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Vision for Optimized Design of Financial Infrastructure using Distributed Ledger Technology

**Scrambling of Information and Partitioned Ledgers, Delegation to the
Contract, Limiting Access to the Outside Market, Single and Multiple-Colored
Tokens as Decentralized Partitioned Ledgers, Commitment to Optimized
Sequential Service to Mitigate Runs**

(Lecture 5)

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Information-Constrained Optimal Allocations

- ❖ Optimal learning as optimal scrambling and partitioned ledgers
- ❖ Delegation of resources to a “third party” or computer
- ❖ Role of commitment
 - Limit access of the individual to the market
- ❖ Tokens
 - As decentralized ledgers
 - Multi-colored coins and partitioned ledgers
- ❖ Implementing solutions to mitigate bank and market runs

Delegation to a Mediator and Scrambling

- Townsend 1982: Multiperiod contracts can facilitate beneficial risk sharing when one agent has private information.
- Townsend (1987) studies a model where all agents have private information in a multiperiod economy
- Central mediator coordinates and implements contracts between agents
 - Collects and pools endowments from agents
 - Redistributes the pooled endowment in each period based on messages received
 - Message space \equiv space of agent types (revelation principle)
- Mediator seeks to construct an allocation rule which maximizes ex ante expected utility subject to truth telling constraints
- Mediator may choose allocation rules which keep certain agents uninformed of the types of the other agents - "Scrambling"

Model

Townsend (1987)

- Two agents a and b and two time periods $t = 0$ and $t = 1$
- In each period, both agents receive a deterministic endowment e_t^i with $e_t = e_t^a + e_t^b$ known by both agents
- preferences are given by the objective function
$$U^i(c_0^i, \theta_0^i) + \beta V^i(c_1^i, \theta_1^i)$$
 - c_0^i, c_1^i is path for agent i 's consumption
 - $(\theta_0^a, \theta_1^a, \theta_0^b, \theta_1^b) \in \Theta$ comes from a finite set Θ which represents a consumption shock profile for both agents
- θ_t^i is revealed only to agent i at time t and is therefore private information
- Each element of $(\theta_0^a, \theta_1^a, \theta_0^b, \theta_1^b) \in \Theta$ occurs with probability $p(\theta_0^a, \theta_1^a, \theta_0^b, \theta_1^b)$

Mediation

Townsend (1987)

- Assume that there exists a mediator in this economy
 - Collect information from the two agents and decide on how to split the endowment in each period based on the agents reports
- Construct an allocation rule that maximizes a weighted sum of the two agents ex ante expected utility
- Direct Revelation: Let the message space at $t = 0$ and $t = 1$ be the set of all possible θ_0^i , and the set of all possible (θ_0^i, θ_1^i) respectively for each agent i
- Revelation Principle: Any equilibrium allocation of an arbitrary mechanism in a truthful equilibrium can be implemented as a truthful equilibrium of a direct revelation mechanism

Example: Optimal Learning as Optimal Scrambling

Townsend (1987)

- The mediator may choose to "scramble" messages
- Mediator may choose a distribution over consumption that, even given both agents reporting truthfully, one or both of the agents may not be able to discern with certainty the $t = 0$ type of the other agent given the observed c_0

Example: Optimal Learning as Optimal Scrambling

Townsend (1987)

- Set θ_0^b and θ_1^a to be constant
 - only agent a is reporting at $t = 0$ and agent b is reporting at $t = 1$
- Preferences are given by

$$U^a(c_0^a, \theta_0^a) = V^a(c_1^a, \theta_1^a) = (c_t^a)^{\theta_t^a}$$

with $\theta_0^a \in \{0.2, 0.9\}$, $\theta_1^a = 0.30$

and

$$U^b(c_0^b, \theta_0^b) = V^b(c_1^b, \theta_1^b) = (c_t^b)^{\theta_t^b}$$

with $\theta_0^b \in 0.90$, $\theta_1^b \in \{0.2, 0.9\}$

Example: Optimal Learning as Optimal Scrambling

Townsend (1987)

