Advanced Economic Growth: Lecture 19: Structural Change

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Introduction

- Changes in composition of employment and production are important part of process of development.
  - Shift of employment and production from agriculture to manufacturing, and then from manufacturing to services.
- Will focus on demand-side and supply-side reasons of structural change.
- Emphasize how structural changes can be reconciled with balanced growth.
- Also present a simple model of industrialization:
  - Pre-industrial agricultural productivity may be a key determinant of industrialization and takeoff.

Non-Balanced Growth: The Demand Side I

- Major changes in structure of production in US economy over past 150 years (see Figure).
- Consumption shares trends similar, though consumption of agricultural products still substantial because of changes in relative prices and productivities (and because of imports of agricultural goods).
- Changes in British economy towards end of 18th century also consistent with US patterns.
- Similar patterns present in all OECD economies.
- Some less-developed economies still largely agricultural but trend towards smaller share of agriculture.
- Kongsamut, Rebelo and Xie (2001): refer to these changes as *Kuznets facts* and provide model to reconcile with *Kaldor facts* (relative constancy of factor shares and interest rate).

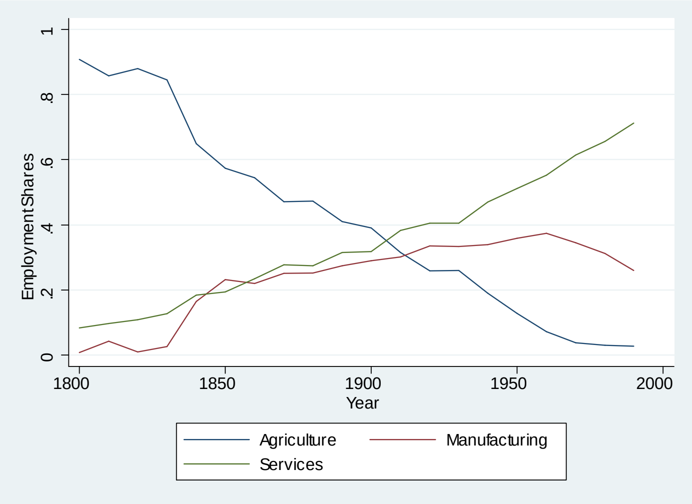

Figure: The share of US employment in agriculture, manufacturing and services, 1800-2000.

Non-Balanced Growth: The Demand Side II

- Figure paints a picture with significant *non-balanced* component.
- Models that depart from Kaldor facts over early stages of development process might be useful.
- But changes in composition of employment and production are present even in relatively advanced economies.
- Start with Kongsamut, Rebelo and Xie: certain degree of non-balanced growth at sectoral level, but Kaldor facts of aggregate balanced growth.
- *Engel's law*: as a household's income increases, fraction that it spends on food (agricultural products) declines.
- Rebelo and Xie extension: household will desire not only to spend less on food, but more on services.

Non-Balanced Growth: The Demand Side III

- Infinite-horizon economy.
- Population grows at exogenous rate $n \geq 0$, so labor supply is:

$$L(t) = \exp(nt) L(0). \tag{1}$$

- Representative household supplies labor inelastically and has preferences

$$U(0) \equiv \int_0^\infty \exp(-(\rho - n) t) \frac{c(t)^{1-\theta} - 1}{1 - \theta} dt, \tag{2}$$

with $\theta \geq 0$ and $c(t)$=consumption aggregate (per capita).
- Aggregate consumption: agricultural (A), manufacturing (M) and services (S)

$$c(t) = \left(c^A(t) - \gamma^A\right)^{\eta^A} c^M(t)^{\eta^M} \left(c^S(t) + \gamma^S\right)^{\eta^S}, \tag{3}$$

Non-Balanced Growth: The Demand Side IV

- *Stone-Geary* preferences:
  - Minimum or *subsistence* level of agricultural (food) consumption, γ^A.
  - After γ^A has been achieved, household starts to demand other items
- γ^S term: household will spend on services only after certain levels of agricultural and manufacturing consumption have been reached.
- Closed economy: agricultural, manufacturing and services consumption must be met by domestic production.
- Production functions for agricultural, manufacturing and service goods:

$$\begin{aligned} Y^A(t) &= B^A F\left(K^A(t), X(t) L^A(t)\right), \qquad (4) \\ Y^M(t) &= B^M F\left(K^M(t), X(t) L^M(t)\right), \\ Y^S(t) &= B^S F\left(K^S(t), X(t) L^S(t)\right), \end{aligned}$$

Non-Balanced Growth: The Demand Side V

- Notation: $Y^j(t)$ for $j \in \{A, M, S\}$=output of agricultural, manufacturing and services, $K^j(t)$ and $L^j(t)$=capital and labor allocated to sectors, B^j =Hicks-neutral productivity term, and $X(t)$=labor-augmenting (Harrod-neutral) productivity term.
- F satisfies usual neoclassical assumptions,
- Note production function for all sectors are identical and same labor-augmenting technology.
  - Isolate demand-side sources of structural change.
- Constant rate of growth $X(t)$:

$$\frac{\dot{X}(t)}{X(t)} = g \tag{5}$$

Non-Balanced Growth: The Demand Side VI

- To ensure transversality condition holds assume $\rho - n > (1 - \theta) g$.
- Market clearing for labor and capital:

$$K^A(t) + K^M(t) + K^S(t) = K(t), \tag{6}$$

and

$$L^A(t) + L^M(t) + L^S(t) = L(t), \tag{7}$$

where $K(t)$ and $L(t)$ are total supplies.

- Manufacturing good is used in production of investment good.
- Thus, market clearing for manufacturing good (we ignore capital depreciation):

$$\dot{K}(t) + c^M(t) L(t) = Y^M(t), \tag{8}$$

Non-Balanced Growth: The Demand Side VII

- Since economy admits a representative household, equations (4)-(8) also represent representative household's budget constraint.
- Market clearing for agricultural and service goods:

$$c^A(t) L(t) = Y^A(t) \text{ and } c^S(t) L(t) = Y^S(t), \tag{9}$$

- All markets are competitive.
- Price of manufacturing good at each date is numeraire.
- Price of agricultural goods, $p^A(t)$, of services, $p^S(t)$, of factor prices $w(t)$ and $r(t)$.
- Consumption aggregator (3) implies prices must satisfy:

$$\frac{p^A(t)\left(c^A(t) - \gamma^A\right)}{\eta^A} = \frac{c^M(t)}{\eta^M}, \tag{10}$$

and

$$\frac{p^S(t)\left(c^S(t) + \gamma^S\right)}{\eta^S} = \frac{c^M(t)}{\eta^M}. \tag{11}$$

