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Money, Cryptocurrency, and Public Debt as Bubbles: Implications for Indeterminacy, Fiscal Policy, Liquidity Pricing and the Interest Rate (Lecture 10)

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The Value of Bitcoin and Cryptocurrency

1 Bitcoin equals

9,085.67 United States Dollar

Mar 6, 5:09 PM UTC · Disclaimer

1	Bitcoin
9085.67	United States Dolla

Data provided by Morningstar for Currency and Coinbase for Cryptocurrency

OLG Model

Bubbles in Overlapping Generations Models

- Tirole (1985), based on Diamond (1965)'s OLG model.
  - 2 period-lived agents. Work only in the first.
  - Population $L_t = (1+n)^t$
  - One physical good per period.
  - CRS technology $Y_t = F(K_t, L_t) = L_t f(k_t)$, operated by competitive firms:

$$r_t = f'(k_t) \quad w_t = f(k_t) - k_t f'(k_t) = \phi(r_t)$$

  - Program of agent born at t

$$\begin{aligned} \max \quad & u(c_{1t}) + \beta u(c_{2t+1}) \\ \text{s.t.} \quad & c_{1t} + s_t = w_t \\ & c_{2t+1} = (1 + r_{t+1}) s_t \end{aligned}$$

    implies a savings function $s_t = s(w_t, r_{t+1})$

Equilibrium with no bubbles

- Asset market clearing:

$$K_{t+1} = L_t s\left(w_t, r_{t+1}\right)$$

dividing by L_t

$$(1+n)\, k_{t+1} = s\left(w_t, r_{t+1}\right) = s\left(w\left(k_t\right), r\left(k_{t+1}\right)\right) \equiv \mathcal{S}\left(k_t, k_{t+1}\right)$$

- Assume $\frac{dk_{t+1}}{dk_t} = \frac{\mathcal{S}_1}{1+n-\mathcal{S}_2} \in [0,1]$: then we have Solow dynamics

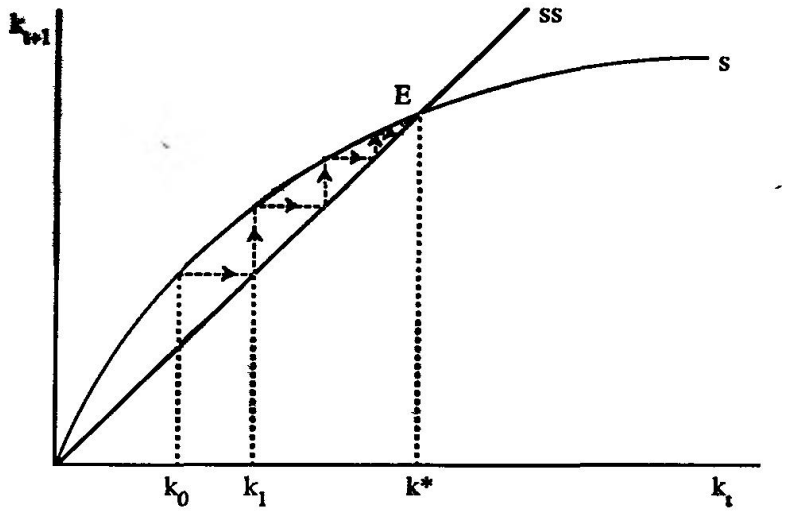


Figure 3.2

Equilibrium with bubbles

Note it's consumption per unit money, not the other way around.

- Introduce M pieces of paper. Let us look for an equilibrium in which these are valued at price p_t each period.
- The gross rate of return on bubbles is then $\frac{p_{t+1}}{p_t}$. No arbitrage with capital implies

$$1 + f'(k_{t+1}) = \frac{p_{t+1}}{p_t} \tag{2}$$

- Write $B_t = Mp_t$ for the aggregate value of the bubble, and $b_t = \frac{B_t}{L_t}$. Then (2) implies

$$B_{t+1} = B_t \left(1 + f'(k_{t+1})\right) \quad \Rightarrow \quad b_{t+1} = b_t \left(\frac{1 + f'(k_{t+1})}{1 + n}\right)$$

Equilibrium with bubbles

- Savings can now be done in capital and bubbles:

$$K_{t+1} + B_t = L_t s\left(w_t, r_{t+1}\right)$$

so

$$k_{t+1} = \frac{s\left(k_t, k_{t+1}\right) - \mathbf{b_t}}{1+n} \tag{3}$$

the bubble reduces capital accumulation. We have $k_t \geq 0$ and assume free disposal of bubbles: $b_t \geq 0$.

