# Tying with Network Effects*

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**Abstract**

We develop a leverage theory of tying in markets with network effects. When a monopolist in one market cannot perfectly extract surplus from consumers, tying can be a mechanism through which unexploited consumer surplus is used as a demand-side leverage to create a "quasi-installed base" advantage in another market characterized by network effects. Our mechanism does not require any precommitment to tying; rather, tying emerges as a best response that lowers the quality of tied-market rivals. While tying can lead to exclusion of tied-market rivals, it can also expand use of the tying product, leading to ambiguous welfare effects.

## 1 Introduction

The leverage theory of tying typically considers the following scenario: There is a monopolistic firm in one market (say $A$). This firm, however, faces competition in another market (say $B$). According to this theory, the monopolistic firm in market $A$ can monopolize market $B$ using the leverage provided by its monopoly power in market $A$ through tying or bundling arrangements. The Chicago School, however, criticized this theory and proposed instead price discrimination as the main motivation for tying. The gist of the Chicago school criticism is based on the so-called "one monopoly theorem," which states that "[a] seller cannot get two monopoly profits from one monopoly." (Blair and Kaserman, 1985).

We demonstrate that in the presence of imperfect rent extraction in a monopolized market and network effects in a market where the monopolist faces competition, tying can be a mechanism through which unexploited consumer surplus in the monopolized market is used as a demand-side leverage to create a strategic "quasi- installed base" advantage in the

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competing market, raising the perceived quality of the monopolist's tied product and lowering the perceived quality of its tied market rivals.

In markets with network effects, consumer utility consists of stand-alone benefits and network benefits (Katz and Shapiro (1986)). Under independent pricing, all firms compete on a level playing field with respect to network effects. Even though markets with strong network effects are typically characterized by tipping equilibria in which all consumers choose the same product, yielding maximal network benefits, the network-benefits component can be competed away in equilibrium to consumers' benefit. With tying, however, the tying firm can use unexploited consumer surplus in the tying market in competition against a rival firm in a tied market. We show that this advantage allows the tying firm to lock in consumers who have a high value for the tying product, ensuring that it captures the network effect and enabling it to win in the tied market even against a more efficient rival.

More precisely, consider a situation in which there are two markets, $A$ and $B$. Firm 1 is a monopolist of product $A$ and sells its product $B1$ in market $B$ against a rival, firm 2, that produces product $B2$. Consumers in the monopolized market $A$ are heterogeneous and some consumers receive surplus in this market under independent pricing. In such a scenario, if firm 1 offers only a bundle that ties purchase of its product $A$ to purchase of its product $B1$, consumers with high valuations for product $A$ may prefer to purchase the bundle even if all other consumers purchase the rival firm's product $B2$. The existence of such consumers ensures a guaranteed market share in market $B$ for firm 1, which is akin to firm 1 having an installed base. This advantage in terms of the quasi-installed base can in turn induce low valuation consumers to purchase the bundle instead of buying $B2$. We show that a process of iterated elimination of dominated strategies can lead to tipping toward the monopolist's bundle.

Notably, and in contrast to much of the literature on the strategic use of tying (which we discuss below), this mechanism does not rely on the ability to precommit to tying, such as through technological bundling. Rather, the incentive to leverage unexploited consumer surplus in the tying market to degrade the relative quality of rivals in the tied market makes tying a best response by the monopolist absent any precommitment.[1]

We first develop our theory in the context of independent products to illustrate how network effects in the tied good market may provide incentives to tie. Specifically, we provide a sufficient condition for the monopolist to tie in equilibrium and thereby monopolize the tied-good market. In situations in which the tying good market is covered (i.e., all consumers purchase the tying product under independent pricing), we show that pure bundling is an optimal strategy for the monopolist. When the tying good market is not covered under independent pricing, firm 1 may instead find it optimal to tie using a mixed bundling strategy in which consumers can choose between buying an $A/B1$ bundle and buying product $B1$ only. This mixed bundling enables firm 1 to screen consumers with respect to their willingness to pay for the monopolized product $A$ while maximizing the network effects for its product $B1$. When the number of consumers buying the bundle is large enough, firm 1 is able to sell even

[1]Our mechanism can still be effective and attractive for the monopolist when precommitment is possible, but precommitment is not necessary (nor assumed in our analysis).

its inferior $B1$ at a profit as a stand-alone product against product $B2$.

We then extend our analysis to the case of complementary products. With pure monopoly in the tying product market, we confirm the Chicago School critique that tying cannot be a leverage mechanism even with network effects. However, pure monopoly with absolutely no competitive products is rare. We show that in the presence of an inferior alternative to the tying good we can restore our mechanism with parallel results to the independent-products case; we formally demonstrate the equivalence of the complementary-products case to the independent-products case, with the inferior alternative in the tying market playing the same role as does the no-purchase option in the independent products case.

Our analysis can be used to develop a theory of harm for tying cases when network effects are critical in the determination of the market winner, and sheds light on some recent cases involving tying.

As we noted above, the literature on tying as an anticompetitive foreclosure mechanism has focused most on situations in which a monopolist firm commits to use of a tying strategy, as first developed in Whinston (1990).[2] If the market structure in the tied-good market is oligopolistic with scale economies, tying can be an effective and profitable strategy to alter market structure by making continued operation unprofitable for tied-good rivals. This effect occurs because a commitment to tying leads the monopolist to price aggressively in order to ensure sales of the valuable product $A$. However, in the main model in Whinston (1990), inducing the exit of the rival firm is essential for the profitability of tying arrangements. Thus, if the competitor has already paid the sunk cost of entry and there is no avoidable fixed cost, tying cannot be a profitable strategy. In contrast, our mechanism requires neither commitment power of the tying firm nor exit of the rival.[3]

While the commitment assumption makes sense when firms employ technological ties, in many tying cases the tie is a pricing choice that seems to involve little commitment.[4] For example, in the recent EU *Android* case, Google was found guilty of requiring Android OEMs to pre-install the Google search app as a condition of gaining a license to Google's app store (the Play Store). The tie was not technological, but rather purely contractual, and raises the question of why Google would not have simply paid the OEMs for Android pre-installation. While the literature on bundling provides an answer by showing that with heterogeneous valuations tying can indeed be a best response as a price discrimination mechanism (as the Chicago School claimed), in these cases tying might be viewed as "innocent" and any effects on rivals inadvertent.[5] What differs in our theory, however, is that tying can be a best

[2]Fumagalli et al. (2018) provide an excellent survey of tying as an exclusionary practice along with discussions of major antitrust cases.

[3]If network effects are absent, we replicate his result that bundling is not profitable if firm 2's exit is not induced.

[4]In some cases, reputational concerns might lead to an element of commitment.

[5]See Section IIC of Whinston (1990) and Peitz (2008) for examples. Peitz's(2008) model differs in several ways from ours. First, in his article the market $A$ monopolist commits to its pricing structure (pure bundling versus independent pricing) prior to a price choice stage (but only after the potential rival's entry decision, in contrast to the main commitment model in Whinston (1990)). Second, in Peitz (2008) bundling reduces welfare only when it affects the potential entrant's entry decision and affects the market structure whereas our model does not require any market structure change. Third, bundling is profitable irrespective of entry in his model, i.e., bundling is always used. In our model, in contrast, bundling is profitable only when there is

response precisely because it lowers the perceived quality of the monopolist's tied market rivals by reducing the network benefits they can provide.

The idea of using unexploited consumer surplus as a leverage mechanism appears in some other papers. Burstein (1960) and Greenlee et al. (2008) analyze a setting in which the monopolist in the tying product market sells to consumers with multiunit demands and is unable to fully extract consumer surplus with linear pricing. By tying, even to competitively-supplied tied goods, the monopolist can require buyers to purchase additional products at elevated prices. In essence, tying serves as a substitute for a fixed fee.[6] In contrast, in our model consumers have single-unit demands for the tying good and so tying cannot serve this function.

Calzolari and Denicolò's (2015) theory of exclusive dealing is also based on uncaptured consumer surplus with multiunit buyers. They consider a single-market situation in which there is a dominant firm with a competitive advantage over a competitive fringe of rivals, but buyers are able to obtain information rents due to private information even if they deal exclusively with the dominant firm. Without exclusive dealing, the dominant firm needs to compete for each marginal unit of a buyer's demand; in contrast with exclusive dealing, the dominant firm competes for the entire volume demanded by a buyer. This change enables the dominant firm to exclude rivals by leveraging the information rents left on inframarginal units. Thus, the dominant firm is able to exclude rivals with a lower discount with the imposition of exclusive deals. Exclusive dealing serves as a more profitable pricing mechanism despite the fact that it has no effects on the prices or qualities offered by the dominant firm's rivals. In contrast, in our model with heterogeneous consumers with single-unit demands, our mechanism leverages the network effect provided by inframarginal tying good consumers who are "committed" to the bundle in order to lower the network benefits provided by the tied-good rivals and thereby monopolize the tied-good market.

Carlton and Waldman (2002) is the paper most closely related to ours. They also consider a model in which network effects exist in the tied-good market and they note that tying can be an effective strategy in such cases without commitment. They consider a dynamic two-period model in which the tying and tied goods are perfect complements. In period 1 there is a monopoly in the tying "primary" market, while in period 2 the tied ("complementary good") market rival can potentially enter the primary market. In contrast to our focus on tying as a mechanism to profitably monopolize the tied-good market, the purpose of tying by the primary good monopolist in their model is to preserve its market power by preventing entry into the primary market, even though it may entail short-run losses. As well, in our model these effects operate within a single period.[7]

Our analysis is also related to work by Caillaud and Jullien (2003) on competition between two-sided platforms. Caillaud and Jullien show how a firm can offer customers on one side

competition in the tied-good market. Otherwise, independent pricing is more profitable. Thus, in our model bundling is useful only as an exclusionary strategy.

[6]However, tying is less efficient than a fixed fee since it causes distortions in the tied markets.

[7]On pp. 206-7, Carlton and Waldman suggest that tying could be profitable in the presence of network effects even "if the alternative producer cannot enter the primary good market," although without providing any analysis. Our paper can be viewed as following up on this point of theirs.

a sufficient discount to lock in their business, thereby leading customers on the other side to join as well (a "divide-and-conquer" strategy). In our analysis, by tying a dominant firm can lock in consumers with a high willingness-to-pay for its monopolized product to the use of its competitive product, with the resulting network effects leading to monopolization of the latter market. In contrast to the offering of a discount in Caillaud and Jullien (2003), the leveraging of unexploited surplus in our analysis is free for the dominant firm.

The rest of the paper is organized in the following way. In Section 2, we illustrate the main intuitions behind our tying mechanism through a simple example with discrete consumer types. In Section 3, we describe our model for independent products with a more general demand structure. In Section 4, we analyze this model and provide conditions under which tying (offering only a bundle and possibly product $B1$ for sale, and not offering $A$ by itself) emerges in equilibrium and leads to monopolization of market $B$. In Section 5, we consider complementary products, and show that we can derive parallel results to the independent products case if we assume an inferior alternative to the monopolized tying good. We discuss the application of our results to recent antitrust cases and offer concluding remarks in Section 6.

## 2 An Illustrative Example

To explain the main mechanism and intuition behind our model, we provide an illustrative example. There are two markets $A$ and $B$. Market $A$ is served by a monopolist, firm 1. In market $B$, firm 1 and firm 2 compete. These two product markets are independent. Firms' production costs are normalized to zero in all markets. There are two consumers.

### 2.1 Market A

The two consumers are heterogeneous in terms of their valuations for product $A$. One is a high ($H$) type consumer and the other is a low ($L$) type consumer. Each consumer's willingness to pay for product $A$ is given by $u_k$, where $k = H, L$, with $u_H = u_L + s > u_L > 0$ so $s > 0$. We assume that $u_L > s$. This implies that the optimal monopoly price in market $A$ is $p_A^* = u_L$ and consumer $H$ receives a surplus of $s$. An important feature of market $A$ is that there is variation in consumers' willingnesses to pay for $A$ so that firm 1 is unable to extract the whole surplus in that market despite its monopoly power.

### 2.2 Market B

Market $B$ is characterized by network effects and firm 1's product $B1$ is inferior to firm 2's product $B2$. Products $B1$ and $B2$ are not compatible with each other. In this market, we assume that the two consumers have the same preferences. More specifically, firm $i$'s product $Bi$ provides a stand-alone value of $v_i$ to consumers, where $v_2 > v_1 > 0$. We define the quality difference $\Delta \equiv v_2 - v_1$. If the two consumers purchase the same product $Bi$, there are additional network benefits of $n > 0$, yielding a total value of $v_i + n$. In other words, given that a consumer buys product $Bi$, her gross surplus is $v_i$ if she is the only consumer buying the product and is the larger amount $v_i + n$ if the other consumer buys the same product. We assume that these potential network effects are larger than the quality difference $n > \Delta$.