Non-Balanced Growth: The Demand Side VIII

- Competitive factor markets imply:

$$w(t) = \frac{\partial B^M F\left(K^M(t), X(t) L^M(t)\right)}{\partial L^M}, \tag{12}$$

and

$$r(t) = \frac{\partial B^M F\left(K^M(t), X(t) L^M(t)\right)}{\partial K^M}, \tag{13}$$

or equivalently marginal products from other sectors.
- Competitive equilibrium: $\left[K^A(t), K^M(t), K^S(t), L^A(t), L^M(t), L^S(t)\right]_{t=0}^{\infty}$ that maximize profits given $[K(t), L(t)]_{t=0}^{\infty}$ and $\left[p^A(t), p^M(t), w(t), r(t)\right]_{t=0}^{\infty}$; $\left[p^A(t), p^M(t), w(t), r(t)\right]_{t=0}^{\infty}$ that satisfy (10)-(13) given $\left[K^A(t), K^M(t), K^S(t), L^A(t), L^M(t), L^S(t)\right]_{t=0}^{\infty}$; and $\left[c^A(t), c^M(t), c^S(t), K(t)\right]_{t=0}^{\infty}$ that maximize (2) subject to (4)-(8); and $[L(t)]_{t=0}^{\infty}$ that satisfies (1).
- Assume

$$B^A F\left(K^A(0), X(0) L^A(0)\right) > \gamma^A L(0), \tag{14}$$

Non-Balanced Growth: The Demand Side IX

Proposition Suppose (14) holds. Then, in any equilibrium, the following conditions are satisfied (omit time suscripts):

$$\frac{K^A}{XL^A} = \frac{K^M}{XL^M} = \frac{K^S}{XL^S} = \frac{K}{XL} \equiv k(t) \quad (15)$$

for all t, where last equality defines $k(t)$ as aggregate effective capital-labor ratio of economy;

$$p^A(t) = \frac{B^M}{B^A} \quad (16)$$

for all t;

$$p^S(t) = \frac{B^M}{B^S} \quad (17)$$

for all t.

Non-Balanced Growth: The Demand Side X

- Intuition:
  - Since production functions are identical capital-labor ratios allocated to three sectors must be equalized.
  - Given (15), equilibrium price relationships (16) and (17) follow from the fact that marginal products of capital and labor have to be equalized in all three sectors.
- Proposition above does not make use of preference side.

Non-Balanced Growth: The Demand Side XI

- Deriving standard Euler equation for representative consumer and using equations (10)-(11), we obtain following.

Proposition Suppose (14) holds. Then, in any equilibrium, we have that

$$\frac{\dot{c}^M(t)}{c^M(t)} = \frac{1}{\theta}(r(t) - \rho) \tag{18}$$

for all t and moreover, provided that $\rho - n > (1-\theta) g$ holds, transversality condition of representative household is satisfied. In addition, we have that for all t

$$\frac{p^A(t)\left(c^A(t) - \gamma^A\right)}{\eta^A} = \frac{c^M(t)}{\eta^M} \tag{19}$$

and

$$\frac{p^s(t)\left(c^S(t) + \gamma^S\right)}{\eta^S} = \frac{c^M(t)}{\eta^M}. \tag{20}$$

Lack of a Balanced Growth Path

- Define a *balanced growth path* in this economy as an equilibrium path in which output and consumption of all three sectors grow at same constant rate.

Proposition Suppose that either $\gamma^A > 0$ and/or $\gamma^S > 0$. Then a balanced growth path does not exist.

- Since preferences incorporate Engel's law, household would always like to change composition of its consumption, and this will be reflected in a change in composition of production.

Constant Growth Path

- Define a weaker notion, *constant growth path* (CGP).
- CGP requires that rate of growth of aggregate consumption must be asymptotically constant.
- Given (2), constant growth rate of consumption implies interest rate must also be constant asymptotically.
- In a CGP, output, consumption and employment in three sectors may grow at different rates.

Existence of Constant Growth Path

Proposition Suppose (14) holds. Then, in the above-described economy a CGP exists if and only if

$$\frac{\gamma^A}{B^A} = \frac{\gamma^S}{B^S}. \tag{21}$$

In a CGP $k(t) = k^*$ for all t, and moreover (omit time suscripts):

$$\frac{\dot{c}^A}{c^A} = g\frac{c^A - \gamma^A}{c^A},\ \frac{\dot{c}^M}{c^M} = g,\ \frac{\dot{c}^S}{c^S} = g\frac{c^S + \gamma^S}{c^S}, \tag{22}$$

$$\frac{\dot{L}^A}{L^A} = n - g\frac{\gamma^A / L^A}{B^A XF(k^*, 1)},\ \frac{\dot{L}^M}{L^M} = n,\ \frac{\dot{L}^S}{L^S} = n + g\frac{\gamma^S / L^S}{B^S XF(k^*, 1)}$$

Non-Balanced Growth: Discussion

- Analysis of structural change that has potential relevance both for early stages of development and relatively advanced countries.
- Engel's law (augmented with highly income elastic demand for services) generates demand-side force towards non-balanced growth.
- As their incomes grow, consumers wish to spend a greater fraction on services and a smaller on food (agricultural goods).
- Thus equilibrium with fully balanced growth is impossible.
- Different sectors grow at different rates and there is reallocation of labor and capital across sectors.
- But under (21) a constant growth path (CGP) exists and structural change takes place despite the fact that interest rate and share of capital in national income are constant.
- Equilibrium path can be consistent with Kaldor facts and a continuous process of structural change.

Shortcomings

1. Process of structural change here falls short of sweeping transformations of by Kuznets, even if looking at transitional dynamics.
2. Some restrictive assumptions:
   1. That all sectors have same production function, though it can be somewhat relaxed.
   2. That investments for all sectors use only manufacturing good:
      - Similar to assumption that only capital is used to be produce capital (investment) goods in Rebelo (1991).
      - If relaxed, no longer possible to reconcile Kuznets and Kaldor facts in this model.
3. Model has constant share of employment in manufacturing, broadly consistent with US experience over past 150 years but not with earlier stages of development.
4. Condition necessary for a CGP, (21), is a "knife-edge" condition.
   - But even when not satisfied, model may approximate structural change we observe.

Non-Balanced Growth: The Supply Side

- Baumol's (1967) seminal work: "uneven growth" (non-balanced growth) will be a general feature of growth process because different sectors will grow at different rates owing to different rates of technological progress
- Review some ideas based on Acemoglu and Guerrieri (2006), who emphasize supply-side causes of non-balanced growth.
- Rich patterns of structural change during early stages of development and those in more advanced economies today require models that combine supply-side and demand-side factors.
- Isolating these factors is both more tractable and also conceptually more transparent.

General Insights I

- There are more subtle and compelling reasons for supply-side non-balanced growth than those originally emphasized by Baumol.
- In particular, industries differ considerably in terms of their capital intensity and also in terms of intensity with which they use other factors.
- In short, different industries have different *factor proportions*.
- Proportion differences across sectors combined with *capital deepening* will lead to non-balanced economic growth.
- Environment with two sectors, each with constant returns to scale production function and arbitrary preferences over goods produced in these two sectors.
- Both sectors employ capital, K, and labor, L.
- Take sequence (process) of capital and labor supplies, $[K(t), L(t)]_{t=0}^{\infty}$, as given.

