Optimal Design of Tokenized Markets*

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Abstract

Trades in today's financial system are inherently subject to settlement uncertainty. This paper explores tokenization as a potential technological solution. A token system, by enabling programmability of assets, can be designed to eradicate settlement uncertainty. We study the allocations achieved in a decentralized market with either the legacy settlement system or a token system. Tokenization can improve efficiency in markets subject to a limited commitment problem. However, it also materially alters the information environment, which in turn aggravates a hold-up problem. This limits potential gains from resolving settlement uncertainty, particularly for markets that depend on intermediaries. We show that optimal design hinges on joint design of settlement and trading systems, and in particular, that token systems work best when matched with direct trading.

Keywords:
Tokenization, programmability, settlement uncertainty, asymmetric information

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1 Introduction

Two traders agree on an asset sale. How can each party ensure that, when the time comes to settle the trade, the other will keep their side of the bargain? Markets have adopted various solutions to resolve the age-old problem of limited commitment. Third-party intermediaries and platforms, such as exchanges or sponsors, facilitate the orderly settlement of transactions. Margin requirements and other uses of collateral ensure that future payments tied to contractual obligations are serviceable. Traders build long-term relationships and a reputation for credibility.

Despite these practices, trades commonly fail to be settled (Fleming and Garbade, 2005). The potential for systematic settlement fails was put on full display during the Global Financial Crisis. In 2008, settlement fails in Treasury markets reached a daily volume of 400 billion dollars per day. Chronic settlement fails in the Treasury market lead the Treasury Market's Practices Group (TMPG) to introduce a "fails charge" to decrease traders' incentives to fail (see (Garbade et al., 2010)).[1] While the fails charge was effective at reducing the incidence of chronic fails, some fails continue to occur as described by Fleming and Keane (2016). Fails reflect the institutional and technological feature of the current settlement system – settlement depends on traders individually submitting settlement instructions that correspond to their contractual obligations from trading activity. When incentives break down, so does settlement.

This paper explores the potential for a settlement system based on distributed ledger technology (DLT) as a technological solution to commitment problems inherent in the current settlement system. In this paper, security tokenization refers to the representation of traditional financial assets and collateral on a distributed ledger.[2] The main innovation of tokenization we focus on is the *programmability* of assets.[3] Programmability allows traders to commit to settlement, thereby eliminating the potential for fails.

Does a token system strictly improve upon a legacy settlement system because it can eliminate settlement risk? In a setting where trading is endogenous, we show that it is not the case. With the gain of eliminating settlement risk, an information problem emerges. This is because eliminating settlement risk requires traders to reveal more

[1] The Treasury Market Practices Group (TMPG) is a group of market professionals committed to supporting the integrity and efficiency of the Treasury, agency debt, and agency mortgage-backed securities markets. See: https://www.newyorkfed.org/tmpg.

[2] This is also often referred to as tokenizing "real world assets."

[3] The idea of programmability is closely related to "smart contracts." The Financial Stability Board notes that "Smart contracts use computer protocols to execute, verify, and constrain the performance of a contract. In doing so, they can automate decision-making, by allowing self-executing computer code to take actions at specified times and/or based on reference to the occurrence (or non-occurrence) of an action or event." (FSB, 2019)

information to their counterparties regarding their positions, and this information can materially impact equilibrium trade.

The intuition is simple but powerful. Suppose that A and B agree on a trade. To guarantee settlement, A and B must jointly write a program that governs the change of ownership of assets. In order to program an asset, a trader must have the ownership right to that asset at the time the settlement must take place. Indeed, if the trader does not already own the rights to the asset at the time of settlement, it is possible that the trader will never acquire these rights, which would make ensuring settlement impossible. Knowing that a trader must own the asset she is trying to sell reveals information that can be exploited by the buyer. This information can lead to a hold-up problem and even break down trade altogether.

In our model, there is a market where traders meet bilaterally in sequential meetings, and must enter inter-dependent trades in order to achieve the optimal allocation of a long-lived asset. In this market, intermediary traders facilitate trade between end-sellers and buyers and are crucial to attaining the optimal set of trades. A key friction is limited commitment. After all contracts between traders are negotiated, traders must deliver assets at a later time in accordance to their agreed upon contractual obligations. Traders are tempted to break their contracts when they learn that the private value of holding onto an asset is high ex-post. We compare the effects of the two representative settlement systems on equilibrium trade: the "legacy system," which represents the current settlement system; and the "token system," which allows for programming of assets.

In the legacy system, traders must individually take settlement actions in order to fulfill contractional obligations. As a result, commitment problems sometimes result in traders choosing to strategically fail to transfer assets as promised. In addition, individual fails have negative spillover effects on others when it forces others to fail on their trades as well (e.g. daisy-chain fails) due to traders' obligations being interdependent. In contrast, the token system trivially resolves settlement uncertainty arising from commitment issues by equipping traders with technology to commit to future settlement actions ex ante. Despite this obvious advantage of the token system, other issues become problematic in the market because some traders serve the role of bridging end sellers and buyers, i.e. act as an intermediary. An intermediary trader may not value the asset themselves, but may buy an asset in advance in anticipation of a future sale. A wedge between their private valuation and the purchase price creates circumstances ripe for a hold-up problem. In particular, having to reveal whether they own the asset exacerbates a hold-up problem, as potential buyers may now condition their offer based on whether intermediaries already purchased assets that are not privately desirable. When the hold-up problem binds, intermediaries have no incentive to "make" markets, and certain trades may altogether fail to occur in equilibrium.

While tokenization is sought as a solution for over-the-counter markets, which often rely heavily on intermediaries, our results show that trading in a token system designed to resolve settlement uncertainty can inadvertently exacerbate other impediments to trade that are "latent" under the legacy system. In addition to the hold-up problem described above, another latent issue that emerges with the token system is an "asynchronicity" problem, whereby the transfer of an asset from end-seller to buyer is only feasible when a specific sequence of meetings are realized. This is because, in the token system, a trader can only enter transactions involving assets that she (state-contingently) owns. In other words, the intermediary trader cannot intermediate an asset on behalf of a prospective buyer without first meeting with a seller and entering a trade in advance. The requirement of a specific sequence of meetings in order for intermediation to occur can be prohibitive. Both issues arise through the interaction between the decentralized nature of trading, and primitive conditions required by the token system.

In contrast, trading in the legacy system is less likely to be subject to hold-up problems because settlement in the legacy system ensures *maximum* privacy by decoupling trade execution and settlement. In particular, in the legacy system, execution of a trade does not require possession of the asset being sold, and so a buyer cannot assume that the seller has already obtained the security she is selling. This, combined with the asynchronous nature of meetings, obfuscates the intermediary trader's trading history, and actually strengthens her ability to intermediate trade in equilibrium.

Not only that, we show that in the legacy system, optimal trading is further strengthened when the intermediary trader is also subject to limited commitment. A commitment problem of the intermediary trader enhances her ability to intermediate trades through two channels. First, it strengthens her bargaining position with counterparties tempted to hold her up, because outside opportunities presented ex-post pose a credible threat of fail (in the case that the offer price is too low). Second, it creates gains from trade from intermediation – because all fails are ex-ante costly, an intermediary trader entering a trade increases costs she faces with failing herself, thereby weakening her incentives to fail ex-post. In this way, intermediated trade acts as a commitment device.

Our results demonstrate a stark contrast in the compatibility between intermediated trading, as assumed in our baseline environment, and each type of settlement system. The dichotomy between legacy and token systems highlights the need for a joint consideration of trading and settlement in the design of financial market systems. To this end, we consider an alternative trading system involving "direct trading," whereby the end seller is offered the opportunity to receive (simultaneous) offers from all potential end-buyers for each state-contingent ownership of the asset. In effect, with direct trading, the optimal allocation of the asset can potentially be achieved without the involvement of an intermediary.

With direct trading, the end seller enters trades directly with the buyer with the highest private valuation of the state-ownership of the asset. As a consequence, with the token system, direct trading resolves both asynchronicity and hold-up problems, as information regarding ownership and valuations become irrelevant in trades involving end sellers and buyers. In contrast, for the legacy system, moving from intermediated to direct trading can potentially magnify commitment problems. This is because settlement issues are shifted from the intermediary trader, who is better positioned to manage settlement risk, to the end-seller, who now directly faces settlement risk, as well as the potential for daisy-chain fails. Consequently, with direct trading, the token system unambiguously dominates the legacy system. In a broader comparison across trading and settlement system pairs, we show that the relative efficiency between direct-token systems and intermediated-legacy systems ultimately depends on the severity of the commitment problem. Overall, our results point to the co-existence in multiple types of financial system designs.

Programming assets to guarantee settlement is closely linked to the idea of "instant settlement." Instant settlement removes the time gap between trading and settlement, thereby eliminating settlement uncertainty. This idea generalizes to a setting where the commitment to future settlement, through programs, and trading happens simultaneously (See Lee et al. (2022)). The information problem that arises from instant settlement is highly relevant given that instant settlement is an explicit goal of several industry projects. SIX, the company that runs the Swiss central securities depository as well as the large value payment and repo trading system is building a "digital exchange" that will have tokenized assets and cash on a blockchain to facilitate trading.[4] SIX states "the most fundamental of these changes is that trading and settlement will no longer be separated. Instead, they will operate in the same cycle. We call this riskless trading." Similarly, Fnality, a project led by some large global banks, aims to provide instant settlement. Finally, the Deutsche Börse is working with R3 to build a blockchain securities platform $HQLA^x$, which would allow instant settlement.

As in our model, the knowledge of ownership of an asset at the time of trade is a starting point for all the existing designs of smart contract protocols. Thus, the implications

[4] Another relevant project is Broadridge's Distributed Repo.

of our paper are orthogonal to other important design considerations, including consensus mechanisms, privacy features, and commitment tools. In particular, our insight on limitations of token systems apply to ongoing developments in cryptography aimed at increasing privacy, which typically take as given an agreement to transfer, and examine whether the transfer can be accomplished without revealing more detailed information regarding identity.

Our results are also orthogonal to the prevalent use of collateral observed in decentralized finance (DeFi). Collateral is commonly used in environments of limited commitment, and may not only protect lenders from credit risk, and but also improve debtors' incentives to fulfill obligations. In our model, programmability resolves settlement risk, eliminating the need to provide debtors with incentives to fulfill their obligations. An added benefit is that programmability enables traders in the tokenized market to enter trades that are, de-facto insulated from credit and counterparty risk. Specifically, the immutable transfer of assets in future periods ensures that future transfers will occur regardless of a counter party's solvency. In this respect, token systems are "liquidity-efficient" – they require minimal tie-up of liquidity, such as collateral, commonly needed to secure transactions. This, of course, does not rule out arrangements where credit-risk sharing is desirable.[5]

A new and growing literature examines the implications of blockchain technology in financial settings (Townsend (2019)). This paper is the first, to our knowledge, to theoretically examines the impact of tokenization on markets with inter-dependent trades. At heart, our paper provides a novel consideration in key design features widely shared by initiatives to develop tokenized markets. Several papers examine the use of blockchain technology in financial markets. A key focus in these papers is the potential for decentralization, whether in the context of cryptocurrencies (Chiu and Koeppl (2017)), settlement (Chiu and Koeppl (2019)), or applications of smart contracts (Cong and He (2019)). In this context, important consideration are costs and incentives for validators (Abadi and Brunnermeier (2018), Easley et al. (2019)). Several consider how greater transparency of a public ledger environment affects informational problems, including the potential for front-running from identity revelation (Malinova and Park (2016)) or collusion (Cong and He (2019)). While the protocols considered in our paper are potentially implementable in a decentralized manner, this is not the contribution nor the focus. The primary application is to tokenization of traditional financial markets, in which adoption likely involves strong guarantees on the preservation of private information, including ownership. In our context, we highlight the informational impact of

[5]For examples of credit risk in settlement risk free systems, see Lee et al. (2022).

key design features of tokenized markets, and outline how these can adversely affect market efficiency. We do so by taking as given a token system that resolves settlement risk, and consider how trade is endogenously determined. Lee et al. (2022) explicitly studies the design problem of zero settlement risk token systems, taking as given a fixed set of trades.

