Deterministic Dynamic Choice

- Today 2 periods, $t = 0,1$.

- Use fairly general representations to clarify some conceptual points; the next lecture will look at more structured representations (e.g. additively separable) for choice over multiple time periods.

- *Warning: the model covered today has a lot of notation we don't usually both with; its purpose is to let make assumptions explicit and not hidden when they are implicit in the notation.*

- Spaces Z_0, Z_1 of alternatives.

- In period 0 the agent may face a choice of period-1 menus, and also choose a period-0 alternative, and these two may be linked. So chooses an element of $X_0 := Z_0 \times M(Z_1)$. (*recall that* $M(Z) = 2^Z - \{\varnothing\}$.)

- In period 1 the agent chooses an element of $X_1 := Z_1$.

- *Example*: agent has initial wealth w_0. Can spend some of it but not borrow; can save at real rate *r.* So period-0 menu is

  $$\left\{(c_0, M(c_0)) : c_0 \in [0, w_0], M(c_0) = \left\{c_1 \in [0, (1+r)(w_0 - c_0)]\right\}\right\} .$$

  Note that not all second period menus are consistent with all first-period choices: if the agent consumes all her wealth in period 0 she can't consume in period 1.

- In general, period- 0 choices are described by a choice correspondence

  $c_0 : M\left(X_0\right) \to M\left(X_0\right)$ s.t. $c_0(A_0) \subseteq A_0$ for all $A_0 \in M(X_0)$.   (0)

  Here $c_0(A_0)$ is a finite collection of pairs $\left\{(z_0', A_1'), (z_0'', A_1''), ..\right\}$.

- Period 1 choices might depend on period-0 consumption.

- And (for now) let's allow the possibility they also depend on the choice problem the agent faced in period 0.

- So define the period-0 histories h_0 to be pairs (A_0, x_0), where A_0 is the menu the agent faced, and x_0 is the choice she made.

- The set of all possible period-0 histories is then

$$H_0 := \{(A_0, x_0) \in M(X_0) \times X_0 : x_0 \in c_0(A_0)\}.$$

- Note the restriction to "intended choices" $x_0 \in c_0(A_0)$): can't observe choice in period 1 when the agent didn't get their intended period-0 outcome.

- Would learn more about preferences if agents were forced to "tremble" as in Frick, Iijima, and Strzalecki [2017]; won't cover that here.

For each period-1 menu A_1, let $H_0(A_1)$ be the period-0 histories consistent with it:

$$H_0(A_1) := \left\{ \left(A_0, x_0 \right) \in H_0 : x_0 = (z_0, A_1) \text{ for some } z_0 \in Z_0 \right\} .$$

The domain of period 1 choice correspondence- the set it's defined on- is the current menu A_1 and the history that preceded it. Denote this as

$$\mathcal{D}_1 := \left\{ (A_1, A_0, x_0) \in M(X_1) \times H_0 : (A_0, x_0) \in H_0(A_1) \right\} .$$

- *The period 1 choice correspondence* is a

$$c_1 : \mathcal{D}_1 :\to M(X_1) \text{ with } c_1\left(A_1, A_0, x_0 \right) \subseteq A_1 . \qquad (1)$$

- A *dynamic choice correspondence* is a pair (c_0, c_1) that satisfies (0) and (1).

- **Definition:** A dynamic choice correspondence is *consequentialist* if $c_1(A_1, A_0, x_0) = c_1(A_1, B_0, x_0)$ for all A_0, B_0 s.t. for all $(A_0, x_0), (B_0, x_0) \in H_0(A_1)$ s.t. $x_0 = (z_0, A_1)$ for some $z_0 \in Z_0$.

  -Note that this <u>does</u> let period-1 choice vary with the period-0 choice x_0 *.*

  -It requires that the agent makes the same choice from A_1 *regardless of the period- 0 <u>menu</u>* A_0 *or* B_0 *. The "s.t." part says to only look at situations where the period-0 menu allows picking* x_0*.*

Suppose that in period 0 you eat lunch and pick a restaurant for dinner, in period 1 you order dinner.

- Consequentialism allows what you order at dinner (period 1) to depend on the z_0 you had for lunch. But it requires that what you order at dinner in an Italian restaurant doesn't depend on whether the alternative restaurant you considered was Greek or Spanish.

- Same idea as the "consequentialism" I defined for static choice under risk, which said that if first Nature decides whether to implement lottery *r* or give you a choice between *p* and *q*, your choice doesn't depend on what *r* was.

