Optimal Design of Tokenized Markets[*]

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Abstract

Trades in today's financial system are inherently subject to settlement uncertainty. This paper explores tokenization as a potential technological solution. A token system, by enabling programmability of assets, can be designed to eradicate settlement uncertainty. We study the allocations achieved in a decentralized market with either the legacy settlement system or a token system. Tokenization can improve efficiency in markets subject to a limited commitment problem. However, it also materially alters the information environment, which in turn aggravates a hold-up problem. This limits potential gains from resolving settlement uncertainty, particularly for markets that depend on intermediaries.

Keywords:

Tokenization, programmability, settlement uncertainty, asymmetric information

JEL Classification Numbers: D82, D86, D47, G29

1 Introduction

Two traders agree on an asset sale. How can each party ensure that, when the time comes to settle the trade, the other will keep their side of the bargain? Markets have adopted various solutions to resolve the age-old problem of limited commitment. Third-party intermediaries and platforms, such as exchanges or sponsors, facilitate the orderly settlement of transactions. Margin requirements and other uses of collateral ensure that future payments tied to contractual obligations are serviceable. Traders build long-term relationships and a reputation for credibility.

Despite these practices, trades commonly fail to be settled (Fleming and Garbade, 2005). The potential for systematic settlement fails was put on full display during the Global Financial Crisis. In 2008, settlement fails in Treasury markets reached a daily volume of 400 billion dollars per day. Chronic settlement fails in the Treasury market lead the Treasury Market's Practices Group (TMPG) to introduce a "fails charge" to decrease traders' incentives to fail (see (Garbade et al., 2010)).[1] While the fails charge was effective at reducing the incidence of chronic fails, some fails continue to occur as described by Fleming and Keane (2016).

Settlement fails reflect the institutional and technological feature of the current settlement system – settlement depends on traders individually submitting settlement instructions that correspond to their contractual obligations from trading activity. When incentives break down, so does settlement.

This paper explores the potential for a settlement system based on distributed ledger technology (DLT) as a potential technological solution to the inability to commit inherent in the current settlement system. In this paper, security tokenization refers to the representation of traditional financial assets and collateral on a distributed ledger. The innovation of tokenization we focus on is the *programmability* of assets.[2] Programmability allows traders to commit to settlement, thereby eliminating the potential for fails.

Does a token system strictly improve upon a legacy settlement system because it can eliminate settlement risk? In a setting where trading is endogenous, we show that it is not the case. With the gain of eliminating settlement risk, an information problem

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[1] The Treasury Market Practices Group (TMPG) is a group of market professionals committed to supporting the integrity and efficiency of the Treasury, agency debt, and agency mortgage-backed securities markets. See: https://www.newyorkfed.org/tmpg.

[2] The idea of programmability is closely related to "smart contracts." The Financial Stability Board notes that "Smart contracts use computer protocols to execute, verify, and constrain the performance of a contract. In doing so, they can automate decision-making, by allowing self-executing computer code to take actions at specified times and/or based on reference to the occurrence (or non-occurrence) of an action or event." (Board, 2019)

emerges. This is because eliminating settlement risk requires traders to reveal more information to their counterparties regarding their positions, and this information can materially impact equilibrium trade.

The intuition is simple but powerful. Suppose that *A* and *B* agree on a trade. To guarantee settlement, *A* and *B* must jointly write a program that governs the change of ownership of assets. In order to program an asset, a trader must have the ownership right to that asset at the time the settlement must take place. Indeed, if the trader does not already own the rights to the asset at the time of settlement, it is possible that the trader will never acquire these rights, which would make ensuring settlement impossible. Knowing that a trader must own the asset she is trying to sell reveals information that can be exploited by the buyer.

This information can lead to a hold-up problem and even breakdown trade altogether. We consider a market where traders must enter inter-dependent trades in order to achieve the optimal allocation of a long-lived asset. A key friction is limited commitment. Traders are tempted to break their contracts when they learn that the private value of holding onto an asset is high ex-post. We compare the effects of the two representative settlement systems on equilibrium trade: the "legacy system," which represents the current settlement system; and the "token system," which allows for programming of assets.

In a legacy system, when the commitment problem is too severe, some trades will fail to settle. In contrast, a token system perfectly resolves settlement uncertainty arising from limited commitment by equipping traders with technology to commit to future settlement actions ex ante. However, some traders serve the role of an intermediary, by bridging the transfer of assets across end sellers and buyers. Intermediaries may not value the asset themselves, and buy and sell primarily to facilitate trade. Having to reveal whether they own the asset exacerbates a hold-up problem because intermediaries must purchase assets in advance to facilitate a transaction. When the hold-up problem binds, trades may altogether fail to occur in equilibrium.

While tokenization is sought as a solution for decentralized markets, which are often heavily reliant on intermediaries to make markets, designing a token system to resolve settlement uncertainty may inadvertently exacerbate other frictions that are less significant under the legacy system. Notably, under the same conditions, we show that the hold-up problem may not preclude trading in the legacy system. This is because the legacy system ensures *maximum* privacy by decoupling trade execution and settlement. In the legacy system, execution of a trade does not require possession of the asset being sold, and so a buyer cannot assume that the seller has already obtained the security she

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is selling.

Programming assets to guarantee settlement is closely linked to the idea of "instant settlement." Instant settlement removes the time gap between trading and settlement, thereby eliminating settlement uncertainty. This idea generalizes to a setting where the commitment to future settlement, through programs, and trading happens simultaneously (See Lee et al. (2021)). The information problem that arises from instant settlement is highly relevant given that instant settlement is an explicit goal of several industry projects. SIX, the company that runs the Swiss central securities depository as well as the large value payment and repo trading system is building a "digital exchange" that will have tokenized assets and cash on a blockchain to facilitate trading.[3] SIX states "the most fundamental of these changes is that trading and settlement will no longer be separated. Instead, they will operate in the same cycle. We call this riskless trading." Similarly, Fnality, a project led by some large global banks, aims to provide instant settlement. Finally, the Deutsche Börse is working with R3 to build a blockchain securities platform $HQLA^x$, which would allow instant settlement.

As in our model, the knowledge of ownership of an asset at the time of trade is a starting point for all the existing designs of smart contract protocols. Thus, the implications of our paper are orthogonal to other important design considerations, including consensus mechanisms, privacy features, and commitment tools. In particular, our insight on limitations of token systems apply to ongoing developments in cryptography aimed at increasing privacy, which typically take as given an agreement to transfer, and examine whether the transfer can be accomplished without revealing more detailed information regarding identity.

Notably, our results are also orthogonal to the prevalent use of collateral observed in decentralized finance (DeFi). Collateral is commonly used in environments of limited commitment, and may not only protect lenders from credit risk, and but also improve debtors' incentives to fulfill obligations. In our model, programmability resolves settlement risk, eliminating the need to provide debtor's with incentives to fulfill their obligations. An added benefit is that programmability enables traders in the tokenized market to enter trades that are, de-facto insulated from credit and counterparty risk. Specifically, the immutable transfer of assets in future periods ensures that future transfers will occur regardless of a counter party's solvency. In this respect, token systems are "liquidity-efficient" – they require minimal tie-up of liquidity, such as collateral, commonly needed to secure transactions. This, of course, does not rule out arrangements

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[3]Another relevant project is Broadridge's Distributed Repo.

where credit-risk sharing is desirable.[4]

The remainder of the paper is organized as follows. In Section 2, we relate our paper to past works. Section 3 introduces our theoretical environment. Equilibrium analysis is provided in 4. We make concluding remarks in Section 5. Proofs not provided in the text can be found in the Appendix.

2 Related Literature

A new and growing literature examines the implications of blockchain technology in financial settings (Townsend (2019)). This paper is the first to our knowledge that theoretically examines the impact of tokenization on markets with inter-dependent trades. At heart, our paper provides a novel consideration in key design features widely shared by initiatives to develop tokenized markets.

Several papers examine the use of blockchain technology in financial markets. A key focus in these papers is the potential for decentralization, whether in the context of cryptocurrencies (Chiu and Koeppl (2017)), settlement (Chiu and Koeppl (2019)), or applications of smart contracts (Cong and He (2019)). In this context, important consideration are costs and incentives for validators (Abadi and Brunnermeier (2018), Easley et al. (2019)). While the protocols considered in our paper are potentially implementable in a decentralized manner, this is not the contribution nor the focus. Rather, we highlight the informational impact of key design features of tokenized securities, and outline how these can adversely affect market efficiency. We do so by taking as given a token system that resolves settlement risk, and consider how trade is endogenously determined. Lee et al. (2021) explicitly studies the design problem of zero settlement risk token systems, taking as given a fixed set of trades.

Our paper contributes to studies of how the post-trade environment affects markets. Tokenized securities share properties of real-time gross settlement, which have been studied extensively in the context of wholesale payments. Martin and McAndrews (2008) explores how liquidity-saving mechanisms can enhance real-time gross settlement systems, which resolve counterparty and credit risk but can be taxing on liquidity. This tradeoff is explored in the context of clearing by Koeppl et al. (2012). Khapko and Zoican (2020) explore how the option to choose faster settlement can lead to inefficiencies. We highlight a novel concern that arises in the context of real-time gross settlement – the implicit requirement that underlying assets must be owned at the time of settlement. This novel form of inefficiency only arises when trade and commitment to settlement

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[4]For examples of credit risk in settlement risk free systems, see Lee et al. (2021).

happen simultaneously – something that has become an increasingly relevant design consideration with tokenization.