Our paper contributes to studies of how the post-trade environment affects markets. Tokenized securities share properties of real-time gross settlement, which have been studied extensively in the context of wholesale payments. Martin and McAndrews (2008) explores how liquidity-saving mechanisms can enhance real-time gross settlement systems, which resolve counterparty and credit risk but can be taxing on liquidity. This tradeoff is explored in the context of clearing by Koeppl et al. (2012). Khapko and Zoican (2020) explore how the option to choose faster settlement can lead to inefficiencies. We highlight a novel concern that arises in the context of real-time gross settlement – the implicit requirement that underlying assets must be owned at the time of settlement. This novel form of inefficiency only arises when trade and commitment to settlement happen simultaneously – something that has become an increasingly relevant design consideration with tokenization.

Our paper is related to the potential impact of post-trade information disclosure on markets. Garratt et al. (2019) analyzes post-trade disclosure in the context of inter-dealer markets, and shows that strategic platforms may choose inefficient disclosure policies. Our paper shows that even though both the legacy and token system, within the context of our framework, do not have access to timely, sensitive information, tokenization can exhibit dramatic difference in equilibrium trade.

The remainder of the paper is organized as follows. Section 2 introduces our theoretical environment. Equilibrium analysis is provided in 3. In Section 4, we explore direct trading and further consider the joint determination of trading and settlement systems. We make concluding remarks in Section 5. Proofs not provided in the text can be found in the Appendix.

2 Model

We consider an asset market where traders enter bilateral trades that are interdependent, in the sense that an asset may be sold from one trader to another, and then sold further to a third trader. Whether such trades maximize surplus and are successfully settled, can depend on the underlying settlement environment. We consider two settlement systems: a legacy system, which represents the current system where trade and settlement happen sequentially and independently; and a token system, which uses programs to put in place irrevocable settlement instructions concurrently with trade.

Agents and Asset. There are three risk-neutral traders, $i = \{A, B, C\}$, and one indivisible asset, which is initially owned by A. The model is divided into two stages: the trading stage and the settlement stage. The model begins with the trading stage, during which traders bilaterally meet with each other and negotiate trades. There are two meetings that occur sequentially, which we represent as $t = m1, m2$. One of these meeting is between A and B and the other is between B and C. The order of these meetings is random. A and C never meet. In this sense, B is a potential intermediary that can facilitate transfers of the asset between A, who owns the asset in the beginning of the model, and C, who can make better use of the asset in certain future periods. While the role of B as an intermediary is assumed, for simplicity, it is important to recognize that intermediaries can play an essential role in facilitating transactions that might not otherwise occur. Specifically, in a more complex model, trades may not occur even if A and C can meet. We expect that our results would extend to an environment where B arises as an intermediary endogenously.

After the trading stage, the settlement stage starts. In the settlement stage, assets are transferred between traders over three dates $t = 1, 2, 3$. In essence, the trades made in $t = m1, m2$ consist of promises to exchanges ownership of the asset, with the actual exchange scheduled to occur at dates $t = 1, 2, 3$.

Trader i derives some payoff v_t^i from holding the asset at dates $t = 1, 2, 3$. Each trader is endowed with multiple accounts where the asset can be held, and the ownership and contents of accounts are assumed to be private. At any date $t = 1, 2, 3$, the asset must be in the account of one of the traders and can be in only one account. We say that trader i owns the asset at date t, and derives the associated payoff, if the asset is in one of trader i's account on that date.

Payoffs (summarized in Figure 1) vary between traders and across periods, and can take values H, M, L, or 0, where $H > M > L > 0$. A derives a payoff of L, L, and H for holding the asset in $t = 1, 2, 3$, respectively. B derives a payoff of H, $\tilde{M} \in \{0, M\}$, and 0 for holding the asset in $t = 1, 2, 3$, respectively. $\tilde{M} = M$ with probability λ_B and 0 otherwise. B privately learns $\tilde{M}$ in the beginning of date $t = 2$. C derives a payoff 0, H, and $\tilde{H} \in \{0, H\}$ for holding the asset in $t = 1, 2, 3$, respectively. $\tilde{H} = H$ with probability λ_C and 0 otherwise. C privately learns $\tilde{H}$ at the beginning of $t = 3$. Importantly, $\tilde{M}$ and $\tilde{H}$ are both revealed to B and C, respectively, in the settlement stage, after trading has occurred. This forms the basis for B and/or C wanting to sometimes break agreements made in the trading stage.

Traders' time-dependent payoffs motivate trade. In each period, a different traders has the highest ex-ante expected payoff from owning the asset. In $t = 1$, B obtains H; in $t = 2$, C obtains H; in $t = 3$, A obtains the H. Figure 2 depicts the transfer of the asset that maximizes the joint expected payoffs of the traders from an ex ante perspective.

Meetings. All trading occurs in pair-wise meetings, which take place sequentially in the trading stage $t = m1, m2$. B is matched with A and C, sequentially, but not necessarily in that order. Matches between A and C never occur.[6] With probability $\frac{1}{2}$, B is matched with A first and then C; with equal probability, B is matched with C first. The order of realized matches is known only to B, who participates in both matches. Communication between traders is assumed to only occur during meetings. In other words, at any point outside of meetings, traders are unable to send messages regarding who they met, the contents of their accounts, or their private realizations.

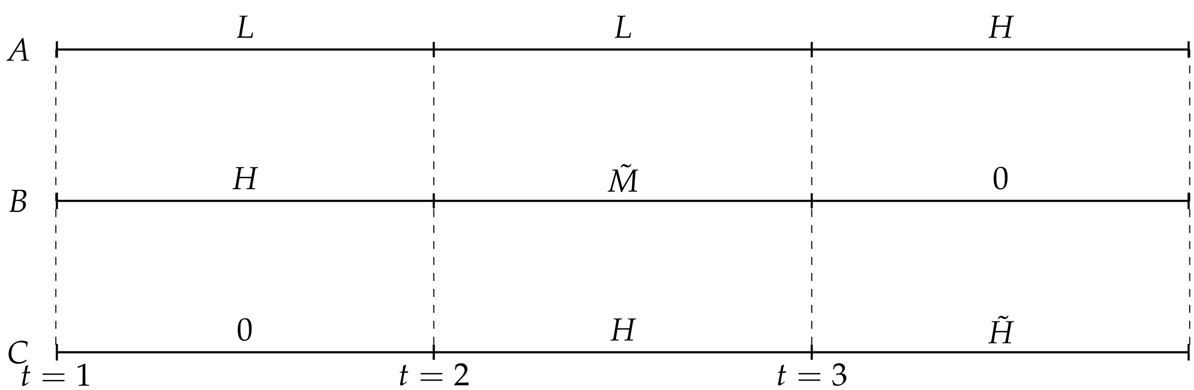

Figure 1: **Traders' payoffs.** This figure shows the payoffs of traders A, B, and C over $t = 1, 2, 3$.

During a meeting, traders negotiate a contract. Each trader knows only their history and the current state of their own accounts. Given some price P, a *contract* $C_{ij}^{\tau_1\tau_2}(P)$ is a securities lending agreement that specifies the lender, trader i; the borrower, trader j; the date τ_1 at which the asset is transferred from i to j; and the date τ_2 at which the asset is transferred back from j to i. We use $P_{ij}^{\tau_1\tau_2}$ for shorthand to denote the price corresponding to contract $C_{ij}^{\tau_1\tau_2}(P)$, where j pays i at τ_1, the date when the asset is first transferred, for borrowing the asset. Note that an agreement to trade according to a contract occurs during meetings $t = m1, m2$, whereas the actual exchange of the asset

[6] As a result, we preclude any multilateral trading scheme.

and payments occur later in the settlement stage at $t = 1, 2, 3$. For example, if A agrees to lend the asset to B for one period at price P, starting at $t = 1$, the contract is $\mathcal{C}_{AB}^{12}(P)$. To summarize, three obligations arise under contract $\mathcal{C}_{ij}^{\tau_1\tau_2}$ negotiated at some price $P_{ij}^{\tau_1\tau_2}$:

- at τ_1, trader i transfers the ownership of the asset to trader j;
- at τ_1, trader j transfers $P_{ij}^{\tau_1\tau_2}$ to trader i; and
- at τ_2, trader j returns asset to trader i.

We assume there exists some DvP settlement between assets and some numeraire used for the exchange between traders i and j at τ_1.[7]

For simplicity, the borrower j of the asset is assumed to make a take-it-or-leave-it offer.[8] This means that for the $A - B$ pair, B makes a take-it-or-leave-it offer to A, and for the $B - C$ pair, C makes a take-it-or-leave-it offer to B. As a tie-breaking rule, we assume that all else equal, traders prefer to trade.

Trades are "promises" made between traders. Whether these promises are kept depends on whether settlement, the transfer of the asset from an trader to another, takes place as stipulated in agreements made in the trading stage. Hence, the set of trades agreed upon at the trading stage determines the transfers required in the settlement stage. Depending on the settlement technology, traders face a set of actions required in order to fulfill their settlement obligations. We turn now to the description of the settlement technology.

Settlement. We consider two different type of settlement technology, which represent the legacy system and the token system.

Under the legacy system, the asset moves out of an account only if the owner of the account initiates a transfer. Formally, at dates $t = 1, 2, 3$, a trader currently in possession of the asset unilaterally decides whether to transfer the asset to the account of another trader or to keep it. The legacy system does not offer traders the ability to commit, so the option not to transfer the asset holds *regardless* of existing contractual obligations. As such, at each date in which a settlement action is required, traders explicitly choose whether to execute the transfer of the asset pertaining to an outstanding trade or strategically fail. To fix ideas, suppose that traders A and B entered a trade at either $t = m1$

[7]In a settlement system like Fedwire securities, settlement is initiated by the seller of the securities. Upon sending securities, cash is automatically transferred from the account of the buyer to the account of the seller.

[8]As will be evident, the lender in the sequence of trades will be privately informed. We adopt the convention that the party without private information is making the offer, which simplifies the analysis and is not crucial for the main results.

or $m2$ that specifies that the asset must be transferred from A's account to B's account at date 1. Then, at date 1, A can choose to initiate the transfer to B's account, as specified in the trade, or can choose not to do so and "fail." A trader that fails to settle suffers a cost Δ, which can be thought of as a reputational cost or penalty. We make several parametric assumptions held throughout the paper. First, to allow for strategic fails to sometimes be attractive, we assume an intermediate range of Δ, which motivates a commitment problem:[9] Second, we assume that throughout that $M \leq \Delta + L$.

Assumption 1. $\Delta \in (2L, \frac{1}{2}H)$ *and* $M \leq \Delta + L$.

The second condition on M ensures that there are no gains from trade that arise between A and B from B failing on A at $t = 2$ for better ex-post allocation.[10]

Under a token system, an asset can be "programmed" during the trading stage with transfer instructions to be completed in the settlement stage at future dates. This allows traders to commit to settlement taking place as specified in the contract. The transfer instructions associated with a contract are self-executing, so that the asset moves from account to account without the need for any trader to take an action. Moreover, a trader is unable to prevent a programmed transfer from occurring.

To add a transfer instruction to an asset, however, a trader must be the current holder of that asset at the time the contract specifies it is to be transferred, and new instructions must be feasible given all instructions already programmed in the asset. In the context of our model, a trader making an agreement to lend the asset at the trading stage, as per endowments or previous trade agreements, can program the asset to also make sure that the asset will be returned at a specific date, as specified by the contract . This eliminates the commitment problem discussed above for the legacy system. Importantly, this requires that bargaining over the terms of the trade and programming the asset occur simultaneously. In this sense, programming assets according to contracts at the time of negotiation achieves the same effect as if trades are immediately settled. As such, while the actual settlement takes place in the future, we refer to the process by which assets are irrevocably programmed to move in the future as constituting "immediate settlement." The operational features of a token system free of settlement risk is studied formally by Lee et al. (2022). An implication is that both parties of a negotiated trade confirm that

[9] If Δ is sufficiently high, no trader would enter a contract that they do not intend to honor. For similar reasons, we do not consider collateralized contracts, which could remedy commitment issues if traders are unconstrained. The lower bound on Δ reduces the number of cases to consider, but our core results do not depend on this assumption.

[10] This assumption is made purely to simplify the analysis, and lets us focus on strategic fails that arise primarily due to the commitment problem.

the conditions of the trade are satisfied.[11]

This requires that a contract must be *feasible*, as defined below:

Definition 1 (Feasibility Condition)**.** *A contract is* feasible *if at the time of agreement, the terms of the contract can be settled immediately.*

Equilibrium. Given a settlement system, a Perfect Bayesian equilibrium is a set of traders' offer strategies, acceptance strategies, and settlement strategies such that:

1. Traders' offer and acceptance strategies maximize their expected payoffs;
2. Traders' settlement strategies maximize their conditional expected payoffs;
3. Traders' beliefs are consistent with Bayes' Rule.

