ON THE JOINT DESIGN OF INFORMATION AND TRANSFERS

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ABSTRACT. This note demonstrates how the insights from Morris et al. (2020) can be applied to the problem of optimal joint design of information and transfers in team production.

1. INTRODUCTION

In Morris et al. (2020), we studied implementation by information design in binary-action supermodular (BAS) games. An outcome is smallest equilibrium implementable if there exists an information structure such that that outcome is induced by the smallest equilibrium of the game with that information structure. We characterized the set of smallest equilibrium implementable outcomes of a given BAS game. If the outcome to be implemented is fixed, our characterization result can be read as characterizing the set of games for which that outcome is smallest equilibrium implementable. In this note, we demonstrate how the methodology developed in Morris et al. (2020) is applied to the problem of joint design of information and transfers in the context of moral hazard in team production due to Winter (2004), where the game payoffs are endogenously determined by a smallest amount of transfers to induce all the agents to always exert effort in the smallest-effort, hence unique, equilibrium. Specifically, we study the settings of Halac et al. (2021) and Moriya and Yamashita (2020) and derive their results, generalizing some of them, by using our solution method. In particular, we show that the comparative statics results obtained by Halac et al. (2021) (under symmetric production technology)

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extend to the general (possibly asymmetric) case, by appealing to interesting connections between our characterizing conditions (in the presence of a potential) and some well-known concepts from cooperative game theory.

Here we summarize the framework and results of Morris et al. (2020) relevant for the analysis in this note. The finite set of players is denoted by $I = \{1, \dots, |I|\}$. The finite state space is denoted by Θ, and the prior distribution over Θ by μ, where we assume that $\mu(\theta) > 0$ for all $\theta \in \Theta$. Each player $i \in I$ has binary actions, $A_i = \{0, 1\}$. A subset S of players in I (resp. players in $I \setminus \{i\}$) is identified with the action profile of all players (resp. opponents of player i) where player j chooses action 1 if and only if $j \in S$. The payoff gain function for player $i \in I$ is given by $d_i \colon 2^{I\setminus\{i\}} \times \Theta \to \mathbb{R}$, where $d_i(S, \theta)$ is player i's payoff gain from playing action 1 over action 0 when the subset $S \subset I \setminus \{i\}$ of players play action 1 and the state is $\theta \in \Theta$. With I, $(A_i)_{i\in I}$, and Θ fixed, the profile $(d_i)_{i\in I}$ is called the base game. We assume *supermodular payoffs*: for each $\theta \in \Theta$, $d_i(S, \theta)$ is nondecreasing in S (i.e., $d_i(S, \theta) \leq d_i(S', \theta)$ whenever $S \subset S'$). The base game $(d_i)_{i\in I}$ satisfies the *dominance state assumption* if there exists $\overline{\theta} \in \Theta$ such that $d_i(\emptyset, \overline{\theta}) > 0$ for all $i \in I$.

An information structure is represented by a type space $\mathcal{T} = ((T_i)_{i\in I}, \pi)$, where each T_i is a countable set of types of player $i \in I$, and $\pi \in \Delta(T \times \Theta)$ is a common prior over $T \times \Theta$, where we write $T = \prod_{i\in I} T_i$ and $T_{-i} = \prod_{j \in i} T_j$. We require an information structure to be consistent with the prior μ: $\sum_{t\in T} \pi(t, \theta) = \mu(\theta)$ for all $\theta \in \Theta$.

Together with the base game $(d_i)_{i\in I}$, an information structure $\mathcal{T}$ defines a supermodular incomplete information game, which we refer to simply as $\mathcal{T}$. In the incomplete information game $\mathcal{T}$, a (pure) strategy for player $i \in I$ is a mapping $\sigma_i \colon T_i \to A_i$, and the expected payoff gain for type $t_i \in T_i$ against opponents' strategy profile $\sigma_{-i} = (\sigma_j)_{j\neq i}$ is

$$D_i(\sigma_{-i}|t_i) = \sum_{t_{-i}\in T_{-i}, \theta\in\Theta} \pi(t_{-i}, \theta|t_i) d_i(\{j \in I \setminus \{i\} \mid \sigma_j(t_j) = 1\}, \theta),$$

where $\pi(t_{-i}, \theta|t_i) = \frac{\pi(t,\theta)}{\sum_{t'_{-i},\theta'} \pi((t_i,t'_{-i}),\theta')}$. A strategy profile $\sigma = (\sigma_i)_{i\in I}$ is an equilibrium of $\mathcal{T}$ if for all $i \in I$, $D_i(\sigma_{-i}|t_i) \geq 0$ whenever $\sigma_i(t_i) = 1$ and $D_i(\sigma_{-i}|t_i) \leq 0$ whenever $\sigma_i(t_i) = 0$. By the supermodularity of payoffs, there always exists a smallest equilibrium.

A strategy profile $\sigma = (\sigma_i)_{i\in I}$ in an incomplete information game $\mathcal{T}$ induces an *outcome*, i.e., a distribution $\nu \in \Delta(2^I \times \Theta)$ over action profiles and states, by

$$\nu(S, \theta) = \sum_{t\in T : \{i\in I | \sigma_i(t_i)=1\}=S} \pi(t, \theta),$$

which, due to the consistency of $\mathcal{T}$, is necessarily consistent with μ: $\sum_{S\in 2^I}\nu(S,\theta)=\mu(\theta)$ for all $\theta\in\Theta$. An outcome $\nu\in\Delta(2^I\times\Theta)$ is *smallest equilibrium implementable* (*S-implementable*) in $(d_i)_{i\in I}$ if there exists an information structure such that the smallest equilibrium induces ν. Let $SI\subset\Delta(2^I\times\Theta)$ denote the set of S-implementable outcomes. In the current analysis, we will focus on the "always play action 1" outcome, i.e., the (consistent) outcome $\bar\nu\in\Delta(2^I\times\Theta)$ such that $\bar\nu(I,\theta)=\mu(\theta)$ (and $\bar\nu(S,\theta)=0$ for $S\neq I$) for all $\theta\in\Theta$. Clearly, $\bar\nu$ is S-implementable in $(d_i)_{i\in I}$ if and only if it is *fully implementable* in $(d_i)_{i\in I}$: there exists an information structure with a unique equilibrium, which induces $\bar\nu$. We will sometimes say that the action profile I is fully implementable to mean that the outcome $\bar\nu$ is fully implementable.