Our paper is related to the potential impact of post-trade information disclosure on markets. Garratt et al. (2019) analyzes post-trade disclosure in the context of inter-dealer markets, and shows that strategic platforms may choose inefficient disclosure policies. Our paper shows that even though both the legacy and token system, within the context of our framework, do not have access to timely, sensitive information, tokenization can exhibit dramatic difference in equilibrium trade.

3 Model

We consider an asset market where traders enter bilateral trades that are interdependent, in the sense that an asset may be sold from one trader to another, and then sold further to a third trader. Whether such trades are successfully settled, and whether traders enter trades that maximize gains from trade can depend on the underlying settlement environment. We consider two settlement systems: a legacy system, which represents the current system where trade and settlement happen sequentially and independently; and a token system, which uses programs to put in place irrevocable settlement instructions concurrently with trade.

Agents and Asset. There are three risk-neutral traders, $i = \{A, B, C\}$, and one indivisible asset, which is initially owned by A. The model is divided into two stages: the trading stage and the settlement stage. The model begins with the trading stage, during which traders bilaterally meet with each other and negotiate trades. There are two meetings that occur sequentially, which we represent as $t = m1, m2$. These meeting are between A and B and between B and C. A and C never meet. In this sense, B is an intermediary that facilitates transfers of the asset between A, who owns the asset in the beginning of the model, and C, who can make better use of the asset in certain future periods. While the role of B as an intermediary is assumed, for simplicity, it is important to recognize that intermediaries can play an essential role in facilitating transactions that might not otherwise occur. Specifically, in a more complex model, trades may not occur even if A and C can meet. We expect that our results would extend to an environment where B arises as an intermediary endogenously.

After the trading stage, the settlement stage starts. In the settlement stage, assets are transferred between traders over three dates $t = 1, 2, 3$. In essence, the trades made in $t = m1, m2$ consist of promises of exchanges in ownership of the asset, with the actual

exchange scheduled to occur at dates $t = 1, 2, 3$.

Trader i derives some payoff v_t^i from holding the asset at dates $t = 1, 2, 3$. Each trader is endowed with multiple accounts where the asset can be held, and the ownership and contents of accounts are assumed to be private. At any date $t = 1, 2, 3$, the asset must be in the account of one of the traders and can be in only one account. We say that trader i owns the asset at date t, and derives the associated payoff, if the asset is in one of trader i's account on that date.

Payoffs (summarized in Figure 1) vary between traders and across periods, and can take values H, M, or L, where $H > M > L > 0$. A derives a payoff of L, L, and H for holding the asset in $t = 1, 2, 3$, respectively. B derives a payoff of H for holding the asset in $t = 1$, $\tilde{M} \in \{0, M\}$ in $t = 2$, and 0 in $t = 3$, where $\tilde{M} = M$ with probability λ_B and 0 otherwise. B privately learns $\tilde{M}$ in the beginning of date $t = 2$. C derives a payoff 0 in $t = 1$, H for holding the asset in $t = 2$, and $\tilde{H} \in \{0, H\}$ in $t = 3$, where $\tilde{H} = H$ with probability λ_C and 0 otherwise. C privately learns $\tilde{H}$ at the beginning of $t = 3$. Importantly, $\tilde{M}$ and $\tilde{H}$ are both revealed to B and C, respectively, in the settlement stage, after trading has occurred. The revelation of $\tilde{M}$ and $\tilde{H}$ in the settlement stage, will be the basis for B and/or C wanting to sometimes break agreements made in the trading stage.

Traders' time-dependent payoffs motivate trade. In each period, one of the three traders gains the highest ex-ante payoff from owning the asset. In $t = 1$, B obtains H; in $t = 2$, C obtains H; in $t = 3$, A obtains the H. Figure 2 depicts the optimal transfer of the asset.

Meetings. All trading occurs in pair-wise meetings, which take place sequentially in the trading stage $t = m1, m2$. B is matched with A and C, sequentially, but not necessarily in that order. Matches between A and C never occur.[5] With probability $\frac{1}{2}$, B is matched with A first and then C; with equal probability, B is matched with C first. The order of realized matches is known only to B, who participates in both matches. Communication between traders is assumed to only occur during meetings. In other words, at any point outside of meetings, traders are unable to send messages regarding who they met, the contents of their accounts, or their private realizations.

During a meeting, traders negotiate a contract. Each trader knows only their history and the current state of their own accounts. Given some price P, a *contract* $C_{ij}^{\tau_1 \tau_2}(P)$ is a securities lending agreement that specifies the lender, trader i; the borrower, trader j; the date τ_1 at which the asset is transferred from i to j; and the date τ_2 at which the

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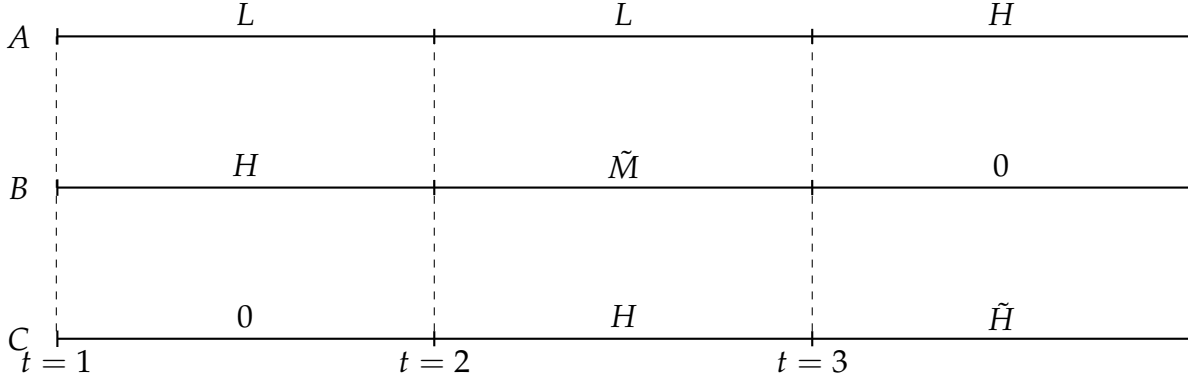[5]As a result, we preclude any multilateral trading scheme.

Figure 1: **Traders' payoffs.** This figure shows the payoffs of traders A, B, and C over $t = 1, 2, 3$.

asset is transferred back from j to i. We use $P_{ij}^{\tau_1 \tau_2}$ for shorthand to denote the price corresponding to contract $\mathcal{C}_{ij}^{\tau_1 \tau_2}(P)$, where j pays i at τ_1, the date when the asset is first transferred, for borrowing the asset. Note that an agreement to trade according to a contract occurs during meetings $t = m1, m2$, whereas the actual exchange of the asset and payments occur later in the settlement stage at $t = 1, 2, 3$. For example, if A agrees to lend the asset to B for one period at price P, starting at $t = 1$, the contract is $\mathcal{C}_{AB}^{12}(P)$. To summarize, three obligations arise under contract $\mathcal{C}_{ij}^{\tau_1 \tau_2}$ negotiated at some price $P_{ij}^{\tau_1 \tau_2}$:

- at τ_1, trader i transfers the ownership of the asset to trader j;

- at τ_1, trader j transfers $P_{ij}^{\tau_1 \tau_2}$ to trader i; and

- at τ_2, trader j returns asset to trader i.

We assume there exists some DvP settlement between assets and some numeraire used for the exchange between traders i and j at τ_1.[6]

For simplicity, the borrower j of the asset is assumed to make a take-it-or-leave-it offer.[7] This means that for the $A - B$ pair, B makes a take-it-or-leave-it offer to A, and for the $B - C$ pair, C makes a take-it-or-leave-it offer to B. As a tie-breaking rule, we assume that all else equal, traders prefer to trade.

Trades are "promises" made between traders. Whether these promises are kept depends on whether settlement, the transfer of the asset from an trader to another, takes

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[6]In a settlement system like Fedwire securities, settlement is initiated by the seller of the securities. Upon sending securities, cash is automatically transferred from the account of the buyer to the account of the seller.

[7]As will be evident, the lender in the sequence of trades will be privately informed. We adopt the convention that the party without private information is making the offer, which simplifies the analysis and is not crucial for the main results.

place as stipulated in agreements made in the trading stage. We turn now to the description of the settlement technology.

Settlement. We consider two different type of settlement technology, which represent the legacy system and the token system.

Under the legacy system, the asset moves out of an account only if the owner of the account initiates a transfer. Formally, at dates $t = 1, 2, 3$, a trader currently in possession of the asset unilaterally decides whether to transfer the asset to the account of another trader or to keep it. The legacy system does not offer traders the ability to commit, so the option not to transfer the asset holds *regardless* of existing contractual obligations. As such, at each date in which a settlement action is required, traders explicitly choose whether to execute the transfer of the asset pertaining to an outstanding trade or strategically fail. To fix ideas, suppose that traders A and B entered a trade at either $t = m1$ or $m2$ that specifies that the asset must be transferred from A's account to B's account at date 1. Then, at date 1, A can choose to initiate the transfer to B's account, as specified in the trade, or can choose not to do so and "fail."