- Parameters θ_0^a and θ_1^b have the following joint distribution

$$(\theta_0^a, \theta_1^b) = \left\{ \begin{array}{ll} (0.2, 0.9) & \text{with prob. } 0.3 \\ (0.9, 0.9) & \text{with prob. } 0.2 \\ (0.2, 0.2) & \text{with prob. } 0.1 \\ (0.9, 0.2) & \text{with prob. } 0.4 \end{array} \right\}$$

- $e_t^a = e_t^b = 5$ for $t = 0, 1$, $\beta = .95$, and $\lambda^a = \lambda^b = 0.5$

Example: Optimal Learning as Optimal Scrambling

Townsend (1987)

Private Information Solution					
θ_0^a	(c_0^a, c_0^b)	$\pi(c_0, \theta_0^a)$	θ_0^a, θ_1^b	(c_1^a, c_1^b)	$\pi(c_1, \theta_1^b)$
0.2	(1.75, 8.25)	1.0	(0.2, 0.2)	(4.75, 5.25)	1.0
	(0.0, 10.0)		(0.2, 0.9)	(2.0, 8.0)	1.0
0.9	(1.75, 8.25)	0.1159346	(0.9, 0.2)	(3.75, 6.25)	1.0
	(3.25, 6.75)	0.0339681		(1.0, 9.0)	0.86100
		0.8544384	(0.9, 0.9)	(10.0, 0.0)	0.13839

- The allocation $(c_0^a, c_0^b) = (1.75, 8.25)$ occurs with positive probability for either report of agent a
 - When $(c_0^a, c_0^b) = (1.75, 8.25)$ is observed, agent b remains uncertain of the type agent a reported in $t = 0$

Example: Optimal Learning as Optimal Scrambling

Townsend (1987)

Consider the full information solution:

Full information Solution					
θ_0^a	(c_0^a, c_0^b)	$\pi(c_0, \theta_0^a)$	θ_0^a, θ_1^b	(c_1^a, c_1^b)	$\pi(c_1, \theta_0^a)$
0.2	(1.75, 8.25)	1.0	(0.2, 0.2)	(2, 5, 7.5)	1.0
			(0.2, 0.9)	(2.5, 7.5)	1.0
0.9	(0.0, 10.0)	0.1042598	(0.9, 0.2)	(3.75, 6.25)	1.0
	(3.25, 6.75)	0.8957402			
			(0.9, 0.9)	(1.0, 9.0)	0.891898
				(10.0, 0.0)	0.108101

- The $(c_0^a, c_0^b) = (1.75, 8.25)$ outcome is eliminated in the $\theta_0^a = 0.9$ branch
 - b already knows agent a 's type and thus scrambling is unhelpful

Example: Optimal Learning as Optimal Scrambling

Townsend (1987)

- Presence of the scrambling in the private information case keeps agent b uninformed of agent a 's type making her more hesitant to lie when $(c_0^a, c_0^b) = (1.75, 8.25)$ is observed and $\theta_1^b = 0.2$ is her type
 - She may be in the $\theta_0^a = 0.9$ case which has a positive probability of a very bad outcome of $c_1^b = 0$

Garratt, Lee, Martin and Townsend (2019) “Who Sees the Trades? The Effect of Information on Liquidity in Inter-dealer Markets”

Abstract

Dealers, who strategically supply liquidity to traders, are subject to both liquidity and adverse selection costs. While liquidity costs can be mitigated through inter-dealer trading, individual dealers' private motives to acquire information compromise inter-dealer market liquidity. Post-trade information disclosure can improve market liquidity by counteracting dealers' incentives to become better informed through their market-making activities. Asymmetric disclosure, however, exacerbates the adverse selection problem in inter-dealer markets, in turn decreasing equilibrium liquidity provision. A non-monotonic relationship may arise between the partial release of post-trade information and market liquidity. This points to a practical concern: a strategic post-trade platform has incentives to maximize adverse selection and may choose to release information in a way that minimizes equilibrium liquidity provision.