Let Π denote the set of permutations of all players. An *ordered outcome* is a distribution $\nu_\Pi\in\Delta(\Pi\times\Theta)$ over permutations of players and states. An ordered outcome ν_Π is *consistent* (with μ) if $\sum_{\gamma\in\Pi}\nu_\Pi(\gamma,\theta)=\mu(\theta)$ for all $\theta\in\Theta$. An ordered outcome ν_Π satisfies *sequential obedience* (resp. *weak sequential obedience*) in $(d_i)_{i\in I}$ if

$$\sum_{\gamma\in\Pi,\theta\in\Theta}\nu_\Pi(\gamma,\theta)d_i(S_{-i}(\gamma),\theta)>\text{(resp. }\geq\text{) }0 \tag{1}$$

for all $i\in I$, where $S_{-i}(\gamma)\subset I\backslash\{i\}$ is the set of players who appear before i in permutation $\gamma\in\Pi$. The outcome $\bar\nu\in\Delta(2^I\times\Theta)$ satisfies sequential obedience (resp. weak sequential obedience) in $(d_i)_{i\in I}$ if there exists a consistent ordered outcome $\nu_\Pi\in\Delta(\Pi\times\Theta)$ that satisfies sequential obedience (resp. weak sequential obedience) in $(d_i)_{i\in I}$.

Morris et al. (2020, Theorem 1) provided a characterization of S-implementability in terms of sequential obedience. Applied to the outcome $\bar\nu$, it implies:

Proposition 1. (1) *If the outcome $\bar\nu$ is fully implementable in $(d_i)_{i\in I}$, then it satisfies sequential obedience in $(d_i)_{i\in I}$.*

(2) *Suppose that $(d_i)_{i\in I}$ satisfies the dominance state assumption. If the outcome $\bar\nu$ satisfies sequential obedience in $(d_i)_{i\in I}$, then it is fully implementable in $(d_i)_{i\in I}$.*

The games we consider in this note will have a potential. The base game $(d_i)_{i\in I}$ is a *potential game* if there exists a function $\Phi\colon 2^I\times\Theta\to\mathbb{R}$ such that for each $\theta\in\Theta$,

$$d_i(S,\theta)=\Phi(S\cup\{i\},\theta)-\Phi(S,\theta)$$

for all $i\in I$ and $S\subset I\setminus\{i\}$. Such a function Φ is called a *potential* of $(d_i)_{i\in I}$. By normalization, we assume $\Phi(\emptyset,\theta)=0$ for all $\theta\in\Theta$. By the monotonicity of $d_i(S,\theta)$ in S, the set function $\Phi(\cdot,\theta)$ is supermodular: $\Phi(S,\theta)+\Phi(S',\theta)\leq\Phi(S\cup S',\theta)+\Phi(S\cap S',\theta)$

for all $S, S' \subset I$. In potential games, sequential obedience has a simple characterization (Morris et al. (2020, Proposition 3)).

Proposition 2. *Suppose that $(d_i)_{i \in I}$ has a potential Φ. The outcome $\bar{\nu}$ satisfies sequential obedience (resp. weak sequential obedience) in $(d_i)_{i \in I}$ if and only if*

$$\sum_{\theta \in \Theta} \mu(\theta)\Phi(I, \theta) > (\textit{resp.} \geq) \sum_{\theta \in \Theta} \mu(\theta)\Phi(S, \theta) \tag{2}$$

for all $S \subsetneqq I$.

This is a duality result. The special case of this result where $|\Theta| = 1$ is explained in the Appendix, in relation to some well-known concepts from cooperative game theory.

The latter condition in this proposition is further simplified if the potential satisfies a convexity condition (Morris et al. (2020, Proposition 4)). Potential Φ satisfies *convexity* if for each $\theta \in \Theta$,

$$\Phi(S, \theta) \leq \frac{|S|}{|I|}\Phi(I, \theta) \tag{3}$$

for all $S \subset I$.

Proposition 3. *Suppose that $(d_i)_{i \in I}$ has a convex potential Φ. The outcome $\bar{\nu}$ satisfies sequential obedience (resp. weak sequential obedience) in $(d_i)_{i \in I}$ if and only if*

$$\sum_{\theta \in \Theta} \mu(\theta)\Phi(I, \theta) > (\textit{resp.} \geq)\ 0. \tag{4}$$

Under supermodularity, convexity is satisfied in particular when payoffs are symmetric so that $\Phi(S, \theta)$ depends on S only through its cardinality $|S|$. In general, it is a restriction on the degree of asymmetry in payoffs.

2. Information Design with Transfers in Team Production

In this note, we apply the arguments from Morris et al. (2020) to two extensions, by Halac et al. (2021) and by Moriya and Yamashita (2020), of the model of moral hazard in teams by Winter (2004).

There is a team I of agents who are engaged in a joint project. Each agent decides whether to exert effort (action 1) or not (action 0), where the effort cost for each agent $i \in I$ is $c_i > 0$. The probability of success of the project depends on the set of agents who exert effort as well as the state of the world, drawn from a finite set Θ according to a probability distribution μ on Θ. The project's technology is given by the function $P \colon 2^I \times \Theta \to [0, 1]$, where $P(S, \theta)$ is the success probability when the agents in set $S \subset I$

exert effort at state $\theta \in \Theta$. We assume strict monotonicity and increasing returns to scale (IRS) on $P(\cdot, \theta)$: i.e., for each $\theta \in \Theta$, $P(\cdot, \theta)$ is strictly increasing, $P(S, \theta) < P(S', \theta)$ if $S \subsetneqq S'$, and is supermodular, $P(S, \theta) + P(S', \theta) \leq P(S \cup S', \theta) + P(S \cap S', \theta)$ for all $S, S' \in 2^I$. For each $i \in I$, we denote

$$\Delta_i P(S, \theta) = P(S \cup \{i\}, \theta) - P(S, \theta)$$

for $S \subset I \setminus \{i\}$.

