Optimal Policy Rules in HANK[†]

Alisdair McKay
FRB Minneapolis

Christian K. Wolf
MIT & NBER

March 13, 2023

Abstract: We characterize optimal policy rules in business-cycle models with nominal rigidities and heterogeneous households. The derived rules are expressed in terms of the causal effects of policy instruments on policymaker targets. Our first result is that the optimal policy rule of a "dual mandate" central banker—a policymaker that only cares about inflation and output—is unaffected by household heterogeneity. The optimal rule of a Ramsey planner contains an additional distributional term that incorporates the effects of her available policy instruments on consumption inequality. We quantify the role of this term in a structural model that incorporates and is consistent with evidence on the key distributional channels of monetary policy, and we find that the concern for inequality only has a moderate effect on optimal interest rate policy. Fiscal stimulus payments, on the other hand, have strongly progressive effects and are thus well-suited to cushion the distributional effects of cyclical fluctuations.

[†]Email: alisdair.mckay@mpls.frb.org and ckwolf@mit.edu. We thank Adrien Auclert, Anmol Bhandari, Jordi Galí, Émilien Gouin-Bonenfant, and Ludwig Straub for helpful comments. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1 Introduction

Should household inequality affect the conduct of cyclical stabilization policy? Recent years have seen a surge of interest in this question, with several prominent studies arguing that the design of optimal monetary policy is substantially altered by distributional considerations (e.g., Bhandari et al., 2021; Acharya et al., 2022). In principle, household heterogeneity can affect optimal policy design in two separate ways. First, household inequality could alter the transmission from policy instruments to any given target (e.g., inflation and output). This change in transmission may affect whether or not a policymaker can attain her given targets, and how policy instruments need to be set to do so. Second, household heterogeneity may also alter the targets themselves. For example, with market incompleteness, policymakers may want to dampen the distributional effects of cyclical fluctuations.

We provide new insights into this question by expressing optimal policy rules in business-cycle models with household heterogeneity in terms of empirically measurable "sufficient statistics." Our first contribution is to cast optimal policy problems in a rich heterogeneous-agent New Keynesian (HANK) environment in linear-quadratic form, closely mirroring the canonical representative-agent New Keynesian (RANK) literature. As familiar from this literature, the solution to the linear-quadratic policy problem takes the form of a forecast target criterion, providing a general characterization of optimal policy completely independently of the shocks hitting the economy (Giannoni & Woodford, 2002). Drawing on McKay & Wolf (2023), we furthermore show that these optimal policy rules can be expressed in terms of the dynamic causal effects of policy instruments on policymaker targets. This "sufficient statistics" characterization of optimal policy rules has two benefits. First, it provides a convenient organizing device for understanding the literature—differences in optimal policy design can be traced back to differences in the causal effects of the policy instrument on policymaker targets. Second, it suggests a natural avenue for model discipline—connect as tightly as possible with existing empirical evidence on the causal effects of policy interventions.

We then use our sufficient statistics framework to revisit the two ways in which household heterogeneity can in principle affect optimal policy design. On *transmission* (i.e., fixing policymaker targets), we show that the optimal policy rule of a dual mandate central banker does not change with household heterogeneity. Such heterogeneity may affect the path of interest rates required to implement the dual mandate optimum, but our sufficient statistics perspective reveals that the scope even for this is rather limited. On *targets*, in principle, distributional concerns can have large effects on optimal stabilization policy. In practice, however, empirical evidence suggests that monetary policy has broad-based effects in the cross-section of households, with consumption of all households rising following a monetary easing, and vice-versa. Monetary policy is thus a rather blunt distributional tool, and as a result a Ramsey planner in an environment consistent with such broad-based effects does not choose to deviate much from dual mandate policy prescriptions.

Environment. Our analysis is set in a business-cycle model with nominal rigidities and household heterogeneity. Households face idiosyncratic income risk and self-insure by borrowing and saving in capital, short-term bonds, and long-term bonds. The policymaker sets short-term nominal interest rates, pays transfers to households, and finances its expenditure through taxation as well as bond issuance. As we discuss later, the environment is designed to be rich enough to speak to the core channels of how monetary policy affects consumption across the cross-section of households.

In this environment we study optimal policy problems cast in linear-quadratic form (e.g., Giannoni & Woodford, 2002; Benigno & Woodford, 2012). Linearization of the private-sector relations of our economy yields the linear constraints. Our quadratic objective is then either *ad hoc*—a conventional dual mandate objective—or derived as an approximation to a particular Ramsey policy problem.

Dual-mandate central banker. We begin our analysis by studying the optimal policy problem of a conventional "dual-mandate" central banker that seeks to close the output gap and stabilize inflation. We find this *ad hoc* loss function useful as a starting point, for two reasons. First, conceptually, it allows us to explore the effects of changes in policy instrument propagation while fixing policymaker targets. Second, practically, it is arguably the relevant objective function for real-world central banks.

Our main result is that the optimal interest rate target criterion for such a dual mandate central banker is *exactly the same* as in a standard representative-agent environment. The logic is as follows. Household heterogeneity in our environment only affects the demand side of the economy (i.e., the model's "IS" curve). In the optimal policy problem, however, this demand block is a slack constraint: the policymaker can pick an output-inflation allocation subject to the Phillips curve, and then set nominal interest rates as necessary to generate demand consistent with the desired allocation. Household heterogeneity thus may matter for the path of the policy instrument, but not for optimal output and inflation outcomes.

Leveraging our sufficient statistics perspective, we then connect these theoretical results on optimal dual-mandate policy to empirical evidence on monetary policy shock transmission. The empirical literature has tried to identify the causal effects of nominal rate changes on output and inflation (e.g. see Ramey, 2016). While silent on transmission mechanisms and so in particular on the importance of heterogeneity-related channels, aggregate data *are* informative about the size of interest rate movements required to move output and inflation by a given amount. But it then follows that *quantitatively relevant* HANK and RANK models—that is, models that are consistent with the empirical evidence on monetary shock propagation—will not only share the same optimal dual-mandate policy rule, but they will in fact also tend to agree quite closely on the interest rate path needed to implement the optimal inflation and output outcomes. Our quantitative analysis confirms this insight: our HANK model—which is calibrated to agree with this empirical evidence on monetary shock propagation—agrees closely with a RANK model calibrated to the same evidence.

Ramsey problem. We next study a Ramsey problem in which the planner seeks to maximize a weighted sum of individual household utilities, allowing us to explore the role of changes in policymaker objectives. Applying a second order approximation, we derive a loss function that adds to the usual output gap and inflation objectives a third term reflecting distributional concerns. We then solve the policymaker problem of minimizing this loss function subject to linearized private-sector optimality conditions.

The policymaker's problem now yields an optimal policy rule consisting of three terms, trading off the policymaker's ability to use her policy instrument to stabilize output, inflation, and the cross-sectional consumption distribution. The third term of this rule—which, importantly, is the sole difference from the optimal standard dual mandate policy rule—is governed by the distributional incidence of the instrument. If, for example, interest rate movements do not affect consumption shares (e.g., as in Werning, 2015), then the third term is zero and the Ramsey rule is identical to the optimal dual-mandate rule; if, on the other hand, monetary policy has large distributional effects (e.g., as in Bhandari et al., 2021), then distributional concerns may swamp price and output stability considerations.

To characterize the quantitatively relevant Ramsey policy, we again leverage our sufficient statistics perspective. As we have discussed, our conclusions on optimal policy will be governed by the causal effects of interest rate policy on consumption inequality. Since prior empirical work has delivered the sharpest results on the cross-sectional *income* incidence of monetary policy, our strategy is to calibrate our model to match these distributional channels of monetary policy, and then rely on the structure of the model to infer the ultimate effects on *consumption* inequality. Two features of our model turn out to be particularly relevant. First, our model features long-duration assets, allowing us to capture an important redistribution channel of monetary policy (see Auclert, 2019). Second, the model is designed to generate only a modest response of aggregate labor income following a monetary expansion, consistent with empirical evidence. For our purposes, the key takeaway is that the model generates fairly evenly distributed effects of policy, with consumption responding by similar percentage amounts across the wealth and income distribution. Given this particular incidence, we find that monetary policy is rather ill-suited as a tool to deal with the distributional implications of business-cycle shocks. For example, if—in response to a shock that largely affects the consumption of the poor—interest rates were set to stabilize consumption at the bottom, then consumption of other households and so aggregate output and inflation would overshoot. The Ramsey policymaker does not find such overshooting optimal, and instead responds to the shock similarly to conventional optimal dual-mandate policy. These quantitative findings contrast with recent work on optimal monetary policy with heterogeneous households that tends to find an important role for distributional considerations (e.g., Bhandari et al., 2021; Acharya et al., 2022; Dávila & Schaab, 2022).

Finally, we also briefly consider a Ramsey policymaker that jointly sets interest rates and stimulus checks, consistent with recent U.S. policy practice. While equivalent in their effects on output and inflation (Wolf, 2021), the two instruments differ significantly in their distributional incidence, with fiscal stimulus payments sharply compressing consumption inequality. The two instruments are thus highly complementary, with monetary policy primarily aimed at aggregate stabilization while fiscal policy manages distributional concerns.

Literature. We contribute to the literature on optimal policy in business-cycle models with rich microeconomic heterogeneity (Acharya et al., 2022; Bhandari et al., 2021; Le Grand et al., 2022; Dávila & Schaab, 2022). Conceptually, our key contribution is to characterize optimal policy through forecast target criteria expressed in terms of easily interpretable sufficient statistics. Our computation of these optimal policy rules leverages sequence-space representations of equilibria and thus the recent work of Auclert et al. (2021).[1] A contemporaneous paper that does the same is Dávila & Schaab (2022). Those authors do not rely on linear-quadratic approximations, thus providing more general optimal policy results, but without our sufficient statistics perspective. Our discussion of the mapping between

[1] By the equivalence of perfect-foresight sequence-space and stochastic linear state-space methods, our targeting criterion also applies to the analogous stochastic linear-quadratic optimal control problem. Sequence-space linear-quadratic policy problems have been used in prior work to derive "optimal policy projections" (Svensson, 2005; De Groot et al., 2021; Hebden & Winker, 2021).

policy rules and empirical evidence builds on our earlier work in McKay & Wolf (2023). These conceptual innovations allow us to revisit several substantive results in the prior heterogeneous-household optimal policy literature.

First, we derive a sharp set of irrelevance results for the effects of inequality on optimal policy design. Prior work has emphasized that inequality will affect policy propagation by altering the demand side of the economy (Kaplan et al., 2018; Auclert, 2019); our analysis, however, reveals that these changes do not affect the optimal "dual mandate" output and inflation outcomes. If furthermore matched to the same evidence on policy shock propagation, then HANK and RANK models will also (roughly) agree on the required rate paths.

Second, our sufficient statistics perspective provides novel insights on the insurance role of stabilization policy. Our formulation of the Ramsey problem objective isolates the policymaker's desire to provide insurance against aggregate shocks with unequal incidence on consumption across households.[2] In particular, our second-order approximation to the social welfare function extends a similar finding by Acharya et al. to a more traditional HANK model that does not permit analytical aggregation. The key benefit of our formulation is that it delivers a transparent characterization of optimal policy, with the extent to which household heterogeneity affects the optimal Ramsey policy fully governed by the distributional effects of the policy instrument. Guided by these insights, and differently from prior work (e.g., Bhandari et al., 2021; Le Grand et al., 2022), we consider a rich quantitative model that is consistent with the main distributional channels of monetary stabilization policy.

Finally, our analysis of optimal joint fiscal-monetary policy extends results in Wolf (2021) and Bilbiie et al. (2021). Wolf (2021) shows that, in a standard HANK setting, transfers and monetary policy can implement the same aggregate allocations, but differ in their distributional implications. In a two-agent environment, Bilbiie et al. (2021) argue that monetary and fiscal policy can together stabilize both the aggregate activity as well as the consumption shares of the two household types. Our work is complementary: we analyze the optimal monetary-fiscal policy mix in an environment with rich heterogeneity.

Outline. In Section 2, we present a general linear-quadratic sequence-space problem and its solution. Section 3 outlines our HANK model, and Sections 4 and 5 discuss our results for optimal dual-mandate and Ramsey policy, respectively. We conclude in Section 6.

[2] As such, our formulation of the problem abstracts from many of the issues of time consistency and inflation bias discussed in Dávila & Schaab (2022).

2 Linear-quadratic problems in the sequence space

Throughout this paper we study optimal policy problems that can be recast as deterministic linear-quadratic control problems. We begin in Section 2.1 by first stating the problem and presenting its solution. Section 2.2 then places our analysis in the broader context of the literature, focussing in particular on the tight connection between our expressions and the (in principle estimable) causal effects of policy shocks.

2.1 Problem & solution

We consider a policymaker that faces a linear-quadratic optimal control problem.

Preferences. The policymaker targets I variables, indexed by i. We let x_{it} be the deviation of the ith target variable from its target value at date t. We consider a policymaker with quadratic loss function

$$\mathcal{L} \;\equiv\; \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\sum_{i=1}^{I}\lambda_i x_{it}^2 \;=\; \frac{1}{2}\boldsymbol{x}'\left(\Lambda\otimes W\right)\boldsymbol{x}, \tag{1}$$

where $\boldsymbol{x}_i \equiv (x_{i0}, x_{i1}, \dots)'$ is the perfect-foresight sequence of the ith target variable through time and $\boldsymbol{x} \equiv (\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_I)'$ stacks those paths for all of the I targets. The λ_i's denote the weights associated with the different policy targets, with $\Lambda \equiv \mathrm{diag}(\lambda_1, \lambda_2, \dots, \lambda_I)$. Finally $W = \mathrm{diag}(1, \beta, \beta^2, \dots)$ summarizes the effects of discounting in the policymaker preferences, with discount factor $\beta \in (0, 1)$.

Constraints. The policymaker faces constraints imposed by the equilibrium relationships between variables. These linear constraints are expressed compactly as

$$\mathcal{H}_x\boldsymbol{x} + \mathcal{H}_z\boldsymbol{z} + \mathcal{H}_\varepsilon\boldsymbol{\varepsilon} \;=\; \boldsymbol{0}, \tag{2}$$

where $\boldsymbol{z} \equiv (\boldsymbol{z}_1, \boldsymbol{z}_2, \dots, \boldsymbol{z}_J)'$ stacks time paths for the J policy instruments available to the policymaker, and $\boldsymbol{\varepsilon} \equiv (\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \dots, \boldsymbol{\varepsilon}_Q)'$ similarly stacks the paths for Q exogenous shocks. $\{\mathcal{H}_x, \mathcal{H}_z, \mathcal{H}_\varepsilon\}$ are then conformable linear maps.

While the structural models considered in the remainder of this paper directly map into constraints of the general form (2), it follows from the discussion in McKay & Wolf (2023) that these constraints can equivalently and more conveniently be expressed as

$$\boldsymbol{x}_i = \sum_{j=1}^{J} \Theta_{x_i,z_j} \boldsymbol{z}_j + \sum_{q=1}^{Q} \Theta_{x_i,\varepsilon_q} \boldsymbol{\varepsilon}_q, \quad i = 1, 2, \ldots, I \tag{3}$$

where the Θ's are linear maps that capture the dynamic causal effects of a policy instrument path $\boldsymbol{z}_j$ or shock path $\boldsymbol{\varepsilon}_q$ on a target variable path $\boldsymbol{x}_i$. The alternative constraint (3) thus expresses the policy targets directly in terms of impulse responses to policy instruments and exogenous shocks, as opposed to imposing implicit relationships as in (2).[3]

Problem & solution. The optimal policy problem is to choose the instrument paths $\boldsymbol{z}$ to minimize (1) subject either to (2) (for the original constraint formulation) or (3) (for the re-cast constraint). The policymaker thus minimizes a convex objective subject to linear constraints, and so the first-order conditions are necessary and sufficient for a solution to the optimal policy problem.

For intuition, it is simpler and more instructive to use the constraint (3). Minimizing (1) subject to (3) yields:

1. **Optimal policy rule.** For each policy instrument $\boldsymbol{z}_j$, the paths of the policy targets satisfy the "policy criterion"

$$\sum_{i=1}^{I} \lambda_i \cdot \underbrace{\Theta'_{x_i,z_j} W}_{\text{(discounted) causal effect of } z_j \text{ on } x_i} \cdot \boldsymbol{x}_i = \mathbf{0}, \quad j = 1, 2, \ldots, J \tag{4}$$

(4) is simply the first-order condition of the optimal policy problem. It says that, for each instrument $\boldsymbol{z}_j$, the paths of the policy targets $\boldsymbol{x}_i$ must be at an optimum within the space implementable through $\boldsymbol{z}_j$. In the language of Svensson (1997) and Woodford (2003), this rule is an example of a so-called implicit "target policy criterion": the policymaker sets the available instruments to align projections (i.e., future paths) of macro aggregates as well as possible with its targets, given what is achievable through the available instruments.

[3]The equivalence of (2) and (3) would be immediate for invertible $\mathcal{H}_x$. In typical macroeconomic models, however, $\mathcal{H}_x$ is not invertible, so recasting the constraint as (3) requires additional arguments. McKay & Wolf (2023) provide those arguments; briefly, the core intuition is that the optimal policy problem can be shown to be equivalent to the alternative, artificial problem of picking shocks to a given baseline, determinacy-inducing policy rule. Policy and non-policy shocks relative to this arbitrary baseline policy rule then yield the impulse response matrices Θ. See Appendix C.1 for further details.

We emphasize two important features of such rules. First, they are derived without reference to and so apply independently of the non-policy shocks hitting the economy. This robustness property is one of the main virtues of target policy criteria (Giannoni & Woodford, 2002). Second, note that the optimal policy rule for instrument j places no weight on a policy target i that cannot be moved by instrument j (i.e., $\Theta_{x_i,z_j} = \mathbf{0}$), even if $\lambda_i > 0$—intuitively, if an instrument does not affect a target, then this target plays no role in setting the instrument.

2. **Optimal policy path.** Given the exogenous shock paths $\boldsymbol{\varepsilon}$, the policy rule (4) together with the constraints (3) characterizes the evolution of the dynamic system. In particular, the optimal instrument path $\boldsymbol{z}^*$ satisfies

$$\boldsymbol{z}^* \;\equiv\; -\left(\Theta_{x,z}'(\Lambda \otimes W)\Theta_{x,z}\right)^{-1} \times \left(\Theta_{x,z}'(\Lambda \otimes W)\Theta_{x,\varepsilon}\,\boldsymbol{\varepsilon}\right), \tag{5}$$

where $\Theta_{x,z}$ and $\Theta_{x,\varepsilon}$ suitably stack the individual Θ_{x_i,z_j}'s and $\Theta_{x_i,\varepsilon_q}$'s. The optimal path of the policy instruments thus has an intuitive regression interpretation: the instruments $\boldsymbol{z}$ are set to offset as well as possible—in a weighted least-squares sense—the perturbation to the policy targets $\boldsymbol{x}$ caused by the exogenous shocks, given as $\Theta_{x,\varepsilon} \times \boldsymbol{\varepsilon}$. In particular, the policymaker will rely most heavily on the tools z_j that are best suited to offset the perturbation to its targets induced by a particular shock path $\boldsymbol{\varepsilon}$.

2.2 Discussion

Equations (4) and (5) will guide our analysis in much of the remainder of the paper. In this section we briefly relate our results to prior work on: first, stochastic linear-quadratic problems; and second, empirical measurement of the propagation of policy shocks.

Deterministic transitions vs. aggregate risk. It is well-established that, by certainty equivalence, the first-order perturbation solution of models with aggregate risk is mathematically identical to linearized perfect-foresight transition paths.[4] This insight implies the following connections between our linear-quadratic perfect foresight problem and the canonical linear-quadratic stochastic problem (as in Benigno & Woodford, 2012). First, the policy target criterion (4) corresponds to a *forecast* targeting criterion in a stochastic

[4]For detailed discussions of this point see for example Fernández-Villaverde et al. (2016), Boppart et al. (2018) or Auclert et al. (2021).

economy. For a time-0 problem with commitment, that forecast targeting criterion is simply

$$\mathbb{E}_0\left[\sum_{i=1}^{I}\lambda_i\cdot\Theta'_{x_i,z_j}W\cdot\boldsymbol{x}_i\right]=\mathbf{0},\quad j=1,2,\ldots,J \tag{6}$$

This is an implicit rule that determines the expected evolution of the economy as of date 0. In a stochastic environment, new shocks will occur as time goes by, causing the evolution of the economy to deviate from what was expected at date 0. In this case, (6) gives a rule for how to *revise* forecasts at each date. Second, by the same logic, the optimal instrument path $\boldsymbol{z}^*$ in (5) corresponds to the instrument *impulse response* to a time-0 shock that changes expectations of the exogenous shifters from $\mathbf{0}$ to $\boldsymbol{\varepsilon}$. The exact same impulse response interpretation applies to our solution for the paths of the policy targets $\boldsymbol{x}^*$.

Measurement and interpreation. The matrices stacked in $\Theta_{x,z}$ collect the dynamic causal effects of variations in the policy instruments z onto the policymaker targets x. In other words, the effects of policy instruments on policymaker targets are sufficient statistics for the characterization of optimal policy rules. This observation will guide much of our analysis Sections 4.3 and 5.4, and in particular will allow us to transparently relate our findings to those reported in previous work. We also note a useful connection to measurement: as we show formally in McKay & Wolf (2023), entries of the causal effect maps $\Theta_{x,z}$ can be estimated using semi-structural time series methods applied to identified policy *shocks* (e.g., as in Ramey, 2016). We will thus in the subsequent analysis relate our findings as much as possible to empirical evidence on the effects of (monetary) stabilization policy.

Outlook. In the remainder of this paper we will first show that optimal policy problems in models with household heterogeneity can be represented in the form of our linear-quadratic control problem, and then use (4) and (5) to characterize optimal policy rules. Section 3 begins the analysis with a description of our model environment.

3 Model

We consider a HANK economy with two special features. First, labor supply is intermediated by labor unions. This will allow us to summarize the supply block through a standard Phillips curve and thus cleanly focus on the demand-side implications of household heterogeneity. Second, households invest in a menu of assets, which allows us to capture several empirically relevant channels of redistribution induced by monetary policy. Previewing our applications in Sections 4.3 and 5.4, we note that the model is designed to be just rich enough to be able to speak to evidence on the aggregate and distributional implications of monetary policy.

Time is discrete and runs forever, $t = 0, 1, 2, \dots$. Consistent with our linear-quadratic framework in Section 2, we will consider linearized perfect-foresight transition sequences. By certainty equivalence, our solutions will be identical to the analogous economy with aggregate risk and solved using conventional first-order perturbation techniques with respect to aggregate variables. Throughout this section, boldface denotes time paths (so e.g., $\boldsymbol{x} \equiv (x_0, x_1, x_2, \dots)'$), bars indicate the model's deterministic steady state ($\bar{x}$), and hats denote (log-)deviations from the steady state ($\widehat{x}$).[5]

3.1 Households

The economy is populated by a unit continuum of ex-ante identical households indexed by $i \in [0, 1]$. Household preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{it}^{1-\gamma} - 1}{1-\gamma} - \nu\left(\ell_{it}\right) \right], \tag{7}$$

where c_{it} is the consumption of household i and ℓ_{it} is its labor supply.

Households face uninsurable risk to their individual incomes. Let ζ_{it} be an idiosyncratic stochastic event that determines the idiosyncratic component of household i's income at date t. The idiosyncratic event ζ_{it} follows a stationary Markov process. Following Werning (2015) and Alves et al. (2020), we assume there is an incidence function Φ that maps aggregate labor income to individual labor income as a function of ζ_{it}. Letting e_{it} be the labor earnings of the household, we have

$$e_{it} = \Phi(\zeta_{it}, m_t, (1-\alpha)y_t),$$

where m_t is a distributional shock that tilts the incidence function towards or away from high-income households, and $(1-\alpha)y_t$ is aggregate labor income. As we describe in greater detail below, labor as a whole receives a share $1-\alpha$ of total income y_t. The incidence function Φ thus satisfies $\int e_{it} di = (1-\alpha)y_t$ for any value of m_t and y_t, and so the shock m_t only affects the distribution of labor income, not the total amount. For the quantitative analysis in Sections 4.3 and 5.4, the shock m_t will be our example of an inequality shock—a shock

[5] To be precise, we use log deviations for the variables $\{y, c, \ell, w, b, a, 1{+}r, 1{+}i, q_b, q_k, \eta\}$ and level deviations for the variables $\{\pi, \tau_x, \tau_e, m\}$.

that affects aggregate demand through redistribution and precautionary savings motives. Next, labor supply is determined by a labor market union (described further below), so hours worked ℓ_{it} are taken as given by the household. Total labor income is taxed at some constant proportional rate τ_y. Finally households also receive a time-varying lump-sum transfer $\tau_{x,t} + \tau_{e,t}e_{it}$. The first component of the transfer, $\tau_{x,t}$, is the same for all households and will be manipulated as part of the optimal policy problem; we will refer to it as a "fiscal stimulus payment" as it resembles the real-world stimulus checks that have been used in recent recessions in the U.S. The second component, $\tau_{e,t}e_{it}$, is the "endogenous" component, adjusting slowly over time to maintain long-run budget balance. This component of transfers is proportional to the household's productivity.