Steady-state

- Write (3) in steady-state: $b = s(k,k) - (1+n)k$
- On the other hand

Substituting from equation 2

$$b_{t+1} = b_t \left(\frac{1 + f'(k_{t+1})}{1+n} \right)$$

implies that if $b_t \neq 0$ is to be constant, then we must have $f'(k) = n$ at the steady-state

- Recall that with a neoclassical production function and no depreciation, the steady-state resource constraint is

$$f(k) = c + k(1+n) - k = c + nk$$

Steady-state per-capita consumption is maximal for the golden rule level of capital:

$$f'(k^*) = n$$

Dynamic inefficiency

- Moreover, for $k > k^*$, a *decrease* in steady-state capital increases steady-state per-capita consumption
  - the economy has "overaccumulated" capital. Productivity is insufficient to cover the resources used each period to provide the newborn with the current level of capital per person
- So, when $k > k^* \Leftrightarrow f'(k) < n$ the economy is **dynamically inefficient**: a Pareto-improvement can be reached by increasing the consumption of the current generation, reducing the stock of capital, and therefore increasing the consumption of all future generations

Dynamics

- Assume dynamic inefficiency. Let's look at the "phase diagram" (this is not completely exact, since the dynamical system is discrete)
  - $k_{t+1} - k_t = \frac{s(k_t, k_{t+1}) - b_t}{1+n} - k_t \equiv g(k_t, b_t)$
  - $b_{t+1} - b_t = \frac{b_t\left(f'(k_{t+1}) - n\right)}{1+n} = \frac{b_t\left(f'(k_t + g(k_t, b_t)) - n\right)}{1+n}$

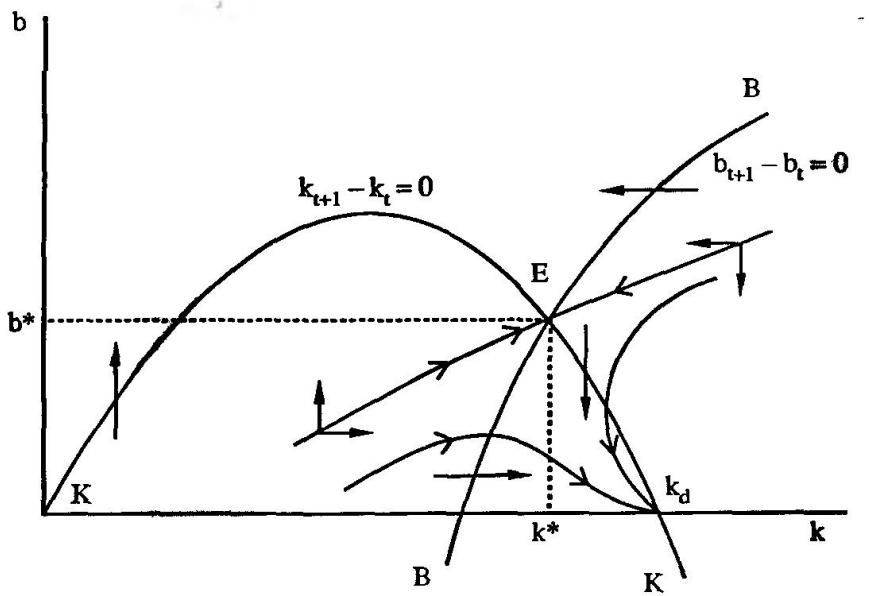


Figure 5.3
Dynamics of the Diamond model with bubbles

Bubbles in infinite-horizon models: Bewley (1980)

- There is a continuum of agents with utility

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[u\left(c_t\right)\right] \tag{4}$$

- Each agent is endowed with $\{y_t\}$:
  - i.i.d. over time and across agents (no aggregate uncertainty)
  - $y_t \in Y$ finite with probability $\pi(y) > 0$, $\sum \pi(y) = 1$
  - We assume $0 \in Y$ with $\pi(0) > 0$. Then, the natural borrowing limit is $\phi_{t+1}\left(y^t\right) = 0 \ \forall y^t$
- Incomplete markets: starting from $a_0 = 0$, each agent maximises (4) subject to the sequence of budget constraints

$$c\left(y^t\right) + a_{t+1}\left(y^t\right) \leq y_t + R_t a_t\left(y^{t-1}\right)$$

and the borrowing constraint

$$a_{t+1}\left(y^t\right) \geq 0$$

Bewley (1980): equilibrium with no money

- In equilibrium we need to have

$$\sum_{y^t \in Y^{t+1}} a_{t+1}\left(y^t\right) \Pr\left(y^t\right) = 0$$

- Since any agent with assets must be owed by someone, and everywhere $\phi = 0$, **equilibrium is autarky**

$$a_{t+1}\left(y^t\right) = 0 \quad c_t\left(y^t\right) = y_t$$

- The interest rate is such that

$$u'\left(y_t\right) \geq \beta R_{t+1} \mathbb{E}\left[u'\left(y_{t+1}\right)\right]$$

hence

$$R_{t+1} \leq \min_y \frac{u'\left(y\right)}{\beta \mathbb{E}_{\widetilde{y}}\left[u'\left(\widetilde{y}\right)\right]} \equiv \overline{R_a}$$

If gross interest is less than 1 then net interest is less than zero = n

- Assume that $\overline{R_a} < 1$: **dynamic inefficiency condition** (here $n = 0$)

Bewley (1980): equilibrium with money

- Introduce fiat money (individuals can hold it, but not issue it). Given M_0, budget constraints become

$$c\left(y^t\right) + P_t M_{t+1}\left(y^t\right) \leq y_t + P_t M_t\left(y^{t-1}\right)$$

and the borrowing constraint

$$M_{t+1}\left(y^t\right) \geq 0$$

- Guess that an equilibrium has constant price appreciation:

$$\frac{P_{t+1}}{P_t} = \bar{R}$$

and write $a_{t+1}\left(y^t\right) = P_t M_{t+1}\left(y^t\right)$ then we fall back on the income fluctuations problem, with budget constraint

$$c\left(y^t\right) + a_{t+1}\left(y^t\right) \leq y_t + \bar{R} a_t\left(y^{t-1}\right)$$

Bewley (1980): equilibrium with money

- Since the interest rate is constant, the solution is well-known: write g^a for the policy function for next-period assets, and assume that conditions for the existence of a unique invariant distribution $\mu(da, R)$ are satisfied
- Average savings are

$$\int \sum_y g^a(y, a, R)\, \pi(y) \mu(da, R) \equiv A(R)$$

- Assume money supply is a constant $\overline{M}$. In a steady-state equilibrium P is a constant, so $R = 1$, and aggregate savings is

$$A(1) = P\overline{M}$$

- So fiat money (a bubble) can be valued here, even though agents have infinite lives
- Note: if we assume dynamic efficiency in autarky ($R_a > 1$) then the bubbly equilibrium disappears.