General Insights II

- Labor is supplied inelastically.
- Preferences defined over final output or a consumption aggregator as in (3).
- Final output denoted by Y and produced as an aggregate of output of two sectors, Y_1 and Y_2,

$$Y(t) = F(Y_1(t), Y_2(t)).$$

- F satisfies usual assumptions, in particular, it exhibits constant returns to scale and is twice continuously differentiable.
- Sectoral production functions:

$$Y_1(t) = A_1(t) G_1(K_1(t), L_1(t)) \tag{23}$$

and

$$Y_2(t) = A_2(t) G_2(K_2(t), L_2(t)), \tag{24}$$

General Insights III

- G_1 and G_2 are also assumed to satisfy usual assumptions.
- $A_1(t)$ and $A_2(t)$ are Hicks-neutral technology terms.
- Market clearing for capital and labor implies:

$$\begin{aligned} K_1(t) + K_2(t) &= K(t), \\ L_1(t) + L_2(t) &= L(t), \end{aligned} \tag{25}$$

- Ignore capital depreciation.
- Final good is numeraire in every period and, prices of Y_1 and Y_2 are p_1 and p_2, and wage and rental rate of capital (interest rate) are w and r.

General Insights IV

- Product and factor markets are competitive, so product and factor prices satisfy:

$$\frac{p_1(t)}{p_2(t)} = \frac{\partial F(Y_1(t), Y_2(t))/\partial Y_1}{\partial F(Y_1(t), Y_2(t))/\partial Y_2} \tag{26}$$

and

$$\begin{aligned} w(t) &= \frac{\partial A_1(t) G_1(K_1(t), L_1(t))}{\partial L_1} = \frac{\partial A_2(t) G_2(K_2(t), L_2(t))}{\partial L_2} \\ r(t) &= \frac{\partial A_1 G_1(K_1(t), L_1(t))}{\partial K_1} = \frac{\partial A_2 G_2(K_2(t), L_2(t))}{\partial K_2}. \end{aligned} \tag{27}$$

- Equilibrium: Sequence $[p_1(t), p_2(t), w(t), r(t)]_{t=0}^{\infty}$ and $[K_1(t), K_2(t), L_1(t), L_2(t)]_{t=0}^{\infty}$, such that given $[K(t), L(t)]_{t=0}^{\infty}$, (25), (26) and (27) are satisfied.

General Insights V

- Shares of capital in two sectors:

$$\sigma_1(t) \equiv \frac{r(t) K_1(t)}{p_1(t) Y_1(t)} \text{ and } \sigma_2(t) \equiv \frac{r(t) K_2(t)}{p_2(t) Y_2(t)}. \tag{28}$$

- There is *capital deepening* at time t if $\dot{K}(t)/K(t) > \dot{L}(t)/L(t)$.
- There are *factor proportion differences* at time t if $\sigma_1(t) \neq \sigma_2(t)$.
- Technological progress is *balanced* at time t if $\dot{A}_1(t)/A_1(t) = \dot{A}_2(t)/A_2(t)$.
- Note factor proportion differences, $\sigma_1(t) \neq \sigma_2(t)$, refers to equilibrium factor proportions in two sectors at time t, so may be equal at some future date.

Proposition: Non-Balanced Growth

Proposition Suppose that at time t, there are factor proportion differences between two sectors, technological progress is balanced, and there is capital deepening, then growth is not balanced, that is, $\dot{Y}_1(t) / Y_1(t) \neq \dot{Y}_2(t) / Y_2(t)$.

Proof of Proposition: Non-Balanced Growth I

- Define capital to labor ratio in two sectors as

$$k_1(t) \equiv \frac{K_1(t)}{L_1(t)} \text{ and } k_2(t) \equiv \frac{K_2(t)}{L_2(t)},$$

and "per capita production functions" (without Hicks-neutral technology term) as

$$\begin{aligned} g_1(k_1(t)) &\equiv \frac{G_1(K_1(t), L_1(t))}{L_1(t)} \qquad (29) \\ \text{and } g_2(k_2(t)) &\equiv \frac{G_2(K_2(t), L_2(t))}{L_2(t)}. \end{aligned}$$

- Since G_1 and G_2 are twice continuously differentiable by assumption, so are g_1 and g_2 and denote their first and second derivatives by g_1', g_2', g_1'' and g_2''.

Proof of Proposition: Non-Balanced Growth II

- Differentiating production functions for two sectors,

$$\frac{\dot{Y}_1(t)}{Y_1(t)} = \frac{\dot{A}_1(t)}{A_1(t)} + \sigma_1(t)\frac{\dot{K}_1(t)}{K_1(t)} + (1-\sigma_1(t))\frac{\dot{L}_1(t)}{L_1(t)}$$

and

$$\frac{\dot{Y}_2(t)}{Y_2(t)} = \frac{\dot{A}_2(t)}{A_2(t)} + \sigma_2(t)\frac{\dot{K}_2(t)}{K_2(t)} + (1-\sigma_2(t))\frac{\dot{L}_2(t)}{L_2(t)}.$$

- Suppose, to obtain a contradiction, that $\dot{Y}_1/Y_1 = \dot{Y}_2/Y_2$. Since F exhibits constant returns to scale, $\dot{Y}_1/Y_1 = \dot{Y}_2/Y_2$ together with (26) implies

$$\frac{\dot{p}_1}{p_1} = \frac{\dot{p}_2}{p_2} = 0. \tag{30}$$

Proof of Proposition: Non-Balanced Growth III

- Given (29), (27) gives equilibrium interest rate and wage:

$$\begin{aligned} r &= p_1 A_1 g_1'(k_1) \\ &= p_2 A_2 g_2'(k_2), \end{aligned} \tag{31}$$

$$\begin{aligned} w &= p_1 A_1 \left(g_1(k_1) - g_1'(k_1)\, k_1\right) \\ &= p_2 A_2 \left(g_2(k_2) - g_2'(k_2)\, k_2\right). \end{aligned} \tag{32}$$

- Differentiating (31), with respect to time and using (30):

$$\frac{\dot{A}_1}{A_1} + \varepsilon_{g_1'} \frac{\dot{k}_1}{k_1} = \frac{\dot{A}_2}{A_2} + \varepsilon_{g_2'} \frac{\dot{k}_2}{k_2}$$

where

$$\varepsilon_{g_1'} \equiv \frac{g_1''(k_1)\, k_1}{g_1'(k_1)} \text{ and } \varepsilon_{g_2'} \equiv \frac{g_2''(k_2)\, k_2}{g_2'(k_2)}.$$

Proof of Proposition: Non-Balanced Growth IV

- Since $\dot{A}_1/A_1 = \dot{A}_2/A_2$,

$$\varepsilon_{g_1'} \frac{\dot{k}_1}{k_1} = \varepsilon_{g_2'} \frac{\dot{k}_2}{k_2}. \tag{33}$$

- Differentiating wage condition, (32), with respect to time, using (30) and some algebra gives:

$$\frac{\dot{A}_1}{A_1} - \frac{\sigma_1}{1-\sigma_1} \varepsilon_{g_1'} \frac{\dot{k}_1}{k_1} = \frac{\dot{A}_2}{A_2} - \frac{\sigma_2}{1-\sigma_2} \varepsilon_{g_2'} \frac{\dot{k}_2}{k_2}.$$

- Since $\dot{A}_1/A_1 = \dot{A}_2/A_2$ and $\sigma_1 \neq \sigma_2$, this equation is inconsistent with (33), yielding a contradiction and proving the claim.

