Can Deficits Finance Themselves? Online Supplement

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July 19, 2024

This document contains further supplemental material for the article "Can Deficits Finance Themselves?". Any references to equations or sections that are not preceded by "E." refer to the main article.

E.1 A sequence-space perspective

All results in the main part of this paper are stated and proved using a standard state-space approach to equilibrium characterization. We can, however, develop some additional insights by instead adopting a sequence-space perspective.

In the context of the paper as a whole the purpose of the sequence-space analysis in this section is twofold. First, by adopting this sequence-space perspective (Auclert et al. (2021)), we will be able to easily substantiate a claim made in Section 6—that intertemporal MPCs fully characterize limiting self-financing equilibria. Second, we provide a different perspective on Assumption 2, re-phrasing it as a sufficient condition ensuring that the intertemporal MPCs decay "sufficiently quickly."

Equilibrium. For the analysis in this section, we substantially generalize the aggregate demand relation (12) to the following linearized sequence-space relation:

$$\boldsymbol{c} = \mathscr{M} \times \left(\boldsymbol{y} - \boldsymbol{t}\right) + \mathscr{M}_i \times \boldsymbol{i} + \mathscr{M}_\pi \times \boldsymbol{\pi} \tag{E.1}$$

where boldface denotes sequences, and $\{\mathscr{M}, \mathscr{M}_i, \mathscr{M}_\pi\}$ are linear maps translating sequences of income (and taxes), nominal rates, and inflation into household consumption demand (e.g., Wolf, 2021a).

Our objective in this section is to shed further light on the possibility of full self-financing in the limit with infinite delay in fiscal adjustment; i.e., we will consider the limiting case of $\tau_d = 0$. We will

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furthermore maintain the assumption of a neutral monetary policy and, for simplicity, assume that prices are rigid ($\kappa = 0$), thus focusing on the tax base channel.[1]

Imposing market-clearing and constant real rates, equation (E.1) becomes

$$\boldsymbol{y} = \mathscr{M} \times \left(\boldsymbol{y} - \boldsymbol{t}\right) \tag{E.2}$$

Now note that, under our assumptions on fiscal policy, taxes are given

$$\boldsymbol{t} = \tau_y \times \boldsymbol{y} - \boldsymbol{\varepsilon} \tag{E.3}$$

where $\boldsymbol{\varepsilon}$ captures the exogenous policy intervention. For our "stimulus check" experiment, this is a vector that has zeros everywhere but in its first entry. Combining (E.2) and (E.3), we find that output in the limiting, full self-financing equilibrium is characterized through the following system of dynamic equations:

$$\boldsymbol{y} = (1 - \tau_y)\mathscr{M} \times \boldsymbol{y} + \mathscr{M} \times \boldsymbol{\varepsilon} \tag{E.4}$$

(E.4) is a variant of the intertemporal Keynesian cross studied previously in Auclert et al. (2023), but with a crucial difference: automatic tax financing is embedded in the tax revenue term $\tau_y \times \boldsymbol{y}$, rather than being specified directly as part of the policy intervention (here $\boldsymbol{\varepsilon}$). This seemingly subtle distinction has important implications and in particular connects tightly with our self-financing results in Sections 4 and 5.2.

Discussion. The above analysis substantiates the claim made in Section 6: for a large family of models (including in particular our spender-OLG hybrid), the matrix of the intertemporal MPCs together with the value of τ_y pin down the dynamics of output in the limiting self-financing equilibrium. It remains to further characterize the solution of (E.4), allowing us to connect with the economic intuitions offered in Sections 4 and 5.2.

The remainder of the discussion here will leverage a crucial property of the intertemporal MPC matrix $\mathscr{M}$. Letting $\boldsymbol{r} \equiv (1, \frac{1}{R^{ss}}, \frac{1}{(R^{ss})^2}, \dots)$, we have that $\boldsymbol{r}' \cdot \mathscr{M}(\bullet, h) = \frac{1}{(R^{ss})^h}$—i.e., every dollar of income is spent at some point. It follows from this property that any solution $\boldsymbol{y}$ of (E.4) necessarily has net present value equal to $\frac{1}{\tau_y}$ times the net present value of the fiscal stimulus:

$$\boldsymbol{r}'\boldsymbol{y} = (1 - \tau_y)\boldsymbol{r}'\mathscr{M} \times \boldsymbol{y} + \boldsymbol{r}'\mathscr{M} \times \boldsymbol{\varepsilon}$$

[1] By an argument analogous to that surrounding Theorem 1, the extension to the partially sticky price case is conceptually straightforward. The only delicate part of the ensuring discussion is that we directly set $\tau_d = 0$, instead of taking the limit as $\tau_d \to 0^+$ from above, or proving the equivalence to $H \to \infty$. These details are of course fully taken care of in our main analysis.

and so from the properties of $\mathscr{M}$ we obtain that indeed

$$\tau_y \times \boldsymbol{r}'\boldsymbol{y} = \boldsymbol{r}'\boldsymbol{\varepsilon}, \tag{E.5}$$

as claimed. Next we note that the solution of (E.4) takes the simple form

$$\boldsymbol{y} = \left[I - (1-\tau_y)\mathscr{M}\right]^{-1} \times \mathscr{M} \times \boldsymbol{\varepsilon} \tag{E.6}$$

where for the purpose of the discussion here we simply assume that the stated inverse exists.[2] Our self-financing results in Theorems 1 and 3 concern the question of whether, as fiscal financing is gradually delayed further and further, we indeed converge to the general self-financing equilibrium characterized by (E.6). As discussed following Theorem 1, the condition required for such convergence to occur is that the Keynesian boom is sufficiently front-loaded, raising all required revenue before fiscal adjustment is ever actually necessary. In (E.6), the "front-loadedness" of the Keynesian boom is entirely governed by the properties of $\left[I - (1-\tau_y)\mathscr{M}\right]^{-1}$: if the off-diagonal entries of $\mathscr{M}$ decay to zero sufficiently quickly along each column, then the same is true for the off-diagonal entries of $\left[I - (1-\tau_y)\mathscr{M}\right]^{-1}$ (e.g., see Bickel and Lindner, 2012). This then ensures that the solution $\boldsymbol{y}$ and thus the debt path $\boldsymbol{d}$ converge to zero, which in turn is what is needed for self-financing to obtain as fiscal adjustment is delayed further and further. For our general aggregate demand relation (30), the condition stated in Assumption 2 is what is needed to ensure that indeed the off-diagonal entries of $\mathscr{M}$ and thus $\left[I - (1-\tau_y)\mathscr{M}\right]^{-1}$ decay to zero sufficiently quickly along each column.

E.2 More on model extensions

We here elaborate on the remaining extensions discussed in Section 5: (i) fiscal adjustment through distortionary taxes; (ii) stimulus in the form of government purchases; (iii) a model with investment; and (iv) more general aggregate demand with our variant fiscal rule.

E.2.1 Distortionary tax hikes

We begin by showing how the equilibrium relations of the model change with fiscal adjustments taking the form of time-varying distortionary taxes. We then discuss implications for our limiting self-financing equilibria.