Households can save and possibly borrow in a variety of assets with different durations and different exposures to surprise inflation. Due to certainty equivalence and no-arbitrage, the returns on all of these assets must be equal at all dates along the equilibrium transition path *except possibly at* $t = 0$, where revaluation effects can lead to heterogeneous realized returns across households. Let r_t denote the (equalized) return between t and $t + 1$, and furthermore let a_{it} denote the net worth of household i at the beginning of period t (inclusive of interest). The overall budget constraint of household i is then

$$\frac{1}{1 + r_t} a_{it+1} + c_{it} = a_{it} + (1 - \tau_y + \tau_{e,t})e_{it} + \tau_{x,t}. \tag{8}$$

All date-0 revaluation effects will be captured by the initial asset position a_{i0}. We will discuss these revaluation effects later. For now, we note that the distribution of households over initial states (ζ_{i0}, a_{i0}) is endogenous, and we write it as Ψ_0. Finally, we impose a constraint on total household net worth: $a_{it+1} \geq \underline{a}$, where $\underline{a} \leq 0$.

The solution to each individual household i's consumption-savings problem gives a mapping from paths of aggregate income $\boldsymbol{y}$, real returns $\boldsymbol{r}$, transfers $\boldsymbol{\tau}_x$ and $\boldsymbol{\tau}_e$, shocks $\boldsymbol{m}$, and the initial asset position a_{i0} to the path of consumption $\boldsymbol{c}_i$. Aggregating consumption decisions across all households, we thus obtain an aggregate consumption function $\mathcal{C}(\bullet)$:

$$\boldsymbol{c} = \mathcal{C}(\boldsymbol{y}, \boldsymbol{r}, \boldsymbol{\tau}_x, \boldsymbol{\tau}_e, \boldsymbol{m}; \Psi_0). \tag{9}$$

3.2 Technology, unions, and firms

Final goods in our economy are produced by a representative firm that combines intermediate inputs. Those intermediate goods in turn are produced out of capital and labor, with labor supply intermediated by a union and the total capital stock of the economy fixed—an extreme form of capital adjustment costs.

Final goods production. The production function of the final goods producer is

$$y_t = \left(\int_j y_{jt}^{\frac{\eta_t - 1}{\eta_t}} dj \right)^{\frac{\eta_t}{\eta_t - 1}}.$$

The elasticity of substitution between different intermediate inputs, η_t, is allowed to vary (exogenously) over time. This "aggregate supply"-type shock will be important in our discussion of optimal policy. The final good is sold at nominal price p_t.

Factor supply. A labor market union intermediates household labor supply. We assume that hours worked are rationed equally across households, so $\ell_{it} = \ell_t$ for all i. Given separable preferences and with everyone supplying an equal amount of hours worked, it follows that all households share a common marginal disutility of labor. The marginal utility of consumption, however, is generally not equalized. For reasons that we will discuss in detail later, we assume that the union evaluates the benefits of higher after-tax income using the marginal utility of average consumption ($c_t^{-\gamma}$) rather than the average of marginal utilities ($\int_0^1 c_{it}^{-\gamma} di$), as also done in Wolf (2021) and Auclert et al. (2021). Under those assumptions, the labor union agrees to supply ℓ_t total units of labor at real wage

$$(1 - \tau_y) w_t = \frac{\nu'(\ell_t)}{c_t^{-\gamma}}. \tag{10}$$

The aggregate capital stock is fixed at k. We assume that capital owners must pay $p_t \delta$ units of the final good in order to maintain each unit of the capital stock.

Intermediate goods producers. A unit continuum of intermediate goods producers combine capital and labor according to the production technology

$$y_{jt} = A k_{jt}^{\alpha} \ell_{jt}^{1-\alpha}. \tag{11}$$

Let w_t be the real wage and let r_t^k be the real rental rate for capital. Cost minimization by the intermediate producers implies they all use the same capital-labor ratio, with

$$\ell_{jt} = \frac{r_t^k}{w_t} \frac{1 - \alpha}{\alpha} k_{jt}$$

Integrating across firms and using input market clearing we have

$$\frac{w_t \ell_t}{1-\alpha} = \frac{r_t^k k_t}{\alpha}.$$

It thus follows that labor receives a share $1-\alpha$ of factor payments, while capital receives the residual share α. Real marginal cost is common across firms and equal to

$$\mu_t = \frac{1}{A}\left(\frac{r_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}.$$

Total income is y_t, with a share μ_t of that total income going to factor payments and the residual share $1-\mu_t$ going to monopoly profits. We then furthermore assume that a share $1-\alpha$ of the monopoly profits is paid to workers in the form of profit sharing, which implies that labor receives a share $1-\alpha$ of total aggregate income y_t (see Appendix A.1). The union then distributes this total labor income to households according to the incidence function Φ introduced above. The remaining share of profits is distributed to capital owners, so capital owners receive a share α of aggregate income.

Our assumption on profit sharing fixes the labor share at $1-\alpha$. In making this assumption we have removed one potential source of redistribution from policy. The alternative, and seemingly more natural, assumption that all monopoly profits are paid to capital owners implies a sharp rise in the labor share following expansionary monetary policy. However, as we discuss further below, empirical evidence suggests that monetary expansions have little effect on the labor share (or perhaps even lower it), and so we have designed our model to be consistent with that evidence.

Each intermediate goods firm sets its price in standard Calvo fashion, with probability $1-\theta$ of updating the price each period. We show in Appendix A.1 that the solution to this problem gives rise to a standard linearized perfect-foresight New Keynesian Phillips curve:

$$\widehat{\pi}_t = \kappa \widehat{y}_t + \beta \widehat{\pi}_{t+1} + \psi \widehat{\eta}_t, \tag{12}$$

where the composite parameters κ and ψ are functions of model primitives and defined in Appendix A.1. We furthermore allow for a (time-invariant) subsidy on production financed with lump-sum taxes on the intermediate goods producers; this subsidy will matter in Section 5, where we require efficiency of the deterministic steady state to write our optimal policy problem in a form consistent with the linear-quadratic set-up of Section 2, exactly as in prior work (e.g. Woodford, 2003).

Aggregate production function. Combining the production functions of final goods producers and intermediate goods producers, we can write aggregate production as

$$y_t = \frac{Ak^\alpha}{d_t} \ell_t^{1-\alpha}, \tag{13}$$

where $d_t \geq 1$ captures efficiency losses from price dispersion.

3.3 Asset structure

There are three different assets: a short-term nominal bond, a long-term nominal bond, and capital. As discussed above, by no-arbitrage, all assets will provide the same returns along any equilibrium transition path *except possibly at* $t = 0$. This section presents the date-$t \geq 1$ no-arbitrage relations as well as the date-0 revaluation effects.

Asset returns. The three assets pay out the following real returns.

1. *Short-term bond.* The short-term nominal interest rate is denoted i_t. If p_t dollars are invested in the short-term bond at date t, then the payoff is valued at $(1 + i_t)p_t/p_{t+1} = (1 + i_t)/(1 + \pi_{t+1})$ units of goods in the next period.

2. *Long-term bond.* Households can purchase a unit of the long-term bond for a real price of q_t^b. At time $t + 1$, the household receives a real coupon of $(\bar{r} + \sigma_b)(1 + \pi_{t+1})^{-1}$ and furthermore retains a fraction $(1 - \sigma_b)(1 + \pi_{t+1})^{-1}$ of the asset position, now valued at $(1-\sigma_b)(1+\pi_{t+1})^{-1}q_{t+1}^b$. The parameter σ_b controls the maturity of the asset, with coupons decaying at rate σ_b, while the coupon scaling factor $(\bar{r} + \sigma_b)$ normalizes the steady-state price of the bond to one. The inflation term captures the fact that inflation reduces the real value of all future nominal payouts.

3. *Capital.* A unit of capital can be purchased for q_t^k units of the final good. It follows from our discussion in Section 3.2 that the real payoff of a unit of capital is $r_{t+1}^k - \delta + (1 - \mu_{t+1})\alpha y_{t+1}/k + q_{t+1}^k = \alpha y_{t+1}/k - \delta + q_{t+1}^k$.

Arbitrage relations. By no-arbitrage, all assets must yield the same expected return at all dates $t \geq 1$. Letting r_t denote the common return on these assets for $t \geq 1$, it then follows from the above discussion that in equilibrium we must have

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}} \tag{14}$$

$$1 + r_t = \frac{(\bar{r} + \sigma_b)(1 + \pi_{t+1})^{-1} + (1 - \sigma_b)(1 + \pi_{t+1})^{-1} q^b_{t+1}}{q^b_t} \tag{15}$$

$$1 + r_t = \frac{\alpha y_{t+1}/k - \delta + q^k_{t+1}}{q^k_t} \tag{16}$$

Revaluation effects. At time-0, the returns are not necessarily equalized, reflecting the arrival of surprise shocks. The expressions on the right-hand sides of (14), (15), and (16) then give us realized returns at date-0 with i_{-1}, q^b_{-1}, and q^k_{-1} at their steady state values.

At the end of period $t = -1$, the households in our economy had positions in short-term bonds, long-term bonds, and equities—positions that are then revalued as the date-0 news arrives. We take this initial distribution of portfolios as given; in fact, for our later quantitative analysis, we will match it directly to empirical evidence on household portfolios. It follows from the above discussion of revaluation effects that the date-0 distribution of household asset holdings inclusive of returns, Ψ_0, depends on π_0, q^b_0, q^k_0, and y_0. We will write this mapping as

$$\Psi_0 = \mathcal{H}(y_0, \pi_0, q^b_0, q^k_0). \tag{17}$$

Note that we can use (17) to substitute out for Ψ_0 in the consumption function (9). Linearizing around the deterministic steady state yields the aggregate consumption function

$$\widehat{\boldsymbol{c}} = \mathcal{C}_y \widehat{\boldsymbol{y}} + \mathcal{C}_r \widehat{\boldsymbol{r}} + \mathcal{C}_x \widehat{\boldsymbol{\tau}}_x + \mathcal{C}_e \widehat{\boldsymbol{\tau}}_e + \mathcal{C}_m \widehat{\boldsymbol{m}} + \mathcal{C}_\Psi \left(\mathcal{H}_y \widehat{y_0} + \mathcal{H}_\pi \widehat{\pi_0} + \mathcal{H}_b \widehat{q_0}^b + \mathcal{H}_k \widehat{q_0}^k \right), \tag{18}$$

where all of the derivative matrices $\boldsymbol{\mathcal{C}}_\bullet$ and $\boldsymbol{\mathcal{H}}_\bullet$ are evaluated at the economy's deterministic steady state.

3.4 Government

The final actor in our model is the government. The government collects tax revenue, pays out lump-sum transfers, sets the nominal interest rate on the short-term bond, and issues short- as well as long-term bonds. Letting a^g_t denote the value of claims on the government entering period t (inclusive of returns), the government budget constraint becomes

$$\frac{a^g_{t+1}}{1 + r_t} = a^g_t + \underbrace{\tau_{x,t} - (\tau_y - \tau_{e,t})(1 - \alpha) y_t}_{\text{tax revenue}}. \tag{19}$$

Government debt maturity. We assume that the government issues both long-term as well as short-term bonds. For all dates $t \geq 1$ along the perfect foresight transition path, the returns on the two bonds are equalized and so the maturity structure of the government debt is irrelevant. At date 0, on the other hand, the maturity structure matters via revaluation, perfectly analogously to our earlier discussion of valuation effects in household portfolios. In particular, the revaluation effects will again be embedded in the date-0 portfolio value a_0^g. The aggregate bond holdings of the household sector are the liabilities of the government, and so the revaluation of the government debt is necessarily the mirror image of the revaluation of the household bond positions.

Policy instruments. We consider the nominal rate of interest i_t and the exogenous component of transfers $\tau_{x,t}$ (i.e., "stimulus checks") as the independent policy instruments of the government, used for business-cycle stabilization policy. We assume that the endogenous component of transfers $\tau_{e,t}$ adjusts gradually to ensure long-term budget balance:

$$\tau_{e,t} = (\bar{r} + \sigma_\tau)(a_t^g - \bar{a}^g), \tag{20}$$

When the stock of bonds outstanding exceeds its steady state level, taxes are raised to pay interest and a portion σ_τ of the outstanding bonds. Given this fiscal feedback rule for taxes, government debt then evolves according to (19).

3.5 Equilibrium

We can now define a linearized perfect-foresight transition equilibrium in this economy.[6]

Definition 1. *Given paths of exogenous shocks $\{m_t, \eta_t\}_{t=0}^\infty$, a linearized perfect foresight equilibrium is a set of government policies $\{i_t, \tau_{x,t}, \tau_{e,t}, a_t^g\}_{t=0}^\infty$ and a set of aggregates $\{c_t, y_t, a_t, \pi_t, r_t, q_t^b, q_t^k, w_t, \ell_t\}_{t=0}^\infty$ such that:*

1. *The path of aggregate consumption $\{c_t\}_{t=0}^\infty$ is consistent with the linearized aggregate consumption function* (18)*, and the path of household asset holdings $\{a_t\}_{t=1}^\infty$ is consistent with the budget constraint* (8)*, aggregated across households. a_0 is determined by the existing portfolios of households now valued at date-0 asset prices.*

2. *The real wage satisfies* (10).

3. *The paths $\{\pi_t, y_t, \eta_t\}_{t=0}^\infty$ are consistent with the Phillips curve* (12).

[6] All statements in Definition 1 thus refer to the linearized versions of the relevant model equations.

4. The paths of $\{\ell_t, y_t\}_{t=0}^{\infty}$ satisfy the aggregate production function (13).[7]

5. The asset returns $\{r_t, q_t^b, q_t^k\}_{t=0}^{\infty}$ satisfy (14), (15), *and* (16).

6. The evolution of government debt a_t^g and the endogenous component of transfers $\tau_{e,t}$ are consistent with the budget constraint (19) *and fiscal rule* (20).

7. The output and asset markets clear, so $y_t = c_t + \delta k$ and $(a_{t+1} - a_{t+1}^g)/(1 + r_t) = q_t^k k$.

In this definition, the policy instrument paths $\{i_t, \tau_{x,t}\}$ are taken as given. Sections 4 and 5 will describe optimal policy problems and so discuss how these instruments are determined.

EQUILIBRIUM CHARACTERIZATION. We can reduce Definition 1 to a small number of linear relations. Lemma 1 provides this more compact characterization of equilibrium dynamics.

Lemma 1. *Given paths of shocks $\{m_t, \eta_t\}_{t=0}^{\infty}$ and government policy instruments $\{i_t, \tau_{x,t}\}_{t=0}^{\infty}$, paths of aggregate output and inflation $\{y_t, \pi_t\}_{t=0}^{\infty}$ are part of a linearized equilibrium if and only if*

$$\begin{aligned} \widehat{\boldsymbol{\pi}} &= \kappa\widehat{\boldsymbol{y}} + \beta\widehat{\boldsymbol{\pi}}_{+1} + \psi\widehat{\boldsymbol{\eta}}, && (21) \\ \widehat{\boldsymbol{y}} &= \tilde{\mathcal{C}}_y\widehat{\boldsymbol{y}} + \tilde{\mathcal{C}}_\pi\widehat{\boldsymbol{\pi}} + \tilde{\mathcal{C}}_i\widehat{\boldsymbol{i}} + \tilde{\mathcal{C}}_x\widehat{\boldsymbol{\tau}}_x + \mathcal{C}_m\widehat{\boldsymbol{m}}, && (22) \end{aligned}$$

where the linear maps $\{\tilde{\mathcal{C}}_y, \tilde{\mathcal{C}}_\pi, \tilde{\mathcal{C}}_i, \tilde{\mathcal{C}}_x\}$ are defined in Appendix D.1, and $\widehat{\boldsymbol{\pi}}_{+1} = (\pi_1, \pi_2, \dots)$.

Lemma 1 reduces the complexity of the equilibrium definition in Definition 1 to two equations: the Phillips curve (21) (which is simply a stacked perfect-foresight version of the original relation (12)); and the IS curve (22), which differs from the consumption function (18) chiefly in that it imposes output market-clearing, asset revaluation effects, and feedback effects through the government budget to the endogenous component of transfers, $\tau_{e,t}$. Together, these two equations fully characterize the evolution of output and inflation given exogenous non-policy shocks $\{m_t, \eta_t\}_{t=0}^{\infty}$ and policy choices $\{i_t, \tau_{x,t}\}_{t=0}^{\infty}$.

DISCUSSION. How does our model differ from the canonical representative agent New Keynesian models (Galí, 2015; Woodford, 2003)? *Positively*, the main change is that a simple

[7] Note that we drop the efficiency loss term d_t since it is of second order, and thus does not affect a first-order approximation of the production function around a zero inflation steady state (see Galí, 2015). Price dispersion will, however, affect the social welfare function in Section 5.

aggregate Euler equation,

$$\widehat{y}_t = -\frac{1}{\tilde{\gamma}}(\widehat{i}_t - \widehat{\pi}_{t+1}) + \widehat{y}_{t+1}, \tag{23}$$

is now replaced by the more general aggregate demand block (22).[8] Inequality thus affects the aggregate dynamics of our economy in response to shocks and policy actions only through the demand side, with supply—in particular the Phillips curve (21)—kept as in standard representative-agent models. To arrive at this clean separation, our assumptions on union-intermediated labor supply (see Section 3.2) are central. We adopt this approach because the demand-side effects of household heterogeneity are the focus of the recent HANK literature (e.g., see Kaplan et al., 2018; Auclert et al., 2018).[9] *Normatively*, household inequality may affect social welfare functions and thus change policymaker objectives.

The remainder of the paper studies the implications of these two changes for optimal policy design. First, in Section 4, we isolate the role of changes in propagation by studying a dual-mandate optimal policy problem. Then, in Section 5, we turn to the Ramsey problem, thus also allowing the planner objective to change.

4 Optimal dual-mandate policy

We begin our optimal policy analysis by studying the problem of a conventional dual-mandate policymaker; that is, a policymaker that simply seeks to stabilize fluctuations in inflation and the output gap. In the context of the structural model of Section 3, such a loss function is *ad hoc*, but we nevertheless find it interesting, for two reasons. First, it is conceptually useful—it allows us to split the effects of inequality on policy design into the role of propagation (which we study here) and loss function (which we will study in Section 5). Second, it is arguably policy relevant, as real-world central banks are often mandated to achieve these types of simple objectives.

We begin in Section 4.1 by stating the optimal policy problem in linear-quadratic form. We then in Section 4.2 characterize the problem's solution in the form of a forecast target criterion. Finally, in Section 4.3, we present some quantitative explorations, leveraging in particular the connection between our optimal policy "sufficient statistic" formulas and empirical evidence on the propagation of identified policy shocks.

[8] Equation (23) combines the standard consumption Euler equation with EIS $1/\gamma$ with the log-linearized aggregate resource constraint $\hat{c}_t = (\bar{c}/\bar{y})\hat{y}_t$. The parameter $\tilde{\gamma}$ that appears in (23) is then $\gamma\bar{c}/\bar{y}$.

[9] As an added benefit, our union structure with uniform labor rationing allows us to sidestep counterfactual cross-sectional labor supply responses to policy interventions (see Auclert et al., 2020).

4.1 The optimal policy problem

We consider a policymaker with *ad hoc* objective function

$$\mathcal{L}^{DM} \equiv \sum_{t=0}^{\infty} \beta^t \left[\lambda_\pi \widehat{\pi}_t^2 + \lambda_y \widehat{y}_t^2\right]. \tag{24}$$

(24) is a *dual-mandate* loss function: the policymaker wishes to stabilize inflation and output around the deterministic steady state, with weights λ_π and λ_y, respectively.

For most of this section, we focus on the optimal setting of nominal interest rates i_t, with only brief reference to optimal stimulus check policy.[10] The policymaker sets nominal interest rates to minimize (24) subject to the equilibrium constraints embedded in Definition 1. By Lemma 1, we can reduce these two constraints to two simple relationships: the Phillips curve (21) and the IS curve (22). This optimal policy problem is thus a minimal departure from optimal policy analysis in conventional representative-agent environments: the loss function (by assumption) and the supply side are unaffected, while the demand constraint changes from a simple Euler equation as in (23) to the richer demand relation (22).

Note that so far the constraints of this policy problem take the form of our general linear constraint (2). By the arguments in McKay & Wolf (2023), we can equivalently re-write this constraint set in impulse response space, thus giving our alternative formulation (3).[11] For future reference, we write the constraints in impulse response space as

$$\widehat{\boldsymbol{\pi}} = \Theta_{\pi,i}\widehat{\boldsymbol{i}} + \Theta_{\pi,x}\widehat{\boldsymbol{\tau}}_x + \Theta_{\pi,\eta}\widehat{\boldsymbol{\eta}} + \Theta_{\pi,m}\widehat{\boldsymbol{m}}, \tag{25}$$

$$\widehat{\boldsymbol{y}} = \Theta_{y,i}\widehat{\boldsymbol{i}} + \Theta_{y,x}\widehat{\boldsymbol{\tau}}_x + \Theta_{y,\eta}\widehat{\boldsymbol{\eta}} + \Theta_{y,m}\widehat{\boldsymbol{m}}. \tag{26}$$

COMPUTATIONAL DETAILS. Solving the dual-mandate optimal policy problem is straightforward. Key to this computational simplicity is that the maps characterizing the linear-quadratic problem—either the $\tilde{\mathcal{C}}$'s in the original constraint formulation or the Θ's in the equivalent impulse response space formulation—can be obtained straightforwardly as a side-product of standard sequence-space solution output, following the methods developed in

[10] To be more precise, we study the optimal monetary policy problem of setting i_t, with $\tau_{x,t} = 0$ in the background. There are, of course, still effects of fiscal adjustments through the endogenous fiscal rule (20).

[11] In McKay & Wolf (2023), we close the model with a determinacy-inducing policy *rule* for the policy instruments, here i and τ_x. The dynamic causal effect matrices in (25) - (26) are then defined as impulse response matrices for shocks to the baseline rule. For example, if the inflation and interest rate impulse response matrices to monetary shocks to the base rule are denoted $\tilde{\Theta}_{\pi,i}$ and $\tilde{\Theta}_{i,i}$, then $\Theta_{\pi,i} \equiv \tilde{\Theta}_{\pi,i}\tilde{\Theta}_{i,i}^{-1}$. By the arguments of McKay & Wolf (2023) this re-writing is without loss of generality.

Auclert et al. (2021). It thus follows that optimal policy analysis in the dual-mandate case comes at essentially zero additional computational cost: if a researcher can solve her HANK model *given* a policy rule, then she is only a trivial linear-quadratic problem away from also obtaining an *optimal* policy rule for a given quadratic loss.

4.2 Policy rule irrelevance

Our first main result is that, under mild regularity conditions on the linear map $\tilde{\mathcal{C}}_i$—i.e., the mapping from nominal interest rate paths to net excess consumption demand in the generalized "IS" curve (22)—, the optimal monetary policy forecast target criterion is completely unaffected by household heterogeneity.

Proposition 1. *Let $\widehat{\boldsymbol{c}}$ be a path of household consumption with zero net present value, i.e., $\sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^t \widehat{c}_t = 0$. If, for* any *such path $\widehat{\boldsymbol{c}}$, we have that*

$$\widehat{\boldsymbol{c}} \in \ image(\tilde{\mathcal{C}}_i), \tag{27}$$

then the optimal monetary policy rule for a dual-mandate policymaker with loss function (24) can be written as the forecast target criterion

$$\lambda_\pi \widehat{\pi}_t + \frac{\lambda_y}{\kappa}\left(\widehat{y}_t - \widehat{y}_{t-1}\right) = 0, \quad \forall t = 0, 1, \ldots \tag{28}$$

Recall that our expressions for general linear-quadratic policy problems derived in Section 2 immediately yield the optimal dual-mandate policy target criterion as

$$\lambda_\pi \cdot \Theta'_{\pi,i} \cdot \widehat{\boldsymbol{\pi}} + \lambda_y \cdot \Theta'_{y,i} \cdot \widehat{\boldsymbol{y}} = \mathbf{0}. \tag{29}$$

The proof of Proposition 1 leverages the structure of our model to turn the general expression (29) into the simple rule (28). Intuitively, while (29) is written in terms of the *absolute* causal effects of interest rate policy on output and inflation, (28) is the analogue in terms of *relative* causal effects: if monetary policy moves inflation at date t by one unit, then it changes output at t and $t-1$ by $\frac{1}{\kappa}$ and $-\beta\frac{1}{\kappa}$ units, respectively. Since the policymaker herself discounts at rate β, we find that the complicated relation (29) collapses to the simple one (28).

Importantly, the rule (28) is *exactly* the same as in conventional representative-agent optimal policy analyses (Galí, 2015; Woodford, 2003). The logic underlying this result is as follows. In the familiar representative-agent policy problem, any desired path of output and inflation that is consistent with the Phillips curve can be implemented through a suitable choice of interest rates. The IS curve is thus a slack constraint: the policymaker picks the best output-inflation pair subject to the NKPC, and then sets interest rates residually to deliver the required time path of demand. Condition (27) is precisely enough to ensure that this logic carries through in our environment with household heterogeneity. In words, the condition says that, through manipulation of short-term nominal interest rates, the policymaker can engineer *any* possible net excess demand path with zero net present value. The proof of Proposition 1 reveals that this is sufficient to ensure that any desired output-inflation pair consistent with the Phillips curve (21) is in fact implementable. But then, with the Phillips curve as the supply side of the economy not depending on household inequality, we find that the target criterion is the same as in conventional representative-agent models.