## 2.3 Independent Pricing Equilibrium

Consider first the market equilibrium when the two products are sold independently by firm 1. In this case, the two markets can be analyzed separately. As noted above, firm 1 will charge $p_A^* = u_L$ in market $A$. In market $B$, given any prices $(p_{B1}, p_{B2})$, there is no continuation equilibrium in which the two consumers choose different products; in equilibrium, either both consumers purchase $B1$ or both purchase $B2$. We assume, as is common in the literature, that consumers coordinate on their Pareto-optimal continuation equilibrium. In addition, we restrict attention to equilibria in undominated price offers. Then, we have a unique equilibrium in which firm 2 charges $p_{B2} = \Delta$, firm 1 charges $p_{B1} = 0$, and both consumers buy product $B2$.[8] Hence, consumers capture the full network benefits for themselves, and firm 2 captures only its quality advantage as profit. In this case, firm 1 earns nothing in market $B$ and an overall profit of

$$\Pi_1^* = 2u_L.$$

## 2.4 Equilibrium with Tying

Now suppose that firm 1 engages in tying: it requires that any consumer who purchases its monopolized product $A$ also buy product $B1$ (and only $B1$), and sells the bundle at price $P$.[9]

We show that if there is sufficient unexploited surplus in market $A$ – specifically if $s > 2n$ – then firm 1 has a profitable deviation to offering only a bundle starting from the independent pricing equilibrium described above.[10] By doing so it leverages the unexploited consumer surplus enjoyed by consumer $H$ in market $A$ to monopolize market $B$. The argument shows that when firm 1 ties in this fashion there is a unique continuation equilibrium in which consumers' choices to buy the bundle are pinned down by iterated dominance.

The leverage mechanism with two discrete-type consumers operates in two steps. First, tying allows firm 1 to leverage the surplus from the monopoly product $A$ that is enjoyed by consumer $H$ to gain purchases of $B1$. Once consumer $H$ is secured to buy the bundle, firm 1 achieves a strategic advantage in selling to consumer $L$ as if it had consumer $H$ as an installed-base of its product. These network benefits allow firm 1 to induce consumer $L$ to buy the bundle as well.

To see this point, suppose that firm 1 offers only a bundle at price $P = u_L + \varepsilon$ where $\varepsilon > 0$. Observe, first, that if both consumers buy the bundle, then firm 1's profit will strictly exceed its profit in an independent pricing equilibrium in which the consumers coordinate

[8] As usual, we break the tie in favor of the more efficient firm.

[9] Our argument requires that consumers will not buy both the bundle and product $B2$ and then use product $B2$ instead of $B1$. Firm 1 can prevent this behavior with a contract that prevents use of $B2$. Alternatively, such a contractual requirement will not be necessary when there are large enough production costs of product $B1$ and $B2$, which would make purchase of the bundle for the purpose of using only product $A$ undesirable.

[10] The assumption $s > 2n$ is a sufficient condition for tying to be profitable, but is stronger than necessary. We make this assumption for simplicity to reduce the number of cases to consider.

on $B2$ and each pay $u_L$ for product $A$, yielding firm 1 a profit of $2u_L$. Moreover, as we now show, both consumers will indeed buy the bundle for small enough $\varepsilon$.

First, with $P = u_L + \varepsilon$, for small enough $\varepsilon$ it is a dominant strategy for consumer $H$ to purchase the bundle: she prefers the bundle even under the most unfavorable condition that $B2$ is offered free and consumer $L$ purchases $B2$ since

$$(u_H + v_1) - P > v_2 + n \Leftrightarrow s > \Delta + n + \varepsilon, \tag{1}$$

which is satisfied for small enough $\varepsilon$ given that $s > 2n > \Delta + n$. This works because of the unextracted surplus consumer $H$ enjoys from buying $A$ at price $p_A = u_L$.

Given that consumer $H$ purchases the bundle, for small enough $\varepsilon$ it is also optimal for consumer $L$ to purchase the bundle even if $B2$ is offered for free since

$$(u_L + v_1) + n - P > v_2 \Leftrightarrow n > \Delta + \varepsilon \tag{2}$$

given that $n > \Delta$. Thus, by deviating in this fashion firm 1 is certain to monopolize market $B$ and increases its profit.

Observe that firm 1's use of tying to monopolize market $B$ does not rely on any commitment by firm 1; rather, tying is simply a best response to firm 2's price for $B2$ that successfully leverages consumer $H$'s desire for $A$. Note also that variation in consumers' willingnesses to pay for $A$ is essential: if both consumers had the same value for $A$, so that $u_H = u_L \equiv u$, firm 1's deviation to offering the bundle at slightly above the market $A$ monopoly price (so that $P = u + \varepsilon$), would lead to no sales of the bundle (consumer purchases of $B2$ would be the same as if firm 1 had offered $B1$ independently at price $p_{B1} = P - u = \varepsilon$).[11]

In the next section, we will examine in a more general model the equilibrium that results when firm 1 can tie in this fashion. When, as here, firm 1 would sell product $A$ to all consumers under independent pricing (the case of "full coverage"), the equilibrium involves firm 1 offering only a bundle and monopolizing both markets. In the example above, that equilibrium outcome involves firm 2 offering to sell product $B2$ at cost ($p^*_{B2} = 0$) and firm 1 selling its bundle at the price $P = u_L + (n - \Delta)$.[12]

[11] Firm 1's use of tying here has the flavor of a "divide and conquer" strategy, reminiscent of that in Caillaud and Jullien (2003), as firm 1's bundle offer makes purchase from firm 1 a dominant strategy for consumer $H$, which then leads consumer $L$ to follow along in buying from firm 1. However, tying is more effective here than would be discriminatory offers by firm 1 for just $B1$ (if possible). While, given $p_{B2} = \Delta$, firm 1 could make purchase of $B1$ a dominant strategy for one consumer by offering that consumer a price slightly less than $p_{B1} = -n$, it could then charge the other consumer at most $p_{B1} = n$, and so it cannot make a profit in this example with such offers. In essence, such a divide-and-conquer attempt just in market $B$ involves use of a costly discount to the first consumer, while the tying strategy costlessly takes advantage of consumer $H$'s unexploited surplus.

[12] By tying firm 1 is therefore able to achieve the same profit as when under independent pricing consumers fail to coordinate on their Pareto-preferred continuation equilibrium, all buying $B1$ instead at price $p^*_{B1} = n - \Delta$.

# 3 The Independent-Products Model

In this section, we lay out a more general model of tying in markets with network effects. As in the illustrative example in the previous section, we study here the case in which products $A$ and $B$ are independent and can be used separately. (We discuss the case of complements in Section 5.)

Market $A$ is monopolized by firm 1. In market $B$, there are direct network effects, firm 1 and firm 2 compete, and consumers have homogeneous valuations: their willingness to pay for each firm's product is given by $v_1 + \beta N_1 > 0$ and $v_2 + \beta N_2 > 0$, respectively, where $v_1 > 0, v_2 > 0$, $\beta > 0$, and $N_i$ represents the number of consumers using firm $i$'s product $Bi$. We normalize the total number of consumers to 1. All marginal costs are normalized to zero.

At the heart of our leverage mechanism is "unexploited consumer surplus" in the tying market which can be used in competition with a competitor in another market. If in market $A$ there are high-valuation consumers who receive sufficiently large consumer surpluses, they may be willing to purchase the bundle (rather than product $B2$ only) even if all other consumers purchase $B2$. The existence of such high-valuation consumers in market $A$ provides a demand-side leverage for firm 1 in market $B$ akin to having an installed base. If network effects are sufficiently strong, this strategic advantage more than makes up for any quality disadvantage of firm 1 and enables firm 1 to monopolize market $B$, extracting the resulting network effects as profit.

More specifically, we assume that consumers' valuations for product $A$, denoted $u$, are distributed on $[\alpha, \overline{u}]$, where $\alpha$ represents the lower bound for the consumers' valuations.[13] It will be convenient to define a consumer's "type" as $x = u - \alpha$, which we assume is distributed on $[0, \overline{x}]$, where $\overline{x} \equiv \overline{u} - \alpha$, according to a c.d.f. $G(\cdot)$ with a strictly positive density $g(\cdot)$.[14] Hence, a consumer of type $x$'s valuation for $A$ is $\alpha + x$.

**Remark 1** *Our model can also be applied to two-sided markets where in market A firm 1 is a two-sided platform that receives advertising revenue whenever it is chosen by a consumer. If we assume that there is an associated advertising revenue of $\alpha > 0$ for each consumer in market A and consumers' valuations for product A are distributed on $[0, \overline{x}]$, then our one-sided market model is isomorphic to a two-sided model with additional advertising revenue per consumer.*

Let $p_A$ be the price of product $A$. With a change of variables of $\widehat{p}_A \equiv p_A - \alpha$, we have a demand function $D(\widehat{p}_A) = 1 - G(\widehat{p}_A)$ in market $A$. We assume that $G(\cdot)$ satisfies the monotone hazard rate condition, that is, $\frac{g(\cdot)}{1-G(\cdot)}$ is strictly increasing. In market $A$, with independent pricing firm 1 chooses $\widehat{p}_A$ to maximize

$$\max_{\widehat{p}_A \geq 0} \Pi_A(\widehat{p}_A) \equiv (\widehat{p}_A + \alpha)\left[1 - G(\widehat{p}_A)\right]. \tag{3}$$

[13] For example, consider a product that has a basic functionality plus some additional features. We can imagine a situation in which the basic functionality provides the same utility of $\alpha$ to all consumers, but additional features may generate different levels of extra utility to consumers, which is distributed on $[0, \overline{u} - \alpha]$.

[14] We admit the possibility that $\overline{x} = \infty$.

The monotone hazard rate assumption implies that the solution to (3) is unique and satisfies the first-order condition if it is interior.

**Remark 2** *In the case of a two-sided market with advertising revenues $\alpha$ earned by firm 1, the price $\widehat{p}_A = p_A - \alpha$ is the price paid by consumers while $\alpha$ is a negative marginal cost of the firm, as reflected in problem (3).*

In market $B$, we make the following assumption:

**Assumption 1:** $\Delta \equiv v_2 - v_1 > 0$ and $\Delta < \beta < \frac{1}{2g(x)}$ for all $x \in [0, \overline{x}]$.

As in Section 2, the condition that $\Delta > 0$ means that firm 2's product $B2$ has higher quality than firm 1's product $B1$. The assumption that $\beta > \Delta$ means that network effects are sufficiently important relative to the quality differential $\Delta$: if all consumers buy product $B1$ from firm 1, then its (network-augmented) quality $v_1 + \beta$ becomes higher than that of product $B2$, $v_2$. Last, the assumption that $\beta < 1/[2g(x)]$ for all $x$ is a stability condition for interior equilibrium in the tying regime[15] and guarantees a unique cut-off type when firm 1 offers a pure bundle (see the proof of Lemma 1).

We will consider two simultaneous-pricing games and compare them. In one firm 1 can offer only single-product prices $p_A$ and $p_{B1}$ ("independent-pricing" game), while in the other it is free to also offer a bundle at price $P$. In the latter case, no pre-commitment is involved; rather, if firm 1 offers a bundle, it does so only because it is a best response to firm 2's price $p_{B2}$.

# 4 Analysis of the Independent-Products Model

In this section, we examine the impact of allowing tying in the independent-products model. In the rest of the paper, we restrict attention to Pareto-undominated Nash equilibrium (NE) consumer responses.[16] In addition, when multiple undominated NE consumer responses exist to the firms' price offers, we assume that the worst such consumer response for firm 1 arises. Because the presence of multiple undominated responses matters only when firm 1 ties, in this way we "stack the deck" against firm 1's ability to succeed when it utilizes a tying strategy. We continue to restrict attention to equilibria in which the firms make undominated price offers (e.g., firm 2 will never set $p_{B2} < 0$).[17]

[15] If $G(\cdot)$ is uniform, $\beta < \frac{1}{2g(x)} = \frac{\overline{x}}{2}$ is the necessary and sufficient condition for an interior equilibrium to be stable. For general distributions, $\beta < \frac{1}{2g(x)}$ is a sufficient, but not necessary, condition for the stability of an interior equilibrium because the violation of the condition implies only local instability.

[16] Specifically, we say that a NE response dominates another NE response if all consumers are weakly better off in the former NE and some are strictly better off.

[17] We can derive identical results by instead employing coalition-proof Nash equilibrium (CPNE) of the consumer response (Bernheim, Peleg, and Whinston, 1987) as a refinement. Alternatively, we can derive similar results with the use of only iterated dominance under tying, an approach that does not rely on any coordination assumptions. See the Appendix for more details on this alternative approach.

### 4.1 Independent-Pricing Game

In the absence of tying, as in Section 2 the two markets can be analyzed separately as we assume independent products.

In market $B$, all consumers have the same preference. Because we restrict attention to Pareto-undominated NE consumer responses to the firms' price offers, in the unique equilibrium all consumers purchase product $B2$, firm 1 sets a zero price ($p^*_{B1} = 0$), and firm 2 charges $p^*_{B2} = \Delta$.[18] Thus, when consumers coordinate their purchase responses with neither firm having an advantage in network effects, all consumers end up purchasing product $B2$ at a price at which the consumers capture the network benefits.