The principal offers incentive contracts to the agents to implement action profile I (all agents exerting effort) as a smallest, hence unique, equilibrium outcome, where the situation is one of hidden action, so that only the final outcome (whether success or failure) of the project is contractible. Ex post, if the bonus payment to agent i is $b_i \geq 0$ and a subset $S \subset I \setminus \{i\}$ of agents exert effort, this agent's payoff is thus given by $P(S \cup \{i\}, \theta)b_i - c_i$ for $a_i = 1$ and $P(S, \theta)b_i$ for $a_i = 0$. Thus, the payoff gain function is given by

$$d_i(S, \theta; b_i) = \Delta_i P(S, \theta)b_i - c_i.$$

By the assumption of IRS, d_i is nondecreasing in S.

In the original model of Winter (2004), there is no uncertainty (i.e., $|\Theta| = 1$), and the contracts are public, so that in the complete information game induced by a bonus profile, the necessary and sufficient condition for full implementability of I is the "divide-and-conquer" condition that there exists $\gamma \in \Pi$ such that

$$d_i(S_{-i}(\gamma); b_i) > 0$$

for all $i \in I$ (where the dependence on the state is dropped since $|\Theta| = 1$).[1] Under the symmetry in the technology and the costs as assumed in Winter (2004), the optimal (i.e., achieving the infimum of the total payment) bonus profile is given by binding the divide-and-conquer condition sequentially along an arbitrarily chosen permutation of all agents. In the following, we demonstrate how the methodology developed in Morris et al. (2020) applies to this context in the alternative settings in which the bonuses may be private with correlation across agents (Halac et al. (2021)) and information about the state may also be transmitted to the agents via private signals (Moriya and Yamashita (2020)), respectively.

[1]See also Segal (2003) for a related model where the action of an agent is contractible.

2.1. **Halac et al. (2021).** Suppose that $|\Theta| = 1$ as in Winter (2004), so that we drop the dependence on θ within this subsection. Without loss, we assume that $P(\emptyset) = 0$. Different from Winter (2004), the principal in Halac et al. (2021) is allowed to offer a private contract to each agent. Formally, an incentive scheme $\varphi = (\mathcal{T}, B)$ consists of an information structure $\mathcal{T} = ((T_i)_{i\in I}, \pi)$, where T_i is a (countable) type set for each agent $i \in I$ and π is a common prior on $T = \prod_{i\in I} T_i$, and a profile of bonus rules $B = (B_i)$, where $B_i \colon T_i \to \mathbb{R}_+$ determines the bonus payment offered to each type of agent $i \in I$. Write $TB(\varphi)$ for the expected total bonus payment under incentive scheme φ:

$$TB(\varphi) = \sum_{t\in T} \pi(t)P(I) \sum_{i\in I} B_i(t_i) = P(I) \sum_{i\in I} \sum_{t_i\in T_i} \pi(t_i)B_i(t_i),$$

where $\pi(t_i) = \sum_{t_{-i}\in T_{-i}} \pi(t_i, t_{-i})$. The optimization problem of the principal is:

$$TB^* = \inf_{\varphi} TB(\varphi)$$

subject to the constraint that the profile of the "always play 1" strategies be a unique equilibrium of the incomplete information game induced by incentive scheme φ.

Halac et al. (2021, Theorems 1 and 2) have shown, although not stated in this language, that this problem reduces to the problem of minimizing the total bonus payment $P(I) \sum_{i\in I} b_i$ subject to the constraint that I satisfies weak sequential obedience in the complete information game given by $b = (b_i)_{i\in I}$:

$$\min_{b\in\mathbb{R}^I_+} \sum_{i\in I} b_i \tag{5a}$$

subject to

$$\exists\, \rho \in \Delta(\Pi) : \sum_{\gamma\in\Pi} \rho(\gamma) d_i(S_{-i}(\gamma); b_i) \geq 0 \text{ for all } i \in I. \tag{5b}$$

Observe that, by the (continuity and) strict quasi-concavity of the function $(q, b_i) \mapsto qb_i$ for $q > 0$ and $b_i > 0$, an optimal solution to this problem (exists and) is unique, denoted $b^* = (b^*_i)_{i\in I}$. Note also that by strict monotonicity in b_i, the constraint (5b) must bind for all $i \in I$ at the optimum.

Proposition 4. *$TB^* = P(I) \sum_{i\in I} b^*_i$, where $b^* = (b^*_i)_{i\in I}$ is the unique solution to* (5)*.*

Below we show how the methods from Morris et al. (2020) help derive this result.

First, the argument in the proof of Theorem 1(1) in Morris et al. (2020) establishes that this problem gives a lower bound of $TB(\varphi)$.

Step 1. $TB^* \geq P(I) \sum_{i\in I} b^*_i$.

Proof. Fix any incentive scheme $\varphi = (\mathcal{T}, B)$ under which "always play 1" is the unique equilibrium. It can be cast into our framework as follows. View each realization of bonus profile $B(t) = (B_1(t_1), \dots, B_{|I|}(t_{|I|}))$ as a state. Thus, the (endogenously determined) state space is $\hat{\Theta} = \{B(t) \mid t \in T\}$. The base game $(\hat{d}_i)_{i \in I}$ is then given by $\hat{d}_i(S, \hat{\theta}) = d_i(S; \hat{\theta}_i) = \varDelta_i P(S)\hat{\theta}_i - c_i$, where the dependence on $\hat{\theta}$ (bonus profile) is only through its ith component $\hat{\theta}_i$ (bonus paid to i), and the prior $\hat{\pi}$ on $T \times \hat{\Theta}$ is by $\hat{\pi}(t, \hat{\theta}) = \pi(t)$ if $B(t) = \hat{\theta}$ and $\hat{\pi}(t, \hat{\theta}) = 0$ otherwise. As in the proof of Theorem 1(1) in Morris et al. (2020), consider the sequential best response process from the smallest strategies. Then the state space T is partitioned into $\{T(\gamma) \mid \gamma \in \Pi\}$ according to the order γ in which agents switch actions from 0 to 1 in the process so that it holds that for each $i \in I$ and $t_i \in T_i$,

$$\sum_{\gamma \in \Pi} \sum_{t_{-i} \in T_{-i} : (t_i, t_{-i}) \in T(\gamma)} \sum_{\hat{\theta} \in \hat{\Theta}} \hat{\pi}(t, \hat{\theta}) \hat{d}_i(S_{-i}(\gamma), \hat{\theta}) > 0,$$

that is,

$$\sum_{\gamma \in \Pi} \sum_{t_{-i} \in T_{-i} : (t_i, t_{-i}) \in T(\gamma)} \pi(t)(\varDelta_i P(S_{-i}(\gamma)) B_i(t_i) - c_i) > 0. \tag{6}$$