The implementability condition (27) is discussed further in Wolf (2021). That paper shows—analytically in simple models, and numerically in heterogeneous-agent environments—that interest rate policies are indeed generally flexible enough to induce every possible zero net present value path of aggregate net excess demand. The irrelevance of household heterogeneity for optimal forecast criteria is thus a robust feature of HANK-type environments.

Implications for monetary policy practice. The upshot of Proposition 1 is that, independently of the non-policy shocks hitting the economy, under optimal dual-mandate policy, the equilibrium paths of output and inflation will be unaffected by household heterogeneity and thus equal to those in a standard representative-agent economy. The only possible effect of heterogeneity is to change the instrument paths—i.e., the current and future values of nominal interest rates—required to attain those desired output and inflation paths. We conclude that the positive implications of household heterogeneity have a rather limited effect on the practice of conventional dual-mandate policymakers: they can continue to set their instruments to bring projections of macroeconomic outcomes in line with target, exactly as done in standard practice of flexible inflation targeting.[12]

Optimal stimulus checks. Proposition 1 only considers the first instrument available to our policymaker: nominal interest rates. Results for stimulus checks follow immediately from

[12] Bernanke (2015) succinctly summarizes the salience of this perspective for Federal Reserve policymaking practice: *"The Fed has a rule. The Fed's rule is that we will go for a 2 percent inflation rate. We will go for the natural rate of unemployment. We put equal weight on those two things. We will give you information about our projections about our interest rates. That is a rule and that is a framework that should clarify exactly what the Fed is doing."*

Wolf (2021), who identifies conditions under which interest rate and stimulus check policies can implement the same sequences of aggregate output and inflation. More formally, it follows from his results that, if

$$\widehat{\boldsymbol{c}} \in \text{image}(\tilde{\mathcal{C}}_\tau) \tag{30}$$

for all sequences $\widehat{\boldsymbol{c}}$ with zero net present value, then stimulus check policies can also implement the target criterion (28), just like conventional monetary policy. This alternative implementability condition (30) is again generally satisfied in HANK-type environments.

It follows from the previous discussion that the two policy instruments are perfect substitutes, and so that the solution to the problem of choosing both instruments to minimize (24) is indeterminate—multiple paths of the two policy instruments are consistent with the optimal outcomes for output and inflation. One way to break this indeterminacy is to introduce further constraints on instruments, e.g. a lower bound on nominal interest rates. Section 5 considers an alternative resolution to this indeterminacy: a richer loss function.

4.3 Quantitative analysis & connection to empirical evidence

We have seen that household heterogeneity does not affect the optimal inflation and output gap outcomes implemented by a dual-mandate policymaker. Heterogeneity could, however, in principle affect the time paths of nominal rates required to achieve those optimal outcomes. This section leverages the close connection between our "sufficient statistics" characterization of optimal policy and empirical evidence on policy shock propagation to further argue that, *in quantitatively relevant models*, the effects of household heterogeneity on optimal rate paths are likely to be modest in scope.

Exact instrument path irrelevance. We begin with another exact irrelevance result, building closely on McKay & Wolf (2023). For this result, it will prove convenient to re-state (5) for the optimal policy instrument path, here specialized to the policy instrument i and for the policy targets $x = (y, \pi)'$, in response to some generic set of shocks $\boldsymbol{\varepsilon}$:

$$\widehat{\boldsymbol{i}}^* \equiv -\left(\Theta_{x,i}'(\Lambda \otimes W)\Theta_{x,i}\right)^{-1} \times \left(\Theta_{x,i}'(\Lambda \otimes W)\Theta_{x,\varepsilon} \cdot \boldsymbol{\varepsilon}\right). \tag{31}$$

Equation (31) states that we can recover the interest rate path required to *implement* the optimal policy rule (28) from the *absolute* causal effects of interest rate policy on output and inflation. It thus has the following important implication. Any two models—say HANK and RANK—that agree on (i) the effects of a given shock $\boldsymbol{\varepsilon}$ on output and inflation, $\Theta_{y,\varepsilon} \cdot \boldsymbol{\varepsilon}$ and

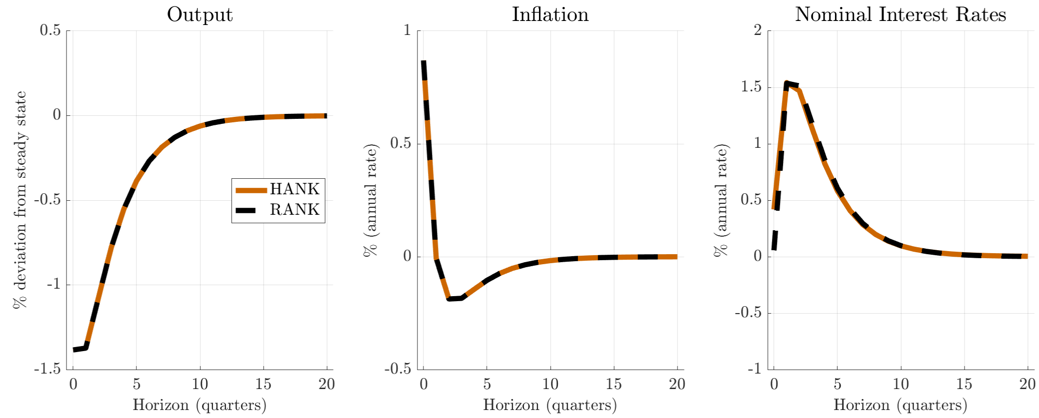


Figure 1: Optimal dual-mandate policy response to a cost-push shock. The RANK model replaces our general "IS" curve (22) with the simple textbook Euler equation (23). We then calibrate all parameters as in the headline HANK model (see Section 5.3), with one exception: we set the elasticity of intertemporal substitution (EIS) to generate the same peak response of output to an identified monetary shock as in the HANK model. In practice, the EIS is little changed.

$\Theta_{\pi,\varepsilon} \cdot \boldsymbol{\varepsilon}$, and (ii) the effects of interest rate changes on output and inflation, $\Theta_{y,i}$ and $\Theta_{\pi,i}$, will *necessarily* agree on the optimal interest rate path $\widehat{\boldsymbol{i}}^*$.

We view this irrelevance result as informative because its ingredients are measurable. In particular, the dynamic causal effects of interest rate changes on aggregate outcomes—that is, elements of $\Theta_{y,i}$ and $\Theta_{\pi,i}$—are the estimands of a large empirical literature on monetary policy shocks (Ramey, 2016). Structural models are often calibrated or estimated to be consistent with estimates from this literature, which leads them to approximately agree for the entire matrices $\Theta_{y,i}$ and $\Theta_{\pi,i}$. The degree to which such quantitative models can disagree on optimal dual-mandate interest rate paths is thus limited by the empirical evidence.

Quantitative illustration. We close with a quantitative illustration of the analytical results presented so far. To this end, we study optimal dual-mandate monetary policy in response to a cost-push shock η_t in a calibrated version of our HANK model. Details of the calibration are postponed until Section 5.3; for purposes of the discussion here, it suffices to note that the model has been parameterized to in particular be consistent with empirical evidence on the output gap and inflation effects of monetary policy shocks. We then contrast aggregate outcomes in this economy with those in an analogous RANK model, calibrated similarly to be consistent with empirical policy shock evidence.

Results are displayed in Figure 1. We emphasize the following two main takeaways, both consistent with our analytical discussion in the earlier parts of this section. First, as shown in Proposition 1, the output gap and inflation paths are exactly the same. The optimal trade-off between inflation and output is fully governed by the Phillips curve, which itself is not affected by household inequality. Second, the nominal interest rate path required to implement the optimal outcome is quite similar across the two models. Intuitively, since by construction both models match the same evidence on the transmission from interest rate changes to output and inflation, the interest rate movements that achieve the policymaker's desired output and inflation responses to the shock cannot be too dissimilar.[13]

5 Optimal Ramsey policy

We now turn to the optimal policy problem of a Ramsey planner. Unlike our *ad hoc* dual-mandate loss function of Section 4, this planner's objective is directly affected by household inequality, reflecting a desire to dampen the distributional consequences of aggregate shocks. Most of our analysis in this section again focuses on optimal monetary policy.

The remainder of the section proceeds in three steps. First, in Section 5.1, we show how to express the optimal policy problem in our linear-quadratic form. Second, in Section 5.2 we present general analytical results and discuss some instructive analytical special cases. Finally, in Sections 5.3 and 5.4, we turn to quantitative analysis, leveraging our analytical "sufficient statistics" formulas to connect to empirical evidence to compare our findings with those of prior work.

5.1 The optimal policy problem

We consider a conventional Ramsey planner that aggregates utilities of the households populating the economy. This section presents a linear-quadratic version of this policy problem. As is typical in the optimal policy literature (e.g. see the discussion in Giannoni & Woodford, 2002), doing so requires efficiency of the steady state. In the representative agent context, efficiency requires that the level of production is optimal. Here we also require that the planner finds the steady-state cross-sectional distribution of consumption desirable. Our optimal policy analysis will therefore capture an insurance motive against fluctuations in consumption shares, but it will not seek to change the long-run distribution of consumption.

[13] As discussed above, the two interest rate paths would agree exactly if the two models were to agree on all of $\Theta_{y,i}$ and $\Theta_{\pi,i}$. Our calibration instead only ensures that peak output and inflation responses in response to a particular, transitory interest rate movement align. This materially limits the extent of—but does not fully eliminate—possible disagreement in nominal rate paths.

Loss function. To state the loss function, it will prove convenient to describe an individual's outcomes in terms of their history of idiosyncratic shocks; that is, we replace c_{it} with $\omega_t(\zeta_i^t)c_t$ where $\zeta_i^t \equiv (\zeta_{it}, \zeta_{it-1}, \zeta_{it-2}, \cdots)$ is individual i's history of idiosyncratic shocks and $\omega_t(\zeta_i^t) \equiv c_{it}/c_t$ is their share of aggregate consumption.[14] Letting $\Gamma(\zeta)$ denote the (stationary) distribution of such histories (with ζ denoting a generic realization of a history), we can write the social welfare function as

$$\mathcal{V}^{HA} = \sum_{t=0}^{\infty} \beta^t \int \varphi(\zeta) \left[\frac{(\omega_t(\zeta)c_t)^{1-\gamma} - 1}{1-\gamma} - \nu(\ell_t) \right] d\Gamma(\zeta), \tag{32}$$

where $\varphi(\zeta)$ is a Pareto weight on the utility of households with history ζ.

In keeping with optimal (monetary) policy analysis in standard representative-agent environments (Woodford, 2003), our objective is to evaluate the social welfare function (32) to second order. To this end, a first-order approximation to aggregate equilibrium dynamics suffices only if the expansion point (i.e., the deterministic steady state) is efficient. Without household heterogeneity, a simple production subsidy is sufficient to ensure this. With household heterogeneity, however, we now also require the consumption *shares* of all households to be optimal. In principle there are two ways of ensuring this: either the steady-state fiscal tax-and-transfer system achieves the optimal level of insurance given the planner weights $\varphi(\bullet)$, or the planner weights are set residually so that the implied steady-state distribution of consumption given a tax-and-transfer system is optimal. We adopt the second approach, following the inverse optimal taxation literature (e.g. Heathcote & Tsujiyama, 2021). Our preference for this approach reflects the overarching focus of this paper: we ask how cyclical policy tools should be manipulated to respond to cyclical changes in inequality, leaving the long-run steady state outside of the purview of our analysis.

Appendix D.3 presents our assumptions on the production subsidy and policymaker preference weights that ensure efficiency of the deterministic steady state. Given those assumptions, a second-order approximation of (32) around the efficient steady state then gives the following characterization of the policymaker loss function.

Proposition 2. *To second order, the social welfare function $\mathcal{V}^{HA}$ is proportional to $-\mathcal{L}^{HA}$, given by*

$$\mathcal{L}^{HA} \equiv \sum_{t=0}^{\infty} \beta^t \left[\widehat{\pi}_t^2 + \frac{\tilde{\kappa}}{\bar{\eta}} \widehat{y}_t^2 + \lambda_\omega \int \frac{\widehat{\omega}_t(\zeta)^2}{\bar{\omega}(\zeta)} d\Gamma(\zeta) \right], \tag{33}$$

[14] Note that this is without loss of generality, as individuals in our model are *ex ante* identical, so their outcomes only differ due to different histories of shocks.

where $\widehat{\omega}_t(\zeta) = \omega_t(\zeta) - \bar{\omega}(\zeta)$ and $\bar{\omega}(\zeta)$ is the steady-state consumption share of an individual with history ζ and $\tilde{\kappa}$ and λ_ω are composite parameters defined in Appendix D.3.

Note that, in the representative-agent analogue of our economy (as discussed in Section 3.5), the loss function would feature the same first two terms, as already well-known from prior work (Woodford, 2003).[15] Our analysis reveals that household heterogeneity adds a third, inequality-related term, with the planner wishing to stabilize the consumption *shares* of everyone in the economy.

How does the inequality term in (33) fit into the linear quadratic framework in Section 2? Moving to a sequence-space formulation, we can write the loss as

$$\mathcal{L}^{HA} = \lambda_\pi \widehat{\boldsymbol{\pi}}' W \widehat{\boldsymbol{\pi}} + \lambda_y \widehat{\boldsymbol{y}}' W \widehat{\boldsymbol{y}} + \int \lambda_\zeta \widehat{\boldsymbol{\omega}}(\zeta)' W \widehat{\boldsymbol{\omega}}(\zeta) d\Gamma(\zeta), \tag{34}$$

where $\lambda_\pi = 1$, $\lambda_y = \frac{\tilde{\kappa}}{\bar{\eta}}$, and $\lambda_\zeta \equiv \lambda_\omega / \bar{\omega}(\zeta)$. The consumption share for each idiosyncratic household history thus emerges as a separate target variable for the policymaker. We will discuss our approach to computation of (34) later in this section.[16]

Constraints. The constraints of the optimal policy problem characterize the evolution of policymaker targets—π, y, and the consumption shares $\omega(\zeta)$—as a function of exogenous shocks and policy choices. As we discussed in Section 4.1, the evolution of aggregate output and inflation are governed by the Phillips curve (21) and the IS curve (22). It then remains to describe the evolution of the inequality term in (33). In Appendix C.2, we establish that, to first order, we can write the consumption share for a household with specific history ζ as

$$\widehat{\boldsymbol{\omega}}(\zeta) = \Omega_{\omega(\zeta),x} \widehat{\boldsymbol{x}}, \qquad \forall \zeta \tag{35}$$

where $\boldsymbol{x} = (\boldsymbol{y}, \boldsymbol{r}, \boldsymbol{\tau}_x, \boldsymbol{\tau}_{\boldsymbol{e}}, \boldsymbol{m}, \pi_0, q_0^k, q_0^b)$ stacks the prices and shocks that affect the household decision problem, and the maps $\Omega_{\omega(\zeta),x}$ give the derivatives of consumption shares with respect to these aggregate variables. The intuition for (35) is the same as that for the aggregate consumption function—an individual household's consumption, given their history of idiosyncratic shocks, evolves over time as a function of the aggregate inputs to the household

[15] When $\delta = 0$ (so there is no investment spending) the composite term $\tilde{\kappa}$ reduces to κ, and so the first two terms in the loss function are exactly the same as the familiar ones from the literature.

[16] Note that, technically, (34) does not immediately fit into our framework in Section 2.1 since the objective here features an integral (rather than a simple sum). Our approach to computation will consider an equivalent formulation of the problem with finitely many policy targets.

consumption-savings problem. By the proof of Lemma 1, we can obtain these inputs as a function of exogenous shocks, policies, and aggregate output and inflation paths.

For future reference it will be useful to note that, again following McKay & Wolf (2023), we can alternatively re-write this constraint in impulse response space, solving out the dependence of consumption shares on endogenous aggregates:

$$\widehat{\boldsymbol{\omega}}(\zeta) = \Theta_{\omega(\zeta),i}\widehat{\boldsymbol{i}} + \Theta_{\omega(\zeta),x}\widehat{\boldsymbol{\tau}}_x + \Theta_{\omega(\zeta),\eta}\widehat{\boldsymbol{\eta}} + \Theta_{\omega(\zeta),m}\widehat{\boldsymbol{m}}. \qquad \forall \zeta \tag{36}$$

Summary & computational details. The Ramsey planner chooses paths of the two available policy instruments—nominal interest rates $\boldsymbol{i}$ and the exogenous component of transfers $\boldsymbol{\tau}_x$—to minimize the derived loss function $\mathcal{L}^{HA}$ subject to the same two constraints as before, (25) and (26), as well as the evolution of the inequality term, (36).

To computationally evaluate the more complicated loss function (34) and the associated constraints, we show in Appendix C.2 that the Ramsey loss can be re-written as

$$\mathcal{L}^{HA} = \widehat{\boldsymbol{\pi}}'W\widehat{\boldsymbol{\pi}} + \frac{\tilde{\kappa}}{\bar{\eta}}\widehat{\boldsymbol{y}}'W\widehat{\boldsymbol{y}} + \lambda_\omega \widehat{\boldsymbol{x}}'Q\widehat{\boldsymbol{x}} \tag{37}$$

where Q is a linear map, defined in Appendix C.2.[17] The alternative formulation in (37) reflects the simple insight that it is always possible to re-write the loss coming from cross-sectional inequality as a function of the (small number of) inputs to the consumption-savings problem—income, interest rates, asset prices, taxes, and shocks. With the re-written loss function (37), the relevant constraints are then simply the equilibrium dynamics of $\boldsymbol{x}$. Computation is thus again straightforward: Appendix C.2 discusses how to recover the map Q, while the coefficient matrices for all constraints are again immediate from standard sequence-space solution output, exactly as in the dual-mandate problem considered before. To summarize, relative to solving a HANK model *given* a policy rule, the only additional computational work needed to solve a Ramsey *optimal* policy problem is the one-time computation of the auxiliary linear map Q.

5.2 Optimal policy rules

Having expressed the optimal Ramsey problem in linear-quadratic form, we can now leverage the results of Section 2 to provide a general characterization of optimal Ramsey policy rules.

[17] We note that the linear map Q in (37) is in general not diagonal. Thus Appendix C.2 also extends the results from Section 2 to the case with such interaction terms.

The optimal Ramsey monetary policy rule is given as

$$\Theta'_{\pi,i}W\widehat{\boldsymbol{\pi}} + \frac{\tilde{\kappa}}{\bar{\eta}}\Theta'_{y,i}W\widehat{\boldsymbol{y}} + \int \lambda_\zeta \Theta'_{\omega(\zeta),i}W\widehat{\boldsymbol{\omega}}(\zeta)d\Gamma(\zeta) = \mathbf{0}, \tag{38}$$

where as before the matrices $\Theta_{\bullet,i}$ collect the dynamic causal effects of interest rate movements on the various policymaker targets—our "sufficient statistics" for optimal policy. In particular, we see that the first two terms in (38) are identical to the optimal dual-mandate rule (29), just now with weights derived from policymaker preferences (rather than exogenously assumed). The novel third term—which reflects the planner's distributional insurance concerns—collects the causal effects of interest rate movements on consumption shares. Proceeding analogously for stimulus checks we find the optimal rule

$$\Theta'_{\pi,\tau_x}W\widehat{\boldsymbol{\pi}} + \frac{\tilde{\kappa}}{\bar{\eta}}\Theta'_{y,\tau_x}W\widehat{\boldsymbol{y}} + \int \lambda_\zeta \Theta'_{\omega(\zeta),\tau_x}W\widehat{\boldsymbol{\omega}}(\zeta)d\Gamma(\zeta) = \mathbf{0}, \tag{39}$$

As discussed in Wolf (2021) and also briefly in Section 4.2, nominal interest rate and stimulus check policy in our environment can implement exactly the same output-inflation allocations, so the first two terms in (38) and (39) will reflect identical aggregate stabilization objectives. The distributional terms $\Theta_{\omega(\zeta),i}$ and $\Theta_{\omega(\zeta),\tau_x}$, on the other hand, will generally differ, thereby opening the door for interest rate and stimulus check policies to be useful complementary tools for aggregate stabilization. We will return to this observation in Section 5.4. Overall, (38) and (39) fully characterize joint optimal monetary-fiscal policy.

The key takeaway from the characterizations in (38) and (39) is that optimal policy rules in the general Ramsey problem deviate from the dual-mandate rules discussed in Section 4 if and only if the policy instruments affect cross-sectional consumption inequality. On the one hand, if monetary policy is distributionally neutral—as is for example the case in the environment of Werning (2015) (see Appendix A.2 for details)—then household inequality does not affect the optimal rule: setting $\Theta_{\omega(\zeta),i} = \mathbf{0}$ and simplifying (38), we find that the optimal Ramsey rule continues to take a standard dual-mandate form[18]

$$\widehat{\pi}_t + \frac{1}{\bar{\eta}}\left(\widehat{y}_t - \widehat{y}_{t-1}\right) = 0, \quad \forall t = 0, 1, \ldots \tag{40}$$

[18]Bilbiie (2021) makes an analogous argument in a two-agent context, showing that, if monetary policy does not redistribute between spenders and savers, then the optimal policy rule is not affected by inequality concerns. Our analysis reveals that the conditions underlying Werning's aggregation result are precisely enough to extend this insight to our heterogeneous-agent setting.

In this case, monetary policy optimally focuses on aggregate objectives while fiscal policy takes sole responsibility for all distributional considerations. On the other hand, if interest rate cuts are strongly progressive (as is the case in Bhandari et al., 2021; Dávila & Schaab, 2022), then the concerns about inequality as embedded in the policymaker objective (33) may materially change optimal monetary policy conduct, with insurance concerns swamping the usual price and output stabilization motives. This discussion suggests the way forward: to provide a convincing answer to the question of whether household inequality affects the optimal Ramsey policy, researchers should consider structural models that agree with empirical evidence on the distributional incidence of policy, as embedded in our sufficient statistics formulas in (38) - (39). The next subsection discusses how we do so.

5.3 Calibration

We seek to calibrate our structural model to deliver empirically realistic distributional effects of monetary and fiscal stabilization policy. In particular, as revealed by our formulas in (38) and (39), what ultimately matters are the effects of policy on *consumption* inequality. Some prior empirical work has tried to directly estimate those causal effects. The results so far, however, are not conclusive: while Holm et al. (2021) find that expansionary monetary policy has moderately pronounced U-shaped effects on consumption in the wealth cross-section (in Norway), Coibion et al. (2017) report declines in consumption inequality, and Chang & Schorfheide (2022) conclude that inequality actually even increases somewhat (both in the U.S.). In addition to cross-country institutional differences, this lack of consensus is likely to reflect the lack of high-quality, high-frequency consumption data.

In light of these challenges with direct measurement, we instead proceed as follows. We calibrate the model to empirical evidence on how monetary policy affects the various components of household income—for which there is much more consensus as we describe below. We then rely on the structure of our model's consumption-savings problem to map these effects on income into effects on consumption—the sufficient statistics that ultimately enter our optimal policy formulas (38) and (39). The remainder of this section implements this approach by proceeding in three steps. First, we review the empirical evidence on how monetary policy affects income and wealth in the cross-section of households. Second, we calibrate our model so that it captures those channels of redistribution as well as possible. And third, we use the model to infer the total effects of monetary policy on the consumption distribution, bringing together the effects from the various channels.

THE DISTRIBUTIONAL EFFECTS OF MONETARY POLICY. We begin with a brief review of the most important transmission channels from monetary policy to household balance sheets.

a) *Labor income.* There is extensive evidence that those with less education, low past earnings, and racial minorities tend to be more exposed to cyclical fluctuations in labor market conditions (e.g. Okun, 1973; Hoynes, 2000; Guvenen et al., 2014; Patterson, 2022). Turning to monetary policy more specifically, Guvenen et al. (2014) study earnings dynamics in the 1979–1983 recession—a contraction that was arguably caused by a large monetary intervention. They find that individuals with low previous earnings also suffered the largest earnings losses. Several recent studies combine identified monetary policy shocks with European administrative data and arrive at very similar conclusions (Andersen et al., 2021; Amberg et al., 2021; Holm et al., 2021).

   In addition to the *distribution* of labor income, there is also the question of how strongly *total* labor income responds to monetary policy. As is well-known, in textbook sticky-price, flexible-wage models, expansionary monetary policy sharply increases labor income, thus giving monetary policy power to redistribute between workers and firm owners. Empirically, on the other hand, studies have found that expansionary policy leads to a modest rise in real wages and profits.[19] Our model, due to our assumptions on the division of monopoly profits between workers and firm owners, implies that total compensation and returns to capital move one-for-one with output, consistent with the evidence.

b) *Asset prices.* Expansionary monetary policy raises the value of long-duration assets and leads to considerable capital gains for the owners of those assets. For example, Bauer & Swanson (2022) estimate that a 100 basis point reduction in short-term nominal interest rates increases the S&P 500 stock market index by about five percentage points. As the distribution of wealth is concentrated, these capital gains are very unequally distributed across the population. Andersen et al. (2021) leverage administrative household-level data from Denmark to quantify this distributional gradient. They find that, in response to a one percentage point decrease in the policy rate, asset values increase by around 70 per cent of total annual disposable household income at the top of the income distribution, and around 30 per cent at the median.[20]

[19]See the summary in Christiano et al. (1999). Relatedly, Cantore et al. (2021) find that expansionary policy leads to a decrease in the labor share—opposite the response in textbook New Keynesian models.