In market $A$, independent pricing may or may not result in all consumers purchasing product $A$. We denote the solution to the monopoly pricing problem (3) by $\widehat{p}^*_A$ (and write the actual monopoly price as $p^*_A \equiv \widehat{p}^*_A + \alpha$). Under the monotone hazard assumption on $G(\cdot)$, when

$$\alpha \geq \frac{1 - G(0)}{g(0)} = \frac{1}{g(0)} \tag{4}$$

firm 1 sets $\widehat{p}^*_A = 0$ (or, equivalently, $p^*_A = \alpha$ ) so that all consumers buy $A$ (there is "full coverage"), while $\widehat{p}^*_A > 0$ (or, equivalently, $p^*_A > \alpha$) otherwise. In the latter case, firm 1 sets a price of $p^*_A = \widehat{p}^*_A + \alpha$, where $\widehat{p}^*_A$ satisfies the following condition:

$$\widehat{p}^*_A = \frac{1 - G(\widehat{p}^*_A)}{g(\widehat{p}^*_A)} - \alpha \ (> 0). \tag{5}$$

In both cases, the mass of consumers buying product A without tying is given by $1 - G(\widehat{p}^*_A)$.

Hence, without tying, firm 1 receives a profit of

$$\Pi^*_1 = \begin{cases} \alpha & \text{if } \widehat{p}^*_A = 0 \\ (\alpha + \widehat{p}^*_A)(1 - G(\widehat{p}^*_A)) = \frac{\left[1 - G(\widehat{p}^*_A)\right]^2}{g(\widehat{p}^*_A)} & \text{if } \widehat{p}^*_A > 0 \end{cases}$$

while firm 2's profit is the same as in Section 2:

$$\Pi^*_2 = \Delta.$$

### 4.2 Tying Equilibrium

We now derive the equilibrium when firm 1 is allowed to tie, requiring purchase (and use) of product $B1$ in order to acquire product $A$.[19] We will show that, under certain conditions, in this equilibrium firm 1 offers for sale either only a bundle (at price $P$) or possibly the bundle and product $B1$ (at price $p_{B1}$), and by doing so monopolizes market $B$. Moreover,

[18] As is standard in the literature, we make the tie-breaking assumption in favor of the firm that can offer the highest consumer surplus to avoid the open set problem.

[19] As we noted earlier, the requirement that the consumer not also purchase and then use instead $B2$ is not necessary if the cost of producing $B1$ and $B2$ (which we have set here to zero) is sufficiently high; if it is, no consumer who has purchased an $A/B1$-bundle will also find it worthwhile to pay a price above cost for product $B2$.

in cases of "full coverage," in which in the independent-pricing game all consumers would buy product $A$ at the monopoly price $p_A^* = \alpha$, firm 1 offers only the bundle. Specifically, we make the following additional assumption, which we will show is a sufficient condition for firm 1 to monopolize market $B$ when tying is permitted:

**Assumption 2:** $\frac{1}{g(0)} - G^{-1}\left(\frac{\beta-\Delta}{2\beta}\right) < \alpha$

Assumption 2 will guarantee that if all consumers of product $A$ under independent pricing purchase the bundle, then product $B1$ offers a higher utility than product $B2$ even if all remaining consumers are expected to purchase $B2$: i.e., $\beta(1-G(\widehat{p}_A^*)) > \Delta + \beta G(\widehat{p}_A^*)$ holds.[20] This fact will imply that firm 1 is able to charge $p_A^*$ plus this resulting utility differential for the bundle and sell it to the same consumers that were buying only product $A$ under independent pricing, increasing its profit. Assumption 2 is more likely to be satisfied if $\alpha$ is large (so $A$ is very valuable to consumers) and $\frac{\Delta}{\beta}$ is small (so network effects are large relative to product differentiation).[21] Note that if the full coverage condition (4) holds, then Assumption 2 necessarily holds since then

$$\frac{1}{g(0)} - G^{-1}\left(\frac{\beta-\Delta}{2\beta}\right) < \frac{1}{g(0)} \leq \alpha.$$

It is useful to first consider how consumers would react to firm 1 offering only a pure bundle. The following lemma shows that the consumer response is pinned down by iterated dominance.[22]

**Lemma 1** *When firm 1 offers only a bundle for sale, given prices of $P$ for the bundle and $p_{B2} < v_2$ for product $B2$, and defining $\widehat{P} \equiv P - \alpha$, the unique outcome in consumers' choices that survives iterated deletion of dominated strategies is as follows:*
*(i) If $(\widehat{P} - p_{B2}) \in (\beta - \Delta, \overline{x} - \beta - \Delta)$, consumers whose valuation for $A$ is higher than $\widetilde{X} \in (0, \overline{x})$ purchase the bundle while consumers whose valuation is lower than $\widetilde{X}$ purchase $B2$, where $\widetilde{X}$ satisfies*

$$\widetilde{X} + v_1 + \beta(1 - G(\widetilde{X})) - \widehat{P} = v_2 + \beta G(\widetilde{X}) - p_{B2}. \tag{6}$$

*(ii) If $(\widehat{P} - p_{B2}) \leq \beta - \Delta$, all consumers purchase the bundle (i.e., $\widetilde{X} = 0$).*
*(iii) If $(\widehat{P} - p_{B2}) \geq \overline{x} - \beta - \Delta$, all consumers purchase $B2$ only (i.e., $\widetilde{X} = \overline{x}$) .*

[20] The proof of Lemma 2 shows that the assumption implies the inequality. While we show Assumption 2 is sufficient for firm 1 to use tying to profitably monopolize market $B$, it is not necessary. For example, if in the example of Section 2, the willingness to pay difference $s$ was slightly larger than $u_L$ so that the monopoly price for $A$ was instead $u_H$, firm 1's deviation to tying with bundle price $P = u_L + \varepsilon$ would still have been profitable despite the fact that $B2$ offers higher utility than $B1$ if consumer $H$ buys the bundle and consumer $L$ buys $B2$. We thank Vincenzo Denicolò for this observation.

[21] If $x$ is uniformly distributed with $g = 1/\overline{x}$ over $[0, \overline{x}]$, the condition in Assumption 2 can be written as $\frac{(\beta+\Delta)}{2\beta}\overline{x} < \alpha$.

[22] When there is a unique consumer response that survives iterated elimination of dominant strategies, it is also the unique Nash equilibrium of consumer responses.

**Remark 3** *Note that* $\beta - \Delta < \overline{x} - \beta - \Delta$ *since* $1 = \int_0^{\overline{x}} g(x)dx \leq [\max_x g(x)]\overline{x}$ *implies that* $\overline{x} \geq 1/[\max_x g(x)] > 2\beta$*, where the last inequality follows from Assumption 1.*

**Proof.** Because $p_{B2} < v_2$, in any equilibrium all consumers will make some purchase. Let $\psi(x, X)$ be the payoff gain from purchasing the bundle over purchasing $B2$ for a type $x$ consumer (i.e., whose willingness to pay for $A$ is $\alpha + x$) if all other players whose types are higher than $X$ choose the bundle while all the remaining consumers choose $B2$:

$$\psi(x, X) \equiv x + \beta[1 - 2G(X)] - \Delta - (\widehat{P} - p_{B2}).$$

Note that $\psi(x, X)$ is continuous in $x$ and $X$, increasing in $x$, and decreasing in $X$. Define as well the function $\Psi(X)$ as follows:

$$\Psi(X) \equiv \psi(X, X) = X + \beta[1 - 2G(X)] - \Delta - (\widehat{P} - p_{B2}). \tag{7}$$

Note that $\Psi(X)$ is increasing in $X$ because by Assumption 1

$$\Psi'(X) = 1 - 2\beta g(X) > 0,$$

and that when $\beta - \Delta < (\widehat{P} - p_{B2}) < \overline{x} - \beta - \Delta$, we have $\Psi(0) < 0 < \Psi(\overline{x})$. Therefore, in any equilibrium response by consumers the "cut-off" type $\widetilde{X}$ who is indifferent between the bundle and product $B2$ (if interior) must be the unique solution to $\Psi(\widetilde{X}) = 0$. As in the analysis of global games, we use an induction argument to show that the choices of types above and below $\widetilde{X}$ are in fact pinned down by iterated dominance.[23] Observe that:

(i) If $(\widehat{P} - p_{B2}) < \overline{x} - \beta - \Delta$, then even when all other consumers are expected to choose $B2$ (i.e., the cut-off type is $\overline{X}^0 = \overline{x}$), it is optimal to choose the bundle for any consumer whose type is higher than $\overline{X}^1 = \beta + \Delta + (\widehat{P} - p_{B2}) < \overline{x} = \overline{X}^0$. Given that at least a measure of $1 - G(\overline{X}^1)$ consumers choose the bundle, we can derive another cut-off value $\overline{X}^2 < \overline{X}^1$. Note that $\overline{X}^n$ is a decreasing sequence. Similarly, if $\beta - \Delta < (\widehat{P} - p_{B2})$, then even when all other consumers are expected to choose the bundle (i.e., the cut-off type $\underline{X}^0 = 0$), it is optimal to choose $B2$ for any consumer whose type is lower than $\underline{X}^1 = -\beta + \Delta + (\widehat{P} - p_{B2}) > 0 = \underline{X}^0$. Given that at least a measure of $G(\underline{X}^1)$ consumers choose $B2$, we can derive another cut-off value $\underline{X}^2 > \underline{X}^1$. Note that $\underline{X}^n$ is an increasing sequence. Thus, when $\beta - \Delta < (\widehat{P} - p_{B2}) < \overline{x} - \beta - \Delta$, the continuity of $\psi(x, X)$ and the way the two sequences $\overline{X}^n$ and $\underline{X}^n$ are constructed imply that $\psi(\underline{X}, \underline{X}) = \psi(\overline{X}, \overline{X}) = 0$, where $\overline{X} = \lim_{n\to\infty} \overline{X}^n$ and $\underline{X} = \lim_{n\to\infty} \underline{X}^n$. Given that there is a unique $\widetilde{X}$ such that $\Psi(\widetilde{X}) = 0$, it must be that $\underline{X} = \overline{X} = \widetilde{X}$.

(ii) If $(\widehat{P} - p_{B2}) \leq \beta - \Delta$, the process of iterated deletion of dominated strategies leads to $\widetilde{X} = 0$ because $\Psi(0) \geq 0$.

(iii) Similarly, if $(\widehat{P} - p_{B2}) \geq \overline{x} - \beta - \Delta$, the process of iterated deletion of dominated strategies leads to $\widetilde{X} = \overline{x}$ because $\Psi(\overline{x}) \leq 0$. ∎

Intuitively, by offering only a bundle firm 1 is able to leverage the presence of consumers

[23]For an excellent survey of global games, see Morris and Shin (2010).

with a high value for product $A$ to gain a network benefit advantage over firm 2 in market $B$. Consumers with a high value for product $A$ want to buy the bundle regardless of what other consumers are doing, which in turn creates network benefits for $B1$ that make consumers with somewhat lower values buy the bundle, and so on.

A consequence of Lemma 1 is that even if $p_{B2} = 0$, by choosing $P = \alpha + \beta - \Delta$, firm 1 can induce all consumers to buy the bundle and realize a profit of $P = \alpha + \beta - \Delta$, which is strictly larger than the profit under independent pricing when full coverage is optimal. More generally, Lemma 1 implies that if tying is allowed and Assumptions 1 and 2 hold, then no independent-pricing equilibrium exists – firm 1 has a profitable deviation in which it offers only a bundle:

**Lemma 2** *If tying is allowed and Assumptions 1 and 2 hold, then no independent-pricing equilibrium exists.*

**Proof.** The argument in the text establishes the result for the case of full coverage, so here we provide the argument when $\widehat{p}_A^* > 0$. In an independent-pricing equilibrium, firm 2 sets price $p_{B2} = \Delta < v_2$ . Using (6), define the bundle price that implements cut-off type $\widetilde{X} \in (0, \overline{x})$ between the bundle and B2 as

$$\widehat{P}(\widetilde{X}|p_{B2}) \equiv p_{B2} + \widetilde{X} + \beta[1 - 2G(\widetilde{X})] - \Delta. \tag{8}$$

Assumption 1 implies that $\widehat{P}(\widetilde{X}|p_{B2}) - p_{B2} < \overline{x} - \beta - \Delta$,[24] so Lemma 1 implies that all types above $\widetilde{X}$ prefer the bundle over $B2$ by iterated dominance.[25] Suppose that firm 1 deviates from independent pricing and instead offers only a bundle at price $\widehat{P}(\widehat{p}_A^*|\Delta)$ where $\widehat{p}_A^* = p_A^* - \alpha$ is the lowest type buying product $A$ in the independent pricing equilibrium. Firm 1 then makes the same number of sales as under independent pricing (since we will have $\widetilde{X} = \widehat{p}_A^*$), but at a higher price since

$$\begin{aligned} \widehat{P}(\widehat{p}_A^*|\Delta) &= \Delta + \widehat{p}_A^* + (\beta - \Delta) - 2\beta G(\widehat{p}_A^*) \\ &> \widehat{p}_A^* + 2\beta \left[ \frac{\beta - \Delta}{2\beta} - G(\widehat{p}_A^*) \right] \\ &> \widehat{p}_A^* \end{aligned}$$

where the first inequality follows from the fact that $\Delta > 0$, while the final inequality follows because

$$G^{-1}\left(\frac{\beta - \Delta}{2\beta}\right) > \frac{1}{g(0)} - \alpha \geq \frac{1 - G(\widehat{p}_A^*)}{g(\widehat{p}_A^*)} - \alpha = \widehat{p}_A^*, \tag{9}$$

where the first inequality in (9) follows from Assumption 2, the second from the monotone hazard rate property, and the equality from (5). Hence, this deviation increases firm 1's profit. ■

[24] This follows because $\widehat{P}(\overline{x}|p_{B2}) - p_{B2} = \overline{x} - \beta - \Delta$ and $\widetilde{X} + \beta[1 - 2G(\widetilde{X})]$ is a strictly increasing function of $\widetilde{X}$ for $\widetilde{X} \in (0, \overline{x})$ by Assumption 1.

