Appendix for "The Economics of Tropical Deforestation"

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A Renewable resource depletion under sole ownership

We assume that forest property owners are price-takers in the market for forest products.[1] The owner's payoff in period t is $py_t - c(y_t, X_t)$, where p is the market price of timber, y_t is the quantity extracted, X_t is the current size of the forest (i.e., the timber stock), and $c(y_t, X_t)$ is the cost to the owner of extracting y_t given current stock X_t.[2] The owner's problem, then, is to choose a path of extraction $\{y_t\}$, given some initial forest stock X_0, that maximizes the net present value of extracted timber:

$$\max_{\{y_t\}} \int_0^\infty [py_t - c(y_t, X_t)]e^{-rt}\, dt$$

subject to

$$\frac{dX_t}{dt} = g(X_t) - y_t$$

where $g(\cdot)$ is known as the *natural growth law* of the forest. Typically, $g(\cdot)$ is assumed to be strictly concave—most commonly using a quadratic function in X_t, which yields the familiar logistic evolution of stock over time.[3] We assume for simplicity that the interest rate r is constant and that extraction costs $c(\cdot, X_t)$ are convex in y_t.[4]

In most expositions of the owner's renewable resource extraction problem, the optimal path of extraction is derived using the Pontryagin maximum principle. The Hamiltonian for the owner's maximization problem is:

$$H = py_t - c(y_t, X_t) + q_t[g(X_t) - y_t]$$

where the co-state variable q_t represents the shadow price of timber: it is the amount by which the net present value of the forest decreases when one unit of timber is extracted today. Peterson and Fisher call this co-state variable the "marginal user cost" of resource extraction. As we will discuss below, a key distinction between sole-ownership and common-property depletion is that in the latter case, agents do not take this marginal user cost into account.

[1] To fix ideas, we simply use "timber" to capture a potentially broader category of products in what follows.

[2] The payoff can be equivalently represented by substituting in place of $c(y_t, X_t)$ a term equal to the total wage paid for extraction effort (as in Peterson and Fisher, 1977). In other words, the process of extraction can be represented using a production function with labor as an input rather than using a cost function. We choose to present the cost function formulation here as it will allow us more clearly to highlight the role of external social costs of deforestation.

[3] Bioeconomic models sometimes assume a growth law that depends on other factors such as inter-species competition and age structure of the resource stock. For the sake of simplicity, we abstract from such forces here.

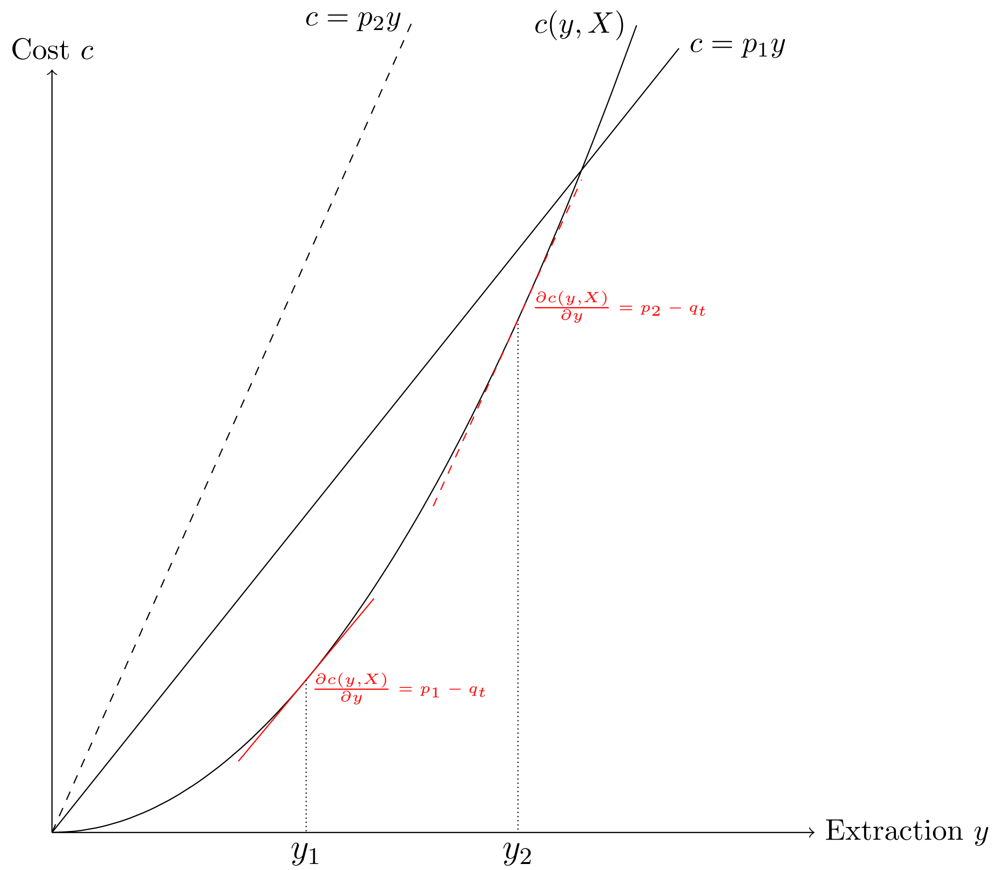
[4] Note that the problems facing the owner of an exhaustible natural resource and the owner of a renewable resource are quite similar: their objective functions are identical, and in the former case, the owner's flow constraint instead reflects the purely exhaustible nature of the resource. In particular, their flow constraint is simply $\frac{dX_t}{dt} = -y_t$. They also face the constraint that $\int_0^\infty y_t\, dt \le X_0$.