Intuition: Non-Balanced Growth I

- Suppose there is capital deepening and sector 2 is more capital-intensive (i.e., $\sigma_1 < \sigma_2$).
  - If capital and labor were allocated to sectors at constant proportions, more capital-intensive sector 2 would grow faster than sector 1.
  - In equilibrium, faster growth in sector 2 would decline relative price of sector 2, and some labor and capital would be reallocated to 1.
  - But reallocation *could not* entirely offset increase in output of sector 2; if it did, relative price change that stimulated reallocation would not take place.
  - Thus equilibrium growth must be non-balanced.

Intuition: Non-Balanced Growth II

- Related Rybczynski's Theorem in international trade:
  - For open economy within "cone of diversification" (where factor prices do not depend on factor endowments), changes in factor endowments will be absorbed by changes in sectoral output mix.
- Can be viewed as a closed-economy analog and as generalization of Rybczynski's Theorem:
  - Changes in factor endowments (capital deepening) will be absorbed by faster growth in one sector, even though relative prices of goods and factors will change in response to the change.

Proposition: Non-Balanced Growth with N sectors

- Straightforward to generalize Proposition above to an economy with $N \geq 2$ sectors.
- Suppose aggregate output is given by constant returns to scale production function:

$$Y = F\left(Y_1(t), Y_2(t), ..., Y_N(t)\right).$$

Defining $\sigma_j(t)$ as capital share in sector $j = 1, ..., N$ as in (28).

Proposition Suppose that at time t, there are factor proportion differences among the N sectors in the sense that there exists i and $j \leq N$ such that $\sigma_i(t) \neq \sigma_j(t)$, technological progress is balanced between i and j, i.e., $\dot{A}_i(t)/A_i(t) = \dot{A}_j(t)/A_j(t)$, and there is capital deepening, i.e., $\dot{K}(t)/K(t) > \dot{L}(t)/L(t)$, then growth is not balanced and $\dot{Y}_i(t)/Y_i(t) \neq \dot{Y}_j(t)/Y_j(t)$.

Balanced Growth and Kuznets Facts I

- Results thus far stated for a given (arbitrary) sequence of capital and labor supplies, $[K(t), L(t)]_{t=0}^{\infty}$.
- Must endogenize path of capital accumulation (and specify pattern of population growth) to address whether supply-side factors provide a useful framework for Kaldor and Kuznets facts.
- Economy again in infinite horizon and population grows at exogenous rate $n > 0$ according to (1).
- Representative consumer, with standard preferences given by (2), who also supplies labor inelastically.
- Capital deepening will now result from exogenous technological progress.

Balanced Growth and Kuznets Facts II

- Assume unique final good is produced with a constant elasticity of substitution aggregator:

$$Y(t) = \left[\gamma Y_1(t)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_2(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad (34)$$

  where $\varepsilon \in [0, \infty)$ is elasticity of substitution between intermediates and $\gamma \in (0, 1)$ determines their relative importance in production.
- Ignore capital depreciation.
- Final good is distributed between consumption and investment:,

$$\dot{K}(t) + L(t) c(t) \leq Y(t), \quad (35)$$

  where $c(t)$ is consumption per capita.

Balanced Growth and Kuznets Facts III

- Y_1 and Y_2 are produced competitively with aggregate production functions

$$\begin{aligned} Y_1(t) &= A_1(t) K_1(t)^{\alpha_1} L_1(t)^{1-\alpha_1} \\ \text{and } Y_2(t) &= A_2(t) K_2(t)^{\alpha_2} L_2(t)^{1-\alpha_2}. \end{aligned} \quad (36)$$

- Throughout, impose

$$\alpha_1 < \alpha_2, \quad (37)$$

i.e., sector 1 is less capital-intensive than sector 2.

- In (36) A_1 and A_2 are Hicks-neutral technology terms that grow at exogenous and potentially different rates:

$$\frac{\dot{A}_1(t)}{A_1(t)} = a_1 > 0 \text{ and } \frac{\dot{A}_2(t)}{A_2(t)} = a_2 > 0. \quad (38)$$

Balanced Growth and Kuznets Facts IV

- Labor and capital market clearing:

$$L_1(t) + L_2(t) = L(t), \tag{39}$$

and

$$K_1(t) + K_2(t) = K(t). \tag{40}$$

- Denote wage and interest rate by $w(t)$ and $r(t)$ and prices of two intermediate goods by $p_1(t)$ and $p_2(t)$.
- Normalize price of final good to 1 at each instant.
- Equilibrium: sequences such that $[K_1(t), K_2(t), L_1(t), L_2(t)]_{t=0}^{\infty}$ maximize intermediate sector profits given $[w(t),\ r(t),\ p_1(t), p_2(t)]_{t=0}^{\infty}$ and $[K(t), L(t)]_{t=0}^{\infty}$; intermediate and factor markets clear at prices $[w(t),\ r(t),\ p_1(t), p_2(t)]_{t=0}^{\infty}$; $[K(t),\ c(t)]_{t=0}^{\infty}$ maximize utility of representative household given $[w(t),\ r(t),\ p_1(t), p_2(t)]_{t=0}^{\infty}$; and population evolves according to (1).

