Automation and Polarization*

Daron Acemoglu† **Jonas Loebbing**‡

February 19, 2024

Abstract

We develop an assignment model of automation. Each of a continuum of tasks of variable complexity is assigned to either capital or one of a continuum of labor skills. We characterize conditions for interior automation, whereby tasks of intermediate complexity are assigned to capital. Interior automation arises when the most skilled workers have a comparative advantage in the most complex tasks relative to capital, and because the wages of the least skilled workers are sufficiently low relative to their own productivity and the effective cost of capital in low-complexity tasks. Minimum wages and other sources of higher wages at the bottom make interior automation less likely. Starting with interior automation, a reduction in the cost of capital (or an increase in capital productivity) causes employment and wage polarization. Specifically, further automation pushes workers into tasks at the lower and upper ends of the task distribution. It also monotonically increases the skill premium above a skill threshold and reduces the skill premium below this threshold. Moreover, automation tends to reduce the real wage of workers with comparative advantage profiles close to that of capital. We show that large enough increases in capital productivity ultimately induce a transition to low-skill automation and qualitatively alter the effects of automation—thereafter inducing monotone increases in skill premia rather than wage polarization.

Keywords: assignment, automation, inequality, polarization, tasks, wages.
JEL Classification: J23, J31, O33.

---

*We thank Juanita Jaramillo for excellent research assistance, David Autor, Chris Busch, Moritz Drechsel-Grau, Pascual Restrepo, four referees and the editor, Lance Lochner, for comments; and the Hewlett Foundation, the Joachim Herz Foundation and the Deutsche Forschungsgemeinschaft through CRC TR 190 (project number 280092119) for financial support.

†Massachusetts Institute of Technology, Department of Economics, E52-380, 50 Memorial Drive, Cambridge MA 02142; E-mail: daron@mit.edu.

‡LMU Munich, Center for Economic Studies, Schackstrasse 4, 80539 Munich; E-mail: jonas.loebbing@econ.lmu.de.

1 Introduction

Automation technologies, including specialized software tools, computerized production equipment, and industrial robots, have been spreading rapidly throughout the industrialized world. For example, the share of information processing equipment and software in overall investment in the US has increased from 3.5% to over 23% between 1950 and 2020 (BEA, 2021a), while the number of industrial robots per thousand workers has risen from 0.38 in 1993 to about 1.8 in 2017 (BEA, 2021b; IFR, 2018). There is growing evidence that these technologies have not just automated a range of tasks previously performed by workers and impacted the wage structure,[1] but also have led to *polarization* of employment and wages—meaning that the negative effects have concentrated on employment and wages in the middle of the wage distribution.[2] This pattern is intimately linked to the fact that many of the tasks that have been automated used to be performed by middle-skill workers.

There is no widespread agreement on why automation has been associated with polarization, however. One explanation, suggested by Autor (2014, 2015), is related to "Polanyi's paradox", as captured by Michael Polanyi's statement that "we can know more than we can tell" (Polanyi, 1966). Put simply, many of the manual and abstract tasks embed rich tacit knowledge, making them non-routine. Because routine tasks are *technologically* easier to automate and are performed by middle-skill workers located in the middle of the wage distribution, new automation technologies have displaced labor from middle-skill occupations and have had their most negative effects on middle-pay worker groups.

In this paper, we provide an alternative, complementary explanation: automation has focused on middle-skill tasks, because these are the most profitable ones to automate. Specifically, low-skill tasks can be performed at lower labor expenses, reducing the cost advantage of machines relative to humans.

To develop this point, we build an assignment model, combining elements from the seminal contributions by Tinbergen (1956), Sattinger (1975), and Teulings (1995, 2005), together with the model of tasks and automation in Acemoglu and Restrepo (2022). Workers are distinguished by a single-dimensional skill index, which is distributed over an interval normalized to $[0, 1]$ and tasks are also distributed over the unit interval. For expositional ease, we refer to higher-index tasks as more "complex" tasks.[3] Following the assignment literature, we assume that high-type workers have a comparative advantage in more complex tasks. Without automation, as in that literature, our model generates a monotone assignment pattern, with higher-skill workers performing more complex tasks. The distinguishing feature of our framework is that some tasks can be assigned to capital.

[1] On the spread of automation technologies over the last 80 years, see Autor (2015), Ford (2015), Acemoglu and Restrepo (2020), Graetz and Michaels (2018) and Acemoglu and Johnson (2023).

[2] The seminal contribution on the inequality and polarization effects of automation is Autor, Levy and Murnane (2003). For a recent study of the effects of automation technologies on US wage inequality, see Acemoglu and Restrepo (2022). Employment polarization from automation is also documented in Goos, Manning and Salomons (2009), Acemoglu and Autor (2011), and Autor and Dorn (2013). Acemoglu and Autor (2011) and Acemoglu and Restrepo (2022) provide evidence for wage polarization.

[3] We show below that more realistic configurations, where manual tasks that require skills that machines and algorithms do not currently fully possess can be incorporated into the model and can still be mapped into our single-dimensional tasks distribution.

Under the assumption that capital does not have a comparative advantage for the most complex task and some additional restrictions on capital productivity, we prove that the equilibrium will take one of two forms: (1) *interior automation*, where capital performs a set of intermediate tasks; or (2) *low-skill automation*, where capital takes over all tasks below a certain threshold.[4]

Interior automation is the configuration that leads to polarization, and we provide conditions under which automation is indeed interior. These conditions depend on the comparative advantage of low-skill workers relative to capital, the cost of capital, and supplies of different types of labor, which together determine the equilibrium wage distribution with and without automation. Intuitively, when equilibrium wages (without automation) are sufficiently low for low-skill workers, tasks in the bottom of the complexity distribution are very cheap and this reduces the profitability of performing them by capital. When this is the case, we also show that further automation leads to both wage and employment polarization. In our model, therefore, polarization is closely linked to the fact that wages are already low at the bottom.

In addition to establishing the existence of a unique competitive equilibrium and characterizing the conditions under which capital takes over tasks from the middle of the skill distribution, we provide a series of comparative static results for marginal (local) and large (global) changes in automation.

Our *first* result, clarifies the conditions under which automation is interior. In the baseline model, interior automation requires that wages at the bottom be sufficiently low relative to the productivity of low-skill workers and the effective cost of capital.[5] To further clarify the role of wages at the bottom, we consider a simple extension in which there is a minimum wage. In this case, interior automation requires that the minimum wage is not too high—otherwise, low-skill labor is too expensive and this induces low-skill automation. We complement this result by showing that a reduction in the supply of skills at the bottom raises low-skill wages and makes a transition to low-skill automation more likely.

Our *second* result is that, as already mentioned above, a further expansion of interior automation—for example, driven by a decline in the price of capital goods—creates employment and wage polarization. Employment polarization here simply means that human workers are squeezed into smaller sets of tasks at the bottom and the top. Wage polarization takes a more specific form: relative wage changes increase as a function of the distance between the task that a skill type performs and the boundaries of the set of automated tasks. As a result, we prove that skill premia *increase* among worker types performing more complex tasks than those that are automated and *decrease* among worker types performing less complex tasks than the automated ones. Put differently, interior automation hurts (relatively) workers that are closer to the set of automated tasks. This is intuitive in view of the fact that workers closer to this set used to have a stronger comparative advantage for tasks that are now automated.

Third, we characterize the effects of automation on the *level* of real wages for different types of workers. As in Acemoglu and Restrepo (2022), whether the real wage of a given skill group declines

[4]A third possibility is no automation, which is not interesting given our focus here and will be ruled out by assuming that capital productivity is sufficiently high to make some automation profitable in equilibrium.

[5]This is the sense in which our explanation is complementary to Autor's (2014, 2015) account—interior automation is also more likely to emerge when low-skill workers are more productive at low-complexity tasks relative to machines.

depends on competing *displacement* and *productivity* effects, though in this paper we will provide more explicit conditions. One noteworthy result in this context is that the larger is the initial set of tasks that are automated, the more likely are the real wages of all skill types to increase. Moreover, we show that the productivity gains from automation are *convex* in the price of capital goods. This implies that, as capital good prices (including costs of algorithmic automation) decline further, the productivity effect strengthens, ultimately eliminating negative wage impacts. These results, together, imply that the most negative consequences of displacement on workers will be at the early stages of the automation process.[6]

We present one more noteworthy result on wages, related to what we call "Wiener's conjecture", after Norbert Wiener's (1950) pioneering study of automation. Wiener claimed, "the automatic machine is the precise economic equivalent of slave labor. Any labor which competes with slave must accept the economic consequences of slave labor." This conjecture does not seem to have been fully borne out by economic models or the data. On the theory side, Zeira (1998) and Acemoglu and Restrepo (2018a) show that real wages will increase in the long run following automation. On the empirical side, although the real wages of low-education groups have declined over the last forty years, automation was also rapid in the 1950s and 1960s and during these decades wages for almost all demographic groups increased robustly. Our analysis suggests that Wiener's conjecture needs to be refined: different workers have different skills, and even if automated machines are like slave labor, they do not perfectly compete against all kinds of labor. Building on this intuition, we show that automation always reduces the real wages of worker types whose productivity schedule over tasks is sufficiently close to capital's productivity schedule (if such worker types exist, but they may not).

Fourth, we use the model to study global—as opposed to local—effects of automation, which result when there are large declines in costs of capital goods. We show that as long as these changes keep us in the region of interior automation, their effects are qualitatively the same as those of local changes. Ultimately, however, automation expands from the interior of the set of tasks to take over all low-skill tasks. When this happens, the pattern of polarization reverses. While an expansion in interior automation hurts workers in the middle of the skill distribution the most (and lowest-skill workers are to some degree sheltered), a switch from interior to low-skill automation has its most adverse effects on lowest-skill workers. Hence, our model predicts that as automation proceeds, its inequality implications may become worse, not just quantitatively but also qualitatively.[7]

Finally, we undertake a preliminary quantitative evaluation of the effects of automation in our framework. We calibrate our model parameters to match the 1980 US wage distribution and the effects of automation on different parts of the wage distribution between 1980 and 2016-17, as estimated in Acemoglu and Restrepo (2022). We show that our model matches these and a number of other moments

[6]Because production in our model exhibits constant returns to scale and the cost of capital is constant, average wages always increase following automation, and this structure is also important for the result that when sufficiently many tasks are automated, the effects on all wages are positive. The consequences of automation on wages are more negative when the price of capital is increasing in the stock of capital, or when we depart from constant returns to scale or competitive markets. See Moll, Rachel and Restrepo (2022) and Acemoglu and Restrepo (2024).

[7]On the other hand, if in this process capital productivity in already-automated tasks continues to increase, this makes negative wage level effects less likely.

in the US data quite well. We then consider (i) the impact of a further wave of automation (driven by a decline in the price of capital of the same magnitude as in 1980-2016); (ii) the implications of advances in artificial intelligence (AI), modeled as improvements in capital productivity in tasks that were previously not automated (as opposed to an across-the-board improvement in capital productivity or decline in the price of capital); and (iii) the effects of a minimum wage of $16 an hour. These exercises suggest that a further wave of automation similar to that of 1980-2016 would increase inequality by even more, because this change would have a bigger impact at the bottom of the wage distribution, given the patterns of comparative advantage implied by 1980-2016 data. AI is predicted to increase inequality by even more, because it expands the set of automated tasks significantly, but does not induce as much "deepening of automation"—meaning productivity improvements in already-automated tasks—as a uniform fall in the price of capital, which tends to increase productivity, benefiting labor of all types. Consequently, low-skill workers are harmed even more by artificial intelligence, while the highest-skill workers continue to benefit, because there is an increase in task services that are complementary to their skills. A sizable minimum wage like $16 an hour, on the other hand, compresses the wage distribution considerably, raising wages at the bottom and reducing them at the top. The wage declines at the top are due to the fact that the minimum wage reduces overall employment and output, which then depresses demand for all tasks.

Our paper is related to several contributions in both the assignment and automation literatures. In the assignment literature, early contributions include Tinbergen (1956), Rosen (1974), Heckman and Sedlacek (1985), and Sattinger (1975, 1993). Our model more closely builds on Teulings (1995, 2005), Teulings and Gautier (2004), Costinot and Vogel (2010), and Stokey (2018). More recently, Lindenlaub (2017) extends assignment models to settings where workers and jobs have multidimensional characteristics and derives comparative statics with respect to the degree of complementarity between skills and manual and cognitive skill requirements of jobs. The major difference between all of these papers and our work is the presence of capital that can perform some of the tasks, which allows for an analysis of automation. From a technical point of view, these papers impose comparative advantage (log supermodularity), which turns the problem into one of monotone assignment. Our analysis of automation relaxes overall supermodularity (though, for simplicity, we maintain log supermodularity between worker and job types).

In the automation literature, we build on earlier models where capital displaces workers in some of the tasks they used to perform. This literature and task-based models started with Zeira's (1998) seminal work and Autor, Levy and Murnane's (2003) empirical study of the polarization and inequality effects of automation. Zeira's model includes only one type of labor and does not focus on inequality implications of automation. Many subsequent works, including Acemoglu and Zilibotti (2001), Acemoglu and Restrepo (2018a,b), Berg, Buffie and Zanna (2018), Jackson and Kanik (2020), Jaimovich et al. (2021) and Hemous and Olsen (2022) allow only two types of workers, making it impossible to study wage polarization. Acemoglu and Autor (2011) study an economy with three types of workers and establish polarization when automation affects the middle group, but this structure does not allow a comprehensive analysis of the implications of different stages of automation on employment and wage patterns. Notable exceptions are Feng and Graetz (2020), Loebbing (2022) and Ales et al. (2024), who study task-based models with a continuum of labor types, though under more restrictive assumptions regarding comparative advantage between capital and labor. In particular, Feng and Graetz (2020) impose that automation is always interior, while Loebbing (2022) focuses on the case in which automation is always low-skill. This contrasts with our focus which is to understand *when* automation is interior and the conditions under which there is a transition to low-skill automation. Ales et al. (2024) study firm-level automation by imposing monotone comparative advantage of labor relative to capital in more complex tasks but also adding indivisibility of machines, which leads to a scale-dependent pattern of automation. At low scales, tasks of medium complexity are automated, whereas at high scales, indivisibilities become less relevant and they obtain low-skill automation because of the monotone comparative advantage pattern. Additionally, these papers do not contain our main characterization and comparative static results.

Acemoglu and Restrepo (2022) develop a general framework with multiple industries and multiple worker types to study the inequality effects of automation. In addition to providing empirical estimates of the effects of automation on US wage inequality, Acemoglu and Restrepo (2022) present a theoretical analysis of the implications of automation. Because their study lacks the specific structure imposed here (with one-dimensional heterogeneity both on the worker and job complexity side and comparative advantage between workers and tasks), it does not contain results on which tasks automation will take over. Rather, they provide equations that specify how wages of different groups will be affected as a function of the total set of tasks that are automated and the "ripple effects", which capture how different skill groups compete over marginal tasks. These ripple effects cannot be explicitly characterized given their assumptions and are studied empirically. Ocampo (2022) studies the assignment of a discrete set of workers and capital to a continuum of tasks. In his framework, matching is one-to-many and occupations emerge endogenously as bundles of tasks assigned to different workers. Like Acemoglu and Restrepo (2022), Ocampo (2022) does not impose enough structure to characterize the pattern of automation and the equilibrium comparative statics as functions of primitives. In contrast to these works, our analysis enables a full characterization of equilibrium and its comparative statics—including how the set of automated tasks expands and how substitution between different types of workers takes place.

The rest of the paper is organized as follows. Section 2 presents our baseline environment and defines a competitive equilibrium. Section 3 first establishes existence and uniqueness of equilibrium and some basic characterization results, and then studies the conditions under which automation is interior. Section 4 presents our main comparative static results for small changes in the cost of capital goods, thus deriving the employment and wage polarization implications of automation. Section 5 considers global changes in automation technology and their equilibrium consequences. Section 6 presents our quantitative analysis and the consequences of various counterfactual technological and institutional changes in our framework. Section 7 concludes. Appendix A includes several of the proofs omitted from the text, while the online Appendix B contains a few additional proofs and information on data and computational methods for our quantitative analysis.

2 Model

In this section, we introduce the basic economic environment, describe some of our assumptions and their motivation, and also define a competitive equilibrium.

2.1 Environment

We consider a static economy with a unique final good, Y, produced from a continuum of workers with skills $s \in [0,1]$ and a continuum of tasks $x \in [0,1]$. The production of the final good is given as a constant elasticity of substitution (CES) aggregate of tasks:

$$Y = \left[\int_0^1 Y_x^{\frac{\lambda-1}{\lambda}} dx \right]^{\frac{\lambda}{\lambda-1}} , \tag{1}$$

where Y_x is the amount of task x and $\lambda > 0$ is the elasticity of substitution.[8]

All labor types are inelastically supplied, with a density function of $l : [0,1] \to \mathbb{R}_{++}$ (which specifies the total endowment of each type of labor) and we assume that this density is continuous and has no mass points. We also assume that capital is produced out of final good with marginal cost $1/q$. We identify increases in q with greater capital productivity or equivalently lower prices of capital goods.

The task production functions are given by

$$Y_x = \int_0^1 \psi_{s,x} L_{s,x} ds + \psi_{k,x} K_x, \tag{2}$$

for all $x \in [0,1]$, where $\psi_{s,x} > 0$ and $\psi_{k,x} > 0$ denote the productivities of different factors in task x, and $L_{s,x}$ and K_x are, respectively, the amounts of labor of type s and capital allocated to the production of task x. We assume that the factor productivities $\psi_{s,x}$ and $\psi_{k,x}$ are twice continuously differentiable. Labor market clearing requires

$$\int_0^1 L_{s,x} dx = l_s \quad \text{for all } s \in [0,1] \tag{3}$$

and net output is

$$NY = Y - \frac{1}{q} \overline{K},$$

where $\overline{K} = \int_0^1 K_x dx$ is the aggregate capital stock and thus $\overline{K}/q$ is total capital expenditure. Net output is also equal to consumption in this economy.

[8] This production function imposes that all tasks have the same productivity/importance. This is without loss of generality, since we allow general task and factor-specific productivity functions $\psi_{s,x}$ and $\psi_{k,x}$ below, and any task-level differences can be subsumed into these.

2.2 Competitive Equilibrium

An allocation in this economy is given by a collection of density functions, $L = \{L_s\}_{s=0}^{1}$, where $L_s : [0,1] \to \mathbb{R}_+$ for each $s \in [0,1]$, and a capital allocation $K : [0,1] \to \mathbb{R}_+$. The density functions allocate labor supply of each type of labor to tasks, and the capital allocation function determines how much capital will be allocated to each task. This definition already incorporates nonnegativity constraints for all factors in all tasks. We describe an allocation with the shorthand $\{L, K\}$. For all $s \in [0,1]$, we define the set $X_s : \{x \,| L_{s,x} > 0\}$ as the set of tasks performed by labor type s in this allocation and $X_k = \{x \,| K_x > 0\}$. We also use the terminology that tasks in the set X_k are *automated*.[9]

We additionally designate two price functions: first, a wage function $w : [0,1] \to \mathbb{R}_+$, which determines the wage level, w_s, for each type of labor $s \in [0,1]$; and second, a task price function $p : [0,1] \to \mathbb{R}_+$, which determines the price p_x of each task $x \in [0,1]$.

A (competitive) equilibrium is defined as an allocation $\{L, K\}$ and prices $\{w, p\}$ such that final good producers maximize profits taking task prices as given, task producers maximize profits taking task and factor prices as given, and all markets clear. Task producers' profit maximization implies that wages equal marginal products of the relevant labor types, i.e.,

$$\begin{aligned} w_s &= p_x \psi_{s,x} \text{ for all } x \in X_s \\ w_s &\geq p_x \psi_{s,x} \text{ for all } x \in [0,1], \end{aligned} \tag{4}$$

while the cost of capital must be equal to the marginal product of capital, i.e.,

$$\begin{aligned} \frac{1}{q} &= p_x \psi_{k,x} \quad \forall x \in X_k \\ \frac{1}{q} &\geq p_x \psi_{k,x} \quad \forall x \in [0,1]. \end{aligned} \tag{5}$$

Final good producers' profit maximization in turn implies that task prices equal the marginal products of tasks in final good production, i.e.,

$$p_x = \left(\frac{Y}{Y_x}\right)^{\frac{1}{\lambda}} \text{ for all } x \in [0,1]. \tag{6}$$

We note that the first welfare theorem holds in our model and the equilibrium allocation maximizes net output (consumption) subject to labor market clearing (3).

2.3 Assumptions and Motivation

We now describe some of the assumptions we will use in our main analysis.

Since the economy exhibits constant returns to scale and can produce output from final goods, in principle its output may be unbounded. Our first assumption ensures that the cost of capital is not so low as to generate infinite output.

---

[9] A more stringent definition of automated tasks might additionally require that these tasks are not also performed by labor, i.e., $L_{s,x} = 0$ for all s. Under our Assumptions 2 and 3, the set of tasks that are performed by both labor and capital in equilibrium is of measure zero, and hence this distinction is not relevant.

Assumption 1 ***(Bounded output)*** *The cost of capital satisfies*

$$\frac{1}{q} > \frac{1}{q_\infty} = \left(\int_0^1 \psi_{k,x}^{\lambda-1}\, dx \right)^{\frac{1}{\lambda-1}}$$

(where the upper bound $1/q_\infty$ *is derived as the marginal product of capital if all tasks are performed by capital, using equations* (1)*,* (5) *and* (6)*).*

Our second assumption follows the assignment literature (e.g., Teulings, 1995, 2005, Costinot and Vogel, 2010) and imposes *comparative advantage.* This means, in particular, that the productivity advantage of higher skilled workers increases more than proportionately with the task index, among workers. We impose this assumption both to simplify the analysis and also to maximize the similarity of our benchmark environment to the previous literature, which will clarify that all of the new results here are driven by the automation margin.

Assumption 2 ***(Comparative advantage among workers)*** *For all* $s > s'$*, we have* $\psi_{s,x}/\psi_{s,x'} > \psi_{s',x}/\psi_{s',x'}$ *for all* $x > x'$*.*

Comparative advantage ensures that, without capital, the equilibrium will assign higher skilled workers to higher-indexed tasks. Note that we do not impose absolute advantage (higher skills are more productive in all tasks), because it is not required for our results.[10]

We next present a motivating example that provides a simple illustration of the comparative advantage aspect. This example has the additional advantage that it shows how multidimensional skills can be mapped into our setup with a one-dimensional skill index.

Example 1: Suppose that each task $x \in [0, 1]$ involves a combination of abstract and manual activities. Specifically, the productivity of a worker with skill level $s \in [0, 1]$ in task x will be a function of this worker's abstract and manual skills, denoted by the vector (a_s, m_s):

$$\psi_{s,x} = a_s^x m_s^{1-x}.$$

In the context of this example, a sufficient condition for comparative advantage is for workers' skill endowments (a_s, m_s) to satisfy

$$\frac{a_s}{m_s} > \frac{a_{s'}}{m_{s'}} \text{ for all } s > s'.$$

To see this, let

$$\Lambda = \log \frac{\psi(x,s)}{\psi(x,s')} = \log \frac{a_s^x m_s^{1-x}}{a_{s'}^x m_{s'}^{1-x}},$$

and by assumption, we have $\partial \Lambda/\partial x = [\log a_s - \log m_s] - [\log a_{s'} - \log m_{s'}] > 0$. A sufficient condition for absolute advantage, on the other hand, is for a_s to be strictly increasing and m_s to be non-decreasing in s.

[10] Since we refer to higher levels of the skill index s as "more skilled" workers, it is natural to think of wages being increasing in s and a simple way to ensure this would be to impose absolute advantage, i.e., $\psi_{s,x} > \psi_{s',x}$ for all $s > s'$ and all tasks x. But we do not formally impose this restriction.

Motivated by this pattern of comparative advantage, we will also refer to higher-index tasks as *more complex tasks*, as in Teulings (1995, 2005).

The other key dimension of our model concerns the productivity of capital relative to different labor types. Crucially, here, we do not assume overall supermodularity. However, it is convenient to put sufficient structure on the comparative advantage of capital to have a simple characterization of the set of tasks, X_k, that are assigned to capital. The next assumption achieves this.