[2] Our analysis in the main text implies that, for standard models of the consumption-savings problem and if $\tau_y > 0$, then this inverse indeed exists.

Environment. We now replace our baseline fiscal rule (6) with the following alternative:

$$T_{i,t} = \tau_{y,t} Y_{i,t} - \mathscr{E}_t \tag{E.7}$$

where the time-varying distortionary tax rate $\tau_{y,t}$ is given by

$$\tau_{y,t} = \tau_y + \tau_d \frac{D_t - D^{ss}}{Y^{ss}}. \tag{E.8}$$

After log-linearization, this extended model variant maps into exactly the same aggregate demand relation and law of motion for public debt as before; intuitively, in those equations, what matters is only how much tax revenue is extracted from the private sector, not whether it is done in a distortionary or lump-sum way. The only model equation that is affected is the NKPC, where the time-varying distortion in labor supply now manifests itself as a cost-push shock:

$$\pi_t = \kappa(y_t + \zeta d_t) + \beta \mathbb{E}_t [\pi_{t+1}] \tag{E.9}$$

where ζ is a function of model primitives such that $\zeta > 0$ if and only if $\tau_d > 0$. Intuitively, higher debt maps into higher distortionary taxes, and thereby to a higher labor wedge and higher real marginal cost for a given level of output.

To derive (E.9), note that, with time-varying distortionary taxes, the labor supply relation is

$$(1 - \tau_{y,t}) W_t = \frac{\iota L_t^{\frac{1}{\varphi}}}{\int_0^1 C_{i,t}^{-1/\sigma} di} \tag{E.10}$$

Log-linearizing, we find that

$$w_t - \frac{1}{\sigma} c_t - \frac{1}{1-\tau_y} \widehat{\tau}_{y,t} = \frac{1}{\varphi} \ell_t \tag{E.11}$$

where $\widehat{\tau}_t \equiv \tau_{y,t} - \tau_y$. Next note that the firm optimal pricing relationship is still

$$\pi_t = \tilde{\kappa} w_t + \beta \mathbb{E}_t [\pi_{t+1}] \tag{E.12}$$

Combining (E.11), (E.12), and the modified baseline fiscal (E.8), we obtain

$$\pi_t = \kappa \left(y_t + \underbrace{\frac{\tau_d}{\left(\frac{1}{\varphi} + \frac{1}{\sigma}\right)\left(1 - \tau_y\right)}}_{\equiv \zeta} d_t \right) + \beta \mathbb{E}_t [\pi_{t+1}], \tag{E.13}$$

where $\kappa = \tilde{\kappa}\left(\frac{1}{\varphi} + \frac{1}{\sigma}\right)$.

Self-financing results. This modification of the model has very limited effect on our headline results. First, if $\kappa = 0$, then nothing changes, and in particular the entirety of Theorem 1 continues to hold. Second, even if $\kappa > 0$, then nothing of essence changes: as we have already emphasized, our characterization of the equilibrium dynamics of y_t and d_t is robust to the specification of the NKPC, and thus in particular to the presence of additional cost-push shocks—all that changes is the split between tax base and inflation self-financing. Finally, in the limit of interest (i.e., as $\tau_d \to 0$), those cost-push terms actually vanish, and so the second part of Theorem 1 remains intact even if $\kappa > 0$. The economics of this limit case are transparent: because our original self-financing limit result guarantees that fiscal adjustment is never needed in equilibrium, it is immaterial whether the adjustment *would have been* distortionary or lump-sum.

E.2.2 Government spending

The only change relative to our baseline economy is that the government now consumes some amount G_t of the final good. We assume that G_t is a stochastic, mean-zero spending shock, and we also shut down the lump-sum transfers featured in our baseline analysis. The linearized government budget constraint becomes

$$d_{t+1} = \frac{1}{\beta}\left[d_t - t_t + g_t + \beta\frac{D^{ss}}{Y^{ss}}(i_t - \pi_{t+1}),\right] \tag{E.14}$$

where $g_t = \frac{G_t}{Y^{ss}}$. We next specify taxes as follows:

$$t_t = \tau_d \cdot (d_t + (1-\tau_y)g_t) + \tau_y y_t, \tag{E.15}$$

where the presence of $(1-\tau_y)$ in front of g_t ensures that $\tau_d = 1$ again corresponds to a period-by-period balanced budget.[3] Finally, the aggregate output market-clearing condition is replaced by

$$y_t = c_t + g_t. \tag{E.16}$$

By standard arguments (e.g., see Galí, 2008), the adjusted NKPC is now given as

$$\pi_t = \kappa y_t + \beta\mathbb{E}_t[\pi_{t+1}] - \kappa\frac{\frac{1}{\sigma}}{\frac{1}{\varphi}+\frac{1}{\sigma}}g_t$$

[3]To see this, one can guess and verify that, under the above specification, $\tau_d = 1$ translates in equilibrium to $y_t = g_t$, $t_t = g_t$, and $d_t = 0$ for all t (i.e., the fiscal multiplier is one, the primary surplus is zero, and debt stays in steady state).