Group Portfolio Management for Borrowing and Lending Contracts

Townsend (1987)

- Consider an economy with one underlying consumption good and a continuum of households on the unit interval
- Each household has endowment of consumption good $y(\varepsilon)$ at $t = 0$ where ε is a random shock
- There is no endowment at $t = 1$ but there is a storage technology that allows consumption to be stored at $t = 0$ for a return of $(1 - \delta)K_1(\varepsilon)$ at $t = 1$
- Planner's problem is to construct a welfare optimizing allocation rule $\pi_0(c_0, \theta_0, \varepsilon_0)$ and $\pi_1(c_1, \theta_0, \theta_1, \varepsilon)$ to maximize ex-ante expected utility
- For simplicity, assume that the $t = 1$ allocation rule must be independent of the realization of $t = 0$ consumption
 - Preference shocks are private, as in previous model and with storage here, as in Diamond-Dybvig

Group Portfolio Management for Borrowing and Lending Contracts

Townsend (1987)

- Numerically solve this model with
 - $U(c, \theta) = [c^\theta - 1]/\theta$
 - $\theta \in \{0.5, 0.9\}$ drawn each period with probability 0.5 for each value
 - $\beta = 1$ and $\delta = 0$
 - study the branch where $y(\varepsilon) = 8$

Group Portfolio Management for Borrowing and Lending Contracts

Townsend (1987)

- The full information solution offers full insurance in both periods independent of history
- The private information solution offers less insurance, especially in $t = 1$
- Higher storage in the full information solution versus the private information solution (4 units stored versus 3.26 units stored)
 - Private information solution displays under storage
- With private information, incentives are more difficult in the second period, so front-end incentives

Group Portfolio Management for Borrowing and Lending Contracts

Townsend (1987)

- If we decrease the productivity of storage and increase the impatience of agents setting $(1 - \delta) = 0.8$ and $\beta = 0.9$
- Private solution over stores the consumption good because consumption in the private information case has intertemporal tie ins, trying to create high-powered incentives
- $\theta_0 = 0.5$ individuals receive more (expected) consumption in $t = 1$ then $\theta_1 = 0.9$ individuals, which allows the incentive constraint to be satisfied

Group Portfolio Management with Many Agents and Hidden Storage

Doepke and Townsend (2004)

- Previous models assume total income is verifiable at any given point in time
 - Contract terms are contingent on the entire wealth of the agent
- Actual financial agents do not tie all of their wealth to one contract
 - Wealth levels are usually private information
- Example from Doepke and Townsend (2004)
 - Optimal information constrained insurance can still provide sizable welfare benefits when agents have access to hidden storage

Group Portfolio Management with Many Agents and Hidden Storage

Doepke and Townsend (2004)

- Set up
- economy with three periods $t = 0, 1,$ and 2 agents
- Unity continuum of risk averse agents with log utility and one risk neutral social planner
- Timeline for each period
 - agents observe income realizations of $e_t \in \{2.1, 2.3\}$
 - agents makes a report to the planner of his income realization
 - Planner offers him a wealth transfer τ_t which is contingent on his current report and his past history of income reports
 - planner also suggests an action $a_t \in \{0, 0.3, 0.6\}$ which is a suggestion of how much of the agents current wealth to store.
 - agent chooses how much to consume and store
- In $t = 1, 2$ the probability of the high outcome increases linearly with storage at a rate R

Group Portfolio Management with Many Agents and Hidden Storage

Doepke and Townsend (2004)

- social planner has access to a credit market which offers a gross return of 1.05.
- planner seeks to choose an incentive compatible allocation rule that minimizes his expected transfers to the agent (subject to individual rationality constraints).