Assumption 3 ***(Comparative advantage of capital)*** *For all $s \in [0,1]$, $\psi_{k,x}/\psi_{s,x}$ is quasi-concave in x.*

This assumption rules out situations in which the direction of comparative advantage for capital changes more than once for any given level of skill. Put differently, Assumption 3 allows some skill types to have a comparative advantage in lower-index tasks relative to capital and then again in higher-index tasks after a certain threshold. But it rules out more than one such switch. We prove in Proposition 2 that this is necessary and sufficient for the set X_k of tasks assigned to capital to be convex. We adopt Assumption 3 in the text for expositional simplicity. In Appendix A.2, we extend our main characterization result, Proposition 3 below, to the case where Assumption 3 is relaxed.

We next illustrate this assumption with the environment considered in Example 1.

Example 1 (continued) Assumption 3 is ensured in this example when $\psi_{k,x}$ is log concave in x. Since the productivity of each labor type is log linear in x, log concavity of capital productivity implies quasi-concavity of all relative productivity schedules $\psi_{k,x}/\psi_{s,x}$.

3 Characterization of Equilibrium and Interior Automation

In this section, we establish existence and uniqueness of a competitive equilibrium and study the conditions under which tasks from the middle of the skill distribution are automated.

3.1 Existence and Uniqueness

Proposition 1 ***(Existence and uniqueness)*** *Suppose Assumption 1 holds. Then, a competitive equilibrium always exists and is essentially unique in the sense that wage and price functions are uniquely determined. If in addition Assumptions 2 and 3 hold, the competitive equilibrium is unique.*

Existence follows from the fact that the competitive equilibrium maximizes net output, which is a continuous function of the allocation. Essential uniqueness, on the other hand, is a consequence of the fact that net output is a concave function of the allocation. The reason why the competitive equilibrium is essentially unique, but not fully unique under Assumption 1, is straightforward to see: some tasks may be produced at the same cost using different factors, creating indeterminacy of equilibrium allocations. Assumptions 2 and 3 rule out such indeterminacy: Assumption 2 imposes strict comparative advantage between any two types of labor and, together with Assumption 3, implies that on any subset of tasks of positive measure, there can be at most one type of labor with a productivity schedule parallel to capital's productivity schedule.[11]

3.2 Interior Automation

The next proposition confirms that, as mentioned above, Assumption 3 is enough to ensure that the set of tasks allocated to capital, X_k, is convex.

Proposition 2 ***(Convexity of assignment)*** *Suppose Assumptions 1 and 2 hold.*

1. *The allocation of labor across tasks is monotone, meaning that for any $s > s'$, if $L_{s,x} > 0$, then $L_{s',x'} = 0$ for all $x' \geq x$.*

2. *The set of automated tasks in equilibrium, X_k, is convex for all labor endowment functions and capital productivity levels* if and only if *Assumption 3 holds.*

The first part of this proposition confirms that the monotonicity obtained in assignment models with log supermodularity continues to hold in our model. The second part implies that we can focus on a convex set of automated tasks. A convex set of automated tasks leaves four feasible configurations:

1. *No automation*, where $X_k = \emptyset$ (because the cost of capital is too high).

2. *Interior automation*, where $X_k = [\underline{x}, \overline{x}]$ with $0 < \underline{x} < \overline{x} < 1$, and thus both the most complex and the least complex tasks are assigned to some labor types.[12]

3. *Low-skill automation*, where $X_k = [0, \overline{x}]$, with $0 < \overline{x} < 1$, and thus all tasks below a certain threshold of complexity are automated.

4. *High-skill automation*, where $X_k = [\underline{x}, 1]$, with $0 < \underline{x} < 1$, such that all tasks above a certain complexity threshold are automated.

We refer to the third configuration as low-skill automation, since capital takes over tasks that used to be performed by lower-skilled workers (as in the first part of Proposition 2), and analogously we refer to the fourth case as high-skill automation. The first case, no automation, is not of great interest given the focus of the current paper and we will assume below that the cost of capital is sufficiently low so as to ensure some automation.

[11] If a single type of labor has a productivity schedule that is parallel to capital's productivity schedule on the set of automated tasks, this does not create any indeterminacy in allocations because a single type of labor has no mass and is hence irrelevant for an allocation. Formally, an allocation is a collection of densities from an L^p space where any two densities that are equal almost everywhere are identified and represent the same allocation.

[12] In this statement, we impose that the boundary tasks $\underline{x}$ and $\overline{x}$ are performed by capital. This is for notational simplicity and is without loss of any generality.

We next study the conditions under which automation will be interior or low-skill. An analogous discussion applies to the distinction between interior and high-skill automation. The least complex task, $x = 0$, is cheaper to produce by the least skilled worker type, $s = 0$, than by capital if

$$\frac{w_0}{\psi_{0,0}} < \frac{1/q}{\psi_{k,0}}. \tag{7}$$

When inequality (7) is satisfied, we cannot have low-skill automation, and hence from Proposition 2, we must have interior automation. This condition is intuitive. It requires that the effective wage of the least skilled workers in the least complex task (wage divided by productivity) is less than the effective cost of capital in that task (cost of capital, $1/q$, divided by the productivity of capital in that task). Whether this condition is satisfied depends on the shape of comparative advantage schedules ψ, capital productivity q and the labor supply profile l (which jointly determine the equilibrium wage for the least skilled type, w_0). The following two conditions allow us to characterize the equilibrium both in cases where inequality (7) is satisfied and those where it is not.

Condition 1 ***(Local comparative advantage of capital)***

1. *The least skilled workers have local comparative advantage relative to capital in the least complex tasks,*
$$\frac{\partial \log \psi_{0,0}}{\partial x} < \frac{\partial \log \psi_{k,0}}{\partial x} .$$
2. *The most skilled workers have local comparative advantage relative to capital in the most complex tasks,*
$$\frac{\partial \log \psi_{1,1}}{\partial x} > \frac{\partial \log \psi_{k,1}}{\partial x} .$$

Condition 2 ***(Global comparative advantage of capital)***

1. *The least skilled workers have global comparative advantage relative to capital in the most complex tasks,*
$$\frac{\psi_{0,0}}{\psi_{k,0}} < \frac{\psi_{0,1}}{\psi_{k,1}} .$$
2. *The most skilled workers have global comparative advantage relative to capital in the most complex tasks,*
$$\frac{\psi_{1,0}}{\psi_{k,0}} < \frac{\psi_{1,1}}{\psi_{k,1}} .$$

Condition 1 imposes only *local* comparative advantage—comparing the least (most) skilled workers to capital around the least (most) complex tasks. This comparative advantage pattern does not have to hold globally. Condition 2 in contrast imposes *global* comparative advantage—comparing the least and most skilled workers to capital in the least versus most complex tasks.

Proposition 3 ***(Interior automation)*** *Suppose Assumptions 1-3 and Condition 1 hold. Then, there exist thresholds $q_0 < q_\infty$ and $q_m \in (q_0, q_\infty]$ such that:*

1. *For $q \leq q_0$, there is no automation, i.e., $X_k = \emptyset$.*

2. *For $q \in (q_0, q_m)$, automation is interior, i.e., $X_k = [\underline{x}, \overline{x}]$ with $0 < \underline{x} < \overline{x} < 1$.*

   Additionally:

3. *If Condition 2 holds, then $q_m < q_\infty$ and, for $q \geq q_m$, automation is low-skill, i.e., $X_k = [0, \overline{x}]$ with $0 < \overline{x} < 1$.*

4. *If Condition 2.1 is strictly violated but Condition 2.2 holds, then $q_m = q_\infty$ such that automation is interior for all $q \in (q_0, q_\infty)$.*

5. *If Conditions 2.1 and 2.2 are both strictly violated, then $q_m < q_\infty$ and, for $q \geq q_m$, automation is high-skill, i.e., $X_k = [\underline{x}, 1]$ with $0 < \underline{x} < 1$.*[13]

Proposition 3 is our first main result and provides a complete characterization of the different patterns of automation that can arise in our model. The first part of the proposition establishes that capital productivity has to cross some threshold level q_0 to induce some automation. The second part is the most important result of the proposition. It establishes that, when capital productivity crosses the automation threshold q_0, automation always starts in the interior (under Assumptions 1-3 and Condition 1).

The next three parts show that, when capital productivity increases further, three different scenarios are possible. Firstly, if Condition 2 holds—which means that both the least and the most skilled workers have global comparative advantage relative to capital in the most versus the least complex tasks—then automation transitions from interior to low-skill at some threshold level of capital productivity, denoted by q_m.[14] Secondly, if only the most skilled workers have global comparative advantage relative to capital in the most complex tasks, then automation remains interior indefinitely. Thirdly, if both the least skilled and the most skilled workers have global comparative advantage relative to capital in the least complex tasks, once capital productivity increases sufficiently automation transitions from interior to high-skill.

Figure 1 shows the assignment of tasks to labor and capital in the case of interior automation. Tasks in the set $X_k = [\underline{x}, \overline{x}]$ are assigned to capital, while the remaining tasks are performed by the labor types indicated on the vertical axis.

Inequality (7) provides further intuition for why automation may affect middle-skill occupations most. Fixing $\psi_{0,0}$ and treating the wage of the least skilled worker, w_0, parametrically, there are two ways in which this inequality is satisfied: either $1/(q\psi_{k,0})$ is high or w_0 is low. The first captures the economic forces proposed by Autor (2014, 2015): many of the tasks performed by lower-skill workers

[13]Note that a configuration where Condition 2.1 holds while Condition 2.2 is violated is not compatible with the comparative advantage among workers imposed in Assumption 2. In knife-edge cases where one of the two conditions is violated weakly, i.e., the respective inequality is replaced by equality, additional information about comparative advantage schedules is needed to determine the pattern of automation and we omit those cases to save space.

In Appendix A.1, we discuss the case where Condition 1 is violated.

[14]We characterize the threshold productivity q_m in Appendix A.3.3.

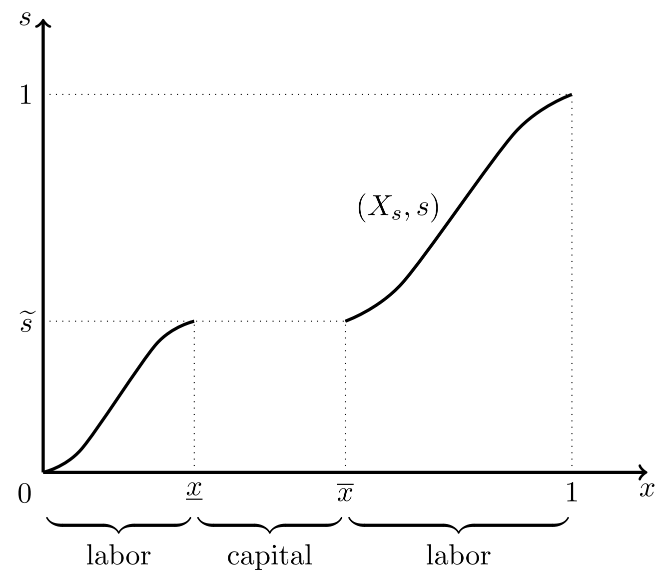

Figure 1: *Assignment of tasks to capital and labor.* Tasks in the set $X_k = [\underline{x}, \overline{x}]$ are assigned to capital, while the remaining tasks are assigned to the labor types indicated by the graph (X_s, s). In particular, tasks $x < \underline{x}$ are assigned to worker types $s < \widetilde{s}$, while tasks $x > \overline{x}$, are assigned to labor types $s > \widetilde{s}$.

may be hard to automate because they require a combination of tacit knowledge and manual dexterity. The second is what we have emphasized in the Introduction: wages at the bottom are too low to make automation economically profitable.

Proposition 3 clarifies that these two explanations are linked, because the wage is endogenous. However, they are also distinct, and one way of illustrating this is to consider variations in the wage at the bottom of the distribution, holding the other parameters of the model constant. The simplest way of doing this is by imposing a minimum wage in the model, which we discuss briefly in the next proposition. In the presence of a binding minimum wage $\underline{w}$, the equilibrium will involve rationing—some worker types may not be hired. This requires an obvious change in the definition of equilibrium, which we omit to save space. It is also straightforward to see that the set of rationed workers will always be of the form $[0, \underline{s}]$ (see Teulings, 2000). Except for rationing, the same equilibrium conditions as in our analysis so far apply. Then we have:

Proposition 4 ***(Minimum wages and automation)*** *Suppose Assumptions 1-3 and Condition 1 hold and let $q \in (q_0, q_m)$, so that in the competitive equilibrium without the minimum wage, we have interior automation. Now consider a minimum wage of $\underline{w} > 0$, which leads to the rationing of workers with skills in $[0, \underline{s}]$. If in addition we have*

$$\frac{\partial \log \psi_{\underline{s},0}}{\partial x} \geq \frac{\partial \log \psi_{k,0}}{\partial x}, \tag{8}$$

then inequality (7) is violated and we transition to low-skill automation.

Intuitively, without the minimum wage, labor performing low-skill tasks tends to be cheap, and this makes automating these tasks unprofitable, ensuring interior automation. When there is a binding minimum wage, skills at the bottom of the distribution become more expensive and this increases the profitability automating some of the tasks previously performed by low-skill workers (and also causes unemployment). It is also useful to observe the role of condition (8): without this condition, some of the workers with skill above $\underline{s}$ may find it profitable to take the lowest-complexity tasks. Notice also that this condition is compatible with Condition 1, since the comparison is for different skill levels. In fact, the juxtaposition of these two conditions highlights that in our model conditions for the comparative advantage of capital relative to labor are endogenous, since which skill type's productivity is compared to capital's productivity is determined in equilibrium.

An alternative way to increase wages at the bottom of the distribution is to reduce the labor supply of low-skill workers relative to the high-skill workers. We consider such a change in labor supply in the following proposition.

Proposition 5 ***(Labor supply and automation)*** *Suppose Assumptions 1, 2 and 3 as well as Conditions 1 and 2 hold, so that automation transitions from interior to low-skill at the threshold $q_m \in (q_0, q_\infty)$.*

Now consider a change in labor supply such that $\Delta \log l_s < \Delta \log l_{s'}$ for all $s < s'$ (an increase in the relative supply of more skilled workers). Then, the threshold q_m declines, $\Delta q_m < 0$. Thus, if $q \in (q_m + \Delta q_m, q_m)$, automation transitions from interior to low-skill in response to the labor supply change.

The proposition shows that an increase in the relative supply of high-skill workers can induce a transition from interior to low-skill automation. This confirms our intuition about the critical role of low-skill workers' wages: an increase in relative skill supply renders low-skill workers scarce and raises their wages, which then makes it more profitable to automate their jobs. We discuss the implications of labor supply changes for wage inequality in Proposition 12 below.

3.3 Characterization of Equilibrium

We next provide a characterization of equilibrium when Assumptions 1-3 hold and $q \geq q_0$ (so that there is automation in equilibrium). Under these assumptions, the set of automated tasks takes the form $X_k = [\underline{x}, \overline{x}]$ with $\underline{x} \leq \overline{x} \leq 1$. This leaves the sets $[0, \underline{x})$ and $(\overline{x}, 1]$ for labor, with workers with skills above a threshold $\widetilde{s} \in [0, 1]$ employed in $(\overline{x}, 1]$ and those below $\widetilde{s}$ employed in $[0, \underline{x})$.

By standard arguments from the assignment literature, the allocation of skills to tasks in either of the two sets $[0, \underline{x})$ and $(\overline{x}, 1]$ can be described by an assignment function $X : [0, \widetilde{s}) \cup (\widetilde{s}, 1] \to [0, \underline{x}) \cup (\overline{x}, 1]$ mapping skills to tasks.[15] The assignment function is differentiable (except at the threshold $\widetilde{s}$), strictly increasing and onto (see Costinot and Vogel, 2010). These properties are illustrated in Figure 1.

Condition (4) implies that every worker type is assigned to the task in which its marginal value product is maximized. Thus:

$$\log w_s = \log p_{X_s} + \log \psi_{s,X_s} = \max_x \{\log p_x + \log \psi_{s,x}\} \ .$$

[15] Recall that X_s was defined as the set of tasks performed by skill s, and thus in general X should be a correspondence. However, under Assumptions 1-3, X_s is a singleton and henceforth we treat X as a function.

An envelope argument then yields the differential equation

$$(\log w_s)' = \frac{\partial \log \psi_{s,X_s}}{\partial s} \quad \forall s \neq \widetilde{s}, \tag{9}$$

which we can think of as determining wages given assignment and a boundary condition (where $(\log w_s)'$ denotes the derivative of the function $\log w_s$ with respect to s). When Condition 1 holds and $q \in (q_0, q_m)$ such that automation is interior, the boundary condition is provided by the requirement that in both tasks $\underline{x}$ and $\overline{x}$, production must be equally costly with capital and skill $\widetilde{s}$:

$$\frac{w_{\widetilde{s}}}{\psi_{\widetilde{s},\underline{x}}} = \frac{1/q}{\psi_{k,\underline{x}}} \quad \text{and} \quad \frac{w_{\widetilde{s}}}{\psi_{\widetilde{s},\overline{x}}} = \frac{1/q}{\psi_{k,\overline{x}}}.$$

When automation is low-skill, we have $\underline{x} = 0$ and $\widetilde{s} = 0$, and the first equality becomes an inequality—$w_{\widetilde{s}}/\psi_{\widetilde{s},\underline{x}} \geq 1/(q\psi_{k,\underline{x}})$, so that it is weakly more costly to produce task $\underline{x} = 0$ with skill $\widetilde{s} = 0$ than with capital. The second equality still provides a boundary condition. So, taking logs, we have

$$\log w_{\widetilde{s}} = \log \psi_{\widetilde{s},\overline{x}} - \log \psi_{k,\overline{x}} - \log q \geq \log \psi_{\widetilde{s},\underline{x}} - \log \psi_{k,\underline{x}} - \log q, \tag{10}$$

with equality when automation is interior and with $\widetilde{s} = \underline{x} = 0$ when automation is low-skill. An analogous condition can be incorporated for the case of high-skill automation. Note further that when automation is interior, the wage function characterized by (9) and (10) has a kink point at $\widetilde{s}$, where the assignment function jumps upwards.[16]

Intuitively, equation (9) ensures that all workers find it optimal to sort into the tasks assigned to them. This requires that the marginal return to skill at any level s is given by the marginal productivity gain in the task assigned to s, X_s.

Next, we can combine the equilibrium conditions for wages (4), task prices (6), and task production (2) to obtain an expression for inverse labor demand,

$$w_s = Y^{\frac{1}{\lambda}} \psi_{s,X_s}^{\frac{\lambda-1}{\lambda}} L_{X_s}^{-\frac{1}{\lambda}},$$

where L_{X_s} is the marginal density of labor over tasks. A change of variable allows us to express this density in terms of the density of labor over skills, $X_s' L_{X_s} = l_s$, for $s \neq \widetilde{s}$. Using this relationship and rearranging, we obtain the labor demand curve as

$$\frac{l_s}{X_s'} = \frac{Y \psi_{s,X_s}^{\lambda-1}}{w_s^{\lambda}} \quad \forall s \neq \widetilde{s}. \tag{11}$$

The labor demand curve here takes the form of a differential equation for the assignment function, given wages. If automation is interior, the assignment function has two branches, one on $[0, \widetilde{s})$ and one on $(\widetilde{s}, 1]$. If automation is low-skill instead, only the upper branch exists. For the lower branch, the boundary condition is

$$\lim_{s \nearrow \widetilde{s}} X_s = \underline{x} \tag{12}$$

[16] The wage function is continuous at $\widetilde{s}$ because the productivity schedule $\psi_{s,x}$ is continuous in s. If there were an upwards jump in wages at $\widetilde{s}$, workers immediately below $\widetilde{s}$ would relocate to the tasks assigned to workers immediately above the threshold and increase their wage.

whereas for the upper branch, the boundary condition is given by

$$\lim_{s \searrow \tilde{s}} X_s = \overline{x}. \tag{13}$$

Intuitively, if labor demand in task X_s is high (e.g., because aggregate output is high or the wage of skill s is low), equation (11) requires that the density of labor supplied to task X_s is high as well. This is achieved by a shallower slope for the assignment function X_s', which means that more workers are squeezed into fewer tasks in the neighborhood of X_s.

Overall, we have a two-dimensional system of differential equations (and boundary conditions) for wages and assignment. This system fully characterizes equilibrium together with the production function (1), the capital allocation rule

$$K = \operatorname{argmax} \left\{ Y - \frac{\overline{K}}{q} \right\},$$

and the requirement that any task is assigned to some production factor. If automation is interior, this requires $X_0 = 0$ and $X_1 = 1$. If automation is low-skill, only $X_1 = 1$ is required.

Our characterization displays the two channels via which automation, as induced by a decline in the price of capital, affects wages and assignment. The first is a *displacement effect* as in Acemoglu and Restrepo (2022): a decline in the price of capital (an increase in q) reduces the boundary condition for wages (10) and hence, given assignment, the wage of worker type $\tilde{s}$. This implies, in particular, that workers directly competing with capital must either relocate to other tasks or accept a wage decline in proportion to the reduction of the price of capital. The second is a *productivity effect*, driven by the fact that a lower price of capital raises aggregate output Y. From equation (11), the productivity effect raises labor demand in all tasks proportionately and, for a given assignment, wages for all skill levels rise proportionately as well.

Finally, we can derive a simple expression for the share of capital in national income, which will be useful when discussing the productivity effects of automation. Combining the task production function (2), equation (5) and task prices (6) for the marginal product of capital, we obtain

$$\frac{1}{q} = Y^{\frac{1}{\lambda}} \psi_{k,x}^{\frac{\lambda-1}{\lambda}} K_x^{-\frac{1}{\lambda}}.$$

Then, solving for capital, integrating over $[\underline{x}, \overline{x}]$ and dividing by qY yields the share of capital in gross output as

$$\alpha_k = \frac{\overline{K}/q}{Y} = \Gamma_k q^{\lambda-1}, \tag{14}$$

where

$$\Gamma_k = \int_{\underline{x}}^{\overline{x}} \psi_{k,x}^{\lambda-1} \, dx$$

is the task share of capital—which is a productivity-weighted measure for the set of automated tasks. Equation (14) shows that a decline in the price of capital has two distinct effects on the capital share: a capital deepening effect (as captured by $q^{\lambda-1}$) the sign of which depends on whether tasks are complements or substitutes; and the effects following from the expansion of the task share of capital—which is the counterpart of the displacement effect on wages discussed above.

4 Local Effects of Automation

In this section, we assume that Assumptions 1-3 as well as Condition 1 hold and that $q \in (q_0, q_m)$, so that automation is interior. We then study the implications of a small decline in the price of capital goods (an increase in q), which will expand the set of automated tasks. Our main results characterize the polarization and inequality consequences of automation.

4.1 Employment Polarization

Proposition 6 ***(Automation and employment polarization)*** *Suppose Assumptions 1-3 and Condition 1 hold, let $q \in (q_0, q_m)$, and consider a small increase in the productivity of capital $d \log q > 0$. Then,*

$$d\underline{x} < 0 \quad \textit{and} \quad d\overline{x} > 0$$

(automation expands in both directions) and

$$dx_s < 0 \textit{ for all } s \in (0, \widetilde{s}) \quad \textit{and} \quad dx_s > 0 \textit{ for all } s \in (\widetilde{s}, 1)$$

(the assignment function shifts down below the set of automated tasks and shifts up above the set).

Moreover, if $\lambda \geq 1$, the labor share always decreases. If $\lambda < 1$, there exists a threshold for capital productivity $\hat{q} > q_0$ such that the labor share decreases if $q \in (q_0, \hat{q})$.

The first part of this proposition establishes that a (small) decline in the price of capital goods (or an increase in the productivity of capital) always expands the set of automated tasks on both sides, and relatedly, it shifts the assignment of workers further towards the two extremes of the task distribution, as shown in Figure 2. This result thus implies that the employment polarization pattern documented in Autor, Levy and Murnane (2003), Goos, Manning and Salomons (2009) and Acemoglu and Autor (2011) always applies so long as we consider a small increase in the set of automated tasks, starting from interior automation.[17]

The second part provides conditions under which the labor share declines. There are two channels via which automation affects the labor share (see also our discussion of equation (14) for the capital share). First, the expansion of the set of automated tasks, established in the first part of the proposition, always decreases the labor share. Second, productivity gains in tasks that are already-automated ("deepening of automation") decreases the labor share when tasks are substitutes ($\lambda \geq 1$) but raises it when tasks are complements ($\lambda < 1$). Yet even when tasks are complements, the proposition establishes that the expansion of the set of automated tasks dominates and the labor share declines in the initial stages of automation (when the productivity of capital is small enough).