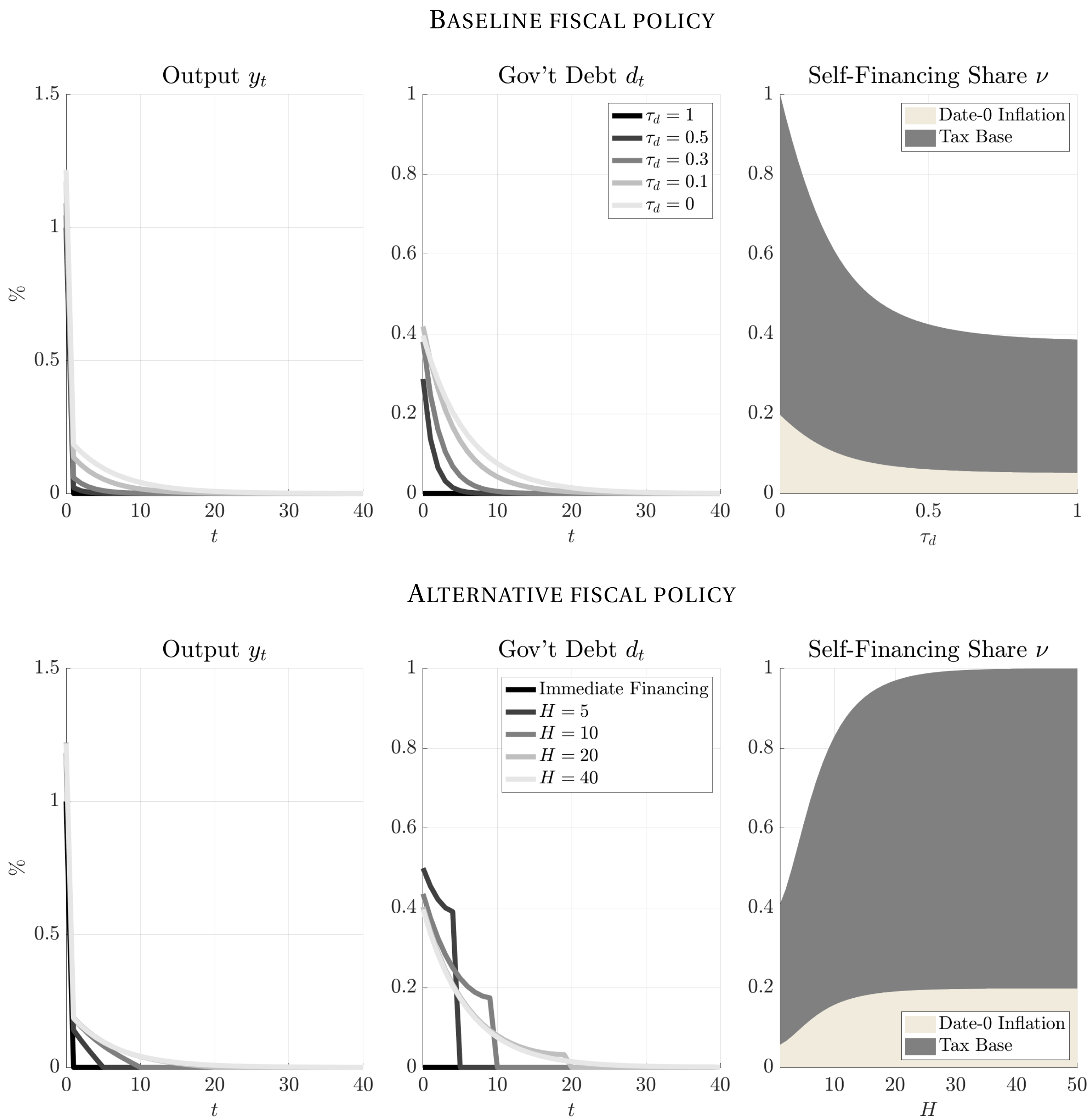

Figure E.1: Top panel: impulse responses of output y_t, government debt d_t, and the total self-financing share ν to a government spending shock ε_0 equal to one per cent of steady-state output, as a function of τ_d. Bottom panel: same as above, but as a function of H.

Intuitively, the last term reflects the fact that, if higher output comes from higher government purchases (rather than higher consumption), then household labor supply is larger for standard wealth effect reasons—i.e., a negative cost-push shock. Since the overall analytics of the self-financing result with government purchases are analogous to our baseline "stimulus checks" case, we do not repeat those derivations here and instead just provide a visual illustration of the self-financing result.

We summarize our results in Figure E.1—the government spending analogue of Figure 1. We emphasize two main takeaways. First, as $\tau_d \to 0$ or $H \to \infty$, we indeed again converge to a full self-financing limit. Second, even immediately tax-financed fiscal purchases actually have a positive spending multiplier, and thus the share of self-financing ν for $\tau_d = 1$ (top panel) and $H = 0$ (bottom panel) is already strictly positive.

E.2.3 Investment

This section provides the missing details on the extension to models with investment discussed in Section 5.3. We begin by stating the (linearized) equations of the extended model before then characterizing its equilibrium.

Model equations. The household block changes very little. Households still receive labor income and dividends; we now denote this total household income by e_t (which in equilibrium will be equal to total household consumption rather than total aggregate income). The linearized household demand relation is now

$$c_t = \left(1-\beta\omega\right)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}\left(\beta\omega\right)^k\left(e_{t+k}-t_{t+k}\right)\right]\right) - \gamma\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k r_{t+k}\right], \tag{E.17}$$

while labor supply still satisfies

$$\frac{1}{\varphi}\ell_t = w_t - \frac{1}{\sigma}c_t \tag{E.18}$$

The firm block on the other hand changes materially relative to our baseline model. Since this production side is entirely standard our discussion here will be brief and only present linearized optimality conditions; a detailed discussion of an almost identical model is offered in Wolf (2021b). The production sector consists of three parts: perfectly competitive intermediate goods producers who accumulate capital and hire labor on spot markets; monopolistically competitive retailers who purchase the intermediate good and costlessly differentiate it, subject to nominal rigidities; and a competitive final goods aggregator. Profits of the corporate sector as a whole are returned to households, subject to the time-invariant tax τ_y. The relevant equilibrium relations follow from the behavior of the intermediate goods producers and the retailers.

1. *Intermediate goods producers.* The production function takes a standard Cobb-Douglas form with capital share α, and capital depreciates at rate δ. We let p_t^I denote the real relative price of the intermediate good. Optimal labor demand gives the static relation

$$w_t = p_t^I + \alpha k_{t-1} - \alpha\ell_t \tag{E.19}$$

while optimal capital accumulation gives[4]

$$\frac{1}{\beta}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right) = \left(\frac{1}{\beta} - 1 + \delta\right) \times \mathbb{E}_t\left[p^I_{t+1} + (\alpha - 1)k_t + (1-\alpha)\ell_{t+1}\right] \tag{E.20}$$

By our assumptions on the production function total output is given as

$$y_t = \alpha k_{t-1} + (1-\alpha)\ell_t \tag{E.21}$$

and finally investment x_t satisfies

$$x_t = \frac{1}{\delta}\left(k_t - (1-\delta)k_{t-1}\right) \tag{E.22}$$

2. *Retailers.* Optimal price-setting as usual relates real marginal costs—here the relative price of the intermediate good, p^I_t—to aggregate inflation:

$$\pi_t = \kappa p^I_t + \beta\mathbb{E}_t\left[\pi_{t+1}\right] \tag{E.23}$$

Aggregating dividend payments from intermediate goods producers and retailers, we obtain (in levels)

$$Q_t = Y_t - W_t L_t - X_t \tag{E.24}$$

which implies that total household income E_t (in levels) is given as

$$E_t = W_t L_t + Q_t = Y_t - X_t \tag{E.25}$$

Aggregate output market-clearing dictates that

$$y_t = \frac{C^{ss}}{Y^{ss}} c_t + \frac{X^{ss}}{Y^{ss}} x_t \tag{E.26}$$

Finally we return to the government. The monetary rule (9) and the government budget constraint (5) are unchanged. The fiscal policy rules (7) or (8) are also unchanged up to the tax base revenue term: since the government taxes labor and dividend income, this term now equals $\tau_y \times E_t$.