[25] The argument can be extended when firm 1 also offers $B1$ for sale: iterated dominance implies that all types above $\widetilde{X}$ buy either the bundle or $B1$.

As in the example of Section 2, firm 1's deviation to tying in the proof of Lemma 2 leverages the unexploited surplus in market $A$ that exists at the monopoly price $p_A^*$ to gain sales in market $B$ that increase its profit. Specifically, firm 1 sets its bundle price equal to $p_A^* = \alpha + \widehat{p}_A^*$ plus the utility advantage that $B1$ has over $B2$ when all consumers above type $\widehat{p}_A^*$ buy the bundle, $\beta(1 - G(\widehat{p}_A^*)) - \beta G(\widehat{p}_A^*) - \Delta$.

**Remark 4** *The argument of Lemma 2 can be extended to establish that firm 1's equilibrium profit when tying is allowed must strictly exceed its profit in the independent-pricing equilibrium: by offering only the bundle at price* $\widehat{P}(\widehat{p}_A^*|0)$ *(which yields sales of* $(1 - G(\widehat{p}_A^*))$ *when firm 2 charges* $p_{B2} = 0$*), firm 1 necessarily sells at least as much of the bundle as it sells of product* $A$ *in the independent-pricing equilibrium (since* $p_{B2} \geq 0$*) and does so at a higher price (since* $\widehat{P}(\widehat{p}_A^*|0) > \widehat{p}_A^*$ *by an argument similar to that in the proof of Lemma 2).*

A further consequence of Lemma 2 is that in any equilibrium (i) firm 1 must offer a bundle discount (i.e., $P < p_A + p_{B1}$) and (ii) make no separate sales of product $A$. A bundle discount is necessary because in its absence consumer purchases and firm profits would be the same as with independent product pricing, but Lemma 2 rules out such an equilibrium outcome.

To see why firm 1 must not be making separate sales of product $A$, we first describe consumer purchase decisions when faced with a bundle discount offer from firm 1. Figure 1 depicts the purchase decisions of three sets of consumers in this case, who are distinguished by their value $u$ for product $A$:[26]

**Consumer Set I ($u < P - p_{B1} < p_A$):** These consumers buy (at most) either $B1$ or $B2$. For these consumers, buying only $B1$ is better than buying the bundle since $u < P - p_{B1}$, and buying either only $A$ or both $A$ and $B2$ is unattractive since $u < p_A$. Note that these consumers' preferences for $B1$ and $B2$, including whether to make a purchase at all, are independent of the level of $u$.

**Consumer Set II ($u \in (P - p_{B1}, p_A)$):** These consumers buy (at most) either $B2$ or the bundle. For these consumers, buying the bundle is better than buying only $B1$ since $u > P - p_{B1}$, and buying only $A$ or both $A$ and $B2$ is unattractive since $u < p_A$. These consumers' preferences for the bundle versus both buying only $B2$ or buying nothing depend on the value of $u$, with a consumer in this set more likely to buy the bundle for higher values of $u$.

**Consumer Set III ($u > p_A > P - p_{B1}$):** These consumers either buy the bundle or buy $A$ and (possibly) $B2$. For these consumers, buying the bundle is better than buying only $B1$ since $u > P - p_{B1}$, while a consumer not buying the bundle will certainly purchase $A$ since $u < P_A$. These consumers' preferences for the bundle versus purchase of $A$ and (possibly) $B2$ are independent of the level of $u$.

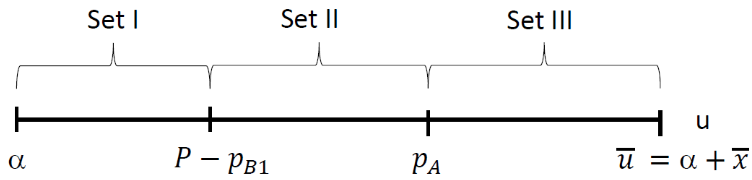


Figure 1: Consumer purchase decisions as a function of their value $u = \alpha + x$ for product $A$

Using this fact, we have:

**Lemma 3** *In any equilibrium, firm 1 makes no separate sales of product $A$.*

**Proof.** Any separate sales of product $A$ to a positive measure of consumers must be to consumers in set III who then all weakly prefer the better of buying only $A$ or buying both $A$ and $B2$ over buying the bundle. Suppose, first, that buying $A$ only is weakly the better option for these consumers – i.e., that purchase of $B2$ yields non-positive surplus. Then the consumer of type $u = p_A$ earns no surplus and must enjoy as well non-positive surplus from the bundle. All types in set II below this type therefore enjoy strictly negative surplus from the bundle, including type $u = P - p_{B1}$. But this then implies that purchase of only product $B1$ gives strictly negative surplus as well. Hence, the sales of $A$ are the *only* sales that firm 1 is making and so its profit is no greater than in the independent-pricing equilibrium, a contradiction to Remark 4.

If, instead, buying both $A$ and $B2$ is best for the consumer who buys $A$ separately, then consumer type $u = p_A$ weakly prefers buying $B2$ only to buying the bundle. All other types in set II then strictly prefer buying $B2$ to buying the bundle. In turn, consumer type $u = P - p_{B1}$ strictly prefers buying $B2$ over buying $B1$, as does every consumer in set I. Thus, again, firm 1's only sales are of product $A$, yielding a contradiction. ■

By Lemma 3, any equilibrium must involve sales of the bundle to all consumers in set III since $u > p_A$ implies that they would otherwise necessarily purchase $A$ separately. As well, a positive measure of consumers in set II must buy the bundle. If not, then all consumers in sets I and II must be buying $B2$ (the conclusion for set I follows because consumer type $u = P - p_{B1}$ then strictly prefers $B2$ over both the bundle and buying $B1$ only); firm 1

[26]We do not specify the purchase decisions of consumer types $u = P - p_{B1}$ and $u = p_A$ as these are measure zero sets and so their purchase decisions do not affect either firm's profit.

would be selling only $A$ and its profit would be no greater than in the independent-pricing equilibrium, a contradiction.

We next state our main result, whose full proof is in the Appendix, which establishes that no sales of *B2* occur in equilibrium – i.e., that firm 1 monopolizes market $B$ in any equilibrium. The equilibrium outcome involves sales of the bundle to consumers above a cut-off type $\widetilde{x}^*$ (which equals $P^* - p^*_{B1}$ in the equilibrium) and sales of $B1$ to consumers below this type. In some cases, the cut-off type is $\widetilde{x}^* = 0$, so firm 1 sells only a bundle.

**Proposition 1** *If tying is allowed and Assumptions 1 and 2 hold, the unique equilibrium involves firm 1 tying (selling only the bundle and possibly product* $B1$*) and fully monopolizing market* $B$*. The cut-off type* $\widetilde{x}^*$ *between sales of the bundle and sales of* $B1$ *solves*

$$\underset{\widetilde{x}}{Max}\ \Pi_A(\widetilde{x}) + \beta[1 - G(\widetilde{X}(\widetilde{x}))], \tag{10}$$

*where* $\Pi_A(\cdot)$ *is the market A profit function defined in* (3) *and* $\widetilde{X}(\cdot)$ *is a strictly increasing function defined by the relation*

$$\widetilde{x} \equiv \widetilde{X} - \beta G(\widetilde{X}). \tag{11}$$

*The set of types buying the bundle contains the set of types that buy product A under independent pricing (i.e.,* $\widetilde{x}^* \leq \widehat{p}^*_A$*). This relation is strict, with the exception that when there is full coverage under independent pricing the two sets coincide (*$\widetilde{x}^* = \widehat{p}^*_A = 0$*). Firm 2 makes no sales and sets price* $p^*_{B2} = 0$*, while firm 1's prices are:*

*(i) When firm 1 sells only the bundle (i.e.,* $\widetilde{x}^* = 0$*):* $P^* = \alpha + (\beta - \Delta)$

*(ii) When firm 1 sells both the bundle and* $B1$ *(i.e.,* $\widetilde{x}^* > 0$*):*

$$\begin{aligned} P^* &= \alpha + \widetilde{X}(\widetilde{x}^*) + \beta[1 - 2G(\widetilde{X}(\widetilde{x}^*))] - \Delta \\ p^*_{B1} &= \beta[1 - G(\widetilde{X}(\widetilde{x}^*))] - \Delta. \end{aligned}$$

*Firm 1's equilibrium profit exceeds its profit under independent pricing.*

**Proof.** See Appendix. ■

To see the idea behind the proof of Proposition 1, recall that $\widetilde{X}$ is the consumer type that is indifferent between the bundle and $B2$ when firm 2 charges $p_{B2}$ and firm 1 charges $\widehat{P}(\widetilde{X}|p_{B2})$ for the bundle. Now consider whether, when charging $\widehat{P}(\widetilde{X}|p_{B2})$ for the bundle, firm 1 can get consumers below $\widetilde{X}$ to buy $B1$ instead of $B2$. Given bundle price $\widehat{P}(\widetilde{X}|p_{B2})$, the highest price $p_{B1}$ that induces all consumers below $\widetilde{X}$ to purchase $B1$ rather than $B2$ (given our assumption about consumers' responses) is given by

$$p_{B1}(\widetilde{X}|p_{B2}) \equiv p_{B2} + \beta(1 - G(\widetilde{X})) - \Delta, \tag{12}$$

since, at this price, consumers below $\widetilde{X}$ are indifferent between the two outcomes in which they all buy $B1$ and they all buy $B2$. (Note that the term $\beta(1-G(\widetilde{X}))$ is the network benefit advantage firm 1 has over firm 2 because of consumers who are buying the bundle, while $\Delta$ is the quality advantage that firm 2 has over firm 1. Firm 1's price for $B1$ can exceed $p_{B2}$ by the difference between these two amounts.) Specifically, at any higher price for $B1$ there is an undominated NE consumer response in which all consumers below $\widetilde{X}$ buy $B2$ and this NE response is worse for firm 1 than any other undominated NE (here we use our "stack-the-deck" assumption). In the proof of the proposition, we show that, as a result, in any equilibrium firm 1 will charge $p_{B1}(\widetilde{X}|p_{B2})$ for $B1$, undercutting firm 2. As a result, firm 2 makes no sales and sets $p^*_{B2} = 0$.

Given the two prices $\widehat{P}(\widetilde{X}|p_{B2})$ and $p_{B1}(\widetilde{X}|p_{B2})$, we denote by $\widetilde{x}(\widetilde{X})$ the consumer type that is indifferent between the bundle and $B1$ when all consumers buy either the bundle or $B1$. This is the type for which

$$\widetilde{x}(\widetilde{X}) = \widehat{P}(\widetilde{X}|p_{B2}) - p_{B1}(\widetilde{X}|p_{B2}) = \widetilde{X} - \beta G(\widetilde{X}) \tag{13}$$

It is given by (11) and is a strictly increasing function by Assumption 1. It is convenient to instead work with the inverse of $\widetilde{x}(\widetilde{X})$, which is the function $\widetilde{X}(\widetilde{x})$ described in the statement of Proposition 1. Figure 2 depicts the resulting purchase regions for a case in which firm 1 optimally prices so that sales of both the bundle and $B1$ occur. In the Appendix, we show that firm 1's optimal choice of prices for the bundle and $B1$ implements the $\widetilde{x}^*$ that solves problem (10).

Since the term $\beta[1 - G(\widetilde{X}(\widetilde{x}))]$ in problem (10) is a weakly decreasing function of $\widetilde{x}$, it follows that at any solution to (10) we must have $\Pi^{'}_A(\widetilde{x}^*) \geq 0$, which implies that $\widetilde{x}^* \leq \widehat{p}^*_A$ – i.e., that the set of types buying the bundle contains the set of types who buy $A$ under independent pricing. (In the Appendix, we show as well that the relationship is strict, except in the case of full coverage.) Intuitively, firm 1 wants to sell more of the bundle than it sells of $A$ under independent pricing because by doing so it gains a network benefit advantage that allows it to charge more for product $B1$.

When firm 1 makes sales of both the bundle and $B1$, its mixed bundling strategy enables it to screen consumers with more price instruments while still maintaining the ability to leverage the presence of high-value consumers for its monopoly product $A$ to the competitive market $B$: as in the case of pure bundling, it is as if firm 1 already had an installed base advantage when competing in market $B$, which ensures firm 1's market dominance and enables it to expropriate the resulting network benefits of consumers in market $B$.

Notice that in this equilibrium, firm 2 lowers its price by $\Delta$ compared to the independent pricing equilibrium. Despite this fact, under our assumptions (that state that network effects are large enough) firm 1's profit is greater in this equilibrium than under independent pricing as we noted in Remark 4: the benefits from leveraging high-value consumers in market $A$ to gain a network benefit advantage in market $B$ are large enough to offset the effect of firm 2 pricing more aggressively.