The Hamiltonian above yields the necessary conditions:

$$\begin{aligned}
\frac{\partial H}{\partial y_t} &= 0 \\
\implies p &= \frac{\partial c(y_t, X_t)}{\partial y_t} + q_t && (1) \\
\frac{dq_t}{dt} &= rq_t - \frac{\partial H}{\partial X} \\
\implies \frac{dq_t}{dt} &= rq_t + \frac{\partial c(y_t, X_t)}{\partial X} - q_t \frac{dg(X_t)}{dX} && (2)
\end{aligned}$$

Equation (1) indicates that the optimal extraction path equates price with the marginal cost of extraction plus the marginal user cost of timber in each period. One insight that emerges from this analysis is that, even in the absence of any market imperfection (e.g., market power on the part of the timber-extracting agent), price does not equal marginal cost—the wedge between the two reflects the rivalrous nature of natural resource consumption, where rivalry in the single-agent case refers to consumption of the resource at different points in time.

Figure A1 presents this optimality condition graphically and illustrates how price changes for timber affect a sole owner's extraction decision within a given period. The convex curve represents the owner's extraction cost as a function of extraction levels y, assuming a given forest stock X. The solid straight line represents the owner's zero-profit curve along which revenues given a price p_1 are exactly equal to extraction costs. Given price p_1, the owner will extract an amount y_1 such that the marginal cost of extraction is equal to the price of output minus the marginal user cost q_t, which is assumed here to be small. Holding q_t constant, increasing the price of output from p_1 to p_2 (and moving from the solid to the dashed zero-profit line) leads the sole owner to extract a greater amount y_2 within that period.



Appendix Figure A1. Sole owner's single-period extraction decision

Equations (1) and (2) define the dynamics of optimal extraction y given any current stock X, and specifying the initial stock X_0 pins down the level of extraction and the size of remaining forest in each period along the equilibrium path. These conditions allow us to analyze the resulting steady state and conduct comparative statics. Let y^*, X^*, and q^* denote the steady-state levels of extraction, forest stock, and marginal user cost, respectively. In this model, a steady state is such that equation (1) and the following additional conditions hold:

$$\frac{dq_t}{dt} = rq^* + \frac{\partial c(y^*, X^*)}{\partial X} - q^* \frac{dg(X^*)}{dX} = 0 \tag{3}$$