Balanced Growth and Kuznets Facts V

- Break characterization of equilibrium: *static* and *dynamic*.
  - Static: take state variables of economy, K, L, A_1 and A_2, as given and determine allocation of capital and labor across sectors and factor and intermediate prices.
  - Dynamic: determine evolution of endogenous state variable, K (dynamics of L given by (1) and of A_1 and A_2 by (38)).
- Choice of numeraire implies:

$$1 = \left[\gamma^{\varepsilon} p_1(t)^{1-\varepsilon} + (1-\gamma)^{\varepsilon} p_2(t)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}},$$

- Profit maximization of final good sector implies:

$$p_1(t) = \gamma \left(\frac{Y_1(t)}{Y(t)}\right)^{-\frac{1}{\varepsilon}} \quad \text{and} \quad p_2(t) = (1-\gamma)\left(\frac{Y_2(t)}{Y(t)}\right)^{-\frac{1}{\varepsilon}}. \tag{41}$$

Balanced Growth and Kuznets Facts VI

- Equilibrium allocation equates marginal product of capital and labor in two sectors:

$$\gamma\left(1-\alpha_1\right)\left(\frac{Y(t)}{Y_1(t)}\right)^{\frac{1}{\varepsilon}}\frac{Y_1(t)}{L_1(t)}=\left(1-\gamma\right)\left(1-\alpha_2\right)\left(\frac{Y(t)}{Y_2(t)}\right)^{\frac{1}{\varepsilon}}\frac{Y_2(t)}{L_2(t)}, \tag{42}$$

$$\gamma\alpha_1\left(\frac{Y(t)}{Y_1(t)}\right)^{\frac{1}{\varepsilon}}\frac{Y_1(t)}{K_1(t)}=\left(1-\gamma\right)\alpha_2\left(\frac{Y(t)}{Y_2(t)}\right)^{\frac{1}{\varepsilon}}\frac{Y_2(t)}{K_2(t)}, \tag{43}$$

- Factor prices:

$$w(t)=\gamma\left(1-\alpha_1\right)\left(\frac{Y(t)}{Y_1(t)}\right)^{\frac{1}{\varepsilon}}\frac{Y_1(t)}{L_1(t)}, \tag{44}$$

$$r(t)=\gamma\alpha_1\left(\frac{Y(t)}{Y_1(t)}\right)^{\frac{1}{\varepsilon}}\frac{Y_1(t)}{K_1(t)}. \tag{45}$$

Balanced Growth and Kuznets Facts VII

- Key for static equilibrium is determine fraction of capital and labor employed in two sectors.
- Define $\kappa(t) \equiv K_1(t)/K(t)$ and $\lambda(t) \equiv L_1(t)/L(t)$.
- Combining (39), (40), (42), and (43):

$$\kappa(t) = \left[1 + \frac{\alpha_2}{\alpha_1}\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{Y_1(t)}{Y_2(t)}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]^{-1}, \tag{46}$$

$$\lambda(t) = \left[1 + \frac{\alpha_1}{\alpha_2}\left(\frac{1-\alpha_2}{1-\alpha_1}\right)\left(\frac{1-\kappa(t)}{\kappa(t)}\right)\right]^{-1}. \tag{47}$$

- Equation (47): λ is monotonically increasing κ.
- Thus in equilibrium capital and labor will be reallocated towards same sector.

Balanced Growth and Kuznets Facts VIII

- Static equilibrium depends on how allocation of capital and labor depends on aggregate amount of capital and labor.

Proposition In equilibrium,

$$\begin{aligned} \frac{d \ln \kappa(t)}{d \ln K(t)} &= -\frac{d \ln \kappa(t)}{d \ln L(t)} \\ &= \frac{(1-\varepsilon)(\alpha_2-\alpha_1)(1-\kappa(t))}{1+(1-\varepsilon)(\alpha_2-\alpha_1)(\kappa(t)-\lambda(t))} \\ &> 0 \text{ if and only if } (\alpha_2-\alpha_1)(1-\varepsilon)>0. \end{aligned} \quad (48)$$

$$\begin{aligned} \frac{d \ln \kappa(t)}{d \ln A_2(t)} &= -\frac{d \ln \kappa(t)}{d \ln A_1(t)} \\ &= \frac{(1-\varepsilon)(1-\kappa(t))}{1+(1-\varepsilon)(\alpha_2-\alpha_1)(\kappa(t)-\lambda(t))} \\ &> 0 \text{ if and only if } \varepsilon<1. \end{aligned} \quad (49)$$

Balanced Growth and Kuznets Facts IX

- Equation (48): when $\varepsilon < 1$, fraction of capital allocated to capital-intensive sector declines in stock of capital (conversely when $\varepsilon > 1$).
  - If K increases and κ remains constant, then capital-intensive sector 2 will grow by *more* than sector 1.
  - Prices in (41): when $\varepsilon < 1$ relative price of capital-intensive sector will fall more than proportionately, thus greater fraction of capital allocated to less capital-intensive sector 1.
  - Intuition when $\varepsilon > 1$ is similar.
- Equation (49): when $\varepsilon < 1$, improvement in technology of a sector causes fall in share of capital going to it (converse when $\varepsilon > 1$)..
  - Increased production of sector causes more than proportional decline in relative price, inducing reallocation of capital away from it towards other sector

Balanced Growth and Kuznets Facts X

- Combining (44) and (45), we also obtain relative factor prices as

$$\frac{w(t)}{r(t)} = \frac{1-\alpha_1}{\alpha_1} \left(\frac{\kappa(t) K(t)}{\lambda(t) L(t)} \right), \tag{50}$$

and capital share in economy as:

$$\sigma_K(t) \equiv \frac{r(t) K(t)}{Y(t)} = \gamma \alpha_1 \left(\frac{Y_1(t)}{Y(t)} \right)^{\frac{\varepsilon-1}{\varepsilon}} \kappa(t)^{-1}. \tag{51}$$

Proposition

In equilibrium,

$$\frac{d\ln\left(w\left(t\right)/r\left(t\right)\right)}{d\ln K\left(t\right)} = -\frac{d\ln\left(w\left(t\right)/r\left(t\right)\right)}{d\ln L\left(t\right)} \tag{52}$$

$$= \frac{1}{1+\left(1-\varepsilon\right)\left(\alpha_2-\alpha_1\right)\left(\kappa\left(t\right)-\lambda\left(t\right)\right)} > 0.$$

$$\frac{d\ln\left(w\left(t\right)/r\left(t\right)\right)}{d\ln A_2\left(t\right)} = -\frac{d\ln\left(w\left(t\right)/r\left(t\right)\right)}{d\ln A_1\left(t\right)} \tag{53}$$

$$= -\frac{\left(1-\varepsilon\right)\left(\kappa\left(t\right)-\lambda\left(t\right)\right)}{1+\left(1-\varepsilon\right)\left(\alpha_2-\alpha_1\right)\left(\kappa\left(t\right)-\lambda\left(t\right)\right)} < 0 \text{ iff } \left(\alpha_2-\alpha_1\right)\left(1-\varepsilon\right) > 0.$$

$$\frac{d\ln\sigma_K\left(t\right)}{d\ln K\left(t\right)} < 0 \text{ iff } \varepsilon < 1. \tag{54}$$

$$\frac{d\ln\sigma_K\left(t\right)}{d\ln A_2\left(t\right)} = -\frac{d\ln\sigma_K\left(t\right)}{d\ln A_1\left(t\right)} \tag{55}$$

$$< 0 \text{ iff } \left(\alpha_2-\alpha_1\right)\left(1-\varepsilon\right) > 0.$$

