Testing Revealed Preference Theory, I: Methodology

- The revealed preference theory developed last time applied to a single agent.

- This corresponds to a "within-subject" experiment: see same subject make choices from different menus.

- How to implement both a choice from the menu {apple, banana, orange} and from {banana, orange, pear}?

- Different days? Only one choice counts? Or don't implement choices, ask hypothetical questions? Each of these poses complications of various degrees of seriousness.

- Many experiments use "between-subject" designs: compare choices of one set of people from menu *A* to those of another set of people from menu *B.*

- Assign subjects randomly to the two conditions so the groups are statistically indistinguishable.

- What are the implications of standard preferences then?

- Let *X* be finite set of alternatives, and let $\Delta(X)$ be the space of probability distributions on *X.*

- Suppose each agent has strict (and complete transitive) preferences on *X* *.*

  - Enumerate the elements of *X,* and represent each preference by a 1-1 map $u : X = \{1, \ldots, |X|\} \to \{1, \ldots, |X|\}$. (*i.e. a permutation*)

- Let *U* be the finite set of all such maps.

- Represent the distribution of preferences in the population by some $\nu \in \Delta(U)$.

- Our data is now a <u>system of choice probabilities</u> (P, X): For each choice set *A* that we observe we see $P_A \in \Delta(A)$. To simplify notation I'll assume we see choice from every non-null subset of *X.*

- We'll say that the choice probabilities are *rationalizable* if they can be generated by some $\nu \in \Delta(U)$. Formally: A system of choice probabilities (P, X) is *rationalizable* if there is $\nu \in \Delta(U)$ s.t. for all nonempty $A \in X$ and all $x \in A$, $P_A(x) = \nu\left(\left\{u \in U \mid u(x) = \max_{y \in A} u(y)\right\}\right)$.

Observation: If (P, X) is rationalizable then for any $x \in X$, if $x \in A$ and $A \subseteq A'$ then $P_A(x) \geq P_{A'}(x)$.

This property is called "*regularity*" in the literature on stochastic choice.

Desired conclusion: If we give people a larger choice set then choice probability/market shares of the originally available items can't increase.

Aside: Regularity is necessary but not sufficient for the choice probabilities to be rationalizable. Will say more about this later if/when we discuss stochastic choice.

Qualifications: not sure how to extend this if a) if preferences aren't strict- then maybe assume uniform randomization? b) if samples are finite, need to do a power calculation.

Testing Revealed Preference Theory, II: Counterxexamples

Decoy Effect: Huber et al *J Cons Research* [1982]

- Six different product categories. Hypothetical choices.

- Each product described by 2 attributes, e.g. quality score and price.

- A “decoy” is a 3rd good that is dominated by one alternative but not the other.

- Adding these decoys can lead to violations of regularity which as we saw implies violations of WARP.

- Decoys can be “range increasing” or “frequency increasing” as in the next figure.

FIGURE B

DIFFERENT DECOY PLACEMENT STRATEGIES[a]