Equilibrium characterization. Our key building block result is that we can reduce the equilibrium of this extended model to a system of equations almost as simple as that of our baseline model in Sec-

[4]Adjustment costs on the capital stock or investment flows would complicate this relation but not affect any of our subsequent arguments.

tion 2. First, combining market-clearing and the policy rules with private-sector demand we obtain

$$c_t = \mathscr{F}_1 \cdot (d_t + \varepsilon_t) + \mathscr{F}_2 \cdot \mathbb{E}_t \left[(1-\beta\omega) \sum_{k=0}^{\infty} (\beta\omega)^k c_{t+k} \right]. \tag{E.27}$$

Relative to our baseline model, the only change is that this equilibrium demand relationship is in aggregate consumption c_t rather than aggregate output y_t. We emphasize that this is possible precisely because the government taxes dividend and labor income, which as discussed above in equilibrium is equal to total consumption. Second, the law of motion for aggregate debt is now

$$d_{t+1} = \beta^{-1} \left(d_t + \varepsilon_t - \underbrace{\tau_d \cdot (d_t + \varepsilon_t)}_{\text{fiscal adjustment}} - \underbrace{\tau_y c_t}_{\text{tax base}} \right) - \underbrace{\frac{D^{ss}}{Y^{ss}} (\pi_{t+1} - \mathbb{E}_t [\pi_{t+1}])}_{\text{debt erosion}}, \tag{E.28}$$

with real debt at date 0 given as

$$d_0 = b_0 - \frac{D^{ss}}{Y^{ss}} (\pi_0 - E_{-1} [\pi_0]) = -\frac{D^{ss}}{Y^{ss}} \pi_0. \tag{E.29}$$

Again, relative to the baseline model, the only change is that now it is aggregate consumption rather than aggregate output appearing in (E.28).

We note that (E.27) - (E.28) is a system in $\{c_t, d_t\}_{t=0}^{\infty}$ that depends on the rest of the economy—and so in particular the investment block—*only* through the presence of π_0. π_0 on the other hand can be obtained as a function of the consumption path $\{c_t\}_{t=0}^{\infty}$ by solving the system (E.18), (E.19), (E.20), (E.21), (E.22), (E.23) and (E.26) given consumption, and with the monetary policy rule (9) imposed. We write this function as

$$\pi_0 = \Pi_0 \left(\{c_t\}_{t=0}^{\infty} \right) \tag{E.30}$$

The equilibrium described by equations (E.27) - (E.30) is straightforward to characterize given our earlier analysis of the model without investment in Sections 2 and 4. We begin with the case of perfectly rigid prices ($\kappa = 0$), and for simplicity restrict attention to the limiting self-financing case ($\tau_d \to 0$ or $H \to \infty$). In that case $\pi_0 = 0$, so we can focus on the bivariate system (E.27) - (E.28) in $\{c_t, d_t\}_{t=0}^{\infty}$. Crucially, this system is *exactly* the same as that covered in Theorem 1, so the equilibrium characterization underlying that result applies unchanged, with c_t replacing y_t.

We now turn to the case of general κ. To this end let $c_{t,0}$ denote the solution of the rigid-price system, and furthermore let $p^I_{t,0}$ denote the corresponding equilibrium intermediate goods price obtained by solving the system (E.18), (E.19), (E.20), (E.21), (E.22), and (E.26) for p^I given $\{c_{t,0}\}_{t=0}^{\infty}$. Proceeding analogously to the proof of Theorem 1, we will now construct the equilibrium for general κ by simply scaling the $\kappa = 0$ equilibrium. To this end conjecture that equilibrium consumption satisfies $a \times c_{t,0}$, for some scalar a. It is then immediate that then we would also have $p_t^I = a \times p_{t,0}^I$. But then, from (E.23), we have that

$$\pi_0 = a \times \kappa \times \sum_{t=0}^{\infty} \beta^t p_{t,0}^I \tag{E.31}$$

Finally it follows from the government budget constraint that—again in our limiting self-financing equilibrium—we must have

$$\varepsilon_0 = a \times \tau_y \times \sum_{t=0}^{\infty} \beta^t c_{t,0} + a \times \frac{D^{ss}}{Y^{ss}} \times \kappa \times \sum_{t=0}^{\infty} \beta^t p_{t,0}^I$$

Solving this equation for a we obtain consumption and thus inflation as well as government debt in the general sticky-price equilibrium. In particular we see that self-financing yet again obtains exactly as in our baseline economy. We summarize these observations in the following corollary.

Corollary 1. *Consider the extended OLG-NK environment with investment. Full self-financing obtains as fiscal adjustment is indefinitely delayed.—that is, $\nu \to 1$—if the tax response is infinitely delayed, i.e., $\tau_d \to 0$ or $H \to \infty$. These two limits induce the same equilibrium paths $\{c_t, \pi_t, d_t\}_{t=0}^{\infty}$, and in this common limit, self-financing is sufficiently strong to return real government debt to steady state (i.e., $\lim_{k\to\infty} \mathbb{E}_t [d_{t+k}] \to 0$ or $\lim_{H\to\infty} \mathbb{E}_0 [d_H] \to 0$).*

E.2.4 General aggregate demand under the variant fiscal policy (8)

We here prove the analogue of Theorem 3 for the variant fiscal policy rule (8). Our arguments require the additional technical assumption that $M_d \geq \frac{\delta\omega M_y}{1-(1-\delta\omega)M_y}$. This additional restriction is sufficient to materially simplify the argument, and it is satisfied by all model variants discussed in the main text, including the one entertained in our quantitative analysis (i.e., the OLG-spender hybrid).[5]

Proof. With market clearing $c_t = y_t$, we first write the aggregate demand in (30) recursively

$$\begin{aligned}
y_t &= \frac{M_d}{1-M_y} d_t - \frac{M_y}{1-M_y} t_t + \delta\beta\omega \frac{M_y}{1-M_y} \mathbb{E}_t \left[y_{t+1} - t_{t+1} \right] + \beta\omega \mathbb{E}_t \left[y_{t+1} - \frac{M_d}{1-M_y} d_{t+1} + \frac{M_y}{1-M_y} t_{t+1} \right] \\
&= \frac{M_d}{1-M_y} d_t - \frac{M_y}{1-M_y} t_t + \delta\beta\omega \frac{M_y}{1-M_y} \mathbb{E}_t \left[y_{t+1} - t_{t+1} \right] + \beta\omega \mathbb{E}_t \left[y_{t+1} + \frac{M_y}{1-M_y} t_{t+1} \right] - \omega \frac{M_d}{1-M_y} (d_t - t_t) \\
&= \frac{M_d (1-\omega)}{1-M_y} d_t - \frac{M_y - \omega M_d}{1-M_y} t_t + \beta\omega \left(\frac{1-(1-\delta) M_y}{1-M_y} \right) \mathbb{E}_t \left[y_{t+1} \right] + \beta\omega \frac{M_y}{1-M_y} (1-\delta) \mathbb{E}_t \left[t_{t+1} \right].
\end{aligned} \tag{E.32}$$

[5] The restriction is not, however, necessary. The detailed discussion (which reveals that Theorem 3 holds generically under our variant rule (8)) is available upon request.