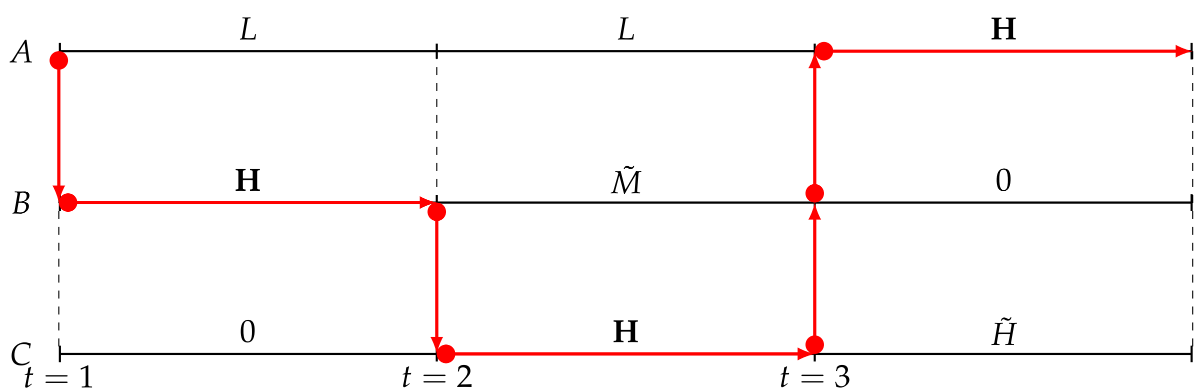

Figure 2: **Optimal allocation of asset.** This figure shows the optimal asset allocation between A, B, and C over $t = 1,2,3$, which is achieved through two contracts: $\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$. The red indicates ownership and transfer of the asset between traders. In the beginning of $t = 1$, A starts with the asset, passes the asset to B, who holds it for one period. Then, B passes the asset to C, who holds it for one period to the end of $t = 2$. At the beginning of $t = 3$, the asset is transferred from C back to B to A, who holds it for the final period.

[11]The requirements for programming asset under the token system free of settlement risk are consistent with those found in Lee et al. (2022), which formally studies operational features of token settlement systems to achieve zero-settlement risk.

3 Equilibrium Trade and Settlement

The first-best allocation is a useful benchmark. The valuations of each trader lend themselves to a clear first-best allocation. Figure 2 depicts the optimal transfer of the asset. This corresponds to the asset being under ownership of B at $t = 1$, C at $t = 2$, and A or C at $t = 3$.

In this section we consider whether the first-best allocation can be achieved in a legacy and in a token settlement system. We derive the equilibrium under the legacy system in section 4.1. We solve the problem by backward induction, analyzing the settlement stage first, in section 4.1.1, and then the trading stage, in section 4.1.2., taking into account the possible settlement outcomes. We consider the equilibrium under the token system in section 4.2. Because the token system eliminates settlement uncertainty, the trading and settlement stage are no longer analyzed sequentially – instead, the incentives to trade directly relate to whether desirable transactions are achieved.

3.1 Equilibrium under the Legacy System

There are two channels through which the first-best allocation may not be attained in equilibrium. The first is a limited commitment problem and the second is a hold-up problem. The commitment problem arises because the legacy system relies on incentive compatibility of settlement actions. The fact that traders may strategically fail on their obligations in the settlement stage must be taken into account by the agents at the trading stage. The hold-up problem arises because the value that B creates by intermediating between A and C can exceed the value she derives from the ownership of the asset. C can exploit this situation by making a "low-ball" offer to B. Taking this risk into account, B may prefer not to intermediate. In the remainder of this section we consider each friction in turn.

In the legacy system, traders may find it ex-post optimal to renege on contractual obligations. In the context of the model, this problem arises when a trader chooses not to return an asset at the designated date for private benefits. We refer to this choice as *failing*. As an example, consider the case of C. Since $H > \Delta$, if C is able to acquire the asset at any date prior or equal to $t = 3$ and her value for the asset is H at $t = 3$, then C will renege on any promise to return the asset at $t = 3$, as the cost Δ is not sufficient to deter a fail:

Lemma 1 (Strategic Fail). *Suppose that C obtains the asset at $t = 2$ with a contractual obligation to return the asset at $t = 3$. C strategically fails on this promise if $\tilde{H} = H$.*

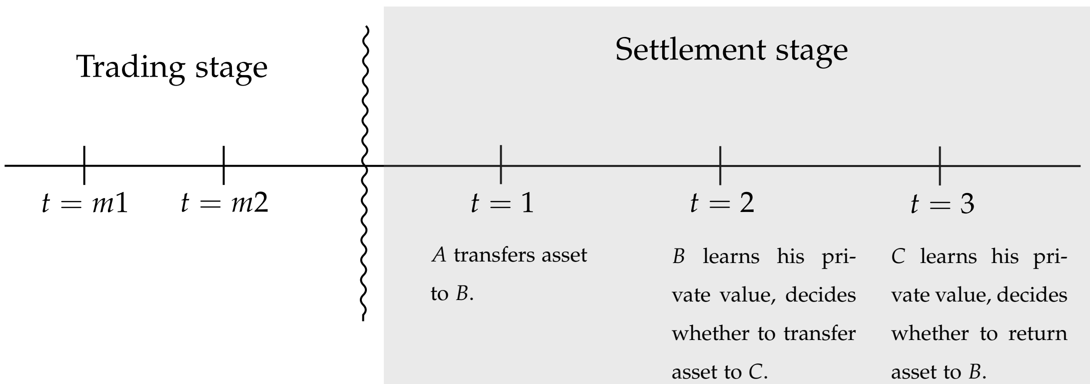

Figure 3: **Settlement in Legacy System.** This figure highlights key decisions that take place in the settlement stage conditional on $\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$. At $t = 3$, $\tilde{H}$ is realized, and C decides on whether to return the asset to B (which is then returned to A). At $t = 2$, $\tilde{M}$ is realized, and B decides on whether to transfer the asset to C.

A owns the asset initially and strictly prefers owning the asset at $t = 3$. As a result, A may only agree to lend the asset to B, knowing that B will lend it to C, if A believes that the likelihood of getting the asset back is sufficiently high, and/or if he is compensated for taking on the risk.

The second channel is a hold-up problem. B is the only trader who is matched with both the lender A and the borrower C.[12] For C to acquire ownership of the asset in $t = 2$, B must successfully negotiate two sides of the intermediation chain. As trades occur asynchronously, this requires B to "make markets" by completing one side of the chain in advance of the other, anticipating the outcome of that other trade.

It is common knowledge that gains from trade arise whenever the asset is transferred from A to C in $t = 2$. Furthermore, when B's valuation of $t = 2$ ownership ($E[\tilde{M}]$) is lower than the valuation of A or C, B's incentives are aligned with acting strictly as an intermediary, by running a "matched book." If the possibility of successfully building a matched book is low, B would be reluctant to make markets on behalf of the other traders.

Herein lies the potential for a hold-up problem. As B privately values the asset less than its owner A, she must pay a price in *excess* of her own valuation in order to acquire the asset on behalf of C. This creates the potential for C to strategically make a discounted offer, based on the possibility that B has already acquired the asset from A.

With these two tensions in mind, let us consider the equilibrium under the legacy system. We solve by backward induction. The optimal allocation requires traders to

[12]This is reminiscent of over-the-counter markets, where intermediaries commonly play an outsized role in reallocating assets between final buyers and sellers.

successfully negotiate two trades in the trading period: $\mathcal{C}_{AB}^{13}$, which transfers the asset from A to B for dates $t = 1, 2$, and $\mathcal{C}_{BC}^{23}$, which transfers the asset from B to C in $t = 2$. In order for these trades to occur in equilibrium, we must verify whether it is incentive compatible for traders to settle accordingly, and to enter such trades in the first place.

3.1.1 Settlement stage in the legacy system

We start by characterizing traders' optimal settlement strategies, taking as given a set of trades agreed upon in the trading stage. Figures 4 summarizes the settlement stage conditional on the different sets of contracts entered at the trading stage. Of particular interest is the case of $\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$, which if settled without fails, achieve the optimal asset allocation over $t = 1, 2, 3$.

Figures 4 and 5 summarizes the settlement actions and terminal payoffs given the sets of contracts arranged at the trading stage. The top panel outlines the case where traders have agreed to $\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$. The two main considerations are if and when B and C choose to fail. By Lemma 1, we already know that C chooses to fail whenever $\tilde{H} = H$. The other key settlement decision is made by B at $t = 2$. In the beginning of $t = 2$, B learns $\tilde{M}$. Just as C reneges on his contract if $\tilde{H} = H$, B may also want to renege on her contract if profitable. If $\mathcal{C}_{AB}^{12}$ has been negotiated, B's decision is whether to return the asset to A at the end of $t = 2$. If $\mathcal{C}_{BC}^{23}$ has been negotiated, B's decision is whether to pass the asset on to C, if $\mathcal{C}_{BC}^{23}$ has been negotiated. Failing is desirable only if holding onto the asset, net of the penalty Δ, is more profitable than transferring the asset.

Starting with the first panel ($\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$), B prefers to fail on C if:

$$\underbrace{\tilde{M} - \Delta}_{B\text{'s valuation}} \quad > \quad \underbrace{P_{BC}^{23} - \lambda_C \Delta.}_{\text{expected payoff from trade}} \tag{1}$$

B's payoff net of the penalty for failing is $\tilde{M} - \Delta$. By delivering the asset to C, B receives agreed upon price P_{BC}^{23} from C. However, because B must deliver the asset back to A at $t = 3$, whenever C fails on B, B has no choice but to fail on A, resulting in a "daisy chain" of settlement fails. Consequently, when B delivers the asset to C, she faces an additional expected cost $\lambda_C \Delta$. This implies that only when the price P_{BC}^{23} is sufficiently high, will B want to honor the trade ex post.

Lemma 2. *Suppose that B obtains the asset at $t = 1$ with a contractual obligation to send the asset to C at $t = 2$. B strategically fails on C if:*