Proof of Proposition I

- Results in (52) and (53) follow from differentiating (50) and previous proposition.
- To prove the remaining:

$$\begin{aligned}
\left(\frac{Y_1}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}} &= \left[\gamma+(1-\gamma)\left(\frac{Y_1}{Y_2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]^{-1} \\
&= \gamma^{-1}\left(1+\frac{\alpha_1}{\alpha_2}\left(\frac{1}{\kappa}-1\right)\right)^{-1}
\end{aligned}$$

Proof of Proposition II

- Using previous proposition and definition of σ_K from (51):

$$\frac{d \ln \sigma_K}{d \ln K} = -\Omega \frac{1-\sigma_K}{\sigma_K} \frac{\alpha_1}{\alpha_2} \frac{(1-\varepsilon)(\alpha_2-\alpha_1)(1-\kappa)/\kappa}{1+(1-\varepsilon)(\alpha_2-\alpha_1)(\kappa-\lambda)} \tag{56}$$

$$\frac{d \ln \sigma_K}{d \ln A_2} = -\frac{d \ln \sigma_K}{d \ln A_1} = \Omega \frac{1-\sigma_K}{\sigma_K} \frac{\alpha_1}{\alpha_2} \frac{(1-\varepsilon)(1-\kappa)/\kappa}{1+(1-\varepsilon)(\alpha_2-\alpha_1)(\kappa-\lambda)}, \tag{57}$$

where

$$\Omega \equiv \left[\left(1+\frac{\alpha_1}{\alpha_2}\left(\frac{1}{\kappa}-1\right)\right)^{-1} - \left(\frac{1-\alpha_1}{1-\alpha_2}+\frac{\alpha_1}{\alpha_2}\left(\frac{1}{\kappa}-1\right)\right)^{-1} \right].$$

- Clearly, $\Omega > 0$ if and only if $\alpha_1 < \alpha_2$, satisfied in view of (37).
- Equations (56) and (57) then imply (54) and (55).

Balanced Growth and Kuznets Facts XI

- Key result is (54): links equilibrium relationship between capital share in national income and capital stock to elasticity of substitution.
- Negative relationship between share of capital in national income and capital stock: equivalent to capital and labor being gross complements in aggregate.
- Hence result also implies that elasticity of substitution between capital and labor is less than one if and only if ε is less than one, as suggested by many approaches.

Balanced Growth and Kuznets Facts XII

- Intuition for Proposition:
  - When $\varepsilon < 1$, increase in capital stock causes output of more capital-intensive sector 2 to increase relative to in other sector (despite share of capital allocated to other sector increases as shown in (48)).
  - This increases production of more capital-intensive sector and reduces relative reward to capital (and its share in national income).
  - Recall when $\varepsilon < 1$, (55) implies that increase in A_1 is "capital biased" and increase in A_2 is "labor biased".
    - ★ When $\varepsilon < 1$ an increase in output of a sector (now driven by change in technology) decreases its price more than proportionately, reducing relative compensation of factor used more intensively in that sector.
  - Converse applies when $\varepsilon > 1$.

Balanced Growth and Kuznets Facts XIII

- Dynamic equilibrium. Start with Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}(r(t) - \rho). \tag{58}$$

- Transversality condition (only asset is capital):

$$\lim_{t\to\infty} K(t) \exp\left(-\int_0^t r(\tau)\, d\tau\right) = 0, \tag{59}$$

- Together with Euler equation (58) and resource constraint (35) determines dynamic behavior of consumption per capita and capital stock, c and K.
- Equations (1) and (38) give behavior of L, A_1 and A_2.
- Dynamic equilibrium: paths of wages, interest rates, labor and capital allocation decisions, w, r, λ and κ, satisfying (44), (42), (45), (43), (46) and (47), and of consumption per capita, c, capital stock, K, employment, L, and technology, A_1 and A_2, satisfying (1), (35), (38), (58), and (59).

Balanced Growth and Kuznets Facts XIV

- Notation for growth rates of key objects:

$$\frac{\dot{L}_s(t)}{L_s(t)} \equiv n_s(t), \frac{\dot{K}_s(t)}{K_s(t)} \equiv z_s(t), \frac{\dot{Y}_s(t)}{Y_s(t)} \equiv g_s(t) \text{ for } s = 1,2$$

$$\text{and } \frac{\dot{K}(t)}{K(t)} \equiv z(t), \frac{\dot{Y}(t)}{Y(t)} \equiv g(t),$$

- Whenever they exist, define corresponding (limiting) asymptotic growth rates:

$$n_s^* = \lim_{t\to\infty} n_s(t) \ , \ z_s^* = \lim_{t\to\infty} z_s(t) \text{ and } g_s^* = \lim_{t\to\infty} g_s(t),$$

for $s = 1, 2$.

- Denote asymptotic capital and labor allocation decisions by

$$\kappa^* = \lim_{t\to\infty} \kappa(t) \text{ and } \lambda^* = \lim_{t\to\infty} \lambda(t).$$

Balanced Growth and Kuznets Facts XV

Proposition
1. If $\varepsilon < 1$, then
$n_1(t) \gtreqless n_2(t) \Leftrightarrow z_1(t) \gtreqless z_2(t) \Leftrightarrow g_1(t) \lesseqgtr g_2(t)$.
2. If $\varepsilon > 1$, then
$n_1(t) \gtreqless n_2(t) \Leftrightarrow z_1(t) \gtreqless z_2(t) \Leftrightarrow g_1(t) \gtreqless g_2(t)$.

- Proof:
  - Omitting time arguments and differentiating (42) with respect to time:

$$\frac{1}{\varepsilon}g + \frac{\varepsilon - 1}{\varepsilon}g_1 - n_1 = \frac{1}{\varepsilon}g + \frac{\varepsilon - 1}{\varepsilon}g_2 - n_2, \tag{60}$$

    which implies that $n_1 - n_2 = (\varepsilon - 1)(g_1 - g_2)/\varepsilon$ and establishes first part of proposition.
  - Similarly differentiating (43) yields

$$\frac{1}{\varepsilon}g + \frac{\varepsilon - 1}{\varepsilon}g_1 - z_1 = \frac{1}{\varepsilon}g + \frac{\varepsilon - 1}{\varepsilon}g_2 - z_2 \tag{61}$$

    and establishes second part of result.

Balanced Growth and Kuznets Facts XVI

- Straightforward but counter-intuitive result: when $\varepsilon < 1$, growth rate of capital stock and labor force in sector that is growing faster must be *less* than in other sector.
- When greater than one, converse result obtains.
- Intuition: terms of trade (relative prices) shift in favor of more slowly growing sector.
    - When elasticity is less than one, change in relative prices is more than proportional with change in quantities and this encourages more of the factors to be allocated towards more slowly growing sector.