From (8), we know that $t_t = d_t$ for all $t \geq H$. As a result, $d_{t+1} = 0$ for all $t \geq H$. Similar to the argument in Appendix A.3, we can then focus on the case that $y_t = d_t = 0$ for $t \geq H+1$. At $t = H$, from (E.32), we have

$$y_H = \frac{-\left(M_y - M_d\right)}{1 - M_y} d_H = \chi_0 d_H \text{ with } \chi_0 = \frac{-\left(M_y - M_d\right)}{1 - M_y}. \tag{E.33}$$

Similar to the main analysis in Appendix A.3, we will now use (E.32) to find the equilibrium path of $\{y_t, d_t\}_{t=0}^{H-1}$ through backward induction. At $t = H-1$, from (8) and (E.32),

$$\begin{aligned}
y_{H-1} &= \frac{\frac{M_d(1-\omega)}{1-M_y}}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y}} d_{H-1} + \beta\omega \frac{\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y}} d_H \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y}}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y}} d_{H-1} + \omega \frac{\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y}} \left(d_{H-1} - \tau_y y_{H-1}\right) \\
y_{H-1} &= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left[\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}\right]}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y} + \omega\tau_y\left[\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}\right]} d_{H-1} \\
&= \chi_1 d_{H-1},
\end{aligned} \tag{E.34}$$

with

$$\begin{aligned}
\chi_1 &= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left[\frac{-\delta M_y(1-M_d)+M_d(1-M_y)}{(1-M_y)^2}\right]}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\tau_y\left[\frac{-\delta M_y(1-M_d)+M_d(1-M_y)}{(1-M_y)^2}\right]} \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} + \frac{\omega M_y}{1-M_y}\left(\frac{M_d}{M_y} - \delta\frac{1-M_d}{1-M_y}\right)}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \frac{\omega M_y}{1-M_y}\left(\frac{M_d}{M_y} - \delta\frac{1-M_d}{1-M_y}\right)\tau_y}.
\end{aligned} \tag{E.35}$$

From $M_y \in (0,1)$, $M_d \in (0,1)$, and $M_d \in \left[\frac{\delta\omega M_y}{1-(1-\delta\omega)M_y}, M_y\right]$, we know that $\frac{M_d(1-\omega)}{1-M_y} + \frac{\omega M_y}{1-M_y}\left(\frac{M_d}{M_y} - \delta\frac{1-M_d}{1-M_y}\right) \geq 0$ and $\frac{\frac{M_d(1-\omega)}{1-M_y} + \frac{\omega M_y}{1-M_y}\left(\frac{M_d}{M_y} - \delta\frac{1-M_d}{1-M_y}\right)}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \frac{\omega M_y}{1-M_y}\left(\frac{M_d}{M_y} - \delta\frac{1-M_d}{1-M_y}\right)\tau_y} \geq 0$. As a result, $\chi_1 \geq 0$.

For $1 \le t \le H-2$, from (8) and (E.32),

$$\begin{aligned}
y_t &= \frac{\frac{M_d(1-\omega)}{1-M_y} d_t}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \beta\omega \frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} \mathbb{E}_t\left[y_{t+1}\right] \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} d_t}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \omega \frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} \left(d_t - \tau_y y_t\right)\chi_{H-t-1} \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-t-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-t-1}} d_t \\
&= \chi_{H-t} d_t \text{ with } \chi_{H-t} = \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-t-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-t-1}}
\end{aligned} \tag{E.36}$$

Finally, for $t = 0$, from (8) and (E.32), we know

$$\begin{aligned}
y_0 &= \frac{\frac{M_d(1-\omega)}{1-M_y} d_0 + \frac{M_y-\omega M_d}{1-M_y}\varepsilon_0}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \beta\omega \frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} \mathbb{E}_0\left[y_1\right] \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} d_0 + \frac{M_y-\omega M_d}{1-M_y}\varepsilon_0}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \omega \frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} \left(d_0 + \varepsilon_0 - \tau_y y_0\right)\chi_{H-1} \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}} d_0 + \frac{\frac{M_y-\omega M_d}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}} \varepsilon_0 \\
&= \chi_H d_0 + \chi_H^{\varepsilon}\varepsilon_0 \text{ with } \chi_H = \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}},
\end{aligned} \tag{E.37}$$

and $\chi_H^{\varepsilon} = \frac{\frac{M_y-\omega M_d}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}}$. Define

$$g\left(\chi\right) \equiv \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi} = \frac{1}{\tau_y} - \frac{\frac{M_y-M_d}{1-M_y} + \frac{1}{\tau_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi}. \tag{E.38}$$

From (E.36) and (E.37) we have $\chi_k = g\left(\chi_{k-1}\right)$ for all $k \in \{2, \cdots, H\}$. We first find the fixed point of $g\left(\chi\right)$:

$$\chi_{MSV} = \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{MSV}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV}}, \tag{E.39}$$

which is equivalent to

$$\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV}^2+\chi_{MSV}\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\right)-\frac{M_d(1-\omega)}{1-M_y}=0. \tag{E.40}$$

Let $\chi_{MSV,1}$ denote the smaller root and $\chi_{MSV,2}$ denote the larger root:

$$\chi_{MSV,1}=\frac{-\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)-\sqrt{\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)^2+4\frac{M_d(1-\omega)}{1-M_y}\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}}{2\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}$$
$$\chi_{MSV,2}=\frac{-\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)+\sqrt{\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)^2+4\frac{M_d(1-\omega)}{1-M_y}\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}}{2\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}. \tag{E.41}$$

If Assumption 1 holds ($\omega<1$), we know that $\chi_{MSV,1}\chi_{MSV,2}<0$ so $\chi_{MSV,1}<0$ and $\chi_{MSV,2}>0$. Note that $1+\frac{M_y-\omega M_d}{1-M_y}\tau_y+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV,1}>0$,[6] we have $g(\chi)>\chi$ if $\chi\in\left(\chi_{MSV,1},\chi_{MSV,2}\right)$ and $g(\chi)<\chi$ if $\chi\in(\chi_{MSV,2},+\infty)$. From (E.38), we also know that $g(\chi)$ increases if $\chi\in[\chi_{MSV,1},+\infty)$. Moreover, from above, we know that $\chi_1\geq 0>\chi_{MSV,1}$. Together with the aforementioned property of $g(\chi)$, we know that $\{\chi_k\}_{k=0}^{\infty}$ is a bounded, monotonic sequence converging to $\lim_{k\to+\infty}\chi_k=\chi_{MSV,2}>0$.