A trader that fails to settle suffers a cost Δ, which can be thought of as a reputational cost or penalty. We make several assumptions on Δ to allow for strategic fails to sometimes be attractive. First, we assume that $\Delta \in (2L, \frac{1}{2}H)$, which motivates a commitment problem.[8] Second, we assume that $M \leq \Delta$.[9]

Under a tokenized system, an asset can be "programmed" during the trading stage with transfer instructions to be completed in the settlement stage at future dates. This allows traders to commit to settlement taking place as specified in the contract. The transfer instructions associated with a contract are self-executing, so that the asset moves from account to account without the need for any trader to take an action. Moreover, a trader is unable to prevent a programmed transfer from occurring.

To add a transfer instruction to an asset, however, a trader must be the current holder of that asset at the time the contract specifies it is to be transferred, and new instructions must be feasible given all instructions already programmed in the asset. In the context of our model, a trader making an agreement to lend the asset at the trading stage, as per endowments or previous trade agreements, can program the asset to also make sure that the asset will be returned at a specific date, as specified by the contract . This

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[8]If Δ is sufficiently high, no trader would enter a contract that they do not intend to honor. For similar reasons, we do not consider collateralized contracts, which could remedy commitment issues if traders are unconstrained. The lower bound on Δ reduces the number of cases to consider, but our core results do not depend on this assumption.

[9]This assumption is made purely to simplify the analysis.

eliminates the commitment problem discussed above for the legacy system. Importantly, this requires that bargaining over the terms of the trade and programming the asset occur simultaneously. In this sense, programming assets according to contracts at the time of negotiation achieves the same effect as if trades are immediately settled. As such, while the actual settlement takes place in the future, we refer to the process by which assets are irrevocably programmed to move in the future as constituting "immediate settlement." The operational features of a token system free of settlement risk is studied formally by Lee et al. (2021). An implication is that both parties of a negotiated trade confirm that the conditions of the trade are satisfied.[10]

This requires that a contract must be *feasible*, as defined below:

Definition 1 (Feasibility Condition). *A contract is* feasible *if at the time of agreement, the terms of the contract can be settled immediately.*

Equilibrium. Given a settlement system, a Perfect Bayesian equilibrium is a set of traders' offer strategies, acceptance strategies, and settlement strategies such that:

1. Traders' offer and acceptance strategies maximize their expected payoffs;

2. Traders' settlement strategies maximize their conditional expected payoffs;

3. Traders' beliefs are consistent with Bayes' Rule.

4 Equilibrium Trade and Settlement

The first-best allocation is a useful benchmark. The valuations of each trader lend themselves to a clear first-best allocation. Figure 2 depicts the optimal transfer of the asset. This corresponds to the asset being under ownership of B at $t = 1$, C at $t = 2$, and A or C at $t = 3$. In this section we consider whether the first-best allocation can be achieved in a legacy and in a token settlement system. We derive the equilibrium under the legacy system in section 4.1. We solve the problem by backward induction, analyzing the settlement stage first, in section 4.1.1, and then the trading stage, in section 4.1.2., taking into account the possible settlement outcomes. We consider the equilibrium under the token

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[10]The requirements for programming asset under the token system free of settlement risk are consistent with those found in Lee et al. (2021), which formally studies operational features of token settlement systems to achieve zero-settlement risk.
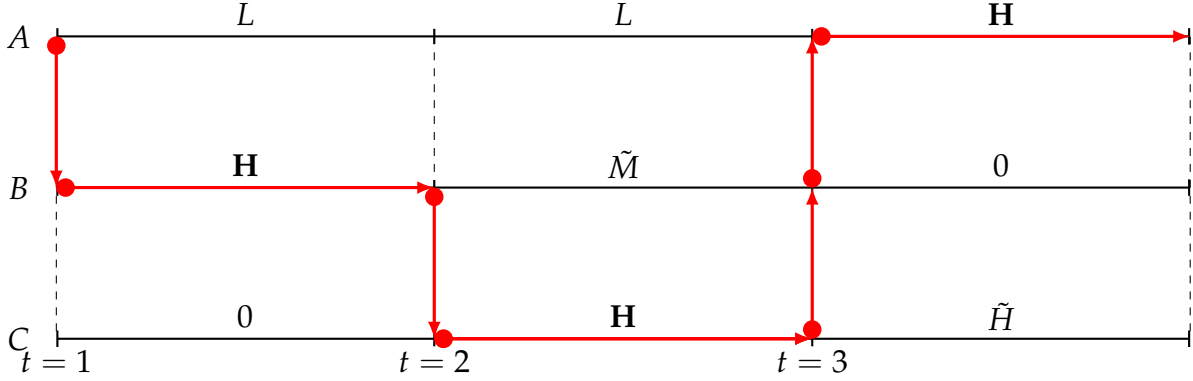
Figure 2: **Optimal allocation of asset.** This figure shows the optimal asset allocation between A, B, and C over $t = 1, 2, 3$, which is achieved through two contracts: $\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$. The red indicates ownership and transfer of the asset between traders. In the beginning of $t = 1$, A starts with the asset, passes the asset to B, who holds it for one period. Then, B passes the asset to C, who holds it for one period to the end of $t = 2$. At the beginning of $t = 3$, the asset is transferred from C back to B to A, who holds it for the final period.

system in section 4.2. Because the token system eliminates settlement uncertainty, the trading and settlement stage are no longer analyze sequentially – instead, the incentives to trade directly relate to whether desirable transactions are achieved.

4.1 Equilibrium under the Legacy System

There are two channels through which the first-best allocation may not be attained in equilibrium. The first is a limited commitment problem and the second is a hold-up problem. The commitment problem arises because the legacy system relies on incentive compatibility of settlement actions. The fact that C may have an incentive to fail to return the asset at date 3 in our model must be taken into account by the agents at the trading stage. The hold-up problem arises because the value that B creates by intermediating between A and C can exceed the value she derives from the ownership of the asset. C can exploit this situation by making a "low-ball" offer to B. Taking this risk into account, B may prefer not to intermediate. In the remainder of this section we consider each friction in turn.

In the legacy system, traders may find it ex-post optimal to renege on contractual obligations. In the context of the model, this problem arises when a trader chooses not to return an asset at the designated date for private benefits. As an example, consider the case of C. Since $H > \Delta$, if C is able to acquire the asset at any date prior or equal
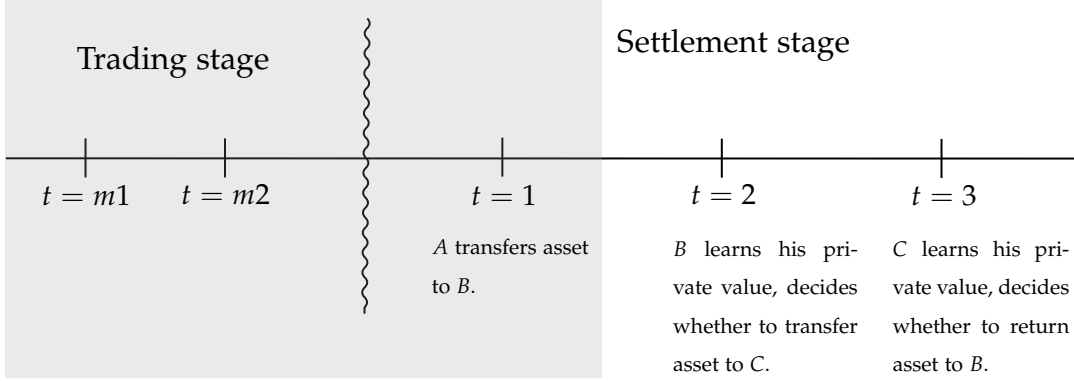
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Figure 3: **Settlement in Legacy System.** This figure highlights key decisions that take place in the settlement stage conditional on $\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$. At $t = 3$, $\tilde{H}$ is realized, and C decides on whether to return the asset to B (which is then returned to A). At $t = 2$, $\tilde{M}$ is realized, and B decides on whether to transfer the asset to C.

to $t = 3$, C will renege on any promise to return the asset at $t = 3$, as the cost Δ is not sufficient to deter a fail:

Lemma 1 (Strategic Fail). *Suppose that C obtains the asset at $t = 2$ with a contractual obligation to return the asset at $t = 3$. C strategically fails on this promise if $\tilde{H} = H$.*

A, who is endowed with the asset initially, strictly prefers owning the asset at $t = 3$. As a result, A may only agree to lend the asset to B, knowing that B will on-lend it to C, if A believes that the likelihood of getting the asset back is sufficiently high, and/or if he is compensated for taking on the risk.

The second channel is a hold-up problem. B is the only trader who is matched with both the lender A and the borrower C.[11] For C to acquire ownership of the asset in $t = 2$, B must successfully negotiate two sides of the intermediation chain. As trades occur asynchronously, this requires B to "make markets" by completing one side of the chain in advance of the other, anticipating the outcome of that other trade.

It is common knowledge that gains from trade arise whenever the asset is transferred from A to C in $t = 2$. Furthermore, when B's expected valuation of $t = 2$ ownership, $E[\tilde{M}]$ is lower than the valuation of A or C, B's incentives are aligned with acting strictly as an intermediary, by running a "matched book." In fact, if the possibility of successfully building a matched book is low, B would be reluctant to make markets on behalf of the other traders.

Herein lies the potential for a hold-up problem. As B privately values the asset less than its owner A, he must pay a price in *excess* of his own valuation in order to

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[11]This is reminiscent of over-the-counter markets, where intermediaries commonly play an outsized role in reallocating assets between final buyers and sellers.

acquire the asset on behalf of C. This creates the potential for C to strategically make a discounted offer, based on the possibility that B has already acquired the asset from A.