Define $\rho \in \Delta(\Pi)$ by

$$\rho(\gamma) = \sum_{t \in T(\gamma)} \pi(t),$$

and for each $i \in I$ and $t_i \in T_i$, define $\rho_i(\cdot|t_i) \in \Delta(\Pi)$ by

$$\rho_i(\gamma|t_i) = \sum_{t_{-i} \in T_{-i} : (t_i, t_{-i}) \in T(\gamma)} \pi(t_{-i}|t_i),$$

where $\pi(t_{-i}|t_i) = \frac{\pi(t_i, t_{-i})}{\pi(t_i)}$. Note that

$$\rho = \sum_{t_i \in T_i} \pi(t_i) \rho_i(\cdot|t_i)$$

holds for all $i \in I$. Thus, by (6), for all $i \in I$ and all $t_i \in T_i$ we have

$$\sum_{\gamma \in \Pi} \rho_i(\gamma|t_i)(\varDelta_i P(S_{-i}(\gamma)) B_i(t_i) - c_i) > 0,$$

and hence

$$B_i(t_i) > h_i(\rho_i(\cdot|t_i)), \tag{7}$$

where the function $h_i \colon \Delta(\Pi) \to \mathbb{R}$ is defined by

$$h_i(\rho') = \frac{c_i}{\sum_{\gamma \in \Pi} \rho'(\gamma) \varDelta_i P(S_{-i}(\gamma))}.$$

Then multiply the inequality by $\pi(t_i)$ and sum it up over all t_i: then we have

$$\sum_{t_i \in T_i} \pi(t_i) B_i(t_i) > \sum_{t_i \in T_i} \pi(t_i) h_i(\rho_i(\cdot|t_i)) \geq h_i\left(\sum_{t_i \in T_i} \pi(t_i) \rho_i(\cdot|t_i)\right) = h_i(\rho), \tag{8}$$

where the second inequality follows from the convexity of h_i (Jensen's inequality). Since $b = (h_i(\rho))_{i \in I}$ trivially satisfies the constraint (5b), we have $\sum_{i \in I} h_i(\rho) \geq \sum_{i \in I} b_i^*$, and hence,

$$TB(\varphi) = P(I) \sum_{i \in I} \sum_{t_i \in T_i} \pi(t_i) B_i(t_i) > P(I) \sum_{i \in I} h_i(\rho) \geq P(I) \sum_{i \in I} b_i^*.$$

Since φ has been taken arbitrarily, this implies that $TB^* \geq P(I) \sum_{i \in I} b_i^*$. □

Second, analogously to the construction in the proof of Theorem 1(2) in Morris et al. (2020), one can construct a sequence of incentive schemes that attains in the limit the lower bound obtained in Step 1. Due to the requirement of known own payoffs, that the value of the bonus be known to each type in the setting here, however, the concrete construction should follow that of Oyama and Takahashi (2020, Theorem 2).

Step 2. $TB^* \leq P(I) \sum_{i \in I} b_i^*$.

Proof. Let $\rho^* \in \Delta(\Pi)$ be an ordered outcome that satisfies (5b) with respect to $b = b^*$. For each $i \in I$, fix any $\bar{b}_i$ strictly larger than $\frac{c_i}{P(\{i\})}$; say, let $\bar{b}_i = \frac{c_i}{P(\{i\})} + 1$.

Fix any sufficiently small $\varepsilon > 0$. We show that there exists an incentive scheme φ such that $TB(\varphi) \leq P(I) \sum_{i \in I}(b_i^* + 2\varepsilon)$. Let $\eta > 0$ be such that

$$\sum_{\gamma \in \Pi} (1-\eta)^{|I|-|S_{-i}(\gamma)|-1} \rho^*(\gamma) d_i(S_{-i}(\gamma); b_i^* + \varepsilon) > 0 \tag{9}$$

for all $i \in I$, and

$$1 - (1-\eta)^{|I|-1} \leq \frac{|I|\varepsilon}{\sum_{i \in I} \bar{b}_i - \sum_{i \in I} b_i^*}. \tag{10}$$

Then construct the incentive scheme $\varphi = ((T, \pi), B)$ as follows. For each $i \in I$, let $T_i = \{1, 2, \ldots\}$, and let $\pi \in \Delta(T)$ be given by

$$\pi(t) = \begin{cases} \eta(1-\eta)^m \rho^*(\gamma) & \text{there exist } m \in \mathbb{N} \text{ and } \gamma \in \Pi \text{ such} \\ & \text{that } t_i = m + \ell(i, \gamma) \text{ for all } i \in I, \\ 0 & \text{otherwise,} \end{cases}$$

where $\ell(i, \gamma)$ is the ranking of $i \in I$ in $\gamma \in \Pi$ (i.e., $\ell(i, (i_1, \ldots, i_{|I|})) = \ell$ if and only if $i_\ell = i$). Let $B_i \colon T_i \to \mathbb{R}$ be given by

$$B_i(t_i) = \begin{cases} \bar{b}_i & \text{if } t_i \leq |I| - 1, \\ b_i^* + \varepsilon & \text{if } t_i \geq |I|. \end{cases}$$

We claim that action 1 is uniquely rationalizable for all players of all types. Indeed, for types $t_i \leq |I| - 1$, action 1 is a strictly dominant action by construction. For $\tau \geq |I|$, if action 1 is uniquely rationalizable for all players of all types $t_i \leq \tau - 1$, then, as in