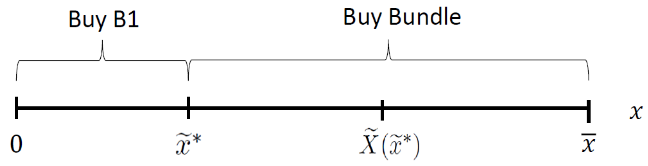


Figure 2: Consumers' equilibrium purchase decisions as a function of their type $x$ for product $A$: types above $\widetilde{X}(\widetilde{x}^*)$ prefer the bundle over $B2$ by iterated dominance, types above $\widetilde{x}^*$ buy the bundle, and types below $\widetilde{x}^*$ buy $B1$.

### 4.3 Welfare Effects of Tying

We now investigate welfare implications of tying in our model. In the case in which market $A$ is covered under independent pricing (i.e., $\widehat{p}_A^* = 0$), bundling is profitable, but always welfare-reducing, lowering aggregate surplus by the amount $\Delta$, as it results in substitution of the inferior product $B1$ for the superior product $B2$. Regarding consumer surplus, as tying reduces aggregate surplus by $\Delta$ while changing total industry profit by $(\alpha+(\beta-\Delta))-(\alpha+\Delta)=\beta-2\Delta$, in the case of full coverage under independent pricing it reduces consumer surplus by $(\beta-\Delta)$: firm 1 rather than consumers now captures the network benefits ($\beta$), but partially offsetting this loss for consumers is the fact that firm 2 lowers its price by $\Delta$.

On the other hand, when market $A$ is instead not covered under independent pricing (i.e., $\widehat{p}_A^* > 0$), there is an opposing welfare effect of tying: it expands the use of product $A$ since $\widetilde{x}^* < \widehat{p}_A^*$. Its welfare impacts thus can be ambiguous. More precisely, the aggregate surplus reduction in market $B$, which equals $\Delta$, must be compared to the increase in aggregate surplus in market $A$:[27]

$$\widetilde{AS} - AS = \underbrace{\int_{\widetilde{x}^*}^{\widehat{p}_A^*} (x+\alpha) g(x) dx}_{\text{Market Expansion Effect in } A} - \underbrace{\Delta}_{\text{Efficiency Loss in } B}$$

To explore further how the market expansion effect depends on key parameters of the model, consider first case (ii) of Proposition 1 in which market $A$ is not fully covered even with tying (i.e., $0 < \widetilde{x}^* < \widehat{p}_A^*$). In this case, note that $\widetilde{x}^*$ is decreasing in $\beta$ as shown in Lemma 4 below, whereas $\widehat{p}_A^*$ is independent of $\beta$. This implies that the market expansion effect is positively related to $\beta$.

[27]We denote by $AS$ the aggregate surplus under independent pricing and by $\widetilde{AS}$ aggregate surplus when tying is allowed; similarly for consumer surplus, denoted in these two situations by $CS$ and $\widetilde{CS}$.

**Lemma 4** *The optimal cutoff $\widetilde{x}^*$ under tying is a nonincreasing function of $\beta$.*

**Proof.** Let $\widetilde{x}^*(\beta)$ be an optimal cutoff as a function of $\beta$. By the definition of $\widetilde{x}^*(\beta)$, we have

$$\begin{aligned}
\Pi_A(\widetilde{x}^*(\beta_1)) + \beta_1 \left[1 - G(\widetilde{X}(\widetilde{x}^*(\beta_1)))\right] &\geq \Pi_A(\widetilde{x}^*(\beta_2)) + \beta_1 \left[1 - G(\widetilde{X}(\widetilde{x}^*(\beta_2)))\right] \\
\Pi_A(\widetilde{x}^*(\beta_2)) + \beta_2 \left[1 - G(\widetilde{X}(\widetilde{x}^*(\beta_2)))\right] &\geq \Pi_A(\widetilde{x}^*(\beta_1)) + \beta_2 \left[1 - G(\widetilde{X}(\widetilde{x}^*(\beta_1)))\right],
\end{aligned}$$

which implies that

$$(\beta_1 - \beta_2)\left[1 - G(\widetilde{X}(\widetilde{x}^*(\beta_1)))\right] \geq (\beta_1 - \beta_2)\left[1 - G(\widetilde{X}(\widetilde{x}^*(\beta_2)))\right]$$

As $\widetilde{X}(.)$ is a strictly increasing function, the inequality above implies that $\widetilde{x}^*(\beta_1) \leq \widetilde{x}^*(\beta_2)$ if $\beta_1 > \beta_2$. ■

An increase in $\alpha$, on the other hand, can affect the aggregate surplus impact of tying in case (ii) because it changes the curvature of the function $\Pi_A(\cdot)$, and thus alters the degree to which $\widetilde{x}^*$ is less than $\widehat{p}_A^*$.

In contrast, in case (i) of Proposition 1 — in which market $A$ is fully covered under tying (i.e., $\widetilde{x}^* = 0$) — $\widetilde{x}^*$ is invariant in both $\alpha$ and $\beta$ whereas $\widehat{p}_A^*$ is decreasing in $\alpha$ and unaffected by $\beta$. As a result, the extent of expansion in purchases of product $A$, $\widehat{p}_A^* - \widetilde{x}^*$, decreases in $\alpha$ and is unaffected by $\beta$. We illustrate these effects with a uniform distribution of $x$ in the following example:

**Example 1** *In the uniform distribution case with $\overline{x} = 1$, we can derive closed-form expressions for welfare analysis. More specifically, Assumptions 1 and 2 can be written as $\Delta < \beta < \frac{1}{2}$ and $\frac{(\beta+\Delta)}{2\beta} < \alpha$, respectively and we assume as well that $\alpha < 1$ so that market $A$ is not covered under independent pricing where we have $\widehat{p}_A^* = (1-\alpha)/2$. Aggregate surplus under independent pricing is then*

$$AS = \underbrace{\frac{3}{8}(\alpha+1)^2}_{AS_A} + \underbrace{v_2 + \beta}_{AS_B}$$

*With bundling, we consider two cases, corresponding to the two cases of Proposition 1.*

*(i) If $\alpha + \frac{\beta}{1-\beta} \geq 1$, all consumers buy the bundle and market $A$ is fully covered (i.e., $\widetilde{x}^* = 0$). We then have*

$$\widetilde{AS} = \underbrace{\alpha + \frac{1}{2}}_{\widetilde{AS}_A} + \underbrace{v_1 + \beta}_{\widetilde{AS}_B}$$

*(ii) If $\alpha + \frac{\beta}{1-\beta} < 1$, consumers with $x \geq \widetilde{x}^* = \frac{1-\alpha-\frac{\beta}{1-\beta}}{2} (< \widehat{p}_A^* = \frac{1-\alpha}{2})$ buy the bundle and*

*those with $x < \widetilde{x}^*$ buy $B1$. We then have*

$$\widetilde{AS} = \underbrace{\frac{(3+3\alpha-\frac{\beta}{1-\beta})(1+\alpha+\frac{\beta}{1-\beta})}{8}}_{\widetilde{AS}_A} + \underbrace{v_1+\beta}_{\widetilde{AS}_B}$$

*Taken together,*

$$\widetilde{AS} - AS = \underbrace{\begin{Bmatrix} \frac{(1+3\alpha)(1-\alpha)}{8} \; \text{if } \alpha + \frac{\beta}{1-\beta} \geq 1 \\ \frac{\frac{\beta}{1-\beta}(2+2\alpha-\frac{\beta}{1-\beta})}{8} \quad \text{if } \alpha + \frac{\beta}{1-\beta} < 1 \end{Bmatrix}}_{\text{Market Expansion Effect in A}} - \underbrace{\Delta}_{\text{Efficiency Loss in B}}$$

*The change in aggregate surplus due to tying decreases in $\alpha$ in case (i)*[28] *and increases in $\beta$ in case (ii).*[29]

We now investigate the effects of tying on consumer welfare when market $A$ is not covered in the independent-pricing equilibrium. Consumer welfare under independent pricing can be written as the sum of consumer surplus in market $A$ and market $B$.

$$CS = \underbrace{\int_{\widehat{p}_A^*}^{\overline{x}} [1 - G(x)]dx}_{CS_A} + \underbrace{v_1 + \beta}_{CS_B = (v_2+\beta)-\Delta}$$

Consider again the two cases:

(i) When $\widetilde{x}^* = 0$, all consumers buy the bundle under tying at price $P^* = \alpha + \beta - \Delta$. In this case, it is useful to decompose the bundle price into two separate (fictitious) prices of $\widetilde{p}_A^* = \alpha$ for product $A$ and $\widetilde{p}_{B1}^* = \beta - \Delta$ for product $B1$, with $P^* = \widetilde{p}_A^* + \widetilde{p}_{B1}^*$. Then,

$$\widetilde{CS} = \underbrace{\int_0^{\overline{x}} [1 - G(x)]dx}_{\widetilde{CS}_A} + \underbrace{v_1 + \Delta}_{\widetilde{CS}_B = (v_1+\beta)-(\beta-\Delta)}$$

We have

$$\widetilde{CS} - CS = \underbrace{\int_0^{\widehat{p}_A^*} [1 - G(x)]dx}_{\widetilde{CS}_A - CS_A > 0} + \underbrace{(\Delta - \beta)}_{\widetilde{CS}_B - CS_B < 0}$$

Tying increases consumer surplus in market $A$ by expanding use of product $A$ with a lower (fictitious) price, but decreases consumer surplus in market $B$. The overall effect depends on the relative magnitude of these two opposing effects. Consumers are more likely to suffer from tying in case (i) if $\alpha$ is higher because it will reduce the positive market expansion effect in market $A$. A higher $\beta$ also makes tying less favorable for consumers. With independent pricing, the network-augmented utility term (represented by $\beta$) is competed away and passed on to consumers, but it is expropriated by firm 1 when it ties. In contrast, an increase in $\Delta$

[28] This follows because $\frac{(\beta+\Delta)}{2\beta} < \alpha$ implies that $\alpha > 1/2$.

[29] This follows because $\beta/(1-\beta) < (1-\alpha)$ in case (ii).

directly increases consumer surplus under tying. The reason is that $\Delta$ (the quality advantage of $B2$ over $B1$) is captured as a profit by firm 2 under independent pricing. However, under tying, as firm 2 charges zero price, $\Delta$ is fully compensated by the tying firm to induce consumers to purchase the inferior product $B1$ as part of the bundle. Thus, the effects of $\Delta$ on social welfare and consumer surplus are opposite.

(ii) When $\widetilde{x}^* > 0$, only a measure $1 - G(\widetilde{x}^*)$ of consumers buy the bundle (at the price of $P^* = \alpha + \widetilde{X}(\widetilde{x}^*) + \beta[1 - 2G(\widetilde{X}(\widetilde{x}^*))] - \Delta$), whereas the remaining consumers buy product $B1$ (at the price of $p^*_{B1} = \beta[1 - G(\widetilde{X}(\widetilde{x}^*))] - \Delta$). In this case, we can define a fictitious price of $A$ by firm 1 as $\widetilde{p}^*_A = P^* - p^*_{B1}$. In other words, we treat consumers who purchase the bundle at the price of $P$ as if they pay an effective price of $\widetilde{p}^*_A$ and $p^*_{B1}$, respectively, for products $A$ and $B1$. With a change of variables, $\widehat{\widetilde{p}}^*_A = \widetilde{p}^*_A - \alpha$, we have $\widehat{\widetilde{p}}^*_A = \widehat{P}^* - p^*_{B1} = \widetilde{x}^*$. Then, we can decompose the total consumer surplus into (fictitious) consumer surplus in market $A$ and consumer surplus in market $B$.

$$\widetilde{CS} = \underbrace{\left[\int_{\widetilde{x}^*}^{\overline{x}} [1 - G(x)]dx\right]}_{\widetilde{CS}_A} + \underbrace{[v_1 + \beta - p^*_{B1}]}_{\widetilde{CS}_B}$$

Thus, we have

$$\widetilde{CS} - CS = \underbrace{\int_{\widetilde{x}^*}^{\widehat{\widetilde{p}}^*_A} [1 - G(x)]dx}_{\widetilde{CS}_A - CS_A > 0} + \underbrace{\left[\Delta - \beta[1 - G(\widetilde{X}(\widetilde{x}^*))]\right]}_{\widetilde{CS}_B - CS_B < 0}$$

Once again, the effects of tying on total consumer surplus depend on the relative magnitudes of two opposite effects in markets $A$ and $B$.