$$\frac{dX_t}{dt} = g(X^*) - y^* = 0 \tag{4}$$

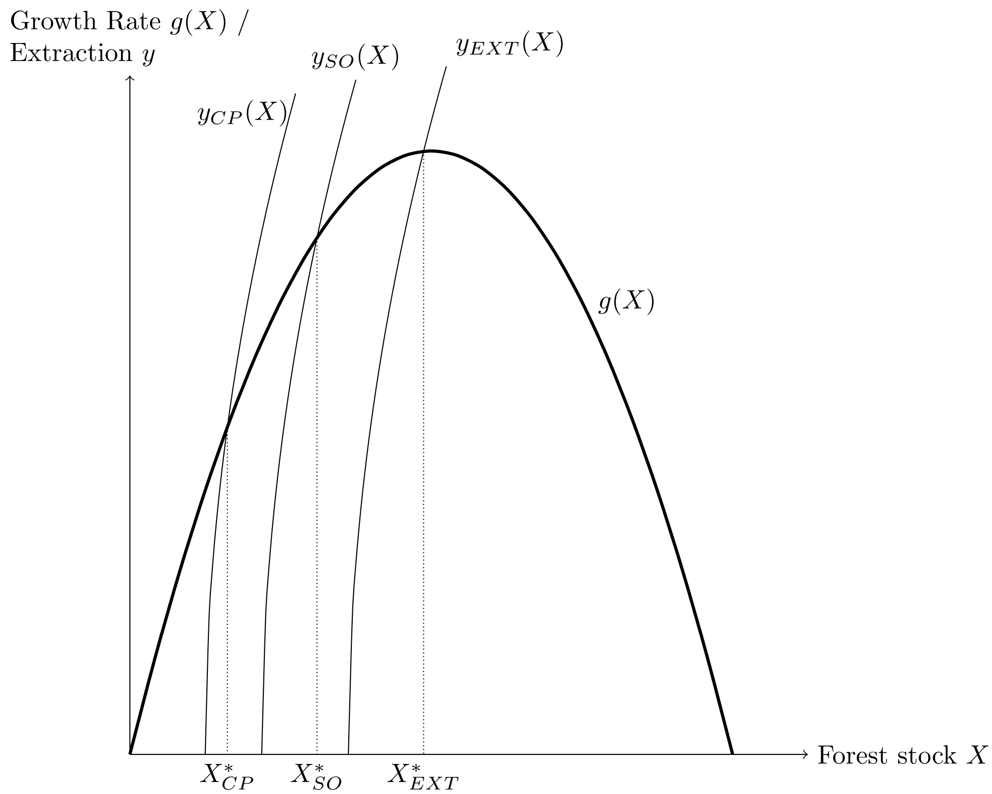
Note that Equation (1) implies that y^*, the steady-state level of extraction, will be such that the marginal cost of extraction equals $p - q^*$. Clearly, higher timber prices will lead to a higher steady-state level of extraction, as examined graphically above. Furthermore, given the convexity of $c(\cdot, X_t)$ in y_t, a negative level shift in marginal extraction costs will lead to higher y^*.

We first examine the case in which an interior solution (a steady state in which $y^* > 0$ and hence $X^* > 0$) exists. Equation (3) then illustrates an important conclusion emerging from the theoretical literature on forest management: the economically optimal path of extraction generically does not coincide with the notion of "maximum sustained yield," i.e., the maximum growth rate of the renewable resource—and hence the maximum rate of extraction—that can be sustained in equilibrium. Given a positive interest rate, $\frac{dg(X^*)}{dX} = 0$ may not be be optimal, i.e., the point of maximum forest growth may be below or above the owner's optimal level of extraction. If we further assume that $\frac{\partial c}{\partial X} = 0$ (the cost of cutting a given number of trees does not depend on the size of the remaining forest), then maximum sustained yield *cannot* be optimal from the standpoint of the owner; instead the steady-state optimum lies below the point of maximum sustained yield. Samuelson (1976) and Peterson and Fisher (1977), in particular, highlight this divergence, which is important because many ecologists and environmental policymakers at the time had tended to advocate for the maximum sustained yield notion by default.[5] However, only with an effective interest rate of zero will the "economists' optimum" coincide with the "foresters' optimum" of maximum sustained yield. Intuitively, economic discounting implies that the agent prefers to cut more trees today rather than to wait for the forest to grow further; the higher the discount rate, the higher the steady-state level of extraction and hence the larger the divergence from maximum sustained yield. In the specific context of tropical forests, agents' discount rates may be especially high (Barbier et al., 1991), due in part to insecure property rights and regulatory uncertainty (we return to this point below).[6]

Figure A2 illustrates one possible steady-state of the model graphically. The natural growth law $g(X)$ is depicted as the inverted parabola in bold. The line labeled $y_{SO}(X)$ represents the locus of single-owner optimal extraction levels as a function of the forest stock. This locus can be traced out by varying the forest stock X (and hence shifting the cost curve $c(y, X)$ outward) in Figure A1. For each stock level X, the optimal level of extraction $y_{SO}(X)$ can be determined according to the optimality condition in equation (1). A steady state X^*_{SO} then occurs when the level of extraction $y_{SO}(X^*_{SO})$ exactly balances the forest's natural rate of growth $g(X^*_{SO})$.