$$P_{BC}^{23} < (\tilde{M} - \Delta) + \lambda_C \Delta. \tag{2}$$

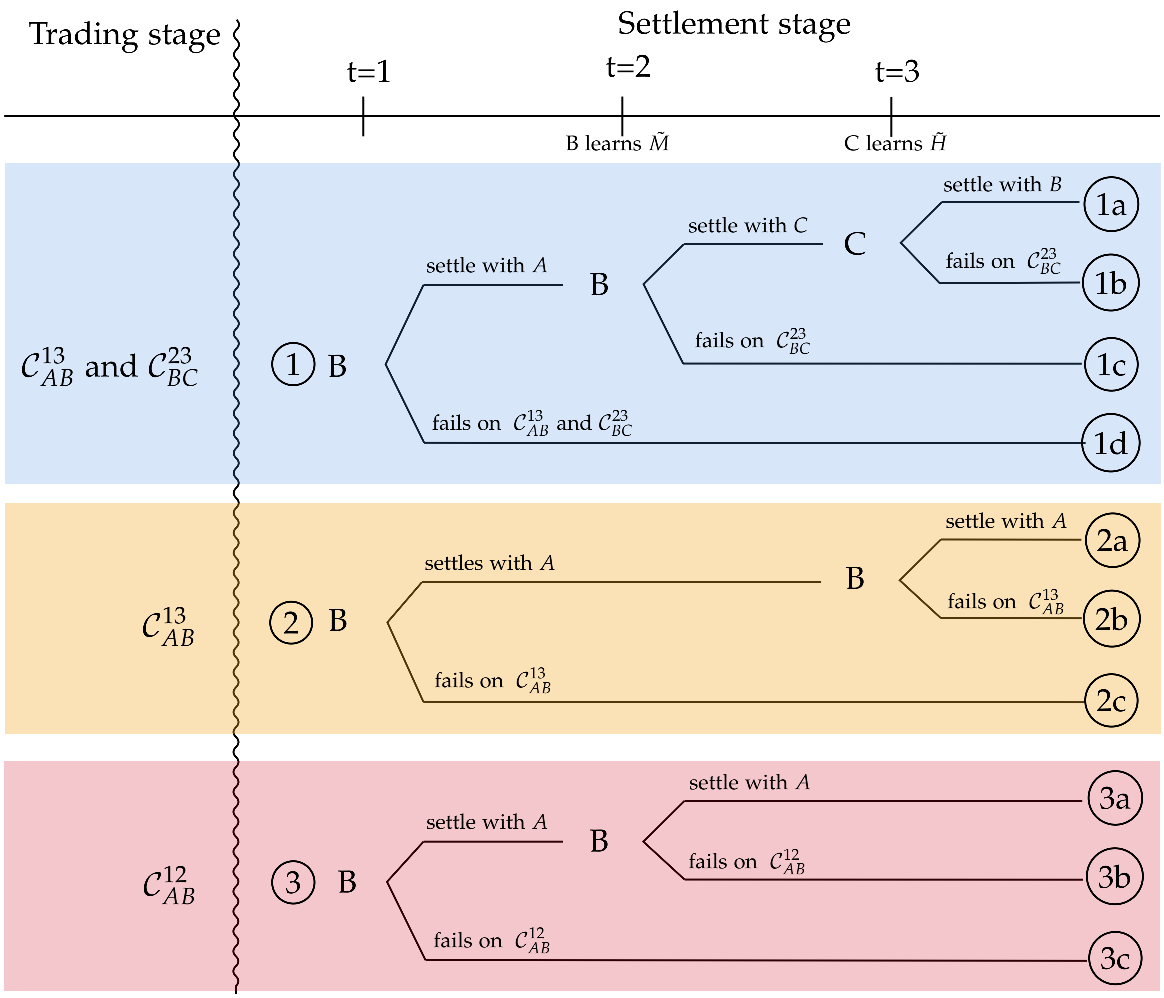

Figure 4: **Contracts and settlement actions under legacy system.** This figure summarizes the key settlement actions that arise at $t = 1, 2, 3$ given a set of contracts, specified on the left. B and C privately learn their $t = 2$ and $t = 3$ payoffs $\tilde{M}$ and $\tilde{H}$ in the beginning of $t = 2$ and $t = 3$, respectively. Private values factor into their settlement strategies, where $\tilde{M} = M$ with probability λ_M, and 0 otherwise; and $\tilde{H} = H$ with probability λ_H and 0 otherwise. The terminal payoffs for A, B, and C at the end of $t = 3$ is provided in Figure 5.

	Node	A's Payoff	B's Payoff	C's Payoff
$\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$	1a	$\mathrm{P}_{AB}^{13}+H$	$-\mathrm{P}_{AB}^{13}+H+P_{BC}^{23}$	$-\mathrm{P}_{BC}^{23}+H$
	1b	P_{AB}^{13}	$-\mathrm{P}_{AB}^{13}+H+P_{BC}^{23}-\Delta$	$-\mathrm{P}_{BC}^{23}+H+\tilde{H}-\Delta$
	1c	$\mathrm{P}_{AB}^{13}+H$	$-\mathrm{P}_{AB}^{13}+H+\tilde{M}-\Delta$	0
	1d	2L + H	-2Δ	0
$\mathcal{C}_{AB}^{13}$ only	2a	$\mathrm{P}_{AB}^{13}+H$	$-\mathrm{P}_{AB}^{13}+H+\tilde{M}$	0
	2b	P_{AB}^{13}	$-\mathrm{P}_{AB}^{13}+H+\tilde{M}-\Delta$	0
	2c	2L + H	-Δ	0
$\mathcal{C}_{AB}^{12}$ only	3a	$\mathrm{P}_{AB}^{12}+L+H$	$-\mathrm{P}_{AB}^{12}+H$	0
	3b	$\mathrm{P}_{AB}^{12}+H$	$-\mathrm{P}_{AB}^{12}+H+\tilde{M}-\Delta$	0
	3c	2L + H	-Δ	0

Figure 5: **Summary of traders' payoffs.** This figure shows the terminal payoffs for the three traders conditional on the set of contracts and settlement actions. Nodes starting with 1 correspond to $\mathcal{C}_{AB}^{13}$ and $\mathcal{C}_{BC}^{23}$; nodes starting with 2 correspond to $\mathcal{C}_{AB}^{13}$ only; and nodes starting with 3 correspond to $\mathcal{C}_{AB}^{12}$. Traders' payoffs from owning the asset at $t=1,2,3$ are as follows: A obtains L, L, H; B obtains $H, \tilde{M}, 0$; and C obtains $0, H, \tilde{H}$.

The third panel in Figure 4 considers the case where B entered contract $\mathcal{C}_{AB}^{12}$ with A. Under the contract, B makes two settlement actions. At $t = 1$, B must pay P_{AB}^{12} to A in order to acquire the asset from A at $t = 1$. At $t = 2$, B must return the asset to A. Failing to do either action constitutes a fail that results in cost Δ.[13] Similar to C's commitment problem, B may fail to return the asset at $t = 2$ if M is large enough, namely:

Lemma 3. *Suppose that B obtains the asset at $t = 1$ with a contractual obligation to return the asset to A at $t = 2$. B strategically fails on this promise if and only if $\tilde{M} = M$ and $M > \Delta$.*

Instead, if $M < \Delta$, retaining the asset at $t = 2$ is not profitable. It is straightforward to see that there exists a price P_{AB}^{12} (e.g. L) that A would accept, and B is strictly better off paying in exchange for the asset at $t = 1$.

We complete the analysis with the middle panel in Figure 4, which considers the case in which B entered contract $\mathcal{C}_{AB}^{13}$ with A but fails to negotiate $\mathcal{C}_{BC}^{23}$. As shown in the figure, B faces a settlement decision at $t = 1$ of whether to execute according to the contract $\mathcal{C}_{AB}^{13}$ or fail.[14] Failing is particularly costly to B, since in addition to the direct cost of failing, B would have to forgo payoff H associated with obtaining the asset at $t = 1$. Honoring the trade associated with $\mathcal{C}_{AB}^{13}$ is optimal if and only if

$$\underbrace{H + E[\tilde{M}] - P_{AB}^{13}}_{\text{expected payoff from settling}} > \underbrace{-\Delta.}_{\text{cost of failing}} \tag{3}$$

It will follow from subsequent analysis that under Assumption 1, this inequality will be always hold.

3.1.2 Trading stage in the legacy system

So far, we characterized traders' strategies in the settlement stage, taking as given possible trading outcomes. Traders bargain in the trading stage anticipating these strategies. As shown in Figure 6, two meetings occur sequentially at $t = m1, m2$, and the order of meetings is only known to B, who participates in both. Additionally, in the trading stage, B and C have not yet learned their valuations (e.g. the realized value of $\tilde{M}$ and $\tilde{H}$).

[13] In a settlement system like Fedwire securities, the seller of a security can fail by choosing not to send the security to the buyer, since all settlements are initiated by the seller. The buyer can fail by returning the security she has received to the buyer. This automatically undoes the transfer of cash that was associated with the initial settlement of the security.

[14] B can fail by returning the asset to A. Since we assumed that the system is DvP, this means that B received his cash back as if no transfer of assets had occurred.

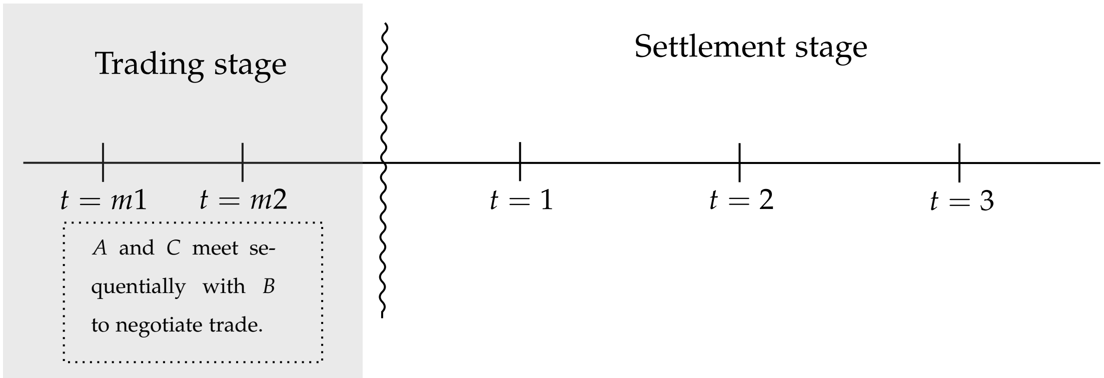

Figure 6: **Trading in Legacy System.** In the trading stage, A and C meet sequentially with B to negotiate trades. However, neither A nor C know the order of meetings.

The main question is if (and when) the optimal asset allocation is achieved in equilibrium. In the context of trading, this requires traders to successfully negotiated $\mathcal{C}^{13}_{AB}$ and $\mathcal{C}^{23}_{BC}$. With this in mind, let us consider a candidate equilibrium in which traders expect two trades, $\mathcal{C}^{13}_{AB}$ and $\mathcal{C}^{23}_{BC}$, to be agreed upon.

As a precursor, note that in the previous section, we showed that both B and C may want to fail if ex-post valuations are high. Since A anticipates the possibility of fails, A's reservation price takes into account expected losses arising from fails. This raises the possibility that traders may want to outright obtain "future" claims on the asset, instead of facing costs associated with fails. Doing so is preferred for B and C, if:

$$\lambda_B M - L \geq \lambda_B (M - L - \Delta) \tag{4}$$

$$\lambda_C H - H \geq \lambda_C (H - H - 2\Delta) \tag{5}$$

Under these cases, traders internalize their commitment problems. As we are interested in cases in which commitment problems persist in the settlement stage, we will preclude these cases by focusing on the parameter space where this is not desirable. This will also make clear that deviations from the set of optimal contracts $\mathcal{C}^{13}_{AB}$ and $\mathcal{C}^{23}_{BC}$ result from the underlying frictions.

Assumption 2. *$\lambda_B < \frac{L}{L+\Delta}$ and $\lambda_C < \frac{H}{H+2\Delta}$.*

With this, we begin by characterizing B's optimal strategy in the trading stage. In her meeting with A, B can either offer $\mathcal{C}^{12}_{AB}$, or $\mathcal{C}^{13}_{AB}$. Let A have beliefs whereby he expects B to intermediate the asset for C if $\mathcal{C}^{13}_{AB}$ is offered. Starting with $\mathcal{C}^{12}_{AB}$, A's reservation price, and hence, B's optimal strategy for P_{AB}^{12} is:

$$P_{AB}^{12} = \begin{cases} L + \lambda_B L & \text{if } M > \Delta \\ L & \text{if } M \leq \Delta \end{cases} \tag{6}$$

It follows from Lemma 3 that A expects B to fail to return the asset only if $M > \Delta$ and $\tilde{M} = M$. As such, A requires a settlement premium of $\lambda_B L$ if $M > \Delta$. B's payoff from P_{AB}^{12}, denoted $\pi_B(P_{AB}^{12})$, is given by:

$$\pi_B(P_{AB}^{12}) = \begin{cases} (H - L) + \lambda_B(M - L - \Delta) & \text{if } M > \Delta \\ H - L & \text{if } M \leq \Delta \end{cases} \tag{7}$$

For $\mathcal{C}_{AB}^{13}$, A expects that B will transfer the asset to C at $t = 2$. Hence, A's reservation price incorporates his expectation that if C obtains the asset, C will sometimes fail in accordance to Lemma 1, resulting in a loss of H with probability λ_C. Consequently, the lowest offer that is accepted by A for P_{AB}^{13} is:

$$P_{AB}^{13} = \underbrace{L}_{A\text{'s } t=1 \text{ valuation}} + \underbrace{L}_{A\text{'s } t=2 \text{ valuation}} + \underbrace{\lambda_C H}_{\text{settlement risk premium}} \tag{8}$$

This expression represents the reservation price at which A is willing to lend the asset for $t = 1, 2$. The first two term are A's payoffs from holding onto the asset at $t = 1, 2$, respectively. A also anticipates that B lends the asset to C, in which case he may not reacquire the asset. The last term represents the *settlement risk premium* that A requires for this possibility. B's payoff from entering P_{AB}^{13}, denoted $\pi_B(P_{AB}^{13}, \cdot)$, depends on whether she chooses to *trade* or *not trade* with C (e.g. $\mathcal{C}_{BC}^{23}$), denoted T, NT:

$$\pi_B(P_{AB}^{13}, \cdot) = \begin{cases} H - P_{AB}^{13} + \max\{M - \Delta, P_{BC}^{23} - \lambda_C \Delta\} & \text{if } T \\ H + \lambda_B M - P_{AB}^{13} \Delta & \text{if } NT \end{cases} \tag{9}$$

There are two things to note with respect to $\pi_B(P_{AB}^{13}, T)$. First, the max operator takes into account B's option to fail on C later in the settlement stage. Second, whenever B trades with C, B expects C to fail sometimes. The final term incorporates the cost $\lambda_C \Delta$ associated with a "daisy chain" fail, i.e. the expected loss incurred by B when C fails.