**Example 2** *For the uniform distribution case with $\overline{x} = 1$,*

$$CS = \underbrace{\frac{(\alpha + 1)^2}{8}}_{CS_A} + \underbrace{v_1 + \beta}_{CS_B}$$

*(i) If $\alpha + \frac{\beta}{1-\beta} \geq 1$,*

$$\widetilde{CS} = \frac{1}{2} + v_1 + \Delta$$

*In this case, we have*

$$\widetilde{CS} - CS = \underbrace{\left[\frac{1}{2} - \frac{(\alpha + 1)^2}{8}\right]}_{\widetilde{CS}_A - CS_A > 0} + \underbrace{[\Delta - \beta]}_{\widetilde{CS}_B - CS_B < 0}$$

*(ii) If* $\alpha + \frac{\beta}{1-\beta} < 1$,

$$\widetilde{CS} = \underbrace{\frac{\left(1+\alpha+\frac{\beta}{1-\beta}\right)^2}{8}}_{\widetilde{CS}_A} + \underbrace{\left[v_1 + \frac{\frac{\beta}{1-\beta}(1-\alpha-\frac{\beta}{1-\beta})}{2} + \Delta\right]}_{\widetilde{CS}_B}$$

*We thus have*

$$\begin{aligned}\widetilde{CS} - CS &= \underbrace{\frac{\frac{\beta}{1-\beta}(2+2\alpha+\frac{\beta}{1-\beta})}{8}}_{\widetilde{CS}_A - CS_A > 0} + \underbrace{\left[\Delta - \frac{\frac{\beta}{1-\beta}(1+\alpha+\frac{\beta}{1-\beta}-2\beta)}{2}\right]}_{\widetilde{CS}_B - CS_B < 0} \\ &= -\frac{\frac{\beta}{1-\beta}(2+2\alpha+3\frac{\beta}{1-\beta}-8\beta)}{8} + \Delta,\end{aligned}$$

*In both cases, it can be shown that* $(\widetilde{CS} - CS)$ *is decreasing in both* $\alpha$ *and* $\beta$ *with the uniform distribution, but increasing in* $\Delta$.

# 5 Complementary Products

In this section, we consider complementary products. In line with the Chicago school logic, we first show that tying is not a profitable strategy as a leverage mechanism to suppress competition in complementary product markets. However, as in Whinston (1990), we show that if there is an inferior competitively-supplied alternative to the tying product then results that parallel those for the independent products case re-emerge. One major difference from Whinston (1990) is, once again, we do not rely on the commitment assumption and subsequent exit of the rival firm in the tied product market.

## 5.1 The Basic Model: The Chicago School Argument

We consider a setting that parallels the baseline model, where firm 1 is a monopolist in market $A$, except that products $A$ and $B$ are now complementary. For the purpose of exposition, consider product $A$ as the primary product whereas $B$ is an add-on product, that is, for the use of product B, product $A$ is necessary; without $A$, product $B$ is of no use.[30] For instance, product $A$ can be considered as an operating system whereas $B$ is application software.

When products are sold independently, consumers can use one of the two system products, $(A, B1)$ and $(A, B2)$, depending on which firm's product $B$ is used, or product $A$ only. Let us denote consumers' valuations for product $A$ by $u$, and their valuations for the combined products $A/B1$ and $A/B2$ are respectively given by $u + (v_1 + \beta N_1)$ and $u + (v_2 + \beta N_2)$, where $u = (\alpha + x)$ with $x$ distributed on $[0, \overline{x}]$ according to a c.d.f. $G(\cdot)$ with a strictly positive density $g(\cdot)$. We assume that $G(\cdot)$ satisfies the monotone hazard rate condition and $\Delta \equiv v_2 - v_1 > 0$ as in the independent products case. To simplify the analysis, we also

[30] Similar points can be made if products $A$ and $B$ are instead perfect complements that must be used in fixed proportions.

assume that condition (4) holds, which guarantees full market coverage in market $A$ under independent pricing.

We first show that for the complementary products case, firm 1 has no incentive to tie as it can benefit from the presence of product $B2$. We maintain the same parametric assumptions (i.e., Assumption 1) made in the independent products model.[31]

Observe that if firm 1 ties then firm 2 is unable to make any sales. In that case, firm 1 would optimally sell a bundle at price $P = \alpha + v_1 + \beta$.[32] Firm 1's profit would then be $\alpha + v_1 + \beta$. However, firm 1 can earn more by adopting independent pricing that induces consumers to use product $B2$ instead of product $B1$, but extracts the resulting increase in aggregate surplus through a higher price of product $A$. In fact, there is a continuum of Nash equilibria due to firm 1's ability to "price squeeze" and extract a portion of the surplus $\Delta$ (Choi and Stefanadis, 2001) which are parameterized by $\lambda \in [0, 1]$, the degree of price squeeze exercised by firm 1:

$$p_A = \alpha + v_1 + \beta + \lambda\Delta,\ p_{B1} = -\lambda\Delta,\ p_{B2} = (1-\lambda)\Delta$$

Firm 1's profit is then given by $\Pi_1 = \alpha + v_1 + \beta + \lambda\Delta$ so that all equilibria under independent pricing yield a higher profit than that under tying unless $\lambda = 0$ (in which case the profits are the same), establishing the Chicago school argument.

### 5.2 An Inferior Alternative Product in the Tying Market

We now suppose that there is an inferior alternative product in market $A$ that is competitively supplied at the marginal cost of zero.[33] We call firm 1's product in the tying market $A1$ while the alternative product is called $A2$. To maintain mathematical isomorphism between the complementary and independent product cases, we normalize consumers' valuations for the combined products that include this alternative $A2/B1$ and $A2/B2$ to $(v_1 + \beta N_1)$ and $(v_2 + \beta N_2)$, respectively.[34] Thus, $\alpha + x$ represents the added value that product $A1$ brings to a system over use of product $A2$ for a consumer of type $x$.

#### 5.2.1 Independent-Pricing Game

In the presence of product $A2$, a consumer of type $x$ chooses $A1$ over $A2$ if and only if

$$\alpha + x - p_{A1} \geq 0.$$

Thus, firm 1's sales of product $A$ are positive only if $p_{A1} < \overline{u}$ and, as in the independent products case, equal $1 - G(\widehat{p}_{A1})$ where $\widehat{p}_{A1} \equiv p_{A1} - \alpha$. We then get the following result:

[31] Recall that condition (4) implies Assumption 2.

[32] For this result, we actually need a less stringent assumption than (4), namely $\alpha + v_1 + \beta > 1/g(0)$.

[33] The assumption of a competitively-supplied alternative is for simplicity. It can be supplied by a firm with market power.

[34] We can allow a more general utility specification by assuming that consumers' valuations for the combined products $A2/Bj$ are given by $u' + (v_j + \beta N_j)$ for $j = 1, 2$ with $u' < u$. For instance, we can assume that $u' = \alpha' + (1-\theta)x$ with $\alpha' < \alpha$ and $1 > \theta \geq 0$ without qualitatively changing any results, where $(\alpha - \alpha')$ and $\theta$ represent the degree of quality inferiority for the alternative product $A2$.

**Proposition 2** *Suppose Assumption 1 and condition (4) hold. When tying is prohibited with complementary products and there is an inferior competitively-supplied alternative in market A, the equilibrium is identical to the one for the independent products case. Specifically:*

*(i) In market A, firm 1 charges* $p^*_{A1} = \alpha$ *and sells* $A1$ *to all consumers*

*(ii) In market B, firm 1 charges* $p^*_{B1} = 0$ *and firm 2 charges* $p^*_{B2} = \Delta$ *and all consumers buy product* $B2$.

*Firm 1 earns* $\Pi^*_1 = \alpha$ *and firm 2 earns* $\Pi^*_2 = \Delta$.

### 5.2.2 Tying

In the presence of tying, let $P$ be the price of the $A1/B1$ bundle of firm 1 and let $p_{B2}$ be the price of firm 2's product $B2$. As above, product $A2$ is provided competitively at the price of zero. As in the analysis of the independent-products model, we restrict attention to Pareto-undominated NE consumer responses and, when multiple undominated NE responses exist, to the worst such consumer response for firm 1. In the presence of product $A2$ in the tying market, the case of complementary products is isomorphic to the case of independent products and there is a unique equilibrium, which involves all consumers purchasing the $A1/B1$ bundle, as described in the following proposition that parallels case (i) of Proposition 1 for the independent products case:

**Proposition 3** *Suppose Assumption 1 and condition (4) hold. When tying is allowed with complementary products and there is an inferior competitively-supplied alternative in market A, there is a unique equilibrium in which all consumers purchase the* $A1/B1$ *bundle and the equilibrium prices are given by*

$$P^* = \alpha + \widehat{P}^* = \alpha + (\beta - \Delta),\ p^*_{B2} = 0.$$

*Moreover, firm 1's profit, equal to* $\alpha + (\beta - \Delta)$ *exceeds that under independent pricing. Both consumer surplus and social welfare decrease:*

$$\begin{aligned} \widetilde{CS} &= CS^* - (\beta - \Delta) < CS^* \\ \widetilde{AS} &= AS^* - \Delta < AS^*. \end{aligned}$$

## 6 Applications and Conclusion

In this paper, we have developed a leverage theory of tying in markets with network effects. We first analyze incentives to tie for independent products. When a monopolist in one market cannot fully extract the whole surplus from consumers, tying can be a mechanism through which unexploited consumer surpluses in one market are used as a demand-side leverage to create a strategic "quasi-installed base" advantage in another market characterized by network effects. Our mechanism does not require any pre-commitment to tying, and hence can apply to cases in which a firm employs a purely contractual tie. Tying can lead to the exclusion of more efficient rival firms in the tied market, but can also in some cases expand purchase of the tying good if the tied market is not fully covered with independent pricing. We also extend our analysis to the complementary products case. By allowing the existence of inferior alternatives as in Whinston (1990), we show that the setup of complementary products is mathematically identical to that of independent products. We also discuss welfare implications of tying.

Our analysis can provide a theory of harm for instances of tying where network effects are critical in the determination of the market winner. For instance, our model can shed light on the recent antitrust investigation concerning Google's practices in its MADA (Mobile Application Distribution Agreement) contracts. In particular, the EC decision has concluded that Google has engaged in illegal tying by requiring Android OEM "manufacturers to pre-install the Google search app ..., as a condition for licensing Google's app store (the Play Store)," a contractual tie.[35] Google's Play Store can be considered as the tying product as a "must-have" app, with other third-party app stores being inferior alternatives. In the search market, Google faces competition from other search engines and there is evidence of network effects, in particular, stemming from the fact that search results quality increases with the scale of queries received by a search engine.[36] One might wonder why Google's tie would be any different from Google simply paying for preinstallation of Google's search app. Our model suggests that by leveraging what would otherwise be unexploited surplus from OEMs' use of the app store, this tying may be a way for Google to lock in part of the search market, reducing the quality of rivals.[37]

The model also has relevance for the *Microsoft* case in Europe (IP/04/382) in 2004. The European Commission held Microsoft guilty of an abuse of dominant position by "tying its Windows Media Player (WMP), a product where it faced competition, with its ubiquitous Windows operating system."[38] Microsoft had a near monopoly position in the PC operating system market with over 90 percent market share. We can consider Linux as an inferior alternative to Microsoft's Windows OS in the tying market. The media player market can be considered as the tied market in which Microsoft faced competition (from firms such as RealPlayer) and network effects are critical. More precisely, the media player market can be considered a two-sided market with indirect network effects. If more content is provided in the format of a particular company, then more consumers will use the company's Media Player to access such content. Moreover, if more consumers select a particular company's Media

[35]See the European Commission Press Release "Antitrust: Commission fines Google €4.34 billion for illegal practices regarding Android mobile devices to strengthen dominance of Google's search engine," released on July 18, 2018. Available at http://europa.eu/rapid/press-release_IP-18-4581_en.htm.

[36]See, for example, He et al. (2017), Schäfer and Sapi (2020), and Klein et al. (2023) for empirical evidence of network effects in Internet search. The complaint in *US v. Google* also alleges the presence of such effects.

[37]One difference from our model, however, is that the "buyers" are distributors (phone OEMs/carriers) not consumers. Thus, while the search "purchase" can be interpreted as determining which search engine gets to be the pre-installed default on the distributor's device, distributors are not consumers with single-unit demands and more complicated pricing than linear pricing may be possible.

[38]Microsoft's tie of its media player to Windows may have involved a technological tie, but as we noted above the effects we highlight would apply in that case as well. The case also involved Microsoft's conduct of "deliberately restricting interoperability between Windows PCs and non-Microsoft work group servers." https://ec.europa.eu/commission/presscorner/detail/en/IP_04_382

Player, then content providers have a greater incentive to make their content available in the format of the company.[39,40] Our model assumes direct network effects in the tied market, but can be considered as capturing such feedback effects of two-sided markets in a reduced form.[41]

We conclude with comments about three possible extensions of our analysis. First, in our model firm 2 faces no fixed costs of either entry or remaining in the market. If it did, the fact that firm 1 would monopolize market $B$ through tying if firm 2 is in the market would mean that firm 2 would choose not to be active (either not enter, or exit if it already had). Firm 1 would then monopolize both markets but, in the absence of firm 2, would use independent pricing. If so, the fact that firm 1 *could* tie would unambiguously reduce both aggregate and consumer surplus regardless of whether market $A$ is covered under independent pricing.

Second, we developed our model in the context of one-sided markets. However, as we noted, our model is mathematically equivalent to one with a two-sided tying market with advertising revenues (with a reinterpretation of $\alpha$ as per-consumer advertising revenue). This may have important implications for recent antitrust debates on two-sided digital platforms. We showed that welfare impacts of tying depend on the relative magnitudes of positive market expansion effects and negative market foreclosure effects of more efficient firms. When advertising revenue is important (i.e., $\alpha$ is high) and services would under independent pricing be provided for free (hence, the market would be covered), as is common for many digital platforms, our model indicates that there are greater incentives to engage in tying to leverage unexploited consumer surplus. In that case, however, there are no socially beneficial market expansion effects. Therefore, the effects of tying are more likely to be welfare-reducing in such a case. In addition, the negative effects on consumer surplus will be more pronounced as network effects in the tied market become more important. This implies that careful scrutiny may be warranted when ad-financed digital platforms engage in tying with other products or services characterized by network effects.