Dynamic inefficiency tests

- Since in competitive equilibrium

$$r = MPK$$

the empirical dynamic inefficiency question is: have we overaccumulated capital?

- Problem: we may agree on g (real growth rate, including population), but what r should we use to test $r \lessgtr g$?
  - Using real returns to safe assets like Treasuries might lead us to conclude $r < g$
  - Using real returns to stocks will lead us to $r > g$
- Answers:
  - Naïve approach: compute r from national accounts
  - Abel et al. (1989) approach: use a criterion which holds in stochastic economies, and involves comparing investment flows and capital returns
- Both of these require a good understanding of national accounts data.

A first test of r vs g

- Problems with this approach:
  - π may be mismeasured: net operating surplus includes returns to labor
  - K may be mismeasured: permanent-inventory method very sensitive to assumptions on depreciation rates, revaluation to current cost is imperfect,...
- Next: the Abel et al. (1989) cash flow criterion

Abel et al (1989) conclusion

- For all the economies considered, and over the time period 1960-1989

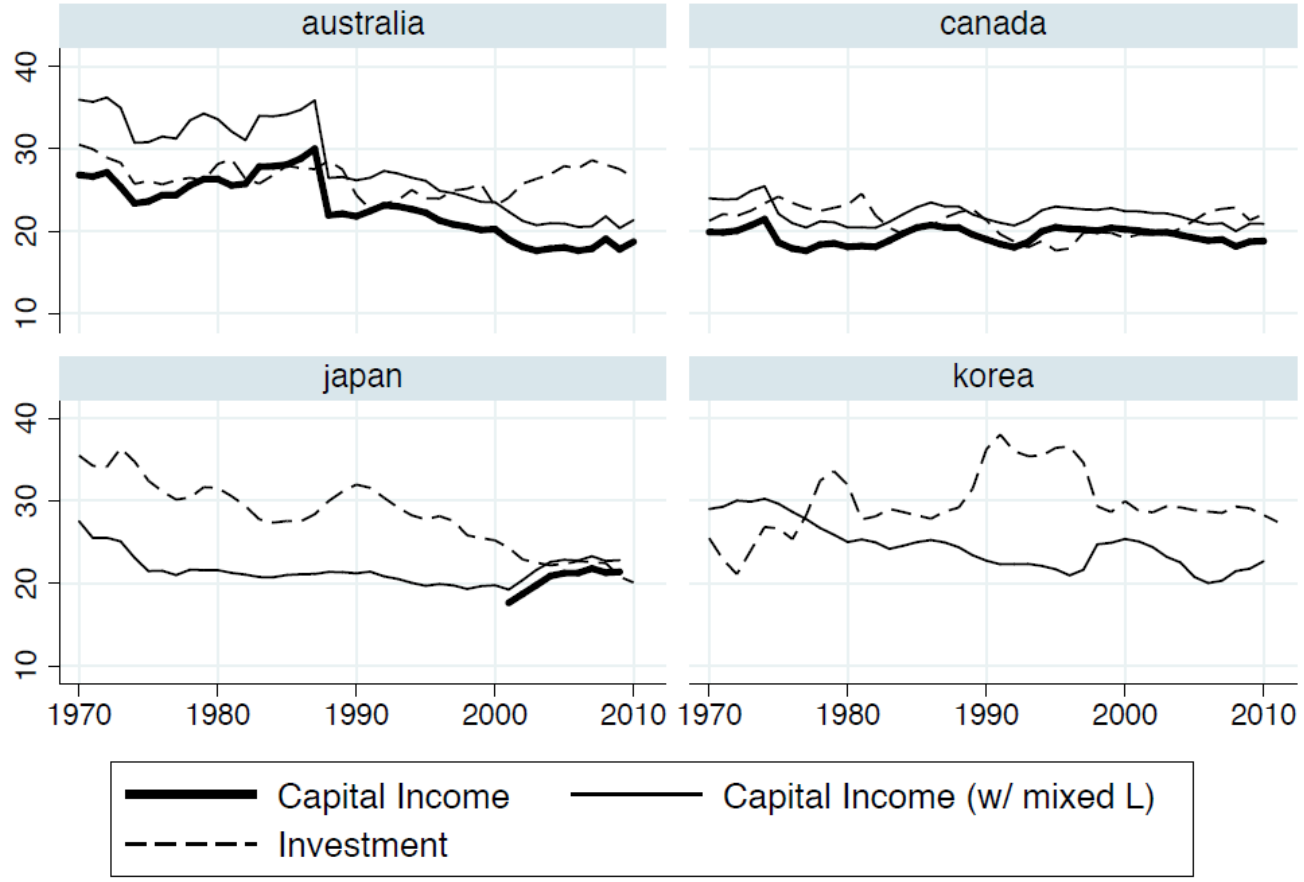
$$\frac{\pi_t}{Y_t} > \frac{I_t}{Y_t}$$

  $\rightarrow$ dynamic efficiency cannot be rejected

- Caveats:
  1. We can only be certain of dynamic efficiency "at eternity" (the criterion needs to hold $\forall t$)
  2. π_t includes land rents which theoretically do not belong in the calculation, since land is not an accumulated factor.
- **Next**: Geerolf (2013) revisits the argument, correcting 2) and updating the imputation of mixed income to profits.