- In both settings, consequentialism is a form of "no regret" condition.

- It allows dependence on things that have happened in the past, but not on things that "might have happened."

- From here on assume consequentialism and write $c_1(A_1, x_0)$.

- Note that w/o consequentialism period-1 choice at A_1 can be different for each A_0 that leads to it- so hard to have non-vacuous consistency conditions on period-1 choice.

- The next condition is not needed to define "rational" dynamic choice, and in some applications such as habit formation it is relaxed. But it is commonly assumed to simplify:

  Definition: Choice is *history-independent* if $c_1(A_1, z_0) = c_1(A_1, z_0^{'})$ for all $z_0, z_0^{'} \in Z, A_1 \in M(X_1)$.

- Will assume history independence (aka *time separability*) for the rest of this lecture and write period-1 choice as $c_1(A_1)$.

- In fact will now go further and suppose there is no period-0 consumption choice, and set $X_0 = M(Z_1)$.

- Assume that both c_0 and c_1 satisfy WARP.

- So they correspond to maximizing complete transitive preferences $\succsim_0$ and $\succsim_1$, and (because Z_1 is finite) to maximizing utility functions u_0 and u_1.

- Note: u_0 and $\succsim_0$ are defined on $X_0 = M(Z_1)$.

- They induce a utility function and preference on Z_1 by looking at singleton menus $\{z_1\}$.

- But the domain of singleton menus is too small to determine period-0 preferences without additional conditions.

Definition: Preferences $(\succsim_0, \succsim_1)$ have a *recursively consistent representation* if they can be represented by u_0, u_1 s.t. $u_0(A_1) = \max_{z_1 \in A_1} u_1(z_1)$.

(*Note: Strzalecki calls this a "dynamically consistent representation" but then calls another condition "dynamic consistency of preferences."*)

- Economists usually (but not always!) use recursively consistent representations.

- Let's try to understand them better by seeing when they apply.

Definition: Period-0 preference $\succsim_0$ is *strategically rational* if $A_1 \succsim_0 B_1 \Rightarrow A_1 \sim_0 A_1 \cup B_1$.

- Note that this needn't be true if the period-1 choice isn't made to maximize period-0 preferences.

- For example B_1 might be a "temptation": Let B_1 be $\{\text{Scotch}\}$ and $A_1 = \{\text{Pellegrino}\}$.

- If Drew thinks that if he buys Scotch now (period 0) he will drink more of it tonight than he ought to, then we could have $A_1 \succ_0 B_1$ and $A_1 \succ_0 A_1 \cup B_1$.

- OTOH if period-1 preference is stochastic, then it might be that $A_1 \succsim_0 B_1$ and $A_1 \prec_0 A_1 \cup B_1$; the larger menu adds an "option value."

Lemma (Kreps *Ema* [1979]): Period-0 preference $\succsim_0$ is *strategically rational* iff the function $u_0(A_1) = \max_{z_1 \in A_1} u_0(\{z_1\})$ represents $\succsim_0$.

Proof sketch: For each A_1 list its elements in decreasing preference order, $\{z_1^1\} \succsim_0 \{z_1^2\} \succsim_0 \ldots \succsim_0 \{z_1^{\# A_1}\}$.

By strategic rationality $\{z_1^1\} \sim_0 \{z_1^1, z_1^2\}$, and by induction $\{z_1^1\} \sim_0 A_1$ so $u_0(A_1) = \max_{z_1 \in A_1} u_0(\{z_1\})$.

Conversely if $u_0(A_1) = \max_{z_1 \in A_1} u_0(\{z_1\})$,

then $u_0(A_1 \cup B_1) = u(A_1)$ so $A_1 \sim A_1 \cup B_1$. ■

- Strategic rationality is a condition only on period-0 choice, so it can't link period-0 and period-1 choices. (Although it can suggest possibilities that are consistent with the period-0 choice and a given way of linking the two periods together.)

Definition: Preferences $(\succsim_0, \succsim_1)$ are *dynamically stable* if
$\{z_1\} \succsim_0 \{w_1\} \Leftrightarrow z_1 \succsim_1 w_1$.

Theorem: Preferences $(\succsim_0, \succsim_1)$ have a *recursively consistent representation* iff they are strategically rational and dynamically stable.

Proof: easy HW.

