector (S^b, y^b) is feasible and the resulting consumption allocation $\{c_*^j;\, j\in S_0^b\}$ yields $V^j(c_*^j)>V^j(c^j)$ for all $j\in S_0^b$. S^b is a *blocking strategy*.

To be emphasized in this definition is that agent b finds it is in his *own* interest to announce a blocking strategy. Of course in the process of undercutting an active intermediary of a specified market or in the process of forming a new market, all the agents with whom agent b deals are made better off than initially. Roughly speaking agent b may be viewed as a firm who is aware of demand curves and seeks to exploit profitable markets. This type of free entry will be crucial in determining the allocation of resources. In particular an equilibrium will be described in part as an allocation for which there exist no blocking strategies for any agent.

An equilibrium for the non-cooperative game is now defined as follows: *An equilibrium* is a set of actions $\{D_*^{ij}, d_*^{ji}, A_*^{ji};\ \ i\in I\}$ and a strategy S_*^j for each agent $j\in I$, an allocation $\{c_*^j, y_*^j;\, j\in I\}$, and a set of markets $\mathscr{A}$ which satisfy the following conditions:

(i) If $d_*^{ij}=0$ for each $i\in I$, then $\{y_*^j, D_*^{hj}, d_*^{jh}, A_*^{jh}\}$ maximize (2) for agent j subject to constraints (3)–(5) and to $S^h=S_*^h,\ \ h\neq j$. ($c_*^j(\omega)$ is determined by (4)).

(ii) $\cup_{M\in\mathscr{A}}M=I$. For each $M\in\mathscr{A}$ there exists an $h\in M$ such that $S_{0*}^h=M$. For each $j\in M-h$, $A^{jh}=1$. For every such h, S^h is feasible, with y^h chosen to maximize (6) subject to (7) and (8). ($c_*^h(\omega)$ is determined by (8).)

(iii) There does not exist a blocking strategy for any agent of I.

Roughly speaking, condition (i) states that any agent who is not an active intermediary maximizes expected utility by choosing an intermediary with whom to trade, the number of shares in his own project to be sold to that intermediary, the number of shares in the portfolio of that intermediary to be purchased, and the amount to invest in his own project, regarding as parameters the announced intermediation strategies of all other agents. Condition (ii) states that agents in effect partition themselves into markets. For each market there is one active intermediary with a strategy and a maximizing input choice which support the maximizing choices of inactive agents. Free entry condition (iii) was motivated above.

The equivalence of core allocations and equilibrium allocations can now be established.

Proposition 4. *If there exists an equilibrium, the equilibrium allocation is in the core.*

Proof. Clearly an equilibrium allocation $\{c_*^j, y_*^j; j \in I\}$ is feasible for I. Suppose it were not in the core. Then suppose as in the proof of Proposition 1 that there exists a blocking market B in which exchange, production, and consumption decisions result in a Pareto optimal allocation for B. This yields an allocation

$$y_{**}^j = K^j - \frac{(2\alpha)(\# B-1)}{\# B}, \quad c_{**}^j = \theta^j[(\# B)(K^j)-(2\alpha)(\# B-1)][\Sigma_{i\in B}\lambda^i/\# B]$$

with $\Sigma_{j\in B}\theta^j = 1$ and $V^j(c_{**}^j) > V^j(c_*^j)$ for each $j \in B$. Let some agent $b \in B$ adopt the following strategy:

$$S_0^b = B,\ S_1^b = \Sigma_{i\in B}\lambda^i/\# B,\ S_2^b = S_4^b = 1,\ S_3^{bj} = \alpha[2(\# B-1)\theta^j-1]+K^j[1-(\# B)\theta^j]$$

for each $j \in B-b$. If each agent $j \in B-b$ maximizes (2) subject to constraints (3)–(5) with $A^{jb} = 1$, then the allocation $\{c_{**}^j;\ j \in B\}$ will be achieved. This contradicts property (iii) of the equilibrium. Hence, the equilibrium allocation must be in the core. ||

Proposition 5. *All core allocations can be supported as equilibria.*

Proof. By Proposition 3 any core allocation is also an allocation of the cooperative economy with $\Gamma \neq \varnothing$. The following actions, strategies, and markets support such an allocation:

(i) $\mathscr{A}$ is a set of markets with each $M \in \mathscr{A}$ a market of the core allocation. Hence, each $M \in \mathscr{A}$ has some $m \in \Gamma$ agents, and $\cup_{M\in\mathscr{A}}M = I$.