Other countries

- Other countries show similar results, for example



Source: OECD, Kuznets (1985), and author's calculations

r vs g and government debt

- Write $D_t \equiv$debt, $Y_t \equiv$GDP, $S_t \equiv$government primary surplus. Assume $\dot{Y}_t = gY_t$ and constant r (all quantities are in real terms)
- Define $d_t = \frac{D_t}{Y_t}$, and $s_t = \frac{S_t}{Y_t}$. Then the debt dynamics of the government are

$$\dot{D}_t = rD_t - S_t \quad \Rightarrow \quad \dot{d}_t = (r-g)\, d_t - s_t$$

  - When $r > g$ this implies the standard intertemporal government budget constraint : debt=PV of future primary surpluses

$$d_t = \int_t^\infty e^{-(r-g)(u-t)} s_u \mathrm{d}u$$

  - When $r < g$, on the other hand, the debt dynamics can be solved backwards:

$$d_t = d_0 e^{-(g-r)t} - \int_0^t e^{-(g-r)(t-u)} s_u \mathrm{d}u$$

    for example, any constant primary deficit ratio $s < 0$ is "sustainable" since

$$d_t = \frac{(-s)}{g-r} + e^{-(g-r)t}\left(d_0 - \frac{(-s)}{g-r}\right) \underset{t\to\infty}{\to} \frac{(-s)}{g-r} < \infty$$

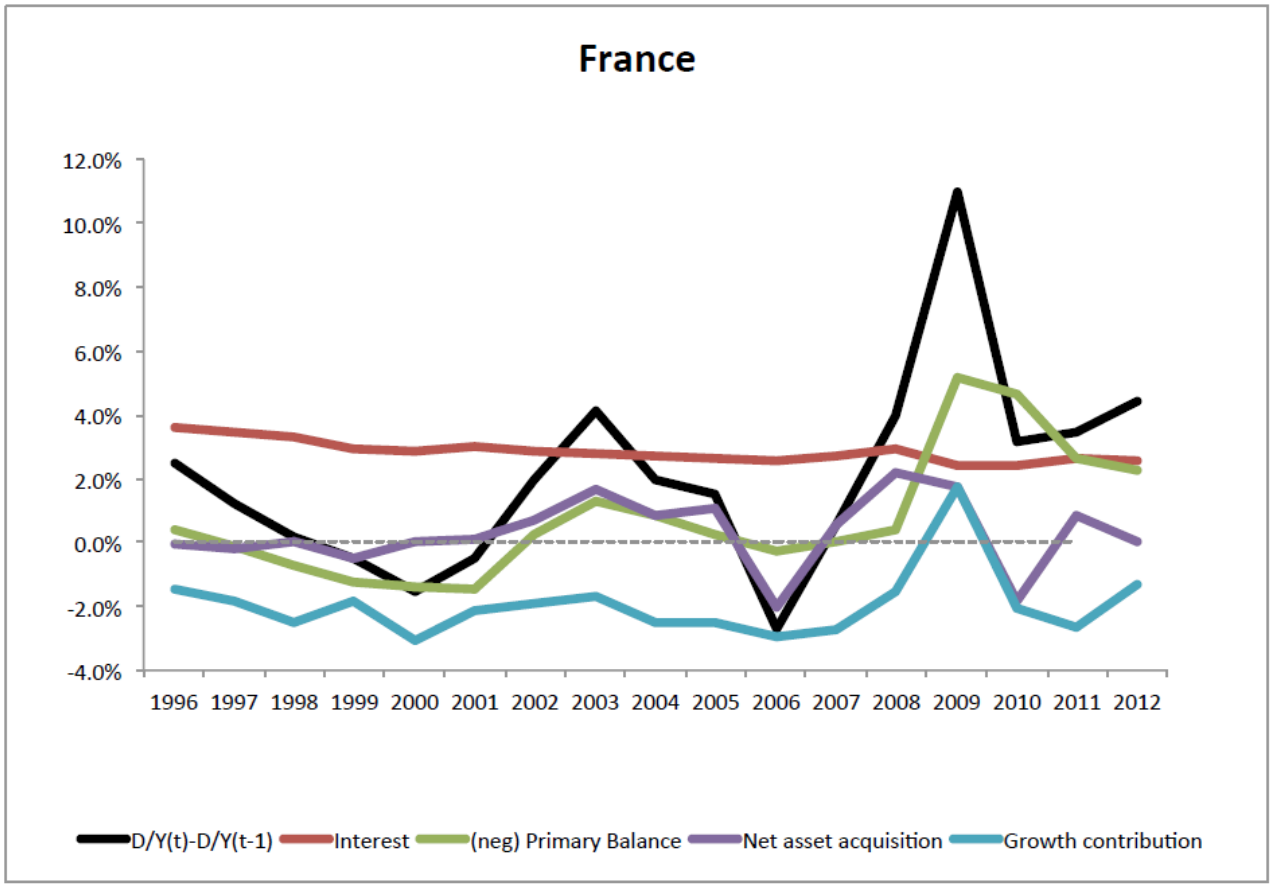
Debt dynamics decomposition: an example

- Example: from Eurostat we can obtain annual data, for any country covered on:
  - general government debt D_t
  - nominal GDP Y_t
  - government primary surplus S_t
  - interest payments IntPay_t
- Decompose yearly:

$$\frac{D_{t+1}}{Y_{t+1}} - \frac{D_t}{Y_t} = \underbrace{-\left(\frac{1}{Y_t} - \frac{1}{Y_{t+1}}\right) D_{t+1}}_{\text{Growth contribution}} + \frac{\mathrm{IntPay}_t}{Y_t} - \frac{S_t}{Y_t} + \frac{\mathrm{Adj}_t}{Y_t} \qquad (1)$$

- Adjustments Adj_t include mostly net acquisitions of assets (e.g. bailouts)
- Between 1995 and 2012, in France, $\frac{D_t}{Y_t}$ increased from 55% to 90%.
  - How important were the first two terms in (1)?

Debt dynamics decomposition: an example

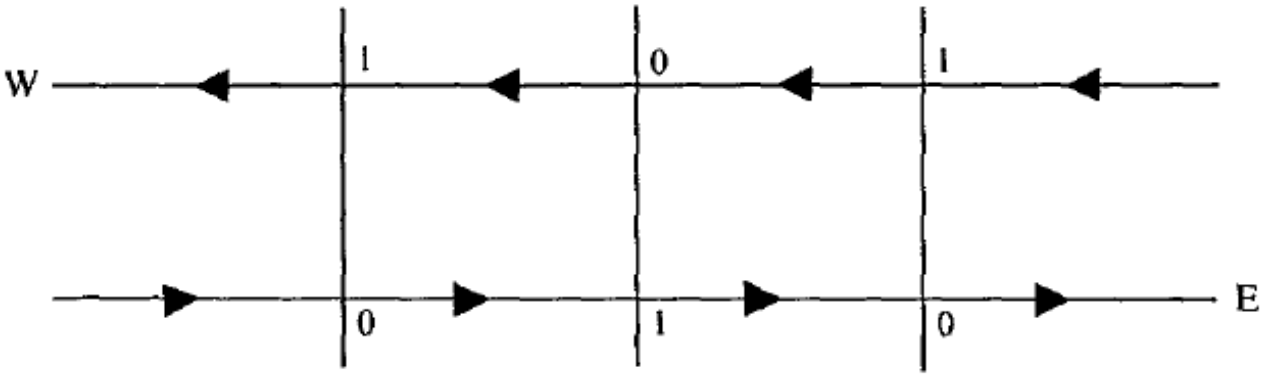


- Notice the contribution of (negative) growth to the 11p increase in D/Y in 2008
- Importantly for us, before the crisis, the interest and the growth contributions were roughly balanced; the former slightly higher (95-06 average: 3% vs -2.2%)

Turnpike Model of Exchange: From Fiat Money

- Valued outside fiat money
  - Townsend

Figure 1
The Turnpike Model



To Woodford (1990) "Public Debt as Private Liquidity"

I. Public Debt in a Liquidity-Constrained Economy

Consider an economy made up of two types of infinite lived households, with the number of each normalized as one. Type A households have endowment $e_1(1+g)^t$ in all even periods and $e_2(1+g)^t$ in all odd periods, while type B households have endowment $e_2(1+g)^t$ in even periods and $e_1(1+g)^t$ in odd periods, where $e_1 > e_2 \geq 0$. Both types ($i = A, B$) seek to maximize an infinite horizon objective function

$$\sum_{t=0}^{\infty} \beta^t (1+g)^t v\left(C_t^i/(1+g)^t\right),$$

where v is an increasing, strictly concave function, and where C_t^i denotes consumption in period t by each household of type i. (We may suppose that each household is an infinite lived family whose members increase at the rate g, with per capita endowment remaining constant; the family pursues a joint consumption and savings program to maximize a discounted sum of individual family members' utilities.) Total lump sum

Blanchard (2019) "Public Debt and Low Interest Rates"

Conclusions

- Historically, US safe rate less than growth rate.
- If the past is like the future or even more so, fiscal cost of debt small or zero.
- Low rates also send a strong signal about the risk-adjusted rate of return to capital
- Welfare costs of debt depend both on the safe rate and the risky rate

Turning to current policies

- High public debt is not catastrophic.
- More debt however has to be justified by clear benefits (output gap reduction, public investment).
- If worried about bad equilibrium, better to have a contingent fiscal rule (which may not need to be used) rather than steady fiscal austerity.