With these two tensions in mind, let us consider the equilibrium under the legacy system. We solve by backward induction. The optimal allocation requires traders to successfully negotiate two trades in the trading period: $\mathcal{C}_{AB}^{13}$, which transfers the asset from A to B for dates $t = 1, 2$, and $\mathcal{C}_{BC}^{23}$, which transfers the asset from B to C for dates $t = 2$. In order for these trades to occur in equilibrium, we must verify whether it is incentive compatible for traders to settle accordingly, and to enter such trades in the first place.

4.1.1 Settlement stage in the legacy system

As a starting point, suppose that in the trading stage, B entered contracts $\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$ with A and C, respectively, for some prices P_{AB}^{13}, P_{BC}^{23}. In the settlement stage, there are two key settlement decisions to consider that are associated with $\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$ (See Figure 3). The first settlement decision arises at $t = 3$ regarding C's incentives to return the asset to B as promised in $\mathcal{C}_{BC}^{23}$, which B needs in order to return it to A. In the beginning of $t = 3$, C is supposed to return the asset back to B. By Lemma 1, C will choose not to return the asset if $\tilde{H} = H$ is realized. If C fails to return the asset to B, then B also fail to return the asset to A, resulting in a "daisy chain" of settlement fails. By sometimes failing to return the asset to A, B incurs an expected cost $\lambda_C \Delta$.

The second settlement decision relates to B's incentives to give the asset to C after realizing $\tilde{M} = M$. In the beginning of $t = 2$, B learns $\tilde{M}$, i.e. his updated valuation of the asset in $t = 2$. Just as C reneges on his contract if $\tilde{H} = H$, B may also want to renege on his contract with C if profitable.

Figures 5 and 6 summarizes the settlement actions and terminal payoffs under $\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$. By Lemma 1, we have already characterized C's settlement strategy at $t = 3$. Given this, consider B's decision in the beginning of $t = 2$ of whether to keep or break his contract $\mathcal{C}_{BC}^{23}$ with C. B reneges on his agreement to transfer the asset to C if holding onto the asset, net of the penalty Δ, is more profitable than transferring the asset to C as promised:

$$\underbrace{\tilde{M} - \Delta}_{B\text{'s valuation}} > \underbrace{P_{BC}^{23} - \lambda_C \Delta}_{\text{expected payoff from trade}}, \tag{1}$$

which we can rewrite as $P_{BC}^{23} < \tilde{M} - (1 - \lambda_C)\Delta$. Only when the price P_{BC}^{23} offered by C to B for $\mathcal{C}_{BC}^{23}$ is sufficiently high, will B want to honor the trade ex post.
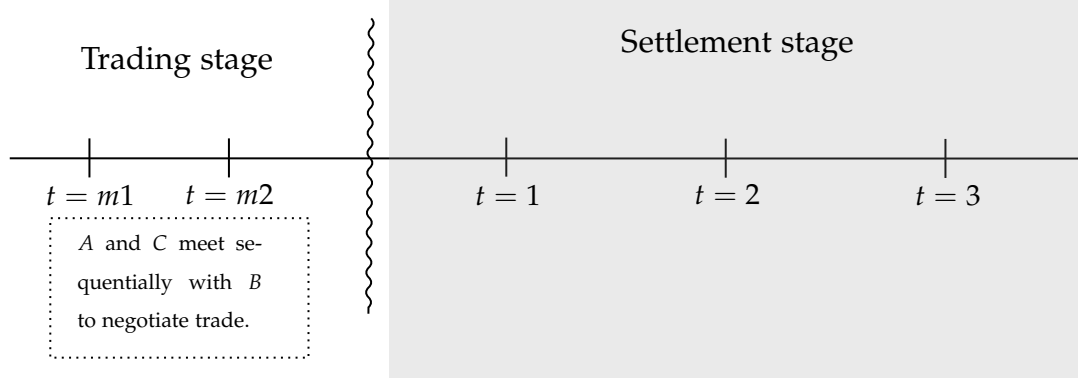
Figure 4: **Trading in Legacy System.** In the trading stage, A and C meet sequentially with B to negotiate trades. However, neither A nor C know the order of meetings.

Alternatively, suppose that B entered contract $\mathcal{C}_{AB}^{13}$ with A but fails to negotiate $\mathcal{C}_{BC}^{23}$. As shown in Figure 5, B faces a settlement decision at $t = 1$ of whether to execute according to the contract $\mathcal{C}_{AB}^{13}$ or fail.[12] Failing is particularly costly to B, since in addition to the direct cost of failing, B would have to forgo payoff H associated with obtaining the asset at $t = 1$. Honoring the trade associated with $\mathcal{C}_{AB}^{13}$ is optimal if and only if

$$\underbrace{H + E[\tilde{M}] - P_{AB}^{13}}_{\text{expected payoff from settling}} > \underbrace{-\Delta.}_{\text{cost of failing}} \tag{2}$$

Finally, B could have entered contract $\mathcal{C}_{AB}^{12}$ with A. This corresponds to the third tree in Figure 5. Under the contract, B makes two settlement actions. At $t = 1$, B must pay P_{AB}^{12} to A in order to acquire the asset from A at $t = 1$. At $t = 2$, B must return the asset to A. Failing to do either action constitutes a fail that results in cost Δ.[13] Since $M < \Delta$, retaining the asset at $t = 2$ is not profitable. It is straightforward to see that there exists a price P_{AB}^{12} (e.g. L) that A would accept, and B is strictly better off paying in exchange for the asset at $t = 1$.

4.1.2 Trading stage in the legacy system

So far, we characterized traders' strategies in the settlement stage. Anticipating these strategies, traders bargain in the trading stage. As shown in Figure 4, two meetings

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[12] B can fail by returning the asset to A. Since we assumed that the system is DvP, this means that B received his cash back as if no transfer of assets had occurred.

[13] In a settlement system like Fedwire securities, the seller of a security can fail by choosing not to send the security to the buyer, since all settlements are initiated by the seller. The buyer can fail by returning the security she has received to the buyer. This automatically undoes the transfer of cash that was associated with the initial settlement of the security.

occur sequentially at $t = m1, m2$, and the order of meetings is only known to B, who participates in both.

Let us start with the bargaining problem between B and C in the trading stage. Note, at this stage, neither B nor C know their valuations (e.g. the realized value of $\tilde{M}$ and $\tilde{H}$). C does not know whether B has already traded with A, but knows that B's reservation price will depends on whether B and C trade in $t = m1$ or $t = m2$.

With probability $\frac{1}{2}$, B matches with C first at $t = m1$. In this case, B can adapt his trading strategy with A conditional on the trading outcome with C. B accepts an offer P^{23}_{BC} from C only if:

$$P^{23}_{BC} \geq \max\{ \underbrace{E[P^{13}_{AB} - P^{12}_{AB}]}_{\text{expected price of } t = 2 \text{ ownership}}, E[\tilde{M}]\} + \lambda_C \Delta. \tag{3}$$

The first term in the max operator represents the price at which B expects to acquire $t = 2$ ownership. The second term represents the expected value of retaining ownership of the asset at $t = 2$. If $E[\tilde{M}] > L$, then B's private valuation of $t = 2$ ownership of the asset may become relevant. The final term $\lambda_C \Delta$ represents the daisy chain premium.

Suppose instead that B matched with C at $t = m2$, and had already agreed to $\mathcal{C}^{13}_{AB}$ at $t = m1$ at some price P^{13}_{AB}. In this case B has already acquired $t = 2$ ownership of the asset from A by the time he meets with C at $t = m2$. B accepts an offer P^{23}_{BC} only if:

$$E[\max\{H + P^{23}_{BC} - P^{13}_{AB} - \lambda_C \Delta, H + \tilde{M} - P^{13}_{AB} - \Delta, -2\Delta\}]$$
$$\geq E[\max\{H + \tilde{M} - P^{13}_{AB}, -\Delta\}]. \tag{4}$$

The term on the left hand side of the inequality represents B's payoff from accepting C's offer and corresponds to B's expected payoff at node ① in Figure 5. The first term in the max operator is B's payoff if he settles both trades (reaching either node ⓐ or ①b), the second term is the payoff when B accepts but fails to deliver the asset to C at $t = 2$ (node ①c), and the third term is the payoff when B fails on both contracts (node ①d). The term on the right hand side of the inequality represents B's payoff from rejecting C's offer and corresponds to B's expected payoff at node ② in Figure 5. The first term in the max operator is B's payoff from retaining the asset (node ②a) and the second term is the payoff when B fails on the contract with A at $t = 1$ (node ②b).