Balanced Growth and Kuznets Facts XVII

Proposition Suppose the asymptotic growth rates g_1^* and g_2^* exist. If $\varepsilon < 1$, then $g^* = \min\{g_1^*, g_2^*\}$. If $\varepsilon > 1$, then $g^* = \max\{g_1^*, g_2^*\}$.

- Proof:
  - Differentiating the production function for final good (34):

$$g(t) = \frac{\gamma Y_1(t)^{\frac{\varepsilon-1}{\varepsilon}} g_1(t) + (1-\gamma) Y_2(t)^{\frac{\varepsilon-1}{\varepsilon}} g_2(t)}{\gamma Y_1(t)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_2(t)^{\frac{\varepsilon-1}{\varepsilon}}}. \tag{62}$$

  - Combined with $\varepsilon < 1$, implies that as $t \to \infty$, $g^* = \min\{g_1^*, g_2^*\}$.
  - Combined with $\varepsilon > 1$, it implies that as $t \to \infty$, $g^* = \max\{g_1^*, g_2^*\}$.
- Thus when $\varepsilon < 1$, asymptotic growth rate will be determined by sector that is growing more slowly, and converse when $\varepsilon > 1$.

Balanced Growth and Kuznets Facts XVIII

- Focus on a *constant growth path* (CGP), where asymptotic growth rate of consumption per capita exists and is constant, i.e.,

$$\lim_{t\to\infty} \frac{\dot{c}(t)}{c(t)} = g_c^*.$$

- Define growth rate of total consumption as $\dot{C}(t)/C(t) \equiv g_C^* = g_c^* + n$.
- From Euler equation (58), the fact that growth rate of consumption or consumption per capita are asymptotically constant implies $\lim_{t\to\infty} \dot{r} = 0$.
- To establish existence of a CGP, impose parameter restriction:

$$\rho - n \geq (1-\theta) \max\left\{ \frac{a_1}{1-\alpha_1}, \frac{a_2}{1-\alpha_2} \right\}. \tag{63}$$

- Ensures that transversality condition (59) holds.

Balanced Growth and Kuznets Facts XIX

- Terms $a_1 / (1 - \alpha_1)$ or $a_2 / (1 - \alpha_2)$ capture "augmented" rate of technological progress.
  - Overall effect on labor productivity (and growth) depend on rate of technological progress augmented with capital deepening.
  - Lower α_s: greater share of capital in sector $s = 1, 2$, and thus higher rate of augmented technological progress given rate of Hicks-neutral technological change.
  - Thus (63): augmented rate of technological progress should be low enough to satisfy transversality condition (59).
- Impose:

$$\begin{aligned} \text{either (i) } a_1 / (1 - \alpha_1) &< a_2 / (1 - \alpha_2) \text{ and } \varepsilon < 1; \\ \text{or (ii) } a_1 / (1 - \alpha_1) &> a_2 / (1 - \alpha_2) \text{ and } \varepsilon > 1, \end{aligned} \tag{64}$$

- $\Longrightarrow$ sector 1 is *asymptotically dominant*, either because it has slower technological progress and $\varepsilon < 1$, or more rapid and $\varepsilon > 1$.
- Comparison is not between a_1 and a_2, but between $a_1 / (1 - \alpha_1)$ and $a_2 / (1 - \alpha_2)$.

Proposition: Constant Growth Path

Proposition Suppose that conditions (37), (63) and (64) hold. Then there exists a unique CGP such that

$$g^* = g_C^* = g_1^* = z_1^* = n + g_c^* = n + \frac{1}{1-\alpha_1} a_1, \tag{65}$$

$$z_2^* = n - (1-\varepsilon) a_2 + (1 + (1-\varepsilon)(1-\alpha_2)) \frac{a_1}{1-\alpha_1} < g^*, \tag{66}$$

$$g_2^* = n + \varepsilon a_2 + (1 - \varepsilon (1-\alpha_2)) \frac{a_1}{1-\alpha_1} > g^*, \tag{67}$$

$$n_1^* = n \tag{68}$$

$$\text{and } n_2^* = n - (1-\varepsilon)(1-\alpha_2)\left(\frac{a_2}{1-\alpha_2} - \frac{a_1}{1-\alpha_1}\right) < n_1^*.$$

Proof of Proposition: Constant Growth Path I

- Suppose first that $g_2^* \geq g_1^* > 0$ and $\varepsilon > 1$. Then equations (46) and (47) imply that $\lambda^* = \kappa^* = 1$. In view of this, previous Proposition implies $g^* = g_1^*$.
  - This condition together with equations, (36) , (60) and (61), solves uniquely for n_1^*, n_2^*, z_1^*, z_2^*, g_1^* and g_2^* as given in equations (65), (66), (67) and (68).
  - This solution is consistent with $g_2^* > g_1^* > 0$, since conditions (37) and (63) imply that $g_2^* > g_1^*$ and $g_1^* > 0$.
  - $C(t) \equiv c(t) L(t) \leq Y(t)$, (35) and (59) imply that consumption growth rate, g_C^*, is equal to growth rate of output, g^*.
  - Suppose that this last claim were not correct, then since $C(t) / Y(t) \to 0$ as $t \to \infty$, resource constraint (35) would imply that asymptotically $\dot{K}(t) = Y(t)$.
  - Integrating this we obtain $K(t) \to \int_0^t Y(s)\, ds$, and since Y is growing exponentially, this implies that capital stock grows more than exponentially, thus violating transversality condition (59).

Proof of Proposition: Constant Growth Path II

- Finally, verify that an equilibrium with z_1^*, z_2^*, m_1^*, m_2^*, g_1^* and g_2^* satisfies transversality condition (59).
- Note that transversality condition (59) will be satisfied if

$$\lim_{t\to\infty} \frac{\dot{K}(t)}{K(t)} < r^*, \tag{69}$$

  where r^* is the constant asymptotic interest rate.
- Since from the Euler equation (58) $r^* = \theta g^* + \rho$, (69) will be satisfied when $g^*(1-\theta) < \rho$.
- Condition (63) ensures that this is the case with $g^* = n + a_1/(1-\alpha_1)$.

- Argument for case in which $g_1^* \geq g_2^* > 0$ and $\varepsilon > 1$ is similar.
- To complete proof, we need to establish that in all CGPs $g_2^* \geq g_1^* > 0$ when $\varepsilon < 1$ ($g_1^* \geq g_2^* > 0$ when $\varepsilon > 1$ is again similar).

Proof of Proposition: Constant Growth Path III

- Derive a contradiction separately for two configurations, (1) $g_1^* \geq g_2^*$, or (2) $g_2^* \geq g_1^*$ but $g_1^* \leq 0$.

(1) Suppose $g_1^* \geq g_2^*$ and $\varepsilon < 1$.