Group Portfolio Management with Many Agents and Hidden Storage

Doepke and Townsend (2004)

- Results:

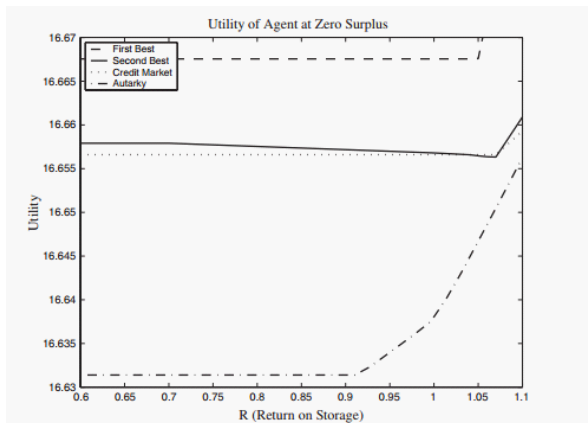


Fig. 4. Utility of the agent with full information, private information, credit-market access, and autarky.

Group Portfolio Management with Many Agents and Hidden Storage

Doepke and Townsend (2004)

- Second best solution
- offers significant benefits over autarky when storage return is low
- decreasing in the return on storage for moderate R
 - incentive compatibility constraints become tighter as the return on storage becomes larger and the optimal storage level is 0
- gain from the second best contract over the credit market outcome increases when R is further away from the credit market borrowing rate
 - credit market rate is close to the storage rate \implies agent has access to private insurance options which are very similar to what the planner can feasibly provide

Decentralized Tokens Implementing Private Partitioned Ledgers

- ❖ Risk-averse agent a is paired with a risk-neutral agent b initially, in the first period at one of the two locations, and likewise for agents a' and b' at a second location in the first period. Townsend (1987)
- ❖ Agents a and a' make announcements to partners of their urgency to consume, and to induce truth telling, if urgent today, they receive the good but at the expense of getting less of it in the second period. if patient today, vice versa.
- ❖ In the second period, a and paired with b' and vice versa, a' with b .
- ❖ Record all messages of all parties and history on distributed ledger, centralized and illustrative of the scaling problem
- ❖ Solution: Tokens. Announced patient agents in the first period give up goods and receive more tokens. Have incentives to reveal tokens in second period, to get more consumption. Patient have less tokens
- ❖ Equivalently: Journal entries on ledgers, but partitioned and private, when to share own accounts with selected others

Multiperiod Private Information Solution

TABLE 3—MULTIPERIOD PRIVATE INFORMATION SOLUTION, ONE GOOD^a

Values for θ_1^a	c^a at Date 1	Values for θ_2^a	c^a at Date 2
.4	3.1	{ .4	6.55
		{ .5	6.55
		{ .6	6.55
.5	5.1	{ .4	4.87
		{ .5	4.87
		{ .6	4.87
.6	7.25	{ .4	3.1
		{ .5	3.1
		{ .6	3.1

^aSee fn. 1.

Townsend (1987)

4 Agents, 2 Locations

TABLE 5—AGENT PAIRINGS IN THE FOUR-AGENT,
TWO-LOCATION MODEL

	Location	1	2
Date	1	(a, b)	(a', b')
	2	(a, b')	(a', b)

Trade with Insurance (Townsend 1987)

The utility function of agent a at date 1 is of the form

$$U^a(c_x, c_y, \theta_1^a) = (c_x)^{\theta_{1x}^a} + (c_y)^{\theta_{1y}^a}$$

with $(\theta_{1x}^a, \theta_{1y}^a) \in \{(.4, .6), (.6, .4)\}$,

each with equal probability, and at date 2 of the form

$$U^a(c_x, c_y, \theta_2^a) = (c_x)^{(1-\theta_{1x}^a)\delta_x} + (c_y)^{(1-\theta_{1y}^a)\delta_y}$$

$$= (c_x)^{\theta_{2x}^a} + (c_y)^{\theta_{2y}^a}$$

with

$$(\delta_x, \delta_y) = \begin{cases} (1, 1) & \text{with Prob .96} \\ (.5, 1.5) & \text{with Prob .02} \\ (1.5, .5) & \text{with Prob .02} \end{cases}$$