Claim A.4 in the proof of Theorem 2 in Oyama and Takahashi (2020) (or Claim A.2 in the proof of Theorem 1(2) in Morris et al. (2020)), for a player $i \in I$ of type $t_i = \tau$ the expected payoff is no smaller than

$$\sum_{S \subset I \setminus \{i\}} \pi(\{j \in I \setminus \{i\} \mid t_j < \tau\} = S | t_i = \tau) d_i(S; b_i^* + \varepsilon)$$
$$= \sum_{\gamma \in \Pi} (1-\eta)^{|I| - |S_{-i}(\gamma)| - 1} \rho^*(\gamma) d_i(S_{-i}(\gamma); b_i^* + \varepsilon) / C_i' > 0$$

by (9), where $C_i' = \sum_{\ell=1}^{|I|} (1-\eta)^{|I|-\ell} \rho^*(\{\gamma = (i_1, \ldots, i_{|I|}) \in \Pi \mid i_\ell = i\}) > 0$, so that action 1 is uniquely rationalizable. By induction, "always play 1" is uniquely rationalizable, hence the unique equilibrium.

Finally, since $\pi(\{t \in T \mid t_i \geq |I| \text{ for all } i \in I\}) = \sum_{m \geq |I|-1} \eta(1-\eta)^m = (1-\eta)^{|I|-1}$, we have

$$TB(\varphi) \leq (1-\eta)^{|I|-1} P(I) \sum_{i \in I} (b_i^* + \varepsilon) + \left(1 - (1-\eta)^{|I|-1}\right) P(I) \sum_{i \in I} \bar{b}_i$$
$$\leq P(I) \sum_{i \in I} (b_i^* + 2\varepsilon)$$

by (10), as desired. □

Halac et al. (2021, Theorem 2) also show that for any sequence of incentive schemes that attains TB^* in the limit, the corresponding sequence of bonus profile distributions converges weakly to the degenerate distribution on b^*. In our proof of Proposition 4 above, this follows from the inequalities (7) and (8), which will be binding in the limit, along with the strict convexity of h_i.

Now, let us discuss the structure of the limit optimal bonus profile b^*. The constraint (5b), which is written as $\sum_{\gamma \in \Pi} \rho(\gamma) \Delta_i P(S_{-i}(\gamma)) \geq \frac{c_i}{b_i}$, must all bind at the optimum. For $\gamma \in \Gamma$, let $\alpha^\gamma(P) = (\alpha_i^\gamma(P))_{i \in I}$ be the vector ("marginal contribution vector") defined by $\alpha_i^\gamma(P) = \Delta_i P(S_{-i}(\gamma))$. By substitution $x_i = \frac{c_i}{b_i}$, the optimization problem (5) is rewritten as

$$\min_x \sum_{i \in I} \frac{c_i}{x_i} \text{ subject to } x \in W(P), \tag{11}$$

where

$$W(P) = \text{conv}\{\alpha^\gamma(P) \in \mathbb{R}^I \mid \gamma \in \Pi\}.$$

Since, as discussed in the Appendix, the set $W(P)$, which is called the *Weber set* of P (Weber (1988)), is equal to the *core* of the set function P by supermodularity (Shapley (1971)),

$$C(P) = \{x \in \mathbb{R}^I \mid x(I) = P(I),\ x(S) \geq P(S) \text{ for all } S \subset I\},$$

where $x(S) = \sum_{i \in S} x_i$, the problem (11), hence the original problem (5), is equivalent to

$$\min_x \sum_{i \in I} \frac{c_i}{x_i} \text{ subject to } x \in C(P). \tag{12}$$

(This also follows from Proposition 2 in Section 1 since $(d_i(\cdot; b_i))_{i \in I}$ is a (weighted) potential game with potential $P - x$.) With the expression (12), one can exploit known results from cooperative game theory to extend to the case of general P the comparative statics results derived in Halac et al. (2021, Section IV) for symmetric P (where $P(S)$ depends only on $|S|$). The geometry of the core of a supermodular function is well known by Shapley (1971). For $S \subset I$, denote $C_S(P) = \{x \in C(P) \mid x(S) = P(S)\}$ with $C_\emptyset(P) = C(P)$. These sets $C_S(P)$, $S \subset I$, are the faces of the core $C(P)$. The vertices of each $C_S(P)$ are precisely the vectors $\alpha^\gamma(P)$ with the permutations $\gamma \in \Pi$ that rank the agents in S earlier than those in $I \setminus S$, i.e., the permutations γ written as $\gamma = (S, I \setminus S)$. If P is strictly supermodular (i.e., $P(S, \theta) + P(S', \theta) < P(S \cup S', \theta) + P(S \cap S', \theta)$ whenever neither $S \subset S'$ nor $S' \subset S$), then the vectors $\alpha^\gamma(P)$ are all distinct. Let $x^* = \left(\frac{c_i}{b_i^*}\right)_{i \in I}$ be the solution to (12), and let $\mathcal{S}_{x^*} = \{S \subset I \mid x^* \in C_S(P)\}$, which corresponds to the set of binding constraints in (12). By supermodularity, $\mathcal{S}_{x^*}$ is closed under union and intersection, and in fact the members of $\mathcal{S}_{x^*}$ are nested if P is strictly supmermodular. Suppose that P is strictly supmermodular, and write $\mathcal{S}_{x^*} = \{S_0^*, S_1^*, \ldots, S_L^*\}$ with $\emptyset = S_0^* \subset S_1^* \subset \cdots \subset S_L^* = I$. Then, any $\rho^* \in \Delta(\Pi)$ such that $x^* = \sum_{\gamma \in \Pi} \rho^*(\gamma) \alpha^\gamma(P)$ assigns strictly positive weights only to permutations γ written in the form $\gamma = (S_1^*, S_2^* \setminus S_1^*, \ldots, S_L^* \setminus S_{L-1}^*)$. Thus define the weak order on I by $i \succ i'$ if and only if $i \in S_\ell^* \setminus S_{\ell-1}^*$ and $i' \in S_{\ell'}^* \setminus S_{\ell'-1}^*$ for some $\ell < \ell'$, and $i \sim i'$ if and only if $i, i' \in S_\ell^* \setminus S_{\ell-1}^*$ for some ℓ. This order is the order identified by Halac et al. (2021, Proposition 1) for the case of (strictly supermodular and) symmetric technology. It can be readily verified that $i \sim i'$ for all $i, i' \in I$ (i.e., $L = 1$) if and only if