If Assumptions 1 and 2 hold, $\chi_{MSV,2}\in\left(\frac{1-\beta}{\tau_y},\frac{1}{\tau_y}\right)$. To see this, define the left-hand side of (E.40) as

$$h(\chi)=\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi^2+\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\right)\chi-\frac{M_d(1-\omega)}{1-M_y}.$$

We have

$$\begin{aligned}
h\left(\frac{1-\beta}{\tau_y}\right)&=\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\frac{(1-\beta)^2}{\tau_y}+\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\right)\frac{1-\beta}{\tau_y}-\frac{M_d(1-\omega)}{1-M_y}\\
&=-\beta\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\frac{1-\beta}{\tau_y}+\left(1+\frac{M_y}{1-M_y}\tau_y\right)\frac{1-\beta}{\tau_y}-\frac{(1-\beta\omega)M_d}{1-M_y}\\
&<-\beta\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\frac{1-\beta}{\tau_y}+\left(1+\frac{M_y}{1-M_y}\tau_y-\frac{(1-\beta\omega)\left[1-\left(1-\tau_y\right)M_y\left(1+\delta\frac{\beta\omega}{1-\beta\omega}\right)\right]}{1-M_y}\right)\frac{1-\beta}{\tau_y}\\
&=-\beta\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\frac{1-\beta}{\tau_y}+\beta\omega\left(\frac{1-\left(1-\tau_y\right)M_y(1-\delta)}{1-M_y}\right)\frac{1-\beta}{\tau_y}=0,
\end{aligned}$$

[6] If $1+\frac{M_y-\omega M_d}{1-M_y}\tau_y+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV,1}<0$, $\frac{M_d(1-\omega)}{1-M_y}+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{MSV,1}<0$, and $\chi_{MSV,1}>0$ from (E.39), a contradiction.

and

$$h\left(\frac{1}{\tau_y}\right) = \omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\frac{1}{\tau_y} + \left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y - \omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\right)\frac{1}{\tau_y} - \frac{M_d(1-\omega)}{1-M_y}$$
$$= \left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y\right)\frac{1}{\tau_y} - \frac{M_d(1-\omega)}{1-M_y} > 0.$$

Similar to (A.9),

$$\mathbb{E}_0[d_t] = \frac{1}{\beta^{t-1}}\Pi_{j=1}^{t-1}\left(1-\tau_y\chi_{H-j}\right)\mathbb{E}_0[d_1].$$

Since $\lim_{k\to+\infty}\chi_k = \chi_{MSV,2} \in \left(\frac{1-\beta}{\tau_y}, \frac{1}{\tau_y}\right)$, we know that $\lim_{H\to\infty}\mathbb{E}_0[d_H] \to 0$. From (21) and (22), we also know that $\nu \to 1$ as $H \to \infty$. This finishes the proof of Theorem 3 with the alternative fiscal policy (8).

□

E.3 Properties of the consumption function

We here formalize our claims in Section 4.3 on the discounting and front-loading properties of aggregate consumer demand.

Lemma E.1. *Let $\mathcal{M}$ denote the the matrix of intertemporal MPCs corresponding to our consumption function* (12)*. Then, if and only if $\omega < 1$:*

1. As ℓ increases, one unit anticipated income changes at date $t+\ell$ (in terms of present value at t) have a vanishing effect on consumer demand at date t:

$$\lim_{\ell\to\infty}\beta^{-\ell}\mathcal{M}_{t,t+\ell} = 0$$

2. As ℓ increases, one unit income changes at date t have a vanishing effect on consumer demand at date $t+\ell$:

$$\lim_{\ell\to\infty}\mathcal{M}_{t+\ell,t} = 0$$

We prove the two parts of the lemma in turn. The proof leverages results on the properties of the intertemporal MPC matrix $\mathcal{M}$ in OLG models from Wolf (2021a).

1. The proof is by induction. First of all we have

$$\mathcal{M}_{0,\ell}\beta^{-\ell} = (1-\beta\omega)\omega^{\ell}$$

   Thus the claim holds for $t=0$. Now suppose the claim holds for some $t-1$ (where $t\geq 1$), and consider horizon t. Here we have, for $\ell \geq 0$,

$$\mathscr{M}_{t,t+\ell}\beta^{-\ell} = -(1-\beta\omega)^2\beta^{t-1}\omega^{2t+\ell-1} + \mathscr{M}_{t-1,t-1+\ell}\beta^{-(\ell-1)}\beta^{-1}$$

As $\ell \to \infty$ the first term converges to zero since $\omega < 1$ while the second term converges to zero by the inductive assumption, completing the argument.

2. The proof is again by induction. Begin again with $t = 0$. Here we have

$$\mathscr{M}_{\ell,0} = (1-\beta\omega)\omega^{\ell}$$

and so the statement holds. Now suppose it holds for some $t-1$ (where $t \geq 1$), and consider horizon t. Here we have, for $\ell \geq 0$,

$$\mathscr{M}_{t+\ell,t} = -(1-\beta\omega)^2\beta^{t-1}\omega^{2t+\ell-1} + \mathscr{M}_{t-1+\ell,t-1}$$

The first term converges to zero as $\ell \to \infty$, for any t. The second term furthermore also converges to zero (by the inductive hypothesis), completing the argument.

E.4 Empirical evidence on fiscal adjustment

Notable prior work that has estimated fiscal financing rules and thus in particular the speed of fiscal adjustment in response to deficits includes Galí et al. (2007), Bianchi and Melosi (2017), and Auclert and Rognlie (2020). Auclert et al. (2020) (Appendix D.1) survey this literature and conclude that the *annual* tax adjustment parameter—ψ in their notation—lies between 0.015 and 0.3, with their preferred estimate equal to 0.1. Our displayed values for τ_d correspond to the quarterly analogues of these values. We note that all of our values strictly exceed $\bar{r}$ and thus correspond to "passive" fiscal rules in the terminology of Leeper (1991).

E.5 Alternative calibration approaches for the household block

For our baseline analysis in Section 6 we discipline our model's departure from permanent-income behavior by requiring consistency with empirical evidence on the *level* and *slope* of (short-run) household consumption behavior following lump-sum income receipt, as in Auclert et al. (2023) and Wolf (2021a). We here discuss two different approaches: one based on farther-out spending responses, and one based on long-run interest rate elasticities of household asset demand.

Calibration via tail MPCs. This alternative calibration strategy was discussed briefly in Section 6.2: the generalized three-type model is parameterized to match the five-year cumulative MPC path as well as possible, in a standard least-squares sense. For this three-type model, we set $\omega_1 = 0.97$ and $\omega_2 = 0.83$, with the fractions equal to $\chi_1 = 0.22$ and $\chi_2 = 0.63$ (and both groups holding government bonds). The residual fraction $1-\chi_1-\chi_2$ are hand-to-mouth. All other model parameters are as before.

Calibration via asset demand elasticities. For this approach we combine evidence on level MPCs with long-run interest rate elasticities of household asset demand. This calibration strategy is promising because models with permanent-income savers invariably imply a (counterfactual) infinite interest rate elasticity of household asset demand (e.g., see Kaplan and Violante, 2018).

Our main building block result for this calibration approach is Proposition E.1. We there express the long-run elasticity of household asset demand as a function of model primitives.

Proposition E.1. *Consider the spender-OLG hybrid model. Let η denote the long-run interest rate elasticity of household asset demand—that is, the long-run response of asset demand to a permanent change in real interest rates. It is given as*

$$\eta \;=\; (1-\mu)\times\frac{\sigma}{1-\beta}\times\left(\frac{1}{1-\omega}-\frac{1}{1-\beta\omega}\right) \tag{E.42}$$

Proof. We note that the proof heavily leverages results from Wolf (2021a). Following that paper, all arguments are established using sequence-space notation, with boldface denoting time paths.