TABLE 4—MULTIPERIOD PRIVATE AND FULL INFORMATION SOLUTION, TWO GOODS

Values for $(\theta_{1x}^a, \theta_{1y}^a)$	Values for (c_x^a, c_y^a)	Values for (δ_x, δ_y)	Values for $\theta_{2x}^a, \theta_{2y}^a$	(c_x^a, c_y^a)	
(.4, .6)	(2, 8)	(1, 1)	(.6, .4)	8.01	2.0
		(.5, 1.5)	(.3, .6)	1.0	8.0
		(1.5, .5)	(.9, .2)	10.0	0.82
(.6, .4)	(8, 2)	(1, 1)	(.4, .6)	2.0	8.0
		(.5, 1.5)	(.2, .9)	0.82	10.0
		(1.5, .5)	(.6, .3)	8.0	1.0

In Table 4 there are two goods, and agent a can be a “borrower” or a “lender” in either good. Still, “preference reversal” shocks at date 2 can cause agent a to want to pretend to have been a lender in the commodity he did not lend.

Multiple Colored Coins and Partitions

- ❖ Multiple colored tokens on a private ledger, so that one can keep track of the direction of trade in each good
- ❖ Intuition: Keep track of more dimensions
- ❖ Links to cryptography and the idea of colored coins
 - Narayanan et al (2016): Stamping on \$ dollars bills a purchase for Yankee's tickets, or code on \$ dollar bill to access info on a common ledger
- ❖ Coins are not necessarily fungible in the sense that coins can have public verified histories, to trace ownership
 - Colored tokens are a metaphor for “colored” entries on ledgers distinguishing histories
 - Free exchange of crypto currency with fiat money can undercut these mechanisms

Diamond and Dybvig (1983)

- Seminal model of bank runs
- The bank serves to provide insurance against liquidity shocks.
- The model features several equilibria. One efficient equilibrium where everyone is truthful about their liquidity needs and one inefficient equilibrium in which a panic occurs and everyone tries to withdraw their money early.

Green and Lin (2003)

- Take sequential servicing constraint seriously. People can condition their actions on their arrival time at the bank and on their type, and the bank must make an allocation decision for each agent knowing only his stated type and the stated types of the agents who arrived before him.
- Look at a wider class of potential contracts than Diamond and Dybvig
- Finite number of agents with types drawn independently from one another. Even this creates aggregate uncertainty (the fraction of impatient agents is unknown ex ante). Note this contrasts with Diamond and Dybvig.

Model Setup

- Finite set of players $\mathbf{I} = \{1, \dots, I\}$ $I < \infty$
- 2 dates $T = 0, 1$
- Each agent has an endowment of 1 unit that they deposit in the bank at the beginning of period 0.
- Agents learn their types privately at the beginning of period 0
- Preferences $u(c_0^i, c_1^i; \omega^i)$ are as in Diamond and Dybvig
- Probability of impatience ($\omega^i = 0$) is P drawn independently
- Set of possible states is $\Omega = \{0, 1\}^I$ with generic element $\omega = (\omega^1, \dots, \omega^I) \in \Omega$
- As before, a generic allocation vector a has an element a_j^i which denotes i 's allocation in period j

Case 2: Sequential Service Constraint

- Now players will show up one at a time, and the bank will be forced to make a decision about their period 0 consumption for each player only knowing the reported types of the players who came before them.
- Assuming the bank can recalculate how much to give depositors after each depositor shows up (and placing some restrictions on preferences v), Green and Lin show that the ex interim efficient mechanism has a unique, truthful equilibrium.