[17] It is also straightforward to show that if there were technological constraints on what tasks could be automated (as in Acemoglu and Restrepo, 2018a) and these dictated that only tasks in the set $[\underline{x}, \overline{x}]$ could be automated, and we considered an expansion of the set with $d\underline{x} < 0$ and $d\overline{x} > 0$, then the same employment polarization result would hold.

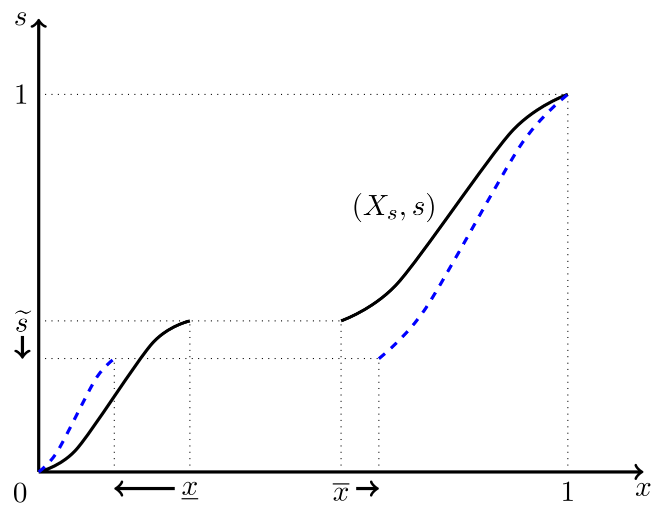

Figure 2: *Employment Polarization.* In response to a small increase in capital productivity, the set of automated tasks expands in both directions and workers move towards the extremes of the task distribution, here illustrated for the case with $d\widetilde{s} < 0$ and shown with the dashed blue curve.

4.2 Wage Polarization

The next proposition gives one of our most important results:

Proposition 7 ***(Automation and wage polarization)*** *Suppose Assumptions 1-3 and Condition 1 hold, let $q \in (q_0, q_m)$, and consider a small increase in the productivity of capital $d\log q > 0$. Then, there is wage polarization in the sense that skill premia increase above the threshold task $\widetilde{s}$ and decrease below this threshold. Or equivalently,*

$$d\log w_s > d\log w_{s'} \text{ for all } s < s' \in (0, \widetilde{s}] \quad \text{and} \quad d\log w_s < d\log w_{s'} \text{ for all } s' > s \in [\widetilde{s}, 1).$$

The wage polarization result contained in Proposition 7 is similar to the finding in Acemoglu and Autor (2011), but as discussed in the Introduction, their result was a direct consequence of the fact that there were three types of workers, and automation was assumed to affect the middle type. Here, we see that wage polarization reflects more general forces and applies throughout the distribution, and regardless of exactly where automation is taking place (provided that we start from interior automation). We are not aware of other results of this sort in the literature.

The economics of this result is again related to the competing displacement and productivity effects. The former directly harms the earnings of workers who used to perform the previously-automated tasks, while the latter benefits all workers symmetrically. Notably, the displacement effect does not just impact directly-affected workers (whose previous tasks are taken over by capital), but *all* workers, because of the general pattern of substitutability between worker types. These "ripple effects" are also present in Acemoglu and Restrepo (2022), but in our setting, they only depend on the distance of a skill group to the threshold type $\widetilde{s}$. This, combined with the symmetric productivity effects, yields the result in Proposition 7.

Proposition 7 establishes how skill premia change, generating a pattern of wage polarization. Other important questions are whether the real wage *level* of some worker types will decline following the expansion in automation and whether the top or the bottom of the wage distribution will be more heavily impacted. The next proposition answers these questions.

Proposition 8 ***(Automation and wage levels)*** *Suppose Assumptions 1-3 and Condition 1 hold, let $q \in (q_0, q_m)$, and consider a small increase in the productivity of capital $d \log q > 0$.*

1. *The average wage in the economy always increases.*

2. *There exists a threshold for capital productivity $\hat{q} > q_0$ such that if $q \in (q_0, \hat{q})$, then for some $\delta_1, \delta_2 > 0$, we have $d \log w_s < 0$ for all $s \in (\widetilde{s} - \delta_1, \widetilde{s} + \delta_2)$.*

3. *Suppose that there exists s' such that $\psi_{s',x}/\psi_{k,x}$ is constant in x. Then for some $\delta_2, \delta_2 > 0$, we have $d \log w_s < 0$ for all $s \in (s' - \delta_1, s' + \delta_2)$.*

4. *Suppose that $\psi_{0,0}/\psi_{k,0} < \psi_{0,1}/\psi_{k,1}$. Then, there exists a threshold for capital productivity $\widetilde{q} < q_m$ such that if $q \in (\widetilde{q}, q_m)$, the inequality between the top and the bottom of the skill space increases. That is,*

$$d \log w_0 < d \log w_1 \ .$$

The first part follows immediately from Euler's theorem given constant returns to scale, since in our economy net output equals the aggregate wage bill:

$$NY = \int_0^1 w_s l_s \, ds.$$

Because labor supply is unchanged, log differentiating this equation yields

$$\int_0^1 \frac{\alpha_s}{1 - \alpha_k} d \log w_s \, ds = d \log NY = \frac{\alpha_k}{1 - \alpha_k} d \log q > 0,$$

where α_s and α_k are the income shares of skill s and capital, respectively, and so the left-most term is the change in the average wage in the economy, while the second equality is a direct implication of Hulten's theorem. Hence, the average wage always increases following an expansion in automation.[18]

The second part is also intuitive. When the initial level of capital productivity is low, the set of automated tasks is small. This implies that a marginal increase in q generates only a small productivity effect, and the most affected worker type, $\widetilde{s}$, necessarily experiences a real wage decline (due to the displacement effect). In fact, the decline in the real wage extends to a set of workers around $\widetilde{s}$, because the wage level effects are continuous in skills. This result highlights the central role of the magnitude of the productivity effect, which we characterize in the next subsection.

[18] As discussed in footnote 6, this result is itself a consequence of some of the special assumptions that are typically imposed in these types of models, including ours, and can be relaxed. Since this is not our main focus, we do not explore this issue further in this paper.

The third part provides a refinement of Norbert Wiener's conjecture discussed in the Introduction. Namely, if a worker type has a productivity profile very similar to that of capital, then Wiener's intuition that production using capital will cause the impoverishment of this worker type is correct.[19] However, even though all labor types are competing against capital, wages will not fall for all workers, but only for worker types whose overall comparative advantage is very similar to that of capital. Indeed, we know from the first part that average wages, and hence the wages of some skill types, have to increase (and in fact, it is possible for all wages to increase).

Finally, the fourth part shows that automation widens the inequality between high- and low-skill workers, at least if the productivity of capital is high enough. The intuition for this result is as follows. If $\psi_{0,0}/\psi_{k,0} < \psi_{0,1}/\psi_{k,1}$, then automation will proceed in an unbalanced way, approaching the bottom of the task space as q grows large (see next section). As automation tilts towards the bottom, so do its displacement effects on wages, reducing wages at the bottom relative to the top of the skill space.

4.3 Productivity

The next proposition provides a characterization of the productivity effects of automation, using TFP as a measure for productivity. We follow Greenwood, Hercowitz and Krusell (1997) and define aggregate TFP growth in the presence of investment-specific technological change as

$$\begin{aligned}\Delta \log TFP &= \Delta \log Y - \alpha_k \Delta \log\left(\overline{K}/q\right) - (1-\alpha_k)\Delta \log\left(\int_0^1 l_s\, ds\right)\\ &= \Delta \log Y - \alpha_k\left(\overline{K}/q\right),\end{aligned}$$

which subtracts the growth in the value of the capital stock, $\Delta \log(\overline{K}/q)$, not the change in the quantity of machines, $\Delta \log \overline{K}$, from gross output growth.

With this definition of TFP, we obtain the following second-order Taylor expansion.

Proposition 9 ***(Productivity effects)*** *Suppose Assumptions 1-3 and Condition 1 hold, let $q \in (q_0, q_m)$, and consider a small increase in the productivity of capital $\Delta \log q > 0$. Then we have*

$$\Delta \log TFP \approx \alpha_k \Delta \log q + \left[\lambda - 1 + \frac{\partial \log \Gamma_k}{\partial \overline{x}}\frac{d\overline{x}}{d\log q} + \frac{\partial \log \Gamma_k}{\partial \underline{x}}\frac{d\underline{x}}{d\log q}\right](\Delta \log q)^2 .$$

The first term in the approximation is an immediate consequence of Hulten's theorem. The first-order effect of an increase in capital productivity is equal to its income share. This first term indicates that the TFP gain will be smaller when α_k is small, thus confirming the result in Proposition 8.2: when the productivity of capital is low to start with, or equivalently only a few tasks are initially automated, then the productivity effect is small (which is the reason why negative wage effects are more likely in this case).

[19]Strictly speaking, Proposition 8 requires the worker type to have exactly the same productivity profile as capital. In Proposition 10, we extend this to worker types whose productivity profile is sufficiently similar—but not exactly equal—to that of capital. This extension is easier to formalize when studying global changes in capital productivity, so we defer it to Proposition 10.

The second term captures two distinct but related effects. First, the expansion of the task share of capital Γ_k tends to make the TFP effects of lower capital prices (higher capital productivity) convex: a lower price for capital expands the set of automated tasks, generating a bigger base on which additional productivity gains can be obtained. Second, when holding the set of automated tasks fixed, lower capital prices increase or decrease the share of capital in national income, depending on whether tasks are complements ($\lambda < 1$) or substitutes ($\lambda > 1$). If tasks are complements, a lower price for capital leads to an increase in the labor share and a decrease in the capital share. This partially counters the effect from the expansion of the set of automated tasks and the implied convexity of TFP effects. In contrast, if tasks are substitutes, a lower price for capital leads to a decrease in the labor share and an increase in the capital share, amplifying convexity.

5 Global Effects of Automation

In this section, we consider non-infinitesimal (potentially large) changes in the productivity of capital. We distinguish two cases. In the first, studied in the next subsection, after this change, automation still remains interior. In the second, studied in the subsequent subsection, we transition from interior to low-skill automation. Finally, we also discuss additional comparative statics with respect to labor supply changes.

5.1 Non-Local Changes with Interior Automation

Proposition 10 ***(Polarization with large changes in automation)*** *Suppose Assumptions 1-3 and Condition 1 hold, let $q \in (q_0, q_m)$, and consider a potentially large increase in the productivity of capital $\Delta \log q > 0$, which still satisfies $\log q + \Delta \log q < \log q_m$. Let $\widetilde{s}' = \widetilde{s} + \Delta\widetilde{s} \in (0,1)$ be the new threshold skill level.*

1. *Then, automation expands in both directions and causes employment polarization. That is,*

$$\Delta \underline{x} < 0 \quad \text{and} \quad \Delta \overline{x} > 0,$$

and

$$\Delta x_s < 0 \text{ for all } s \in (0, \widetilde{s}') \quad \text{and} \quad \Delta x_s > 0 \text{ for all } s \in (\widetilde{s}', 1).$$

2. *There is wage polarization in the sense that skill premia increase above the threshold skill $\widetilde{s}'$ and decrease below this threshold. Or equivalently,*

$$\Delta \log w_s > \Delta \log w_{s'} \text{ for all } s < s' \in (0, \widetilde{s}'] \quad \text{and} \quad \Delta \log w_s < \Delta \log w_{s'} \text{ for all } s' > s \in [\widetilde{s}', 1).$$

3. *The average wage always increases, and moreover, there exists a threshold for capital productivity $\hat{q} > q_0$ such that if $q + \Delta q \in (q_0, \hat{q})$, then for some $\delta_1, \delta_2 > 0$, we have $d \log w_s < 0$ for all $s \in [\widetilde{s} - \delta_1, \widetilde{s} + \delta_2]$.*

4. Let $\gamma_{s,k}^{max} = \max_x \{\log \psi_{s,x} - \log \psi_{k,x}\} - \min_x \{\log \psi_{s,x} - \log \psi_{k,x}\}$ *for some* s*. Then,*

$$\Delta \log w_s \leq \gamma_{s,k}^{max} - \Delta \log q.$$

In particular, if $\gamma_{s,k}^{max} < \epsilon$*, then any* $\Delta \log q > \epsilon$ *will reduce the wage of workers with skill level* s*.*

5. Suppose that $\psi_{0,0}/\psi_{k,0} < \psi_{0,1}/\psi_{k,1}$*. Then, if* $q + \Delta q$ *is sufficiently close to* q_m*, the inequality between the top and the bottom of the skill space increases, i.e.,*

$$\Delta \log w_0 < \Delta \log w_1.$$

In summary, this proposition establishes that our main employment and wage polarization results do not depend on whether we consider small or large changes in capital productivity—provided that automation starts out and remains interior. Moreover, as before, when we initially have relatively few tasks automated (or the productivity of capital is still relatively low), an expansion in automation hurts workers around the skill threshold $\widetilde{s}$. Our refinement of Wiener's conjecture also extends to this case: the wages of worker types with productivity profiles sufficiently similar to capital's will decline (but, as before, wages cannot decline for all worker types). Finally, under the same conditions as in the local analysis, the impact of automation on the wage distribution is asymmetric: inequality increases between high-skill and low-skill workers. The proof of this proposition and of the remaining results are in the online Appendix B.

We will next see that, in contrast to this case, when automation ceases to be interior, we obtain very different comparative statics.

5.2 Transition to Low-Skill Automation

We now consider a non-local change in capital productivity inducing a transition from interior to low-skill automation.

Proposition 11 ***(Transition to low-skill automation)*** *Suppose Assumptions 1-3 and Condition 1 hold. Suppose also that Condition 2 holds and that* $q \in (q_0, q_m)$ *initially. Now consider a potentially large increase in the productivity of capital* $\Delta \log q > 0$ *such that* $\log q + \Delta \log q > \log q_m$*. Then:*

1. Automation transitions from interior to low-skill, so all low-complexity tasks are taken over by capital. That is, $\Delta \underline{x} = -\underline{x}$*.*

2. This transition does not induce employment polarization. Instead, the assignment function shifts up everywhere, $\Delta X_s > 0$ *for all* $s < 1$*.*

3. It does not induce wage polarization either. Instead, skill premia increase over the entire skill space. That is,

$$\Delta \log w_s < \Delta \log w_{s'} \quad \text{for all } s < s'.$$

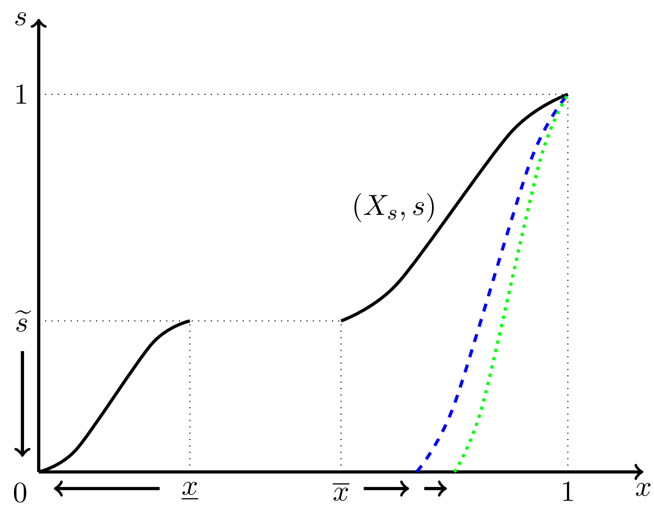

Figure 3: *Transition to low-skill automation.* If capital productivity grows sufficiently, automation becomes low-skill (as shown by the dashed blue curve). A further increase in capital productivity then pushes all workers towards the upper end of the task distribution (shown by the dotted green curve).

Next, suppose Assumptions 1-3 and Conditions 1 and 2 hold but we start from $q \geq q_m$ already. Then, a further increase in the productivity of capital shifts up the assignment function everywhere and skill premia increase over the entire skill space (i.e., there is no longer any employment or wage polarization).

The most important result in this proposition is that for a sufficiently large capital productivity, automation transitions from interior to low-skill and this transition changes the wage effects of automation qualitatively. In contrast to the wage polarization pattern we have seen so far, once automation becomes low-skill it induces monotone increases in wage inequality—whereby automation impacts the lowest-skill workers most negatively (or least positively). In this case, the employment polarization effects of automation vanish as well: further automation now pushes all workers towards more complex tasks. Figure 3 diagrammatically illustrates this transition.

5.3 Implications of Labor Supply Changes

Finally, we consider the wage implications of changes in the labor supply profile. Although this could have been studied with local analysis, it is more convenient to discuss these comparative statics in the case of global changes. The main comparative static is given in the next proposition.

Proposition 12 ***(Labor Supply Changes and Automation)*** *Suppose Assumptions 1-3 as well as Conditions 1 and 2 hold and let $q \in (q_0, q_m)$ under the initial labor supply l. Now consider a change in labor supply such that $\Delta \log l_s < \Delta \log l_{s'}$ for all $s < s'$ (an increase in the relative supply of more skilled workers) and suppose that the resulting decline in q_m, $\Delta q_m < 0$, leads to $q_m + \Delta q_m < q$, such that automation transitions from interior to low-skill. Then, skill premia increase in the bottom part of* *the wage distribution and decrease in the upper part. Specifically, there exists $\hat{s} \in (\widetilde{s}, 1)$ (where $\widetilde{s}$ is the threshold skill before the labor supply change) such that*

$$\Delta \log w_s < \Delta \log w_{s'} \text{ for all } s < s' \in (0, \hat{s}] \quad \text{and} \quad \Delta \log w_s \geq \Delta \log w_{s'} \text{ for all } s' > s \in [\hat{s}, 1).$$

The proposition complements our analysis of labor supply changes in Proposition 5. There, we show that an increase in relative skill supply can trigger a transition from interior to low-skill automation by raising low-skill workers' wages and making it more profitable to automate their jobs. In this case, Proposition 12 establishes a type of upward-sloping relative demand for skills. Given a fixed assignment of workers and capital to tasks, the monotone increase in the supply of skills would have reduced skill premia. However, the response of equilibrium assignment qualitatively changes this pattern. Low-skill automation becomes more likely and this reduces the relative wages of low-skill workers and raises skill premia at the bottom. Other instances of greater relative supply of skills leading to higher skill premia are present in models of directed technological change (because greater abundance of skilled workers encourages more skill-biased technological change, as in Acemoglu, 1998, 2007), and in models of search and matching (because with more skilled workers around, more employers make investments complementary to skilled workers and search for them, as in Acemoglu, 1999). In the model here, a similar outcome arises, even though there is no endogenous innovation and all markets are competitive. Rather, this result is driven by the response of the equilibrium assignment of tasks between capital and labor.

6 Quantitative Analysis

In this section, we undertake a preliminary quantitative analysis of the consequences of different types of automation and policies in our framework.

6.1 Calibration

We normalize labor supply to one for all skill types, i.e., $l_s \equiv 1$, which implies that the skill index s represents percentiles of the wage distribution. For labor productivity, $\psi_{s,x}$, we follow Teulings (1995, 2005), where productivity is assumed to be log linear in the product of s and x. We extend this by including a quadratic in s:

$$\log \psi_{s,x} = ms + ns^2 + asx + A,$$

where $a > 0$ corresponds to more skilled workers having comparative advantage in more complex tasks. The quadratic in s allows us to flexibly match the empirical wage distribution. In particular, without the ns^2 term, this functional form generates too much inequality at the top of the wage distribution, because of the linear absolute advantage of more skilled workers.

The simplest specification for capital productivity, $\psi_{k,x}$, consistent with Assumption 3, is:

$$\log \psi_{k,x} = a_k x + bx^2 \ ,$$

Panel A. Calibrated Parameters

Parameter	Value	Rationale/Target
λ	0.5	Humlum (2021)
A	-0.14	Log median wage
m	2.5	Jointly calibrated to match the remaining seven moments
n	-2.4	
a	7.8	
a_k	100	
b	-310	
$\log q$	-9.2	

Panel B. Comparison of Moments

Moment	Data	Model
Log 50-10 wage percentile ratio, $\log(w_{0.5}/w_{0.1})$	0.60	0.60
Log 90-50 wage percentile ratio, $\log(w_{0.9}/w_{0.5})$	0.67	0.67
Income share of equipment and software, α_k	0.16	0.17
Change in log 30-10 wage percentile ratio due to automation, $\Delta\log(w_{0.3}/w_{0.1})$	-0.02	-0.01
Change in log 50-30 wage percentile ratio due to automation, $\Delta\log(w_{0.5}/w_{0.3})$	0.03	0.03
Change in log 90-50 wage percentile ratio due to automation, $\Delta\log(w_{0.9}/w_{0.5})$	0.16	0.16
Change in income share of equipment and software from 1980 to 2016, $\Delta\alpha_k$	0.01	0.03
Log median wage level (in 2008 dollars), $\log w_{0.5}$	2.6	2.6

Table 1: *Calibration Results.* Panel A shows the calibrated parameter values and the targets used in their calibration. Panel B presents the data moments used in the calibration (left column) and their model counterparts (right column). See Appendix B.2.2 for data sources.

with $b < 0$.

Given these specifications, we have to choose eight parameters: m, n, a, a_k, b, the capital productivity parameter q, the elasticity of substitution between tasks λ, and the labor-augmenting technology parameter A. We set $\lambda = 0.5$ externally, using the estimate of the firm-level elasticity of substitution between production workers, tech workers and other workers from Humlum (2021), which we interpret following Humlum as the firm-level elasticity of substitution between tasks. Next, we choose m, n, a, a_k, b and q to match two sets of empirical moments. The first are moments from the US income distribution in 1980, including the 90-50 and the 50-10 differences in log hourly wage, and the factor income share of equipment and software capital in 1980. We focus on the share of equipment and software capital because all capital is used for automation in our model. The second set of moments concerns the impact of automation on the US wage distribution between 1980 and 2016-2017 from Acemoglu and Restrepo (2022). Specifically, we take from that paper the estimates of the impact of automation on the 30-10, 50-30 and 90-50 differences in log hourly wages. We include the 30th percentile because this is where automation had its least positive (most negative) impact on wages according to their estimates (see Figure 5). Additionally, we target the change in the income share of equipment capital and software

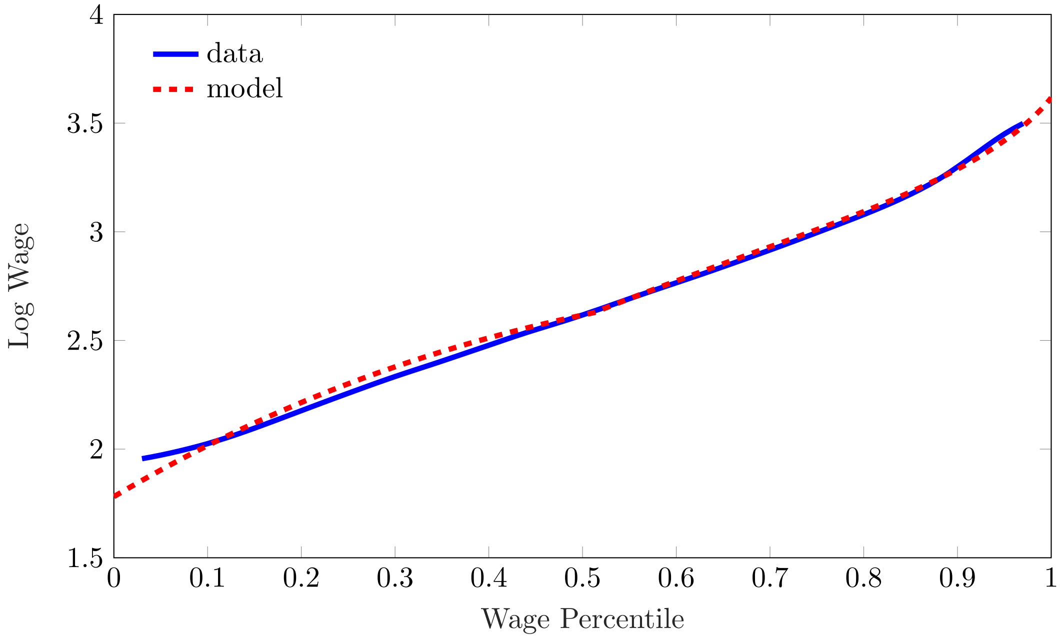

Figure 4: *Log wage distribution in 1980.* The graph depicts log wages by percentile of the wage distribution. The solid blue curve corresponds to the data, based on Acemoglu and Autor (2011), while the red dashed curve is computed from our calibration, using the parameter values shown in Table 1. See Appendix B.2.2 for data sources.

between 1980 and 2016 in the US. To match these moments, we reduce the cost of capital $1/q$ in the model by 79%, which corresponds to the decline in the real prices of equipment and software in the US between 1980 and 2016. Finally, we choose the labor-augmenting technology parameter A to match the level of wages in 1980, since this parameter does not affect any of the relative equilibrium quantities.