[5]Indeed, Peterson and Fisher (1977) and Goundrey (1960) note that the concept of maximum sustained yield was, at the time of writing, codified in US and Canadian forestry policies.

[6]For the purposes of considering social optimality, Samuelson (1976) relates his analysis to several other works (e.g., Ramsey, 1928; Diamond and Mirrlees, 1971) on the appropriate social discount rate.



Appendix Figure A2. Steady-state extraction and forest stocks

Equations (1) and (4) yield another important insight: if the price of timber is very high, or if the level of extraction costs are very low, then there may exist no positive X^* such that $g(X^*) = y^*$: the forest cannot grow at a rate that compensates for the high economic returns to extraction that the agent faces today. In this case, the only steady state is complete extinction of the forest: $y^* = X^* = g(X^*) = 0$. An analogous insight arises from Equation (3): if the interest rate r is very high, then again there may be no positive X^* such that $\frac{dg(X^*)}{dX}$ is large enough to sustain an equilibrium. As Peterson and Fisher (1977) note, forest stocks in this case "do not grow fast enough to justify waiting around for them." Extinction can occur even under sole ownership of the forest, a point developed rigorously in the work of Smith (1968), Clark (1973), and Neher (1974), among others. In Figure A2, such a scenario would be represented by an extraction locus $y_{SO}(X)$ that is shifted so far leftward that it intersects the growth curve only at $X = 0$. This insight has important implications for policy solutions to common-property resource issues we discuss below.

Similarly, if timber is valued at a very low price or if costs of extraction are high, then there may exist no stock X^* such that a positive level of extraction $y^* > 0$ is optimal: in this case, $y^* = g(X^*) = 0$, while the equilibrium stock equals the "carrying capacity" of the forest, i.e., $X^* = X_{max}$ such that $g(X_{max}) = 0$ and $g(X_t) < 0$ for any $X_t > X_{max}$. Timber production is not lucrative, so there will be no extraction whatsoever, and the forest will remain at the maximum size that natural growth constraints allow.

A.1 Alternative land uses

Directly incorporating alternative uses for forested land into the above model of resource depletion complicates the analysis substantially. One parsimonious approach taken by Bohn and Deacon

(2000) is to assume that demand for alternative land uses (e.g., agriculture) is perfectly inelastic and exogenously determined by food needs. Permanent shifts in demand for agricultural land can then be reflected in shifts in the carrying capacity of a given forest: greater food demand reduces the amount of forest land by re-designating a portion for agricultural production. In order to relax this assumption, one can additionally incorporate the quantity of cleared forest land (i.e., land that has been converted for agricultural use) as another state variable in the above dynamic system.

B Optimal resource depletion with deforestation externalities

There are several possible avenues through which the externalities of deforestation may be incorporated into our simple theoretical framework of resource extraction. Insofar as externalities arise from static misallocation in each period, the external costs of deforestation can simply be incorporated into the term $c(y_t, X_t)$ representing the costs of extraction (Fisher, 1981).[7]

To see how the presence of externalities can shift the socially optimal steady-state level of deforestation, we return to the set of equilibrium conditions derived in Section 4.1. Assume that, in addition to the private cost of extracting y_t units in period t, the owner also considers a static external cost $E(y_t)$ that does not depend on the current forest stock X_t.[8] The sole owner's maximization problem is then:

$$\max_{\{y_t\}} \int_0^\infty [py_t - c(y_t, X_t) - E(y_t)]e^{-rt}\, dt$$

subject to

$$\frac{dX_t}{dt} = g(X_t) - y_t$$

Equations (3) and (4) governing the resulting steady state are unchanged from above, but equation (1) now becomes

$$p = \frac{\partial c(y_t, X_t)}{\partial y_t} + \frac{dE(y_t)}{dy_t} + q_t \tag{5}$$