Recursive consistency rules out stochastic period-1 preferences.
To allow stochastic preference, Kreps (*Ema* [1979]) provides a representation theorem for the representation

$u_0(A_1) = \sum_{s \in S} p(s)[\max_{a_1 \in A_1} u_1(a_1, s)]$ for some *S, p,* and u_1. (3)

Here u_1 can depend on *s* but neither it nor *p* can depend on A_1.
(*Note this reduces to recursive consistency when S is a singleton.*)

With representation (3), if $\{a,b\} \sim_0 \{a\}$ then the agent can never strictly prefer to add *b* to any menu that contains *a:* if adding *b* helps menu *C* it means sometimes *b* is better than *a* so we would have $\{a,b\} \succ_0 \{a\}$.

More generally, representation (3) satisfies *modularity:*

If $A_1 \sim_0 A_1 \cup B_1$ then $A_1 \cup C_1 \sim_0 A_1 \cup C_1 \cup B_1$ for all C_1.

Kreps shows that period-0 menu preferences have representation (3) iff they satisfy modularity and

"Preference for Flexibility" : If $A_1 \supseteq B_1$ then $A_1 \succsim_0 B_1$. (*)

Aside: I prefer to call (*) *monotonicity* as it can have other interpretations.

Note that this result is about the representation of choice in period 0, and doesn't say anything about period-1 choice; see Ahn and Sarver *Ema* [2013].

Temptation and Self Control

A strategically rational agent never strictly prefers a smaller menu, and wouldn't sign up for a monitoring program that has only fines and no positive payments.

Many experiments where a non-trivial (though sometimes small) fraction of participants prefer a smaller menu:
Ashraf, Karlan and Yin *QJE* [2006] 28% of subjects choose a commitment savings account; Gine, Karlan and Zimmerman *AER* [2010] 11% of smokers agree to be fined if they don't quit; Kaur, Kremer and Mullainathan *JPE* [2015] commitment contracts chosen 35% of the time (averaging over days and workers); Houser et al [2010] 36% of subjects use commitment device to keep from web surfing during a lab experiment; Augenblick, Nierderle, and Sprenger *QJE* [2015] 58% of subjects prefer costless commitment in a work now/work later task (though only 9% will pay more than $0.25 for it.

Period-0 preference $\succsim_0$ has *a* *Strotz representation* (Strotz *REStud* [1955]) if there are u_0, u_1 s.t.

$$c_1(A_1) = \arg\max_{z_1 \in A_1} u_1(z_1) \text{ and } c_0(A_0) = \arg\max_{A_1 \in A_0, z_1 \in c(A_1)} u_0(z_1).$$

- Here the agent is "sophisticated": she knows not only that her period-1 preferences will be different but what they will be, and picks a menu accordingly- it's as if the observed choices are the equilibria of a game between these two selves.

- Strotz preferences aren't concerned by items in the menu that won't be chosen. This rules out a preference to avoid temptations that would be resisted.

Definition Period-0 preference $\succsim_0$ satisfies **no compromise** if for all $A_1, B_1 \in M(Z_1)$ either $A_1 \sim_0 A_1 \cup B_1$ or $B_1 \sim_0 A_1 \cup B_1$.

Theorem (Gul and Pesendorfer *REStud* [2005]): Period-0 preference $\succsim_0$ satisfies no compromise iff it has a Strotz representation.

Not-very-revealing proof outline: *define a revealed preference on subsets, and a separate revealed preference on singletons, and then relate them.*

- The Strotz representation is closely related to quasi-hyperbolic discounting, where preferences represented by $U_0 = u_0 + \beta\delta u_1 + \beta\delta^2 u_2$ and $U_1 = u_1 + \beta\delta u_2$
- Here the period-1 utility function U_1 differs from period-0 utility U_0 in its tradeoffs between immediate rewards in period 1 and rewards that will arrive later, say in a period 2 where the agent has no decision to make. The idea is that the period-1 agent will sacrifice period 2 consumption for consumption in period 1 at worse terms than the period-0 agent would like.

- O'Donoghue and Rabin *AER* [1999] define "partially naïve" Strotz models: agent realizes that his future self will be present biased but mis-forecasts how large that bias will be.

- Ahn, Iijima, Le Yaouonq, and Sarver mimeo [2016]: representation theorem for partially naïve Strotz.

- Gul and Pesendorfer *Ema* [2001]: the value of a menu is $u_0(A_1) = \max_{z_1 \in A_1} u_1(z_1) - \left(\max_{x_1 \in A_1} v_1(x_1) - v_1(z_1)\right)$.

- Here v_1 is an arbitrary "temptation function," and the control cost of choosing z_1 from menu A_1 is the resisted temptation $\left(\max_{z_1' \in A_1} v_1(z_1') - v_1(z_1)\right)$.