Finally, and more conceptually, the lack of any needed commitment for tying to be a profitable monopolization strategy in our model with network effects contrasts with models in which economies of scale are central, such as Whinston (1990). However, we conjecture that tying could be a profitable best response without commitment in dynamic models in which consumer choices are sticky and production involves decreasing marginal costs. Specifically, by leveraging unexploited consumer surplus a dominant firm could shift consumers to its bundled offering, raising rivals' marginal costs, and shifting additional consumers in the following periods, mimicking over time the iterated shift in consumers' expectations of

[39] As described in the *Official Journal of the European Union* (6.2.2007): "The decision then explains why tying in this particular case is liable to foreclose competition....WMP's ubiquitous presence induces content providers and software developers to rely primarily on Windows Media technology. Consumers will in turn prefer to use WMP, since a wider array of complimentary software and content will be available for that product. Microsoft's tying reinforces and distorts these 'network effects' to its advantage, thereby seriously undermining the competitive process in the media player market."

[40] The Korean Fair Trade Commission also fined Microsoft 33 billion won (US$32 million) for abusing its market dominant position by bundling Windows OS with its instant messaging (IM) program as well as WMP. For the messenger case, the tied market market is characterized by direct network effects, as in our model.

[41] See Choi and Jeon (2021) for an analysis of tying that explicitly accounts for indirect network effects in two-sided markets.

network benefits that happens in our static model here.

# References


[1] Bernheim, B. Douglas, Bezalel Peleg, and Michael Whinston (1987), "Coalition-Proof Nash Equilibria I. Concepts," *Journal of Economic Theory*, 42: 1-12.

[2] Blair, Roger D. and David L. Kaserman (1985), *Antitrust Economics*, R. D. Irwin

[3] Burstein, Meyer L. (1960), "The Economics of Tie-in Sales," *Review of Economics and Statistics*, 42: 68-73.

[4] Caillaud, Bernard and Bruno Jullien (2003), "Chicken & Egg: Competition among Intermediation Service Providers," *RAND Journal of Economics*, 34: 309-28.

[5] Carlton, Dennis, W. and Michael Waldman (2002), "The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries," *RAND Journal of Economics*, 33: 194-220.

[6] Calzolari, Giacomo and Vincenzo Denicolò (2015), "Exclusive Contracts and Market Dominance," *American Economic Review*, 105: 3321-51.

[7] Choi, Jay Pil and Doh-Shin Jeon (2021), "A Leverage Theory of Tying in Two-Sided Markets with Non-Negative Price Constraints," *American Economic Journal: Microeconomics*, 13: 283-337.

[8] Choi, Jay Pil and Christodoulos Stefanadis (2001), "Tying, Investment, and the Dynamic Leverage Theory," *RAND Journal if Economics*, 32: 52-71

[9] He, Di, Aadharsh Kannan, Tie-Yan Liu, R. Preston McAfee, Tao Qin, and Justin M.Rao (2017), "Scale Effects in Web Search." In: R. Devanur, N., Lu, P. (eds) Web and Internet Economics. WINE. *Lecture Notes in Computer Science*, vol 10660. Springer, Cham.

[10] Fumagalli, Chiara, Massimo Motta, and Claudio Calcagno (2018), *Exclusionary Practices: The Economics of Monopolisation and Abuse of Dominance*, Cambridge University Press.

[11] Greenlee, Patrick, David Reitman, and David S. Sibley (2008), "An Antitrust Analysis of Bundled Loyalty Discounts," *International Journal of Industrial Organization*, 26: 1132-52.

[12] Katz, Michael and Carl Shapiro (1986), "Technology Adoption in the Presence of Network Externalities," *Journal of Political Economy*, 94: 822-41.

[13] Klein, Tobias J., Madina Kurmangaliyeva, Jens Prüfer and Patricia Prüfer (2023), "How Important Are User-Generated Data For Search Result Quality?," working paper.

[14] Morris, Stephen and Hyun Song Shin (2010), "Global Games: Theory and Applications," in *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress*, edited by Mathias Dewatripont, Lars Peter Hansen and Stephen J. Turnovsky, Cambridge University Press, 56 - 114.

[15] Peitz, Martin (2008), "Bundling May Blockade Entry," *International Journal of Industrial Organization*, 26: 41-58.

[16] Schaefer, Maximilian and Geza Sapi (2020), "Learning from Data and Network Effects: The Example of Internet Search," DIW Berlin Discussion Paper No. 1894.

[17] Whinston, Michael D. (1990), "Tying, Foreclosure and Exclusion," *American Economic Review*, 80: 837-59.

# Appendix

## Proof of Proposition 1

As we noted in the main text, a positive measure of consumers in set II must be buying the bundle in any equilibrium and all consumers in set III must be doing so (since there are no separate sales of $A$). The set II consumers buying the bundle must also be the ones with the highest values for product $A$. Moreover, in a Pareto-undominated NE consumer response to prices that involve sales of the bundle, all lower-type consumers not purchasing the bundle must be making the same choice between product $B1$ and product $B2$: otherwise, all consumers not purchasing the bundle are indifferent between $B1$ and $B2$, implying that there is a Pareto-superior NE consumer response in which all consumers not purchasing the bundle buy $B1$, resulting in larger network benefits for all consumers.

In the remainder of the proof, we again denote by $\widetilde{X}$ the consumer type that is indifferent between the bundle and $B2$ when consumer types above $\widetilde{X}$ buy the bundle and those below $\widetilde{X}$ buy $B2$, given by $\Psi(\widetilde{X}) = 0$ (see (7)). We denote by $\widetilde{x}$ the consumer type that is indifferent between the bundle and $B1$ when consumer types above $\widetilde{x}$ buy the bundle and those below $\widetilde{x}$ buy $B1$. That is, $\widetilde{x}$ satisfies

$$\widetilde{x} + v_1 - \widehat{P} = v_1 - p_{B1}$$

or equivalently

$$\widetilde{x} = \widehat{P} - p_{B1}$$

We next argue that no sales of $B2$ occur in equilibrium. To see this, suppose instead that all of the consumers who do not buy the bundle buy $B2$ and that $\widetilde{X}^*$ is the cut-off consumer type in the equilibrium. In that case, we can show that firm 1 would have a profitable deviation to undercut firm 2's sales of $B2$, switching firm 2's customers to product $B1$.

To establish this fact, note first that if given $p_{B2}$ we have $\widehat{P} \leq \widehat{P}(0|p_{B2})$, it follows immediately that firm 2 makes no sales. So suppose instead that $\widehat{P} > \widehat{P}(0|p_{B2})$ and consider the consumer responses for various prices of $p_{B1}$ when firm 1 offers both the bundle and $B1$ for sale with a bundle price of $\widehat{P}$ and price for $B1$ of $p_{B1} \geq 0$:

- $\mathbf{p_{B1} < p_{B2} + \beta(1 - 2G(\widetilde{X}^*)) - \Delta}$: Since all consumer types above $\widetilde{X}^*$ prefer the bundle over $B2$ by iterated dominance (see footnote 25), even if all other consumer types below $\widetilde{X}^*$ buy $B2$ the payoff to a consumer whose type is below $\widetilde{X}^*$ from buying $B1$

$(v_1 + \beta(1 - G(\widetilde{X}^*)) - p_{B1})$ exceeds the payoff from buying $B2$ $(v_2 + \beta G(\widetilde{X}^*) - p_{B2})$.[42] Thus, the unique NE consumer response has all such consumers buy $B1$.

- $\mathbf{p_{B1}} \in [\mathbf{p_{B2}} + \beta(\mathbf{1} - \mathbf{2G}(\widetilde{\mathbf{X}}^*)) - \boldsymbol{\Delta}, \mathbf{p_{B2}} + \beta(\mathbf{1} - \mathbf{G}(\widetilde{\mathbf{X}}^*)) - \boldsymbol{\Delta})$: In this case there is a NE consumer response in which all consumer types below $\widetilde{X}^*$ buy either $B1$ or the bundle (consumers below a cut-off type $\widetilde{x}$ purchase $B1$) and another NE in which they all buy $B2$. The former NE response gives a larger payoff to all consumers than the latter NE response: in the former NE response, consumer-types below $\widetilde{X}^*$ have a payoff of at least $(v_1 + \beta - p_{B1})$, exceeding the largest possible payoff from buying $B2$ in any NE, $(v_2 + \beta G(\widetilde{X}^*) - p_{B2})$, since no types above $\widetilde{X}^*$ buy $B2$, while all consumer types above $\widetilde{X}^*$ have a larger payoff in the former NE because of greater network benefits. So the former NE consumer response Pareto dominates the one in which types below $\widetilde{X}^*$ buy $B2$.

- $\mathbf{p_{B1}} = \mathbf{p_{B2}} + \beta(\mathbf{1} - \mathbf{G}(\widetilde{\mathbf{X}}^*)) - \boldsymbol{\Delta}$: In this case there is a NE consumer response in which all consumer types below $\widetilde{X}^*$ buy either $B1$ or the bundle, where the type $\widetilde{x}$ that is indifferent between buying the bundle and buying $B1$ satisfies relation (11) in the text.

  There is also another NE consumer response in which all consumer types below $\widetilde{X}^*$ buy $B2$. While these two NE give consumer types below $\widetilde{x}$ the same payoff, consumers with types above $\widetilde{x}$ strictly prefer the former NE response, so the NE in which all consumers buy either the bundle or $B1$ Pareto dominates the one in which types below $\widetilde{X}^*$ buy $B2$.[43]

- $\mathbf{p_{B1}} > \mathbf{p_{B2}} + \beta(\mathbf{1} - \mathbf{G}(\widetilde{\mathbf{X}}^*)) - \boldsymbol{\Delta}$: In this case there is a NE consumer response in which all consumers with types below $\widetilde{X}^*$ buy $B2$ and, by iterated dominance and the definition of $\widetilde{X}^*$, it is the unique consumer response in which no sales of $B1$ occur. If there is another NE consumer response in which some consumers buy $B1$ (these must be consumer types for which $x \leq \widetilde{x}(\widetilde{X}^*) \leq \widetilde{X}^*$ since otherwise these consumers prefer the bundle over purchase of $B1$) the payoff of these consumers would be no greater than $(v_1 + \beta - p_{B1})$, which is strictly less than their payoff in the NE response where all consumer-types below $\widetilde{X}^*$ buy $B2$, $(v_2 + \beta G(\widetilde{X}^*) - p_{B2})$. Hence, the NE response in which all consumer-types below $\widetilde{X}^*$ buy $B2$ is not Pareto dominated. Moreover, because firm 1 still sells the bundle to all consumer types above $\widetilde{X}^*$ and (we show below) $p_{B1} > 0$, firm 1's profits are lower in the NE response in which all consumer-types below $\widetilde{X}^*$ buy $B2$. By our ("stack-the-deck") assumption that consumers coordinate on the worst undominated NE response for firm 1, all consumer types below $\widetilde{X}^*$ would buy $B2$.

[42] Note that the fact that firm 2 is making sales of $B2$ implies that $p_{B2} \leq v_2 + \beta G(\widetilde{X}^*)$ so consumer surplus is non-negative.

[43] If we instead used a criterion of *strict* Pareto dominance so that the NE consumer response in which all consumer types below $\widetilde{X}^*$ buy $B2$ was not dominated, then our selection of the worst equilibrium for firm 1 among undominated consumer responses would create an openness problem when we consider firm 1's optimal prices for $B1$.

A first implication of these consumer responses is that

$$p_{B2} < v_2 + \beta G(\widetilde{X}^*).$$

To see this, note first that the fact that consumers are buying $B2$ implies that $p_{B2} \leq v_2 + \beta G(\widetilde{X}^*)$. Moreover, if instead $p_{B2} = v_2 + \beta G(\widetilde{X}^*)$ then firm 1 would have a profitable deviation that undercuts firm 2 by setting a price for $B1$ satisfying

$$p_{B1} = p_{B2} + \beta(1 - G(\widetilde{X}^*)) - \Delta = v_2 + \beta - \Delta = v_1 + \beta > 0.$$

With this deviation, consumers buy either $B1$ or the bundle, with the indifferent consumer being type $\widetilde{x} < \widetilde{X}^*$ satisfying (11).[44] So the deviation expands sales of the bundle and also makes sales of $B1$ at a strictly positive price.