We can now derive C's equilibrium offer strategy. Recall, C chooses his offer strategy, without knowing whether his meeting with B is taking place at $t = m1$ or $t = m2$. Given the fact that B could reject an offer from C and, even if the offer is accepted, may fail to
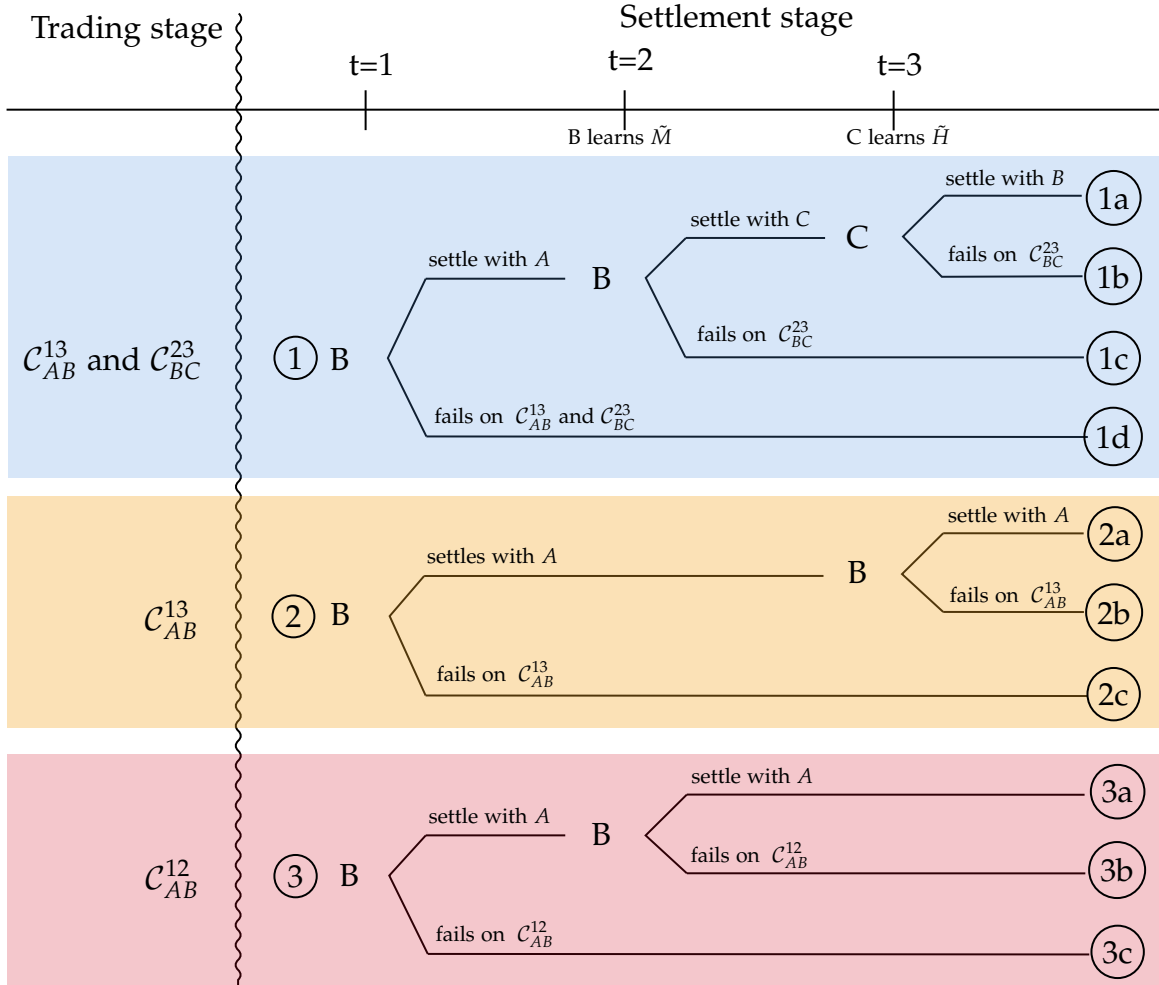
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Figure 5: **Contracts and settlement actions under legacy system.** This figure summarizes the key settlement actions that arise at $t = 1, 2, 3$ given a set of contracts, specified on the left. B and C privately learn their $t = 2$ and $t = 3$ payoffs $\tilde{M}$ and $\tilde{H}$ in the beginning of $t = 2$ and $t = 3$, respectively. Private values factor into their settlement strategies, where $\tilde{M} = M$ with probability λ_M, and 0 otherwise; and $\tilde{H} = H$ with probability λ_H and 0 otherwise. The terminal payoffs for A, B, and C at the end of $t = 3$ is provided in Figure 6.

	Node	A's Payoff	B's Payoff	C's Payoff
$\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$	(1a)	$\mathrm{P}_{AB}^{13} + H$	$-\mathrm{P}_{AB}^{13} + H + P_{BC}^{23}$	$-\mathrm{P}_{BC}^{23} + H$
	(1b)	P_{AB}^{13}	$-\mathrm{P}_{AB}^{13} + H + P_{BC}^{23} - \Delta$	$-\mathrm{P}_{BC}^{23} + H + \tilde{H} - \Delta$
	(1c)	$\mathrm{P}_{AB}^{13} + H$	$-\mathrm{P}_{AB}^{13} + H + \tilde{M} - \Delta$	0
	(1d)	$2L + H$	-2Δ	0
$\mathcal{C}_{AB}^{13}$ only	(2a)	$\mathrm{P}_{AB}^{13} + H$	$-\mathrm{P}_{AB}^{13} + H + \tilde{M}$	0
	(2b)	P_{AB}^{13}	$-\mathrm{P}_{AB}^{13} + H + \tilde{M} - \Delta$	0
	(2c)	$2L + H$	$-\Delta$	0
$\mathcal{C}_{AB}^{12}$ only	(3a)	$\mathrm{P}_{AB}^{12} + L + H$	$-\mathrm{P}_{AB}^{12} + H$	0
	(3b)	$\mathrm{P}_{AB}^{12} + H$	$-\mathrm{P}_{AB}^{12} + H + \tilde{M} - \Delta$	0
	(3c)	$2L + H$	$-\Delta$	0

Figure 6: **Summary of traders' payoffs.** This figure shows the terminal payoffs for the three traders conditional on the set of contracts and settlement actions. Nodes starting with 1 correspond to $\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$; nodes starting with 2 correspond to $\mathcal{C}_{AB}^{13}$ only; and nodes starting with 3 correspond to $\mathcal{C}_{AB}^{12}$. Traders' payoffs from owning the asset at $t = 1, 2, 3$ are as follows: A obtains L, L, H; B obtains $H, \tilde{M}, 0$; and C obtains $0, H, \tilde{H}$.

settle the trade with C, we can express C's expected payoff as:

$$Prob(B \text{ accepts})Prob(B \text{ honors}) \left(H - P_{BC}^{23} + \underbrace{\lambda_C(H - \Delta)}_{\text{net gain from retaining asset at } t = 3} \right)$$

(5)

In this expression, the likelihood of B accepting the trade, and the probability that B settles as specified in the contract increase (weakly) in offer price P_{BC}^{23}, while the payoff conditional on successful trade and settlement decreases in P_{BC}^{23}. The lowest possible price that is accepted with positive probability must satisfy Condition (4), which corresponds to the case where B has already acquired the asset from from A.

At what price P_{BC}^{23} does B always accept *and* honor his agreement with C? First, note that whenever B meets with C first at $t = m1$, B can choose his trading strategy with A, *conditional* on the outcome of his trade with C. Specifically, B has the option to enter C_{AB}^{12} with A instead of C_{AB}^{13}, if C makes an unattractive offer P_{BC}^{23}. Only when C makes a sufficiently attractive offer for C_{BC}^{23}, would B also choose to enter C_{AB}^{13} with A at $t = m2$. As such, Condition 3 is (weakly) stricter than Condition 4. Second, if A expects that B will enter contract C_{BC}^{23} with C, A requires a *settlement risk premium* of $\lambda_C H$ associated with the risk of C failing to return the asset at $t = 3$. Third, if $E[\tilde{M}] < L$, B does not have an incentive to obtain ownership of the asset beyond $t = 1$ without arranging a contract with C. As such, A's reservation price for C_{AB}^{12} is his valuation L. Together, under B's expected cost of acquiring $t = 2$ ownership of the asset from A is given by:

$$E[P_{AB}^{13} - P_{AB}^{12}] = \underbrace{L}_{A\text{'s } t = 2 \text{ valuation}} + \underbrace{\lambda_C H}_{\text{settlement risk premium}}$$

(6)

This expression represents the reservation price at which A is willing to lend the asset at $t = 2$, which also corresponds to the price difference between P_{AB}^{13} and P_{AB}^{12}. The first term is A's payoff from holding onto the asset at $t = 2$, L. A also anticipates that B may lend the asset to C at $t = 2$, in which case he may not reacquire the asset. The second term represents the compensation A requires for that possibility.

A special case is when B's valuation is sufficiently high, i.e. when $E[\tilde{M}] > E[P_{AB}^{13} - P_{AB}^{12}]$. This occurs when λ_B is large enough that $E[\tilde{M}] > L$, and λ_C is sufficiently small. In this case, B's valuation is binding regardless of the order of trades, and B accepts a contract C_{BC}^{23} if and only if $P_{BC}^{23} \geq E[\tilde{M}] + \lambda_C \Delta$:

Lemma 2. *Suppose that B's $t = 2$ valuation of the asset is greater than that of A's $t = 2$ valuation ($E[\tilde{M}] > L$), and C's limited commitment problem is not too severe ($\lambda_C < \frac{E[\tilde{M}]-L}{H+\Delta}$). B accepts offer C_{BC}^{23} only if $P_{BC}^{23} \geq E[\tilde{M}] + \lambda_C\Delta$.*

When the conditions for Lemma 2 are violated, because λ_C is too large, the cost associated with intermediating between A and C is strictly greater than B's private valuation of the asset. Hence, Condition 3 reduces to:

$$P_{BC}^{23} \geq L + \lambda_C H + \lambda_C\Delta. \tag{7}$$

The first two terms on the right hand side represent the expected opportunity cost for A to lend the asset to B, given that B will lend the asset to C. The third term represents the expected cost of a daisy chain fail for B. As λ_C increases, C must pay a steeper price in order to acquire the asset with certainty, because of the expected cost of a fail on both A and B.