$$\frac{P(I)}{\sum_{i \in I} \sqrt{c_i}} > \frac{P(S)}{\sum_{i \in S} \sqrt{c_i}}$$

for all $S \neq I$, while $i \succ i'$ or $i' \succ i$ whenever $i \neq i'$ (i.e., $L = |I|$, or $x^* = \alpha^\gamma(P)$ for some $\gamma \in \Pi$) if and only if there exists $\gamma = (i_1, \ldots, i_{|I|}) \in \Pi$ such that

$$\frac{\mathit{\Delta}_{i_{k+1}} P(S_{-i_{k+1}}(\gamma))}{\sqrt{c_{i_{k+1}}}} \leq \frac{\mathit{\Delta}_{i_k} P(S_{-i_k}(\gamma))}{\sqrt{c_{i_k}}}$$

for all $k = 1, \ldots, |I| - 1$ (compare Halac et al. (2021, Proposition 2)). In particular, if costs are symmetric (i.e., $c_i = c_j$ for all $i, j \in I$) and technology is not too asymmetric in the sense that P satisfies the convexity condition (3), then $i \sim i'$ for all $i, i' \in I$

and the symmetric bonus profile (i.e., $x_i^* = \frac{P(I)}{|I|}$ for all $i \in I$) is the solution, so that the discriminatory, deterministic divide-and-conquer contract of Winter (2004) is strictly suboptimal.

The problem (12) is a special case of a class of well-studied problems in cooperative game theory. If costs are symmetric, the solution x^* coincides with the constrained egalitarian allocation of Dutta and Ray (1989), or the Dutta-Ray solution, which is the unique element of $C(P)$ that Lorenz-dominates every other element of $C(P)$.[2] For general c_i's, x^* is the unique element of $C(P)$ such that $\left(\frac{x_i^*}{\sqrt{c_i}}\right)_{i\in I}$ Lorenz-dominates $\left(\frac{y_i}{\sqrt{c_i}}\right)_{i\in I}$ for all $y \in C(P)$, which is a specific form of a generalized Dutta-Ray solution (e.g., Hokari (2002)).[3] It follows from Hokari (2002, Proposition 1) (with the "monotone-path function" $f_i(S, i) = \frac{\sqrt{c_i}t}{\sum_{j\in S}\sqrt{c_j}}$) that the solution x^* is explicitly written as

$$x_i^* = \max_{S\subset I, S\ni i} \min_{T\subset S\setminus\{i\}} \frac{\sqrt{c_i}(P(S) - P(T))}{\sum_{j\in S\setminus T}\sqrt{c_j}}.$$

Therefore, we have:

Proposition 5. *The unique limit optimal bonus profile $b^* = (b_i^*)_{i\in I}$ is given by*

$$b_i^* = \min_{S\subset I, S\ni i} \max_{T\subset S\setminus\{i\}} \frac{\sqrt{c_i}\sum_{j\in S\setminus T}\sqrt{c_j}}{P(S) - P(T)}. \tag{13}$$

If P is strictly supermodular, then for $i \in S_\ell^* \setminus S_{\ell-1}^*$, the "min" and the "max" in the formula (13) are attained with $S = S_\ell^* \setminus S_{\ell-1}^*$ and $T = S_{\ell-1}^*$, respectively. Immediately from Proposition 5, one can see that the comparative statics of the solution with respect to the costs c_i derived by Halac et al. (2021, Proposition 3) for the case of symmetric P in fact holds for general P. Also, some comparative statics properties with respect to P are known (e.g., Hokari (2002, Propositions 2–5)).

2.2. **Moriya and Yamashita (2020).** Now let the production technology depend on the state of the world $\theta \in \Theta$ which is known only to the principal. As in Winter (2004), we assume a symmetric environment: technology is symmetric so that it depends on $S \subset I$ only through its cardinality $|S|$, and costs are symmetric: $c_i = c$ for all $i \in I$. Here we allow for any finite number of states as well as agents, while Moriya and Yamashita

[2]For $x, y \in \mathbb{R}^I$, x Lorenz-dominates y if $(x_{(1)}, \ldots, x_{(n)}) \geq (y_{(1)}, \ldots, y_{(n)})$ for all $n = 1, \ldots, |I|$, where $x_{(k)}$ denotes the kth component of x in the increasing order.

[3]It can also be shown that x^* is an optimal solution to (12) if and only if it is "lexicographically optimal" in $C(P)$ with respect to weight vector $(\sqrt{c_i})_{i\in I}$ (Fujishige (2005, Chapter V)), where several algorithms for computing the solution have been proposed.

(2020) only consider the case where $|\Theta| = 2$ and $I = 2$. Write $P(S,\theta) = p(|S|,\theta)$, where $p(n,\theta)$ is strictly increasing in n, and

$$\Delta p(n,\theta) = p(n+1,\theta) - p(n,\theta)$$

is nondecreasing in n (increasing returns to scale, or IRS). Let $\overline{\theta} \in \Theta$ be a state such that $\Delta p(0,\bar{\theta}) \geq \Delta p(0,\theta)$ for all $\theta \in \Theta$. We impose the following assumption:

$$\Delta p(0,\overline{\theta}) \geq \sum_{\theta\in\Theta} \mu(\theta) \frac{p(|I|,\theta) - p(0,\theta)}{|I|}. \tag{14}$$

This is a richness assumption that corresponds to the dominance state assumption. It says that the marginal productivity by a single agent's effort at $\overline{\theta}$ (left hand side) is large enough that it exceeds the expected average productivity (right hand side). In particular, under the strictly monotonicity of $p(n,\theta)$ in n, it excludes the case of $|\Theta| = 1$.