High Velocity Public Debt as a Means of Payment

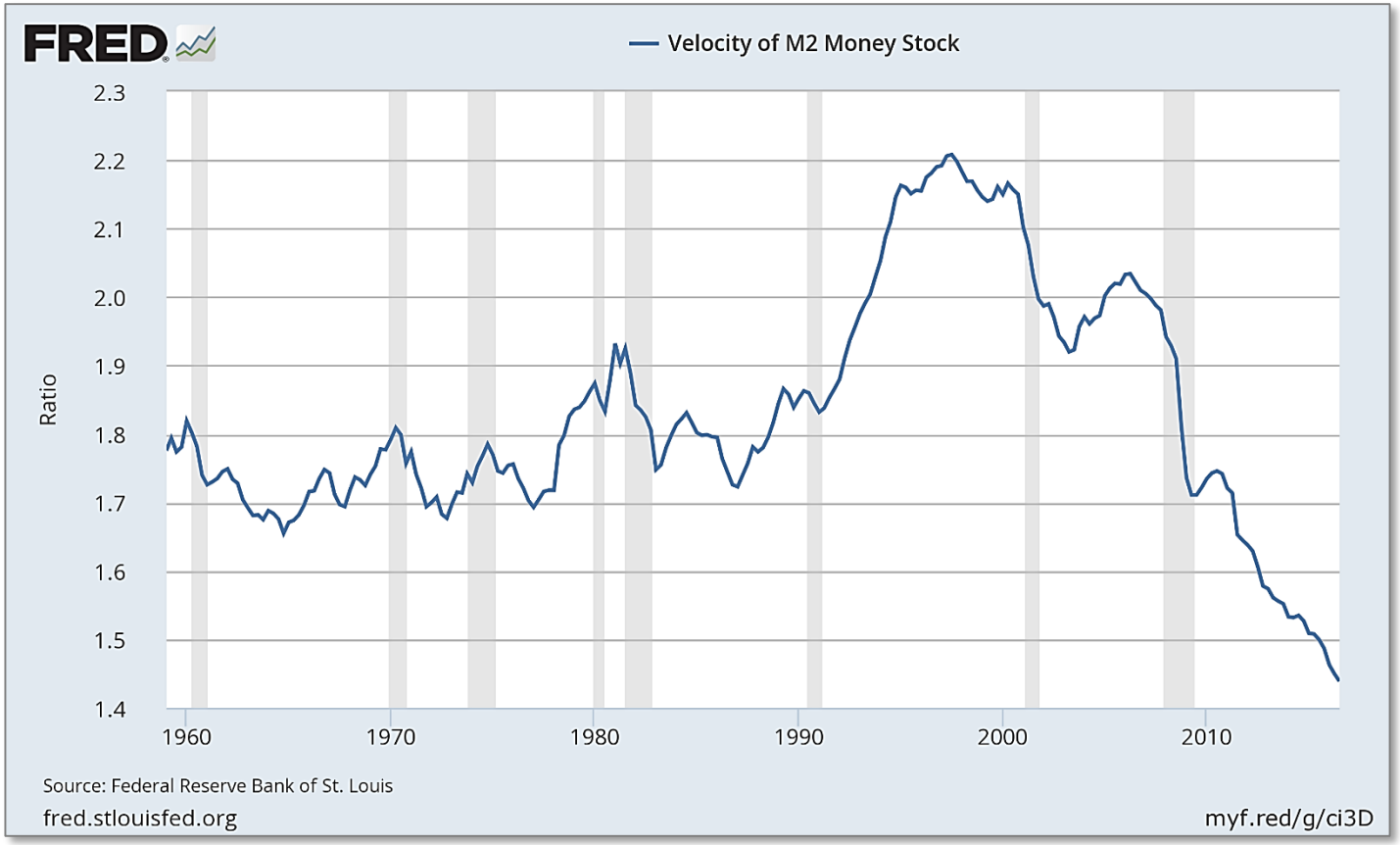
Rehypothecation: Contemporary Example

M. Singh (2011): "Velocity of Pledged Collateral: Analysis and Implications"

- It highlights the role of securities as money, potentially with high velocity.
- The suppliers of collateral to the 'Street': hedge funds (HFs), securities lending via custodians (on behalf of pension, insurers, etc.) and commercial banks that liaise with dealers. The 'supply' of pledged collateral is received by the central collateral desk of dealers that re-use the collateral to meet the 'demand' from the financial system.
- There is a demand for securities by posting cash as collateral.
- Note the language: lending and borrowing of securities in exchange for cash
- **Velocity of collateral** is the ratio of total collateral received (the flow) over primary sources of collateral (the stock outstanding).
  - 10-14 large banks active in collateral management globally, they pick up over 90% plus of the pledged collateral that is received from primary sources such as hedge funds, pension funds and insurers, and official accounts. take total collateral received as of end-2007 (almost $10 trillion)
  - and compares it to the primary sources of collateral issued, the stock (around $3.3 trillion).
  - Thus, velocity is $10/3.3 \approx 3$