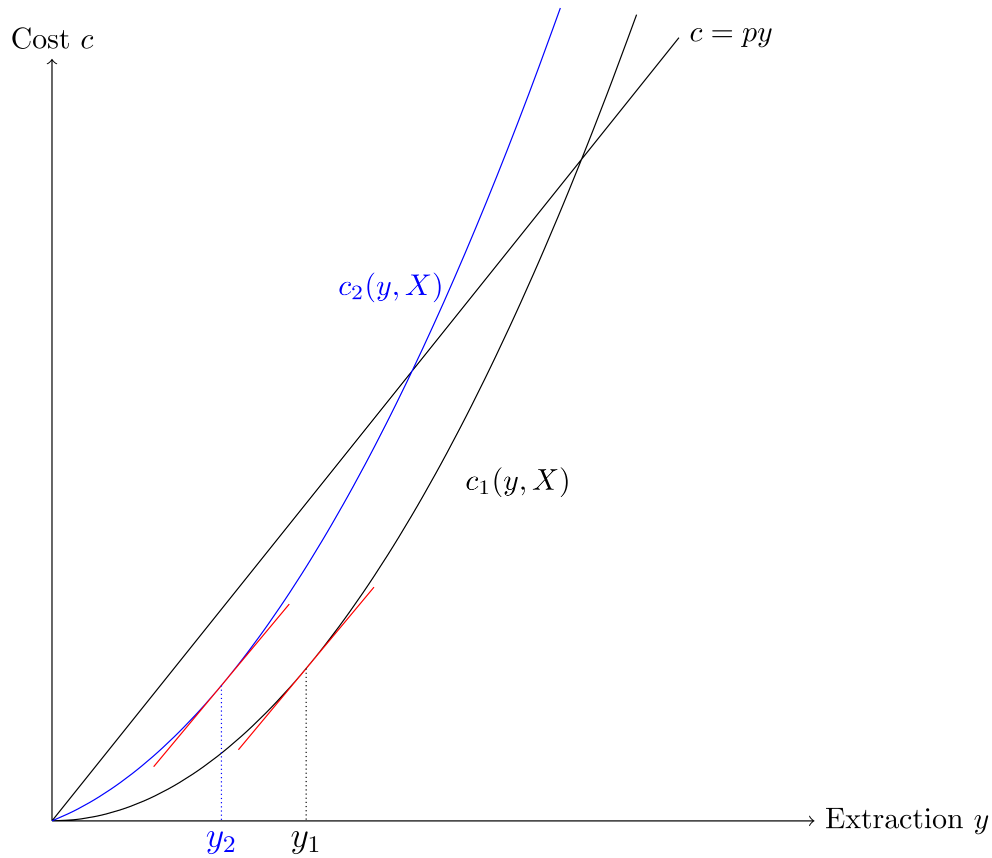
As long as the external cost of extraction is not decreasing in the amount extracted, the presence of deforestation externalities widens the wedge between the price of timber and the marginal cost of extraction. In the case of constant marginal external costs (such as when invoking the social cost of carbon), externalities represent a level shift upward in the marginal costs of extraction. The higher these external costs, the farther will be the socially optimal extraction level below the sole-ownership steady state level.

The influence of externalities on the socially optimal deforestation level is depicted graphically in Figure B1. The upward level shift in marginal costs of extraction are depicted as a shift from cost curve $c_1(y, X)$ (the cost curve faced by a sole owner depicted in Figure A1) to $c_2(y, X)$, drawn

[7] Alternatively, other models have explicitly accounted for the dynamics of waste accumulation as a byproduct of natural resource extraction (e.g., D'Arge and Kogiku, 1973; Rausser and Lapan, 1979) and have considered the optimal intertemporal control of pollution (e.g., Plourde, 1972; Keeler et al., 1971). Many of these models consider extractive natural resources as inputs into production of both goods and bads (e.g., factories use coal as an input and produce local pollutants in addition to consumer goods). To the extent that the main externalities of tropical deforestation are byproducts of the extraction process *per se*, rather than of production from forest resources, the "static" approach of incorporating external social costs in $c(y_t, X_t)$ captures the principal forces of interest while retaining tractability.

[8] Of course, a social planner might also consider the dynamics of carbon accumulation in the atmosphere and other environmental harm resulting from a given amount of extraction today. The assumption of static external costs, however, captures the qualitative insights of our model in a more straightforward fashion.