[20]Of course, while long-duration real assets increase in value following an expansionary monetary shock, so too do the values of long-duration liabilities. In particular, the future consumption plans of households

c) *Nominal wealth redistribution.* Surprise inflation will lower the real value of nominal positions. Doepke & Schneider (2006) report that wealthy households have about 10-15% of their net worth in nominal positions, while young middle-class households have considerable nominal liabilities in the form of mortgages. The magnitude of the redistribution that occurs through this channel of course depends on the strength of the inflation response to monetary policy. Many empirical studies point to a relatively flat Phillips curve (see e.g. the review in Mavroeidis et al., 2014), thus implying relatively small inflation responses to changes in policy. However, we note that the resulting distributional effects can be permanent if monetary policy allows permanent changes in the price level.

d) *Net interest income.* Expansionary policy lowers interest rates, thus leading to lower debt service payments for debtor households and to lower interest income for creditor households. This channel benefits middle-class households that own a home with a large mortgage relative to their net worth (see Cloyne et al., 2020; Wong, 2021).

MODEL CALIBRATION. We calibrate our model to capture the just-summarized channels of monetary policy propagation to consumption inequality.

- *Income process.* Since many of the distributional effects of monetary policy involve changes in asset values, it is important that the model generates a concentrated distribution of wealth, mirroring the data. Since standard incomplete-markets models of the consumption-savings decision struggle to generate sufficiently concentrated wealth holdings in the top tail of the distribution, we follow Castaneda et al. (2003), Boar & Midrigan (2020), and Greenwald et al. (2021) in specifying an income process with superstar earners, allowing us to generate a realistic concentration of wealth.

  We assume that the underlying household income state, ζ_{it}, follows a two-component process, with households being either regular workers or high earners. The function Φ that maps ζ_{it} to labor productivity e_{it} is parameterized as

$$\log e_{it} = \log\left[(1-\alpha)y_t\right] + \log\left(\zeta_{it}\right)\left(1 + m_t + \chi \log y_t\right) + \log \bar{e}_t, \tag{41}$$

  where χ controls the sensitivity of income dispersion to the aggregate business cycle, with a negative χ implying that low-ζ households are more exposed to the cycle, and

---

become more costly as real interest rates fall. The theoretical ideal to measure the net exposure to interest rates, accounting for the difference between asset income and planned consumption (see e.g. the discussion in Auclert, 2019). Our model will embed these effects on the present value of consumption liabilities.

	Wealth		Income	
	Data	Model	Data	Model
Top 1%	37	27	17	20
Top 5%	65	66	32	32
Top 10%	76	82	43	44
Top 25%	91	96	64	60
Top 50%	99	101	84	77

Table 1: Shares (%) of wealth and income concentrated in the top x% of the distribution. Data are from the 2019 Survey of Consumer Finance.

$\bar{e}_t$ is a normalization constant ensuring that $\int e_{it} di = (1-\alpha) y_t$ at all dates.[21] For regular workers, the state $\log \zeta_{it}$ follows an AR(1) process. High-earning households instead can receive one of two (high) levels of earnings. There are thus six parameters associated with the high-earnings states: the probability of becoming a high earner of either kind, the earnings level associated with each state, and the probabilities of transitioning back to being a regular worker.[22]

To estimate the full income process we extend the estimation procedure of Guvenen et al. (2022). Those authors propose a rich parametric income process that allows for differential exposure to aggregate business-cycle conditions, and then estimate it using a variety of moments taken from the Social Security Administration data. This strategy pins down χ in (41) using data on the differential impact of business-cycle episodes across the income distribution. We extend their procedure by also targeting moments of the wealth and income distribution, as reported in Table 1. Further details on the calibration of the income process and its parameters are presented in Appendix B.1.

- *Asset structure.* In the model, households have positions in capital, short-term bonds, and long-term bonds. We need to calibrate the total supply of each asset class and then how these assets are distributed in steady state. The steady state portfolios determine the distribution of capital gains at $t = 0$, when the economy is subjected to shocks.

  We use data on the balance sheets of U.S. households from the Financial Accounts and the 2019 SCF to impute the holdings of particular asset classes across the household

[21] Regular workers share a common value of χ while high earners have $\chi = 0$.

[22] To reduce the number of parameters, we assume that there are no transitions between the high-earning states and that upon exit the individual receives a random draw from the unconditional distribution of regular worker ζ's.

Line	Category	Total	Holdings by net worth group			
			Top 1%	Next 9%	Next 40%	Bottom 50%
1	Real estate and durables	167	24	48	72	23
2	Equity and mutual funds	191	101	66	23	2
3	Currency, deposits, and similar	60	16	23	19	2
4	Govt. and corp. bonds and similar	29	10	11	7	1
5	Pension assets	131	6	63	58	4
6	Mortgage liabilities	49	2	12	24	11
7	Consumer credit and loans	24	1	2	8	12
8	Net worth excluding pension assets	374	147	135	89	4
9	Capital	419	157	135	101	25
10	Short-term bonds	-12	1	7	-3	-16
11	Long-term bonds	-33	-11	-8	-9	-5
12	Total	374	147	135	89	4

Table 2: Assets and liabilities in percent of GDP with decomposition by net-worth. Lines 1-8 report the observed asset and liability categories from the Distributional Financial Accounts. Lines 9-12 map these assets and liabilities into their model counterparts.

distribution of net worth. The top panel of Table 2 reports data on asset holdings in coarse net worth bins, taken from the Distributional Financial Accounts (DFAs).[23] An important takeaway from Table 2 is that pension assets represent 22% of total household assets. Of these pension assets, 61% are defined benefit entitlements. Given that we do not explicitly model pensions we must make a choice about how to incorporate these assets into our calibration. As households are quite insulated from the returns on these pension assets, we choose to incorporate the pension funds on to the government balance sheet. The returns on these pension assets are then gradually paid out to households through adjustments in taxes.

The next step is to map the observed asset categories into the corresponding asset classes in the model. We treat corporate and non-corporate equity as levered claims on capital, so each dollar of equity represents \$1.32 dollars of capital and a \$0.32 debt position. This leverage ratio reflects the ratio of debt to assets in the non-financial business sector. Based on the results in Greenwood et al. (2010) we assume that 62%

[23]Our imputation procedure yields similar results to the DFAs, but we use the SCF to create portfolios that are smooth functions of net worth as opposed to the coarse bins shown in Table 2.

of this debt is long-term debt. We set the duration of the long-term bond in our model to 10 years, in line with results for corporate bonds in Guedes & Opler (1996). Next, we treat mortgages as a 50-50 split of short-term and long-term debt. Even though many mortgages are 30-year loans, the duration of these loans is much shorter due to amortization and prepayment (e.g., Greenwald et al., 2021, report a duration of approximately 5 years). We treat consumer credit as short-term debt. Overall our choices lead to a capital-output ratio of 4.19, as shown in Table 2. In total, households have debts of 12% of GDP in short-term bonds and 33% of GDP in long-term bonds.[24]

We set the capital share to $\alpha = 0.36$ and the depreciation rate to $\delta = 0.01$, allowing us to match a 17% investment rate (the average observed from 1984 to 2019). These values imply a steady state annual interest rate of 4.5%. Lastly, we set the borrowing limit—which restricts net worth and not gross positions—to -0.27 times steady state average income. This value matches the 5th percentile of net worth in our SCF data.

- *Fiscal system.* The U.S. fiscal tax-and-transfer system is reasonably well-approximated by a uniform baseline lump-sum transfer coupled with a constant marginal tax rate (see Kaplan et al., 2018). Data on post-government and pre-government income from the Congressional Budget Office (2019) imply a steady state transfer of 0.17 times average income, pinning down the ratio $\bar{\tau}_x/\bar{y}$ in our model.[25] The speed of fiscal adjustment is controlled by σ_τ, which we set equal to σ_b, implying very gradual fiscal feedback.[26]

- *Other parameters.* We set the Frisch elasticity to one, and the elasticity of substitution between labor varieties to six, based on Basu & Fernald (1997). Finally, we set the coefficient of relative risk aversion γ and the slope of the Phillips curve κ to match the peak responses of output and inflation to an empirically identified monetary policy shock, ensuring that our model is not just consistent with *distributional* incidence, but also with overall *level* responses. Again, the details for this internal calibration are provided in Appendix B.1.

[24] As households (excluding pension funds) collectively have negative credit positions, the pension funds will lend to the households and to the government. As we consolidate the pension funds and government in our model, it is thus as if the government is lending to the households. See Appendix B.2 for further description of how we use the SCF to impute asset positions as smooth functions of household net worth.

[25] We exclude Medicaid and CHIP benefits from the after-tax income. We then regress after-tax incomes on before-tax incomes for the first four quintiles of the income distribution. The intercept of this regression gives us the steady state transfer.

[26] We have found that the results are robust to the choice of σ_τ as long as it takes a low value. Empirically, changes in public debt are highly persistent, thus implying a low value of σ_τ.

Parameter	Description	Value	Calibration target
Households			
$\{\zeta_{it}\}$	Income risk process	–	See text
χ	Income exposure	-2.0	Het. earnings cyclicality
γ	Relative risk aversion	1.2	Monetary shock effects
ϕ	Frisch elasticity	1	Standard
β	Discount factor	0.984	Asset market clearing
Firms			
κ	Phillips curve slope	0.022	Monetary shock effects
$\bar{\eta}$	Labor Substitutability	6	Basu & Fernald (1997)
Asset Structure			
α	Capital share	0.36	Standard
δ	Depreciation rate	0.01	Investment rate
$\underline{a}/\bar{y}$	Borrowing limit	-0.27	Fifth percentile of net worth
σ_b	Long-term bond duration	0.025	Guedes & Opler (1996)
Government			
τ_y	Labor tax rate	0.23	Steady-state budget balance
$\frac{\bar{\tau}_x}{\bar{y}}$	Transfer share	0.17	After-tax vs. before-tax income
σ_τ	Tax-debt responsiveness	$= \sigma_b$	See text

Table 3: Calibration of our quantitative HANK model. The model period is one quarter.

We summarize all parameter values in Table 3.

Consumption responses. Having calibrated our model to be consistent with the most salient transmission channels of monetary policy to households, we now use the structural model to map these income responses into the sufficient statistics that ultimately enter our optimal policy rules—the effects of monetary policy on consumption inequality.

Figure 2 shows the model-implied impact change in consumption across the wealth distribution following a transitory, expansionary monetary policy shock. Since histories ζ map into differences in income and wealth, we note that this figure is directly informative about the $\Theta_{\omega(\zeta),i}$'s that enter our optimal policy rules. Our main finding is that households across the wealth distribution increase their consumption relatively uniformly, implying small effects on consumption *shares*—that is, causal effects $\Theta_{\omega(\zeta),i}$ that are quite close to zero.

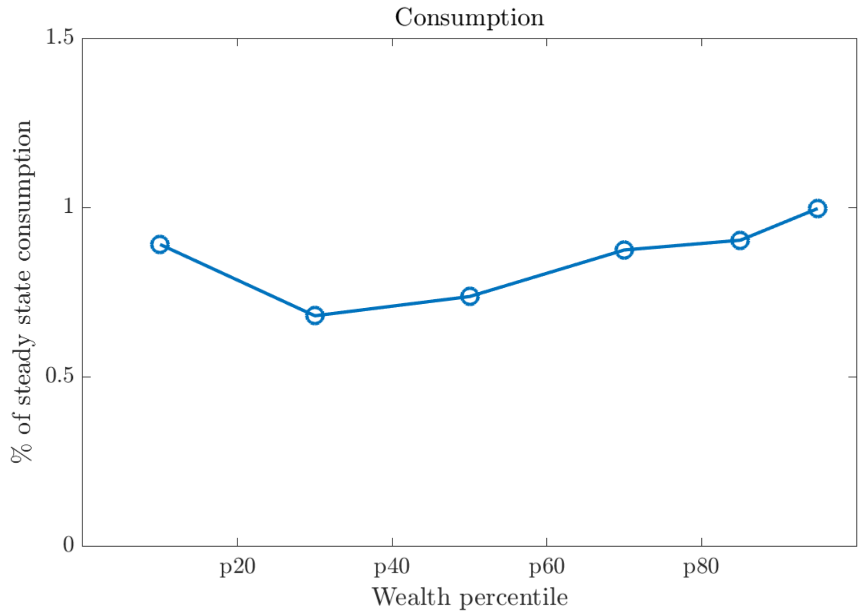


Figure 2: Initial response of consumption to an expansionary monetary policy shock across the distribution of wealth. We simulate the shock we estimate empirically in Appendix B.1.

What explains these rather evenly distributed effects of policy stimulus? On the one end of the spectrum, low-wealth households tend to also have rather low incomes, their labor earnings are more exposed to aggregate income, and they are often borrowing-constrained and so tend to have high marginal propensities to consume (MPCs). As a result, changes in labor earnings pass through strongly into consumption. Households with a moderate amount of wealth instead tend to have substantial real estate and mortgage positions (see the Next 40% column in Table 2), and also relatively moderate MPCs. Finally, high-wealth households benefit from increases in asset values. While these capital gains are very large, the pass-through to consumption is relatively weak, given the rather low model-implied MPCs at the top end of the wealth distribution.[27] Putting all of the pieces together, we arrive at the relatively uniform effects shown in Figure 2.

[27]Note that the earnings of the lowest-wealth group increase by 0.5% of their steady-state income, while the capital gains of the highest-wealth group amount to more than 20% of steady-state income. The model is thus fully consistent with the empirical results of Andersen et al. (2021) and Bartscher et al. (2021), who emphasize that capital gains effects are large relative to labor income effects. While the implications for short-run consumption are more equal, high-wealth consumption does remain elevated for longer. These dynamic effects are not visible in Figure 2, but they are incorporated into our analysis of optimal policy.

5.4 Applications

We now use the calibrated model as a laboratory to explore optimal Ramsey policy in two scenarios. As our first application, we consider an aggregate shock with strong distributional consequences, depressing consumption of the poor relative to that of the rich, akin to the Covid-19 recession. Second, we return to the inflationary cost-push shock from Section 4.3—a shock that has received much attention in the classic RANK literature as well as the recent HANK literature (e.g., Bhandari et al., 2021; Acharya et al., 2022). We will use this second application and the comparison with prior work to in particular highlight the usefulness of our sufficient statistics approach to optimal policy rule characterization.

Distributional shock. Our first shock is an innovation to m_t, the exogenous driver of income dispersion in (41). This shock redistributes labor income from low-income to high-income households, thus depressing aggregate demand: precautionary savings increase due to the increase in risk, and so spending falls as income is redistributed towards lower-MPC households. We study three optimal policy responses to this demand shock: monetary policy for a dual mandate policymaker; monetary policy for a Ramsey planner; and finally joint monetary-fiscal policy for a Ramsey planner. Results are reported in Figure 3.

We begin with the optimal dual mandate policy response, displayed as the grey lines in Figure 3. As aggregate demand falls, the dual-mandate central banker cuts nominal interest rates to perfectly stabilize output and inflation (the classic "divine coincidence"). The bottom panel shows how the consumption distribution changes on impact of the shock: the shock itself redistributes from low-income to high-income households, and monetary policy—precisely because of how we calibrated the model—does very little to offset that.

We then consider the optimal Ramsey monetary policy that also considers distributional objectives, depicted as the orange dashed lines in Figure 3. Our headline finding here is that the optimal policy response is very similar to the dual-mandate policy, with nominal interest rates now cut somewhat more on impact of the shock but then very closely following the dual mandate policy. As a result, aggregate output and inflation continue to be stabilized almost perfectly, and the response of the consumption distribution hardly differs from the dual-mandate outcome. The intuition is as follows. The original shock m_t has a strong distributional tilt, with low-income households losing the most. The Ramsey planner would like to lean against this redistribution and stabilize consumption shares. Monetary policy, however, is a rather blunt tool: if the planner were to cut rates by enough to stabilize consumption at the bottom of the distribution, then consumption of richer households and

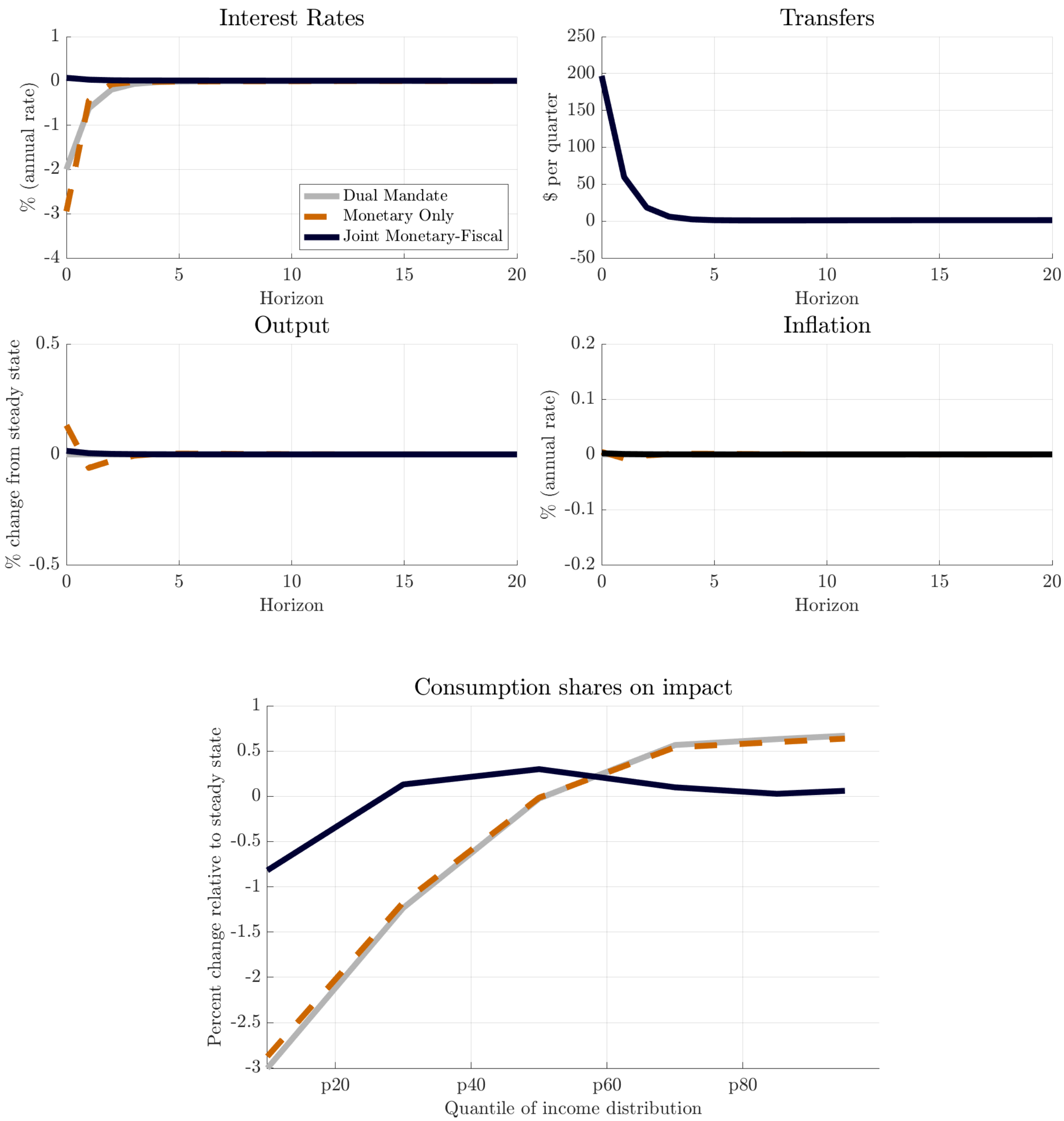


Figure 3: Optimal policy response to an income distribution shock m_t. The figure shows the results from three policy rules: optimal monetary policy for a dual-mandate policymaker (Dual Mandate), optimal monetary policy for a Ramsey planner that is constrained not to use lump-sum transfers (Monetary Only), and optimal policy for a Ramsey planner that can use both tools (Joint Monetary-Fiscal). Transfers are expressed in units of dollars, using the conversion that steady state per capita GDP is $60,000.

so aggregate output and inflation would overshoot significantly—an immediate implication of the flat distributional effects of monetary interventions shown in Figure 2. We provide a visual illustration of this point in Appendix Figure B.4: increasing the size of the interest rate cut to stabilize the consumption of households with low income and low wealth comes at the cost of over-stimulating the consumption of other households (and thus aggregate output and inflation).

Finally we turn to the optimal joint monetary-fiscal Ramsey policy, with a planner using both interest rate policy and stimulus checks to pursue her aggregate and distributional objectives. Results are displayed as the dark blue lines in Figure 3. Compared to monetary policy, fiscal stimulus payments are much more progressive, with consumption of low-income and low-wealth households responding significantly more than that of high-income, high-wealth households (see Wolf (2021) or Appendix Figure B.3). Intuitively, this reflects both differences in MPCs as well as differences in income levels, with any given dollar amount of stimulus checks amounting to a much larger fraction of income at the bottom of the income distribution. As a result, fiscal stimulus payments are particularly well-suited as a tool to address shocks that differently affect low-income and high-income households. The results depicted in Figure 3 are consistent with this intuition: in response to the distributional shock m_t, the Ramsey planner can use checks to almost perfectly stabilize aggregate output, inflation, *and* consumption inequality. Since stimulus payments both compress inequality and stimulate aggregate demand, there is little need for the monetary authority to intervene, with nominal interest rates now responding very little.

Overall, our analysis of optimal Ramsey policy responses to the distributional demand shock m_t suggests two broad lessons for optimal cyclical policy design. First, recall from our prior analysis that a monetary policymaker has an incentive to deviate from the prescriptions of dual mandate policy only if monetary policy can help her to offset the distributional incidence of the underlying business-cycle shock. With empirical evidence suggesting a relatively uniform distributional incidence of monetary interventions, it follows that monetary policy is in general rather ill-suited to attain distributional objectives. In particular, stabilizing consumption at the bottom of the distribution may require significant departures from dual-mandate objectives; if the Ramsey planner is unwilling to accept these departures (as was the case in our example), then optimal policy will tend to stay close to the dual-mandate benchmark, as seen here. Second, fiscal stimulus payments and nominal interest rate policy can be highly complementary policy tools. They are likely to have very different cross-sectional incidence profiles (see Figure 2 and Figure B.3), and so the optimal mix of

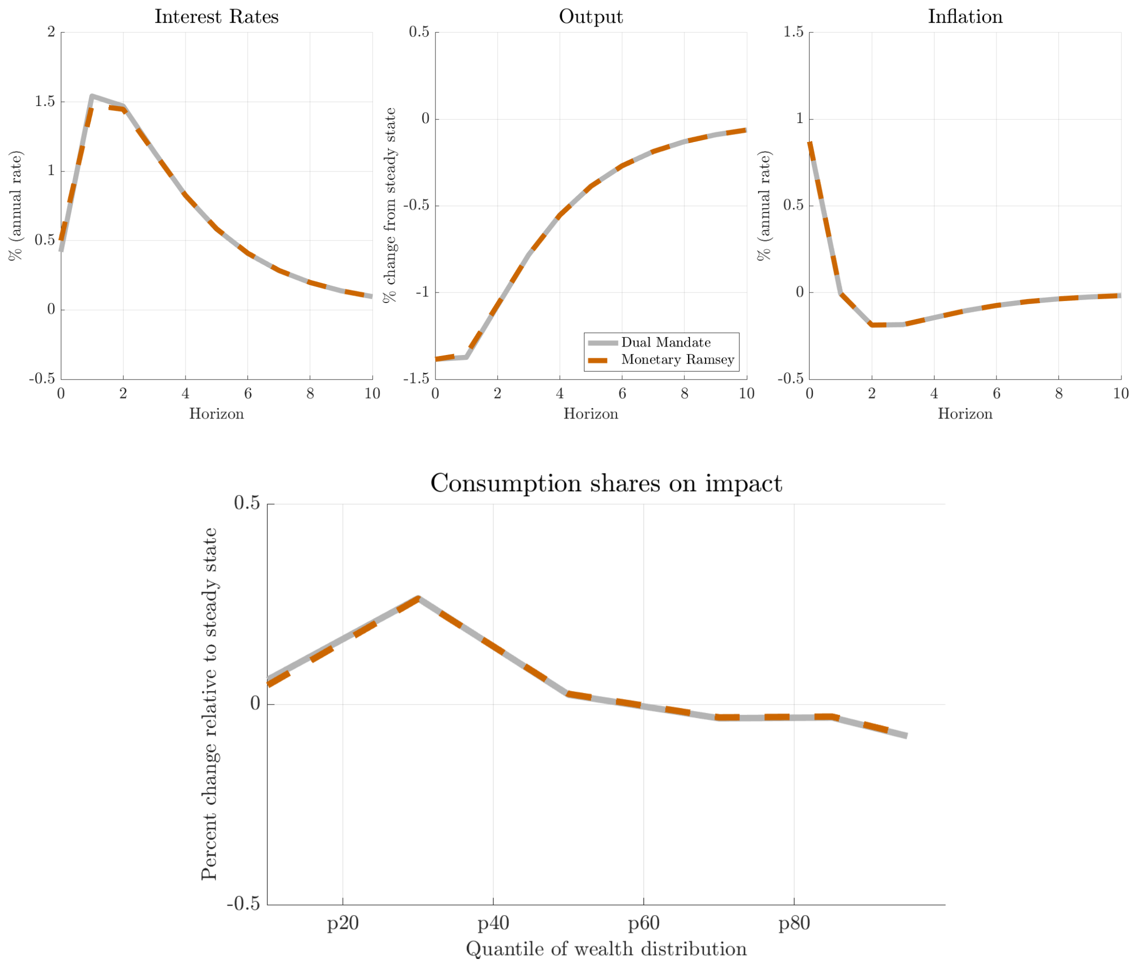


Figure 4: Optimal policy response to an inflationary cost-push shock. The figure shows optimal monetary policy for a planner whose objective is to stabilize output and inflation (Dual Mandate) and for a Ramsey planner (Monetary Ramsey).

monetary and fiscal policy as a stabilization tool will depend on the distributional incidence of the shocks hitting the economy.