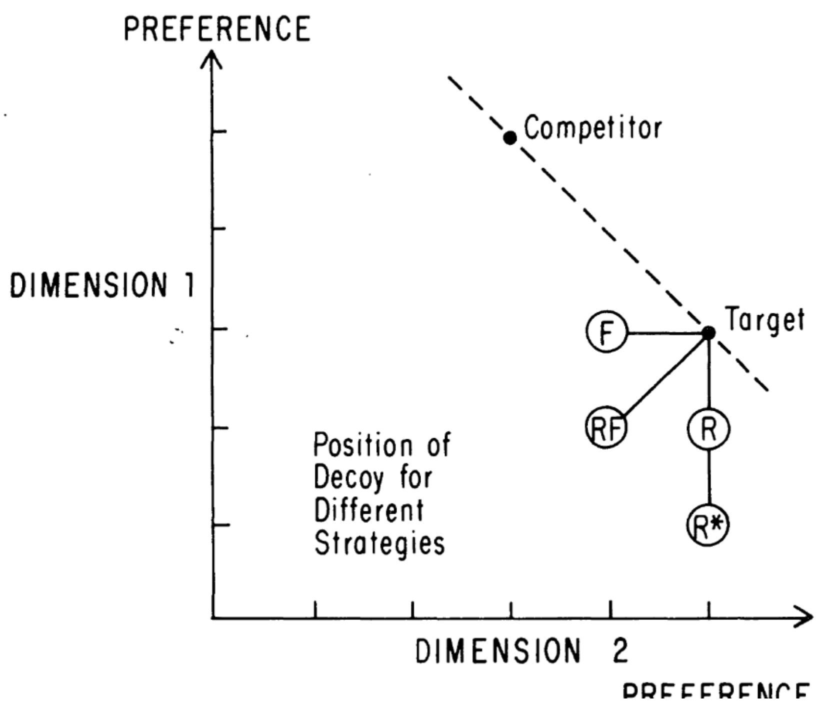

TABLE 1

EXAMPLES OF CHOICE SETS FOR DIFFERENT STRATEGIES

	Price/ sixpack	Quality rating
Range increasing (R)		
Target	$1.80	50
Competitor	$2.60	70
Added decoy	$1.80	40
Extreme range increasing (R*)		
Target	$1.80	50
Competitor	$2.60	70
Added decoy	$1.80	30
Frequency increasing (F)		
Target	$1.80	50
Competitor	$2.60	70
Added decoy	$2.20	50
Range-frequency (RF)		
Target	$1.80	50
Competitor	$2.60	70
Added decoy	$2.20	40

- On average over all 6 product categories the “range-increasing” decoys increased the target’s market share by .13, “frequency-increasing” decoys increased target’s share by .08.

- Cross-subject, weaker effect on individuals (53 vs. 56%) and p=.1.

- Later work calls it an “attraction effect”

- Subsequent work challenged just how often these decoys work.

Huber et al *JMarketingResearch* [2014]:

"..O*ur 1982 article was designed as a demonstration study showing that one could, under certain circumstances, obtain violations of the important theoretical assumption of regularity. We did not set out to suggest a tool for marketing practic*e…. *The most critical condition* (for the attraction effects) *is that people have either very weak or initially unformed preferences between the target and the competitor. They will be the people most affected by the attraction effect. The converse is also true: the greater the heterogeneity in basic trade-off values, the smaller the attraction effect… We suspect that the asymmetric dominance effect occurs rarely in the marketplace today… most market choices have multiple complex attributes rather than two numeric ones. The multiple attributes make it virtually impossible to find an alternative without some unique benefit. More importantly, people may have strong preferences for complex attributes (e.g., brand name, country of origin, product type), but there are situations in which those preferences are reversed…."*

- There is also evidence of a “compromise effect (Simonson *J Cons Research* [1989]): choose the “middle option”.

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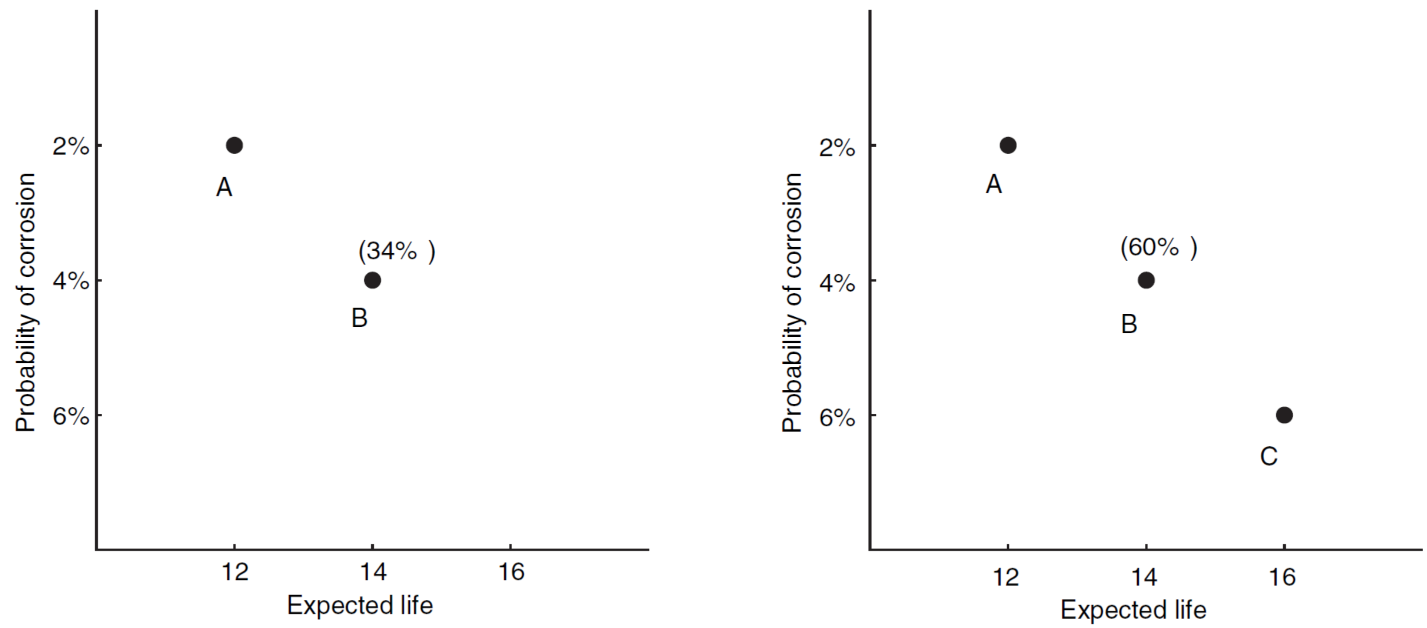