Comparison of Velocities: Rehypothecation Velocity to Velocity of Money M2

- Flow GDP = Money M2 supply x Velocity of money
- Lower than 3 in 2007

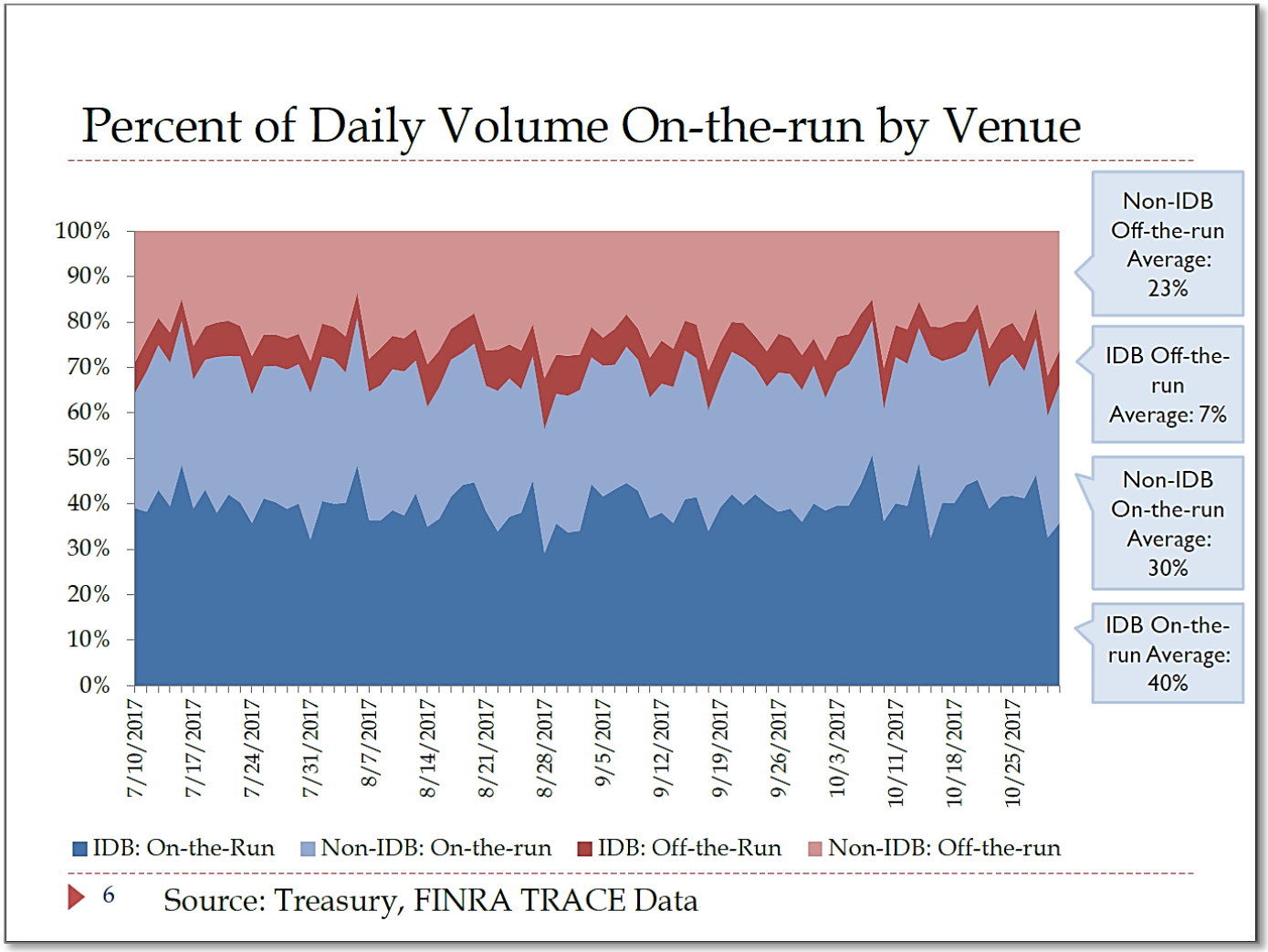


The Aggregate Demand for Treasury Debt: Safety and Liquidity (A. Krishnamurthy and A. Vissing-Jorgensen 2012)

- Money, such as currency or checking accounts, offers a low rate of return relative to other assets. Money is a medium of exchange for buying goods and services, has high liquidity, and has extremely high safety in the sense of offering absolute security of nominal repayment.
- Argue that a similar phenomenon affects prices of Treasury bonds. The high liquidity and safety of Treasuries drive down the yield on Treasuries relative to assets that do not to the same extent share these attributes. Treasuries are in important respects similar to money
- Examine the yield spread between a pair of assets that are different only in terms of their liquidity, as well as the yield spread between a pair of assets that are different only in terms of their safety. Under the hypothesis that liquidity and safety are priced attributes, the yield spread between these pairs of assets should reflect the equilibrium price of liquidity/safety.
- Show that changes in Treasury supply affect each of these yield spreads in these other assets. The results indicate that Treasuries offer liquidity and safety so that changes in the supply of Treasuries separately change the equilibrium prices of liquidity and safety

Treasuries: On-the-run

- Department of the Treasury (2017) "TRACE Data Update"