Knowing that B's private valuation of $t = 2$ ownership is much lower than that required by Condition 7, C may want to offer a price that is so low that it will only be accepted by B if B *already* acquired the asset from A. If making such low offer is attractive, a hold-up problem arises.

Using Condition 4, we can find the lowest price P^{23} that B is willing to accept. To simplify Condition 4, we first show that $-\Delta < H + E[\tilde{M}] - P_{AB}^{13}$. Note that $P_{AB}^{12} = L$ since A's valuation of the asset at $t = 1$ is L, which implies that $P_{AB}^{13} = 2L + \lambda_C H$, from equation (7). Thus, we have

$$-\Delta < H + E[\tilde{M}] - P_{AB}^{13} \tag{8}$$
$$= H + \lambda_B M - (2L + \lambda_C H), \tag{9}$$

which holds since $\Delta > 2L$.[14]

Consequently, we have that conditional on having entered C_{AB}^{13}, the lowest price which B accepts from C is $P_{AB}^{23} = \lambda_B M + \lambda_C\Delta$. So, when would C be tempted to offer a price P_{BC}^{23} that is lower than the price required for B to acquire the asset from A? The main tradeoff arises between the (1) likelihood of trade and (2) the payoff conditional on trade.

Since C does not know whether B has met with A prior to their meeting, C cannot make an offer contingent on the order of matches. B will accept an offer of $P_{BC}^{23} = \lambda_B M + \lambda_C\Delta$ only if he has already entered C_{AB}^{13}, which occurs with probability $\frac{1}{2}$. With probability $\frac{1}{2}$, B has not yet matched with A and will reject such an offer. Despite this, C

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[14]This assumption is made only to economize on cases, and does not qualitatively matter.

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will find it optimal to offer $P_{BC}^{23'} = E[\tilde{M}] + \lambda_C \Delta$ instead of $P_{BC}^{23''} = P_{AB}^{13} + \lambda_C \Delta$ if[15]

$$\frac{1}{2}\left[H - P_{BC}^{23'} + \lambda_C(H - \Delta)\right] \geq H - P_{BC}^{23''} + \lambda_C(H - \Delta). \tag{10}$$

The next result summarizes the optimal offer strategy of C, taking as given that B successfully trades $\mathcal{C}_{AB}^{13}$ with A when matched:

Lemma 3. *Suppose that C's limited commitment problem is severe ($\lambda_C > \frac{E[\tilde{M}]-L}{H+\Delta}$), and C expects that B enters contract $\mathcal{C}_{AB}^{13}$. C's optimal strategy is to offer a contract $\mathcal{C}_{BC}^{23}$ at price $P_{BC}^{23} = L + \lambda_C(H + \Delta)$ if $\lambda_C < \bar{\lambda}$, and $P_{BC}^{23} = E[\tilde{M}] + \lambda_C \Delta$ if $\lambda_C > \bar{\lambda}$, for some threshold $\bar{\lambda}$.*

The offer strategy in Lemma 3 assumes that whenever B matches with A first, B acquires $t = 2$ ownership from A prior to matching with C. However, this is an equilibrium strategy *only* if B at least breaks even by doing so. When C finds it optimal to offer some P_{BC}^{23} such that B's net payoff from intermediating drops below zero, B ex-ante strictly prefers $\mathcal{C}_{AB}^{12}$ (node ③) to $\mathcal{C}_{AB}^{13}$ (node ①) and intermediation breaks down. This is exactly the case when $P_{BC}^{23} = E[\tilde{M}] + \lambda_C \Delta$, implying:

Lemma 4. *Suppose that C's limited commitment problem is severe (i.e. $\lambda_C > \frac{E[\tilde{M}]-L}{H+\Delta}$) and B believes that C will offer the hold-up price of $P_{BC}^{23} = E[\tilde{M}] + \lambda_C \Delta$. Then, B's optimal trading strategy with A is to offer contract $\mathcal{C}_{AB}^{12}$ at price $P_{AB}^{12} = L$.*

We can combine Lemmas 3 and 4 to fully characterize the equilibrium. By Lemma 3, C's optimal offer, expecting that B will intermediate the asset (by entering $\mathcal{C}_{AB}^{13}$ with A), is to hold up B whenever λ_C is greater than $\bar{\lambda}$. In turn, Lemma 4 states that if B anticipates C to attempt a hold-up, B forgoes any attempt to intermediate the asset at all. Together this implies that whenever $\lambda_C > \bar{\lambda}$, the asset cannot be intermediated with probability 1 in equilibrium.

As an intermediary, B can weaken C's incentives to attempt a hold-up by buying the asset on behalf of C with probability less than 1. Specifically, whenever B matches with A first, B can offer $\mathcal{C}_{AB}^{13}$ with some probability $\mu < 1$, and $\mathcal{C}_{AB}^{12}$ otherwise. By doing so, B lowers the expected probability that C believes he will obtain the asset conditional on offering a hold-up price $E[\tilde{M}] + \lambda_C \Delta$ from $\frac{1}{2}$ to $\frac{\mu}{2}$. As long as μ is sufficiently low, C's dominant strategy is to offer $L + \lambda_C H + \lambda_C \Delta$, which lets B break even in expectation from intermediation.

The next proposition summarizes the conditions under which, in equilibrium, the set of optimal trades are achieved under the legacy system:

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[15]It is straightforward to verify that B never finds it optimal to renege ex-post on C after accepting an offer as $M \leq \Delta$.

Proposition 1 (Equilibrium under legacy system). *Suppose that trade and settlement occurs under the legacy system. If C's limited commitment problem is not too severe (i.e. $\lambda_C < \bar{\lambda}$ for some cutoff $\bar{\lambda} \in (0,1)$), there exists an equilibrium in which the optimal trades are achieved with certainty.[16] If C's limited commitment problem is severe (i.e. $\lambda_C < \bar{\lambda}$), C obtains the asset with probability $\frac{1}{2}(1 + \mu^*) < 1$, where μ^* decreases in λ_C and $\mu^* \in [0,1)$.*

In the legacy environment, trades are unrestricted by the feasibility of settlement. Instead, whether settlement actions consistent with the agreed-upon trades are carried out ultimately depends on traders' incentives, given the costs associated with failing to keep promises. As a result, settlement breaks down whenever the costs of failing are not high enough to deter traders from reneging on contractual obligations ex-post.

In the context of the model, C's payoff from retaining the asset at $t = 3$ is sometimes too large for the cost Δ to provide sufficient incentives to return the asset to B. Expecting C's limited commitment problem, both A and B require higher prices to compensate for the possibility of settlement to fail. This, however, in turn makes a "low-ball" offer ever more attractive. When λ_C is too large, intermediation completely breaks down as intermediating is no longer profitable for B.

At the same time, the complete decoupling between trade and settlement enables traders to enter into a contract without having to explicitly prove to their counterparty that they can fulfill the terms of that contract. In the context of the model, B is able to enter $\mathcal{C}_{BC}^{23}$ with C, regardless of whether he has already acquired $t = 2$ ownership from A. As a consequence, when B matches with C, B preserves private information about whether he has met with A or not.

An important takeaway is the *interplay* between the trading system and the settlement system. Trading occurs asynchronously, with little transparency over the history of trades. Taken in isolation, both the opacity of a trading system and the reliance of a settlement system on ex-post incentive compatibility could be viewed as suboptimal, from the standpoint of market design. However, we show that, when paired, this combination is fundamental to facilitating the efficient transfer of ownership between multiple traders. Even with the potential for a hold-up problem, traders successfully agree to an interdependent set of contracts, as long as the limited commitment problem is not too severe (i.e. λ_C is small enough).

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[16]In the legacy system, the trades are optimal, but the allocation may not be optimal ex-post due to the positive probability that C fails in $t = 3$, resulting in a deadweight loss of 2Δ.
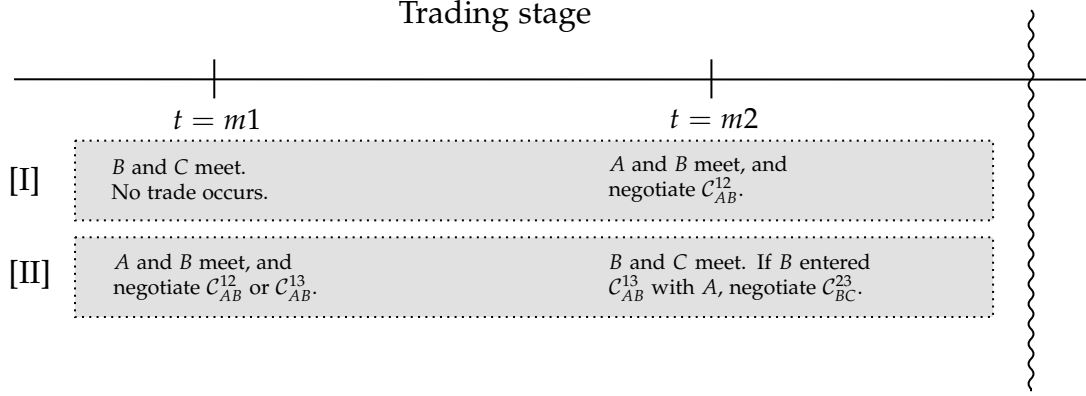
Figure 7: **Trading in Token System.** This figure outlines the events under the two possible sequence of meetings. In sequence [I], B meets with C, then with A. In sequence [II], B meet with A, then with C.