The sequence of wealth holdings associated with an interest rate sequence $\boldsymbol{r}$ (both in deviation from steady state) is given as

$$\boldsymbol{d}(\boldsymbol{r}) = \mathscr{D}_r \times \boldsymbol{r}$$

where $\mathscr{D}_r$ is the sequence-space Jacobian of wealth holdings with respect to interest rates. The desired long-run elasticity η is the long-run response of asset holdings to a permanent change in interest rates; that is, it is given as the limit (if it exists) of the sequence $\boldsymbol{d}(\boldsymbol{1})$.

It follows from the aggregate household budget constraint that the savings matrix $\mathscr{D}_r$ and the analogous consumption matrix $\mathscr{M}_r$ are related as[7]

$$\mathscr{M}_r + \frac{1}{R^{ss}}\mathscr{D}_r = \begin{pmatrix}\boldsymbol{0}' \\ \mathscr{D}_r\end{pmatrix} \tag{E.43}$$

[7] Note that this construction removes income effects related to steady-state wealth holdings.

where $R^{ss} = \beta^{-1}$. Since by definition

$$\eta = \lim_{H\to\infty} \mathscr{D}_r(H,\bullet) \times \mathbf{1}$$

it follows from (E.43) that we have

$$\eta = \frac{R^{ss}}{R^{ss}-1} \lim_{H\to\infty} \mathscr{M}_r(H,\bullet) \times \mathbf{1} \tag{E.44}$$

It thus remains to characterize $\mathscr{M}_r$. For this we momentarily assume that there are no spenders ($\mu = 0$); the extension to the full spender-OLG model is straightforward and will come at the end. It follows from the results in Wolf (2021a) that $\mathscr{M}_r$ has the following limiting properties:

$$\begin{aligned} \lim_{H\to\infty} \mathscr{M}_r(H,H) &= -\sigma\beta\omega\frac{1-\omega}{1-\beta\omega^2} \\ \lim_{H\to\infty} \mathscr{M}_r(H,H-1) &= \sigma\frac{\omega(1-\beta\omega)}{1-\beta\omega^2} \end{aligned}$$

as well as

$$\begin{aligned} \lim_{H\to\infty} \frac{\mathscr{M}_r(H,H-s)}{\mathscr{M}_r(H,H-s+1)} &= \omega\beta, \quad s \geq 2 \\ \lim_{H\to\infty} \frac{\mathscr{M}_r(H,H+s)}{\mathscr{M}_r(H,H+s-1)} &= \omega, \quad s \geq 1 \end{aligned}$$

Plugging those relations into (E.44) and simplifying, we find

$$\eta = \frac{1}{1-\beta}\sigma\left[\frac{1}{1-\theta} - \frac{1}{1-\beta\theta}\right] \tag{E.45}$$

Finally, if there is a margin of spenders, then the elasticity is simply scaled down to correspond to the margin of OLG households $(1-\mu)$, thus giving (E.42). □

Empirical work suggests a range for η of around 1.25 to 35 (see Moll et al., 2022). Setting $\beta = 0.99^{\frac{1}{4}}$, $\sigma = 1$, and requiring the model to generate an impact MPC of 22 per cent (all as in our baseline calibration), we find $\omega \in [0.21, 0.85]$. Our baseline calibration lies somewhat beyond the upper end of this range and is thus conservative.

E.6 Self-financing in other model variants

We here discuss our self-financing result in two further model variants: (i) a quantitative HANK model; and (ii) a model with cognitive discounting.

E.6.1 A full HANK model

This section provides a sketch of the quantitative HANK model that we use to numerically illustrate the generality of our self-financing result. The discussion is brief because the household block of the model is essentially borrowed from Wolf (2021a).

Model sketch & calibration. The model economy is exactly as in Section 2, but with one twist: the OLG household block is replaced by a unit continuum of households $i \in [0,1]$ that face uninsurable income risk. Households have preferences

$$\mathbb{E}_t\left[\sum_{k=0}^{\infty}\beta^t\left[u(C_{i,t+k})-v(L_{i,t+k})\right]\right]$$

Households save and borrow (subject to a constraint) in a nominally risk-free bond, as in our baseline model. They receive labor and dividend income in proportion to their (stochastic) productivity, pay a proportional tax τ_y on that income, and finally pay additional lump-sum uniform taxes $\tilde{T}_t$. We can thus write the household budget constraint in real terms as

$$C_{i,t}+D_{i,t+1}=(1-\tau_y)e_{i,t}Y_t-\tilde{T}_t+\frac{I_{t-1}}{\Pi_t}D_{i,t}, \quad D_{i,t+1}\geq \underline{D}$$

Whenever possible we set parameters as in our baseline model. The remaining HANK-specific parameters are: the income risk process; the borrowing constraint; and the discount factor and steady-state interest rate. The income risk process is taken from Kaplan et al. (2018), just ported to discrete time as in Wolf (2021b). The borrowing constraint $\underline{D}$ is set to zero, and the discount factor β is backed out residually to clear the asset market, with a quarterly real rate of one per cent. Finally, we need to make one more change relative to our baseline model: in the model set-up as described so far, tax revenue $\tau_y \times Y^{ss}$ would far exceed debt servicing costs, so the government would make a substantial uniform transfer, thus materially dampening household MPCs. We instead set the steady-state transfer share as in the data (following Kaplan et al., 2018, which gives $\tilde{T}^{ss}/Y^{ss}=0.06$), and then clear the government budget by additionally allowing for positive (and time-invariant) government purchases.

Results. We use the quantitative HANK model to revisit our numerical exercises in Section 6.2. Exactly as done there, we here compute the aggregate effects of one-off fiscal stimulus for different assumptions on the delay in fiscal financing. Results are reported in Figure E.2.

Our results closely echo those of Section 6.2. We emphasize two main takeaways. First, Figure E.2 is *qualitatively* very similar to Figure 3: output and inflation responses as well as the share of self-financing ν are all increasing in the delay in fiscal adjustment (i.e., decreasing in τ_d). Furthermore, as $\tau_d \to 0$, we again converge to a full self-financing limit. Second, the two figures are also *quantitatively*