With B's payoffs fully characterized, we can now consider B's trading strategy with

C, where the core tension of the model lies. There are two cases to consider: (1) when B matches with C first at $t = m1$, and (2) when B matches with C at $t = m2$, after trading with A.

Starting with the first case in which B matches with C first at $t = m1$, B chooses his trading strategy with A conditional on the trading outcome with C. Suppose that A believes B will enter $\mathcal{C}_{BC}^{23}$ with probability 1. B accepts an offer P_{BC}^{23} from C only if accepting C's offer at least matches her outside option, where B's outside option is given by the payoff she derives from no trade with C:

$$\pi_B(P_{AB}^{13}, T) \geq \max\{\pi_B(P_{AB}^{13}, NT), \pi_B(P_{AB}^{12})\} \tag{10}$$

The payoff from trading with C, which depends on C's offer price P_{BC}^{23}, must be greater than the continuation value of rejecting C's offer. The first term in the max operator represents B's expected payoff from entering $\mathcal{C}_{AB}^{13}$, and the second term represents B's expected payoff from entering $\mathcal{C}_{AB}^{12}$.

In the second case, B matches with C only after trading with A. Suppose that B already agreed with A to $\mathcal{C}_{AB}^{13}$ at $t = m1$ at price at some price P_{AB}^{13}. Since B has already acquired $t = 2$ ownership of the asset from A, B has two outside options: either to hold the asset at $t = 2$ herself, or fail at $t = 1$. B accepts an offer P_{BC}^{23} only if:

$$\pi_B(P_{AB}^{13}, T) \geq \max\{\pi_B(P_{AB}^{13}, NT), -\Delta\} \tag{11}$$

The term on the left hand side of the inequality represents B's payoff from accepting C's offer and corresponds to B's expected payoff in the top panel of Figure 4 (node 1). The term on the right hand side of the inequality represents B's payoff from rejecting C's offer, and corresponds to B's expected payoff in the middle panel of Figure 4 (node 2). The first term in the max operator is B's payoff from retaining the asset (node 2a) and the second term is the payoff when B fails on the contract with A at $t = 1$ (node 2b).

Note, Condition 10 is (weakly) stricter than Condition 11. Intuitively, B's trading strategy with C depends on the order of trades, and importantly, her bargaining position considerably weakens when B enters $\mathcal{C}_{AB}^{13}$ prior to negotiating with C on $\mathcal{C}_{BC}^{23}$. This feature previews a key consideration for C's offer strategy. In principle, C could offer a "fair" price that B would always accept in any order of meetings, by offering a price that satisfies Condition 10. Alternatively, C could offer a "hold-up" price that B would reject if they meet at $t = m1$, but would accept if they meet at $t = m2$, B having already acquired the asset on C's behalf.

We now analyze C's optimal offer strategy. C chooses his offer strategy without knowing whether his meeting with B is taking place at $t = m1$ or $t = m2$. Given the fact that B can reject an offer from C and, even if the offer is accepted, may fail to settle the trade with C, we can express C's expected payoff as:

$$Prob(B \text{ accepts})Prob(B \text{ settles}) \left(H - P_{BC}^{23} + \underbrace{\lambda_C(H - \Delta)}_{\text{net gain from retaining asset at } t=3} \right) \tag{12}$$

In this expression, the likelihood of B accepting the trade, and the probability that B settles as specified in the contract increase (weakly) in offer price P_{BC}^{23}, while the payoff conditional on successful trade and settlement decreases in P_{BC}^{23}. The lowest possible price that is accepted with positive probability must satisfy Condition (11), which corresponds to the case where B has already acquired the asset from from A.

A special case arises when B's $t = 2$ valuation $E[\tilde{M}]$ is sufficiently high. If retaining the asset at $t = 2$ is sufficiently profitable for B, then hold-up is not possible because $\pi_B(P_{AB}^{13}, NT) > \pi_B(P_{AB}^{13}, T)$ for any price $P_{BC}^{13} < E[\tilde{M}] + \lambda_C\Delta$. This corresponds to when λ_C is small and λ_B is large. When this is the case, C must offer a price in excess of the price B pays to A:

Lemma 4. *Suppose that B's $t = 2$ valuation of the asset is greater than that of A's $t = 2$ valuation ($E[\tilde{M}] > L$), and C's limited commitment problem is not too severe ($\lambda_C < \frac{E[\tilde{M}]-L}{H+\Delta}$). B accepts offer $\mathcal{C}_{BC}^{23}$ only if $P_{BC}^{23} \geq E[\tilde{M}] + \lambda_C\Delta$.*

When $E[\tilde{M}] > L$ and $\lambda_C < \frac{E[\tilde{M}]-L}{H+\Delta}$, we have that C's optimal offer is to match B's outside option $E[\tilde{M}]$ plus the daisy chain premium $\lambda_C\Delta$. Furthermore, since $L > M - \Delta$, B does not fail on C at this price point. This pins down equilibrium trade under the conditions laid out in Lemma 4:

Proposition 1 (Equilibrium with no hold-up)**.** *Suppose that $\lambda_C < \frac{E[\tilde{M}]-L}{H+\Delta}$ and $E[\tilde{M}] > L$. In equilibrium, the optimal trades $\mathcal{C}_{AB}^{12}$ and $\mathcal{C}_{BC}^{23}$ are achieved in equilibrium.*

When the conditions for Lemma 4 are violated, the price that B must pay to acquire the asset on behalf of C is strictly greater than his private valuation, i.e. $\pi_B(P_{AB}^{12}) > \pi_B(P_{AB}^{13}, NT)$. This opens up the possibility for C is to hold-up B if profitable. For the remainder of the section, we work through the parameter space where $\lambda_C > \frac{E[\tilde{M}]-L}{H+\Delta}$ and $E[\tilde{M}] < L$.

First, since $\pi_B(P_{AB}^{12}) > \pi_B(P_{AB}^{13}, NT)$, observe that the Condition 10 reduces to:

$$P_{BC}^{23} \geq \begin{cases} L + \lambda_C H + \lambda_C \Delta + \lambda_B(M - L - \Delta) & \text{if } M > \Delta \\ L + \lambda_C H + \lambda_C \Delta & \text{if } M \leq \Delta \end{cases} \tag{13}$$

We show that for any price satisfying Condition 13, B will not find it desirable to fail ex-post:

Lemma 5. *If Condition 10 holds, then Condition 1 is satisfied.*

With this, there are (at most) three candidate offer prices for P_{BC}^{23}, which we summarize below:

1. Fair price (Condition 13);

$$\begin{cases} L + \lambda_C H + \lambda_C \Delta & \text{if } M \leq \Delta \\ L + \lambda_C H + \lambda_C \Delta + \lambda_B(M - L - \Delta) & \text{if } M > \Delta \end{cases} \tag{14}$$

2. Hold-up price with no fail (Condition 11);

$$\begin{cases} \lambda_B M + \lambda_C \Delta & \text{if } M \leq \frac{1}{1-\lambda_B}\Delta \\ (M - \Delta) + \lambda_C \Delta & \text{if } M > \frac{1}{1-\lambda_B}\Delta \end{cases} \tag{15}$$

3. Hold-up price (Conditions 1 and 11);

$$\begin{cases} \lambda_B M + \lambda_C \Delta & \text{if } M \leq \frac{1}{1-\lambda_B}\Delta \\ \frac{\lambda_B}{1-\lambda_B}\Delta + \lambda_C \Delta & \text{if } M > \frac{1}{1-\lambda_B}\Delta \end{cases} \tag{16}$$

The fair price represents the minimum price P_{BC}^{23} that B always accepts. In the fair price (Equation 14), the first two terms on the right hand side represent the expected opportunity cost for A to lend the asset to B, given that B will lend the asset to C. The third term represents the expected cost of a daisy chain fail for B. As λ_C increases, C must pay a steeper price in order to acquire the asset with certainty, because of the expected cost of a fail on both A and B. Notably, when B also has a limited commitment problem (i.e. $M > \Delta$), her reservation price has an additional term $\lambda_B(M - L - \Delta)$, which is strictly negative. By Lemma 3, when A and B trade in isolation, B is sometimes tempted to fail ex-post whenever $\tilde{M} > \Delta$. By virtue of entering an arrangement to intermediate the asset to C (for which if B fails is costly), B credibly commits to settling with A. The gains from commitment are internalized by C's fair price offer.

Hold-up prices (Equations 15 and 16) are prices accepted by B conditional on having already entered $\mathcal{C}^{13}_{AB}$. For $M < \Delta$, Equations 15 and 16 are identical and pinned down by Condition 11, which is simply B's private valuation and daisy chain premium:

$$\lambda_B M + \lambda_C \Delta. \tag{17}$$

When $M > \Delta$, C can potentially offer prices that internalize B's settlement strategy in the settlement stage. The minimum price at which B chooses not to fail even when $\tilde{M} = M$ must satisfy:

$$P^{23}_{BC} - \lambda_C \Delta \geq \lambda_B M \tag{18}$$

$$P^{23}_{BC} - \lambda_C \Delta \geq M - \Delta. \tag{19}$$

The minimum price is $(M - \Delta) + \lambda_C \Delta$ if $M > \frac{1}{1-\lambda_B}\Delta$, and otherwise $\lambda_B M + \lambda_C \Delta$. Alternatively, C could offer a price that takes into account that B fails if $\tilde{M} = M$, which solves:

$$(1 - \lambda_B)(P^{23}_{BC} - \lambda_C \Delta) + \lambda_B (M - \Delta) = \lambda_B M, \tag{20}$$

and yields $P^{23}_{BC} = \frac{\lambda_B}{1-\lambda_B}\Delta + \lambda_C \Delta$. This price is the minimum hold-up price with fails if and only if $\lambda_B M$ only if $M > \frac{1}{1-\lambda_B}\Delta$.

So, when would C be tempted to offer anything other than the fair price that allows B to intermediate the asset for C without incurring losses? The main tradeoff in C's offer strategy is between (1) the likelihood of trade and settlement and (2) the payoff conditional on trade.

Since C does not know whether B has met with A prior to their meeting, C cannot make an offer contingent on the order of matches. In a candidate equilibrium where B always enters $\mathcal{C}^{13}_{AB}$, a hold-up price is only accepted when B meets with A first, which occurs with probability $\frac{1}{2}$. With probability $\frac{1}{2}$, B has not yet met with A and will reject such an offer. Despite this, C will find it optimal to offer the hold-up price instead of the fair price if his commitment problem is too severe.[15] The next result summarizes the optimal offer strategy of C, taking as given that B successfully trades $\mathcal{C}^{13}_{AB}$ with A when matched:

[15]It is straightforward to verify that B never finds it optimal to renege ex-post on C after accepting an offer as $M \leq \Delta$.

Lemma 6. *Suppose that C's limited commitment problem is severe ($\lambda_C > \frac{E[\tilde{M}]-L}{H+\Delta}$), and C expects that B enters contract $\mathcal{C}_{AB}^{13}$. C's optimal strategy is to offer a contract $\mathcal{C}_{BC}^{23}$ at the fair price if $\lambda_C < \bar{\lambda}(\lambda_B, M)$, and a hold-up price $\lambda_C > \bar{\lambda}(\lambda_B, M)$, for some threshold $\bar{\lambda}(\lambda_B, M)$.*

The offer strategy in Lemma 6 assumes that whenever B matches with A first, B acquires $t = 2$ ownership from A prior to matching with C. However, this is an equilibrium strategy *only* if B at least breaks even by doing so. When C finds it optimal to offer a hold-up price, B's net payoff from intermediating drops below zero, and B ex-ante strictly prefers $\mathcal{C}_{AB}^{12}$ (node 3) to $\mathcal{C}_{AB}^{13}$ (node 1) and intermediation breaks down:

Lemma 7. *Suppose that C's limited commitment problem is severe (i.e. $\lambda_C > \frac{E[\tilde{M}]-L}{H+\Delta}$) and B believes that C will offer a hold-up price. Then, B's optimal trading strategy with A is to offer contract $\mathcal{C}_{AB}^{12}$.*

We can combine Lemmas 6 and 7 to fully characterize the equilibrium. By Lemma 6, C's optimal offer, expecting that B will intermediate the asset (by entering $\mathcal{C}_{AB}^{13}$ with A), is to hold up B whenever λ_C is greater than $\bar{\lambda}$. In turn, Lemma 7 states that if B anticipates C to attempt a hold-up, B forgoes any attempt to intermediate the asset at all. Together this implies that whenever $\lambda_C > \bar{\lambda}$, the asset cannot be intermediated with probability 1 in equilibrium.