The principal chooses an information structure and a bonus payment scheme. We assume that the state realization is unverifiable, so that the bonus payment to each agent can depend only on the success of the project, and as in Winter (2004), assume public contracts. If the bonus payment to agent i is $b_i > 0$ and a subset $S \subset I \setminus \{i\}$ of agents exert effort, this agent's payoff is thus given by $p(|S|+1,\theta)b_i - c$ for $a_i = 1$ and $p(|S|,\theta)b_i$ for $a_i = 0$. By normalization, we let the payoff gain function be given by

$$d_i(S,\theta;b_i) = \Delta p(|S|,\theta) - \frac{c}{b_i}.$$

By the assumption of IRS, d_i is nondecreasing in S.

The objective of the principal is to find a bonus scheme $b = (b_i)_{i\in I}$ and an information structure that minimize the total payment while inducing all types of all agents to exert effort in the unique, hence smallest, equilibrium. Thus, the problem becomes:

$$\inf_{b:\bar{\nu}\in SI(b)} \sum_{i\in I} b_i,$$

where $\bar{\nu} \in \Delta(2^I \times \Theta)$ is the "always play 1" outcome, i.e., the outcome such that $\bar{\nu}(I,\theta) = \mu(\theta)$ for all $\theta \in \Theta$, and $SI(b) \subset \Delta(2^I \times \Theta)$ is the set of S-implementable outcomes under the bonus scheme b. We say that a bonus scheme $b^* = (b_i^*)_{i\in I}$ is optimal if $\sum_{i\in I} b_i^*$ is equal to this infimum and $\bar{\nu} \in SI(b^* + \varepsilon)$ for every $\varepsilon > 0$, where $b^* + \varepsilon = (b_i^* + \varepsilon)_{i\in I}$.

By Proposition 1(1), sequential obedience of $\bar{\nu}$ under $(d_i(\cdot;b_i))_{i\in I}$ is a necessary condition for $\bar{\nu} \in SI(b)$, which will give us a condition on payoffs, hence bonuses. But the base game given $b = (b_i)_{i\in I}$ is a potential game with a potential

$$\Phi(S,\theta;b) = p(|S|,\theta) - p(0,\theta) - \sum_{i\in S} \frac{c}{b_i}.$$

Therefore, by Proposition 2, sequential obedience reduces to the simpler condition $\sum_{\theta\in\Theta}\mu(\theta)\Phi(I,\theta;b) > \sum_{\theta\in\Theta}\mu(\theta)\Phi(S,\theta;b)$ for all $S \subsetneqq I$. We consider the minimization problem under the relaxed constraint $\sum_{\theta\in\Theta}\mu(\theta)\Phi(I,\theta;b) \geq \sum_{\theta\in\Theta}\mu(\theta)\Phi(\emptyset,\theta;b) = 0$:

$$\min_{b\in\mathbb{R}^I_+}\sum_{i\in I} b_i$$

subject to

$$\sum_{i\in I}\frac{c}{b_i} \leq \sum_{\theta\in\Theta}\mu(\theta)(p(|I|,\theta) - p(0,\theta)).$$

Since the left hand side of the constraint, which must be binding at the optimum, is (continuous and) strictly quasi-convex in b, an optimal solution to this relaxed problem (exists and) is unique. It is readily verified to be $b^* = (\beta^*,\dots,\beta^*)$ with

$$\beta^* = \frac{|I|c}{\sum_{\theta\in\Theta}\mu(\theta)(p(|I|,\theta) - p(0,\theta))}. \tag{15}$$

This will indeed be a (unique) optimal bonus scheme if $\bar{\nu} \in SI(b^* + \varepsilon)$ for every $\varepsilon > 0$. Under $b^* + \varepsilon$, the potential is now

$$\Phi(S,\theta;b^* + \varepsilon) = p(|S|,\theta) - p(0,\theta) - |S|\frac{c}{\beta^* + \varepsilon},$$

which satisfies convexity by IRS. Therefore, by Proposition 3, sequential obedience is equivalent to the condition that $\sum_{\theta\in\Theta}\mu(\theta)\Phi(I,\theta;b^* + \varepsilon) > 0$, which, by the definition of β^*, is satisfied for any $\varepsilon > 0$. Finally, by the assumption (14), we have $\beta^* \geq \frac{c}{\Delta p(1,\bar{\theta})}$, and therefore,

$$d_i(\emptyset,\bar{\theta};b_i^* + \varepsilon) = \Delta p(1,\bar{\theta}) - \frac{c}{\beta^* + \varepsilon} > 0,$$

so that the dominance state assumption is satisfied for any $\varepsilon > 0$. Hence, from Proposition 1(2), it follows that $\bar{\nu} \in SI(b^* + \varepsilon)$ for any $\varepsilon > 0$. Thus, we have:

Proposition 6. *The unique optimal bonus scheme is given by $b^* = (\beta^*,\dots,\beta^*)$, where β^* is as defined in* (15).

Thus, under the richness assumption (i.e., assumption (14)), a symmetric bonus scheme is optimal, and asymmetric ones, in particular, the deterministic divide-and-conquer schemes of Winter (2004) $\left(\frac{c}{\Delta p(1)},\dots,\frac{c}{\Delta p(|I|)}\right)$ (modulo permutation), are strictly suboptimal.

Appendix

In this section, we report interesting connections between sequential obedience in complete information potential games and some well-known concepts in cooperative game theory, which may help gain additional intuition about sequential obedience and its dual characterizations, Propositions 2 and 3 in Morris et al. (2020). They are also used in the arguments in Section 2.1 of this note.

Consider the complete information case where $\mu(\theta^*) = 1$ for some $\theta^* \in \Theta$ (whereby we will suppress the dependence on θ within this section), and suppose that $(d_i)_{i\in I}$ has a potential $\Phi\colon 2^I \to \mathbb{R}$ at θ^*: $d_i(S) = \Phi(S \cup \{i\}) - \Phi(S)$ for all $i \in I$ and $S \in 2^{I\setminus\{i\}}$, where we normalize $\Phi(\emptyset) = 0$. We restrict attention to the action profile I, or the "all play action 1" outcome.