FIGURE 1. COMPROMISE EFFECT WITH CALCULATOR BATTERIES

Notes: Numbers in parentheses indicate percentage of subjects choosing the alternative. The difference is significant at $p < 0.05$.
Source: Simonson (1989).

- Also anecdotal evidence that the compromise effect makes premium loss-leaders effective, as in the Simonson-Tversky story about Williams of Sonoma bread makers: adding a $429 version increased the sales of the original $275 model.

- One way to model these sorts of regularity violations and context effects is to let utility depend on the menu.

- But without some structure on this dependence the model becomes vacuous at the individual level (so *a fortiori* when aggregating): any choice correspondence *c* can be generated from utility maximization by setting $u(x, A) = 1$ if $x \in c(A)$ and $u(x, A) = 0$ for $x \notin c(A)$.

- There are now many models and representation theorems for various sorts of "menu effects," and topic of ongoing research; I'll just mention 2.

- Ok, Ortoleva, and Riella *AER* [2015] provide an axiomatic characterization of a form of menu dependence where choice can depend on a "reference point."

- They assume that preference on pairs of item is transitive, and also assume that if $\{x, y\} \subseteq c(A)$ for some *A,* then $c(\{x, y\}) = \{x, y\}$.

- Then develop a notion of "revealed reference points," and use conditions on it to prove their characterization.

- The representation says that when a given menu has a reference point, the agent restricts attention to choices that dominate it according to each element of a collection of functions. (*these functions are a priori arbitrary but constrained by the choice data.*) E.g. in the decoy examples, the revealed "consideration set" is the objects that dominate the decoy in both dimensions.

- WARP violations can also arise when the menu contains information-e.g. maybe only restaurants that offer item A are any good at making item B.

- Plausible story but incomplete w/o a model of how firms choose menus.

- Kamenica *AER* [2008]: a monopolist and some consumers know a global preference parameter, other consumers infer it from the menu and the monopolist's strategy in an equilibrium of the incomplete-information game.

- Given the firm's optimal policy it makes sense for some consumers to use the rule "pick the middle item" (in terms of a characteristic like lumens.)

- Pset 1 has a simplified version of this idea.

Static Decisions Under “Risk” (Objectively Known Probabilities)

- Z: finite (*for now*) set of “prizes” or “consequences.”

- $\Delta(Z)$: probability distributions or “lotteries” on *Z.* (*implicit: the decision maker doesn’t care about how this distribution is determined, and in particular “reduces compound lotteries”*: the lottery ¼ chance of an apple, ¾ chance of a banana is the same as “flip two fair coins, get an apple if both heads and otherwise get a banana.)