(ii) For each $M \in \mathscr{A}$ there exist some agent $h \in M$ with strategy:

$$S_0^h = M,\ S_1^h = \Sigma_{i\in M}\lambda^i/\# M,\ S_2^h = S_4^h = 1,$$

$$S_3^{hj} = \begin{cases} 0 \text{ if } \# M = 1 \\ (\alpha)(\# M-2)/\# M \text{ if } \# M \geqq 2 \end{cases} \qquad \text{for each } j \in S_0^h.$$

(iii) For each agent $j \in M-h$, $S_0^j = \varnothing$.

(iv) $d^{jh} = D^{hj} = K^j - \dfrac{(2\alpha)(\# M-1)}{\# M}$ for each $j \in M-h$

$A^{jh} = 1$ for each $j \in M-h$

$y^j = K^j - \dfrac{(2\alpha)(\# M-1)}{\# M}$ for each $j \in M$

$$c^j(\omega) = \left[K^j - \frac{(2\alpha)(\# M-1)}{\# M}\right][\Sigma_{i\in M}\lambda^i(\omega)/\# M] \text{ for each } \omega\in\Omega \text{ and for each } j\in M.$$

Equilibrium properties (i) and (ii) follow immediately. Note that agents of M will not collude in their dealings with h. If one agent were to act as a conduit for the funds of another

2E—45/3

to avoid the fixed fee for one of the agents, an additional transactions cost of $\alpha-[(\alpha)(\#M-2)/\#M]$ would be incurred for the pair.

As the initial allocation is in the core, no market B can be made better off and equilibrium condition (iii) is satisfied. ||

It should be noted that there are other schemes by which intermediaries can recoup fixed costs. For example let S_5^h denote a minimum purchase requirement for shares in the portfolio of intermediary h. Then under condition (ii) in Proposition 5 with $\#M>2$ consider the strategy

$$S_1^h = \Sigma_{i\in M}\lambda^i/\#M,\ S_2^h = 1,\ S_3^{hj} = 0,$$

$$S_4^h = \frac{K^j-\alpha}{K^j-[(2\alpha)(\#M-1)/\#M]} \qquad S_5^h = K^j - \frac{(2\alpha)(\#M-1)}{\#M}$$

Agent j would like to purchase less than S_5^h shares in the intermediary at prices S_4^h greater than one. However, he is constrained to purchase at least that amount and will not purchase more. Expenditures remain as before and the new strategies will support a core allocation. The quantity $S_4^h-S_2^h$ can be interpreted as a type of bid–ask spread. It is on this margin that the intermediary covers transactions costs.

5. CONCLUDING REMARKS

This last section offers some comments on the strengths and limitations of the model. First, one may wish to determine whether the parametric behaviour specified is incentive compatible. Following Hurwicz (1972), a resource allocation mechanism is said to be individually incentive compatible if the behaviour patterns specified are consistent with agents' natural inclinations. If an agent's project were essential to the portfolio offered by an intermediary, then it could be argued that such an agent might not deal passively but attempt to bargain. However, the model is constructed in such a way that each agent acting on his own has no bargaining power. With I countably infinite, the removal of one agent from the model has no effect on the equilibrium allocation.

In a core allocation of the model the number of bilateral exchanges in a market is one less than the number of agents in that market. Though a non-cooperative game in which agents propose markets and adopt intermediation strategies will support such core allocations, the form of organization which could emerge in the model may not be unique. An efficient allocation could be achieved if agents were " lined up " on an interval, each agent trading with adjacent agents. Alternatively, agents could form pyramids. Such hierarchical intermediation structures are common in financial markets. It remains to show that there exist non-cooperative games which yield such structures. In principle the form of market organization should also be explained by the model.

The model is a particularly elementary one. Agents are assumed to have identical tastes which display constant relative risk aversion and identical initial endowments. These facts are known to all agents. An effort to relax the assumption that tastes and endowments are identical would complicate the model. In particular, a restriction to the trading of shares might be binding and hence not satisfactory. Yet, in principle, the kind of analysis suggested above could be carried out. Intermediaries would take into consideration the desires of agents with diverse attitudes toward risk to trade contingent claims. Acting in their own interest, intermediaries may support efficient allocations.