The (supermodular) set function Φ is viewed as a cooperative game, where $\Phi(S)$ is the "worth" of coalition $S \subset I$. Let Π denote the set of permutations of all players, and for $\gamma \in \Pi$, let $\alpha^\gamma(\Phi) = (\alpha_i^\gamma(\Phi))_{i\in I} \in \mathbb{R}^I$ be the vector defined by

$$\alpha_i^\gamma(\Phi) = d_i(S_{-i}(\gamma)) = \Phi(S_{-i}(\gamma) \cup \{i\}) - \Phi(S_{-i}(\gamma)),$$

where $S_{-i}(\gamma)$ is the set of players listed before i in γ. These vectors $\alpha^\gamma(\Phi)$, $\gamma \in \Pi$, are called the *marginal contribution vectors* of Φ. Then the sequential obedience condition for I reads: there exists $\rho \in \Delta(\Pi)$ such that $\sum_{\gamma\in\Pi} \rho(\gamma)\alpha^\gamma(\Phi) \gg 0$, or equivalently,

$$W(\Phi) \cap \mathbb{R}^I_{++} \neq \emptyset, \tag{A.1}$$

where

$$W(\Phi) = \operatorname{conv}\{\alpha^\gamma(\Phi) \in \mathbb{R}^I \mid \gamma \in \Pi\}$$

is called the *Weber set* of Φ (Weber (1988)). By duality (or the separating hyperplane theorem), sequential obedience of I is equivalent to $\max_{\gamma\in\Pi} \lambda \cdot \alpha^\gamma(\Phi) > 0$ for all $\lambda \in \mathbb{R}^I_+ \setminus \{0\}$, or equivalently,

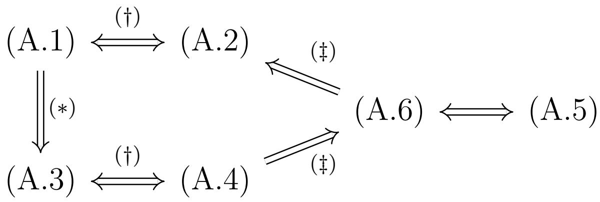
$$\max_{x\in W(\Phi)} \lambda \cdot x > 0 \text{ for all } \lambda \in \mathbb{R}^I_+ \setminus \{0\}. \tag{A.2}$$

This is a special case of Proposition 2 in Morris et al. (2020).

The *core* of Φ is the set

$$C(\Phi) = \{x \in \mathbb{R}^I \mid x(I) = \Phi(I),\ x(S) \geq \Phi(S) \text{ for all } S \subset I\},$$

where $x(S) = \sum_{i\in S} x_i$. In general, $C(\Phi) \subset W(\Phi)$ (by (A.7) below), and when Φ is supermodular, $C(\Phi) = W(\Phi)$, where the marginal contribution vectors $\alpha^\gamma(\Phi)$, $\gamma \in \Pi$,



(∗)By supermodularity (Shapley (1971))
(†)By duality
(‡)By (A.7)

FIGURE 1. Equivalence

are precisely the vertices of $C(\Phi)$ (Shapley (1971)). Thus, under the supermodularity of Φ, (A.1) is equivalent to

$$C(\Phi) \cap \mathbb{R}^I_{++} \neq \emptyset, \tag{A.3}$$

which by duality (or the separation hyperplane theorem) is equivalent to

$$\max_{x \in C(\Phi)} \lambda \cdot x > 0 \text{ for all } \lambda \in \mathbb{R}^I_+ \setminus \{0\}. \tag{A.4}$$

Proposition 3 in Morris et al. (2020), applied to the current special case, proves that (A.2) is equivalent to

$$\Phi(I) > \Phi(S) \text{ for all } S \subsetneqq I. \tag{A.5}$$

The equivalence of this condition to (A.2), and hence to (A.4), can be understood as follows. The (signed) *Choquet integral* with respect to Φ (also known as the *Lovász extension* of Φ) is defined by

$$\begin{aligned}\int \lambda d\Phi &= \sum_{k=1}^{|I|} (\lambda_{i_k} - \lambda_{i_{k-1}}) \Phi(\{i_k, \ldots, i_{|I|}\}) \\ &= \sum_{k=1}^{|I|} \lambda_{i_k} (\Phi(\{i_k, \ldots, i_{|I|}\}) - \Phi(\{i_{k+1}, \ldots, i_{|I|}\})),\end{aligned}$$

where the components of $\lambda = (\lambda_i)_{i \in I} \in \mathbb{R}^I$ are ordered so that $\lambda_{i_1} \leq \cdots \leq \lambda_{i_{|I|}}$, and $\lambda_{i_0} = 0$. Note that

$$-\int (-\lambda) d\Phi = \sum_{k=1}^{|I|} (\lambda_{i_k} - \lambda_{i_{k-1}})(\Phi(I) - \Phi(\{i_k, \ldots, i_{|I|}\})) = \int \lambda d\Phi^{\#},$$

where $\Phi^{\#}(S) = \Phi(I) - \Phi(I \setminus S)$. Hence, (A.5) is equivalent to

$$-\int (-\lambda) d\Phi > 0 \text{ for all } \lambda \in \mathbb{R}^I_+ \setminus \{0\}. \tag{A.6}$$

It is readily verified from the definitions that we have $\min_{x\in W(\Phi)} \lambda\cdot x \leq \int \lambda d\Phi \leq \min_{x\in C(\Phi)} \lambda\cdot x$ for all $\lambda \in \mathbb{R}^I$, and hence

$$\max_{x\in W(\Phi)} \lambda\cdot x \geq -\int(-\lambda)d\Phi \geq \max_{x\in C(\Phi)} \lambda\cdot x \text{ for all } \lambda\in\mathbb{R}^I, \tag{A.7}$$

where the inequalities hold with equality when Φ is supermodular. This establishes, under the supermodularity of Φ, the equivalence of (A.6) to (A.2) and (A.4) (Figure 1).

Thus, Propositions 2 and 3 of Morris et al. (2020) can be viewed as an incomplete information generalization of the equivalence among (A.1)–(A.6).

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