- $\Delta(Z)$ is convex: for any $p,q \in \Delta(Z)$ and $\alpha \in [0,1]$, $\alpha p + (1-\alpha)q \in \Delta(Z)$ is the lottery that assigns probability $\alpha p(z) + (1-\alpha)q(z)$ to each $z \in Z$.

- We assume the agent has continuous, complete, transitive preference $\succsim$ on $\Delta(Z)$- so they can be represented by an ordinal utility function. (*we need continuity because* $\Delta(Z)$ *is infinite, it’s a convex subset of* $\mathbb{R}^{\#Z}$. *as noted last time can’t reject continuity with finite data.)*

- We're after more structure than this- an *expected utility* f*unction.*

- A *Bernoulli utility* is any function $u: Z \to \mathbb{R}$.

- The corresponding *expected utility* of a lottery $p \in \Delta(Z)$ is $U(p) = \sum_{z \in Z} p(z)u(z)$.

  (*once we prove the expected utility representation theorem I will be less careful and call both u and U the utility.*)

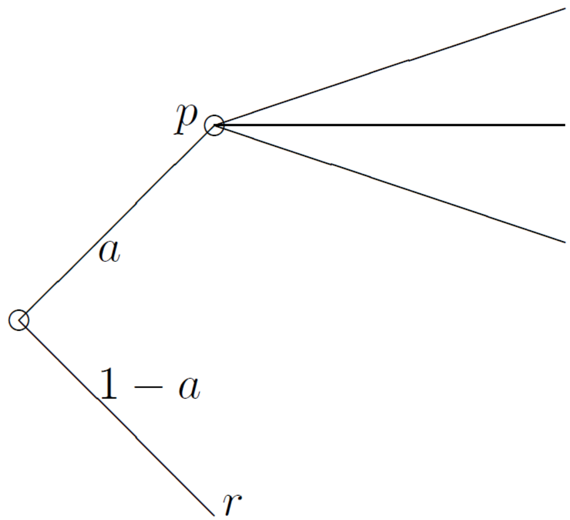
- $\succsim$ has an *expected utility representation* if there is a Bernoulli utility function *u* s.t. $U(p) = \sum_{z \in Z} p(z)u(z)$ represents $\succsim$.

- The key condition for expected utility is the *Independence Axiom*

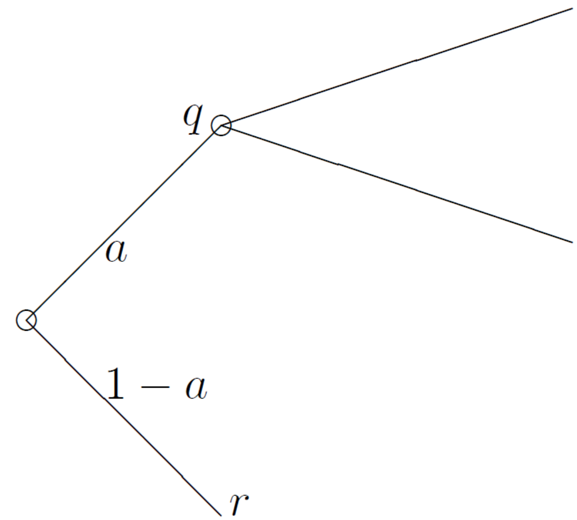
Independence Axiom: For all $p, q, r \in \Delta(Z)$ and $\alpha \in (0,1]$,

$p \succ q$ implies

$\alpha p + (1-\alpha)r \succ \alpha q + (1-\alpha)r$.



$ap + (1 - a)r$



$aq + (1 - a)r$

Compare two situations: agent chooses between *p* and *q* either before or after Nature chooses between *r* and the *p/q* branch.

The independence axiom follows from the combination of

(1) *Consequentialism on lotteries:* When deciding after Nature rules out *r,* the agent makes the same choice between *p* and *q* regardless of *r,*

and

(2) *Dynamic Consistency*: The agent makes the same decisions in the two choice problems.

So (to some people- including me) it's normatively appealing, though as we'll see next time it isn't always satisfied.

Later we'll see related definitions of consequentialism.

Lotteries δ_z that assign probability 1 to a single outcome z are called *Dirac measures.* Since Z is finite, these measures are a finite subset of $\Delta(Z)$.