We describe the procedure for numerically solving for the equilibrium of our model in Appendix B.2.1; our data sources and the construction of empirical moments in Appendix B.2.2; and the details of the calibration procedure in Appendix B.2.3.

The results are displayed in Table 1. The top panel shows the calibrated parameter values, while the bottom panel depicts the above-described moments both in the data and in our calibration. The model fits all data moments closely. Figure 4 additionally shows that the model matches the 1980 US wage distribution very well even beyond the targeted percentiles. In Figure 5, we show the log wage changes induced by automation between 1980 and 2016 as predicted by our model and in the data (left panel) as well as the corresponding change in the assignment function (right panel). As expected, the set of automated tasks expands in both directions while the wage responses lead to wage polarization. The wage effects predicted by our model track the wage changes in the data quite well, but the two curves differ by a constant of about 0.3, on average—the model predicts a median wage increase of 0.3 log points, while the estimated series shows a median wage change close to zero. This discrepancy is likely due to the fact that Acemoglu and Restrepo (2022) isolate the effects of expansions in the set of automated tasks, while an increase in q in our model both expands automation and induces "deepening

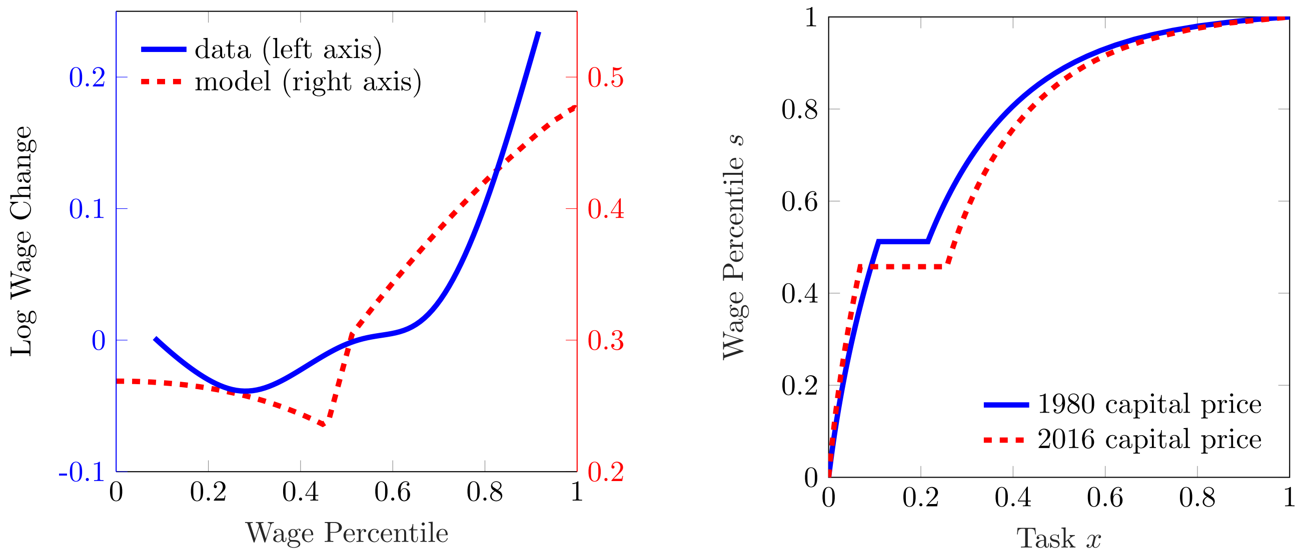

Figure 5: *Change in log wages and assignment due to automation, 1980-2016.* The left panel depicts the estimated change in log wages between 1980 and 2016 due to automation from Acemoglu and Restrepo (2022) (in solid blue) and the corresponding changes from our calibrated model (in dashed red). The right panel shows the corresponding change in the assignment function in our model. The values of parameters used in the calibration are shown in Table 1. See Appendix B.2.2 for data sources. See Appendix B.2.2 for data sources.

of automation", which tends to raise the wages for all skill groups. Consistent with this, the implied median wage increase of 0.3 log points in our model is in the ballpark of the actual change in US median wages—of about 0.26 log points—though the actual median wage change in the data is likely impacted by a variety of other factors, including other types of technological changes, educational upgrading, new tasks, and institutional and norm changes in the US labor markets. In this context, we also note that our calibration does not impose Conditions 1 and 2, but both of these conditions are comfortably satisfied given the implied parameter values.

Although there is no one-to-one mapping between the seven moments we use from the US and the six parameters, each parameter is closely related to a particular equilibrium outcome of the model, which allows us to gain additional intuition about the parameter choices and the consequences of reasonable variations therein. The parameters m and n determine the degree of absolute advantage among workers. A higher m raises skill premia across the entire wage distribution, while increasing n does so mainly at the upper end. Choosing m and n jointly thus allows us to match both the 90-50 and 50-10 wage ratios in 1980. The parameter a determines the degree of comparative advantage among workers and their substitutability. A high value of a corresponds to low substitutability between skill types and implies that the displacement effect of automation is largely concentrated on those workers who are directly displaced. Graphically, this corresponds to the two branches of the red dashed curve in the left panel of Figure 5 becoming steeper. Thus, by varying a, we can scale the effects of automation on the 30-10, 50-10 and 90-50 wage ratios up or down. The parameters a_k and b determine the slope and curvature of the capital productivity curve. A higher slope moves the set of automated tasks towards the top of the wage distribution, and therefore, the slope parameter a_k is set such that automation is interior and has its least positive wage impact in the middle of the wage distribution. Graphically, this means that it regulates where the minimum of the red dashed curve is in the left panel of Figure 5. The curvature parameter b, on the other hand, determines how quickly capital becomes less productive when moving away from the set of automated tasks, and as such controls by how much the set of automated tasks expands when capital prices fall further. The value of b is therefore disciplined directly by the increase in the income share of equipment and software capital in the data. The level of the capital price $1/q$ determines the extent of automation in the baseline and is thus set to match the 1980 equipment and software income share in the initial equilibrium. Finally, as noted above, because the parameter A only scales wages, it is not calibrated jointly with the rest of the parameters and is set at the end of the calibration to match the median wage in 1980 exactly given the other parameters.

6.2 Counterfactuals

We use our calibrated model for three counterfactual exercises in order to shed light on how further automation, different types of technological changes and binding minimum wages might impact assignments and wages. We first consider the consequences of reducing capital prices—increasing capital productivity. To make comparisons easier, we consider a further 79% decline in the price of capital, $1/q$, which is equal to its decline between 1980 and 2016 in the US. Unsurprisingly, the results, depicted in Figure 6, are similar to the empirically-estimated automation-driven wage changes between 1980 and 2016-17. There is again a sizable amount of wage polarization, as shown by the left panel of the figure, while the right panel confirms that the set of automated tasks expands in both directions.[20] Nevertheless, there are some important differences as well. Automation moves further towards the bottom of the wage distribution than it did between 1980 and 2016, and hence, the displacement effects fall more on the shoulders of low-skill workers, boosting inequality even more than in 1980-2016. This result accords with Proposition 10, which shows that further declines in the price of capital that take us closer to the threshold for low-skill automation will tend to increase inequality between the top and the bottom of the wage distribution. Intuitively, given the global comparative advantage of capital in Condition 2, further declines in the price of capital leave lower-skill workers more vulnerable to automation.

There are in principle many ways in which automation opportunities may improve. This raises the possibility that the next wave of automation may have very different consequences than what we experienced between 1980 and 2016. The most important reason for this may be advances in (generative) artificial intelligence, which could expand the reach of automation to a broader set of occupations. On the other hand, generative AI may have less impact on already-automated tasks,

[20] As explained in the previous subsection, reducing the price of capital generates a significant amount of deepening of automation in our model, which is the reason why in these counterfactuals wages increase even at the bottom of the distribution. An alternative, in line with our discussion above, would be to remove these non-automation implications by reducing A by an equivalent amount. In that case, the median equilibrium wage would remain roughly constant following the decline in the price of capital, and there would be sizable real wage declines at the bottom of the distribution.

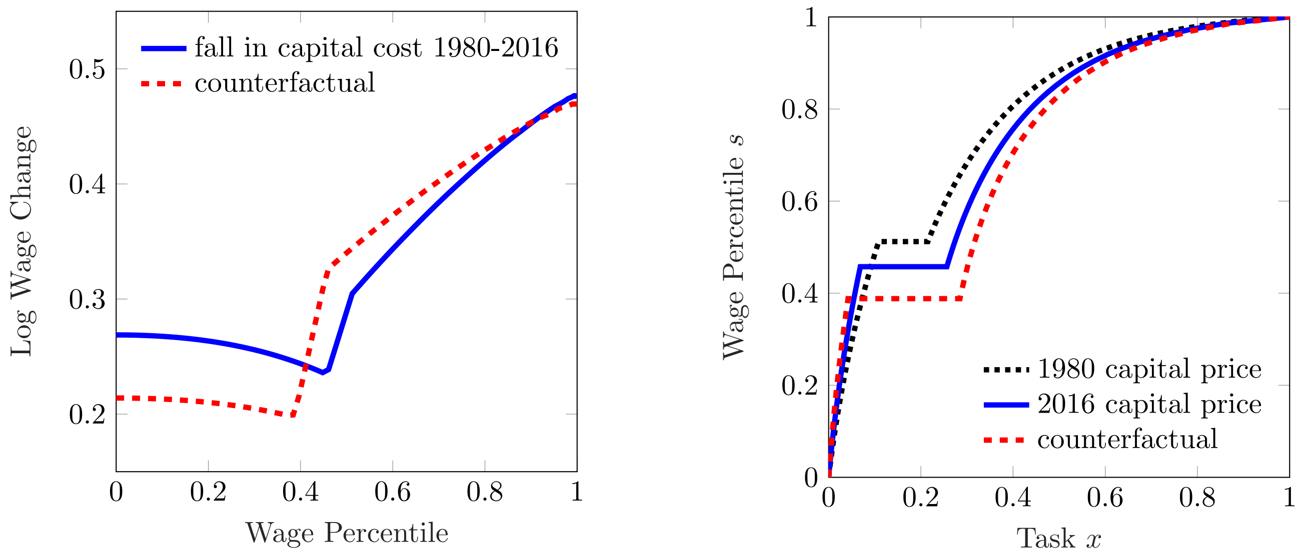

Figure 6: *Change in log wages and assignment due to a counterfactual fall in capital costs.* The left panel depicts the change in log wages in response to a further fall in the capital price by 79% starting from its 2016 level (dashed red curve). For comparison, we also show the change between 1980 and 2016 (solid blue curve). The right panel shows the corresponding assignment functions (dotted black is for the 1980 baseline, solid blue is for the 2016, and dashed red is for the counterfactual). The values of parameters used in the calibration are shown in Table 1. See Appendix B.2.2 for data sources.

such as software-based clerical functions and various production tasks that have been automated by robots and advanced automatic equipment, such as assembly, welding and painting. This motivates us to consider an AI counterfactual where the curvature of the capital productivity schedule $\log \psi_{k,x}$ is reduced, but its maximum remains unaffected, so that capital becomes more productive at the tails of the task distribution, but not so much in already-automated tasks. Formally, we benchmark capital productivity against the productivity of the median worker, $\log \psi_{k,x} - \log \psi_{0.5,x}$, and reduce the curvature of this quadratic function, while keeping its maximum fixed by adjusting parameters a_k and b (see Appendix B.2.4 for details). We further discipline the exact shape of the new productivity scheduled by imposing that the aggregate net output gains, and thus the productivity effects, must be the same as from a uniform capital price reduction of 79%, as in our first counterfactual.

The resulting wage and assignment consequences of the AI calibration are shown in Figure 7. As a benchmark, the figure also includes the results from our first counterfactual where capital productivity increased uniformly. While both counterfactuals result in employment and wage polarization, their wage effects are quite distinct. AI leads to steeper wage declines at the bottom of the wage distribution (because automation expends even further at the bottom) and bigger wage gains at the top (since higher-skilled workers that do not suffer automation benefit from the productivity gains). This is a consequence of the way we have modeled the AI advances—less deepening of automation and productivity gains in already-automated tasks and more aggressive expansions in previously non-automated tasks.

Finally, we explore the implications of a binding minimum wage in this setup. We fix the capital price

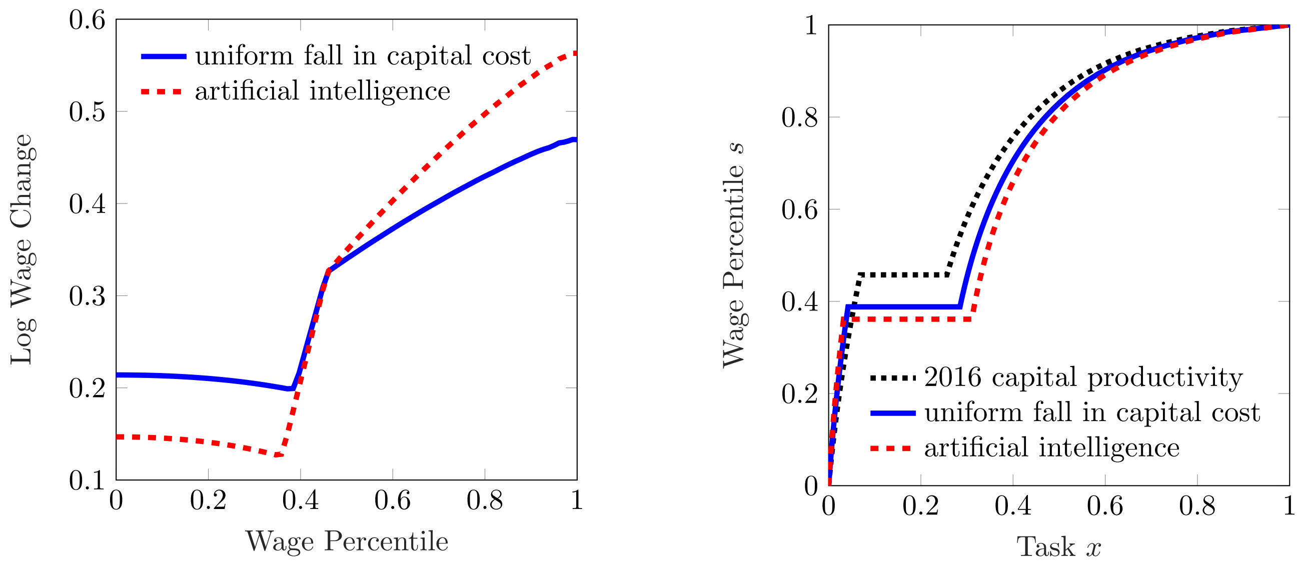

Figure 7: *Change in log wages and assignment due to a counterfactual progress in artificial intelligence.* The left panel shows the change in log wages in response to the change in capital productivity inspired by changes in generative AI, as explained in the text (dashed red curve), and also shows the uniform capital productivity increase counterfactual from Figure 6 (solid blue curve). The right panel shows the corresponding assignment functions. The values of parameters used in the calibration are shown in Table 1.

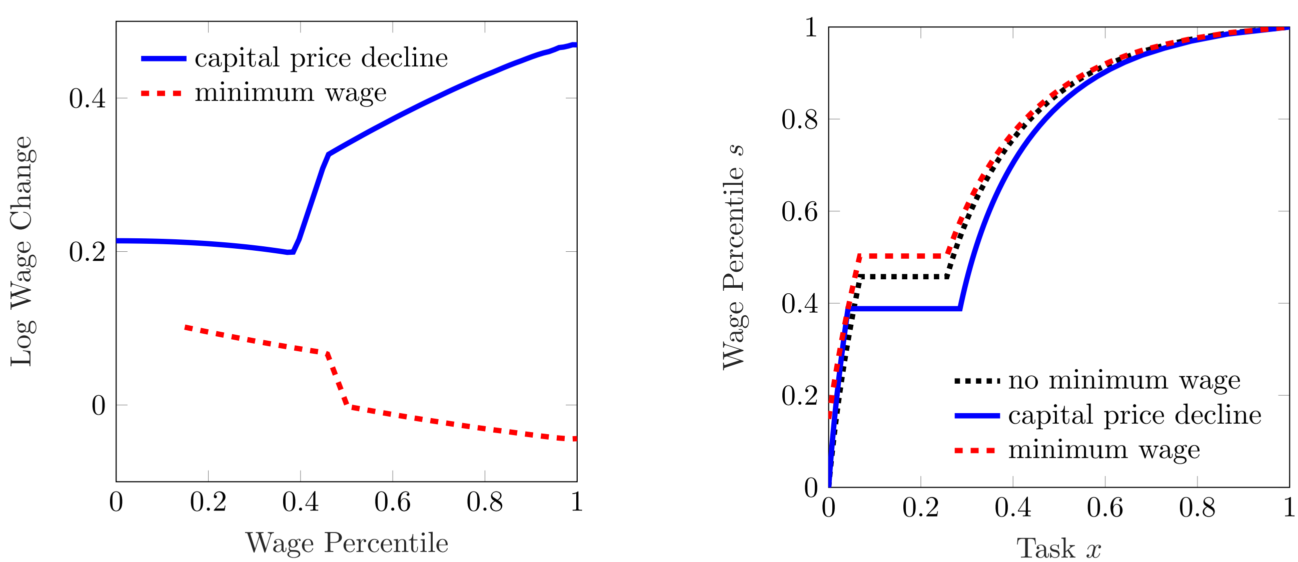

Figure 8: *Change in log wages and assignment when introducing a minimum wage.* The left panel depicts the impact of a minimum wage of \$16 (in 2022 dollars) on the equilibrium wage distribution (dashed red curve), and also shows the uniform capital productivity increase counterfactual from Figure 6 (solid blue curve). The right panel presents the corresponding assignment functions. The values of parameters used in the calibration are shown in Table 1.

at its 2016 level and impose a fairly high minimum wage of $16 (in 2022 dollars), which corresponds to the highest state-level minimum wages in the US in 2022. The effects of this counterfactual are displayed in Figure 8. The left panel shows that the minimum wage has a significant impact on the entire wage distribution, with large wage increases of up to 10% at the bottom and declines of about 3% at the top. The negative impact at the top comes from the fact that the minimum wage leads to an approximately 10% reduction in employment, entirely concentrated at the bottom. This reduces aggregate output and lowers demand for labor across all tasks.

7 Conclusion

There has been rapid automation of a range of tasks across the industrialized world over the last four decades. There is growing evidence that this automation has fueled both inequality and polarization—whereby middle-skilled workers have been displaced from their jobs and have experienced relative wage declines.

To develop a deeper understanding of the causes of polarization, this paper has built an assignment model of automation. In our model, each of a continuum of tasks of variable complexity is assigned to either capital or one of a continuum of labor skills. Our model generalizes existing assignment models, which typically impose global supermodularity conditions that ensure monotone matching between workers and tasks. In contrast, in our model with capital there is no global supermodularity.

We prove existence and uniqueness of competitive equilibria and characterize conditions under which automation is interior, meaning that it is tasks of intermediate complexity that are assigned to capital. Put simply, interior automation arises when the most skilled workers have a comparative advantage in the most complex tasks relative to capital and other labor, and when the wages of the least skilled workers are sufficiently low relative to their productivity and the effective cost of capital in low-complexity tasks, so that it is not profitable to use capital or algorithms instead of low-skill workers. Highlighting the role of wages at the low-end of the wage distribution, we demonstrate that minimum wages and other sources of higher wages at the bottom make interior automation less likely.

We provide a series of local and global comparative statics, showing how further automation impacts wages and assignment patterns. Most importantly, when automation starts and remains interior, a lower cost of capital (or greater capital productivity) causes employment polarization: middle-skill workers are displaced from middle-complexity tasks and are pushed towards higher or lower parts of the complexity distribution. This type of automation also causes wage polarization: the skill premium monotonically increases above a skill threshold and monotonically declines below the same threshold. Moreover, automation tends to reduce the real wage of workers with comparative advantage profiles close to that of capital.

Our global comparative static results additionally establish that large enough increases in capital productivity ultimately induce a transition to low-skill automation, whereby the pattern of comparative statics changes qualitatively. In particular, after this transition to low-skill automation, further declines in the cost of capital no longer cause employment or wage polarization. Rather, they have a monotone effect on the skill premium.

Despite its richness, our framework is tractable and opens the way to further analysis of the changing assignment patterns in modern labor markets. We illustrated this richnes by presenting both a range of comparative static results and a simple calibration exercise where we explore the quantitative implications of various counterfactual technological and institutional changes.

There are many areas for future fruitful inquiries. First, automation has been going on together with a changing structure of tasks and an evolving distribution of skills over at least the last 250 years. This can be introduced into our framework by simultaneously expanding the range of tasks and skills, and would be an important area for future work. Second, the productivity of capital in various tasks should in principle change endogenously, responding to which tasks are being assigned to capital—or are likely to be assigned to capital in the future. This issue can be investigated in an extended version of our framework in which the direction of technological change and capital productivity across tasks are endogenized. Third, in practice multiple tasks may be assigned to a worker because either there are economies of scope or other types of task complementarities, and extending this class of models to "one-to-many" matching is another important area for further inquiry. Fourth, our quantitative exploration can be significantly expanded, for example, by introducing a more general family of comparative advantage schedules and then estimating the parameters of this family using more moments from the baseline wage distribution and the response of the wage distribution to different types of shocks. There are also several other counterfactual exercises to be considered, including those related to changes in supplies and offshoring-type opportunities. Last but not least, the framework here can be used to further refine the empirical investigation of the relationship between automation and inequality, for example, by adding more structure and predictions to studies such as Acemoglu and Restrepo (2022).

A Appendix

A.1 A Converse to Proposition 3

In Proposition 3 in the main text, we show that automation is interior for low levels of capital productivity if Condition 1 holds (together with Assumptions 1 to 3). Here we provide an inverse to this result: if Condition 1 is violated, automation is either high or low-skill (again under Assumptions 1 to 3). In this sense, Condition 1 is necessary for interior automation in our setup.

Proposition A1 ***(Interior automation, converse)*** *Suppose Assumptions 1, 2 and 3 hold. Then, we have the following:*

1. *If Condition 1.1 is violated while Condition 1.2 holds, then automation is low-skill for all* $q \in (q_0, q_\infty)$.

2. *If Condition 1.1 holds while Condition 1.2 is violated, then automation is high-skill for all* $q \in (q_0, q_\infty)$.

Proof. We focus on the case where Condition 1.1 is violated while Condition 1.2 holds, as the proof for the second case is symmetric.

If Condition 1.1 does not hold, we have

$$\frac{\partial \log \psi_{k,0}}{\partial x} - \frac{\partial \log \psi_{0,0}}{\partial x} \leq 0.$$

By log supermodularity of labor productivity (Assumption 2), it follows that

$$\frac{\partial \log \psi_{k,0}}{\partial x} - \frac{\partial \log \psi_{s,0}}{\partial x} < 0 \quad \text{for all } s > 0.$$

Together with quasi-concavity of $\psi_{k,x}/\psi_{s,x}$ (Assumption 3), this implies that $w_s\psi_{k,x}/\psi_{s,x}$ is decreasing in x for all $s > 0$. By continuity of productivity schedules, the same must then hold for $s = 0$. Hence, the minimal effective unit cost of producing the amount $\psi_{k,x}$ of task x with labor, $\omega_x = \min_s \left\{ \frac{w_s \psi_{k,x}}{\psi_{s,x}} \right\}$, is the lower envelope of a family of decreasing functions and must therefore be decreasing itself. This implies that $\{x | \omega_x \geq 1/q\}$ is either empty (if $q < q_0$) or contains zero (if $q \geq q_0$). Using Lemma A2, the same holds for the set of automated tasks X_k. ■

A.2 Interior Automation without a Convex Set of Automated Tasks

In this part of the Appendix, we show that our characterization of the conditions leading to interior, low-skill and high-skill automation (Proposition 3) does not crucially rely on Assumption 3, i.e., on the quasi-concavity of $\psi_{k,x}/\psi_{s,x}$. Assumption 3 guarantees that the set of automated tasks X_k is convex and consequently our definitions of interior, low-skill and high-skill automation are restricted to a convex set of automated tasks. Thus, we first relax these definitions to encompass cases in which the set of automated tasks is not convex.