Next, to show that firm 1 has a profitable deviation that undercuts firm 2 we show that if all lower-type consumers buy $B2$ then in any equilibrium the cut-off type $\widetilde{X}^*$ who is indifferent between the bundle and product $B2$ would have $\widetilde{X}^* \leq \widehat{p}_A^*$. If $\widetilde{X}^* = 0$, this is necessarily the case, so suppose that $\widetilde{X}^* > \widehat{p}_A^* \geq 0$, contrary to our claim. Then for any cut-off type $\widetilde{X} \in [\widetilde{X}^* - \varepsilon, \widetilde{X}^*]$ for some $\varepsilon > 0$, by offering only the bundle for sale at price $\widehat{P}(\widetilde{X}|p_{B2})$ firm 1 can ensure (by iterated dominance[45]) sales of the bundle to all types above $\widetilde{X}$. Thus, it must be that

$$\widetilde{X}^* = \underset{\widetilde{X} \in [\widetilde{X}^* - \varepsilon, \widetilde{X}^*]}{\arg\max} \Pi_1(\widetilde{X}|p_{B2}) \equiv [\widehat{P}(\widetilde{X}|p_{B2}) + \alpha] \cdot (1 - G(\widetilde{X})).$$

Substituting we have:

$$\begin{aligned} \Pi_1(\widetilde{X}|p_{B2}) &= [p_{B2} + \widetilde{X} + \beta(1 - 2G(\widetilde{X})) - \Delta + \alpha](1 - G(\widetilde{X})) \\ &= (\widetilde{X} + \alpha)(1 - G(\widetilde{X})) + [p_{B2} + \beta(1 - 2G(\widetilde{X})) - \Delta](1 - G(\widetilde{X})) \\ &= \Pi_A(\widetilde{X}) + [p_{B2} + \beta(1 - 2G(\widetilde{X})) - \Delta](1 - G(\widetilde{X})) \end{aligned}$$

Observe that by Remark 4 we have $\Pi_1(\widetilde{X}^*|p_{B2}) > \Pi_A(\widetilde{X}^*)$, which implies that

$$[p_{B2} + \beta(1 - 2G(\widetilde{X}^*)) - \Delta] > 0 \tag{14}$$

Then:

$$[d\Pi_1(\widetilde{X}|p_{B2})/d\widetilde{X}]_{\widetilde{X}=\widetilde{X}^*} = \Pi_A'(\widetilde{X}^*) - g(\widetilde{X}^*)[p_{B2} + \beta(1-2G(\widetilde{X}^*)) - \Delta] - 2\beta g(\widetilde{X}^*)(1 - G(\widetilde{X}^*)) < 0$$

where the inequality follows because $\Pi_A'(\widetilde{X}^*) < 0$ when $\widetilde{X}^* > \widehat{p}_A^* \geq 0$ and from (14). Thus,

[44] This follows from the discussion in the main text around (13).

[45] This follows in a similar fashion to the proof of Lemma 1 in which $p_{B2} < v_2 + \beta G(\widetilde{X}^*)$ replaces the assumption that $p_{B2} < v_2$ to ensure that all types $\widetilde{X} \in [\widetilde{X}^* - \varepsilon, \widetilde{X}^*]$ enjoy a positive surplus from purchase of the bundle.

we have a contradiction and we can conclude that $\widetilde{X}^* \leq \widehat{p}_A^*$.

Given the consumer responses we described above, observe that firm 1 can undercut firm 2 and switch the consumers buying $B2$ to all buying $B1$ by offering price $p_{B1} = p_{B2} + \beta(1 - G(\widetilde{X}^*)) - \Delta$. This deviation would be profitable (and would be firm 1's best deviation) because, as above, sales of the bundle expand and sales of $B1$ occur at a positive price since

$$\begin{aligned} p_{B1} &= p_{B2} + \beta[1 - G(\widetilde{X}^*)] - \Delta \\ &\geq \beta - \Delta - 2\beta G(\widehat{p}_A^*) \\ &> 0, \end{aligned}$$

where the first inequality follows because $p_{B2} \geq 0$, $\widetilde{X}^* \leq \widehat{p}_A^*$, and $\beta G(\widehat{p}_A^*) \geq 0$, while the second inequality follows from (9). Thus, we conclude that firm 2 cannot be making any sales of $B2$.

Since firm 1 sets $p_{B1}$ at the highest level such that consumers do not switch to buying $B2$, firm 2 must be setting $p_{B2}^* = 0$ or otherwise it would have a profitable deviation that lowers $p_{B2}$ slightly.

Finally, we identify the equilibrium purchases and prices when firm 1 makes sales of the bundle and also possibly of product $B1$, while firm 2 makes no sales and sets $p_{B2}^* = 0$.

As seen above, when firm 1 charges $\widehat{P}(\widetilde{X}|0) = \widetilde{X} + \beta(1 - 2G(\widetilde{X})) - \Delta$ for the bundle, the most firm 1 can charge for $B1$ and secure low-type consumers' business for $B1$ instead of $B2$ is $p_{B1} = \beta(1 - G(\widetilde{X})) - \Delta$. Among all prices for $B1$ less than or equal to this amount, this price for $B1$ is best for firm 1 since any lower price reduces revenues from sales of $B1$ and shifts sales from the higher-priced bundle to the lower-priced $B1$.

If firm 1 charges this amount, types above the cutoff $\widetilde{x}$ satisfying relation (11) buy the bundle, while lower types buy $B1$. We first observe from (11) that $\widetilde{X} = 0$ implies $\widetilde{x} = 0$ and vice versa and that there is a monotonic relationship between $\widetilde{X}$ and $\widetilde{x}$ as $1 - \beta g(\widetilde{X}) > 0$ from Assumption 1. As defined in the proposition, we denote by $\widetilde{X}(\widetilde{x})$ the level of $\widetilde{X}$ satisfying (11) given $\widetilde{x}$ and we denote by $\widetilde{x}(\widetilde{X})$ its inverse.

We now focus on firm 1's optimal choice of $\widetilde{x}$ which solves

$$\underset{\widetilde{x}}{Max}\ [\alpha + \widetilde{X}(\widetilde{x}) + \beta(1 - 2G(\widetilde{X}(\widetilde{x}))) - \Delta](1 - G(\widetilde{x})) + [\beta(1 - G(\widetilde{X}(\widetilde{x}))) - \Delta]G(\widetilde{x})$$

or equivalently

$$\underset{\widetilde{x}}{Max}\ [\alpha + \widetilde{X}(\widetilde{x}) + \beta(1 - G(\widetilde{X}(\widetilde{x}))) - \beta G(\widetilde{X}(\widetilde{x})) - \Delta)](1 - G(\widetilde{x})) + [\beta(1 - G(\widetilde{X}(\widetilde{x}))) - \Delta]G(\widetilde{x})$$

Using the fact that $\widetilde{x} = \widehat{P} - p_{B1} = \widetilde{X}(\widetilde{x}) - \beta G(\widetilde{X}(\widetilde{x}))$, this becomes

$$\underset{\widetilde{x}}{Max}\ (\alpha + \widetilde{x})(1 - G(\widetilde{x})) + \beta(1 - G(\widetilde{X}(\widetilde{x}))) - \Delta. \tag{15}$$

whose solution is the solution to (10). Observe that the fact that firm 1's equilibrium profit must exceed its independent-pricing profit (Remark 4) implies that we must have

$\beta(1 - G(\widetilde{X}(\widetilde{x}))) > 0$. Hence, we have $\widetilde{X} < \overline{x}$, which (along with the fact that $g(x) > 0$ for all $x$) implies that $\beta(1 - G(\widetilde{X}(\widetilde{x})))$ is strictly decreasing in $\widetilde{x}$. The remaining conclusions of the Proposition follow from the discussion in the main text.

### An Alternative Approach without Any Coordination Assumptions under Tying

To illustrate the robustness of the leverage mechanism with network effects in our model, we show that the same qualitative results can be derived with the use of only iterated dominance under tying. Specifically, we now assume that, when multiple NE consumer responses exist, the consumers' response is the worst such NE for firm 1. Relative to our selection assumption in the main text, this assumption further stacks the deck against firm 1's use of tying: as in the main text, it results in the unique Pareto-undominated NE consumer response under independent pricing, but now the consumer response under tying can be even worse for firm 1 than in the main text (a dominated NE consumer response can result if it yields the lowest profit for firm 1). We demonstrate that profitable market foreclosure with tying is guaranteed even under this more pessimistic scenario against the use of tying by firm 1.

As in the proof of Lemma 2, given $p_{B2} \geq 0$, let us define the bundle price that implements a cut-off type $\widetilde{X} \in [0, \overline{x}]$ between the bundle and *B2* as

$$\widehat{P}(\widetilde{X}|p_{B2}) \equiv p_{B2} + \widetilde{X} + \beta(1 - 2G(\widetilde{x})) - \Delta.$$

Then, all types above $\widetilde{X}$ prefer the bundle over *B2* by iterated dominance. Given that bundle price, the highest price $p_{B1}$ that makes it a *dominant strategy* for a consumer type below $\widetilde{X}$ to purchase *B1* over *B2* is

$$p^D_{B1}(\widetilde{X}|p_{B2}) \equiv p_{B2} + \beta(1 - 2G(\widetilde{X})) - \Delta.$$

At this price, any consumer type below $\widetilde{X}$ prefers *B1* over *B2* even if all other consumers below $\widetilde{X}$ purchase *B2*. Thus, this pricing strategy guarantees the foreclosure of *B2*. Furthermore, from the proof of Lemma 2, we know $\widehat{P}(\widehat{p}^*_A|p_{B2}) > \widehat{p}^*_A$ and $p^D_{B1}(\widehat{p}^*_A|p_{B2}) > 0$: note that $\widehat{P}(\widehat{p}^*_A|p_{B2}) > \widehat{p}^*_A$ is equivalent to $p^D_{B1}(\widehat{p}^*_A|p_{B2}) > 0$. Therefore, firm 2 makes no sales and sets $p_{B2} = 0$ in equilibrium and firm 1 can realize a higher profit than under independent pricing by choosing $\widehat{P} = \widehat{P}(\widehat{p}^*_A|0)$ and $p_{B1} = p^D_{B1}(\widehat{p}^*_A|0)$.[46]

As in the analysis of the main text, given the two prices $\widehat{P}(\widetilde{X}|p_{B2})$ and $p^D_{B1}(\widetilde{X}|p_{B2})$, let $\widetilde{x}(\widetilde{X})$ denote the consumer type that is indifferent between the bundle and $B1$ when all consumers buy the bundle or $B1$. Then, we have $\widetilde{x}(\widetilde{X}) = \widetilde{X}$. When firm 1 implements a

[46]Note that since $\widehat{P}(\widehat{p}^*_A|0) - p^D_{B1}(\widehat{p}^*_A|0) = \widehat{p}^*_A$, firm 1 sells the bundle to all consumer types above $\widehat{p}^*_A$, and $B1$ to those below $\widehat{p}^*_A$.

cutoff $\widetilde{x}$ by prices $\widehat{P}(\widetilde{x}|0)$ and $p^D_{B1}(\widetilde{x}|0)$, its profit is given by

$$\begin{aligned}\Pi_1(\widetilde{x}) &= \underbrace{[\alpha+\widetilde{x}+\beta(1-2G(\widetilde{x}))-\Delta]\,(1-G(\widetilde{x}))}_{\text{Profit from Bundle Sale}}+\underbrace{[\beta(1-2G(\widetilde{x}))-\Delta]\,G(\widetilde{x})}_{\text{Profit from Sale of B1}}\\ &= (\alpha+\widetilde{x})(1-G(\widetilde{x}))+\beta(1-2G(\widetilde{x}))-\Delta\\ &= \Pi_A(\widetilde{x})+\beta(1-2G(\widetilde{x}))-\Delta\end{aligned}$$

As $\beta(1-2G(\widetilde{x}))$ strictly decreases with $\widetilde{x}$, by proceeding in a similar manner as in the proof of Proposition 1, we can establish the following Proposition.

**Proposition 4** *Suppose that firm 2 enjoys favorable beliefs in that whenever there exist multiple NE consumer responses, the worst such response for firm 1 results. If tying is allowed and Assumptions 1 and 2 hold, the unique equilibrium involves firm 1 tying (offering for sale only the bundle and possibly product B1) and fully monopolizing market B. The cut-off type* $\widetilde{x}^*$ *between sales of the bundle and sales of B1 solves*

$$\underset{\widetilde{x}}{Max}\ \Pi_A(\widetilde{x})+\beta(1-2G(\widetilde{x})), \tag{16}$$

*where* $\Pi_A(.)$ *is the market A profit function defined in (3).*

*When there is full coverage under independent pricing, all consumers buy the bundle* ($\widetilde{x}^*=0$) *and when there is not full coverage under independent pricing, the set of types buying the bundle strictly contains the set of types that buy product A under independent pricing (i.e.,* $\widetilde{x}^*<\widehat{p}^*_A$*). The equilibrium prices under tying are:*

*(i) When firm 1 sells only the bundle (i.e.,* $\widetilde{x}^*=0$*), the bundle price is* $P^*=\alpha+(\beta-\Delta)$ *while firm 2 sets* $p^*_{B2}=0$*.*

*(ii) When firm 1 sells both the bundle and* $B1$ *(i.e.,* $\widetilde{x}^*>0$*), prices are*

$$\begin{aligned}P^* &= \alpha+\widetilde{x}^*+\beta(1-2G(\widetilde{x}^*))-\Delta\\ p^*_{B1} &= \beta(1-2G(\widetilde{x}^*))-\Delta\\ p^*_{B2} &= 0.\end{aligned}$$

*Firm 1's equilibrium profit exceeds its profit under independent pricing.*