4.2 Tokenized Market

In a token system, traders can program the asset to guarantee the future settlement of the trade they are negotiating. This commitment technology implies that settlement occur regardless of whether an trader would like to strategically fail ex post.

Let us revisit the limited commitment problem posed by C's ex-post incentive to retain the asset in $t = 3$. Suppose that, during the trading stage, A and B meet at $t = m1$ and agree to some contract $\mathcal{C}_{AB}^{13}$. In a token environment, a corresponding program is jointly submitted by A and B, which instantly transfers $t = 1, 2$ ownership of the asset from A's to B's, while guaranteeing that the ownership of the asset is transferred back to A at $t = 3$. By entering $\mathcal{C}_{AB}^{13}$ A relinquishes any ownership (and thus control) of the asset for dates $t = 1, 2$ the moment they trade; concurrently, B immediately gains the right to enter any transfer of $t = 1, 2$ ownership of the asset.

At $t = m2$, B and C meet and agree to some contract $\mathcal{C}_{BC}^{23}$. Since B acquired control over $t = 2$ ownership of the asset, $\mathcal{C}_{BC}^{23}$ is feasible. C instantly gains the right to enter any transfer of $t = 2$ ownership of the asset. However, the asset is programmed to return to A at date $t = 3$, and C exercises no control over the asset beyond $t = 2$. In this way, the token system resolves settlement uncertainty arising from C's limited commitment problem.

Despite its clear advantage over the legacy system in resolving settlement uncertainty, the token system poses two new issues that were not present under the legacy system. They both arise due to the requirement that contracts must be feasible.

Critically, when B meets with C, B must reveal to C about whether he has $t = 2$ ownership of the asset. In the token system, a contract is feasible only if the seller of the

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asset already holds the ownership rights of the asset for the date at which the asset must settle. As such, when B and C negotiate a contract, both traders can verify whether the terms of negotiation are feasible. In effect, C can verify at the time of trade that B holds the asset, since otherwise the program corresponding to $\mathcal{C}_{BC}^{23}$ would not be permissible.

This information, which was not revealed in the legacy system, exacerbates the possibility of a hold-up. In contrast to the legacy system, C chooses his offer P_{BC}^{23} conditional on verifying that B has already acquired $t = 2$ ownership of the asset. With this certainty, the optimal offer strategy of C is to offer just B's reservation price, $E[\tilde{M}]$. The revelation of information regarding the order of trades magnifies the hold-up problem that was possible in the legacy system but not always binding. The hold-up problem now can directly prevent desired allocations from being achieved in equilibrium:

Theorem 1 (Equilibrium with Tokens). *Suppose that trade and settlement occurs under the token system. For $\lambda_B > \frac{L}{M}$, there exists an equilibrium where B and C enter $\mathcal{C}_{BC}^{23}$ with probability $\frac{1}{2}$. For $\lambda_B < \frac{L}{M}$, C is unable to acquire the asset in equilibrium.*

Additionally, the order of trades directly affects whether the asset can be intermediated by B. In order for B to enter a contract $\mathcal{C}_{BC}^{23}$ with C, B must at the time of their match own the rights to the asset at $t = 2$. This implies that B must have acquired the relevant rights from A prior to trading with C:

Corollary 1. *$\mathcal{C}_{BC}^{23}$ is a feasible contract only if B matches with A first and obtains $t = 2$ ownership of the asset.*

An implication is that the intermediation chain between A, B, and C can only arise with at most probability $\frac{1}{2}$, when B matches with A at $t = m1$, before matching with C at $t = m2$.[17]

There are two things to note. First, due to the requirement of matching orders, the equilibrium with tokens fails to achieve ex-ante first-best allocations. This result, which arises due to the random match sequence, *is not*, in general a problem when trade and settlement are segregated, as in a typical legacy system. This points to an efficiency loss that can arise when immediate settlement is implemented. Second, the hold-up problem becomes acute with C's revelation of B's ownership of the asset (and the order of trades). In fact, when A values $t = 2$ ownership of the asset strictly less than L, C will *always* finds it profitable to offer a low price, which B accepts conditional on owning the

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[17]Of course, this inefficiency is borne directly from our simplifying assumption that the sequence of matches are random. One could consider an environment where B could take (costly) actions to endogenously determine the order of matches. As discussed earlier, this alone will not materially improve the outcomes in token system as it does not address the hold-up problem.
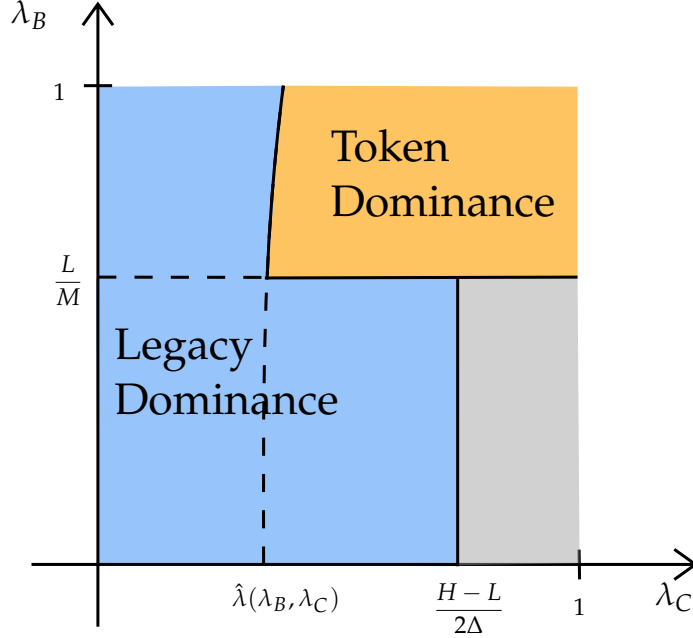
Figure 8: **Relative efficiency of settlement systems.** This plot shows the relative efficiency of the equilibrium under the legacy system and the token system for λ_B and λ_C, which each capture the degree to which a hold-up or limited commitment problem exist. Parameters are set at $L = 3, H = 12, M = 5, \Delta = 6$. Efficiency is greater under the legacy system in the blue region, the token system in the orange region, and equivalent in the grey region.

asset. However, anticipating this, B opts not to acquire the asset in the first place, thus thwarting intermediation. The next theorem summarizes the relative efficiency between a legacy and token system:

Theorem 2 (Relative Efficiency). *Suppose that $\lambda_B < \frac{L}{M}$. Then, efficiency is greater under the legacy system if $\lambda_C < \frac{H-L}{2\Delta}$, and equivalent when $\lambda_C > \frac{H-L}{2\Delta}$. If $\lambda_B > \frac{L}{M}$, then efficiency is greater under the token system if $\lambda_C > \hat{\lambda}$ and lower when $\lambda_C < \hat{\lambda}$, for some threshold $\hat{\lambda} > \bar{\lambda}$.*

Discussion. The main takeaway is that token systems are not unambiguously superior to the legacy system. Rather, our model highlights two key issues that arise in decentralized, heavily intermediated markets. First, the scope of intermediation *drops* under tokenization, as intermediaries' valuations play a much more significant role in their ability to assist with the reallocation of the asset. Second, while tokenization can eradicate settlement uncertainty when two parties can agree to a desirable trade, the fact that each trade must be feasible at the time of trade puts undue emphasis on the need for the sequence of meetings to coincide with the order of intermediation.

Our results motivate a natural question on what solutions could address potential inefficiencies arising with token systems. As our paper shows, token systems may reveal

too much information in environments where optimal allocations depend on intermediaries. Markets involving trading mechanisms with direct matching between sellers and buyers are more attractive candidates for applying token systems. In general, an obvious proposition is to re-design the trading system in conjunction with the settlement system in order to maximize the potential of zero settlement uncertainty. This, however, ignores the frictions, outside of our model, that may support or necessitate the trading environment taken as given in our environment. In the context of our model, while the role of B as an intermediary is assumed, it is important to recognize that intermediaries can play an essential role in facilitating transactions that might not otherwise occur. Specifically, in a more complex model, trades may not occur even if A and C can meet. We expect that our results would extend to an environment where B arises as an intermediary endogenously.

The two key sources of inefficiency arise due to the revelation of information regarding ownership at the time of trade. Our results might suggest that another solution is to relax the conditions of trade, so that traders could enter trades without revealing this information. At first glance, it seems like it could resolve the issues pertinent to the token system. In the context of our model, this would be akin to enabling B and C to enter a state-contingent contract, whereby a trade is executed only if at the end of the trading stage, B's account holds $t = 2$ ownership of the asset.

However, allowing for such a contract re-introduces a commitment problem and with it settlement fails. A small modification of the model suffices using an argument related to that made by (Lee et al., 2021). Suppose that B's valuation of the asset at $t = 2$ for a range between O and H, and B learns it after the meeting at $t = m2$. The state-contingent contract introduces a "gap", whereby B now has the option to forgo his trade with C by failing to satisfy the conditions of trade. In particular, even if B obtains $t = 2$ ownership of the asset from A, if B wants to hold onto it for himself, then B simply needs to hold ownership in, for example, a separate account unknown to C. This means that B honors the trade with C only when his $t = 2$ valuation is realized as low. It is straightforward to see that, in equilibrium, C would offer a price less than H, which means that, generically, trade fails with a positive probability. In other words, there is an implicit form of strategic settlement failure. The re-introduction of a commitment problem can become even more problematic for token systems as, technically, the agreed upon trade, which reference accounts, was executed without fault by the joint program.