As an intermediary, B can weaken C's incentives to attempt a hold-up by buying the asset on behalf of C with probability less than 1. Specifically, whenever B matches with A first, B can offer $\mathcal{C}_{AB}^{13}$ with some probability $\mu < 1$, and $\mathcal{C}_{AB}^{12}$ otherwise. By doing so, B lowers the absolute probability that C can obtain the asset, but increases the relative probability of trade with the fair price (versus a hold-up price). As long as μ is sufficiently low, C's dominant strategy is to offer the fair price, which lets B break even in expectation from intermediation.

The next proposition summarizes the conditions under which, in equilibrium, the set of optimal trades are achieved under the legacy system:

Proposition 2 (Equilibrium under legacy system)**.** *Suppose that trade and settlement occurs under the legacy system. If C's commitment problem is not too severe (i.e. $\lambda_C < \bar{\lambda}(\lambda_B, M)$), there exists an equilibrium in which the optimal trades are achieved with certainty.*[16] *If C's limited commitment problem is severe (i.e. $\lambda_C < \bar{\lambda}(\lambda_B, M)$), C obtains the asset with probability $\frac{1}{2}(1 + \mu^*) < 1$, where μ^* decreases in λ_C and $\mu^* \in [0, 1)$.*

[16] In the legacy system, the trades are optimal, but the allocation may not be optimal ex-post due to the positive probability that C fails in $t = 3$, resulting in a deadweight loss of 2Δ.

In the legacy environment, trades are unrestricted by the feasibility of settlement. Instead, whether settlement actions consistent with the agreed-upon trades are carried out ultimately depends on traders' incentives, given the costs associated with failing to keep promises. As a result, settlement breaks down whenever the costs of failing are not high enough to deter traders from reneging on contractual obligations ex-post.

In the context of the model, C's payoff from retaining the asset at $t = 3$ is sometimes too large for the cost Δ to provide sufficient incentives to return the asset to B. Expecting C's commitment problem, both A and B require higher prices to compensate for the possibility of settlement to fail. This, however, makes a "low-ball" offer even more attractive. When λ_C is too large, intermediation completely breaks down as intermediating is no longer profitable for B.

At the same time, the complete decoupling between trade and settlement enables traders to enter into a contract without having to explicitly prove to their counterparty that they can fulfill the terms of that contract. In the context of the model, B is able to enter $\mathcal{C}_{BC}^{23}$ with C, regardless of whether he has already acquired $t = 2$ ownership from A. As a consequence, when B matches with C, B preserves private information about whether he has met with A or not.

Remarkably, our model illustrates a positive interaction that arises when both B and C are subject to commitment problems.

Proposition 3. *Let the cutoff value $\bar{\lambda}(\lambda_B, M)$ for when $M < \Delta$, $M \in (\Delta, \frac{\Delta}{1-\lambda_B})$, and $M \geq \frac{\Delta}{1-\lambda_B}$ be denoted $\bar{\lambda}^1$, $\bar{\lambda}^2$, and $\bar{\lambda}^3$ respectively.*

$$\bar{\lambda}^1 < \bar{\lambda}^2 < \bar{\lambda}^3. \tag{21}$$

That is, B's commitment problem soothes issues arising from C's commitment problem.

Proposition 3 highlights a striking interaction between B and C's commitment problems. As mentioned earlier, when $M > \Delta$, B must pay A a settlement premium $\lambda_B L$ for $\mathcal{C}_{AB}^{12}$. By intermediating the asset to C, B is able to resolve the temptation to ex-post fail. Intermediation raises the cost of failing to B, and works as an ex-ante commitment mechanism. The gains from trade arising from the commitment mechanism are reflected in the fair price when $M > \Delta$, and thus improves C's payoff from offering the fair price. As a result, $\bar{\lambda}^1 < \bar{\lambda}^2, \bar{\lambda}^3$.

Second, when $M > \frac{\Delta}{1-\lambda_B}$, C must offer a hold-up price that either rises to ensure B does not fail (e.g. hold-up price with no fail), or is accepted but risks a fail with probability λ_B (e.g. hold-up price with fail). In this way, B's commitment problem lowers C's payoff from offering a hold-up price and $\bar{\lambda}^2 < \bar{\lambda}^3$.

Consequently, in the true sense of B acting as an intermediary, we see that B's role as an effective intermediary is enhanced by the ex-post opportunities presented to her, which in isolation normally create issues. In the context of our model, we show it relaxes inefficiencies arising commitment problems in the legacy environment.

Generally, an important takeaway is the *interplay* between the trading system and the settlement system. Trading occurs asynchronously, with little transparency over the history of trades. Taken in isolation, both the opacity of a trading system and the reliance of a settlement system on ex-post incentive compatibility could be viewed as suboptimal, from the standpoint of market design. However, we show that, when paired, this combination is fundamental to facilitating the efficient transfer of ownership between multiple traders. Even with the potential for a hold-up problem, traders successfully agree to an interdependent set of contracts, as long as the limited commitment problem is not too severe (i.e. λ_C is small enough).

3.2 Tokenized Market

In a token system, traders can program the asset to guarantee the future settlement of the trade they are negotiating. This commitment technology implies that settlement occur regardless of whether an trader would like to strategically fail ex post.

Let us revisit the limited commitment problem posed by C's ex-post incentive to retain the asset in $t = 3$. Suppose that, during the trading stage, A and B meet at $t = m1$ and agree to some contract $\mathcal{C}_{AB}^{13}$. In a token environment, a corresponding program is jointly submitted by A and B, which instantly transfers $t = 1, 2$ ownership of the asset from A's to B's, while guaranteeing that the ownership of the asset is transferred back to A at $t = 3$. By entering $\mathcal{C}_{AB}^{13}$ A relinquishes any ownership (and thus control) of the asset for dates $t = 1, 2$ the moment they trade; concurrently, B immediately gains the right to enter any transfer of $t = 1, 2$ ownership of the asset.

At $t = m2$, B and C meet and agree to some contract $\mathcal{C}_{BC}^{23}$. Since B acquired control over $t = 2$ ownership of the asset, $\mathcal{C}_{BC}^{23}$ is feasible. C instantly gains the right to enter any transfer of $t = 2$ ownership of the asset. However, the asset is programmed to return to A at date $t = 3$, and C exercises no control over the asset beyond $t = 2$. In this way, the token system resolves settlement uncertainty arising from C's limited commitment problem. In a similar vein, B's commitment issue is also no longer a problem.

Despite its clear advantage over the legacy system in resolving settlement uncertainty, the token system poses two new issues that were not present under the legacy system. They both arise due to the requirement that contracts must be feasible.

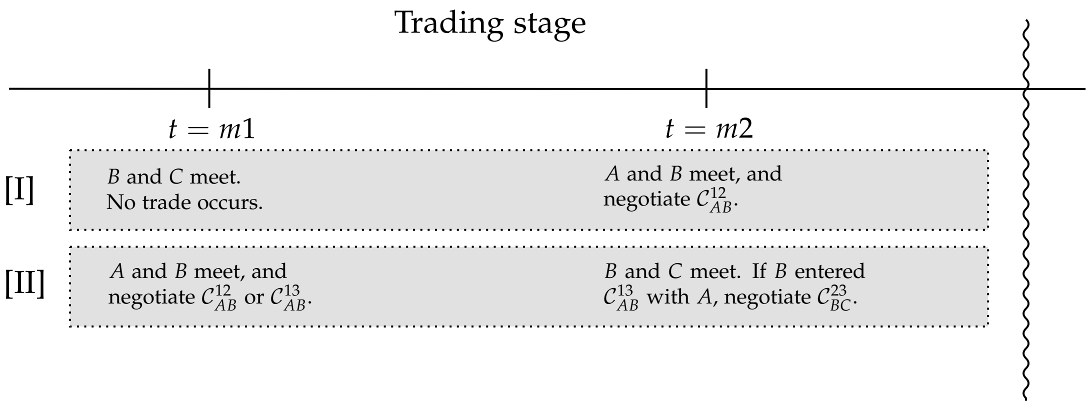

Figure 7: **Trading in Token System.** This figure outlines the events under the two possible sequence of meetings. In sequence [I], B meets with C, then with A. In sequence [II], B meet with A, then with C.

Critically, when B meets with C, B must reveal to C about whether he has $t = 2$ ownership of the asset. In the token system, a contract is feasible only if the seller of the asset already holds the ownership rights of the asset for the date at which the asset must settle. As such, when B and C negotiate a contract, both traders can verify whether the terms of negotiation are feasible. In effect, C can verify at the time of trade that B holds the asset, since otherwise the program corresponding to $\mathcal{C}_{BC}^{23}$ would not be permissible.

This information, which was not revealed in the legacy system, exacerbates the possibility of a hold-up. In contrast to the legacy system, C chooses his offer P_{BC}^{23} conditional on verifying that B has already acquired $t = 2$ ownership of the asset. With this certainty, the optimal offer strategy of C is *always* to offer B's reservation price, $E[\tilde{M}]$. The revelation of information regarding the order of trades magnifies the hold-up problem that was possible in the legacy system but not always binding. The hold-up problem now can directly prevent desired allocations from being achieved in equilibrium:

Theorem 3 (Equilibrium with Tokens)**.** *Suppose that trade and settlement occurs under the token system. For $\lambda_B > \frac{L}{M}$, there exists an equilibrium where B and C enter $\mathcal{C}_{BC}^{23}$ with probability $\frac{1}{2}$. For $\lambda_B < \frac{L}{M}$, C is unable to acquire the asset in equilibrium.*

The second issue is an asynchronicity problem that arises from a dissonance between random matching and the ownership condition required by the token system. In order for B to enter a contract $\mathcal{C}_{BC}^{23}$ with C, B must at the time of their match own the rights to the asset at $t = 2$. The asynchronous nature of trading means that B does not always trade with A in time to trade with C. This implies that the order of trades directly affects whether the asset can be intermediated by B, and namely B and C can only trade with each other $\frac{1}{2}$ of the time:

Corollary 1. *$\mathcal{C}_{BC}^{23}$ is a feasible contract only if B matches with A first and obtains $t = 2$ ownership of the asset.*

An implication is that the intermediation chain between A, B, and C can only arise with at most probability $\frac{1}{2}$, when B matches with A at $t = m1$, before matching with C at $t = m2$.[17]

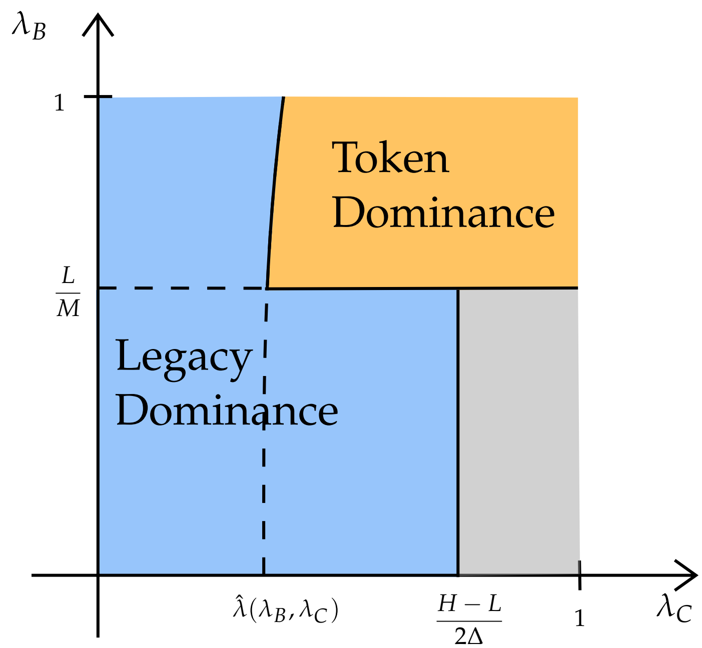

Figure 8: **Relative efficiency of settlement systems.** This plot shows the relative efficiency of the equilibrium under the legacy system and the token system for λ_B and λ_C, which each capture the degree to which a hold-up or limited commitment problem exist. Parameters are set at $L = 3, H = 12, M = 5, \Delta = 6$. Efficiency is greater under the legacy system in the blue region, the token system in the orange region, and equivalent in the grey region.