- Important: only the biggest temptation in A_1 matters- fits with perfectly foreseen preferences.

- This comes from their assumption of set betweenness: $A_1 \succsim B_1 \rightarrow A_1 \succsim A_1 \cup B_1 \succsim B_1$.

- Control cost is linear in foregone utility from a version of the independence axiom, on a different and more complex space. But there is some evidence for convex costs.

- To model period-1 choice GP specify dynamic consistency, so that
$$c_1(A_1) = \arg\max_{z_1} \left(u_1(z_1) - \left(\max_{x_1 \in A_1} v_1(x_1) - v_1(z_1)\right)\right):$$
$$= \arg\max_{z_1} u_0(z_1) + v_1(z_1)$$

Note that the "temptation" $\max_{x_1 \in A_1} v_1(x_1)$ *doesn't alter period-1 choice.*

- Fudenberg and Levine *AER* [2006] show that a similar representation describes the equilibrium of a game between a "long run self" and a sequence of completely myopic short-run selves who have the same per-period utilities (and so differ from the LR only in their discount factors).

- Dual selves: a long-run "planner" can exert effort to change the preferences of a myopic "doer.

- As in GP this control cost depends on foregone utility.

- The subgame-perfect equilibrium is as if LR self maximizes

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t\left(u_t(z_t)-\gamma(\Delta_t)\right)$$

where $\Delta_t = \max_{z_t^{'}\in A_t} u(z_t^{'}) - u(z_t)$ is the difference between the maximum feasible utility in the current period and the utility actually received.

- More restrictive than GP because same utility function for temptation and choice instead of the pair (u, v): the two selves differ only in their discount factors, as in quasi-hyperbolic preferences.

- Less restrictive than GP because γ can be strictly convex.

- When γ strictly convex, it's more than twice as hard/costly to resist twice the temptation.

- Convex control costs allow certain (but not all!) violations of WARP, such as the "compromise effect" : pick fruit from {fruit, small desert} but small desert from {fruit, small desert, large desert.}

- This can't happen with GP's linear cost function because there the action chosen maximizes $w(z_1) := u_0(z_1) + v_1(z_1)$- so choice satisfies WARP.

- Convex costs explain why more self-indulgent in one domain (e.g. diet or exercise) when exerting more self control in another (e.g. hours of work).

- Can also be used to explain the effect of cognitive load (Shiv and Fedorikhin *J. Cons. Res.* [1999]:

- Subjects were asked to memorize either a two- or a seven-digit number, and then walk to a table with a choice of two deserts, chocolate cake and fruit salad.

- Subjects would then pick a ticket for a desert and report the number and their choice in a second room. Longer number to memorize: % choosing cake increases from 41% to 63%.

- Ward and Mann *J Pers. Social Psych.* [2000] report a similar effect of cognitive load.

- Dual-self explanation: control cost depends on the sum of cognitive load and foregone utility.

Toussaert [2016] : Subjects paid for doing a tedious task, face temptation to forego earnings to hear a story.

- Elicit preference ordering over {story}, {no story}, {choose later}, when the preferred choice is implemented *stochastically*: highest ranked menu is most likely but not certain. *Here "choose later" means choose the subsequent menu {story, no story}*

- "self-control types" can rank {no story}$\succ$ {choose later}$\succ${story}.

- Strotz preferences can't do this, as they aren't concerned by items in the menu that won't be chosen.

- Implement (stochastic) distribution over menus: now can observe 2nd period choices of subjects who preferred commitment. (w/o this stochastic element, the 2nd period choice of someone who chose not to have a 2nd period choice is hypothetical/counterfactual.)

- To distinguish indifference from strict preference, offered subjects the chance to pay to get their 1st choice if lottery says get 2nd- either in $ or in extra time to work (using a price list mechanism as in the BDM procedure.)

- Also asked subjects' beliefs about what they'd do w/o commitment. (*and used predictions of other subject's second-period choice as an instrument…*)

Findings:

- 36% of subjects report {no story} $\succ$ {choose later} $\succ$ {story}.

- 58% of this group (so 25% of overall pool) willing to pay for a commitment.

- Only 2.5% of subjects consistent with Strotz preferences.

- Almost all self-control types predicted that they would resist the temptation to learn the story in the absence of commitment- no support for “random temptation” or “random Strotz” models.

- Perceived self-control almost exactly matches observed self control when making the choice: 18%.

Now back to “standard” preferences and work up to more than 2 periods…

Additively separable discounting: $U(z_0, z_1, ..) = \sum_t \delta^t u(z_t)$.