So with complete transitive preferences there is a "best" Dirac measure, that is, a δ_B s.t. $\delta_B \succsim \delta_z \ \forall z \in Z$,and a "worst" Dirac measure δ_W . (*neither need be unique*)

Claim: If $\succsim$ on $\Delta(Z)$ satisfies Independence then it is monotonic in the following sense:

For any $p, q \in \Delta(X)$ with $p \succ q$ and $0 \le b < a \le 1$,

$$ap + (1-a)q \succ bp + (1-b)q \ .$$

Proof:

$$ap + (1-a)q = bp + (a-b)p + (1-a)q \quad \text{(algebra)}$$

$$\succ bp + (a-b)q + (1-a)q \quad \text{(independence axiom)}$$

$$= bp + (1-b)q. \quad \text{(algebra)}$$

Claim: If $\succsim$ on $\Delta(Z)$ is complete, transitive, and satisfies Independence, it has best and worst elements δ_B, δ_W, so that $\delta_B \succsim p \succsim \delta_W$ for all $p \in \Delta(Z)$.

(prove by induction on the cardinality of the support of p: if it holds for all *q* with #support=j, and *p* has #support= j+1, write $p = (1-a)q + a\delta_z$ for some z and appeal to the Independence Axiom:

$$\delta_B = \alpha\delta_B + (1-\alpha)\delta_B \succsim \alpha\delta_B + (1-\alpha)\delta_z$$
$$\succsim \alpha q + (1-\alpha)\delta_z = p$$
$$\succsim \alpha\delta_W + (1-\alpha)\delta_z \succsim \delta_W.$$

Theorem (vN-M [1944]) Preference $\succsim$ on $\Delta(Z)$ has an expected utility representation iff it is complete, transitive, continuous, and satisfies the Independence Axiom.

Proof: Obvious that the representation implies the stated conditions.

Proof that the conditions are sufficient for the representation.

If $\delta_B \sim \delta_W$ then from transitivity $\delta_B \sim \delta_z$ for all *z,* and from Independence $\delta_B \sim p$ for all $p \in \Delta(Z)$, so we can represent the preferences with $u(z) = 0 \forall z$.

If $\delta_B \succ \delta_W$ define $f : [0,1] \to \Delta(Z)$ by $f(\lambda) = \lambda\delta_B + (1-\lambda)\delta_W$.

(*f* maps into $\Delta(Z)$ because $\Delta(Z)$ is convex.)

From monotonicity, $f(\lambda) \succ f(\lambda')$ iff $\lambda > \lambda'$. So

$f(1) \succsim p \succsim f(0) \quad \forall p \in \Delta(Z)$.

Because preferences are continuous and the set of lotteries is connected, $\forall p \in \Delta(Z)$, $\exists \lambda(p)$ s.t. $\lambda(p)\delta_B + (1-\lambda(p))\delta_W = f(\lambda(p)) \sim p$, and from monotonicity this λ is unique.

Now verify that setting $u(z) = \lambda(\delta_z)$ yields an expected utility representation of $\succsim$.

(i) Suppose $\sum_z p(z)\lambda(\delta_z) \geq \sum_z q(z)\lambda(\delta_z)$.

Then from the Independence axiom and monotonicity,

$$
\begin{aligned}
p = \sum_z p(z)\delta_z &\sim \left(\sum_z p(z)\lambda(z)\right)\delta_B + \left(1 - \sum_z p(z)\lambda(z)\right)\delta_W \\
&\succsim \left(\sum_z q(z)\lambda(z)\right)\delta_B + \left(1 - \sum_z q(z)\lambda(z)\right)\delta_W \sim q.
\end{aligned}
$$

(ii) Can similarly show that if

$\sum_z p(z)u(z) < \sum_z q(z)u(z)$, then $q \succ p$.

Cardinal Uniqueness

If Bernoulli utility functions *u* and *u'* both give expected utility representations of $\succsim$, there is $a > 0$ and *b* s.t. for all $z \in Z$, $u(z) = au'(z) + b$, and all such *a,b* give equivalent representations. ("*unique up to affine transformations/ affine uniqueness.*")

Proof: It is clear that the two utility functions generate the same preferences if $u(z) = au'(z) + b$: adding a constant to the utility of all outcomes won't change their ranking and neither does multiplying ever utility by *a* (provided $a > 0$): it's like the conversion from Celsius to Fahrenheit.