1. We say that automation is *weakly interior* if there exist $\epsilon_1, \epsilon_2 > 0$ such that $[0, \epsilon_1] \cap X_k = \emptyset$ and $[1 - \epsilon_2, 1] \cap X_k = \emptyset$ while $X_k \neq \emptyset$, i.e., the most and least complex tasks are performed by labor.

2. Automation is *weakly low-skill* if there exists $\epsilon > 0$ such that $[0, \epsilon] \subset X_k$, i.e., all tasks below a certain threshold of complexity are automated.

3. Automation is *weakly high-skill* if there exists $\epsilon > 0$ such that $[1 - \epsilon, 1] \subset X_k$, i.e., all tasks above a certain threshold are automated.

Note that, together with no automation, these types of automation are exhaustive, i.e., any configuration of automation is of at least one of these types. They are also mutually exclusive, with one exception—automation can be weakly low-skill and weakly high-skill at the same time. This happens if both an interval at the bottom of the task space and an interval at the top of the task space are automated. Such a configuration is impossible if the set of automated tasks is convex, as guaranteed by Assumption 3 in the main text. Since we drop Assumption 3 here, simultaneous low- and high-skill automation becomes a possibility now.

Besides ensuring a convex set of automated tasks, Assumption 3 guarantees uniqueness of equilibrium allocations, see Proposition 1. To avoid multiplicity in allocations, we follow Acemoglu and Restrepo (2022) and impose a tie-breaking rule such that, if a task x can be performed at equal costs by capital and labor, $w_s/\psi_{s,x} = 1/(q\psi_{k,x})$ for some s, then the task is assigned to capital, $x \in X_k$. To ensure existence of an equilibrium under this tie-breaking rule, we also assume that no skill has a productivity schedule parallel to that of capital over any interval, i.e., the level sets of $\psi_{k,x}/\psi_{s,x}$ in x are of measure zero for all s (see Acemoglu and Restrepo, 2022).

Dropping Assumption 3 but keeping Assumptions 1, 2 and Condition 1, we can now restate Proposition 3 as follows.

Proposition A2 ***(Interior automation, extended)*** *Suppose Assumptions 1, 2 and Condition 1 hold. Then, there exist thresholds $q_0 < q_\infty$ and $q_m \in (q_0, q_\infty]$ such that:*

1. *For $q \leq q_0$, there is no automation, i.e., $X_k = \emptyset$.*

2. *For $q \in (q_0, q_m)$, automation is weakly interior.*

3. *If Condition 2 holds, then $q_m < q_\infty$ and, for $q \geq q_m$, automation is weakly low-skill.*

4. *If Condition 2.1 is violated strictly but Condition 2.2 holds, then $q_m < q_\infty$ and automation is weakly low-skill or weakly high-skill for $q \geq q_m$, or we have $q_m = q_\infty$ such that automation is weakly interior for all $q \in (q_0, q_\infty)$.*

5. *If Conditions 2.1 and 2.2 are both strictly violated, then $q_m < q_\infty$ and, for $q \geq q_m$, automation is weakly high-skill.*

Proposition A2 shows that the main insights from Proposition 3 do not depend on Assumption 3. In particular, under Assumptions 1, 2 and Condition 1 alone, automation is weakly interior for low levels of capital productivity. Moreover, under Condition (2), automation encroaches on the least skilled workers and becomes low-skill once capital productivity crosses a threshold $q_m < q_\infty$.

To prove Proposition A2, we start with the following important result.

Lemma A1 *Suppose Assumptions 1 and 2 hold. Then, the set of automated tasks X_k is increasing in q, $X_k(q) \subseteq X_k(q')$ for all $q \le q'$.*

Proof. We start by showing that if q increases to $q' > q$, no wage can fall by more than the capital price $1/q$, that is, $w'_s/w_s \ge q/q'$ for all s. To show this, assume the contrary. In particular, let $\hat{s}$ be the minimizer of w'_s/w_s and assume that $w'_{\hat{s}}/w_{\hat{s}} < q/q'$. Then, if the assignment of $\hat{s}$ changes, $X_{\hat{s}} \neq X'_{\hat{s}}$, some other skill $\dot{s}$ must replace $\hat{s}$, $X'_{\dot{s}} = X_{\hat{s}}$. But this requires $w'_{\dot{s}}/\psi_{\dot{s},X_{\hat{s}}} < w'_{\hat{s}}/\psi_{\hat{s},X_{\hat{s}}}$ while before the increase in q, this inequality holds in opposite direction.[21] Thus, $w'_{\dot{s}}/w_{\dot{s}} < w'_{\hat{s}}/w_{\hat{s}}$, which contradicts $\hat{s}$ being a minimizer of w'_s/w_s. Hence, the assignment of $\hat{s}$ must not change, $X_{\hat{s}} = X'_{\hat{s}}$. But by equation (6), this requires that the mass of labor assigned to task $X_{\hat{s}}$ has increased and, by equation (11), the derivative of the assignment function $dX_{\hat{s}}/ds$ has decreased at $X_{\hat{s}}$.[22] Together with the fact that the assignment at $\hat{s}$ is unchanged, this implies that X_s shifts upwards in a lower neighborhood of $\hat{s}$ and downwards in an upper neighborhood of $\hat{s}$. Using equation (9), this implies that skills both in a lower and an upper neighborhood of $\hat{s}$ experience wage declines relative to $\hat{s}$, which contradicts $\hat{s}$ being a minimizer of w'_s/w_s. Thus, we cannot have $w'_{\hat{s}}/w_{\hat{s}} < q/q'$. Instead, $w'_s/w_s \ge q/q'$ for all s.

Next, our tie-breaking rule implies that $X_k = \{x \mid w_s/\psi_{s,x} \ge 1/(q\psi_{k,x}) \text{ for all } s\}$. Hence, for a task to be in X_k under q but not under q', we would need some wage $w_{\hat{s}}$ to fall by more than the capital price $1/q$, which is not possible as shown previously. So, we must have $X_k(q) \subseteq X_k(q')$. ■

With Lemma A1, we can now prove Proposition A2.

Part 1 For part 1, suppose that we restrict capital usage to zero in all tasks and consider the output-maximizing allocation of labor. Let w^0 be the resulting wage vector and $\omega^0_x = \min_s\{w^0_s \psi_{k,x}/\psi_{s,x}\}$ the corresponding minimum labor cost function. Then, we can define $\frac{1}{q_0} = \max_x \omega^0_x$ as the prohibitive cost of capital. Capital will be used in equilibrium if and only if $q \ge q_0$.

Part 2 We next prove that there exists a right neighborhood of q_0 on which automation is weakly interior. In particular, we prove that task 0 is not automated on this neighborhood. The proof that task 1 is not automated on such a neighborhood either proceeds symmetrically and is thus omitted.

[21] The fact that these inequalities must hold strictly comes from Assumption 2: if it held only weakly (i.e., with equality), there would be some skill level between $\hat{s}$ and $\dot{s}$ with a strictly lower effective cost in either the task assigned to $\hat{s}$ or to $\dot{s}$.

[22] Equation (11) holds on every interval to which only labor is assigned. Since we have assumed that the wage of $\hat{s}$ falls by more than the capital price, continuity of wages implies that there is a neighborhood of skills around $\hat{s}$ all of which have a lower effective cost in some neighborhood of tasks around $X_{\hat{s}}$ than capital. So, there exists an interval containing $X_{\hat{s}}$ to which only labor is assigned and equation (11) applies at $X_{\hat{s}}$.

Suppose, to derive a contradiction, that task 0 is automated for q arbitrarily close to q_0, that is, we can construct a sequence of q converging to q_0 with corresponding equilibrium allocations such that task 0 is automated for any element of the sequence. Then, since equilibrium allocations are upper hemicontinuous by Berge's maximum theorem, task 0 would also have to be automated in (some) equilibrium under q_0. But we know that $X_k \subseteq \{x \mid w_s/\psi_{s,x} \geq 1/(q\psi_{k,x}) \text{ for all } s\}$ and under q_0: $X_k \subseteq \operatorname{argmax}_x \omega_x^0$, where ω_x^0 is the minimum labor cost function without capital defined in the proof of part 1. At the same time, $0 \notin \operatorname{argmax}_x \omega_x^0$ by Condition 1. Hence, task 0 cannot be automated at q_0 and thus, by upper hemicontinuity of equilibrium allocations, there must exist a right neighborhood of q_0 on which task 0 is not automated.

Part 3 For part 3, consider the minimum labor cost function $\omega_x \equiv \min_s \{w_s\psi_{k,x}/\psi_{s,x}\}$. By Assumption 2 and Proposition 2, ω_x is equal to $w_0\gamma_{k,x}/\gamma_{0,x}$ at $x = 0$. Thus, for any level of q, we must have

$$\frac{\omega_0}{\omega_1} \geq \frac{\gamma_{k,0}\gamma_{0,1}}{\gamma_{k,1}\gamma_{0,0}} > 1$$

where the last inequality follows from Condition 2. This implies that whenever task 1 is automated (which requires $\omega_1 \geq 1/q$), we have $\omega_0 > 1/q$ such that task 0 must be automated as well. Hence, whenever automation is not weakly interior, it must be weakly low-skill.

To see that automation becomes weakly low-skill at some $q_m < q_\infty$, i.e., before output becomes infinite, note that as $q \to q_\infty$, we must have $X_k \to [0, 1]$.[23] But the previous reasoning shows that task 0 will be automated before the tasks in some neighborhood of task 1 and hence before q reaches q_∞. Finally, we note that by Lemma A1, if task 0 is automated at q_m, it must also be automated at all $q \geq q_m$.

Part 4 The only restriction imposed on the behavior of X_k in part 4 is that, if automation becomes weakly low (high) skill at some q_m, it must remain so for all $q \geq q_m$. This immediately follows from Lemma A1.

Part 5 The proof of part 5 is entirely symmetric to the proof of part 3 and thus omitted.

A.3 Proofs for Section 3: Equilibrium and Interior Automation

A.3.1 Proof of Proposition 1: Existence and Uniqueness

Existence An equilibrium allocation maximizes net output subject to labor market clearing, given by

$$\int_0^1 L_{s,x} dx \leq l_s \quad \text{for all } s. \tag{A1}$$

To prove existence, it is useful to split the problem of net output maximization into two steps. First, we fix the aggregate capital stock $\overline{K}$ and maximize gross output for given $\overline{K}$ and subject to (A1). This is a

[23] This follows from the fact that the average wage grows without bound as $q \to q_\infty$ and hence labor must be replaced from all tasks asymptotically.

problem of maximizing a continuous function over a compact set, such that a maximizer is guaranteed to exist. Let $F(\overline{K}, l)$ denote the maximal gross output for given $\overline{K}$ and labor supply l.

In the second step, we choose $\overline{K}$ to maximize net output $F(\overline{K}, l) - \overline{K}/q$. This is again a continuous problem, but $\overline{K}$ can be any positive real number, so we have to establish boundedness. For this, note that $\lim_{\overline{K}\to\infty} \frac{\partial F(\overline{K},l)}{\partial \overline{K}} = \left(\int_0^1 \psi_{k,x}^{\lambda-1}\, dx\right)^{\frac{1}{\lambda-1}}$. Thus, Assumption 1 ensures that $\lim_{\overline{K}\to\infty} \frac{\partial F(\overline{K},l)}{\partial \overline{K}} < \frac{1}{q}$ such that net output is bounded and attains its maximum for finite $\overline{K}$.

Essential Uniqueness For essential uniqueness of equilibrium, note that net output is concave in the allocation and the set of feasible allocations is convex. This implies that, while the equilibrium allocation itself may not be unique, the Frechet derivative of net output is constant across all equilibrium allocations.[24] Hence, equilibrium wages are unique. The same argument applies to task prices when writing the maximization of net output as a maximization over task inputs, including task production functions as constraints, i.e.,

$$\max_{\{Y_x\}_{x=0}^1, L, K} \left[\int_0^1 Y_x^{\frac{\lambda-1}{\lambda}}\, dx\right]^{\frac{\lambda}{\lambda-1}} - \frac{1}{q}\int_0^1 K_x\, dx$$

subject to task production (2) and labor market clearing (3).

Uniqueness Our proof of Proposition 2 below shows that under Assumptions 1, 2 and 3, the labor allocation L and the set of tasks performed by capital X_k are uniquely determined given wages. Moreover, given prices, X_k and the labor allocation, choosing the output-maximizing capital allocation K is a strictly concave problem with a unique solution.

Thus, given wages and task prices, the equilibrium allocation is determined uniquely. Since equilibrium wages and task prices are unique, the equilibrium allocation is unique as well.

A.3.2 Proof of Proposition 2: Convexity of Assignment

Monotonicity Monotonicity of the labor allocation under comparative advantage assumptions is a standard result. One way to prove it, which is useful for our argument in the next step below, is presented here. Let

$$S_x^{min} = \underset{s}{\operatorname{argmin}} \{\log w_s - \log \psi_{s,x}\}$$

be the set of skills that produce task x at minimal cost. By Assumption 2, $\log w_s - \log \psi_{s,x}$ is strictly submodular, so Topkis' monotonicity theorem (Topkis 1998) implies that if $s \in S_x^{min}$, $s' \in S_{x'}^{min}$ and $x > x'$, then $s \geq s'$. Moreover, if for some x there exist $s, s' \in S_x^{min}$ with $s > s'$, then all skill levels in (s', s) can only be assigned to x. This creates a mass point in the density of labor over tasks, such that $p_x = 0$, contradicting condition (4).[25] Hence, S_x^{min} is a singleton for all x. Inverting this correspondence,

[24] The set of maximizers of a concave function on a convex set is a face of the hypograph of the function. Thus, there exists a supporting hyperplane of the hypograph that contains the entire set of maximizers. Together with differentiability, this immediately implies that the derivative of the function is constant on the set of maximizers.

[25] Note that this holds independently of whether capital is used in the production of task x or not. In any case, assigning a strictly positive measure of skills to x creates a mass point in task output at x, which leads to $p_x = 0$.

we obtain $\widetilde{X}_s = \{x | s \in S_x^{min}\}$, which is a superset of X_s, $X_s \subseteq \widetilde{X}_s$ for all s. From the properties of S_x^{min} it follows immediately that if $x \in \widetilde{X}_s$, $x' \in \widetilde{X}_{s'}$ and $s > s'$, then $x > x'$. Finally, since $X_s \subseteq \widetilde{X}_s$ for all s, the same implication holds for X_s.

Convexity We start with the following lemma which will be useful to establish properties of X_k throughout the paper.

Lemma A2 *Suppose Assumptions 1, 2 and 3 hold and let*

$$\omega_x = \min_s \left\{ \frac{w_s \psi_{k,x}}{\psi_{s,x}} \right\}$$

be the minimal effective unit cost of producing the amount $\psi_{k,x}$ of task x with labor. Then, the set of automated tasks is equal to the upper level set of ω at level $1/q$,

$$X_k = \{x | \omega_x \geq 1/q\}.$$

Proof. The unit cost of producing the amount $\psi_{k,x}$ of task x with capital is $1/q$. Therefore, we must have $1/q \leq \omega_x$ on X_k. Moreover, if $1/q < \omega_x$ at some x, then x must be in X_k. Hence,

$$\{x | \omega_x > 1/q\} \subseteq X_k \subseteq \{x | \omega_x \geq 1/q\}.$$

Now suppose that X_k and $\{x | \omega_x \geq 1/q\}$ differ by a set of strictly positive measure. Then, since the labor endowment has no mass points, a strictly positive measure of skills must be assigned to a subset of $\{x | \omega_x = 1/q\}$. In particular, there must exist skill levels $s_1 < s_2 < s_3$ assigned to tasks $x_1 < x_2 < x_3$ in $\{x | \omega_x = 1/q\}$. Moreover, since the cost-minimizing skill S_x^{min} is unique for every task (see first step of the proof), we must have

$$\frac{w_{s_2} \psi_{k,x_1}}{\psi_{s_2,x_1}} > \frac{w_{s_1} \psi_{k,x_1}}{\psi_{s_1,x_1}} = \frac{w_{s_2} \psi_{k,x_2}}{\psi_{s_2,x_2}} = \frac{w_{s_3} \psi_{k,x_3}}{\psi_{s_3,x_3}} < \frac{w_{s_2} \psi_{k,x_3}}{\psi_{s_2,x_3}}.$$

But this string of relations contradicts the quasi-concavity of $\psi_{k,x}/\psi_{s_2,x}$. Hence, the difference between X_k and $\{x | \omega_x \geq 1/q\}$ must be of measure zero and we can set $X_k = \{x | \omega_x \geq 1/q\}$ without loss of generality. ■

The "if" part of Proposition 2.2 follows from Lemma A2. By Assumption 3, $w_s \psi_{k,x}/\psi_{s,x}$ is quasi-concave in x for all s. Thus, ω_x is the lower envelope of quasi-concave functions and as such it is quasi-concave itself. Hence, its upper level sets are convex and so is X_k.

Next, consider the "only if" part. We will prove that if $\psi_{k,x}/\psi_{s,x}$ is not quasi-concave in x for some s, then there exists a labor endowment l and capital productivity q such that X_k is not convex. For this it turns out useful to rewrite labor market clearing as

$$\int_0^s \int_0^1 L_{s',x} \, dx \, ds' = H_s \quad \text{for all } s,$$

where H_s is the cumulative distribution function of labor endowments. This specification allows to embed mass points as jumps in H_s.

Suppose now that $\psi_{k,x}/\psi_{s',x}$ is not quasi-concave in x for s' and consider the case where only skill s' is supplied,

$$H_s = \mathbb{I}_{s>s'} = \begin{cases} 0 & \text{if } s < s' \\ 1 & \text{if } s \geq s'. \end{cases}$$

Since $\psi_{k,x}/\psi_{s,x}$ is not quasi-concave, there exist $x_1 < x_2 < x_3$ such that

$$\frac{w_{s'}\psi_{k,x_1}}{\psi_{s',x_1}} > \frac{w_{s'}\psi_{k,x_2}}{\psi_{s',x_2}} < \frac{w_{s'}\psi_{k,x_3}}{\psi_{s',x_3}}.$$

Because only labor of type s' is being supplied, Euler's theorem in this case implies that $w_{s'}$ equals net output. Next note that net output is continuous in the allocation and in q, and hence Berge's maximum theorem applies and implies that equilibrium net output is continuous in q (and equilibrium allocations are upper hemicontinuous in q). Moreover, net output is also increasing in q (see Proposition 8). Thus, the wage $w_{s'}$ is continuously increasing in q and there exists a value for q such that

$$\frac{w_{s'}\psi_{k,x_1}}{\psi_{s',x_1}}, \frac{w_{s'}\psi_{k,x_3}}{\psi_{s',x_3}} > \frac{1}{q} > \frac{w_{s'}\psi_{k,x_2}}{\psi_{s',x_2}},$$

which implies that X_k cannot be convex.

It remains to extend the result to labor endowments without mass points, which is a simple continuity argument. Net output is continuous in allocations while the set of feasible allocations is continuous in the endowment cumulative density function H. Thus by the maximum theorem, the set of equilibrium allocations is upper hemicontinuous in H. Since there is no equilibrium allocation generating a convex X_k under the endowment function $\mathbb{I}_{s>s'}$ considered above, we can construct a sequence of differentiable endowment functions with strictly positive derivative, $\{H^{(n)}\}_{n\in\mathbb{N}}$, that converges to $\mathbb{I}_{s>s'}$; for sufficiently large n, the set of automated tasks cannot be convex.

A.3.3 Characterization of the Interior Automation Threshold

Here, we provide characterize the productivity threshold q_m at which automation transitions from interior to low-skill or high-skill, respectively. For a characterization of the threshold q_0 at which automation starts, see the proof of Proposition A2 in Appendix A.2. We assume that our Assumptions 1-3 and Condition 1 are satisfied.

We distinguish three cases. First, suppose that Condition 2 holds (as in part 3 of Proposition 3). We know from Condition 1 that $\psi_{k,x}/\psi_{0,x}$ is strictly increasing in x on a neighborhood of $x = 0$. Thus, we can define a threshold task $\overline{x}_m$ as the smallest $x \in (0, 1)$ such that $\frac{\psi_{k,0}}{\psi_{0,0}} = \frac{\psi_{k,x}}{\psi_{0,x}}$. That is, the productivity ratio between capital and the least skilled workers is the same at task $\overline{x}_m$ and task 0. Note that such an $\overline{x}_m$ exists if and only if $\frac{\psi_{0,0}}{\psi_{k,0}} < \frac{\psi_{0,1}}{\psi_{k,1}}$, which is Condition 2.1.

Now, suppose that we restrict capital to tasks below $\overline{x}_m$ and labor to tasks above $\overline{x}_m$. Then we choose the allocation that maximizes net output subject to these restrictions. Let w^m_s be the resulting wage function. Note that w^m_s is strictly increasing in q, allowing us to define q_m as the unique value of q that solves $\frac{1}{q_m} = \frac{w^m_0(q_m)\psi_{k,\overline{x}_m}}{\psi_{0,\overline{x}_m}}$, where we wrote $w^m_0(q)$ to emphasize the dependence of w^m_0 on q.

Intuitively, this condition equates the costs of producing task $\overline{x}_m$ with capital and with the least skilled workers.

Finally, note that if $q = q_m$, the restriction of capital to tasks below $\overline{x}_m$ and labor to tasks above $\overline{x}_m$ is not binding, and in this case we have $X_k = [0, \overline{x}_m]$.

For the second case, suppose that both Condition 2.1 and Condition 2.2 are violated strictly (as in part 5 of Proposition 3). This case is completely symmetric to the first case: we define $\overline{x}_m$ as the largest $x \in (0, 1)$ such that $\frac{\psi_{k,1}}{\psi_{1,1}} = \frac{\psi_{k,x}}{\psi_{1,x}}$, and q_m analogously to the first case.

In the third case, Condition 2.1 is violated strictly while Condition 2.2 holds. Then, a threshold task $\overline{x}_m$ as defined in either the first or the second case does not exist, and we simply set $q_m = q_\infty$.

A.3.4 Proof of Proposition 3: Interior Automation

In Appendix A.2, we establish Proposition A2, which is a more general version of Proposition 3. Compared to Proposition A2, in Proposition 3 we additionally impose Assumption 3, which implies a convex set of automated tasks (see Proposition 2). With a convex set of automated tasks, weakly interior automation implies interior automation, weakly low-skill automation implies low-skill automation, and weakly high-skill automation implies high-skill automation. Thus, parts 1, 2, 3 and 5 of Proposition 3 follow immediately from Proposition A2 and the fact that Assumption 3 guarantees a convex set of automated tasks in Proposition 3.

It remains to prove part 4 of Proposition 3, which is the case in which the automation pattern is ambiguous under the assumptions of Proposition A2. We prove part 4 by showing that (i) automation cannot be high-skill under Condition 2.2 and (ii) automation cannot be low-skill if Condition 2.1 is violated strictly. Together, this implies that automation is interior for all $q \in (q_0, q_\infty)$ if Condition 2.1 is violated strictly while Condition 2.2 holds, which is part 4 of Proposition 3.

No High-Skill Automation under Condition 2.2 We first show that there cannot be high-skill automation if Condition 2.2 holds. The proof is by contradiction.

Suppose $1 \in X_k$. Then, it must be cheaper to produce task $x = 1$ with capital than with labor, $w_s\psi_{k,1}/\psi_{s,1} \geq 1/q$ for all s. Moreover, this inequality must hold strictly for all but the most skilled workers $s = 1$, because Assumption 2 implies that among all labor types, task $x = 1$ can be produced at the lowest cost using skill $s = 1$, $S_1^{min} = \{1\}$.

At the same time, combining $w_1\psi_{k,1}/\psi_{1,1} \geq 1/q$ with Condition 2.2 implies that it must be strictly cheaper to produce task $x = 0$ with capital than with the most skilled workers, $w_1\psi_{k,0}/\psi_{1,0} > 1/q$. By continuity of labor productivity $\psi_{s,x}$, this extends to some neighborhood of $s = 1$, i.e., there exists $\epsilon > 0$ such that $w_s\psi_{k,0}/\psi_{s,0} > 1/q$ for all $s \in (1 - \epsilon, 1]$.