However, if tastes and endowments are not known, there are new difficulties. Though an equilibrium may exist, it may not be both optimal and incentive compatible. Agents may understate their willingness to pay fixed fees to support a given market. This is a problem which has arisen in the context of models for the allocation of public goods. Though the works of Groves and Ledyard (1974) suggests that the problem may not be insuperable, the effect may be to limit the number of agents in a market. A model with imperfect information may also be rich enough to account for the relatively limited number of types of assets which are traded in active markets. This is a subject of further research.

This paper was motivated in part by questions of optimal regulation of financial institutions. Obviously the analysis remains too abstract to shed much light on any particular set of arrangements. However, the results do suggest that under some conditions competitive markets with free entry will support efficient allocations. More work is needed in this area.

First version reviewed February 1976; *final version accepted October* 1977 (*Eds.*).

A version of this paper appeared in the author's Ph.D. thesis presented at the University of Minnesota, July 1975. I gratefully acknowledge the advice and encouragement of Neil Wallace and the assistance of John Danforth. I would also like to thank the participants of the market organization workshop at Carnegie-Mellon University and the referees for helpful comments. I claim full responsibility for any errors or ambiguities. Financial assistance from the Federal Reserve Bank of Minneapolis is also gratefully acknowledged. The views expressed herein are solely those of the author and do not necessarily represent the views of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

NOTES

1. Heller (1972) notes that such a transactions technology is quite different from the one assumed in his paper. Heller and Starr (forthcoming) also mention the possibility of a theory of intermediation in a bilateral trade model with costly transactions.
2. This motivates the work of Shapley and Shubik (1966) on the existence of quasi-cores in a model in which non-convex preferences make the core empty.
3. Clearly the condition that $\# M < \infty$ is not essential to the definition of a market. Yet without that condition, feasible allocations for infinite sets of agents would need to be defined. This might be done for example by taking limits in the appropriate way. The alternative adopted here is to find a condition which is sufficient to ensure that such allocations need not be defined.
4. In a core allocation all agents of I must have the same expected utility. For consider some market M of the core and its complement M'. Pick some $i \in M'$. Suppose that $V(c^i) < V(c^j)$ for some $j \in M$. Then agent i could form a blocking coalition by replacing agent j of M. So $V(c^i) \geqq V(c^j)$ for each $j \in M$. Suppose $V(c^i) > V(c^j)$ for some j. Then a symmetric counter-argument applies. Hence $V(c^i) = V(c^j)$ for all $j \in M$. Finally note that the selection of $i \in M'$ was arbitrary.
5. This result rests in part on the definition of the core adopted above. In a model in which feasible allocations for infinite markets were defined, one might anticipate that Proposition 2 would be modified as follows: If $\Gamma = \emptyset$, there do not exist core allocations with a finite number of agents in each market.

REFERENCES

ARROW, K. J. and HAHN, F. H. (1971), *General Competitive Analysis* (San Francisco: Holden Day).

FOLEY, D. K. (1970), " Economic Equilibrium with Costly Marketing ", *Journal of Economic Theory*, **2**, 276–291.

GROVES, T. and LEDYARD, J. (1974), " An Incentive Mechanism for Efficient Resource Allocation in General Equilibrium with Public Goods " (Discussion Paper No. 119, Northwestern University).

HAHN, F. H. (1971), " Equilibrium with Transactions Costs ", *Econometrica*, **39**, 417–440.

HELLER, W. P. (1972), " Transactions with Set-Up Costs ", *Journal of Economic Theory*, **4**, 465–478.

HELLER, W. P. and STARR, R. M. (forthcoming), " Equilibrium with Non-convex Transactions Costs: Monetary and Non-monetary Economies ", *Review of Economic Studies*.

HURWICZ, L. (1972), " On Informationally Decentralized Systems ", Ch. 14 in *Decision and Organization*, ed. by McGuire, C. B. and Radner, R. (Amsterdam).

KURZ, M. (1974), " Equilibrium with Transactions Costs and Money in a Single Market Exchange Economy ", *Journal of Economic Theory*, **7**, 418–452.

SHAPLEY, L. S. and SHUBIK, M. (1966), " Quasi-Cores in a Monetary Economy with Non-convex Preferences ", *Econometrica*, **34** (4).

STARR, R. (1969), " Quasi-Equilibria in Markets with Non-convex Preferences ", *Econometrica*, **36**, 25–38.