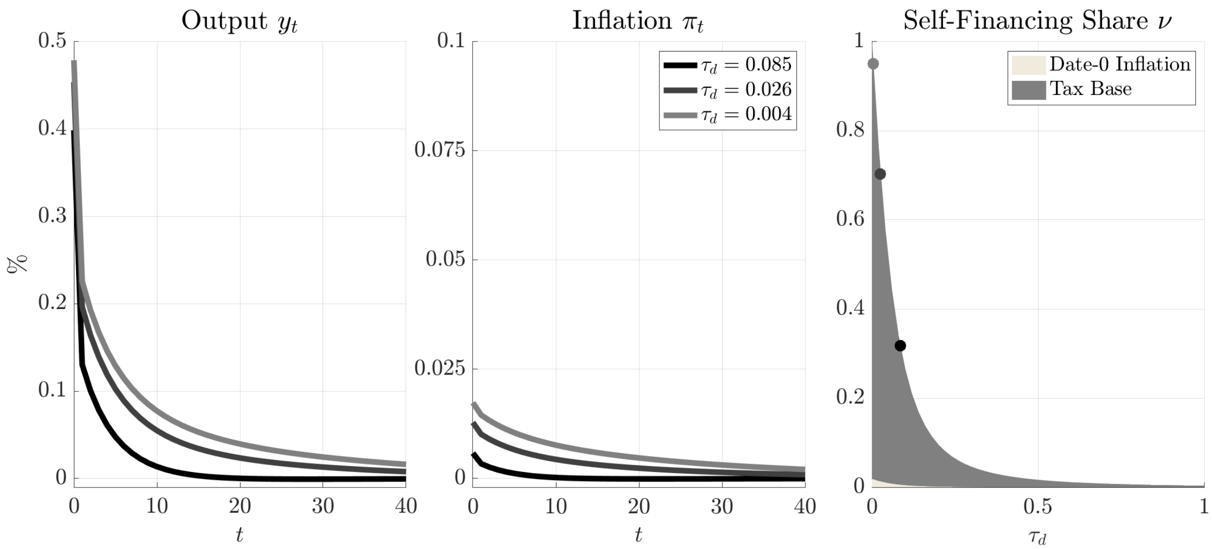

Figure E.2: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to a shock ε_0 equal to one per cent of steady-state output, as a function of τ_d, for the quantitative HANK model. The left and middle panels show the impulse responses for the three particular values of τ_d discussed in Section 6.1. In the right panel these three points are marked with circles.

similar: for our three values of τ_d taken from prior work, the share of self-financing ν is very similar to the spender-OLG hybrid model. This conclusion confirms prior work arguing that, as far the dynamics of macroeconomic aggregates are concerned, spender-OLG hybrid models and fully specified HANK models look extremely similar (e.g., see the discussions in Auclert et al., 2023; Wolf, 2021a)

E.6.2 The effects of cognitive discounting

Figure E.3 repeats our analysis of Section 6.2 in a variant of our spender-OLG hybrid model with cognitive discounting. To illustrate the effects of discounting as clearly as possible we consider a rather significant degree of discounting ($\theta = 0.25$).

The figure illustrates the two effects described in Section 5.2. First, for τ_d close to one, the Keynesian boom and thus the share of self-financing ν are larger than in our baseline model. Intuitively, in this case, the strong discounting of the not-so-distant tax hike meaningfully amplifies the initial boom. Second, for τ_d close to zero, the self-financing limit is approached somewhat more slowly, reflecting a weakening of the intertemporal Keynesian cross.

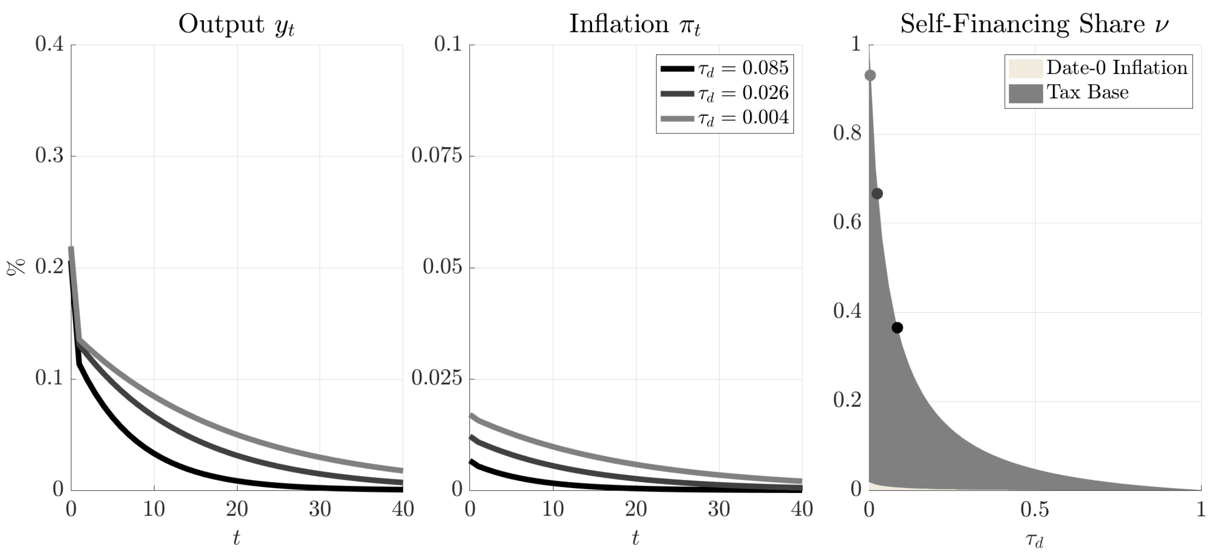

Figure E.3: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to a shock ε_0 equal to one per cent of steady-state output, as a function of τ_d, with cognitive discounting. The left and middle panels show the impulse responses for the three particular values of τ_d discussed in Section 6.1. In the right panel these three points are marked with circles.

References

AUCLERT, A., B. BARDÓCZY, M. ROGNLIE, AND L. STRAUB (2021): "Using the sequence-space Jacobian to solve and estimate heterogeneous-agent models," *Econometrica*, 89, 2375–2408.

AUCLERT, A., AND M. ROGNLIE (2020): "Inequality and aggregate demand," *NBER Working Paper*.

AUCLERT, A., M. ROGNLIE, AND L. STRAUB (2020): "Micro jumps, macro humps: Monetary policy and business cycles in an estimated HANK model," *NBER Working paper*.

——— (2023): "The intertemporal keynesian cross," *Journal of Political Economy*.

BIANCHI, F., AND L. MELOSI (2017): "Escaping the great recession," *American Economic Review*, 107, 1030–58.

BICKEL, P., AND M. LINDNER (2012): "Approximating the inverse of banded matrices by banded matrices with applications to probability and statistics," *Theory of Probability & Its Applications*, 56, 1–20.

GALÍ, J. (2008): *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*: Princeton University Press.

GALÍ, J., J. D. LÓPEZ-SALIDO, AND J. VALLÉS (2007): "Understanding the effects of government spending on consumption," *Journal of the european economic association*, 5, 227–270.

KAPLAN, G., B. MOLL, AND G. L. VIOLANTE (2018): "Monetary policy according to HANK," *American Economic Review*, 108, 697–743.

KAPLAN, G., AND G. L. VIOLANTE (2018): "Microeconomic heterogeneity and macroeconomic shocks," *Journal of Economic Perspectives*, 32, 167–94.

LEEPER, E. M. (1991): "Equilibria under "active" and "passive" monetary and fiscal policies," *Journal of monetary Economics*, 27, 129–147.

MOLL, B., L. RACHEL, AND P. RESTREPO (2022): "Uneven growth: automation's impact on income and wealth inequality," *Econometrica*, 90, 2645–2683.

WOLF, C. K. (2021a): "Interest rate cuts vs. stimulus payments: An equivalence result," *NBER Working Paper*.

——— (2021b): "The missing intercept: A demand equivalence approach," *NBER Working Paper*.