Hence, we have shown that, for every $s \in (1-\epsilon, 1)$, s is strictly more expensive than capital in both the most and the least complex task. Quasi-concavity of $\psi_{k,x}/\psi_{s,x}$ (Assumption 3) requires that this extends to all tasks:

$$\frac{w_s\psi_{k,x}}{\psi_{s,x}} > \frac{1}{q} \quad \text{for all } s \in (1 - \epsilon, 1).$$

But this implies that skill levels $s \in (1-\epsilon, 1)$ cannot be assigned to any task in equilibrium, which is clearly incompatible with an equilibrium allocation maximizing (finite) net output. Hence, we must have $1 \notin X_k$: the most complex task is not automated.

No Low-Skill Automation without Condition 2.1 Now suppose that Condition 2.1 is violated strictly, i.e.,

$$\frac{\psi_{0,0}}{\psi_{k,0}} > \frac{\psi_{0,1}}{\psi_{k,1}} .$$

Then, arguments entirely symmetric to those of the previous paragraph show that there cannot be low-skill automation, such that we must have $1 \notin X_k$.

A.3.5 Proof of Proposition 4: Minimum Wages and Automation

The first part of the proof follows closely the proof of part 1 of Proposition A1. In particular, condition (8) together with comparative advantage across labor types (Assumption 2) implies that $\psi_{k,x}/\psi_{s,x}$ is decreasing in x for all $s > \underline{s}$ and, by continuity, also for $s = \underline{s}$ (where $\underline{s}$ is such that all skills below $\underline{s}$ are non-employed due to the minimum wage). Thus, the minimum labor cost function ω_x is decreasing and the set of automated tasks, which is equal to $\{x|\omega_x \geq 1/q\}$, is either empty or contains zero.

The second part is to show that, if $q \in (q_0, q_m)$, then the set of automated tasks remains non-empty after the introduction of the minimum wage. For this, we compare the assignment problems without capital, with and without the minimum wage.

Without capital, our setting has been studied extensively in the literature (e.g., Costinot and Vogel 2010). The introduction of the minimum wage is equivalent to a shift in the lower bound of the skill space from 0 to $\underline{s}$. Without capital, it leads to a decline in skill premia along the entire skill space, with the wage of the least skilled remaining worker type $\underline{s}$ increasing and the wage of the most skilled worker type ($s = 1$) decreasing (Teulings, 2000; Costinot and Vogel, 2010). Let w_s^0 be the wage function without capital and without minimum wage (and ω_x^0 the associated minimum labor cost function) and $w_s^{0,min}$ the wage function without capital but with minimum wage. Then,

$$\frac{1}{q} < \max_x \omega_x^0 \leq \max_x \frac{w_{\underline{s}}^0 \psi_{k,x}}{\psi_{\underline{s},x}} \leq \frac{w_{\underline{s}}^{0,min} \psi_{k,0}}{\psi_{\underline{s},0}},$$

where the first inequality uses that $q > q_0$, the second follows from the definition of ω_x^0 as the lower envelope of all workers' effective cost, and the last inequality is implied by $w_{\underline{s}}^0 \leq w_{\underline{s}}^{0,min}$ and $\psi_{k,x}/\psi_{\underline{s},x}$ being decreasing in x. The inequalities imply that if X_k were empty, we had $0 \in X_k$ by Lemma A2, a contradiction. Hence, X_k remains non-empty after introduction of the minimum wage.

A.3.6 Proof of Proposition 5: Labor Supply and Automation

We start with a useful lemma on the wage effects of labor supply changes which holds for all settings where production is concave and linear homogeneous in labor and wages equal marginal products.

Lemma A3 *Consider any two labor endowments $l > 0$ and $l^{new} > 0$ with corresponding wage functions w and w^{new}. Then, if $w_s \leq w_s^{new}$ for all s, we must have $w = w^{new}$.*

Proof. By Euler's homogeneous function theorem, we have

$$\begin{aligned} NY^{new} - NY &= \int_0^1 w_s^{new} l_s^{new}\, ds - \int_0^1 w_s l_s\, ds \\ &= \int_0^1 (w_s^{new} - w_s) l_s^{new}\, ds + \int_0^1 w_s (l_s^{new} - l_s)\, ds \\ \Rightarrow \qquad \int_0^1 (w_s^{new} - w_s) l_s^{new}\, ds &= NY^{new} - NY - \int_0^1 w_s (l_s^{new} - l_s)\, ds \leq 0, \end{aligned}$$

where the inequality in the last line follows from concavity of net output in labor inputs. The last line shows that it is impossible to have $w_s \leq w_s^{new}$ for all s and with strict inequality on a subset of skills of strictly positive measure. Hence, if $w_s \leq w_s^{new}$ for all s, then $w = w^{new}$, i.e., the two wage functions are equal almost everywhere. ■

The important implication of Lemma A3 is that a labor supply change alone can never cause all wages to increase or all wages to decrease. Instead there will always be some wages that increase and some that decrease, except in the case where the wage function is completely unchanged.

We can now prove that the threshold where automation transitions from interior to low-skill, q_m, is strictly decreasing in relative skill supply. First, recall from characterization of q_m in Appendix A.3.3 that q_m is defined as the unique solution to

$$\frac{1}{q_m} = \frac{w_0^m(q_m)\psi_{k,\overline{x}_m}}{\psi_{0,\overline{x}_m}}, \tag{A2}$$

where w^m is the wage function obtained under the restriction that capital can only be allocated to tasks below $\overline{x}_m$ and labor only to tasks above $\overline{x}_m$. This equation has a unique solution because the left-hand side is strictly decreasing and the right-hand side strictly increasing in q_m.

Now consider an increase in relative skill supply, $\Delta \log l$ with $\Delta \log l_s$ strictly increasing in s. We know from prior work (e.g., Costinot and Vogel, 2010) that, in the pure assignment model without capital, such a change in labor supply would lower all skill premia. With the restriction that capital must be assigned below and labor above $\overline{x}_m$, the labor allocation is determined as in a pure labor assignment model. Hence, the result from prior work applies and the wage change $\Delta \log w_s^m$ must be strictly decreasing in s. By Lemma A3, the wage change cannot be negative for all skill levels and we must have $\Delta \log w_0^m > 0$.[26] Thus, the right-hand side of equation (A2) increases, such that q_m must decrease to solve equation (A2), so $\Delta q_m < 0$.

A.4 Proofs for Section 4: Local Effects of Automation

We use our characterization of the wage and the assignment function in terms of the differential equation system (9)-(13) to conduct comparative statics with respect to capital productivity. Implicitly, this imposes Assumptions 1 to 3 and $q \geq q_0$.

[26] Note that the proof of Lemma A3 only uses linear homogeneity and concavity of net output in labor, so it also applies to the situation where capital is restricted to tasks below and labor to tasks above $\overline{x}_m$.

We consider a small change in capital productivity $d\log q$ (if $q = q_0$ we impose $d\log q > 0$ such that our equilibrium characterization continues to hold) and study its first-order effects on wages and assignment. From equations (9) and (11), we obtain the variational equations

$$(d\log w_s)' = \frac{\partial^2 \log \psi_{s,X_s}}{\partial s \partial x} dX_s \tag{A3}$$

$$\begin{aligned}(dX_s)' =& \lambda \frac{l_s w_s^{\lambda}}{Y \psi_{s,X_s}^{\lambda-1}} d\log w_s - \frac{l_s w_s^{\lambda}}{Y \psi_{s,X_s}^{\lambda-1}} d\log Y \\ & - (\lambda - 1) \frac{l_s w_s^{\lambda}}{Y \psi_{s,X_s}^{\lambda-1}} \frac{\partial \log \psi_{s,X_s}}{\partial x} dX_s,\end{aligned} \tag{A4}$$

which hold for all $s \neq \widetilde{s}$. The boundary conditions for the upper branch of these variations, i.e., the branch on $(\widetilde{s}, 1]$, are given by

$$\begin{aligned} d\log w_{\widetilde{s}}^{+} &= \left(\frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\overline{x}}}{\partial x} \right) d\overline{x} + \frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial s} d\widetilde{s} - d\log q - (\log w_{\widetilde{s}})'^{+} d\widetilde{s} \\ &= \left(\frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\overline{x}}}{\partial x} \right) d\overline{x} - d\log q \end{aligned} \tag{A5}$$

$$\begin{aligned} dX_{\widetilde{s}}^{+} &= d\overline{x} - (X_{\widetilde{s}})'^{+} d\widetilde{s} \\ &= d\overline{x} - \frac{l_{\widetilde{s}} w_{\widetilde{s}}^{\lambda}}{Y \psi_{\widetilde{s},\overline{x}}^{\lambda-1}} d\widetilde{s}, \end{aligned} \tag{A6}$$

where the superscript '+' denotes the right-side limit of the respective function. The boundary conditions for the lower branch (which exists only if automation is interior) are

$$\begin{aligned} d\log w_{\widetilde{s}}^{-} &= \left(\frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\underline{x}}}{\partial x} \right) d\underline{x} + \frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial s} d\widetilde{s} - d\log q - (\log w_{\widetilde{s}})'^{-} d\widetilde{s} \\ &= \left(\frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\underline{x}}}{\partial x} \right) d\underline{x} - d\log q \end{aligned} \tag{A7}$$

$$\begin{aligned} dX_{\widetilde{s}}^{-} &= d\underline{x} - (X_{\widetilde{s}})'^{-} d\widetilde{s} \\ &= d\underline{x} - \frac{l_{\widetilde{s}} w_{\widetilde{s}}^{\lambda}}{Y \psi_{\widetilde{s},\underline{x}}^{\lambda-1}} d\widetilde{s}, \end{aligned} \tag{A8}$$

with the superscript '−' denoting left-side limits.

From the upper branch of the system, we can obtain the change in assignment of the most skilled workers, $dX_1(d\overline{x}, d\widetilde{s})$, as a function of $d\overline{x}$ and $d\widetilde{s}$. Analogously, if automation is interior, the lower branch yields $dX_0(d\underline{x}, d\widetilde{s})$, the change in assignment of the least skilled workers as a function of $d\underline{x}$ and $d\widetilde{s}$. Both of these changes must be zero in equilibrium, which defines functions $d\overline{x}(d\widetilde{s})$ and $d\underline{x}(d\widetilde{s})$. The following lemma establishes some properties of $d\overline{x}(d\widetilde{s})$ and $d\underline{x}(d\widetilde{s})$.

Lemma A4 *Suppose Assumptions 1, 2 and 3 hold, $q \geq q_0$ and $d\log q > 0$. Then, if $\overline{x} < 1$, the function $d\overline{x}(d\widetilde{s})$ is strictly increasing, satisfies $d\overline{x}(0) > 0$ and*

$$\left(\frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\overline{x}}}{\partial x} \right) d\overline{x}(0) < d\log q + \frac{1}{\lambda} d\log Y.$$

Moreover, if $\underline{x} > 0$*,* $d\underline{x}(d\widetilde{s})$ *is strictly increasing, satisfies* $d\underline{x}(0) < 0$ *and*

$$\left(\frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\underline{x}}}{\partial x}\right) d\underline{x}(0) < d\log q + \frac{1}{\lambda} d\log Y.$$

Proof. We focus on the results for $d\overline{x}(d\widetilde{s})$. The proof for $d\underline{x}(d\widetilde{s})$ proceeds analogously. Define

$$d\log \widetilde{w}_s = d\log w_s - \frac{1}{\lambda} d\log Y$$

as the wage change net of the productivity effect, i.e., the pure displacement effect of automation.

With this, we can write the variational equations (A3) and (A4) more compactly as

$$\begin{aligned}(d\log \widetilde{w}_s)' &= \gamma(s) dX_s \\ (dX_s)' &= \alpha(s) dX_s + \beta(s) d\log \widetilde{w}_s\end{aligned}$$

where $\gamma(s), \beta(s) > 0$ for all s and the initial values are

$$\begin{aligned}d\log \widetilde{w}^+_{\widetilde{s}} &= \left(\frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\overline{x}}}{\partial x}\right) d\overline{x} - d\log q - \frac{1}{\lambda} d\log Y \\ dX^+_{\widetilde{s}} &= d\overline{x} - \frac{l_{\widetilde{s}} w^{\lambda}_{\widetilde{s}}}{Y \psi^{\lambda-1}_{\widetilde{s},\overline{x}}} d\widetilde{s}.\end{aligned}$$

We next show that the implied change dX_1 is strictly increasing in both initial values. First consider $dX^{+(1)}_{\widetilde{s}} < dX^{+(2)}_{\widetilde{s}}$ with $d\log \widetilde{w}^+_{\widetilde{s}}$ equal in both cases. The implied path $dX^{(2)}_s$ starts above $dX^{(1)}_s$. Suppose now $dX^{(2)}_s$ crosses $dX^{(1)}_s$ for the first time at a skill level s_1. Clearly, this crossing must be from above, that is, we need $(dX^{(2)}_{s_1})' \leq (dX^{(1)}_{s_1})'$. But from the differential equations, we obtain

$$(dX^{(1)}_{s_1})' - (dX^{(2)}_{s_1})' = \beta(s_1)(d\log \widetilde{w}^{(1)}_{s_1} - d\log \widetilde{w}^{(2)}_{s_1}) < 0,$$

where the last inequality follows from the fact that $d\log \widetilde{w}^{(2)}_s$ starts from the same value as $d\log \widetilde{w}^{(1)}_s$ but increases at a faster rate until s_1 because $dX^{(2)}_s > dX^{(1)}_s$ for $s < s_1$. It follows that the two paths cannot cross and we have $dX^{(2)}_1 > dX^{(1)}_1$.

The reasoning for the second initial value, $d\log \widetilde{w}^+_{\widetilde{s}}$, follows a similar line. If we increase $d\log \widetilde{w}^+_{\widetilde{s}}$, the path dX_s will have a larger slope initially and hence move upwards for s slightly above $\widetilde{s}$. Then, the argument for why it can't cross its original path again, is the same as above.

We have thus shown that the change dX_1 is strictly increasing in both initial values. Both initial values, in turn, are increasing in $d\overline{x}$. Moreover, $dX^+_{\widetilde{s}}$ is strictly decreasing in $d\widetilde{s}$ while $d\log \widetilde{w}^+_{\widetilde{s}}$ is unaffected by $d\widetilde{s}$. Thus, dX_1 is strictly increasing in $d\overline{x}$ and strictly decreasing in $d\widetilde{s}$. Then, setting $dX_1 = 0$ yields the first claim of the lemma: $d\overline{x}$ is strictly increasing in $d\widetilde{s}$.

Next, we show that if $d\widetilde{s} = 0$, then $dX_1 = 0$ requires to set $d\overline{x}$ such that the initial value $dX^+_{\widetilde{s}}$ is positive while $d\log \widetilde{w}^+_{\widetilde{s}}$ is negative (recall from above that $d\log \widetilde{w}^+_{\widetilde{s}}$ is negative at $d\overline{x} = 0$). To see why this must be the case, suppose we were to start with both initial values negative. Then, dX_s could never attain zero for any $s \geq \widetilde{s}$ by reasoning analogous to that used above: If dX_s approaches zero from below, its derivative will turn negative because $\beta(s) d\log \widetilde{w}_s$ is negative ($d\log \widetilde{w}_s$ starts from a negative initial value and declines from there because dX_s is negative initially). Similarly, if we start with both initial values positive (and at least one strictly positive), dX_s can never attain zero either, because as it approaches zero, its derivative will become positive by reversing the arguments from the case with both initial values negative.

Hence, when setting $d\widetilde{s}$ to zero, then $dX_1 = 0$ requires $d\overline{x}$ to be strictly positive but sufficiently small for $\log \widetilde{w}_{\widetilde{s}}^{+}$ to be strictly negative, which are the second and third claims of the lemma. ■

A.4.1 Proof of Proposition 6: Automation and Employment Polarization

Expansion of Automation Suppose now that Condition 1 holds and $q \in (q_0, q_m)$ such that automation is interior. In this case, condition (10) requires that

$$\underbrace{\left(\frac{\partial \psi_{\widetilde{s},\overline{x}}}{\partial x} + \frac{\partial \log \psi_{k,\overline{x}}}{\partial x}\right)}_{=\gamma_{\widetilde{s},\overline{x}}} d\overline{x}(d\widetilde{s}) + \frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial s} d\widetilde{s} = \underbrace{\left(\frac{\partial \psi_{\widetilde{s},\underline{x}}}{\partial x} + \frac{\partial \log \psi_{k,\underline{x}}}{\partial x}\right)}_{=\gamma_{\widetilde{s},\underline{x}}} d\underline{x}(d\widetilde{s}) + \frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial s} d\widetilde{s}, \tag{A9}$$

where we have already inserted the functions $d\overline{x}(d\widetilde{s})$ and $d\underline{x}(d\widetilde{s})$ derived from equations (A3) to (A8) above. Rearranging and signing terms, we obtain:

$$\underbrace{\gamma_{\widetilde{s},\overline{x}}}_{\geq 0} d\overline{x}(d\widetilde{s}) - \underbrace{\gamma_{\widetilde{s},\underline{x}}}_{\leq 0} d\underline{x}(d\widetilde{s}) = \underbrace{\left(\frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial s} - \frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial s}\right)}_{<0} d\widetilde{s}.$$

By Lemma A4, the left-hand side of this equation is increasing while the right-hand side is strictly decreasing in $d\widetilde{s}$. Thus, the equation determines a unique equilibrium change $d\widetilde{s}^*$.

If $\gamma_{\widetilde{s},\overline{x}} d\overline{x}(0) - \gamma_{\widetilde{s},\underline{x}} d\underline{x}(0)$ is strictly positive, then $d\widetilde{s}^*$ must be strictly negative, which implies that $d\underline{x}(d\widetilde{s}^*) < 0$ (by Lemma A4) and

$$d\overline{x}(d\widetilde{s}^*) = \frac{\gamma_{\widetilde{s},\underline{x}}}{\gamma_{\widetilde{s},\overline{x}}} d\underline{x}(d\widetilde{s}^*) + \frac{1}{\gamma_{\widetilde{s},\overline{x}}} \left(\frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial s} - \frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial s}\right) d\widetilde{s}^* > 0.$$

Note here that $\gamma_{\widetilde{s},\overline{x}} d\overline{x}(0) - \gamma_{\widetilde{s},\underline{x}} d\underline{x}(0) > 0$ can only hold if $\gamma_{\widetilde{s},\overline{x}} > 0$.

Analogously, if $\gamma_{\widetilde{s},\overline{x}} d\overline{x}(0) - \gamma_{\widetilde{s},\underline{x}} d\underline{x}(0)$ is strictly negative, then $d\widetilde{s}^*$ must be strictly positive, which implies that $d\overline{x}(d\widetilde{s}^*) > 0$ and $d\underline{x}(d\widetilde{s}^*) < 0$.

Finally, if $\gamma_{\widetilde{s},\overline{x}} d\overline{x}(0) - \gamma_{\widetilde{s},\underline{x}} d\underline{x}(0)$ equals zero, then $d\widetilde{s}^*$ must be zero as well, such that $d\overline{x}(\widetilde{s}^*) > 0$ and $d\underline{x}(d\widetilde{s}^*) < 0$ follow immediately from Lemma A4.

At this point, note also that

$$\max\{\gamma_{\widetilde{s},\overline{x}} d\overline{x}(d\widetilde{s}^*), \gamma_{\widetilde{s},\underline{x}} d\underline{x}(d\widetilde{s}^*)\} \leq \max\{\gamma_{\widetilde{s},\overline{x}} d\overline{x}(0), \gamma_{\widetilde{s},\underline{x}} d\underline{x}(0)\} < d \log q + \frac{1}{\lambda} d \log Y,$$

where the second inequality follows from Lemma A4. This implies that the initial values $d \log \widetilde{w}_{\widetilde{s}}^{+}$ and $d \log \widetilde{w}_{\widetilde{s}}^{-}$ must both be negative at $d\widetilde{s}^*$. This result will be useful in the next step of the proof.

Employment Polarization We have just shown above that $d\log\widetilde{w}_{\widetilde{s}}^{+}$ is strictly negative in equilibrium. This implies that the initial value $dX_{\widetilde{s}}^{+}$ in the dynamic system for dX_s and $d\log\widetilde{w}_s$ must be strictly positive:

$$dX_{\widetilde{s}}^{+} = d\overline{x} - \frac{l_{\widetilde{s}} w_{\widetilde{s}}^{\lambda}}{Y\psi_{\widetilde{s},\overline{x}}^{\lambda-1}} d\widetilde{s} > 0.$$

If it were negative, we could never attain $dX_1 = 0$ by the reasoning in the proof of Lemma A4.

Suppose now that at some skill $s_1 > \widetilde{s}$, dX_s turns negative, that is, it crosses zero from above: $dX_{s_1} = 0$ and $(dX_{s_1})' \leq 0$. To obtain $dX_1 = 0$, dX_s must at some point $s_2 > s_1$ attain zero again, this time from below: $dX_{s_2} = 0$ and $(dX_{s_2})' \geq 0$. The differential equation for $(dX_s)'$, however, implies that

$$(dX_s)' = \beta(s) d\log\widetilde{w}_s \quad \text{for } s = s_1, s_2.$$

We must therefore have $d\log\widetilde{w}_{s_1} \leq 0$. Since dX_s is negative between s_1 and s_2, we will also have $d\log\widetilde{w}_{s_2} < 0$ by the equation for $(d\log\widetilde{w}_s)'$. This in turn implies $(dX_{s_2})' < 0$, a contradiction.

As a result, dX_s cannot cross zero but stays positive until dX_1.[27] Analogous reasoning yields $dX_s < 0$ for $0 < s < \widetilde{s}$.

Labor Share Instead of proving the results for the labor share directly, we prove that the inverse of these results holds for the capital share. From equation (14), we obtain the response of the capital share to the increase in capital productivity as

$$\begin{aligned} d\alpha_k &= \alpha_k(\lambda-1)d\log q + q^{\lambda-1}\frac{\partial\Gamma_k}{\partial\underline{x}}d\underline{x} + q^{\lambda-1}\frac{\partial\Gamma_k}{\partial\overline{x}}d\overline{x} \\ &= \alpha_k(\lambda-1)d\log q - q^{\lambda-1}\psi_{k,\underline{x}}^{\lambda-1}d\underline{x} + q^{\lambda-1}\psi_{k,\overline{x}}^{\lambda-1}d\overline{x}. \end{aligned} \tag{A10}$$

The last two terms are strictly positive by our employment polarization result, such that the capital share increases if $\lambda \geq 1$.

If $\lambda < 1$, the total effect on the capital share depends on the relative strength of the capital deepening effect (first term) and the expansion of the set of automated tasks (second and third term). If $q = q_0$, we have $\alpha_k = 0$, such that the capital deepening effect vanishes. Moreover, Lemma A4 implies that

$$\max\{|d\overline{x}|, |d\underline{x}|\} \geq \min\{|d\overline{x}(0)|, |d\underline{x}(0)|\} > 0,$$

which means that the expansion of the set of automated tasks does not vanish. So, we must have $d\alpha_k(q_0) > 0$. Finally, note that $d\alpha_k$ (considering the perturbation $d\log q > 0$) is a right-hand derivative and as such it is continuous from the right, i.e.,

$$\lim_{q\searrow q_0} d\alpha_k = d\alpha_k(q_0) > 0.$$

This proves that $d\alpha_k > 0$ in some right neighborhood of q_0.

[27] We can exclude the case where dX_s has a critical zero (a point where dX_s is tangent to zero but does not cross it). This is because a critical zero would imply $(dX_s)' = 0$ and $dX_s = 0$. But then, the entire upper branch of dX_s would be identically zero, which is incompatible with the initial value $dX_{\widetilde{s}}^{+}$ being strictly positive.

A.4.2 Proof of Proposition 7: Automation and Wage Polarization

By (A3), we have

$$d \log w_s - d \log w_{s'} = \int_{s'}^{s} \frac{\partial^2 \log \psi_{t,X_t}}{\partial s \partial x} dX_t \, dt.$$

By Assumption 2 and our employment polarization result in Proposition 6, this expression is strictly positive for all $s > s' \geq \widetilde{s}$ and also for all $s < s' \leq \widetilde{s}$.

A.4.3 Proof of Proposition 8: Automation and Wage Levels

We have already proved part 1 of the proposition in the main text. Here we prove parts 2 to 4.