Cost-push shock. Our second shock is a classic cost-push shock η_t, as already considered in our dual-mandate application in Section 4.3. In our model, this cost-push shock introduces a wedge in the aggregate Phillips curve, thus giving an output-inflation trade-off. Under our assumptions on the incidence of firm profits, however, this shock does not by itself have strong distributional implications. For our analysis we restrict attention to monetary policy alone, comparing only the optimal dual-mandate and Ramsey monetary policy responses. Results are displayed in Figure 4. We find that the Ramsey policy is nearly identical to the dual-mandate policy. Intuitively, neither the shock nor the monetary policy response has strong distributional effects, and so there is little reason to deviate from the dual mandate policy. Furthermore, by the logic discussed above, this result is unlikely to change materially even if the cost-push *did* have meaningful distributional effects, simply because monetary policy would still be ill-suited to offset those.

Our findings contrast with recent arguments made in Bhandari et al. (2021), who conclude that distributional considerations may lead to substantial changes in optimal policy conduct—changing the signs of impulse response functions and the order of magnitude of inflation volatility. Our analysis differs from theirs both methodologically and due to the assumptions on the economic environment. To demonstrate that the difference in conclusions is driven to a considerable extent by the economic assumptions, Appendix B.4 repeats our analysis of the cost-push shock in an alternative model environment that is more similar to that in Bhandari et al.. In particular, for this alternative environment, we assume that (i) all monopoly profits are distributed to wealthy firm owners and (ii) there are no other types of long-duration assets (i.e., the model omits capital and housing-related assets and liabilities). This change in the model environment has two implications. First, cost-push shocks now have strong distributional effects: they increase markups, decrease real wages, and increase the profits flowing to a concentrated group of firm owners. Second, expansionary monetary policy is also redistributive, but in the other direction—it reduces markups, raises real wages, and reduces profits. In light of our previous discussion it is thus unsurprising that we find an optimal Ramsey monetary policy closely echoing the conclusions of Bhandari et al.: while a dual mandate policymaker hikes interest rates following a cost-push shock, the Ramsey planner instead sharply lowers them to reduce cross-sectional inequality.

6 Conclusion

Should household inequality affect the conduct of cyclical stabilization policy? The analysis in this paper suggests the following three main takeaways.

First, for central banks that target standard macroeconomic aggregates (e.g., a traditional "dual mandate"), household inequality—at least as typically modeled in the burgeoning HANK literature—is likely to only have rather moderate effects. Analytically, we have given conditions under which the optimal forecast target criterion of a dual mandate central banker is unaffected by household inequality, implying that output and inflation outcomes in response to any shock will be exactly as in analogous representative-agent environments.

Empirically, a long literature already estimates the causal effects of interest rate changes on output and inflation. Models that are consistent with this evidence will yield similar predictions for the paths of nominal rates necessary to implement a given output and inflation target—irrespective of whether the model features household inequality or not.

Second, the extent to which distributional objectives shape optimal Ramsey monetary policy depends crucially on the causal effects of interest rate changes on consumption inequality. According to our analysis, monetary policy has relatively even effects on consumption across households. Interest rate policy is thus not a particularly sharp tool to deal with shocks that disproportionately affect the poor, at least not without substantial costs in terms of aggregate stabilization.

Third, fiscal stimulus checks—an alternative tool of stabilization policy, used increasingly frequently in recent decades—promise to be more useful for distributional purposes. Such stimulus checks achieve aggregate stabilization *through* insurance at the bottom, thus making them well suited to address cyclical fluctuations that mostly affect poor households.

References

Acharya, S., Challe, E., & Dogra, K. (2022). Optimal monetary policy according to HANK. Forthcoming American Economic Review.

Alves, F., Kaplan, G., Moll, B., & Violante, G. L. (2020). A further look at the propagation of monetary policy shocks in hank. *Journal of Money, Credit and Banking*, *52*(S2), 521–559.

Amberg, N., Jansson, T., Klein, M., & Rogantini Picco, A. (2021). Five facts about the distributional income effects of monetary policy. CESifo Working Paper.

Andersen, A. L., Johannesen, N., Jørgensen, M., & Peydró, J.-L. (2021). Monetary policy and inequality. Working Paper.

Auclert, A. (2019). Monetary policy and the redistribution channel. *American Economic Review*, *109*(6), 2333–67.

Auclert, A., Bardóczy, B., & Rognlie, M. (2020). Mpcs, mpes and multipliers: A trilemma for new keynesian models. Technical report, National Bureau of Economic Research.

Auclert, A., Bardóczy, B., Rognlie, M., & Straub, L. (2021). Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, *89*(5), 2375–2408.

Auclert, A. & Rognlie, M. (2018). Inequality and aggregate demand. Technical report, National Bureau of Economic Research.

Auclert, A., Rognlie, M., Souchier, M., & Straub, L. (2021). Exchange rates and monetary policy with heterogeneous agents: Sizing up the real income channel. Technical report, National Bureau of Economic Research.

Auclert, A., Rognlie, M., & Straub, L. (2018). The intertemporal keynesian cross. Technical report, National Bureau of Economic Research.

Bartscher, A. K., Kuhn, M., Schularick, M., & Wachtel, P. (2021). Monetary policy and racial inequality. CEPR Discussion Paper No. DP15734.

Basu, S. & Fernald, J. G. (1997). Returns to scale in us production: Estimates and implications. *Journal of political economy*, *105*(2), 249–283.

Bauer, M. D. & Swanson, E. T. (2022). A reassessment of monetary policy surprises and high-frequency identification. National Bureau of Economic Research Working Paper.

Benigno, P. & Woodford, M. (2012). Linear-quadratic approximation of optimal policy problems. *Journal of Economic Theory*, *147*(1), 1–42.

Bernanke, B. (2015). Remarks at Brookings Institution: Objections to Federal Reserve Accountability Bill.

Bhandari, A., Evans, D., Golosov, M., & Sargent, T. J. (2021). Inequality, business cycles, and monetary-fiscal policy. *Econometrica*, *89*(6), 2559–2599.

Bilbiie, F. O. (2021). Monetary policy and heterogeneity: An analytical framework. Working Paper.

Bilbiie, F. O., Monacelli, T., & Perotti, R. (2021). Stabilization vs. redistribution: the optimal monetary-fiscal mix. CEPR Discussion Paper No. DP15199.

Boar, C. & Midrigan, V. (2020). Efficient redistribution. National Bureau of Economic Research Working Paper.

Boppart, T., Krusell, P., & Mitman, K. (2018). Exploiting MIT shocks in heterogeneous-agent economies: the impulse response as a numerical derivative. *Journal of Economic Dynamics and Control*, *89*, 68–92.

Cantore, C., Ferroni, F., & León-Ledesma, M. (2021). The missing link: monetary policy and the labor share. *Journal of the European Economic Association*, *19*(3), 1592–1620.

Castaneda, A., Diaz-Gimenez, J., & Rios-Rull, J.-V. (2003). Accounting for the us earnings and wealth inequality. *Journal of Political Economy*, *111*(4), 818–857.

Chang, M. & Schorfheide, F. (2022). On the effects of monetary policy shocks on earnings and consumption heterogeneity. CEPR Discussion Paper No. DP17049.

Christiano, L. J., Eichenbaum, M., & Evans, C. L. (1999). Monetary policy shocks: What have we learned and to what end? *Handbook of Macroeconomics*, *1*, 65–148.

Cloyne, J., Ferreira, C., & Surico, P. (2020). Monetary policy when households have debt: new evidence on the transmission mechanism. *The Review of Economic Studies*, *87*(1), 102–129.

Coibion, O., Gorodnichenko, Y., Kueng, L., & Silvia, J. (2017). Innocent bystanders? monetary policy and inequality. *Journal of Monetary Economics*, *88*, 70–89.

Congressional Budget Office (2019). Projected changes in the distribution of household income, 2016 to 2021. Technical report, Congressional Budget Office. Report.

Dávila, E. & Schaab, A. (2022). Optimal monetary policy with heterogeneous agents: A timeless ramsey approach. Working Paper.

De Groot, O., Mazelis, F., Motto, R., & Ristiniemi, A. (2021). A toolkit for computing constrained optimal policy projections (copps). ECB Working Paper.

Doepke, M. & Schneider, M. (2006). Inflation and the redistribution of nominal wealth. *Journal of Political Economy*, *114*(6), 1069–1097.

Farhi, E. & Werning, I. (2019). Monetary policy, bounded rationality, and incomplete markets. *American Economic Review*, *109*(11), 3887–3928.

Fernández-Villaverde, J., Rubio-Ramírez, J. F., & Schorfheide, F. (2016). Solution and estimation methods for DSGE models. In *Handbook of Macroeconomics*, volume 2 (pp. 527–724). Elsevier.

Galí, J. (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. Princeton University Press.

Gertler, M. & Karadi, P. (2015). Monetary policy surprises, credit costs, and economic activity. *American Economic Journal: Macroeconomics*, *7*(1), 44–76.

Giannoni, M. P. & Woodford, M. (2002). Optimal interest-rate rules. NBER Working Paper.

Greenwald, D. L., Leombroni, M., Lustig, H., & Van Nieuwerburgh, S. (2021). Financial and total wealth inequality with declining interest rates. National Bureau of Economic Research Working Paper.

Greenwood, R., Hanson, S., & Stein, J. C. (2010). A gap-filling theory of corporate debt maturity choice. *The Journal of Finance*, *65*(3), 993–1028.

Guedes, J. & Opler, T. (1996). The determinants of the maturity of corporate debt issues. *the Journal of Finance*, *51*(5), 1809–1833.

Guvenen, F., McKay, A., & Ryan, C. (2022). A tractable income process for business cycle analysis. Working Paper.

Guvenen, F., Ozkan, S., & Song, J. (2014). The nature of countercyclical income risk. *Journal of Political Economy*, *122*(3), 621–660.

Heathcote, J. & Tsujiyama, H. (2021). Optimal income taxation: Mirrlees meets ramsey. *Journal of Political Economy*, *129*(11), 3141–3184.

Hebden, J. & Winker, F. (2021). Impulse-Based Computation of Policy Counterfactuals. Working Paper.

Holm, M. B., Paul, P., & Tischbirek, A. (2021). The transmission of monetary policy under the microscope. *Journal of Political Economy*, *129*(10), 2861–2904.

Hoynes, H. (2000). The employment, earnings, and income of less skilled workers over the business cycle. In B. Rebecca & D. Card (Eds.), *Finding Jobs: Work and Welfare Reform* (pp. 23–71). Russell Sage Foundation.

Kaplan, G., Moll, B., & Violante, G. L. (2018). Monetary policy according to hank. *American Economic Review*, *108*(3), 697–743.

Le Grand, F., Martin-Baillon, A., & Ragot, X. (2022). Should monetary policy care about redistribution? optimal fiscal and monetary policy with heterogeneous agents. Technical report, Working Paper, SciencesPo.

Mavroeidis, S., Plagborg-Møller, M., & Stock, J. H. (2014). Empirical evidence on inflation expectations in the new keynesian phillips curve. *Journal of Economic Literature*, *52*(1), 124–88.

McKay, A. & Wolf, C. K. (2023). What can time-series regressions tell us about policy counterfactuals? MIT Working Paper.

Okun, A. M. (1973). Upward mobility in a high-pressure economy. *Brookings Papers on Economic Activity*, *4*(1), 207–262.

Patterson, C. (2022). The matching multiplier and the amplification of recessions. Working Paper.

Plagborg-Møller, M. & Wolf, C. K. (2021). Local projections and vars estimate the same impulse responses. *Econometrica*, *89*(2), 955–980.

Ramey, V. A. (2016). Macroeconomic shocks and their propagation. *Handbook of macroeconomics*, *2*, 71–162.

Svensson, L. E. (1997). Inflation forecast targeting: Implementing and monitoring inflation targets. *European economic review*, *41*(6), 1111–1146.

Svensson, L. E. (2005). Monetary policy with judgment: Forecast targeting.

Werning, I. (2015). Incomplete markets and aggregate demand. Working Paper 21448, National Bureau of Economic Research.

Wolf, C. K. (2021). Interest rate cuts vs. stimulus payments: An equivalence result. National Bureau of Economic Research Working Paper 29193.

Wong, A. (2021). Refinancing and the transmission of monetary policy to consumption. Working paper.

Woodford, M. (2003). *Interest and prices: Foundations of a theory of monetary policy*. princeton university press.

A Supplementary model details

This appendix provides some supplementary details for our structural model. Appendix A.1 begins by further discussing the price-setting problem and deriving the log-linearized Phillips curve (12), and Appendix A.2 discusses the model's special case in which monetary policy is distributionally neutral, following Werning (2015).

A.1 Phillips Curve

Cost minimization by the final goods firm yields the usual demand curve for intermediate varieties

$$y_{jt} = \left(\frac{p_{jt}}{p_t}\right)^{-\eta} y_t. \tag{A.1}$$

and the usual price index

$$p_t = \left(\int p_{jt}^{1-\eta} dj\right)^{\frac{1}{1-\eta}}. \tag{A.2}$$

We now turn to the price-setting problem of the intermediate goods firm. We assume that intermediate goods production is subsidized at gross rate $\frac{\bar{\eta}\Xi}{(\bar{\eta}-1)(1-\tau_y)}$, where $\bar{\eta}$ is the steady state elasticity of substitution between varieties of labor and the term Ξ accounts for the fact that the social planner may attach a higher value to resources than the labor union (we derive the precise value of Ξ in Appendix D.3); for the purposes of our analysis here, it suffices to note that the labor subsidy takes this general form and that it is financed with a lump-sum tax on the monopolostically competitive firms. The intermediate producer's problem is to choose the reset price p_t^* and the input mix $\{k_{jt+s}, \ell_{jt+s}\}_{s\geq 0}$ to maximize

$$\sum_{s\geq 0} \beta^s \theta^s \left[\frac{\bar{\eta}\Xi}{(\bar{\eta}-1)(1-\tau_y)} \frac{p_t^*}{p_{t+s}} y_{j,t+s} - r_{t+s}^k k_{jt+s} - w_{t+s}\ell_{jt+s}\right]$$

subject to (A.1), (11) and taking r_{t+s}^k and w_{t+s} as given. The Lagrangian is

$$\mathcal{L} = \sum_{s\geq 0} \beta^s \theta^s \left[\frac{\bar{\eta}\Xi}{(\bar{\eta}-1)(1-\tau_y)} \frac{p_t^*}{p_{t+s}} \left(\frac{p_t^*}{p_{t+s}}\right)^{-\eta_{t+s}} y_{t+s} - r_{t+s}^k k_{jt+s} - w_{t+s}\ell_{jt+s}\right.$$

$$
-\mu_{t+s}\left(\left(\frac{p_t^*}{p_{t+s}}\right)^{-\eta_{t+s}} y_{t+s} - Ak_{jt+s}^{\alpha}\ell_{jt+s}^{1-\alpha}\right)\Bigg]
$$

and the first-order conditions are

$$
\sum_{s\geq 0}^{\infty} \frac{\beta^s\theta^s\bar{\eta}\Xi(\eta_{t+s}-1)}{(\bar{\eta}-1)(1-\tau_y)}\left(\frac{p_t^*}{p_{t+s}}\right)^{-\eta_{t+s}} \frac{y_{t+s}}{p_{t+s}} = \sum_{s\geq 0}^{\infty}\beta^s\theta^s\left(\eta_{t+s}\mu_{t+s}\left(\frac{p_t^*}{p_{t+s}}\right)^{-\eta_{t+s}-1}\frac{y_{t+s}}{p_{t+s}}\right) \tag{A.3}
$$

$$
r_{t+s}^k = \mu_{t+s}\alpha\frac{y_{jt+s}}{k_{jt+s}} \tag{A.4}
$$

$$
w_{t+s} = \mu_{t+s}(1-\alpha)\frac{y_{jt+s}}{\ell_{jt+s}}. \tag{A.5}
$$

Taking the ratio of (A.4) and (A.5) we obtain

$$
\frac{r_{t+s}^k}{w_{t+s}} = \frac{\alpha}{1-\alpha}\frac{\ell_{jt+s}}{k_{jt+s}},
$$

which implies that all firms use the same capital-labor ratio, which by market clearing must be the aggregate capital-labor ratio. Equations (A.4) and (A.5) imply that capital receives a share $\mu_t\alpha$ of output and labor receives a share $\mu_t(1-\alpha)$, which leaves a share $(1-\mu_t)$ for monopoly profits. As we assume workers receive a share $1-\alpha$ of the monopoly profits, the total compensation to workers is a share $1-\alpha$ of output.

Using (11) in (A.5) we obtain

$$
w_{t+s} = \mu_{t+s}(1-\alpha)A\left(\frac{k_{jt+s}}{\ell_{jt+s}}\right)^{\alpha} = \mu_{t+s}(1-\alpha)A\left(\frac{k}{\ell_{t+s}}\right)^{\alpha},
$$

where the second equality replaces the firm-specific capital-labor ratio with the aggregate capital-labor ratio by the argument above. Now use (10) to substitute for w_{t+s} and log-linearize to obtain

$$
\hat{\mu}_{t+s} = (\phi+\alpha)\,\hat{\ell}_{t+s} + \gamma\hat{c}_{t+s},
$$

where $\phi = v''(\ell)\ell/v'(\ell)$ is the inverse Frisch elasticity. To a first-order approximation, the aggregate production function gives

$$
\hat{y}_{t+s} = (1-\alpha)\hat{\ell}_{t+s}
$$

and the aggregate resource constraint gives

$$\hat{y}_{t+s} = \frac{\bar{c}}{\bar{y}}\hat{c}_{t+s}.$$

Combining the last three equations gives

$$\hat{\mu}_{t+s} = \left(\frac{\phi+\alpha}{1-\alpha} + \frac{\bar{y}\gamma}{\bar{c}}\right)\hat{y}_{t+s}. \tag{A.6}$$

Log-linearizing the (A.3) around a zero-inflation steady state yields

$$\widehat{p}_t^* - \widehat{p}_t = (1-\beta\theta)\sum_{s\geq 0}\beta^s\theta^s\left(\hat{\mu}_{t+s} - \frac{1}{\bar{\eta}-1}\widehat{\eta}_{t+s} + \widehat{p}_{t+s} - \widehat{p}_t\right)$$

Next, we from the definition of the price index have

$$1+\pi_t \equiv \frac{p_t}{p_{t-1}} = \left(\theta^{-1} - \frac{1-\theta}{\theta}\left(\frac{p_t^*}{p_t}\right)^{1-\eta_t}\right)^{\frac{1}{\eta_t-1}}. \tag{A.7}$$

Log-linearizing around a zero inflation steady state this gives

$$\widehat{\pi}_t = \widehat{p}_t - \widehat{p}_{t-1} = \frac{1-\theta}{\theta}\left(\widehat{p}_t^* - \widehat{p}_t\right).$$

Eliminating $\widehat{p}_t^* - \widehat{p}_t$, using (A.6), and simplifying, we get

$$\widehat{\pi}_t = \kappa\widehat{y}_t + \psi\widehat{\eta}_t + \beta\widehat{\pi}_{t+1}$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\frac{\phi+\alpha}{1-\alpha} + \frac{\bar{y}\gamma}{\bar{c}}\right)$ and $\psi = -\frac{(1-\theta)(1-\beta\theta)}{\theta(\bar{\eta}-1)}$.

A.2 Werning (2015) special case

This section elaborates on our discussion in Section 5.2 of optimal policy in a special case of our model where monetary policy is distributionally neutral, following Werning (2015). We proceed in three steps. First, we present the assumptions required to arrive at this special case. Second, we derive the distributional irrelevance result. And third, we formally state implications for optimal policy design. Proofs are deferred to Appendices D.4 to D.7.

Model assumptions. We consider a special case of our environment, adapted to be to be consistent with the assumptions in Werning (2015).

First, we further restrict the household consumption-savings problem.

Assumption A.1. *Household utility is logarithmic ($\gamma = 1$). The distribution of household earnings e_{it} is acylical (i.e., the incidence function Φ from idiosyncratic events to earnings can be written as $\tilde{\Phi}(\zeta_{it}, m_t) \times (1-\alpha)y_t$ so earnings scale with y_t) and households can self-insure only through saving, not borrowing ($\underline{a} = 0$).*

Second, we assume that the asset supply has particular properties.

Assumption A.2. *There is no depreciation ($\delta = 0$), there is no net supply of bonds at any date, and at date-0 capital is the only asset held by households.*

The third and final assumption restricts the exogenous component of transfers, τ_x.

Assumption A.3. *In steady-state, the exogenous and endogenous components of government transfers to households are zero, i.e. $\bar{\tau}_x = 0$ and $\bar{\tau}_e = 0$.*

Under similar assumptions, Werning (2015) proves that the demand side of the economy responds to monetary policy exactly like the conventional Euler equation (23). Household heterogeneity affects the *split* of the consumption response into indirect income and direct interest rate effects, but leaves the overall *sum* unchanged. We will see that the same logic allows a sharp characterization of optimal policy rules for the Ramsey planner.

The distributional effects of monetary policy. Following steps similar to those in Werning (2015), we can prove the following useful building block result.

Proposition A.1. *Under Assumptions A.1 to A.3, we have that*

$$\Theta_{\omega(\zeta),i} = \mathbf{0} \qquad \forall \zeta. \tag{A.8}$$

In words, changes in nominal interest rates have no effect on the consumption *distribution*, at any horizon.

Optimal policy characterization. Combining Proposition A.1 and our general characterizations of optimal forecast targeting policy rules in Section 2.1, it follows immediately that the presence of the inequality term in the policymaker loss function does not at all affect the target criterion for optimal monetary policy. Corollary A.1 summarizes this conclusion.

Corollary A.1. *Under Assumptions A.1 to A.3 and the conditions of Proposition 1, the optimal monetary policy rule for a Ramsey policymaker with loss function* (33) *can be written as the forecast target criterion*

$$\widehat{\pi}_t + \frac{1}{\bar{\eta}} (\widehat{y}_t - \widehat{y}_{t-1}) = 0, \quad \forall t = 0, 1, \ldots \tag{A.9}$$

Corollary A.1 formalizes our intuitive discussion in Section 5.2. Finally we also note the following implication of these results: since monetary policy stabilizes output and inflation as well as possible, and since fiscal policy has no additional scope to help with this stabilization (recall Section 4), it follows that fiscal transfer policy is exclusively concerned with inequality stabilization, with the optimal transfer target criterion given as

$$\int \lambda_{\omega(\zeta)} \Theta'_{\omega(\zeta),\tau_x} W \widehat{\boldsymbol{\omega}}(\zeta) d\Gamma(\zeta) = \mathbf{0}. \tag{A.10}$$

Corollary A.2 summarizes this conclusion.

Corollary A.2. *Under the optimal joint fiscal-monetary policy, transfers are set following the target criterion* (A.10)*, minimizing the inequality term in the loss function* (33)*. Monetary policy is set residually to enforce the target criterion* (A.9)*, thus attaining the same paths of aggregate inflation and output as under optimal monetary policy alone.*

We thus achieve a strict separation of monetary and fiscal instruments. Corollary A.2 is related to the results in Bilbiie et al. (2021): while those authors find that transfers can be set to perfectly stabilize inequality between two groups of households, transfers in our HANK model are set to stabilize the general inequality term *as well as possible*. In both cases, given fiscal stabilization of inequality, conventional monetary policy then implements the aggregate allocations familiar from standard representative-agent analysis—in their case because inequality is already perfectly stabilized; in our case because conventional monetary policy cannot help any further with the inequality-related loss.

B Supplementary material for Sections 5.3 and 5.4

This appendix presents supplementary material for our quantitative optimal Ramsey policy analysis. Appendix B.1 begins with further details on model calibration, while Appendix B.3 provides various additional results for our particular business-cycle shock experiments.

B.1 Calibration

Income process. We use a similar estimation procedure as Guvenen et al. (2022) in fitting the parameters of the income process. We briefly summarize the moments that are targeted here and describe how we deviate from Guvenen et al. while referring the reader to that paper for most of the details.