There are two things to note. First, due to the requirement of matching orders, the equilibrium with tokens fails to achieve ex-ante first-best allocations. This result, which arises due to the random match sequence, *is not*, in general a problem when trade and settlement are segregated, as in a typical legacy system. This points to an efficiency loss that can arise when immediate settlement is implemented. Second, the hold-up problem becomes acute with C's revelation of B's ownership of the asset (and the order of trades). In fact, when A values $t = 2$ ownership of the asset strictly less than L, C will

[17] Of course, this inefficiency is borne directly from our simplifying assumption that the sequence of matches are random. One could consider an environment where B could take (costly) actions to endogenously determine the order of matches. As discussed earlier, this alone will not materially improve the outcomes in token system as it does not address the hold-up problem.

always finds it profitable to offer a low price, which B accepts conditional on owning the asset. However, anticipating this, B opts not to acquire the asset in the first place, thus thwarting intermediation. The next theorem summarizes the relative efficiency between a legacy and token system:

Theorem 4 (Relative Efficiency)**.** *Suppose that $\lambda_B < \frac{L}{M}$. Then, efficiency is greater under the legacy system if $\lambda_C < \frac{H-L}{2\Delta}$, and equivalent when $\lambda_C > \frac{H-L}{2\Delta}$. If $\lambda_B > \frac{L}{M}$, then efficiency is greater under the token system if $\lambda_C > \hat{\lambda}$ and lower when $\lambda_C < \hat{\lambda}$, for some threshold $\hat{\lambda} > \bar{\lambda}$.*

The main takeaway is that token systems are not unambiguously superior to the legacy system. Rather, our model highlights two key issues that arise in decentralized, heavily intermediated markets. First, the scope of intermediation *drops* under tokenization, as intermediaries' valuations play a much more significant role in their ability to assist with the reallocation of the asset. Second, while tokenization can eradicate settlement uncertainty when two parties can agree to a desirable trade, the fact that each trade must be feasible at the time of trade puts undue emphasis on the need for the sequence of meetings to coincide with the order of intermediation.

Our results motivate a natural question on what solutions could address potential inefficiencies arising with token systems. The two key sources of inefficiency arise due to the revelation of information regarding ownership at the time of trade. Our results might suggest that another solution is to relax the conditions of trade, so that traders could enter trades without revealing this information. At first glance, it seems like it could resolve the issues pertinent to the token system. In the context of our model, this would be akin to enabling B and C to enter a state-contingent contract, whereby a trade is executed only if at the end of the trading stage, B's account holds $t = 2$ ownership of the asset.

However, allowing for such a contract re-introduces a commitment problem and with it settlement fails. A small modification of the model suffices using an argument related to that made by (Lee et al., 2022). Suppose that B's valuation of the asset at $t = 2$ for a range between 0 and H, and B learns it after the meeting at $t = m2$. The state-contingent contract introduces a "gap", whereby B now has the option to forgo his trade with C by failing to satisfy the conditions of trade. In particular, even if B obtains $t = 2$ ownership of the asset from A, if B wants to hold onto it for himself, then B simply needs to hold ownership in a separate account unknown to C. This means that B honors the trade with C only when his $t = 2$ valuation is realized as low. It is straightforward to see that, in equilibrium, C would offer a price less than H, which means that, generically, trade fails with a positive probability. In other words, there is an implicit form of settlement failure that arises due to ex-post strategic considerations.

Proposition 4. *Consider a modified token system that permits state-contingent programs. Efficiency improves only if* $H \geq M(1+\lambda_B)$. *Otherwise, settlement fails occur with probability* λ_B. *Furthermore, if* $\lambda_B > \frac{1}{2}$, *then C is strictly worse off due to the introduction of state-contingent programs.*

An overaching question is whether token systems should incorporate certain levels of settlement risk tolerance.[18] In the exercise above, we demonstrate that relaxing the set of permitted programs, which opens the market to the possibility of settlement fails, can sometimes be beneficial. However, as Proposition 4 highlights, even small changes to the environment can lead to new commitment issues that can worsen overall efficiency.

4 Joint Determination of Trading and Settlement Systems

Our results show that token systems may sometimes reveal too much information in environments where optimal allocations depend on intermediaries. A broad takeaway from our analysis is the importance for settlement to be compatible with the trading environment. In our baseline model, *intermediated trading* is assumed, such that any transactions between A and C require B in between. The underlying rationale is that for markets have adopted intermediated trading to overcome various (unmodeled) frictions.[19] Thus, end-sellers and end-buyers, like A and C, may prefer trading through B rather than directly.

We demonstrated a synergy between intermediated trading and the legacy settlement. In contrast, we find incompatibility with the token system, due to both the trade asynchronicity and information problems. This dichotomous result strongly suggests the need for the joint consideration of trading and settlement in the design of financial market systems.

This section takes a step to make this insight concrete, by considering the optimal design of financial markets in totality. We consider a simple alternative trading system that directly and simultaneously matches end-seller and end-buyers for each unit-period claim, which we refer to as *direct trading*.

Direct Trading. Consider the following alternative trading stage. Instead of sequential matching, we assume that at the trading stage (e.g. $t = m1$), B and C each simultaneously

[18] In a related study, Danos et al. (2020) consider the design of programs to that use flexible settlement to improve allocations.

[19] There is a large literature that specifically studies why intermediation occurs; offering an exact reason in our setting is not necessary for the results, nor within the scope of this paper.

post ultimatum offers on $t = 1$ and $t = 2$ ownership to A, i.e. $\mathcal{C}_{Aj}^{12}$ and $\mathcal{C}_{Aj}^{23}$. After receiving offers, A chooses the offers (if any) to accept at $t = m2$. To account for unmodeled frictions that create a natural preference toward trading through an intermediary, we assume that A incurs a cost $\phi \geq 0$ whenever trading directly with C.[20]

4.1 Equilibrium Efficiency with Direct Trading

The legacy settlement strengthens intermediation through two channels. The legacy system allows B to negotiate C without revealing trading history. Second, the legacy system permits B to fail, and when B's ex-post options improve, so does her ability to intermediate. These two channels dampened the potential for hold-up problems to actualize in equilibrium.

Direct trading offers a direct solution to the potential for breakdown in trade arising from hold-up problems. End-buyers (i.e. B and C) must now offer prices that satisfy the reservation price of the end-seller (i.e. A). As such, one might expect direct trading to unambiguously improve outcomes.

However, in contrast with intermediated trading, a daisy-chain settlement fail can now arise from B's commitment problem. Whenever A enters the pair of contracts $\mathcal{C}_{AB}^{12}$ and $\mathcal{C}_{BC}^{23}$, A must transfer the asset between B and C at $t = 2$, effectively "intermediating" ownership. Then, whenever B fails to return the asset to A at $t = 2$, A also fails on his obligation to C. In contrast to the benefits of B's commitment problem with intermediated trading, B's commitment problem drags efficiency with direct trading.

As a result, direct trading is more efficient in the legacy system only if ϕ is below some cutoff ϕ^{legacy}, and additionally, the cutoff ϕ^{legacy} is lower when B is prone to fail ($M > \Delta$):

Proposition 5 (Direct Trading with Legacy System)**.** *There exists a cutoff ϕ^{legacy}, such that efficiency is greater with intermediated trading than with direct trading with the legacy system if $\phi > \bar{\phi}^{legacy}$. Furthermore, $\bar{\phi}^{legacy}$ is lower when $M > \Delta$.*

Next, consider direct trading in the token system. Direct trading improves outcomes in a token system in two critical ways. First, it resolves asynchronicity issues, since C is able to directly match with A. Second, without any potential for a hold-up problem, C is able to make a credible offer for $t = 2$ ownership of the asset to A, negating potential for issues arising from strategic behavior. As such, direct trading unambiguously improves efficiency in the token system as long as cost ϕ is not too high:

[20]While we view this as the most natural, other cost structures work as well.

Proposition 6 (Direct Trading with Token System)**.** *There exists a cutoff ϕ^{token}, such that efficiency is greater with intermediated trading than with direct trading in the token system if $\phi > \phi^{token}$, where:*

$$\phi^{token} = \begin{cases} H - L & \text{if } \lambda_B M < L \\ \frac{1}{2}(H - \lambda_B M) & \text{if } \lambda_B M \geq L \end{cases} \tag{22}$$

As foreshadowed in the discussion for Propositions 5 and 6, direct trading disproportionately improves outcomes for the token system relative to the legacy system. It also implies generally:

Corollary 2. $\phi^{legacy} \leq \phi^{token}$.

As a final result, we consider the relative efficiency of four systems: (1) the intermediated-legacy system; (2) the direct-legacy system; (3) the intermediated-token system; and (4) the direct-token system. Following Proposition 6, as long as $\phi \leq H - L$, a direct-token system dominates intermediated-token system. Second, even if $\lambda_C < \bar{\lambda}$, a direct-token system dominates a direct-legacy system, because it achieves the same set of trades without ex-post fails. In contrast, relative efficiency between an intermediate-legacy system can dominate a direct-token system is determined by the cost associated with fails $(2\lambda_C \Delta)$ and ϕ, summarized below:

Theorem 5 (Optimal Design)**.** *Let $\phi \leq H - L$ and $\lambda_C < \bar{\lambda}$. Efficiency is greatest for:*

$$\begin{cases} \textit{Intermediated trading with legacy system} & \text{if } \phi \geq 2\lambda_C \Delta \\ \textit{Direct trading with token system} & \text{if } \phi < 2\lambda_C \Delta \end{cases} \tag{23}$$

Theorem 5 offers a rebalanced perspective on the merits of considering a token system. If Theorem 4 showed that gains from direct applications of tokenization to resolve settlement uncertainty in heavily-intermediated markets can be underwhelming and even backfire, Theorem 5 states that gains can be significant when the token system is paired with a compatible trading environment.

5 Conclusion

This paper studies how tokenization affects equilibrium trade in a theoretical model of an over-the-counter market. Tokenization has clear advantages: We illustrate how tokenization it can eliminate a limited commitment problem, by committing settlement actions at the time that contracts are forged. Collapsing trade and settlement, however, comes at a cost. We show that doing so necessitates that traders reveals more private information relative to the traditional environment. This creates a hold-up problem and may destory an intermediation chain necessary for efficient outcomes.

Whether a settlement protocol is efficient is intricately tied to the whether it is paired with a congruent trading mechanism. Due to the potential for decentralization, tokenized markets have been viewed as particularly disruptive for over-the-counter markets. However, some features are not amenable to the current market structure, which depends highly on intermediaries to facilitate complex intermediation chains. Our paper offers a concrete illustration of this problem.