- Used by (most?) models of choice over time.

- Here the same utility function is used in every period, and the discount factor δ is constant.

- This rules out e.g. exogenously changing tastes, habit formation as in Becker-Murphy *JPE* [1988], and a preference for consumption streams that increase over time. Can pick these up with a state variable that tracks the payoff-relevant aspects of past consumption, won't do that here.

- To understand the restrictions that the discounting representation imposes on choice, need to first understand when preferences are *additively separable,* e.g. when can we decompose $u(\text{apples, oranges})$ into $u_a(\text{apples}) + u_o(\text{oranges})$?

Separable Preferences

- Let *I* be a finite (for now) set of indices (e.g. time periods, fruits, states). (*We will see a representation theorem for countably many time periods, it needs more assumptions. And the expected utility representations extend to uncountable state spaces, this also needs more structure.*)
- For each $i \in I$ there is a set X_i, let $X := \times_{i \in I} X_i$.
- Analyst observes complete transitive preference $\succsim$ on *X.*
- **Definition:** $\succsim$ has an *additively separable representation* if there are $u_i : X_i \to \mathbb{R}$ s.t. $U(x_1,\ldots,x_n) = u_1(x_1) + \ldots + u_n(x_n)$ represents $\succsim$.
- In an additively separable representation, the tradeoff between any x_i and x_j is independent of the other components, i.e. of $X_{-i,j}$.

- For any $E \subseteq I$ and any $x, y \in X$ define $x_E y \in X := \begin{cases} x_i & i \in E \\ y_i & i \notin E \end{cases}$.

- **Definition:** $\succsim$ is *singleton separable* if for all $i \in I$ and all $x, y, z, z' \in X$, $x_i z \succsim y_i z \leftrightarrow x_i z' \succsim y_i z'$. (*this should remind you of the independence axiom of expected utility*!)

- Singleton separability implies that for each index *i* we have a complete transitive preference $\succsim_i$ on X_i that is independent of the other components: $x_i \succsim_i y_i$ if $x_i z \succsim y_i z$ for some *z.*

- Already restrictive, but not sufficient, because it doesn't yet imply that the tradeoff between any x_i and x_j is independent of the other components.

- For example if $X_1 = \{1,2,3\}, X_2 = \{1,3,5\}$, the preference induced by $u(x_1.x_2) = x_1 x_2 + x_1^{x_2}$ is strictly increasing in x_1 for each x_2 and vice versa. But it does not have an additive representation (HW).

- In the discounting application, we need an additive representation if we want the tradeoff between consumption in periods *t* and *s* to be independent of consumption in other periods.

- So we use a stronger condition:

  $\succsim$ has *jointly separable indices* if for any $E \subseteq I$ and all $x, y, z, z' \in X$, $x_E z \succsim y_E z \leftrightarrow x_E z' \succsim y_E z'$. (*Strzalecki calls this "separable."*).

- With 3 or more indices this say the tradeoffs between x_i and x_j don't depend on the level of some 3rd index *k.*

- For this to have any bite we need 3 indices that "matter."

- *Reason:* with only 2 indices, jointly separable indices reduces to singleton separability, and as we saw that doesn't imply an additive representation. And having 3 with one that doesn't matter is like having 2.

- **Definition:** An index *i* is *null* if for all $x, y, z \in X$, $x_i z \sim y_i z$.

- The next theorem will ask that every index is non-null, and also that each X_i is connected; together these two conditions mean that each coordinate has a continuum of elements.

- Since additively separable representations have a utility function, we expect to need a continuity condition when X isn't finite.

- “Technical condition”: Assume each X_i is a connected subset of $\mathbb{R}^k$ (or more generally a connected topological space) and that the preference $\succsim$ on $X := \times_{i\in I} X_i$ is continuous w.r.t. the product topology. (*result extends to more general topological spaces)*

Theorem (Debreu [1960], generalized by Wakker *J Math Pyschology* [1988]): Suppose complete transitive preference $\succsim$ satisfies the technical condition and has at least three non-null indices. Then it has jointly separable indices iff it has an additively separable representation by continuous utility functions $u_i : X_i \to \mathbb{R}$ s.t. u_i is constant whenever *i* is null. Moreover, if $v_1,...,v_n$ also represent $\succsim$, then there are $\alpha > 0$ and β_i s.t. $v_i = \alpha u_i + \beta_i$.

Proof: omitted.

- ***Reading for next time***: Strzalecki Ch 3.1-3.3, Ch 4.