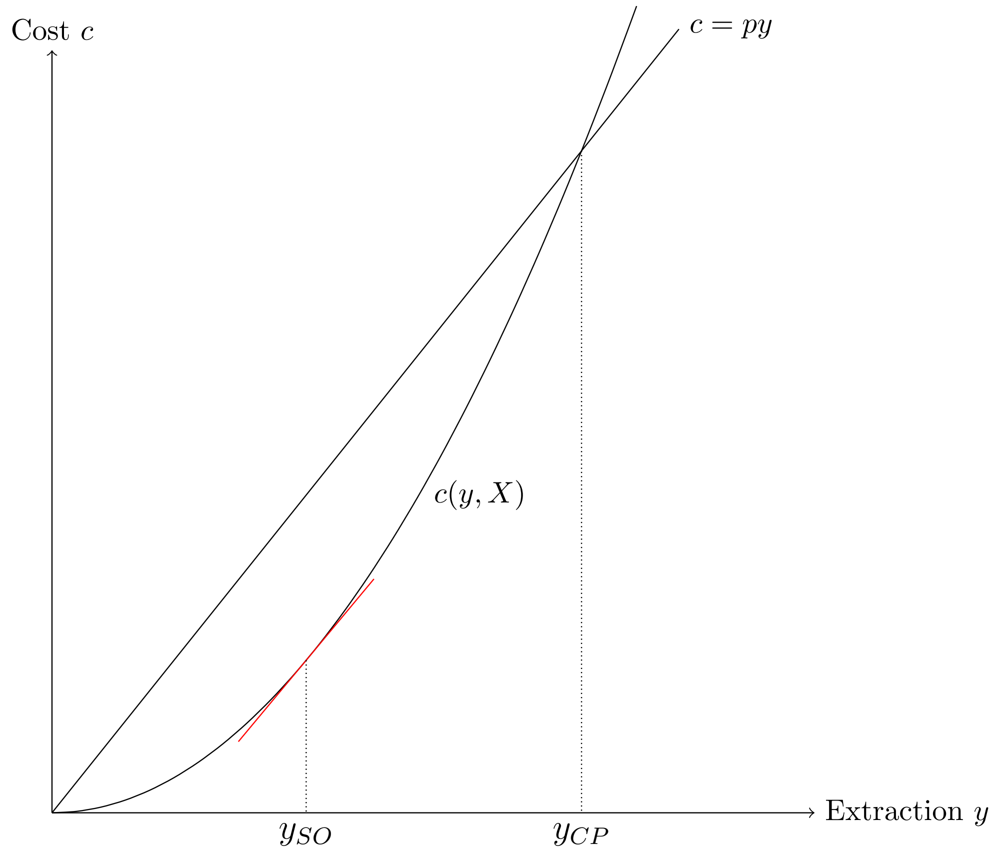
in blue. The optimal level of extraction within each period decreases accordingly, from y_1 to y_2.



Appendix Figure B1. Optimal extraction with externalities

C Resource depletion with common-property access

To illustrate how the free-entry equilibrium leads to over-exploitation relative to sole ownership, we revisit the model of renewable resource extraction developed in Appendix A. The zero-profit condition discussed in the previous paragraph requires that $py_t^{CP} - c(y_t^{CP}, X_t) = 0$, where y_t^{CP} denotes the agent's level of extraction in period t under the common-property regime. Note that extraction y and costs c are now aggregate quantities due to the free entry of foresting firms. The increase in aggregate deforestation is seen easily in Figure C1, as the common-property equilibrium occurs where the aggregate cost curve intersects the line $c = py$.



Appendix Figure C1. Common-property levels of aggregate extraction

A useful comparison arises from differentiating the zero-profit condition with respect to y, yielding:

$$\frac{\partial c(y_t^{CP}, X_t)}{\partial y} = p \tag{6}$$

Under a common property regime, deforesting firms enter until the marginal cost of extraction for any agent exactly offsets the economic return to extraction. By contrast, the single-agent condition (1) yields $\frac{\partial c(y_t^{SA}, X_t)}{\partial y} = p - q_t$, where y_t^{SA} denotes the sole-agent optimal extraction level. Because $q_t > 0$ unless the supply of the resource is truly unlimited, we have $\frac{\partial c(y_t^{SA}, X_t)}{\partial y} < \frac{\partial c(y_t^{CP}, X_t)}{\partial y}$, which, given the assumption of convex extraction costs, implies that $y_t^{SA} < y_t^{CP}$. Not only is aggregate extraction greater than under the sole-agent optimum, each individual agent also extracts more per period under a common property regime than they would under sole ownership of the forest.

Unlike "uni-directional" externalities studied in the previous section, common-property resources feature "reciprocal" externalities in which each agent's actions affect all other agents' yields, including their own. Such externalities arise because agents do not account for the marginal user cost q_t of extraction nor for their effect on the growth rate of a renewable resource through a depletion in stock. Compounding the fact that extraction in each period t is greater under common property than under sole ownership *given* a particular stock X_t, this higher level of extraction will lower the forest stock in the following period, making extinction even more likely than in the single-agent case (Smith, 1968; Peterson and Fisher, 1977).

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