Sequential Service Constraint

- The idea is that agents show up at the bank one at a time and report their types. The bank must determine the allocation of each agent as a function of their stated type and the stated types of those who came before the current agent.
- An allocation rule $\vec{a} : M \rightarrow \mathbb{R}_+^{2I}$ satisfies the sequential service constraint if $\vec{a}_0^i(m)$ is a function only of m^1, \dots, m^i
That is the i 'th component of the period 0 allocation must only be a function of the stated types of the agents who arrived earlier than agent i and i 's own stated type.
- Let F' be the set of all allocation functions that satisfy sequential service and feasibility

The Unique Equilibrium

Theorem (4)

When $M = M^1 \times \dots \times M^I$ is as above and \vec{a} is as in lemmas 2 and 3 that there is a unique BNE and in that BNE everyone tells the truth.

The proof is largely omitted but the intuition is as follows. We will proceed by backwards induction. The last agent is going to have a dominant strategy to tell the truth, as he will get more in period 1 by reporting patience than in period 0 by reporting impatience no matter what the others have reported.

Intuition Continued

Our inductive step is to show that assuming all future agents will tell the truth, agent i will receive more in expectation (in period 1) by reporting patience than he will get for sure in period 0 if he reports impatience. Thus a patient agent will tell the truth. (Recall that impatient agents always tell the truth, as when an agent reports patience his period 0 consumption is 0.)

Thus truth telling is the unique surviving strategy of iterated deletion of strictly dominated strategies, and is therefore the unique BNE.

Recap

- Assuming the bank can recalculate how much to give depositors after each depositor shows up (and placing some restrictions on preferences v), Green and Lin show that the ex interim efficient mechanism has a unique, truthful equilibrium.
- Note though, that their backwards induction proof relies on type independence of the agents. Had types not been independent, then the bank's forecast of the probability of future sequences of types $\pi(s)$ would need to be conditioned on past reports, and hence would be sensitive to lies by previous agents. Then the simple backwards induction argument would not work. Intuition for why this fails is provided in the following paper.

Preliminaries

- They also specify a functional form for v so that they can derive the analytical form of the ex interim efficient allocation.

$$v(a_0^i, a_1^i; \omega_i) = \frac{1}{1-\gamma} (a_0^i + \omega_i a_1^i)^{1-\gamma} \quad \gamma > 1$$

- They specify uncertainty to allow for correlation as follows: P_k is the set of all type vectors with k patient agents.

Formally $P_k = \{\omega \in \Omega : \theta^*(\omega) = k\}$.

- Let π_k be the probability that there are k patient agents, and the probability of some $\omega \in P_k$ is $\frac{\pi_k}{\#P_k}$

That is, all permutations are equally likely conditional on there being k patient agents.

- One special case of this probability distribution is independence. Suppose there are three agents and the probability that any agent is patient is .5, with independent draws. Then $\pi_0 = \pi_3 = .125$
 $\pi_1 = \pi_2 = .375$ $P_0 = \{(0, 0, 0)\}$, $P_1 = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$
 and so on.

The Mechanism

The Calvacanti and Monteiro mechanism is $\vec{\alpha}$ defined as follows:

- For some (arbitrarily small) $\epsilon > 0$ and for each player i , pick $0 < a^i < \epsilon$

- For $i < I$
$$\vec{\alpha}^i(b, t) = \begin{cases} \vec{\alpha}^i(b) & \text{if } b^i = 1 \\ \vec{\alpha}^i(b) & \text{if } b^i = 0 \text{ and } g^i = 0 \\ \vec{\alpha}^i(b) + (-a^i, Ra^i) & \text{if } b^i = 0 \text{ and } g^i = 1 \end{cases}$$

- For $i = I$ $\vec{\alpha}^I = \vec{\alpha}^I(s)$