Part 2 Condition (10) implies that

$$d \log w_{\widetilde{s}} = \begin{cases} \frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial s} d\widetilde{s} + \gamma_{\widetilde{s},\underline{x}} d\underline{x} - d \log q - (d \log w_{\widetilde{s}})'^{-} = \gamma_{\widetilde{s},\underline{x}} d\underline{x} - d \log q & \text{if } d\widetilde{s} \geq 0 \\ \frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial s} d\widetilde{s} + \gamma_{\widetilde{s},\overline{x}} d\overline{x} - d \log q - (d \log w_{\widetilde{s}})'^{+} = \gamma_{\widetilde{s},\overline{x}} d\overline{x} - d \log q & \text{if } d\widetilde{s} \leq 0 \end{cases} \tag{A11}$$

where $\gamma_{\widetilde{s},\underline{x}}$ and $\gamma_{\widetilde{s},\overline{x}}$ are defined as in equation (A9). Now suppose at first that $q = q_0$. Then, $\overline{x} = \underline{x}$ and $\gamma_{\widetilde{s},\underline{x}} = \gamma_{\widetilde{s},\overline{x}} = 0$ because $\underline{x} = \overline{x}$ is a maximizer of the effective labor cost function ω_x. So, we obtain $d \log w_{\widetilde{s}} = -d \log q < 0$.

Next, for $d \log q > 0$, $d \log w_{\widetilde{s}}$ is a right-hand derivative and thus must be continuous from the right. So, $d \log w_{\widetilde{s}} < 0$ in a right neighborhood of q_0. Finally, this extends to skills in some neighborhood around $\widetilde{s}$ because the wage change $d \log w_s$ is continuous in s.

Part 3 If $\psi_{s',x}/\psi_{k,x}$ is constant, we must have

$$\frac{w_{s'}}{\psi_{s',x}} = \frac{1/q}{\psi_{k,x}}$$

for all x.[28] Differentiating this, we obtain

$$d \log w_{s'} = -d \log q < 0.$$

Since the change $d \log w_s$ is continuous in s, we obtain $d \log w_s < 0$ for all s in some neighborhood of s'.

Part 4 Suppose at first that $q = q_m$ and consider $d \log q < 0$. It is easy to check that for $d \log q < 0$, the reasoning of Lemma A4 can be adjusted to imply that $d\overline{x}(d\widetilde{s})$ is still strictly increasing but now $d\overline{x}(0) < 0$ and

$$\left(\frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\overline{x}}}{\partial x} \right) d\overline{x}(0) > d \log q + \frac{1}{\lambda} d \log Y.$$

Since at $q = q_m$ we have $\underline{x} = 0$ and $\widetilde{s} = 0$, the analogous results for $d\underline{x}$ do not apply. Instead, we have $dX_{\widetilde{s}}^{-} = 0$ and hence by equation (A8),

$$d\underline{x}(d\widetilde{s}) = \frac{l_0 w_0^{\lambda}}{Y \psi_{\widetilde{s},\underline{x}}^{\lambda-1}} d\widetilde{s}.$$

[28] Otherwise, either the set of automated tasks were empty (if the equation held with $<$ instead of $=$) or skill s' could not be assigned to any task (if the equation held with $>$ instead of $=$).

So $d\underline{x}(\widetilde{s})$ is strictly increasing and $d\underline{x}(0) = 0$.

As we consider $d\log q < 0$ starting from q_m, equation (10) holds and so does its variational counterpart (A9). Then, by reasoning analogous to that in the first part of the proof of Proposition 6, we can show that $d\overline{x} < 0$ and $d\underline{x} \geq 0$. Next, by the same reasoning as in the second part of the proof of Proposition 6, we obtain that $dX_s < 0$ for all $s \in (0,1)$. By the argument in the proof of Proposition 7, this implies that $d\log w_1 < d\log w_0$.

Finally, note that for $d\log q < 0$, the changes $d\log w_1$ and $d\log w_0$ are left-hand derivatives and as such they are continuous from the left. So, we have that $d\log w_1 < d\log w_0$ in response to $d\log q < 0$ for all q in some left neighborhood of q_m. But for $q \in (q_0, q_m)$, wages are differentiable in q and we obtain the reverse for $d\log q > 0$, i.e., $d\log w_1 > d\log w_0$ in response to $d\log q > 0$ for all q in some left neighborhood of q_m.

A.4.4 Proof of Proposition 9: Productivity Effects

We first derive a second-order approximation of net output NY. Net output is given by

$$NY = \max_{\overline{K}} F(\overline{K}, l) - \frac{\overline{K}}{q}$$

where F is maximal output subject to aggregate factor supplies $\overline{K}$ and l (see proof of Proposition 1). So by the envelope theorem, we obtain $dNY/dq = \overline{K}/q^2$ and hence:

$$\frac{d\log NY}{d\log q} = \frac{\overline{K}/q}{NY} = \frac{\overline{K}/q}{Y - \overline{K}/q} = \frac{\alpha_k}{1-\alpha_k}.$$

The second-order term is then given by

$$\frac{d\log(\alpha_k/(1-\alpha_k))}{d\log q} = \frac{1}{1-\alpha_k}\frac{d\log\alpha_k}{d\log q},$$

which by our previous result (A10) can be written as

$$\frac{1}{1-\alpha_k}\left(\lambda - 1 + \frac{\partial\log\Gamma_k}{\partial\underline{x}}\frac{d\underline{x}}{d\log q} + \frac{\partial\log\Gamma_k}{\partial\overline{x}}\frac{d\overline{x}}{d\log q}\right).$$

Combining these first- and second-order terms yields our second-order Taylor approximation of $\Delta\log NY$:

$$\Delta\log NY \approx \frac{\alpha_k}{1-\alpha_k}\Delta\log q + \frac{1}{1-\alpha_k}\left[\lambda - 1 + \frac{\partial\log\Gamma_k}{\partial\overline{x}}\frac{d\overline{x}}{d\log q} + \frac{\partial\log\Gamma_k}{\partial\underline{x}}\frac{d\underline{x}}{d\log q}\right](\Delta\log q)^2 .$$

Next, we translate this into an expression for TFP. Since $Y = NY + \overline{K}/q$, we have

$$\Delta\log Y \approx (1-\alpha_k)\Delta\log NY + \alpha_k\left(\overline{K}/q\right) .$$

Using this in the definition of TFP, we obtain

$$\Delta\log TFP \approx (1-\alpha_k)\Delta\log NY .$$

Plugging in the second-order approximation of $\Delta\log NY$ derived above, we obtain the expression for TFP from Proposition 9.

References

Acemoglu, D. (1998): "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality," *Quarterly Journal of Economics*, **113** (4), 1055–1089.

——— (1999): "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence," *American Economic Review*, **89** (5), 1259–78.

——— (2007): "Equilibrium Bias of Technology," *Econometrica*, **75** (5), 1371–1409.

Acemoglu, D. and D. H. Autor (2011): "Skills, Tasks and Technologies: Implications for Employment and Earnings," in *Handbook of Labor Economics*, **4** Elsevier, 1043–1171.

Acemoglu, D. and S. Johnson (2023): *Power and Progress: Our Thousand-Year Struggle Over Technology and Prosperity*, Public Affairs.

Acemoglu, D. and P. Restrepo (2018a): "The Race Between Machine and Man: Implications of Technology for Growth, Factor Shares and Employment," *American Economic Review*, **108** (6), 1488–1542.

——— (2018b): "Modeling Automation," *AEA Papers and Proceedings*, **108**, 48–53.

——— (2020): "Robots and Jobs: Evidence from US Labor Markets," *Journal of Political Economy*, **128** (6), 2188–2244.

——— (2022): "Tasks, Automation, and the Rise in US Wage Inequality," *Econometrica*, **90** (5), 1973–2016.

——— (2024): "Distorted Automation: Tasks, Labor Market Rents and Unemployment in Search-Matching Model."

Acemoglu, D. and F. Zilibotti (2001): "Productivity Differences," *Quarterly Journal of Economics*, **116** (2), 563–606.

Ales, L., C. Combemale, E. R. H. Fuchs, and K. S. Whitefoot (2024): "How It's Made: A General Theory of the Labor Implications of Technical Change," Working Paper.

Autor, D. H. (2014): "Polanyi's Paradox and the Shape of Employment Growth," Working Paper 20485, NBER.

——— (2015): "Why Are There Still So Many Jobs? The History and Future of Workplace Automation," *Journal of Economic Perspectives*, **29** (3), 3–30.

Autor, D. H. and D. Dorn (2013): "The Growth of Low-Skill Service Jobs and the Polarization of the U.S. Labor Market," *American Economic Review*, **103** (5), 1553–1597.

Autor, D. H., F. Levy, and R. J. Murnane (2003): "The Skill Content of Recent Technological Change: An Empirical Exploration," *Quarterly Journal of Economics*, **118** (4), 1279–1333.

BEA (2021a): "Fixed Assets Accounts Tables, Table 2.7: Investment in Private Fixed Assets, Equipment, Structures, and Intellectual Property Products by Type," US Bureau of Economic Analysis.

——— (2021b): "National Income and Product Accounts, Table 6.8: Persons Engaged in Production by Industry," US Bureau of Economic Analysis.

Berg, A., E. F. Buffie, and L.-F. Zanna (2018): "Should we fear the robot revolution? (The correct answer is yes)," *Journal of Monetary Economics*, **97**, 117–148.

Costinot, A. and J. Vogel (2010): "Matching and Inequality in the World Economy," *Journal of Political Economy*, **118** (4), 747–786.

DiCecio, R. (2009): "Sticky Wages and Sectoral Labor Comovement," *Journal of Economic Dynamics and Control*, **33** (3), 538–553.

Feng, A. and G. Graetz (2020): "Training Requirements, Automation, and Job Polarisation," *Economic Journal*, **130** (631), 2249–2271.

Ford, M. (2015): *Rise of the Robots: Technology and the Threat of a Jobless Future*, Basic Books.

Gollin, D. (2002): "Getting Income Shares Right," *Journal of Political Economy*, **110** (2), 458–474.

Goos, M., A. Manning, and A. Salomons (2009): "Job Polarization in Europe," *American Economic Review*, **99** (2), 58–63.

Graetz, G. and G. Michaels (2018): "Robots at Work," *Review of Economics and Statistics*, **100** (5), 753–768.

Greenwood, J., Z. Hercowitz, and P. Krusell (1997): "Long-Run Implications of Investment-Specific Technological Change," *American Economic Review*, **87** (3), 342–362.

Heckman, J. J. and G. Sedlacek (1985): "Heterogeneity, Aggregation, and Market Wage Functions: An Empirical Model of Self-Selection in the Labor Market," *Journal of Political Economy*, **93** (6), 1077–1125.

Hémous, D. and M. Olsen (2022): "The Rise of the Machines: Automation, Horizontal Innovation, and Income Inequality," *American Economic Journal: Macroeconomics*, **14** (1), 179–223.

Humlum, A. (2021): "Robot Adoption and Labor Market Dynamics," Working Paper.

IFR (2018): "World Robotics 2018," International Federation of Robotics.

Jackson, M. O. and Z. Kanik (2020): "How Automation that Substitutes for Labor Affects Production Networks, Growth, and Income Inequality," Working Paper.

Jaimovich, N., I. Saporta-Eksten, H. Siu, and Y. Yedid-Levi (2021): "The macroeconomics of automation: Data, theory, and policy analysis," *Journal of Monetary Economics*, **122**, 1–16.

Lindenlaub, I. (2017): "Sorting Multidimensional Types: Theory and Application," *Review of Economic Studies*, **84** (2), 718–789.

Loebbing, J. (2022): "An Elementary Theory of Directed Technical Change and Wage Inequality," *Review of Economic Studies*, **89** (1), 411–451.

Moll, B., L. Rachel, and P. Restrepo (2022): "Uneven Growth: Automation's Impact on Income and Wealth Inequality," *Econometrica*, **90** (6), 2645–2683.

Ocampo, S. (2022): "A Task-Based Theory of Occupations with Multidimensional Heterogeneity," Working Paper.

Polanyi, M. (1966): *The Tacit Dimension*, Doubleday.

Rosen, S. (1974): "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition," *Journal of Political Economy*, **82** (1), 34–55.

Sattinger, M. (1975): "Comparative Advantage and the Distributions of Earnings and Abilities," *Econometrica*, **43** (3), 455–468.

——— (1993): "Assignment Models of the Distribution of Earnings," *Journal of Economic Literature*, **31** (2), 831–880.

Stokey, N. L. (2018): "Technology, Skill, and the Wage Structure," *Journal of Human Capital*, **12** (2), 343–384.

Teulings, C. N. (1995): "The Wage Distribution in a Model of the Assignment of Skills to Jobs," *Journal of Political Economy*, **103** (2), 280–315.

——— (2000): "Aggregation Bias in Elasticities of Substitution and the Minimum Wage Paradox," *International Economic Review*, **41** (2), 359–398.

——— (2005): "Comparative Advantage, Relative Wages, and the Accumulation of Human Capital," *Journal of Political Economy*, **113** (2), 425–461.

Teulings, C. N. and P. A. Gautier (2004): "The Right Man for the Job," *Review of Economic Studies*, **71** (2), 553–580.

Tinbergen, J. (1956): "On the Theory of Income Distribution," *Weltwirtschaftliches Archiv*, **77** (2), 156–173.

Topkis, D. M. (1998): *Supermodularity and Complementarity*, Princeton University Press.

Wiener, N. (1950): *The Human Use of Human Beings: Cybernetics and Society*, Houghton Mifflin.

Zeira, J. (1998): "Workers, Machines, and Economic Growth," *Quarterly Journal of Economics*, **113** (4), 1091–1117.

B Online Appendix

B.1 Proofs for Section 5: Global Effects of Automation

B.1.1 Proof of Proposition 10: Polarization with Large Changes in Automation

Part 1 The first result, the expansion of the set of automated tasks, is a direct consequence of its local counterpart in Proposition 6 and the fundamental theorem of calculus. In particular, we can obtain $\Delta \underline{x}$ by integrating a series of local changes $d\underline{x}$ from q to $q + \Delta q$. Since $[q, q + \Delta q] \subset (q_0, q_m)$, all of these local changes are strictly negative by Proposition 6 and, hence, we have $\Delta \underline{x} < 0$. Analogously, we obtain that $\Delta \overline{x} > 0$.

For the corresponding changes in labor assignment, we split the skill space into three intervals. First on $(0, \min\{\widetilde{s}', \widetilde{s}\})$, all of the local changes dX_s when moving from q to $q + \Delta q$ are strictly negative (by Proposition 6), such that we obtain $\Delta X_s < 0$ for all $s \in (0, \min\{\widetilde{s}', \widetilde{s}\})$. Analogously, all local changes on $(\max\{\widetilde{s}', \widetilde{s}\}, 1)$ are strictly positive by Proposition 6 such that we have $\Delta X_s > 0$ for all $s \in (\max\{\widetilde{s}', \widetilde{s}\}, 1)$.

Finally the skills on $(\min\{\widetilde{s}', \widetilde{s}\}, \max\{\widetilde{s}', \widetilde{s}\})$ switch from one side of the set of automated tasks to the other when capital productivity increases from q to $q + \Delta q$. Suppose at first that $\widetilde{s}' < \widetilde{s}$. In this case, $X_s < \underline{x} < \overline{x}^{new} < X_s^{new}$ for all $s \in (\widetilde{s}', \widetilde{s})$. So, $\Delta X_s > 0$ for all $s \in (\widetilde{s}', \widetilde{s})$. Analogously, if $\widetilde{s} < \widetilde{s}'$, we have $X_s > \overline{x} > \underline{x}^{new} > X_s^{new}$ and therefore $\Delta X_s < 0$ for all $s \in (\widetilde{s}, \widetilde{s}')$.

Part 2 By equation (9) and Assumption 2 (supermodularity of $\log \psi_{s,x}$), the local skill premium $(\log w_s)'$ is strictly increasing in the task assigned to s. With the employment polarization result from part 1, this immediately implies wage polarization as stated in part 2 of the proposition.

Part 3 These two results follow immediately from the fundamental theorem of calculus and their local counterparts in Proposition 8, along the lines of part 1 above.

Part 4 Consider first the equilibrium under $q \geq q_0$ and any skill s. Since there must be some task in which capital is less costly than s, we have

$$w_s \geq \min_x \left\{ \frac{\psi_{s,x}}{\psi_{k,x} q} \right\}.$$

Under the new capital productivity $q^{new} = q + \Delta q$, there must be some task performed by skill s in equilibrium, so we must have

$$w_s^{new} \leq \max_x \left\{ \frac{\psi_{s,x}}{\psi_{k,x} q^{new}} \right\}.$$

Combining the previous two inequalities, we obtain

$$\frac{w_s^{new}}{w_s} \leq \frac{\max_x \{\psi_{s,x}/\psi_{k,x}\}}{\min_x \{\psi_{s,x}/\psi_{k,x}\}} \frac{q}{q^{new}},$$

or, in logs,

$$\Delta \log w_s \leq \max_x \{\log \psi_{s,,x} - \log \psi_{k,x}\} - \min_x \{\log \psi_{s,x} - \log \psi_{k,x}\} - \Delta \log q.$$

Part 5 We have shown in Proposition 3 that $q_m < q_\infty$ if $\psi_{0,0}/\psi_{k,0} < \psi_{0,1}/\psi_{k,1}$. So at first suppose that the new capital productivity $q^{new} = q + \Delta q$ is equal to q_m. Since the equilibrium allocation is continuous in q, the same reasoning as in part 1 of the proof implies that $X_s^{new} > X_s$ for all $s \in (\widetilde{s}', 1)$.[29] Moreover, by definition of q_m we have $\widetilde{s}' = 0$ (see the characterization of q_m in Appendix A.3.3). Thus, $X_s^{new} > X_s$ for all $s \in (0,1)$ and, by the same reasoning as in part 2, $\Delta \log w_1 > \Delta \log w_0$.

Next, since wages are continuous in q, $\Delta \log w_1 > \Delta \log w_0$ must hold for all q^{new} in some lower neighborhood of q_m, which is claim 5 of the proposition.

B.1.2 Proof of Proposition 11: Transition to Low-Skill Automation

Low-Skill Automation We start by showing that if automation is low-skill, it remains low-skill when capital productivity increases further. Moreover, further increases in capital productivity shift all skills towards more complex tasks and raise all skill premia, which is the last part of the proposition.

If automation is low-skill, only the upper branch of the differential system (9) to (13) applies while we have $\widetilde{s} = 0$ and $\underline{x} = 0$. We now conjecture that a marginal increase in capital productivity $d \log q > 0$ leaves the threshold skill and the lower bound of the set of automated tasks unchanged, $d\widetilde{s} = 0$ and $d\underline{x} = 0$, i.e., automation remains low-skill. Then, from Lemma A4 we obtain that $d\overline{x} > 0$, while the first part of equation (10) implies that

$$d \log w_0 = \left(\frac{\partial \log \psi_{0,\overline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\overline{x}}}{\partial x} \right) d\overline{x} - d \log q > -d \log q.$$

To verify our initial conjecture that automation remains low-skill, we can now check that the second part of equation (10) remains satisfied, i.e.,

$$d \log w_0 \geq \left(\frac{\partial \log \psi_{0,\underline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\underline{x}}}{\partial x} \right) d\overline{x} - d \log q = d \log q,$$

which we have shown above. Thus, we have shown that if automation starts low-skill, then it remains low-skill when capital productivity increases. So, automation is low-skill for all $q \geq q_m$.

What are the employment and wage effects of an increase in capital productivity when automation is low-skill? We have already shown that $d\widetilde{s} = 0$ and $d\overline{x} > 0$. Then, the same reasoning as in part 2 ("Employment Polarization") of the proof of Proposition 6 implies that the assignment function shifts up everywhere, i.e., $dX_s > 0$ for all $s \in (0,1)$. By the same reasoning as in the proof of Proposition 7, this in turn implies that all skill premia increase, i.e., $d \log w_s$ is strictly increasing in s.

By the fundamental theorem of calculus, these local effects of low-skill automation extend to increases in capital productivity of any size when starting from $q \geq q_m$.

Transition We have shown above that automation is low-skill for all $q \geq q_m$. This implies that if we start from $q < q_m$ and consider a change Δq such that $q + \Delta q \geq q_m$, then $\Delta \underline{x} = -\underline{x}$ (i.e., automation transitions from interior to low-skill), which is claim 1 of the proposition.

[29] Continuity of the equilibrium allocation in q follows from Berge's maximum theorem (see the proof of Proposition 2 above) and the uniqueness of the allocation (by Proposition 1).

Task Upgrading We have shown in the first part above ("Low-Skill Automation") that $dX_s > 0$ for all $s \in (0,1)$ in response to a marginal increase $d\log q > 0$ starting from $q \geq q_m$. Moreover, in Proposition 6, we have shown that $dX_s > 0$ for all $s \in (\widetilde{s}, 1)$ in response to $d\log q > 0$ when starting from $q \in (q_0, q_m)$, where $\widetilde{s}$ is the threshold skill level at q. Integrating these marginal changes between $q \in (q_0, q_m)$ and $q^{new} \geq q_m$, we obtain that $\Delta X_s > 0$ for all $s \in (\widetilde{s}, 1)$.

For skills $s \leq \widetilde{s}$, which switch to the other side of the set of automated tasks when capital productivity increases to q^{new}, we have

$$X_s < \underline{x} < \overline{x}^{new} < X_s^{new},$$

and hence $\Delta X_s > 0$ as well.

Rise in Skill Premia The task upgrading result in the previous part immediately implies that all skill premia increase, i.e., $\Delta \log w_s$ is strictly increasing in s, by the reasoning of part 2 of Proposition 10.

B.1.3 Proof of Proposition 12: Labor Supply Changes and Automation

We have already shown in Proposition 5 that the interior automation threshold q_m declines in response to an increase in relative skill supply as defined in the proposition. Now we consider an increase in relative skill supply $\Delta \log l$ such that $q \in (q_0, q_m)$ initially but $q > q_m^{new}$ after the change.

We know that before the change, automation is interior and $\overline{x} < \overline{x}_m$, where $\overline{x}_m$ is the value defined in the characterization of q_m in Appendix A.3.3.[30] After the change, by construction of q_m^{new}, we must have $\overline{x}^{new} > \overline{x}_m$. So, for all skills $s \in [0, \widetilde{s}]$, we have

$$X_s < \overline{x} < \overline{x}^{new} \leq X_s^{new}$$

such that the assignment function increases strictly on $[0, \widetilde{s}]$.

Suppose now that the assignment function shifts up everywhere. Then, by the reasoning of part 2 of the proof of Proposition 10, $\Delta \log w_s$ is increasing in s. However, since task $x = 0$ is assigned to skill $s = 0$ before the change and to capital afterwards (while the productivities of capital and labor are unchanged), the wage w_0 must increase strictly. This requires that all wages increase, $\Delta \log w_s > 0$ for all s, in contradiction to Lemma A3. So, there must exist a skill level $\hat{s} \in (\widetilde{s}, 1)$ such that the new assignment function X^{new} crosses the old function X from above at $\hat{s}$.

Next, suppose that X^{new} crosses X again at some skill $s_1 \in (\hat{s}, 1)$, this time from below. Then, we have $X_{s_1}^{new} = X_{s_1}$ and $X_{s_1}'^{new} \geq X_{s_1}'$. Moreover, there is another skill $s_2 > s_1$ (potentially but not necessarily equal to 1) such that $X_{s_2}^{new} = X_{s_2}$ and $X_{s_2}'^{new} \leq X_{s_2}'$ because the two assignment functions must intersect at $s = 1$. Now, since we also have $l_{s_2}^{new}/l_{s_1}^{new} > l_{s_2}/l_{s_1}$, the ratio of labor supply in X_{s_2} over X_{s_1} is strictly greater under l^{new} than under l. By equation (11), this implies

$$\frac{w_{s_2}^{new}}{w_{s_1}^{new}} < \frac{w_{s_2}}{w_{s_1}}.$$

[30] If we had $\overline{x} \geq \overline{x}_m$ before the change, then no skill type would be assigned to any task below $\overline{x}_m$ by construction of $\overline{x}_m$ and hence we would have $\underline{x} = 0$, which contradicts interior automation.