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Proofs

Proof of Lemma 1. Follows from text. □

Proof of Lemma 4. Note that $E[\tilde{M}] > L + \lambda_C(H + \Delta)$ for $\lambda_C > \frac{E[\tilde{M} - L]}{H + \Delta}$, which requires $E[\tilde{M}] > L$ since $\lambda_C > 0$. □

Proof of Lemma 5. First, note that if $M < \Delta$, then Condition 1 trivially holds. Let $M > \Delta$. With Condition 10, we have:

$$
\begin{aligned}
P_{BC}^{23} - \lambda_C \Delta &\geq L + \lambda_C(H + \Delta) + \lambda_B(M - L - \Delta) && (24)\\
&\geq (1 - \lambda_B)\underbrace{(L + \Delta - M)}_{>0} + \lambda_C(H + \Delta) + (M - \Delta) && (25)\\
&> \tilde{M} - \Delta, && (26)
\end{aligned}
$$

which implies Condition 1. □

Proof of Lemma 6. Following the argument in the text, it suffices to consider whether C finds it optimal to make an offer which is accepted with probability 1, or $E[\tilde{M}] + \lambda_C \Delta$, which is accepted with probability $\frac{1}{2}$ conditional on B already having acquired $t = 2$ ownership from A. Consider a price at which B always accepts, which requires

Condition 10. The minimum price satisfying the condition is $L + \lambda_C H + \lambda_C \Delta$. C's payoff for offering $L + \lambda_C H + \lambda_C \Delta$ is greater than offering $E[\tilde{M}] + \lambda_C \Delta$ if:

$$
\begin{aligned}
& H - (L + \lambda_C H + \lambda_C \Delta) + \lambda_C (H - \Delta) && (27) \\
\geq & \frac{1}{2}\left(H - (E[\tilde{M}] + \lambda_C \Delta) + \lambda_C (H - \Delta)\right) && (28)
\end{aligned}
$$

Reorganizing this inequality, we get the inequality holds only if $\lambda_C < \frac{\frac{1}{2}H - L + \frac{1}{2}E[\tilde{M}]}{\frac{1}{2}H + (1-\frac{1}{2})2\Delta}$. Note, C's payoff is positive given price $L + \lambda_C H + \lambda_C \Delta$ only if $\lambda_C < \frac{H-L}{2\Delta}$, and $\frac{(1-\frac{1}{2})H - L + \frac{1}{2}E[\tilde{M}]}{\frac{1}{2}H + (1-\frac{1}{2})2\Delta} < \frac{H-L}{2\Delta}$. Also, note that $\frac{(1-\frac{1}{2})H - L + \frac{1}{2}E[\tilde{M}]}{\frac{1}{2}H + (1-\frac{1}{2})2\Delta} > \frac{E[\tilde{M} - L]}{H + \Delta}$. Hence, for some threshold $\bar{\lambda} \equiv \frac{\frac{1}{2}H - L + \frac{1}{2}E[\tilde{M}]}{\frac{1}{2}H + \Delta}$, C offers $L + \lambda_C(H + \Delta)$ for $\lambda_C < \bar{\lambda}$, and $E[\tilde{M}] + \lambda_C \Delta$ otherwise. □

Proof of Lemma 7. Note that given $P^{23}_{BC} = L + \lambda_C(H + \Delta)$, B breaks even in expectation. This implies that any lower price violates B's participation condition, and the optimal strategy for B is to make offer $P^{12}_{AB} = L$ to A. Hence if $\lambda_C > \bar{\lambda}$, B's optimal offer strategy is $P^{12}_{AB} = L$. □

Proof of Theorem 2. We first show existence of an equilibrium where traders agree to $\mathcal{C}^{13}_{AB}$ and $\mathcal{C}^{23}_{BC}$, independent of the order of matches when $\lambda_C < \bar{\lambda}$. Assume $\lambda_C > \bar{\lambda}$. A always accepts B's offer $P^{13}_{AB} = 2L + \lambda_H$, which is equal to his reservation price taking into account C's strategic fail as outlined in Lemma 1. Under Lemma 6, C's offer strategy is $P^{23}_{BC} = L + \lambda_C(H + \Delta)$, which B accepts conditional on having or expected to enter $\mathcal{C}^{13}_{AB}$, since it is equal to his reservation price and a premium $\lambda_C \Delta$ associated with the daisy chain fail.

Next, we show that when when $\lambda_C > \bar{\lambda}$, there does not exist an equilibrium where C obtains the asset with probability 1. By Lemma 6, conditional on B always entering $\mathcal{C}^{13}_{AB}$ with A, C's optimal offer strategy is $P^{23}_{BC} = E[\tilde{M}] + \lambda_C \Delta$, which violates B's participation condition. Following Lemma 7, B's dominant strategy to choose $P^{12}_{AB} = L$. Together this implies that C fails to obtain the asset with certainty. We show that there is an equilibrium where C obtains the asset with some probability $\mu^* \frac{1}{2} + (1 - \frac{1}{2})$ for some $\mu^* \in [0, 1)$. Consider a candidate equilibrium in which B enters $\mathcal{C}^{13}_{AB}$ with probability μ, and $\mathcal{C}^{12}_{AB}$ otherwise if matched with A first, and enter $\mathcal{C}^{13}_{AB}$ if he accepts $\mathcal{C}^{23}_{BC}$ when matched with C first. C maximizes his payoff from offering $P^{23}_{BC} = L + \lambda_C(H + \Delta)$ if:

$$
(\mu\frac{1}{2} + (1 - \frac{1}{2}))[H - (L + \lambda_C H + \lambda_C \Delta) + \lambda_C(H - \Delta)] \geq \mu\frac{1}{2}[H - (\lambda_B M + \lambda_C \Delta) + \lambda_C(H - \Delta)] \tag{29}
$$

Note that the inequality becomes monotonically tighter as μ increases, and holds when $\mu \to 0$. This implies that there exists some μ' such that equality holds. Let μ^* be given by $\max\left\{0, \frac{H-L-2\lambda_C\Delta}{L+\lambda_C H-\lambda_B M}\right\}$. Since for price $P^{23}_{BC} = L + \lambda_C(H+\Delta)$, B exactly breaks even, B is indifferent between any μ conditional on $P^{23}_{BC} = L + \lambda_C(H+\Delta)$. This establishes existence. □

Proof of Proposition 3. Cutoffs values for each region are given by:

$$\bar{\lambda}^1 = \frac{H - 2L + \lambda_B M}{H + 2\Delta} \tag{30}$$

$$\bar{\lambda}^2 = \frac{H - 2L - 2\lambda_B(M - L - \Delta) + \lambda_B M}{H + 2\Delta} \tag{31}$$

$$\bar{\lambda}^3 = \min\{\frac{H - 2L - 2\lambda_B(M - L - \Delta) + (M - \Delta)}{H + 2\Delta}, \frac{(1+\lambda_B)H - 2L - 2\lambda_B(M - L - \Delta) + \lambda_B\Delta}{(1-\lambda_B)H + 2(1+\lambda_B)\Delta}\} \tag{32}$$

Since $M - L - \Delta < 0$, $\bar{\lambda}^2 > \bar{\lambda}^1$. For $M > \frac{\Delta}{1-\lambda_B}$, $M - \Delta > \lambda_B M$ and $\lambda_B(H+\Delta) > \lambda_B M$, which implies $\bar{\lambda}^3 > \bar{\lambda}^2$. □

Proof of Corollary 1. Follows from text. □

Proof of Theorem 3. Suppose that the order of matches is $B - C$ and $A - B$. Since B does not own any rights to the asset when she matches with C, no contract is feasible. Hence, trade only occurs between $A - B$.

Suppose that the order of matches is $A - B$ and $B - C$. The order of trades is common knowledge, since trade between B and C requires B to own the asset. Suppose that B acquires rights to the asset for $t = 2$ with probability 1. Suppose that C offers $\lambda_C L$. Then, B accepts with probability 1. Note, however, that B must offer A at least L in order to obtain $t = 2$ ownership. Since doing so leads to negative profits, it is optimal for B to only acquire $t = 1$ ownership. Hence, in equilibrium C never obtains the asset if $\lambda_B < 1$, and there exists an equilibrium with intermediation only if $\lambda_B = 1$.

□

Proof of Theorem 4. First, consider when $\lambda_B < \frac{L}{M}$. In the legacy system, if $\lambda_C < \frac{H-L}{2\Delta}$, total payoff is given by $3H - 2\lambda_C\Delta$ for $\lambda_C < \bar{\lambda}$; $2H + (\mu^*\frac{1}{2} + (1-\frac{1}{2}))(H - 2\lambda_C\Delta) + (1-\mu^*)\frac{1}{2}L$ otherwise; In the token system, if $\lambda_C > \frac{H-L}{2\Delta}$, total payoff is $2H + L$. Hence, payoff is greater in the legacy system for $\lambda_C < \frac{H-L}{2\Delta}$ and equal otherwise.

Next, consider when $\lambda_B > \frac{L}{M}$. As before, in the legacy system, if $\lambda_C < \frac{H-L}{2\Delta}$, total payoff is given by $3H - 2\lambda_C\Delta$ for $\lambda_C < \bar{\lambda}$; $2H + (\mu^*\frac{1}{2} + (1-\frac{1}{2}))(H - 2\lambda_C\Delta) + (1-\mu^*)\frac{1}{2}L$

otherwise; if $\lambda_C > \frac{H-L}{2\Delta}$, total payoff is $2H + L$. In the token system, total payoff is $\frac{5}{2}H + \frac{1}{2}L$. Note that μ^* decreases in λ_C and $2H + (\mu^*\frac{1}{2} + (1-\frac{1}{2}))(H - 2\lambda_C\Delta) + (1-\mu^*)\frac{1}{2}L \to 2H + L$ as $\lambda_C \to \frac{H-L}{2\Delta}$. This implies that there exists some cutoff $\hat{\lambda} \equiv \frac{\mu^*}{1+\mu^*}\frac{H-L}{2\Delta} \in (\bar{\lambda}, \frac{H-L}{2\Delta})$ such that $2H + (\mu^*\frac{1}{2} + (1-\frac{1}{2}))(H - 2\lambda_C\Delta) + (1-\mu^*)\frac{1}{2}L = 2H + \frac{1}{2}H + (1-\frac{1}{2})L$. Hence, the token system dominates for $\lambda_C > \hat{\lambda}$.

□

Proof of Proposition 4. With state-contingent contracts, B can renege if $\tilde{M} = M$. C can ensure settlement occurs only with an offer at least as large as M. This is a dominant strategy only if: $H - M > \lambda_B(H - \lambda_B M)$, which requires $H > M(1+\lambda_B)$. Since conditional on settlement fail, C's optimal offer is $\lambda_B M$, C's expected payoff if $H \leq M(1+\lambda_B)$ is $\lambda_B(H - \lambda_B M)$, which is lower than in the baseline token system if $\lambda_B < \frac{1}{2}$. □

Proof of 5. Let $\lambda_B M < L$, and first suppose that $M < \Delta$. B and C's optimal offers are $P^{12}_{AB} = L$ and $P^{23}_{BC} = L + \lambda_C H + \phi$, respectively. Since B never fails, the total payoff is:

$$3H - \lambda_C\Delta - \phi, \tag{33}$$

which is greater than intermediated trading if:

$$\begin{cases} \phi < \lambda_C\Delta & \text{for } \lambda_C < \bar{\lambda} \\ \phi < \left(\frac{1+\mu^*}{2}\right)\lambda_C\Delta + \left(\frac{1-\mu^*}{2}\right)(H - \lambda_C\Delta - L) & \text{for } \lambda_C \geq \bar{\lambda} \end{cases} \tag{34}$$

where $\mu^* = \max\{0, \frac{H-L-2\lambda_C\Delta}{L+\lambda_C H-\lambda_B M}\}$.

Now, suppose that $M > \Delta$. Since B fails if $\tilde{M} = M$, B and C's optimal offers are $P^{12}_{AB} = L + \lambda_B(L+\Delta)$ and $P^{23}_{BC} = L + \lambda_C H + \phi$, respectively. The total payoff is:

$$2H + \lambda_B(M - 2\Delta) + (1-\lambda_B)(H - \lambda_C\Delta) - \phi. \tag{35}$$

which is greater than intermediated trading if:

$$\phi < \lambda_B(M - 2\Delta) - \lambda_B(H - \lambda_C\Delta) + \lambda_C\Delta \qquad \text{for } \lambda_C < \bar{\lambda} \tag{36}$$

$$\phi < \left(\frac{1+\mu^*}{2}\right)(\lambda_B(M-2\Delta) - \lambda_B(H-\lambda_C\Delta) + \lambda_C\Delta) \tag{37}$$

$$+ \left(\frac{1-\mu^*}{2}\right)((1-\lambda_B)(H - \lambda_C\Delta - L) - \lambda_B\Delta) \qquad \text{for } \lambda_C \geq \bar{\lambda} \tag{38}$$

for some $\mu^* \in [0, 1)$. Now suppose that $\lambda_B M > L$, and $M < \Delta$. Since the hold-up problem does not bind, efficiency depends wholly on whether ϕ is less than the daisy chain premium, i.e. $\phi < \lambda_C \Delta$. $\square$

Proof of Proposition 6. Suppose that $\lambda_B M < L$. Then, B and C's optimal offer strategy is L and L for $t = 1$ and $t = 2$ respectively. Furthermore, total payoff is $3H - \phi$, which dominates as long as $\phi < H - L$.

Now suppose that $\lambda_B M \geq L$. With direct trading, B offers L and $\lambda_B M$ for $t = 1$ and $t = 2$, and C offers $\lambda_B M + \phi$ for $t = 2$, yielding total payoff $3H - \phi$. This dominates intermediated trading if $\phi < \frac{1}{2}(H - \lambda_B M)$. Furthermore, this is more profitable for C as long as $H - \lambda_B M - \phi \geq \frac{1}{2}(H - \lambda_B M)$, which is true as long as $\phi < \frac{H - \lambda_B M}{2}$. $\square$