To concisely describe the income process, it is useful to adopt the notation that $\zeta_{it} \in \mathcal{Z}^R$ for regular workers and $\zeta_{it} \in \{z_1^H, z_2^H\}$ for high-earners. We estimate the following income process:

$$\log e_{it} = \log\left[(1-\alpha)y_t\right] + \log\left(\zeta_{it}\right)\left(1 + m_t + \chi \mathbb{I}_{[\zeta_{it} \in \mathcal{Z}^R]} \log y_t\right) + \log \bar{e}_t + \mu_i + \xi_{it}^T$$

As described by Guvenen et al., over the lifecycle, the cross-sectional variance of earnings grows almost linearly in the age of a cohort. In the absence of dispersion in the deterministic component of individual income profiles, this implies near-random walk behavior in individual earnings. To have a stationary distribution of income, we fix ρ_ζ at a value less than one, setting $\rho_\zeta = 0.9^{1/4}$.[28] ξ_{it}^T is a transitory income shock that follows a two-state Markov chain. We fix the parameters of this Markov chain to mimic the "non-employment" shock in Guvenen et al..[29] Finally μ_i is an individual fixed effect.

If $\zeta_{it-1} \in \mathcal{Z}^R$, with probability Π_{RR} the worker remains a regular worker and ζ_{it} follows

$$\begin{aligned} \log \zeta_{it} &= \rho_\zeta \log \zeta_{it-1} + \xi_{it} \\ \xi_{it} &\sim N(0, \sigma_\xi^2). \end{aligned}$$

In principal, the Gaussian shocks imply the support for $\log \zeta_{it}$ for regular workers is all

[28] Allowing for heterogeneous income profiles, one finds a lower value of ρ_ζ.

[29] We include this shock in our income process estimation so that the persistent shocks ξ_{it} are not forced to account for all features of the income data. For the sake of parsimony, however, we do not include this shock in quantitative model analysis. We have experimented with including this shock and found the results to be similar; the main difference is that the transitory shock leads to a lower average MPC in steady state.

of $\mathbb{R}$, but, for the sake of discussion, suppose the support of this process $\mathcal{Z}^R$ is a subset of $\mathbb{R}$ that is disjoint from $\{z_1^H, z_2^H\}$ allowing us to use a single variable to describe the state.[30] With probability Π_j^{RH} the worker transitions to state $\zeta_{it} = z_j^H$ where $j \in \{1,2\}$. If $\zeta_{it-1} \in \{z_1^H, z_2^H\}$, with probability Π_j^{HH} they remain a high-earner with $\zeta_{it} = \zeta_{it-1}$ and with the remaining probability they draw from $\zeta_{it} \sim N(0, \sigma_\xi^2/(1-\rho_\xi^2))$.

The parameters to estimate are χ, the variance of μ_i, σ_ξ^2, the levels of high earnings z_1^H and z_2^H, and the transition probabilities $\{\Pi_j^{RH}, \Pi_j^{HH}\}_{j=1}^2$. We do this by simulating the income process as in Guvenen et al. (2022). This procedure takes an observed times series of average earnings as an input and then seeks to match a variety of moments on earnings dynamics as measured in the Social Security Administration data. We simulate the process at a quarterly frequency and time aggregate to annual observations. Many of the moments we target reflect the shape of the earnings growth distribution for earnings growth at 1-year, 3-year, and 5-year horizons. For the purposes of this paper, the main question is how we identify χ. Here we proceed as follows. For each business cycle episode between 1979 and 2010, Guvenen et al. (2014) construct a figure analogous to Figure B.1, using average income over the five years prior to the business cycle episode to rank individuals in the distribution. For each recession and expansion, we fit a trend line between the 11th and 80th percentiles of Figure B.1 to obtain a target "slope." We then seek to replicate these slopes in our simulated process. To get the model-implied analogue, we feed in average earnings as our measure of $(1-\alpha)y_t$, from there recover household earnings, and then finally construct the model-implied slope. One component of our objective function is then simply the percentage deviation between the model and empirical slopes of the incidence of the business cycle.

In addition to the Guvenen et al. (2022) moments, we also solve for the steady state wealth and income distributions (assuming a constant level of aggregate earnings) and evaluate the moments in Table 1. The estimation procedure seeks to minimize the deviation of all of these moments from their target values.

The estimated parameters that enter our model are $\sigma_\xi = 0.064$, $\chi = -2.0$, $(z_1^H, z_2^H) = (2.9, 24.8)$ relative to an average ζ of 1, $(\Pi_1^{RH}, \Pi_2^{RH}) = (6.2\times10^{-3}, 4.1\times10^{-4})$, and $(\Pi_1^{HH}, \Pi_2^{HH}) = (0.91, 0.95)$.

Monetary policy shocks for internal calibration. Two parameters of our model are set by internal calibration, to target estimated impulse response functions to identified

[30]In our computations we discretize the process for regular workers allowing us to use a single state for the idiosyncratic shock.

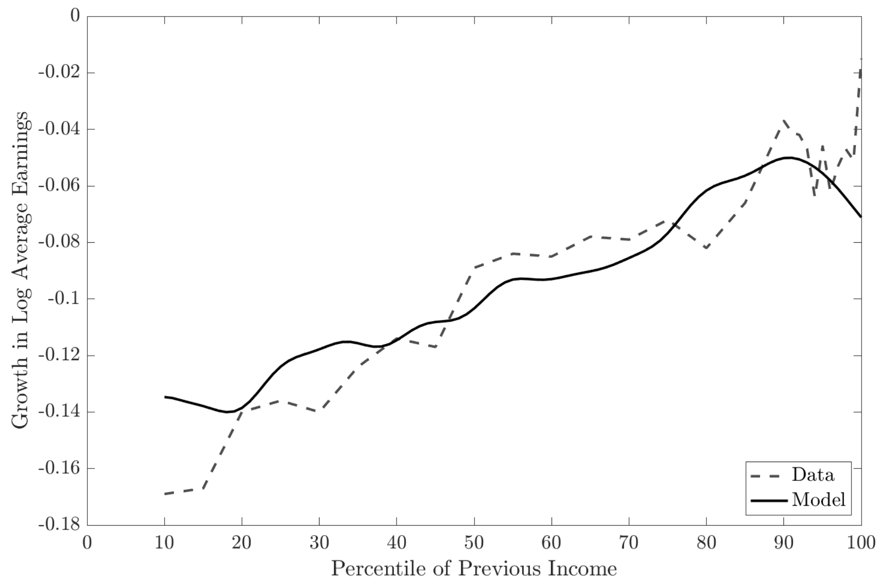


Figure B.1: Incidence of earnings losses during the 1979-1983 recession. The horizontal axis ranks individuals according to their earnings during 1974-1978. For each percentile of this distribution, the vertical axis shows the change from 1979 to 1983 in the log of average earnings for that group. The upward slope implies that lower-income individuals suffered larger earnings losses on average in the recession. Data are taken from Guvenen et al. (2014). The model results are from a simulation of the fitted income process, feeding in the time series of average (aggregate) earnings growth.

monetary policy shocks: the relative risk aversion γ and the Phillips curve slope κ. We use the high-frequency monetary shocks identified by Gertler & Karadi (2015) to estimate the response of key macro aggregates to identified monetary shocks. We first describe the results and internal calibration procedure and then give the details of the empirical implementation.

Figure B.2 shows point estimates for four estimated impulse response functions. In the left panel we show the time path of real interest rates, which we construct as $i_t - \pi_{t+1}$. As expected, the expansionary monetary shock leads to a persistent decline in real interest rates. The center panel shows that consumption increases, eventually reaching a peak of 0.9% above steady state. The right panel reveals that inflation rises quite persistently, with a peak increase of about 0.3%.

Our internal calibration procedure assumes that monetary policy is set according to a standard Taylor rule subject to shocks. We find a path of current and anticipated monetary policy shocks that, when announced at date 0, leads to a change in real rate expectations that

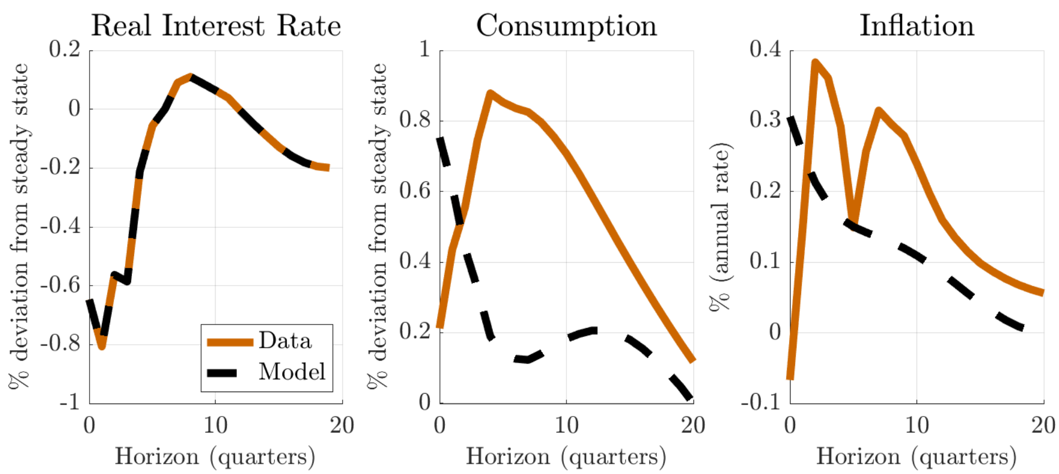


Figure B.2: Empirical (orange) and model-implied (blue-dashed) impulse response following an expansionary monetary policy shock.

perfectly replicates the empirical estimate in Figure B.2.[31] We then choose $\{\gamma, \kappa\}$ to match the peak responses of output and inflation. Figure B.2 shows the model-implied impulse response functions. For output and inflation, the model generates the correct peak response, but unsurprisingly does not generate the hump-shaped pattern of the empirical estimates.

Our empirical analysis uses the following series: the 3-month Treasury rate, the inflation rate constructed from the log difference in the GDP deflator, and log real GDP per capita.[32] Our monetary shock series is equal to the OLS point estimates of monetary shocks as implied by the Gertler & Karadi SVAR-IV, estimated only for scheduled FOMC meetings. We use data from 1980Q3 to 2015Q3, and estimate the impulse response functions using a recursive VAR with the identified shock ordered first (Plagborg-Møller & Wolf, 2021).

B.2 Initial asset positions and revaluation effects

At date 0, news arrives that changes the returns on capital, long-term bonds, and capital bonds away from the steady state return. To allocate these revaluation effects across households we need to know their steady state gross positions in these three asset classes. Our approach is to impute these as functions of household net worth based on data from the 2019

[31]By the results in McKay & Wolf (2023), the output of this procedure is independent of the baseline policy rule, as long as it induces a determinate, sunspot-free equilibrium. Our choice of baseline rule is thus a purely computational device.

[32]In Figure B.2 we plot $W \equiv \log(PW/PY) + \log(Y)$, where PW is nominal net worth, PY is nominal GDP and Y is real GDP.

SCF. Auclert & Rognlie (2018) use a related approach.

To give an overview, for each household in the SCF we do a calculation that is similar to the one reported in Table 2. We take the "real world" asset classes and map them into model assets. We then create fine bins of households by net worth and compute average portfolios at different levels of net worth. We then fit piece-wise polynomials through these functions of net worth and use these to impute the portfolios of the households in the model according to their net worth.

In our model, all capital holdings are liquid while in reality consumer durables and real estate is illiquid. We assume that all households with negative net worth simply have short-term debt and no long-term debt or capital. As these households tend to be liquidity constrained in our model, this assumption avoids counterfactually strong spending responses out of changes in asset values for these households. What follows then applies to households with positive net worth.

Turning to the details, we use the summary extract of the SCF produced by the Federal Reserve and the variable names here refer to the variable names there. Define $equity = stocks + stmutf + 0.5 * comutf + bus$, where we assume combination and other mutual funds are 50-50 stocks and bonds. Note that *equity* includes both public and private equity. Define $mortgage = nh\_mort + resdbt + heloc$. Define $capital = (nfin - bus + 1.32 \times equity)$, where 1.32 is the leverage ratio for equity and $nfin - bus$ is real estate and consumer durables (non-financial assets less business holdings). Define short term bond holdings as $short = liq + cds - ccbal - install - 0.38 * 0.32 \times equity - 0.5 * mortgage)$. This sums cash-like accounts less credit card and installment loans. We then subtract off the short-term debt associated with the leveraged position in capital[33] and half of the mortgage balance. Finally, we define $long = bond + tfbmutf + gbmutf + obmutf + 0.5 * comutf - 0.62 * 0.32 * equity - 0.5 * mortgage$. We rescale all dollar values so that average household income is 1. We then take averages within fine bins of net worth. We then perform a piecewise regression of the portfolios on a quadratic function of net worth. We perform this regression separately for households with net worth above and below one year's average income.

Like the Distributional Financial Accounts, we use the Financial Accounts for aggregate quantities of each asset. Therefore, the total stocks of capital, short-term bonds, and long-term bonds are the totals from Table 2. Our imputation procedure will not exactly match these totals both because the SCF does not aggregate to the same totals as the Financial Ac-

[33] Every dollar of equity is 1.32 dollars of capital and -0.32 dollars of bonds. Those bonds are 62% long-term and 38% short term.

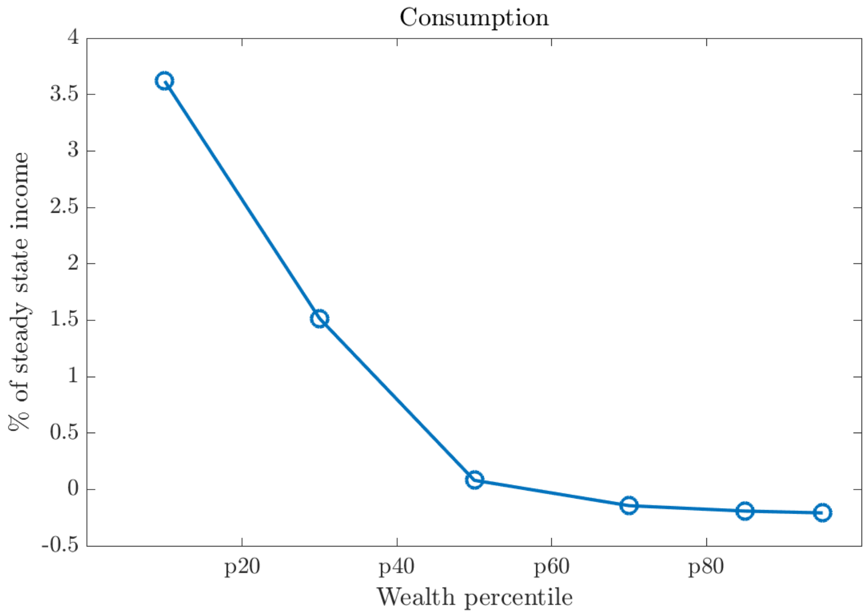


Figure B.3: Initial response of consumption to a \$500 fiscal stimulus payment across the income distribution. The figure includes general equilibrium effects where monetary policy is assumed to follow a simple interest rate rule $i_t = \frac{3}{2}\pi_t + \frac{1}{8}y_t$.

counts and because our model-generated distribution of net worth does not perfectly match the SCF distribution of net worth. We therefore rescale the bond positions to match the total stocks of these assets. We then set capital as net worth less bond holdings. The resulting distribution of capital holdings aggregates to the correct total because in the stationary equilibrium of the economy the total holdings of assets matches the total from Table 2 and if we have the correct total bond holdings we residually have the correct total capital holdings.

B.3 Additional optimal policy results

We here collect some figures that supplement the quantitative optimal policy analysis presented in Section 5.4.

Distributional effects of stimulus checks. Stimulus checks and interest rate cuts have the same effects on aggregate output and inflation (Wolf, 2021), but they differ substantially in their cross-sectional incidence. Figure 2 in the main text shows the model-implied incidence profile of interest rate cuts; Figure B.3 here does the same for stimulus checks. As expected, the incidence profile here is strongly downward-sloping: the consumption of poor households responds the most, reflecting their high MPCs and low overall income.

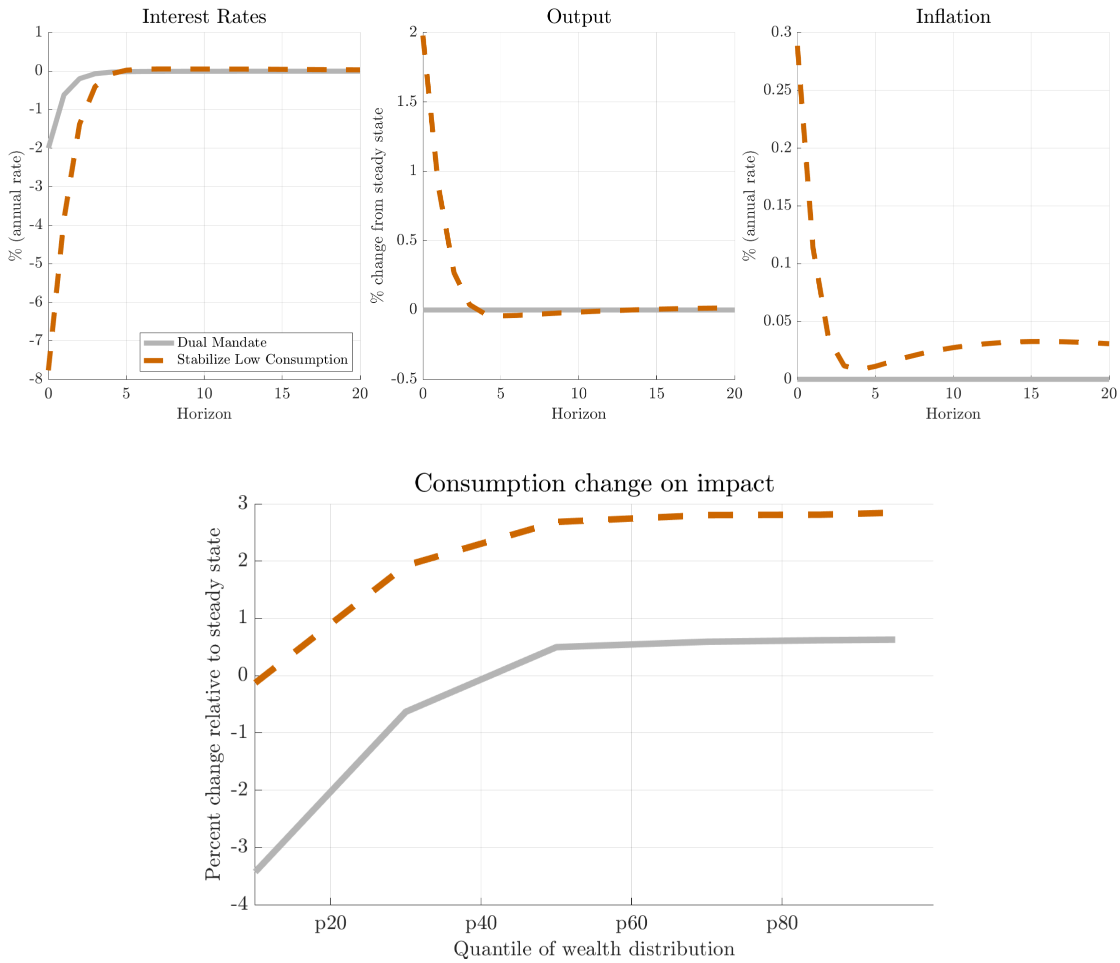


Figure B.4: Simulation of income distribution shock using ad hoc policy rule that stabilizes the consumption of households with low wealth and low income. The bottom panel shows the consumption of wealthy households increases substantially under this rule leading output and inflation to exceed their targets.

STABILIZING CONSUMPTION AT THE BOTTOM. What happens if monetary policy is used to stabilize consumption of poor households in the face of our distributional shock m_t? In Figure B.4 we plot the implications of following a monetary policy targeting rule that adds a distributional consideration to the dual mandate targeting rule that seeks to stabilize the consumption of households with low wealth and low income.[34] As the figure shows, output and inflation exceed the dual-mandate targets and the consumption of high-wealth households is substantially above the steady state level.

[34]The targeting rule is $\hat{\pi}_t + \frac{1}{\eta}\left(\hat{y}_t - \hat{y}_{t-1}\right) + \frac{1}{2}\hat{c}_t^{\text{Low}} = 0$, where c_t^{Low} is the aggregated consumption of households with income in the bottom third of the income distribution and wealth below the median.

B.4 Comparison with Bhandari et al. (2021)

In this section we explore further the reasons for why our conclusions on optimal Ramsey monetary policy following a cost-push shock differ from those of Bhandari et al. (2021). We argue that the differences in policy conclusions are due to differences in the economic environment. To do so we proceed in two steps. First, we present a model closer to that of Bhandari et al.. Second, we repeat our optimal policy analysis in that environment.

Environment. We alter our baseline model to arrive at the following alternative environment, designed to mimic key properties of the model of Bhandari et al.. The changes to the environment are as follows:

- *Profits.* We now assume that profits are distributed to households as a function of their histories of idiosyncratic shocks. In particular, a household receives a share of their profits equal to the steady state wealth share of a household with that history of shocks. These claims on future profits are not traded.
- *Other assets.* The alternative model omits capital in production and includes only short-term nominal bonds, so there are no traded long-term assets. As we omit capital, we reduce the total supply of assets, which we now set equal to the steady-state supply of bonds in Bhandari et al..
- *Income process.* With fewer assets, our income process—which was calibrated to match the wealth distribution in our full model—now produces far too many borrowing constrained agents. Therefore we use a simpler income process without super-star earners.
- *Business-cycle incidence.* The alternative model omits the unequal incidence of the aggregate business cycle across levels of labor income, again in keeping with the baseline model in Bhandari et al..

Results. We begin by showing that, in this alternative model, monetary policy has strong distributional effects. Mirroring Figure 2 from our main analysis, Figure B.5 displays the initial consumption response to an expansionary monetary policy shock across levels of wealth. We see that, in the alternative environment, there is a clear difference in shock incidence across wealth levels, with low-wealth households increasing consumption strongly and high-wealth households leaving consumption unchanged. This pattern stands in sharp contrast to Figure 2, where consumption increases evenly across wealth levels.

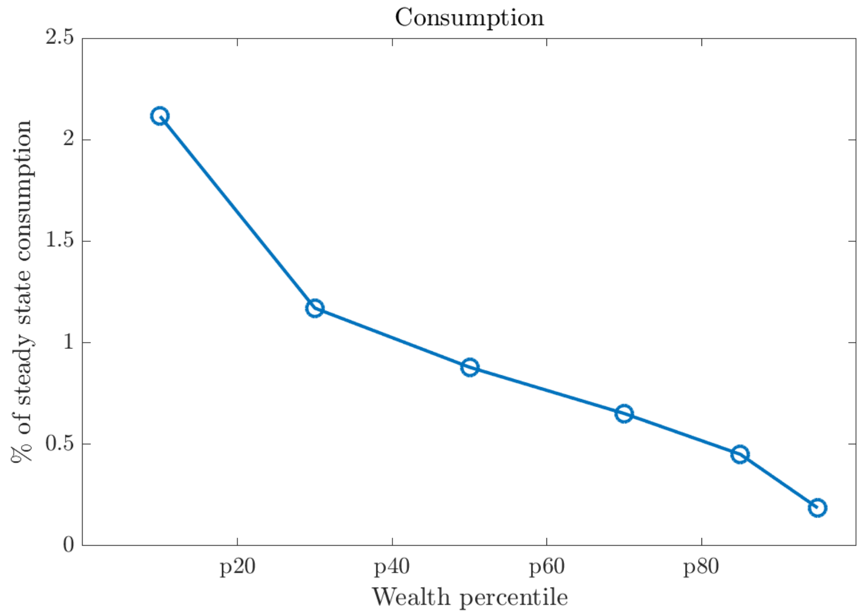
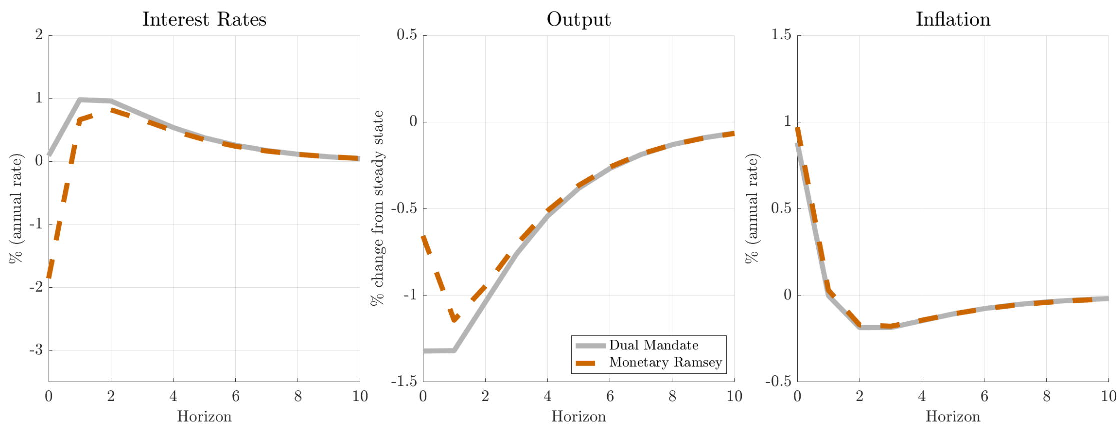


Figure B.5: Initial response of consumption to an expansionary monetary policy shock across the distribution of wealth in the alternative model described in Appendix B.4.

Next, Figure B.6 shows the optimal policy response to an inflationary cost-push shock in the alternative model. Crucially, and mirroring the findings of Bhandari et al., we see that here short-term interest rates *fall* on impact following the inflationary shock. Overall, the policy response is thus notably more accommodative than the dual mandate policy, with a shallower fall in output and —given our flat Phillips curve—a modest increase in inflation. In the lower panel we see why the Ramsey policymaker finds such an aggressive rate cut optimal: the cost push shock has a distinct distributional gradient, with high-wealth households increasing consumption and low-wealth households decreasing consumption, and a rate cut can mitigate this distributional gradient relative to the dual-mandate outcome.