5 Conclusion

This paper studies how tokenization affects equilibrium trade in a theoretical model of an over-the-counter market. Tokenization has clear advantages: We illustrate how tokenization it can eliminate a limited commitment problem, by committing settlement actions at the time that contracts are forged. Collapsing trade and settlement, however, comes at a cost. We show that doing so necessitates that traders reveals more private information relative to the traditional environment. This creates a hold-up problem and may destory an intermediation chain necessary for efficient outcomes.

Whether a settlement protocol is efficient is intricately tied to the whether it is paired with a congruent trading mechanism. Due to the potential for decentralization, tokenized markets have been viewed as particularly disruptive for over-the-counter markets. However, some features are not amenable to the current market structure, which depends highly on intermediaries to facilitate complex intermediation chains. Our paper offers a concrete illustration of this problem.

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Proofs

Proof of Lemma 1. Follows from text. □

Proof of Lemma 2. Note that $E[\tilde{M}] > L + \lambda_C(H + \Delta)$ for $\lambda_C > \frac{E[\tilde{M}-L]}{H+\Delta}$, which requires $E[\tilde{M}] > L$ since $\lambda_C > 0$. □

Proof of Lemma 3. Following the argument in the text, it suffices to consider whether C finds it optimal to make an offer which is accepted with probability 1, or $E[\tilde{M}] + \lambda_C\Delta$, which is accepted with probability $\frac{1}{2}$ conditional on B already having acquired $t = 2$ ownership from A. Consider a price at which B always accepts, which requires Condition 3. The minimum price satisfying the condition is $L + \lambda_C H + \lambda_C\Delta$. C's payoff for offering $L + \lambda_C H + \lambda_C\Delta$ is greater than offering $E[\tilde{M}] + \lambda_C\Delta$ if:

$$H - (L + \lambda_C H + \lambda_C\Delta) + \lambda_C(H - \Delta) \tag{11}$$

$$\geq \frac{1}{2}\left(H - (E[\tilde{M}] + \lambda_C\Delta) + \lambda_C(H - \Delta)\right) \tag{12}$$

Reorganizing this inequality, we get the inequality holds only if $\lambda_C < \frac{\frac{1}{2}H - L + \frac{1}{2}E[\tilde{M}]}{\frac{1}{2}H + (1-\frac{1}{2})2\Delta}$. Note, C's payoff is positive given price $L + \lambda_C H + \lambda_C\Delta$ only if $\lambda_C < \frac{H-L}{2\Delta}$, and $\frac{(1-\frac{1}{2})H - L + \frac{1}{2}E[\tilde{M}]}{\frac{1}{2}H + (1-\frac{1}{2})2\Delta} < \frac{H-L}{2\Delta}$. Also, note that $\frac{(1-\frac{1}{2})H - L + \frac{1}{2}E[\tilde{M}]}{\frac{1}{2}H + (1-\frac{1}{2})2\Delta} > \frac{E[\tilde{M}-L]}{H+\Delta}$. Hence, for some threshold $\bar{\lambda} \equiv \frac{\frac{1}{2}H - L + \frac{1}{2}E[\tilde{M}]}{\frac{1}{2}H + \Delta}$, C offers $L + \lambda_C(H + \Delta)$ for $\lambda_C < \bar{\lambda}$, and $E[\tilde{M}] + \lambda_C\Delta$ otherwise. □

Proof of Lemma 4. Note that given $P_{BC}^{23} = L + \lambda_C(H + \Delta)$, B breaks even in expectation. This implies that any lower price violates B's participation condition, and the optimal strategy for B is to make offer $P_{AB}^{12} = L$ to A. Hence if $\lambda_C > \bar{\lambda}$, B's optimal offer strategy is $P_{AB}^{12} = L$. □

Proof of Theorem 1. We first show existence of an equilibrium where traders agree to $\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$, independent of the order of matches when $\lambda_C < \bar{\lambda}$. Assume $\lambda_C > \bar{\lambda}$. A always accepts B's offer $P_{AB}^{13} = 2L + \lambda_H$, which is equal to his reservation price taking into account C's strategic fail as outlined in Lemma 1. Under Lemma 3, C's offer strategy is $P_{BC}^{23} = L + \lambda_C(H + \Delta)$, which B accepts conditional on having or expected to enter $\mathcal{C}_{AB}^{13}$, since it is equal to his reservation price and a premium $\lambda_C\Delta$ associated with the daisy chain fail.

Next, we show that when when $\lambda_C > \bar{\lambda}$, there does not exist an equilibrium where C obtains the asset with probability 1. By Lemma 3, conditional on B always entering $\mathcal{C}_{AB}^{13}$ with A, C's optimal offer strategy is $P_{BC}^{23} = E[\tilde{M}] + \lambda_C\Delta$, which violates B's participation condition. Following Lemma 4, B's dominant strategy to choose $P_{AB}^{12} = L$. Together this implies that C fails to obtain the asset with certainty. we show that there is an equilibrium where C obtains the asset with some probability $\mu^* \frac{1}{2} + (1 - \frac{1}{2})$ for some $\mu^* \in [0,1)$. Consider a candidate equilibrium in which B enters $\mathcal{C}_{AB}^{13}$ with probability μ, and $\mathcal{C}_{AB}^{12}$ otherwise if matched with A first, and enter $\mathcal{C}_{AB}^{13}$ if he accepts $\mathcal{C}_{BC}^{23}$ when matched with C first. C maximizes his payoff from offering $P_{BC}^{23} = L + \lambda_C(H + \Delta)$ if:

$$(\mu\frac{1}{2} + (1 - \frac{1}{2}))[H - (L + \lambda_C H + \lambda_C\Delta) + \lambda_C(H - \Delta)] \geq \mu\frac{1}{2}[H - (\lambda_B L + \lambda_C\Delta) + \lambda_C(H - \Delta)] \tag{13}$$

Note that the inequality becomes monotonically tighter as μ increases, and holds when $\mu \to 0$. This implies that there exists some μ' such that equality holds. Let μ^* be given by $\max\left\{0, \frac{H - L - 2\lambda_C\Delta}{L + \lambda_C H - \lambda_B L}\right\}$. Since for price $P_{BC}^{23} = L + \lambda_C(H + \Delta)$, B exactly breaks even, B is indifferent between any μ conditional on $P_{BC}^{23} = L + \lambda_C(H + \Delta)$. This establishes existence. $\qquad\square$

Proof of Corollary 1. Follows from text. $\qquad\square$

Proof of Theorem 1. Suppose that the order of matches is $B - C$ and $A - B$. Since B does not own any rights to the asset when she matches with C, no contract is feasible. Hence, trade only occurs between $A - B$.

Suppose that the order of matches is $A - B$ and $B - C$. The order of trades is common knowledge, since trade between B and C requires B to own the asset. Suppose that B acquires rights to the asset for $t = 2$ with probability 1. Suppose that C offers $\lambda_C L$. Then, B accepts with probability 1. Note, however, that B must offer A at least L in order to obtain $t = 2$ ownership. Since doing so leads to negative profits, it is optimal for B to only acquire $t = 1$ ownership. Hence, in equilibrium C never obtains the asset if $\lambda_B < 1$,

and there exists an equilibrium with intermediation only if $\lambda_B = 1$.

$\square$

Proof of Theorem 2. First, consider when $\lambda_B < \frac{L}{M}$. In the legacy system, if $\lambda_C < \frac{H-L}{2\Delta}$, total payoff is given by $3H - 2\lambda_C\Delta$ for $\lambda_C < \bar{\lambda}$; $2H + (\mu^*\frac{1}{2} + (1 - \frac{1}{2}))(H - 2\lambda_C\Delta) + (1 - \mu^*)\frac{1}{2}L$ otherwise; if $\lambda_C > \frac{H-L}{2\Delta}$, total payoff is $2H + L$. In the token system, total payoff is $2H + L$. Hence, payoff is greater in the legacy system for $\lambda_C < \frac{H-L}{2\Delta}$ and equal otherwise.

Next, consider when $\lambda_B > \frac{L}{M}$. As before, in the legacy system, if $\lambda_C < \frac{H-L}{2\Delta}$, total payoff is given by $3H - 2\lambda_C\Delta$ for $\lambda_C < \bar{\lambda}$; $2H + (\mu^*\frac{1}{2} + (1 - \frac{1}{2}))(H - 2\lambda_C\Delta) + (1 - \mu^*)\frac{1}{2}L$ otherwise; if $\lambda_C > \frac{H-L}{2\Delta}$, total payoff is $2H + L$. In the token system, total payoff is $\frac{5}{2}H + \frac{1}{2}L$. Note that μ^* decreases in λ_C and $2H + (\mu^*\frac{1}{2} + (1 - \frac{1}{2}))(H - 2\lambda_C\Delta) + (1 - \mu^*)\frac{1}{2}L \to 2H + L$ as $\lambda_C \to \frac{H-L}{2\Delta}$. This implies that there exists some cutoff $\hat{\lambda} \equiv \frac{\mu^*}{1+\mu^*}\frac{H-L}{2\Delta} \in (\bar{\lambda}, \frac{H-L}{2\Delta})$ such that $2H + (\mu^*\frac{1}{2} + (1 - \frac{1}{2}))(H - 2\lambda_C\Delta) + (1 - \mu^*)\frac{1}{2}L = 2H + \frac{1}{2}H + (1 - \frac{1}{2})L$. Hence, the token system dominates for $\lambda_C > \hat{\lambda}$.

$\square$