The converse- that we have to have $u(z) = au'(z) + b$ for *u* and u' to represent the same preferences, is immediate if *u* is constant.

Suppose *u* is not constant.

As above there is a unique function λ s.t. $\lambda(p)\delta_B + (1-\lambda(p))\delta_W \sim p$.

So
$$u(z) = \lambda(z)u(B) + (1-\lambda(z))u(W)$$
$$u'(z) = \lambda(z)u'(B) + (1-\lambda(z))u'(W)$$

Rearranging terms:

$$\frac{u(z)-u(W)}{u(B)-u(W)} = \frac{u'(z)-u'(W)}{u'(B)-u'(W)}$$

So $u(z)-u(W) = \frac{u(B)-u(W)}{u'(B)-u'(W)}\left(u'(z)-u'(W)\right) = au'(z)+b$, $a>0$. ■

Note: if we only see preferences on a finite subset of lotteries we can't pin down the utility functions this precisely.

Now suppose prizes are money that you might receive at the end of class today.

The *certain equivalent (or certainty equivalent)* of a gamble is the certain amount that leaves you just indifferent

Write down your own certain equivalents for:

(a) ½ 0, ½ $10,000. Call it CE(a)
(b) what is your CE of (1/2 0, ½ CE(a))?
(c) What is your CE of (1/2 CE(a), ½ $10,000)

Lotteries with Money Payoffs

- Suppose that the prizes are money, so $Z \subseteq \mathbb{R}$, and possibly infinite.

- Assume expected utility representation, with Bernoulli utility *u* that is continuous and strictly increasing- more money is preferred to less.

  (*Strzalecki 5.3, optional, gives conditions for the EU theorem to hold with infinite Z*)

  (*continuous and strictly increasing functions are differentiable almost everywhere, to simplify assume u is twice differentiable.*)

- Consider only Lebesgue-measurable lotteries $p \in \Delta(Z)$ and identify them with their cumulative distribution functions: $F_p(x) = \Pr(z \le x)$.

- Let $Ep = \int z dF(z)$ be expected value of *p.* (*restrict attention to lotteries where this is finite*)

Let δ_{Ep} be the deterministic lottery (Dirac delta measure) on the expected value of lottery *p.*

Definition Preference $\succsim$ is

- *risk averse* if for all *p* $\delta_{Ep} \succsim p$
- *risk neutral* if for all *p* $\delta_{Ep} \sim p$
- *risk loving* if for all *p* $p \succsim \delta_{EP}$

Theorem (Jensen's inequality): An expected utility preference is risk averse/risk neutral/risk loving if *u* is concave/affine/convex.

Definition: Preference $\succsim_1$ is *more risk averse* than $\succsim_2$ if for all $z \in Z$ and $p \in \Delta(Z)$, $\delta_z \succsim_2 p$ implies $\delta_z \succsim_1 p$ and $\delta_z \succ_2 p$ implies $\delta_z \succ_1 p$.

Definition: The *coefficient of absolute risk aversion* at *z* is $A(z) := -u''(z)/u'(z)$.
(a "scale-free" measure of local concavity.

Theorem: Suppose $\succsim_1$ and $\succsim_2$ are EU preferences represented by strictly increasing, twice-differentiable and concave Bernoulli utility functions u_1, u_2 , with associated absolute risk aversions A_1, A_2. Then the following three conditions are equivalent:

1. $\succsim_1$ is *more risk averse* than $\succsim_2$.

2. There is a concave function $g : range(u_2) \to \mathbb{R}$ s.t. $u_1 = g \circ u_2$.
(*because* u_1, u_2 *are both continuous and differentiable so is* g.)

3. $A_1(z) \geq A_2(z) \forall z$.

Idea of Proof: Use Jensen's inequality to show equivalence of 1 and 2, use calculus to show equivalence of 2 and 3; *Details are homework.*

Reading for next time: MWG 6C,D,E Rabin *Ema* [2000].