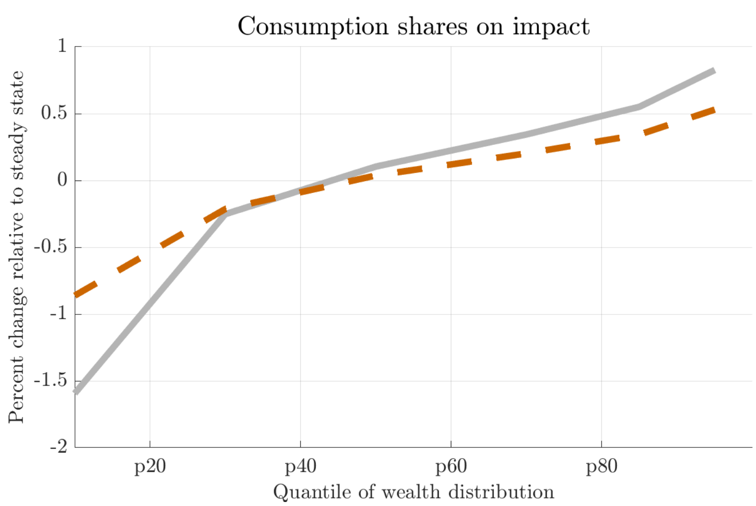


Figure B.6: Optimal policy response to an inflationary cost-push shock in the alternative model described in Appendix B.4.

C Computational appendix

This appendix provides supplementary information on our computational approach. We compute sequence-space transition paths using the methodology developed in Auclert et al. (2021), as discussed further in Appendix C.1. Our computation of the inequality term in the full Ramsey loss function is described in Appendix C.2.

C.1 General equilibrium transition paths

The baseline constraints (2) in our linear-quadratic policy problem in Section 2 are expressed in sequence space. To compute the corresponding constraints (21) - (22) for our "HANK" model, we thus follow the computational techniques of Auclert et al. (2021) to compute the required sequence-space Jacobian matrices. In particular, our computation of the $\boldsymbol{\mathcal{C}}_{\bullet}$ maps in the augmented HANK "IS curve" (22) leverages the so-called "fake news" algorithm.

For computation of the alternative (but equivalent) constraint formulations (25) - (36) in impulse response space, we follow McKay & Wolf (2023) and proceed as follows. First, we close the model with arbitrary policy rules for the two instruments $\boldsymbol{i}$ and $\boldsymbol{\tau}_x$, subject only to the requirement that the two rules induce a unique equilibrium. We then compute impulse responses of all policy targets to the full menu of contemporaneous and news shocks to those two policy rules. Now denote the impulse response matrix of some variable x_i to shocks to the rule for instrument z_j by $\tilde{\Theta}_{x_i,z_j}$, and similarly write $\tilde{\Theta}_{x_i,\varepsilon_q}$ for responses to non-policy shocks ε_q under the (arbitrary) baseline rule. Finally write $\tilde{\Theta}_{z_j,z_j}$ for the impulse response matrix of the instrument itself. We then define $\Theta_{x_i,z_j} \equiv \tilde{\Theta}_{x_i,z_j}\tilde{\Theta}_{z_j,z_j}^{-1}$ and similarly $\Theta_{x_i,\varepsilon_q} \equiv \tilde{\Theta}_{x_i,\varepsilon_q}\tilde{\Theta}_{z_j,z_j}^{-1}$. The results in McKay & Wolf (2023) imply that the resulting impulse responses are independent of the chosen baseline policy rule.

C.2 Inequality term

The inequality term says that the planner would like to stabilize a very large number of targets—one for each history ζ. Both for intuition and for computation, it is useful to observe that these consumption shares will only fluctuate if the inputs to the household's decision problem fluctuate. By our discussion of the consumption-savings problem in Section 3.1, those inputs include total labor income (equal to $\boldsymbol{y}$), the return on savings ($\boldsymbol{r}$), transfers ($\boldsymbol{\tau}_x$ and $\boldsymbol{\tau}_e$), the inequality shock ($\boldsymbol{m}$), the initial inflation rate π_0, and the initial asset prices q_0^k and q_0^b. In this section we leverage this insight to recast the problem of stabilizing consumption shares as one of stabilizing the inputs to the household consumption-savings decision, thus giving the representation in (37), couched in terms of a small number of aggregates rather than a distribution of histories.

Reformulating the inequality term. Let $\boldsymbol{x} = (\boldsymbol{y}, \boldsymbol{r}, \boldsymbol{\tau}_x, \boldsymbol{\tau}_{\boldsymbol{e}}, \boldsymbol{m}, \pi_0, q_0^k, q_0^b)$ be the stacked sequences of inputs to the household problem. Our goal is to show that there is symmetric matrix Q such that

$$\sum_{t=0}^{\infty} \beta^t \int \frac{\widehat{\omega}_t(\zeta, \boldsymbol{x})^2}{\bar{\omega}(\zeta)} d\Gamma(\zeta) = \widehat{\boldsymbol{x}}' Q \widehat{\boldsymbol{x}} + \mathcal{O}(||\widehat{\boldsymbol{x}}||^3),$$

where here we have been explicit that the consumption shares at date t depend on the full sequences of inputs $\boldsymbol{x}$. To arrive at this representation, consider a first-order approximation to the time-t consumption share of individuals with history ζ:

$$\widehat{\omega}_t(\zeta, \boldsymbol{x}) \approx \Omega_t(\zeta) \widehat{\boldsymbol{x}}$$

where the derivative $\Omega_t(\zeta)$ will be defined formally below. This yields

$$\frac{\widehat{\omega}_t(\zeta^t, \boldsymbol{x})^2}{\bar{\omega}(\zeta^t)} = \widehat{\boldsymbol{x}}' \underbrace{\frac{\Omega_t(\zeta^t)' \Omega_t(\zeta^t)}{\bar{\omega}(\zeta^t)}}_{\equiv Q_t(\zeta^t)} \widehat{\boldsymbol{x}} + \mathcal{O}(||\widehat{\boldsymbol{x}}||^3).$$

We then integrate across histories and take the discounted sum across time to arrive at

$$\sum_{t=0}^{\infty} \beta^t \int \frac{\widehat{\omega}_t(\zeta^t, \boldsymbol{x})^2}{\bar{\omega}(\zeta^t)} d\Gamma(\zeta^t) = \widehat{\boldsymbol{x}}' \underbrace{\left(\sum_{t=0}^{\infty} \beta^t \int Q_t(\zeta^t) d\Gamma(\zeta^t) \right)}_{\equiv Q} \widehat{\boldsymbol{x}} + \mathcal{O}(||\widehat{\boldsymbol{x}}||^3),$$

giving the desired representation. We have thus arrived at a representation that almost fits into our general linear-quadratic set-up of Section 2.1, the sole difference being that the objective function features a non-diagonal quadratic form. To extend our optimal policy results to this more general case, we consider the same problem as in Section 2.1, but replacing the diagonal loss function (1) by the more general (non-diagonal) expression

$$\mathcal{L} \equiv \frac{1}{2} \boldsymbol{x}' P \boldsymbol{x}, \tag{C.1}$$

The corresponding necessary and sufficient first-order conditions yield the more general optimal policy targeting rule

$$\Theta'_{xz} P \boldsymbol{x} = 0$$

and the optimal instrument path

$$\boldsymbol{z}^* \equiv -\left(\Theta'_{x,z} P \Theta_{x,z}\right)^{-1} \times \left(\Theta'_{x,z} P \Theta_{x,\varepsilon} \cdot \boldsymbol{\varepsilon}\right)$$

Evolution of consumption shares. We now explain the first-order approximation of consumption shares

$$\widehat{\omega}_t(\zeta^t) \approx \Omega_t(\zeta^t)\boldsymbol{x}$$

that we used above. Let $c_t(\zeta^t, \boldsymbol{x})$ be the consumption in date t after history ζ^t with the input sequences given by $\boldsymbol{x}$, and similarly let $a_{t+1}(\zeta^t, \boldsymbol{x})$ be the savings chosen in date t. Also let ζ_t be the date-t value of the idiosyncratic state. Using the standard recursive representation of the household's problem, we can write these choices in terms policy functions, f and g, that take as their arguments assets $a_t(\zeta^{t-1})$ and the current shock ζ_t, so we have

$$c_t(\zeta^t, \boldsymbol{x}) = f_t\left(a_t(\zeta^{t-1}, \boldsymbol{x}), \zeta_t, \boldsymbol{x}\right) \tag{C.2}$$

$$a_{t+1}(\zeta^t, \boldsymbol{x}) = g_t\left(a_t(\zeta^{t-1}, \boldsymbol{x}), \zeta_t, \boldsymbol{x}\right). \tag{C.3}$$

We now consider a first-order approximation to f and g around $\boldsymbol{x} = \bar{\boldsymbol{x}}$:

$$\begin{aligned} c_t(\zeta^t, \boldsymbol{x}) &\approx \bar{c}(\zeta^t) + \frac{dc_t(\zeta^t, \boldsymbol{x})}{d\boldsymbol{x}} \widehat{\boldsymbol{x}} \\ a_{t+1}(\zeta^t, \boldsymbol{x}) &\approx \bar{a}(\zeta^t) + \frac{da_{t+1}(\zeta^t, \bar{\boldsymbol{x}})}{d\boldsymbol{x}} \widehat{\boldsymbol{x}}. \end{aligned}$$

The derivatives that appear here are total derivatives with respect to $\boldsymbol{x}$, including both the effect on the policy rule at date t and the effect on assets $a_t(\zeta^{t-1}, \boldsymbol{x})$. The derivatives are evaluated at the steady-state inputs $\bar{\boldsymbol{x}}$ and the level of assets that an individual with history ζ^t would have if the inputs x remained at steady state forever, which we denote by $\bar{a}(\zeta^{t-1})$. To calculate these derivatives, we differentiate (C.2) and (C.3):

$$\frac{dc_t(\zeta^t, \bar{\boldsymbol{x}})}{d\boldsymbol{x}} = \frac{\partial f_t\left(\bar{a}_t(\zeta^{t-1}), \zeta_t, \bar{\boldsymbol{x}}\right)}{\partial a} \frac{da_t(\zeta^{t-1}, \bar{\boldsymbol{x}})}{d\boldsymbol{x}} + \frac{\partial f_t\left(\bar{a}_t(\zeta^{t-1}), \zeta_t, \bar{\boldsymbol{x}}\right)}{\partial \boldsymbol{x}} \tag{C.4}$$

$$\frac{da_{t+1}(\zeta^t, \bar{\boldsymbol{x}})}{d\boldsymbol{x}} = \frac{\partial g_t\left(\bar{a}_t(\zeta^{t-1}), \zeta_t, \bar{\boldsymbol{x}}\right)}{\partial a} \frac{da_t(\zeta^{t-1}, \bar{\boldsymbol{x}})}{d\boldsymbol{x}} + \frac{\partial g_t\left(\bar{a}_t(\zeta^{t-1}), \zeta_t, \bar{\boldsymbol{x}}\right)}{\partial \boldsymbol{x}}. \tag{C.5}$$

The partial derivative $\frac{\partial f_t\left(\bar{a}_t(\zeta^{t-1}),\zeta_t,\bar{\boldsymbol{x}}\right)}{\partial a}$ is the marginal propensity to consume for an individual with states $(\bar{a}_t(\zeta^{t-1}),\zeta_t)$ in the stationary equilibrium, and $\frac{\partial g_t\left(\bar{a}_t(\zeta^{t-1}),\zeta_t,\bar{\boldsymbol{x}}\right)}{\partial a}$ is the marginal propensity to save. Similarly, the partial derivative $\frac{\partial f_t\left(\bar{a}_t(\zeta^{t-1}),\zeta_t,\bar{\boldsymbol{x}}\right)}{\partial \boldsymbol{x}}$ is the derivative of the consumption policy rule with respect to the input sequences for an individual with the same states, and $\frac{\partial g_t\left(\bar{a}_t(\zeta^{t-1}),\zeta_t,\bar{\boldsymbol{x}}\right)}{\partial \boldsymbol{x}}$ is the analogous derivative of the savings policy rule.

We discuss below how to compute these derivatives. Given that they have been recovered, it remains to move from consumption *levels* to consumption *shares*:

$$\omega_t(\zeta^t,\boldsymbol{x}) \equiv \frac{c_t(\zeta^t,\boldsymbol{x})}{\int c_t(\zeta^t,\boldsymbol{x})d\Gamma_t(\zeta)} \approx \bar{\omega}_t(\zeta^t) + \frac{1}{\bar{c}}\frac{dc_t(\zeta^t,\boldsymbol{x})}{d\boldsymbol{x}}\widehat{\boldsymbol{x}} - \int \frac{\bar{c}(\zeta^t)}{\bar{c}^2}\frac{dc_t(\zeta^t,\boldsymbol{x})}{d\boldsymbol{x}}d\Gamma(\zeta^t)\widehat{\boldsymbol{x}}.$$

$\Omega_t(\zeta^t)$ is then given by

$$\Omega_t(\zeta^t) \equiv \frac{1}{\bar{c}}\frac{dc_t(\zeta^t,\boldsymbol{x})}{d\boldsymbol{x}} - \int \frac{\bar{c}(\zeta^t)}{\bar{c}^2}\frac{dc_t(\zeta^t,\boldsymbol{x})}{d\boldsymbol{x}}d\Gamma(\zeta^t). \tag{C.6}$$

Computing Q. To compute $\Omega_t(\zeta^t)$ and so Q, the key challenge is to arrive at the derivatives in (C.4) - (C.5). To do so we begin by simulating a history ζ^t for $t = 0, 1, ..., T$ in a stationary equilibrium (i.e. with $\boldsymbol{x} = \bar{\boldsymbol{x}}$). At each date along this simulation, we recover the required partial derivatives as follows. The marginal propensities to consume and save can be computed by standard methods. For the derivatives of the policy rules, we use the fact that the derivatives with respect to past prices are zero and the derivatives with respect to current and future prices only depends on the number of periods until the price change occurs. This allows us to compute all the derivatives by perturbing prices at a single date and iterating backwards in time using a single loop from T to 0 (see Auclert et al., 2021). With the partial derivatives in hand, we then construct $\frac{dc_t(\zeta^t,\bar{\boldsymbol{x}})}{d\boldsymbol{x}}$ and $\frac{da_{t+1}(\zeta^t,\bar{\boldsymbol{x}})}{d\boldsymbol{x}}$ by iterating (C.4)-(C.5) forward starting with $\frac{da_0(\zeta^{-1},\bar{\boldsymbol{x}})}{d\boldsymbol{x}}$ given by the change in the households assets with the initial asset prices (the revaluation effects). Given those derivatives, we can recover $\Omega_t(\zeta)$ and thus get $Q_t(\zeta^t)$ as well as Q itself.

Impulse response representation. Using (C.4), (C.5), and (C.6), we can write $\omega_t(\zeta)$ as a linear function of the aggregate variables contained in $\boldsymbol{x}$. Since each $\omega_t(\zeta)$ is linearly related to $\boldsymbol{x}$, and since we can recover the entries of $\boldsymbol{x}$ as a function of $(\boldsymbol{y},\boldsymbol{\pi},\boldsymbol{i},\boldsymbol{\tau}_x,\boldsymbol{m})$ (by Lemma 1), we recover the impulse response representation in (35).

D Proofs and auxiliary lemmas

D.1 Proof of Lemma 1

We begin the proof by re-stating and slightly simplifying the definition of an equilibrium in Definition 1. We first repeat the Phillips curve in stacked form:

$$\Pi_\pi \widehat{\boldsymbol{\pi}} = \Pi_y \widehat{\boldsymbol{y}} + \psi \boldsymbol{\eta}, \tag{D.1}$$

where

$$\Pi_\pi = \begin{pmatrix} 1 & -\beta & 0 & \cdots \\ 0 & 1 & -\beta & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad \Pi_y = \kappa I.$$

Turning to the demand side, we re-write (18) and goods market clearing as

$$\widehat{\boldsymbol{y}} = \frac{\bar{c}}{\bar{y}} \left[(\mathcal{C}_y + \mathcal{C}_\Psi \mathcal{H}_y e_1') \widehat{\boldsymbol{y}} + \mathcal{C}_r \widehat{\boldsymbol{r}} + \mathcal{C}_x \widehat{\boldsymbol{\tau}}_x + \mathcal{C}_e \widehat{\boldsymbol{\tau}}_e + \mathcal{C}_m \widehat{\boldsymbol{m}} + \mathcal{C}_\Psi \left(\mathcal{H}_\pi e_1' \widehat{\boldsymbol{\pi}} + \mathcal{H}_b e_1' \widehat{\boldsymbol{q}}^b + \mathcal{H}_k e_1' \widehat{\boldsymbol{q}}^k \right) \right], \tag{D.2}$$

where $e_1 = (1, 0, 0, \dots)'$ and we have used that $\widehat{y}_t = (\bar{c}/\bar{y})\widehat{c}_t$. Next, using (14), (15), and (16), we write the relationships between asset prices and rates of return as

$$\widehat{\boldsymbol{r}} = r(\widehat{\boldsymbol{i}}, \widehat{\boldsymbol{\pi}}) \tag{D.3}$$

$$\widehat{\boldsymbol{q}^b} = q^b(\widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{r}}) \tag{D.4}$$

$$\widehat{\boldsymbol{q}^k} = q^k(\widehat{\boldsymbol{y}}, \widehat{\boldsymbol{r}}) \tag{D.5}$$

Finally, we combine the government budget constraint (19) and the law of motion for government debt (20) and solve for $\tau_{e,t}$ to obtain

$$\widehat{\boldsymbol{\tau}}_e = \tau_e(\widehat{\boldsymbol{y}}, \widehat{\boldsymbol{\tau}}_x, \widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{r}}). \tag{D.6}$$

Our first auxiliary result is that, given the shocks $(\boldsymbol{m}, \boldsymbol{\eta})$ and policy choices $(\boldsymbol{i}, \boldsymbol{\tau}_x)$, a list $(\boldsymbol{y}, \boldsymbol{r}, \boldsymbol{\tau}_e, \boldsymbol{\pi}, \boldsymbol{q})$ is part of an equilibrium if and only if (D.1) - (D.6) hold. To show this we need to check the conditions of Definition 1: requirement 1 and goods market clearing hold by the construction of (D.2); requirement 3 is re-stated in (D.1); requirement 6 is imposed by (D.6); requirement 5 is imposed by (D.3)-(D.5); and asset market clearing holds by Walras' Law.

Finally, to complete the equilibrium let $\widehat{w}$ and $\widehat{\ell}$ be determined by (10) and the linearized version of (13) so requirements 2 and 4 hold by construction.

We now simplify this characterization of equilibria further to arrive at Lemma 1. The key step in the argument is to solve out the asset-pricing and government financing rules and plug them into (D.2). First, given $\widehat{\boldsymbol{\pi}}$ and $\widehat{\boldsymbol{i}}$, we can recover $\widehat{\boldsymbol{r}}$ from (D.3). We can recover $\widehat{\boldsymbol{q}}^b$ from (D.4) and $\widehat{\boldsymbol{q}}^k$ from (D.5). Second, given $\widehat{\boldsymbol{y}}$, $\widehat{\boldsymbol{\tau}}_x$, $\widehat{\boldsymbol{\pi}}$ and $\widehat{\boldsymbol{q}}^b$, we can recover $\widehat{\boldsymbol{\tau}}_e$ from (D.6). We can thus write

$$\begin{aligned}\widehat{\boldsymbol{y}} = \frac{\bar{c}}{\bar{y}}\Big[&(\mathcal{C}_y + \mathcal{C}_\Psi \mathcal{H}_y e_1')\widehat{\boldsymbol{y}} + \mathcal{C}_r r(\widehat{\boldsymbol{i}},\widehat{\boldsymbol{\pi}}) + \mathcal{C}_x \widehat{\boldsymbol{\tau}}_x + \mathcal{C}_e \tau_e(\widehat{\boldsymbol{y}},\widehat{\boldsymbol{\tau}}_x,\widehat{\boldsymbol{\pi}},\widehat{\boldsymbol{i}}) \\ &+\mathcal{C}_m \widehat{\boldsymbol{m}} + \mathcal{C}_\Psi\Big(\mathcal{H}_\pi e_1'\widehat{\boldsymbol{\pi}} + \mathcal{H}_b e_1' q^b(\widehat{\boldsymbol{\pi}},\widehat{\boldsymbol{i}}) + \mathcal{H}_k e_1' q^k(\widehat{\boldsymbol{y}},\widehat{\boldsymbol{i}},\widehat{\boldsymbol{\pi}})\Big)\Big],\end{aligned}$$

and so

$$\begin{aligned}\widehat{\boldsymbol{y}} =& \underbrace{\frac{\bar{c}}{\bar{y}}\left[\mathcal{C}_y + \mathcal{C}_\Psi\left(\mathcal{H}_y e_1' + \mathcal{H}_k e_1' \mathcal{Q}_y^k\right) + \mathcal{C}_e \mathcal{T}_y\right]}_{\tilde{\mathcal{C}}_y}\widehat{\boldsymbol{y}} + \underbrace{\frac{\bar{c}}{\bar{y}}\left[\mathcal{C}_r \mathcal{R}_\pi + \mathcal{C}_e \mathcal{T}_\pi + \mathcal{C}_\Psi\left(\mathcal{H}_\pi e_1' + \mathcal{H}_b e_1' \mathcal{Q}_\pi^b + \mathcal{H}_k e_1' \mathcal{Q}_\pi^k\right)\right]}_{\tilde{\mathcal{C}}_\pi}\widehat{\boldsymbol{\pi}} \\ &+ \underbrace{\frac{\bar{c}}{\bar{y}}\left[\mathcal{C}_r \mathcal{R}_i + \mathcal{C}_e \mathcal{T}_i + \mathcal{C}_\Psi\left(\mathcal{H}_b e_1' \mathcal{Q}_i^b + \mathcal{H}_k e_1' \mathcal{Q}_i^k\right)\right]}_{\tilde{\mathcal{C}}_i}\widehat{\boldsymbol{i}} + \underbrace{\frac{\bar{c}}{\bar{y}}\left[\mathcal{C}_x + \mathcal{C}_e \mathcal{T}_x\right]}_{\tilde{\mathcal{C}}_x}\widehat{\boldsymbol{\tau}}_x + \frac{\bar{c}}{\bar{y}}\mathcal{C}_m \boldsymbol{m}\end{aligned} \tag{D.7}$$

where $\mathcal{T}_\bullet$, $\mathcal{R}_\bullet$, $\mathcal{Q}^b_\bullet$, and $\mathcal{Q}^k_\bullet$ are derivative matrices for $\widehat{\boldsymbol{r}}(\bullet)$, $\widehat{\boldsymbol{\tau}}_e(\bullet)$, $\widehat{\boldsymbol{q}}^b(\bullet)$, and $\widehat{\boldsymbol{q}}^k(\bullet)$. (D.7) embeds (14), (15), (16), and (D.6). We have thus reduced the equilibrium characterization from statements about $(\boldsymbol{y},\boldsymbol{r},\boldsymbol{\tau}_e,\boldsymbol{\pi},\boldsymbol{q}^b,\boldsymbol{q}^k)$ to statements about $(\boldsymbol{y},\boldsymbol{\pi})$, establishing the claim.

□

D.2 Proof of Proposition 1

In light of Lemma 1, we can re-state the optimal policy problem as minimizing (24) subject to the two constraints (D.1) and (D.7). This problem gives the following necessary and sufficient first-order conditions:

$$\lambda_\pi W\widehat{\boldsymbol{\pi}} + \Pi_\pi' W\boldsymbol{\varphi}_\pi - \tilde{\mathcal{C}}_\pi' W\boldsymbol{\varphi}_y = \mathbf{0} \tag{D.8}$$

$$\lambda_y W\widehat{\boldsymbol{y}} - \Pi_y' W\boldsymbol{\varphi}_\pi + (I - \tilde{\mathcal{C}}_y')W\boldsymbol{\varphi}_y = \mathbf{0} \tag{D.9}$$

$$-\tilde{\mathcal{C}}_i' W\boldsymbol{\varphi}_y = \mathbf{0}, \tag{D.10}$$

where $\boldsymbol{\varphi}_\pi$ and $\boldsymbol{\varphi}_y$ are sequences of Lagrange multipliers on the two constraints. The proof of Proposition 1 proceeds by guessing (and then verifying) that $\boldsymbol{\varphi}_y = \mathbf{0}$. Under this assumption, we can combine (D.8) - (D.9) to get

$$\lambda_\pi \widehat{\boldsymbol{\pi}} + \lambda_y W^{-1} \Pi'_\pi (\Pi'_y)^{-1} W \widehat{\boldsymbol{y}} = \mathbf{0}$$

It is straightforward to verify that this can be re-written as

$$\lambda_\pi \widehat{\boldsymbol{\pi}} + \frac{\lambda_y}{\kappa} \begin{pmatrix} 1 & 0 & 0 & \dots \\ -1 & 1 & 0 & \dots \\ 0 & -1 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \widehat{\boldsymbol{y}} = \mathbf{0} \tag{D.11}$$

But this is just (28), with the conclusion following for $t = 0$ since $\widehat{y}_{-1} = 0$ (as the economy starts from steady state). It now remains to verify the guess that $\boldsymbol{\varphi}_y = \mathbf{0}$. For this, consider some arbitrary $(\boldsymbol{m}, \boldsymbol{\eta})$, and let $(\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*)$ denote the solution of the system (D.1) and (D.11) given $(\boldsymbol{m}, \boldsymbol{\eta})$. It remains to show that condition (27) is precisely sufficient to ensure that we can always find $\widehat{\boldsymbol{i}}$ such that (D.7) holds.