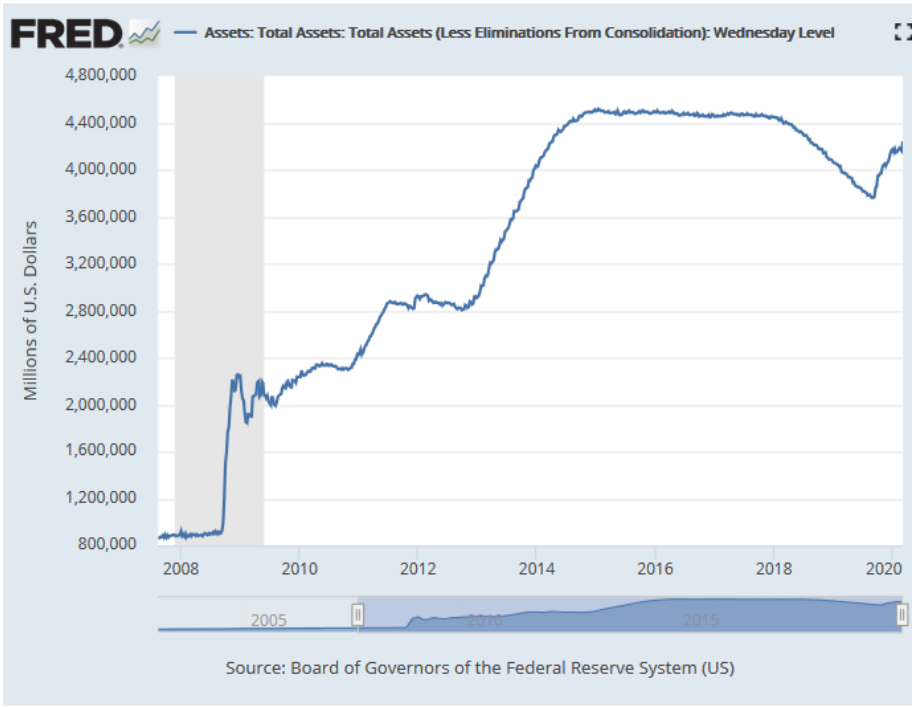
Concerns about Federal Reserve Liabilities: Expanded Fed Balance Sheet

- Expanded Fed balance sheet and excess reserves may cause problems
- Fiscal theory of money: "Central Bank Solvency and Inflation" (M. Del Negro, C. A. Sims 2015)
  - Monetary theory: empirically, nominal income is proportional to monetary supply
  - Only applies to non-interest-bearing central bank liabilities, so maybe no problem?
  - Since 2008 a large fraction of monetary base consisted of reserves paying interest: substitutes for Treasury bills
  - It changed maturity structure: more short term assets held by the private sector, long term by CB
  - QE resulted in a sizable mismatch between asset and liability side of CB balance sheet
  - If inflation expectations are high, interest rates go up. Fed takes asset losses and, if there is no fiscal support, it must rely on seigniorage, which in turn creates inflation
- Money multiplier: "Speculative Runs on Interest Rate Pegs" (Bassetto and Phelan, JME 2015)
  - Related to multiple equilibria
  - A class of equilibria emerges when a central bank conducts monetary policy by setting an interest rate and letting the private sector set the quantity traded
  - Particularly dangerous when banks hold large excess reserves, such as is the case following periods of QE
  - Sudden rise in inflation and interest rate can become self-fulfilling if reserves are lent out via money multiplier on excess reserves
  - Freezing excess reserves or fiscal-policy intervention may be needed to fend off adverse expectations

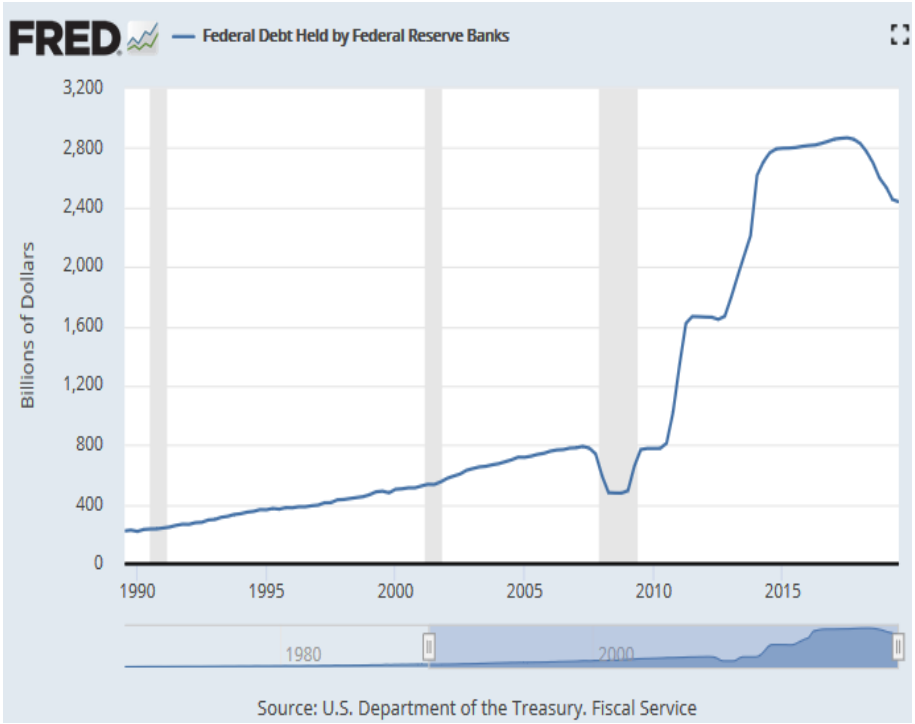
Central Bank Intermediation: Financial Repression and Lack of Separation with Treasury

- General concerns about whether the Fed should intermediate so much credit

On the Asset Side



Federal Debt held by FR banks as % of Total Debt



Optimal Interest Rate?

- For Samuelson, when $g = 0$, then $r = 0$ and fiat money without interest is optimal
- For Bewley (1983) "A Difficulty with the Optimum Quantity of Money", money may carry interest but only if interest rate is sufficiently small, otherwise non-existence and Pareto optimality cannot be assured