But since X^{new} crosses X from below at s_1, we have $X_s^{new} \geq X_s$ for $s \in [s_1, s_2]$ such that equation (9) implies that

$$\frac{w_{s_2}^{new}}{w_{s_1}^{new}} \geq \frac{w_{s_2}}{w_{s_1}},$$

which yields a contradiction.

We have therefore established that there exists $\hat{s} \in (\widetilde{s}, 1)$ such that $\Delta X_s > 0$ for all $s \in (0, \hat{s})$ and $\Delta X_s \leq 0$ for all $s \in [\hat{s}, 1]$. With the reasoning from part 2 of the proof of Proposition 10, this implies that $\Delta \log w_s$ is strictly increasing on $[0, \hat{s}]$ and decreasing on $[\hat{s}, 1]$.

B.2 Details of the Quantitative Analysis

In this section, we provide details on and extensions of our quantitative analysis. We describe the algorithm we use to solve our model numerically, data sources and the construction of the empirical moments used in our calibration, and calibration results for the version of our model with a minimum wage.

B.2.1 Numerical Solution Procedure

To solve our model numerically, we proceed in two steps. First, we solve for the assignment function X_s, wages w_s, gross output Y and net output NY, taking the automation thresholds $\underline{x}$, $\overline{x}$ and $\widetilde{s}$ as given. Then, we choose $\underline{x}$, $\overline{x}$ and $\widetilde{s}$ to maximize net output NY.

For the first step, we solve the system of differential equations given by (9) and (11). Because this system contains the unknown variable Y, we first eliminate it by defining

$$\chi_s \equiv \frac{w_s^{\lambda}}{Y}$$

and rewriting the two differential equations as

$$(\log \chi_s)' = \lambda \frac{\partial \log \psi_{s,X_s}}{\partial s}$$
$$X_s' = \frac{\chi_s l_s}{\psi_{s,X_s}^{\lambda-1}} \ .$$

We solve this system from $\widetilde{s}$ to 0 and from $\widetilde{s}$ to 1, with initial values $[\chi_{\widetilde{s}}^{-}, \underline{x}]$ and $[\chi_{\widetilde{s}}^{+}, \overline{x}]$, respectively, and choose the initial values for χ to obtain $X_0 = 0$ and $X_1 = 1$.

Once we have solved for X_s and χ_s, we can use Euler's theorem,

$$\left(1 - \Gamma_k q^{\lambda-1}\right) Y = \int_0^1 w_s l_s \, ds \ ,$$

to compute gross output:

$$Y = \left[\frac{\int_0^1 \chi_s^{\frac{1}{\lambda}} l_s \, ds}{1 - \Gamma_k q^{\lambda-1}} \right]^{\frac{\lambda}{\lambda-1}} \ .$$

Net output $NY = (1 - \Gamma_k q^{\lambda-1})Y$ and wages, $w_s = (\chi_s Y)^{1/\lambda}$ can then be computed from Y.

In the second step, we maximize net output with respect to $\underline{x}$, $\overline{x}$ and $\widetilde{s}$ subject to the conditions $0 \leq \underline{x} \leq \overline{x} \leq 1$ and $0 \leq \widetilde{s} \leq 1$. This maximization uses an interior-point method (implemented by Matlab's fmincon function), which terminates once the problem's first-order conditions are satisfied up to tolerance levels. The first-order conditions of the problem correspond to equilibrium condition (10) (for $\underline{x}$ and $\overline{x}$) and to the condition that $\lim_{s \nearrow \widetilde{s}} w_s = \lim_{s \searrow \widetilde{s}} w_s$ (for $\widetilde{s}$). This implies that once a solution is found, this solution satisfies all our equilibrium conditions and thus represents the unique equilibrium of our model.

When there is a minimum wage in the model, we add a third step to the solution algorithm. In particular, we complete the first two steps for a given value of $\underline{s}$, which denotes the cutoff skill level below which workers are rationed. Then, we choose $\underline{s}$ to solve

$$\min \left\{ \frac{w_{\underline{s}} - \underline{w}}{w_{\underline{s}}}, \underline{s} \right\} = 0 \ .$$

B.2.2 Data Sources and Construction of Empirical Moments

Here, we describe the data we use in the calibration and how we construct the empirical moments that we target with our model.

1980 US Wage Distribution We use data on the US wage distribution in 1980 from the Current Population Survey Outgoing Rotation Group (CPS May/ORG) samples, which we take from the replication files of Acemoglu and Autor (2011), thus adopting their sample restriction criteria. We use data on log hourly wages by wage percentile and measure wage levels in terms of 2008 dollars using the Personal Consumption Expenditure (PCE) deflator from the Bureau of Economic Analysis (BEA).

Wage Changes due to Automation To measure the effects of automation on the wage distribution, we use the estimates provided by Acemoglu and Restrepo (2022). We use their most comprehensive estimates, including general equilibrium effects (see their Table VIII and Figure 7). They estimate the effect of automation on log wages for 500 demographic groups, defined by education, age, gender, race and native/foreign born, between 1980 and 2016-17. To map the estimated log wage changes from the level of demographic groups to wage percentiles, we use the average hourly wage of each demographic group, map these average wages to percentiles in the 1980 US hourly wage distribution (from Acemoglu and Autor, 2011, see previous paragraph) and run a local polynomial regression to obtain a smooth relationship between log wage change due to automation and wage percentile. This procedure is illustrated in Figure 9. Each circle represents the estimate of Acemoglu and Restrepo (2022) for a specific demographic group, with the size of the circle proportional to the population weight of the group. The black curve is our local polynomial regression, mapping wage percentiles to log wage changes due to automation. We use the fitted values at the 10th, 30th, 50th and 90th percentile as targets in our calibration.

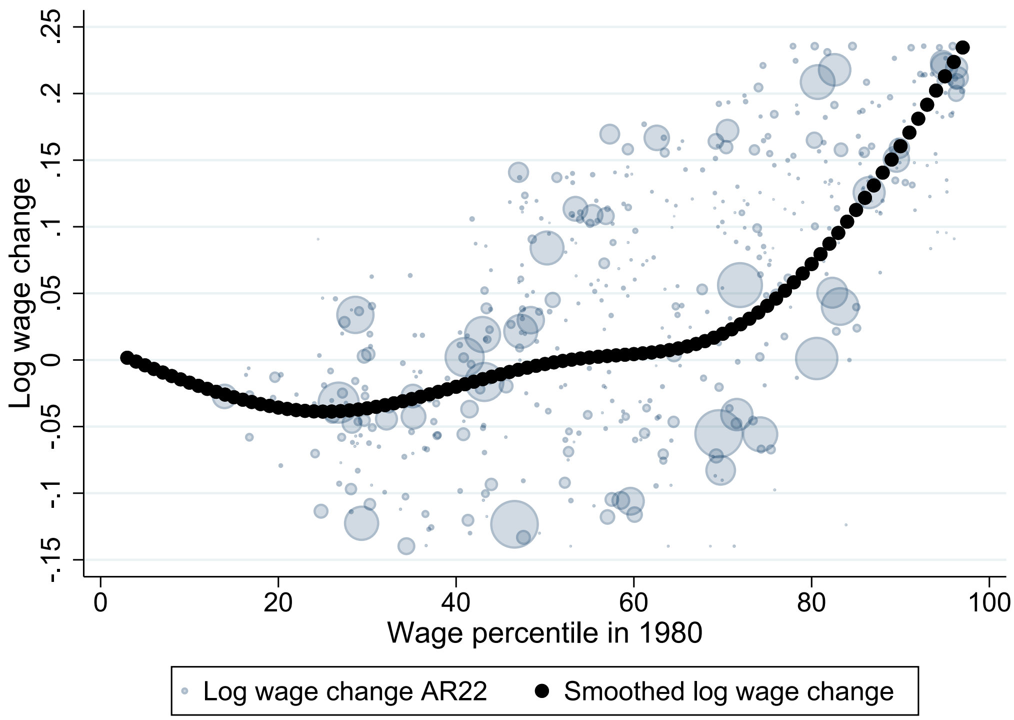

Figure 9: *Log wage changes due to automation.* The graph shows the estimated effects of automation on log wages of 500 demographic groups between 1980 and 2016 by Acemoglu and Restrepo (2022). Each gray circle represents the estimate for a specific demographic group with the size of the circle being proportional to the population size of the group. Groups are sorted into wage percentiles according to their average hourly wages in 1980. The black dots are fitted values from a local polynomial regression.

Equipment and Software Price Index We identify capital in our model with the sum of non-residential equipment capital and software as defined in the BEA Fixed Asset Tables (FAT). The capital price $1/q$ thus corresponds to the gross return per unit of non-residential equipment and software, which we compute as

$$(r_t + \delta_t)\frac{p_{k,t}}{p_{c,t}} \ ,$$

where r_t is the net rate of return, δ_t the depreciation rate of non-residential equipment and software, $p_{k,t}$ its unit price and $p_{c,t}$ is the consumption price index.

For the net rate of return, we first compute gross domestic income of capital in the private sector using data from the BEA National Income and Product Accounts (NIPA), Table I.10. In particular, we sum up the net operating surplus of private enterprises and the private consumption of fixed capital, and add a share of proprietor's income plus taxes minus subsidies. When imputing the capital share of proprietor's income and taxes, we follow Gollin (2002) and assume this to be the same as the capital share in the rest of the private economy—that is, net operating surplus of private enterprises divided by the sum of itself and the compensation of employees minus compensation of employees by government enterprises and the general government. We then subtract from this gross domestic income of private-sector capital the private fixed asset depreciation (from BEA FAT Table 2.4) and divide it by the current-cost net stock of private fixed assets (from BEA FAT Table 2.1).

The depreciation rate is obtained by summing the depreciation of non-residential equipment and software (BEA FAT Table 2.4) and dividing it by the sum of their current-cost net stocks (BEA FAT Table 2.1).

For the price of non-residential equipment capital and software $p_{k,t}$, we use the price index proposed by Di Cecio (2009), provided via Federal Reserve Economic Data (FRED). The index for non-residential equipment and software is available only until 2011, so we impute the remaining years until 2016 with the growth rates of the index for non-residential equipment only.

With this procedure, we obtain an index for the required return per unit of non-residential equipment and software. Dividing its 2016 value by its 1980 value yields 0.21, indicating a 79% decline in the required return. This is the basis of our 79% decline in the price of capital, $1/q$.

Equipment and Software Income Share For the income share of non-residential equipment and software, we first compute gross domestic income for non-residential equipment and software. We compute this quantity by multiplying its gross return $r_t + \delta_t$ (see previous paragraph) by its current-cost net stock from BEA FAT Table 2.1.[31] We do the same for software and sum up the gross domestic income of non-residential equipment and of software.

For the denominator of the income share, we compute private-sector labor income from BEA NIPA Table I.10. For this, we take the compensation of employees and add shares of proprietor's income and taxes minus subsidies. As when computing capital income, we assume that the share of proprietor's income and taxes minus subsidies pertaining to labor equals the share of labor income in the rest of the private economy, that is, the compensation of employees minus compensation of employees by government enterprises and the general government divided by the sum of itself and the net operating surplus of private enterprises. In this way, we obtain gross domestic income of labor. We subtract the compensation of employees of government enterprises and of general government to obtain gross domestic income of labor in the private sector.

Finally, we compute the income share of non-residential equipment and software as gross domestic income of non-residential equipment and software divided by the sum of itself and gross domestic income of private-sector labor. We understand the resulting income share as an empirical counterpart of the capital share α_k in our model. We do not include the income of other types of capital in the denominator of the income share because, when interpreting capital in our model as equipment and software, our model does not include these other types of capital either.

[31] Note that we have constructed the net return r_t such that, if we were to follow this procedure for the entirety of private fixed assets, we would exactly obtain the gross domestic income of private-sector capital computed from BEA NIPA Table I.10. An alternative approach would be to assign the gross domestic income of private-sector capital to the various types of capital (e.g., structures, equipment, software) in proportion to their current-cost net stocks reported in BEA FAT Table 2.1. This would assume that the gross return to all types of capital is equal. Our approach instead assumes that the net return is equal, but allows for differences in the gross returns via differences in depreciation rates. Since depreciation rates vary hugely between structures, equipment and software, we consider our approach preferable.

Minimum Wages In Appendix B.2.5, we calibrate the version of our model that includes a minimum wage. For this calibration, we need minimum wage levels in 1980 and 2016. We take all information on minimum wages from the US Department of Labor website.

In 1980, Alaska was the only state with a state-level minimum wage that significantly exceeded the federal minimum wage. Hence, for 1980, we simply use the federal minimum wage of 3.1 dollars for our calibration. Like all wage levels, we express minimum wages in 2008 dollars using the Personal Consumption Expenditure (PCE) deflator from BEA NIPA Table 1.1.4. This leads to a minimum wage of 7.1 dollars in 1980.

In 2016, many state-level minimum wages exceeded the federal minimum wage, some of them by a lot. Thus, for 2016, we compute an employment-weighted average minimum wage across states. In particular, we first compute for each state the binding minimum wage as the maximum of the state-level and the federal minimum wage. Then, we compute a weighted average of these state-specific minimum wages, with weights given by the state-level total employment (number of jobs) data from the BEA Regional Data, Table SASUMMARY. The result is an average minimum wage of 8.24 in 2016 dollars, which we convert into 7.46 2008 dollars using the PCE deflator.

We use the values of 7.1 for the 1980 minimum wage and 7.46 for the 2016 minimum wage in our calibration in Appendix B.2.5.

B.2.3 Calibration Procedure

Here, we explain in more detail how we proceed to calibrate our model. As described in the main text, we first set the elasticity of substitution between tasks, λ, to 0.5, using an estimate from Humlum (2021). Then, we choose parameters m, n, a, a_k, b and q to match the first seven empirical moments listed in Table 1: the log 50-10 and 90-50 percentile ratios of hourly wages in the US in 1980; the income share of non-residential equipment capital and software in the US in 1980; the changes in the log 30-10, 50-30 and 90-50 percentile ratios of hourly wages in the US between 1980 and 2016 due to automation as estimated by Acemoglu and Restrepo (2022); and the change in the income share of non-residential equipment capital and software in the US between 1980 and 2016.

We choose the parameters such as to minimize the sum of absolute deviations between the empirical moments and their model counterparts. The minimization proceeds in two steps. First, we perform a coarse grid search, with parameters taking on the following values: $m \in \{0.5, 2.5, 5\}$, $n \in \{0, -2.35, -4.7\}$, $a \in \{1, 7, 14\}$, $a_k \in \{1, 110, 220\}$, $b \in \{-1, -300, -600\}$ and $\widetilde{q} \in \{0.005, 0.1\}$, where $\widetilde{q}$ is an auxiliary parameter that determines q given values for the other parameters. In particular, q is obtained as

$$q = \min\{q_{max}, q_0 + \widetilde{q}(q_{max} - q_0)\}$$

where q_0 is the threshold below which there is no automation and q_{max} is the value for q such that $q/0.21 = q_\infty$ (q_∞ being the threshold above which output becomes unbounded). In words, $\widetilde{q}$ determines q as a convex combination between the automation threshold and the maximum value q can take such that, even after applying the 79% fall in capital prices observed between 1980 and 2016, output is still bounded under the 2016 value of q. The advantage of defining q via $\widetilde{q}$ in this way is that, as long as we choose $\widetilde{q} \in (0,1)$ (which is the case on our grid), we always get some automation and finite output, both of which is required to fit our empirical moments. If alternatively we were to define grid values for q directly, many vectors of the grid would lead to no automation or infinite output. The exact values for our grid were chosen after extensive numerical experimentation identifying broadly suitable parameter ranges.

In the second step, we use the grid vector that minimizes the sum of absolute deviations from targets on the grid as the starting vector for a local minimization routine. In this step, we rely on Matlab's fminsearch function, which implements the Nelder-Mead simplex algorithm. We have experimented with the starting vector for this local minimization procedure within a reasonable range and found the results to be robust.

To further corroborate that we indeed identify a global minimum, we have also employed a global minimization routine, Matlab's bayesopt function, which explores the parameter space in an automated way. We did not find any improvement over the result of our grid search combined with local minimization.

In the final step, we choose the parameter A to exactly match the log median wage in the US in 1980. The only role of A is to scale wages, such that we can choose it independently of all other parameters.

This separability of A no longer holds once a binding minimum wage is included in the model upon calibration, as we do in Appendix B.2.5. In that case, we include A in the vector of parameters to be calibrated jointly, and include the log median wage in the US in 1980 as an additional empirical moment to match. For this calibration, we do not re-run the grid search, as the model with minimum wage is computationally costly to solve. Instead, we use the final parameter vector from our calibration without minimum wage as a starting point for a local minimization (again using Matlab's fminsearch), now including the minimum wage as well as parameter A and the additional target. We consider this approach appropriate as long as the minimum wage does not induce extreme levels of worker rationing, which is not the case in our calibration.

B.2.4 Artificial Intelligence Counterfactual

In this section, we provide a formal explanation of how we construct our artificial intelligence counterfactual simulation. As described in the main text, we benchmark capital productivity by the productivity schedule of the median worker,

$$\log \psi_{k,x} + \log q - \log \psi_{0.5,x} = a_k x + bx^2 + \log q - 0.5ax - 0.5m - 0.25n - A \ ,$$

where we have included the general capital productivity level q. We now reduce the curvature of capital productivity without changing its productivity relative to the median worker at its peak level, i.e., in the task where capital already has its strongest comparative advantage. In particular, the maximum of the relative productivity schedule is obtained at

$$\widehat{x} = \frac{0.5a - a_k}{2b}$$

Panel A. Calibrated Parameters

Parameter	Value	Target
λ	0.5	Humlum (2021)
A	-0.20	jointly calibrated to match the eight data moments
m	2.5	
n	-2.1	
a	7.2	
a_k	110	
b	-300	
$\log q$	-9.2	

Panel B. Comparison of Moments

Moment	Data	Model
Log 50-10 wage percentile ratio, $\log(w_{0.5}/w_{0.1})$	0.60	0.49
Log 90-50 wage percentile ratio, $\log(w_{0.9}/w_{0.5})$	0.67	0.67
Income share of equipment and software, α_k	0.16	0.16
Change in log 30-10 wage percentile ratio due to automation, $\Delta \log(w_{0.3}/w_{0.1})$	-0.02	0.00
Change in log 50-30 wage percentile ratio due to automation, $\Delta \log(w_{0.5}/w_{0.3})$	0.03	0.03
Change in log 90-50 wage percentile ratio due to automation, $\Delta \log(w_{0.9}/w_{0.5})$	0.16	0.16
Change in income share of equipment and software from 1980 to 2016, $\Delta\alpha_k$	0.01	0.03
Log median wage level (in 2008 dollars), $\log w_{0.5}$	2.6	2.6

Table 2: *Calibration Results with minimum wage.* Panel A shows the calibrated parameter values and the respective targets used for their calibration. Panel B shows the data moments used in the calibration (left column) and their model counterparts (right column) as explained in the text.

and amounts to

$$\widehat{\psi} = \log q - \frac{(0.5a - a_k)^2}{4b} - 0.5m - 0.25n - A \, .$$

We reduce the curvature parameter b while adjusting a_k and q such as to keep $\widehat{x}$ and $\widehat{\psi}$ unchanged. In our simulation, we do this for a range of values of b and report results for the one that leads to aggregate net output gains (approximately) equal to those obtained from reducing $1/q$ by 79% (our first counterfactual simulation).

B.2.5 Calibration with Minimum Wage

In this section, we report of our calibration when including a minimum wage into the model. Apart from including the minimum wage, we use the same parameterization of the model as in the baseline case without minimum wage.

Again, we first set of the elasticity of substitution between tasks to $\lambda = 0.5$. Then, we calibrate the remaining parameters m, n, a, A, a_k, b and q jointly by matching the same eight moments as in the baseline calibration: the log 50-10 and 90-50 percentile ratios of hourly wages in the US in 1980, the income share of non-residential equipment capital and software in the US in 1980, the changes in the log

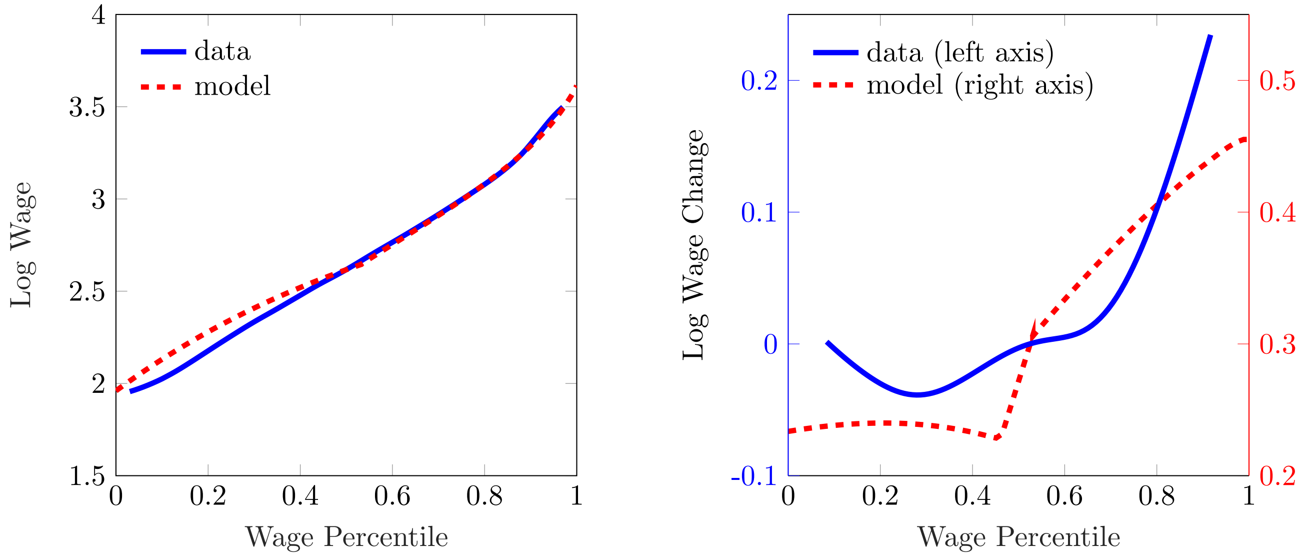

Figure 10: *Log wage distribution in 1980 and change in log wages, 1980-2016.* The left panel shows log wages by percentile of the wage distribution, as in the data (blue) and produced by our calibrated model with minimum wage (red dashed). The right panel shows the change in log wages between 1980 and 2016 due to automation, as estimated by Acemoglu and Restrepo (2022) (in blue) and predicted by our calibrated model with minimum wage (in red, dashed).

30-10, 50-30 and 90-50 percentile ratios of hourly wages due to automation in the US between 1980 and 2016 (estimates from Acemoglu and Restrepo, 2022), the change in the income share of non-residential equipment and software in the US between 1980 and 2016, and the US log median hourly wage in 1980. When matching the moments of the US 1980 wage distribution and the 1980 income share of equipment and software, we include a minimum wage of 7.1 dollars (measured in terms of 2008 dollars, see Section B.2.2) as it was present in the US in 1980. When matching the changes in the wage distribution and the change in the income share of equipment and software, we reduce the capital cost $1/q$ by 79% and increase the minimum wage from 7.1 to 7.46 dollars, which is the employment-weighted average minimum wage across US states in 2016 (see Section B.2.2).

The results are displayed in Table 2, with the calibrated parameter values given in the top panel and the matched moments in the bottom panel. The model matches most of the empirical moments very well, with two exceptions. First, the model understates inequality in the bottom of the wage distribution, as measured by the 50-10 percentile ratio, by a significant amount. Inspecting the left panel of Figure 10, this is mostly due to the fact that in the data, there is a large mass of individuals located at or close to the minimum wage threshold. Our model with a perfectly competitive labor market cannot replicate this bunching at the minimum wage, even when including the minimum wage explicitly in the model. The second moment that is not matched well is the reduction in the 30-10 wage percentile ratio due to automation. The right panel of Figure 10 shows that our model produces a decline in wages around the 40th relative to wages around the 30th percentile, but not a decline of the 30th relative to the 10th percentile. In fact, at the very lower end of the wage distribution, log wage changes are smaller than around the 20th or 30th percentile, such that our model does not produce pure wage polarization. The reason is that, when reducing the cost of capital by 79%, the bite of the minimum wage in our model falls drastically. This induces an increase in wage inequality along the entire distribution, which mingles with the automation-induced polarization. The minimum wage effects are particularly powerful at the lower end of the wage distribution and dominate at the very bottom.