Plugging into (D.7) and re-arranging:

$$\underbrace{\widehat{\boldsymbol{y}}^* - \tilde{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* - \tilde{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \mathcal{C}_m \boldsymbol{m}}_{\text{demand target}} = \tilde{\mathcal{C}}_i \widehat{\boldsymbol{i}}. \tag{D.12}$$

Note that the left-hand side of (D.12) is an excess demand term: supply $\widehat{\boldsymbol{y}}^*$ vs. demand $\tilde{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \tilde{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* + \mathcal{C}_m \boldsymbol{m}$. At this point, as long as $\tilde{\mathcal{C}}_i$ is invertible we could stop and say that whatever the demand target, there is a $\widehat{\boldsymbol{i}}$ that satisfies (D.12). Invertibility of $\tilde{\mathcal{C}}_i$ is too restrictive, however, as we can say that the demand target will satisfy certain conditions so $\tilde{\mathcal{C}}_i \widehat{\boldsymbol{i}}$ does not need to be able to generate any possible sequence.

By construction of the $\widetilde{C}$ function, $\widetilde{\mathcal{C}}_\pi$, $\widetilde{\mathcal{C}}_i$, and $\widetilde{\mathcal{C}}_m$ map into zero-NPV sequences and $\widetilde{\mathcal{C}}_y$ preserves the NPV of $\widehat{\boldsymbol{y}}$. So equation (D.12) has a particular structure: both sides are zero-NPV sequences. Provided that all zero-NPV sequences are in the image of $\widetilde{\mathcal{C}}_i$ there exists a $\widehat{\boldsymbol{i}}$ that satisfies (D.12). □

D.3 Proof of Proposition 2

Let U_t denote the time-t flow utility of the Ramsey planner. To derive the second-order approximation to the social welfare function, it is convenient to begin by writing U_t in terms of log deviations of c_t and ℓ_t from steady state:

$$U_t = \int \varphi(\zeta) \frac{\left(\bar{c} e^{\widehat{c}_t} \omega_t(\zeta)\right)^{1-\gamma} - 1}{1-\gamma} d\Gamma(\zeta) - \nu \left(\bar{\ell} e^{\widehat{\ell}_t}\right). \tag{D.13}$$

Our objective is to construct a second-order approximation of (D.13). Similar to the analysis in Woodford (2003), our strategy is to consider an efficient steady state, allowing evaluation of Equation (D.13) to second order using only a first-order approximation of aggregate equilibrium dynamics.

Optimality of the steady state requires that the weighted marginal utility of consumption is equalized across histories:

$$\varphi(\zeta)\bar{c}^{1-\gamma}\bar{\omega}(\zeta)^{-\gamma} = \bar{u}_c \bar{c} \qquad \forall \zeta$$

for some constant $\bar{u}_c$. Rearranging, we can write this as

$$\varphi(\zeta)^{1/\gamma} = \bar{c}\bar{\omega}(\zeta)\bar{u}_c^{1/\gamma} \qquad \forall \zeta$$

Furthermore imposing that consumption shares integrate to 1 yields

$$\int \varphi(\zeta)^{1/\gamma} d\Gamma(\zeta) = \bar{c}\bar{u}_c^{1/\gamma}. \tag{D.14}$$

Combining the previous two equations, we can recover consumption shares as a function of planner weights:

$$\bar{\omega}(\zeta) = \frac{\varphi(\zeta)^{1/\gamma}}{\int \varphi(\zeta)^{1/\gamma} d\Gamma(\zeta)} \qquad \forall \zeta$$

For future reference it will furthermore be useful to define

$$\Xi \equiv \left(\int \varphi(\zeta)^{1/\gamma} d\Gamma(\zeta)\right)^{\gamma} = \varphi(\zeta)\bar{\omega}(\zeta)^{-\gamma} \qquad \forall \zeta. \tag{D.15}$$

With these preliminary definitions out of the way, we can begin constructing the second-order approximation of (D.13). Differentiating U_t with respect to $\widehat{c}_t$, we find that

$$\begin{aligned}\frac{\partial U}{\partial \widehat{c}_t} &= \int \varphi(\zeta)(\bar{c}\bar{\omega}(\zeta))^{1-\gamma} d\Gamma(\zeta) \\ &= \bar{c}^{1-\gamma}\Xi\end{aligned}$$

where the second line follows from the definition of Ξ and some algebra. Notice that the definition of Ξ and (D.14) together imply that $\Xi = \bar{u}_c/\bar{c}^{-\gamma}$ so Ξ is the ratio of the (common) marginal utility of consumption as evaluated by the planner and the marginal utility of aggregate consumption used by the labor union to value income gains. Next we have

$$\frac{\partial U}{\partial \widehat{\ell}_t} = -\nu_\ell(\bar{\ell})\bar{\ell}.$$

The total level of production in steady state is efficient by virtue of the assumed production subsidy. To see this, first note that the planner's problem is to choose c and ℓ to maximizing welfare subject to $\bar{c} = Ak^\alpha \bar{\ell}^{1-\alpha} - \delta k$. This leads to the efficiency condition

$$\bar{c}^{-\gamma}\Xi\,(1-\alpha)\,Ak^\alpha \bar{\ell}^{-\alpha} = \nu_\ell(\bar{\ell}).$$

For future reference, it is useful to re-express this as

$$\bar{c}^{1-\gamma}\Xi = (1-\alpha)^{-1}\,\frac{\bar{c}}{\bar{y}}\nu_\ell(\bar{\ell})\bar{\ell}. \tag{D.16}$$

Turning to the decentralized equilibrium, it follows from the steady state versions of equations (13), (10), and (A.3)-(A.5) that $\Xi\bar{c}^{-\gamma} = \nu_\ell$. That the same efficiency condition holds.

Differentiating U_t with respect to the consumption shares $\omega_t(\zeta)$ we have

$$\begin{aligned}\frac{\partial U}{\partial \omega_t(\zeta)} &= \varphi(\zeta)\bar{c}^{1-\gamma}\bar{\omega}(\zeta)^{-\gamma} d\Gamma(\zeta) \\ &= \bar{c}^{1-\gamma}\Xi d\Gamma(\zeta)\end{aligned}$$

Turning to second order terms, we begin again with the total level and cross-sectional split of consumption. We find

$$\frac{\partial^2 U_t}{\partial \widehat{c}_t^2} = (1-\gamma)\Xi\bar{c}^{1-\gamma}$$

$$
\begin{aligned}
\frac{\partial^2 U_t}{\partial \omega_t(\zeta)^2} &= -\gamma \bar{c}^{1-\gamma} \frac{\Xi}{\bar{\omega}(\zeta)} d\Gamma(\zeta) \\
\frac{\partial^2 U_t}{\partial \widehat{c}_t \partial \omega_t(\zeta)} &= (1-\gamma)\Xi \bar{c}^{1-\gamma} d\Gamma(\zeta)
\end{aligned}
$$

For hours worked we have

$$
\frac{\partial^2 U}{\partial \widehat{\ell}_t^2} = -\nu_{\ell\ell}(\bar{\ell})\bar{\ell}^2 - \nu_\ell(\bar{\ell})\bar{\ell}
$$

We can now put everything together, giving the following second-order approximation of time-t planner utility (D.13):

$$
\begin{aligned}
U_t \approx\ & \bar{U} + \bar{c}^{1-\gamma}\Xi \widehat{c}_t - \nu_\ell(\bar{\ell})\bar{\ell}\widehat{\ell_t} \\
& + \frac{1}{2}(1-\gamma)\Xi \bar{c}^{1-\gamma}\widehat{c}_t^2 - \frac{1}{2}\left[\nu_{\ell\ell}(\bar{\ell})\bar{\ell}^2 + \nu_\ell(\bar{\ell})\bar{\ell}\right]\widehat{\ell}_t^2 - \frac{1}{2}\gamma \bar{c}^{1-\gamma}\Xi \int \frac{\widehat{\omega}(\zeta)^2}{\bar{\omega}(\zeta)} d\Gamma(\zeta) \\
& + \bar{c}^{1-\gamma}\Xi \int \widehat{\omega}_t(\zeta) d\Gamma(\zeta) + (1-\gamma)\bar{c}^{1-\gamma}\Xi \widehat{c}_t \int \widehat{\omega}_t(\zeta) d\Gamma(\zeta)
\end{aligned}
$$

Since consumption shares integrate to 1, it follows that $\int \widehat{\omega}_t(\zeta) d\Gamma(\zeta) = 0$, and so all terms in the last row are zero. We now wish to evaluate the remaining terms to second order. To begin, note that the resource constraint and production function give

$$
\widehat{c}_t = \frac{\bar{y}}{\bar{c}}\widehat{y}_t = \frac{\bar{y}}{\bar{c}}\left[(1-\alpha)\widehat{\ell}_t - \widehat{d}_t\right]
$$

$$
(1-\alpha)^{-1}\left(\frac{\bar{c}}{\bar{y}}\widehat{c}_t + \widehat{d}_t\right) = \widehat{\ell}_t.
$$

Substituting this in for $\widehat{\ell}_t$ everywhere we have

$$
\begin{aligned}
U_t \approx\ & \bar{U} + \bar{c}^{1-\gamma}\Xi \widehat{c}_t - \nu_\ell(\bar{\ell})\bar{\ell}(1-\alpha)^{-1}\left(\frac{\bar{c}}{\bar{y}}\widehat{c}_t + \widehat{d}_t\right) \\
& + \frac{1}{2}(1-\gamma)\Xi \bar{c}^{1-\gamma}\widehat{c}_t^2 - \frac{1}{2}(\phi+1)\nu_\ell(\bar{\ell})\bar{\ell}(1-\alpha)^{-2}\left(\frac{\bar{c}}{\bar{y}}\widehat{c}_t + \widehat{d}_t\right)^2 - \frac{1}{2}\gamma \bar{c}^{1-\gamma}\Xi \int \frac{\widehat{\omega}(\zeta)^2}{\bar{\omega}(\zeta)} d\Gamma(\zeta)
\end{aligned}
$$

where we have used the definition of ϕ. To simplify this expression further, impose the aggregate resource constraint $\widehat{c}_t = \frac{\bar{y}}{\bar{c}}\widehat{y}_t$, use (D.16), and finally note that all higher-order price dispersion terms can be ignored to second order. We thus get

$$U_t \approx \bar{U} + \nu_\ell(\bar{\ell})\bar{\ell}(1-\alpha)^{-1}\left[-\widehat{d}_t + \frac{1}{2}(1-\alpha)^{-1}\left[(1-\gamma)(1-\alpha)\frac{\bar{y}}{\bar{c}} - (\phi+1)\right]\widehat{y}_t^2 - \frac{1}{2}\gamma\frac{\bar{c}}{\bar{y}}\int\frac{\widehat{\omega}(\zeta)^2}{\bar{\omega}(\zeta)}d\Gamma(\zeta)\right]$$

The last step in the derivation is to express $\widehat{d}_t$ in terms of the history of inflation, closely following the arguments in Woodford (2003). The dispersion term is defined as[35]

$$\begin{aligned} d_t &\equiv \int\left(\frac{p_{jt}}{p_t}\right)^{-\eta_t}dj \\ &= \int\left(\frac{e^{\widehat{p}_{jt}}}{p_t}\right)^{-\eta_t}dj, \end{aligned}$$

where we have defined $\widehat{p}_{jt}$ as the log of p_{jt}. Taking a second-order approximation around $\widehat{p}_{jt} = \bar{p}_t \equiv \mathbb{E}_j[\log p_{jt}]$ and $\eta_t = \bar{\eta}$ yields

$$\begin{aligned} \widehat{d}_t &\approx \int -\bar{\eta}(\widehat{p}_{jt} - \bar{p}_t) + \frac{1}{2}\left[\bar{\eta}^2(\widehat{p}_{jt} - \bar{p}_t)^2 - 2(\widehat{p}_{jt} - \bar{p}_t)\widehat{\eta}_t\right]dj \\ &= \frac{\bar{\eta}^2}{2}\mathrm{Var}_j\left[\widehat{p}_{jt}\right] \end{aligned}$$

where we have simplified using that fact that, at our expansion point, there is no dispersion in p_{jt}, so $e^{\widehat{p}_{jt}} = \bar{p}_t \forall j$. Next we use the Calvo structure to rewrite the definition of d_t as

$$d_t = \theta\int\left(\frac{p_{jt-1}}{p_{t-1}}\right)^{-\eta_t}dj\,(1+\pi_t)^{\eta_t} + (1-\theta)\left(\frac{1}{1-\theta} - \frac{\theta}{1-\theta}(1+\pi_t)^{\eta_t-1}\right)^{\eta_t/(\eta_t-1)}$$

A second-order approximation of this expression (around a zero-inflation steady state) yields

$$\begin{aligned} \widehat{d}_t &\approx \theta\frac{\bar{\eta}^2}{2}\mathrm{Var}_j\left[\widehat{p}_{jt-1}\right] + \frac{\theta\bar{\eta}}{2(1-\theta)}\widehat{\pi}_t^2 \\ &\approx \theta\widehat{d}_{t-1} + \frac{\theta\bar{\eta}}{2(1-\theta)}\widehat{\pi}_t^2. \end{aligned}$$

Solving backwards:

$$\widehat{d}_t \approx \theta^{t+1}\widehat{d}_{-1} + \frac{\theta\bar{\eta}}{2(1-\theta)}\sum_{s=0}^{t}\theta^{t-s}\widehat{\pi}_s^2 \tag{D.17}$$

[35]The aggregate production function is obtained from integrating (A.1) across j and using (13), the fact that all firms choose the same capital-labor ratio, and labor market clearing.

We can now return to the problem of the planner. Using our results so far, we can write planner preferences as

$$\sum_{t=0}^{\infty}\beta^t U_t \approx -\frac{\nu_\ell(\bar{\ell})\bar{\ell}}{1-\alpha}\sum_{t=0}^{\infty}\beta^t\left[\widehat{d}_t+\frac{1}{2}\left[(\gamma-1)\frac{\bar{y}}{\bar{c}}+\frac{\phi+1}{1-\alpha}\right]\widehat{y}_t^2+\frac{\gamma}{2}\frac{\bar{c}}{\bar{y}}\int\frac{\widehat{\omega}(\zeta^t)^2}{\bar{\omega}(\zeta^t)}d\Gamma(\zeta^t)\right]$$

Note that $\widehat{\pi}_t^2$ affects $\widehat{d}_t, \beta\widehat{d}_{t+1}, \cdots$ by $\frac{\theta\bar{\eta}}{2(1-\theta)}(1, \beta\theta, \cdots)$ so the discounted sum is $\frac{\theta\bar{\eta}}{2(1-\theta)(1-\beta\theta)}$. Using this we have

$$\begin{aligned}
\sum_{t=0}^{\infty}\beta^t U_t &\approx -\frac{\nu_\ell(\bar{\ell})\bar{\ell}}{1-\alpha}\sum_{t=0}^{\infty}\beta^t\left[\frac{\theta\bar{\eta}}{2(1-\theta)(1-\beta\theta)}\widehat{\pi}_t^2+\frac{1}{2}\left[(\gamma-1)\frac{\bar{y}}{\bar{c}}+\frac{\phi+1}{1-\alpha}\right]\widehat{y}_t^2+\frac{\gamma}{2}\frac{\bar{c}}{\bar{y}}\int\frac{\widehat{\omega}(\zeta^t)^2}{\bar{\omega}(\zeta^t)}d\Gamma(\zeta^t)\right] \\
&= -\frac{\nu_\ell(\bar{\ell})\bar{\ell}\theta\bar{\eta}}{2(1-\alpha)(1-\theta)(1-\beta\theta)} \\
&\quad\times\sum_{t=0}^{\infty}\beta^t\left[\widehat{\pi}_t^2+\frac{\tilde{\kappa}}{\bar{\eta}}\widehat{y}_t^2+\frac{(1-\theta)(1-\beta\theta)\gamma}{\bar{\eta}\theta}\frac{\bar{c}}{\bar{y}}\int\frac{\widehat{\omega}(\zeta^t)^2}{\bar{\omega}(\zeta^t)}d\Gamma(\zeta^t)\right]
\end{aligned}\tag{D.18}$$

where

$$\tilde{\kappa} \equiv \left[(\gamma-1)\frac{\bar{y}}{\bar{c}}+\frac{\phi+1}{1-\alpha}\right]\frac{(1-\theta)(1-\beta\theta)}{\theta}.$$

□

D.4 Auxiliary lemma for Proposition A.1

We here establish that, under Assumptions A.1 to A.3, changes in nominal interest rates do not affect the distribution of consumption shares. We proceed in two steps. First, we show that consumption dispersion is unaffected by changes in real interest rates and output that satisfy particular conditions. Second, we establish that changes in the monetary policy stance induce changes in real interest rates and output that satisfy precisely those conditions. Throughout, our arguments closely follow Werning (2015).

Lemma D.1. *Suppose that Assumptions A.1 to A.3 hold, and consider paths* $(\boldsymbol{r}, \boldsymbol{y}, \boldsymbol{m}, \boldsymbol{\tau}_x)$ *such that* $\widehat{\boldsymbol{m}} = \boldsymbol{\tau}_x = \boldsymbol{0}$ *and, for all* $t = 0, 1, 2, \ldots,$

$$y_t^{-1} = \tilde{\beta}(1+r_{t+1})y_{t+1}^{-1}, \tag{D.19}$$

where $\tilde{\beta} \equiv (1+\bar{r})^{-1}$. *Then the distribution of consumption shares remains constant at its* *steady state distribution:* $\omega_t(\zeta) = \bar{\omega}(\zeta)$ *for all* t *and* ζ.

Proof. We will rewrite the household budget constraint using several substitutions. First, there are no transfers and no government debt to service so the government budget implies $\tau_y = 0$. Second, we use the acyclical earnings risk from Assumption A.1. Third, as capital is the only asset traded, the value of assets at the start of period t is $a_{i,t} = \left(\alpha y_t/k + q_t^k\right) k_{i,t}$ where $k_{i,t}$ is the units of capital purchased by the household at date $t-1$. Similarly, the value of assets at the end of period t is $a_{i,t+1}/(1+r) = q_t^k k_{i,t+1}$. Putting the pieces together, the household budget constraint (8) becomes

$$q_t^k k_{i,t+1} + c_{it} = \left(\alpha y_t/k + q_t^k\right) k_{i,t} + \tilde{\Phi}(\zeta_{it}, \bar{m})(1-\alpha)y_t. \tag{D.20}$$

Combining the asset-pricing relationship (16) and the aggregate Euler equation (D.19), we have

$$\frac{q_t^k}{y_t} = \tilde{\beta}\left(\frac{\alpha}{k} + \frac{q_{t+1}^k}{y_{t+1}}\right).$$

Provided that steady state real rates are positive, we have that $\tilde{\beta} < 1$ and so we can solve forward to find $q_t^k/y_t = \tilde{\beta}\alpha/\left[k(1-\tilde{\beta})\right]$. In steady state, the return on capital implies $\bar{r}\bar{q}^k = \alpha\bar{y}/k$ so we have (using $\tilde{\beta} = 1/(1+\bar{r})$)

$$q_t^k = \frac{\bar{q}^k}{\bar{y}} y_t.$$

The household budget constraint becomes

$$\frac{y_t}{\bar{y}}\bar{q}^k k_{i,t+1} + c_{it} = (1+\bar{r})\,\bar{q}^k \frac{y_t}{\bar{y}} k_{i,t} + \tilde{\Phi}(\zeta_{it}, \bar{m})(1-\alpha)\bar{y}\frac{y_t}{\bar{y}}. \tag{D.21}$$

Letting z denote an arbitrary realization of ζ_{it}, we will now re-state the household consumption-savings problem in recursive form. We have

$$V_t(k,z) = \max_{k'\geq 0} \log\left[(1+\bar{r})k\bar{q}^k + \tilde{\Phi}(z,\bar{m})(1-\alpha)\bar{y} - \bar{q}^k k'\right] + \log\left(\frac{y_t}{\bar{y}}\right) + \beta\mathbb{E}\left[V_{t+1}(k',z')\right]$$

If V_{t+1} is of the form $V_{t+1}(k,z) = \tilde{V}(k,z) + B_{t+1}$ for some sequence B_{t+1}, then the Bellman equation above can be written as

$$V_t(k,z) = \max_{k'\geq 0} \log\left[(1+\bar{r})k\bar{q}^k + \tilde{\Phi}(z,\bar{m})(1-\alpha)\bar{y} - \bar{q}^k k'\right] + \beta\mathbb{E}\left[\tilde{V}(k',z')\right] + B_t, \tag{D.22}$$

where $B_t = \log(y_t/\bar{y}) + \beta B_{t+1}$. As there is no time-varying aggregate variable apart from B_t, V_t satisfies the same functional form as V_{t+1}. By induction, all previous value functions satisfy this form. Using the steady-state value function to start the induction (i.e., we start at the steady-state value function $\tilde{V}(k, z)$ and $B_t = 0$), we can conclude from (D.22) that the optimal decision rule for k' as a function of (k, z) will be constant and equal to the steady state decision rule. This constant decision rule and a stable process for the evolution of z' implies the distribution of (k', z') is stationary. It follows from (D.21) that the optimal consumption decision rule will scale with $y_t = c_t$. This scaling implies consumption shares are constant and equal to their steady state values.

As a final step, it remains to relate this recursive formulation of the household decision problem to the histories of idiosyncratic events. To this end, note that we can write the consumption share as a function of the state variables associated with that history:

$$\omega_t(\zeta) = \frac{c(k(\zeta), z(\zeta))}{\bar{c}}, \tag{D.23}$$

where $c(k, z)$ is the steady state consumption function, $k(\zeta)$ is the steady state capital holdings of a household with history ζ and $z(\zeta)$ is the most recent event in the history ζ. (D.23) holds for any paths $(\boldsymbol{r}, \boldsymbol{y}, \boldsymbol{m}, \boldsymbol{\tau}_x)$ such that (D.19) holds and $\widehat{\boldsymbol{m}} = \boldsymbol{\tau}_x = \mathbf{0}$. □

D.5 Proof of Proposition A.1

It remains to show that changes in nominal interest rates induce paths of $\boldsymbol{r}$ and $\boldsymbol{y}$ that satisfy (D.19) (since by linearity we already have $\widehat{\boldsymbol{m}} = \boldsymbol{\tau}_x = \mathbf{0}$). But this follows directly from Werning (2015), as our model economy with Assumptions A.1 to A.3 satisfies the conditions of his result (i.e., acyclical risk and acyclical liquidity). We furthermore note that our special case is isomorphic to the incomplete markets model that appears in Section IIIB of Farhi & Werning (2019).[36] We refer the reader to Werning (2015) for the formal proof. □

[36]The model in Farhi & Werning (2019) specifies a particular AR(1) process for idiosyncratic income risk for the sake of computing numerical solutions. We leave the process more general. The important aspect is that risk is not affected by monetary policy (see Werning, 2015).

D.6 Proof of Corollary A.1

By (4) we can write the optimal monetary policy rule as

$$\Theta'_{\pi,i} W \widehat{\boldsymbol{\pi}} + \frac{\kappa}{\bar{\eta}} \Theta'_{y,i} W \widehat{\boldsymbol{y}} = \mathbf{0} \tag{D.24}$$

where we have used the fact that with $\delta = 0$ we have $\tilde{\kappa} = \kappa$. It follows from (D.1) that

$$\Pi_\pi \Theta_{\pi,i} = \Pi_y \Theta_{y,i} \tag{D.25}$$

and so we can re-write (D.24) as

$$\widehat{\boldsymbol{\pi}} + \frac{\kappa}{\bar{\eta}} \frac{1}{\kappa} \begin{pmatrix} 1 & 0 & 0 & \dots \\ -1 & 1 & 0 & \dots \\ 0 & -1 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \widehat{\boldsymbol{y}} = \mathbf{0} \tag{D.26}$$

giving (A.9). □

D.7 Proof of Corollary A.2

We have already shown that the optimal monetary rule is given as (A.9). Since (D.1) also implies that

$$\Pi_\pi \Theta_{\pi,\tau_x} = \Pi_y \Theta_{y,\tau_x} \tag{D.27}$$

we can use the same steps as in the proof of Corollary A.1 to rewrite the first two terms in (38) as

$$\Theta'_{\pi,\tau_x} W \widehat{\boldsymbol{\pi}} + \frac{\kappa}{\bar{\eta}} \Theta'_{y,\tau_x} W \widehat{\boldsymbol{y}} = \widehat{\boldsymbol{\pi}} + \frac{1}{\bar{\eta}} \begin{pmatrix} 1 & 0 & 0 & \dots \\ -1 & 1 & 0 & \dots \\ 0 & -1 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \widehat{\boldsymbol{y}}.$$

Imposing (A.9) sets these terms to zero, so Corollary A.2 follows. □