-
  - Then, following same reasoning as above, unique solution to equilibrium conditions (36), (60) and (61), when $\varepsilon < 1$ is:

$$g^* = g_C^* = g_2^* = z_2^* = n + a_2 / (1 - \alpha_2),$$

$$z_1^* = n - (1 - \varepsilon) a_1 + (1 + (1 - \varepsilon)(1 - \alpha_1)) \frac{a_1}{1 - \alpha_1},$$

$$g_1^* = n + \varepsilon a_1 + (1 - \varepsilon (1 - \alpha_1)) \frac{a_1}{1 - \alpha_1} \tag{70}$$

    and also similar expressions for n_1^* and n_2^*.
  - Combining these equations implies that $g_1^* < g_2^*$.
  - This contradicts hypothesis $g_1^* \geq g_2^* > 0$.
  - The argument for $\varepsilon > 1$ is analogous.

Proof of Proposition: Constant Growth Path IV

(2) Suppose $g_2^* \geq g_1^*$ and $g_1^* \leq 0$.

-
  - First suppose that $\varepsilon < 1$.
  - The same steps as above imply that there is a unique solution to equilibrium conditions (36), (60) and (61), which are given by equations (65), (66), (67) and (68).
  - But now (65) directly contradicts $g_1^* \leq 0$, and shows that it cannot be in case 1.
  - Next suppose $g_2^* \geq g_1^*$ and $\varepsilon > 1$, then unique solution is given by equations in subpart 1 above.
  - But in this case, (70) directly contradicts hypothesis that $g_1^* \leq 0$, completing proof.

Discussion: Constant Growth Path I

- As long as $a_1 / (1 - \alpha_1) \neq a_2 / (1 - \alpha_2)$, growth is non-balanced.
  - Intuition: suppose $a_1 / (1 - \alpha_1) < a_2 / (1 - \alpha_2)$ (e.g. if $a_1 \approx a_2$).
  - Differential capital intensities combined with capital deepening (itself from technological progress) ensures faster growth in more capital-intensive sector 2.
  - If capital were allocated proportionately to the two sectors, sector 2 would grow faster.
  - Because of changes in prices, capital and labor reallocated in favor of sector 1, thus relative employment in 1 increases.
  - But not enough to fully offset faster growth of real output in more capital-intensive sector.
- Assumption of balanced technological progress ($a_1 = a_2$) was not necessary: needed to rule out knife-edge case where relative rates of technological progress between sectors were exactly in right proportion to ensure balanced growth ($a_1 / (1 - \alpha_1) = a_2 / (1 - \alpha_2)$).

Discussion: Constant Growth Path II

- CGP simple because restricted attention to parameters such that sector 1 is asymptotically dominant (cf., condition (64)).
  - If also $\varepsilon < 1$, richest set of dynamics: more slowly growing sector determines long-run growth rate, while more rapidly growing continually sheds capital and labor at right rate to ensure it grows faster.
- In limiting equilibrium share of capital and labor allocated to one of sector tends to one (e.g., when 1 is asymptotically dominant, $\lambda^* = \kappa^* = 1$).
  - But at all points both sectors produce positive amounts, so limit point is never reached.
  - In fact, at all times both sectors grow at rates *greater* than population.
  - Moreover, when $\varepsilon < 1$, sector shrinking grows *faster* than rest of economy even asymptotically.
  - Thus rate at which capital and labor are allocated away from it is *exactly* such that grows faster than rest of economy.
  - Non-balanced growth is not a trivial outcome (with one sector shutting down), but results from positive but differential growth of the two sectors.

Discussion: Constant Growth Path III

- Capital share in national income and interest rate are constant in CGP.
  - Asymptotic capital share in national income reflects share of dominant sector:
    - ★ when condition (64) holds $\sigma_K^* = \alpha_1$. When it does not hold, $\sigma_K^* = \alpha_2$
  - Since limiting interest rate is constant, model also consistent with Kaldor facts.
  - Also CGP is asymptotically stable.

Agricultural Productivity and Industrialization I

- Industrialization process, beginning at end of 18th century in Europe, lies at root of modern economic growth and cross-country income differences.
- Why industrialization started and then progressed rapidly in some countries while it did not in others?
- In view of the stylized facts motivating our investigation, this question might hold important clues about cross-country differences in income per capita today.
- Number of different approaches to this question:
  - Acemoglu and Zilibotti (1997): takeoff in general, based on whether investments in different sectors undertaken by different societies turned out to be successful.
  - *The big push* suggested by Rosenstein-Rodan. Murphy, Shleifer and Vishny (1989) formalize this notion and show how, in the presence of technologies with fixed costs and monopolistic competition, coordination failures might prevent industrialization.

Agricultural Productivity and Industrialization II

- Economic history literature: 18th-century England was well-placed for industrialization because of its high agricultural productivity (e.g., Nurske, 1953, Rostow, 1960, Mokyr, 1989, or Overton, 2001).
  - Societies with a high agricultural productivity can afford to shift part of their labor force to industrial activities.
  - Increasing returns from technology or demand: shift a critical fraction of labor force to industry is key in early industrial experience.
- Matsuyama (1992) formalizes this intuition and presents a number of comparative static results that are useful
  - Combines Engel's law and learning-by-doing externalities in industrial sector.
  - Also enables an insightful analysis of impact of international trade on industrialization.

Agricultural Productivity and Industrialization III

- Infinite-horizon continuous time economy with constant population normalized to 1.
- Representative household with preferences:

$$U(0) \equiv \int_0^{\infty} \exp(-\rho t) \left(c^A(t) - \gamma^A\right)^{\eta} c^M(t)\, dt, \qquad (71)$$

- Household supplies labor inelastically.
- Closed economy.
- Output in two sectors:

$$Y^M(t) = X(t) F(L^M(t)) \qquad (72)$$

and

$$Y^A(t) = B^A G(L^A(t)), \qquad (73)$$

- F and G are continuously differentiable and strictly concave. In particular, $F(0) = 0$, $F'(\cdot) > 0$, $F''(\cdot) < 0$, $G(0) = 0$, $G'(\cdot) > 0$, and $G''(\cdot) < 0$.

Agricultural Productivity and Industrialization IV

- Diminishing returns to labor:
  - Might arise because they both use land or some other factor of production.
  - Implies that when labor is priced competitively, there will be equilibrium profits.
- Key: there is no technological progress in agriculture but (72) includes term $X(t)$, which will allow for technological progress in manufacturing.
- Productivity parameter B^A potentially differs across countries, reflecting either previous technological progress in terms of new agricultural methods or differences in land quality
- Evidence shows very large (perhaps too large) differences in labor productivity and TFP of agricultural activities among countries even today.
- Image of agricultural sector as a quasi-stagnant sector is not accurate: experiences both substantial capital-labor substitution and technological change.

Agricultural Productivity and Industrialization V

- Labor market clearing:

$$L^{M}(t) + L^{A}(t) \leq 1,$$

- $n(t)$=fraction of labor employed in manufacturing as of time t.
- Full employment in this economy implies $L^{M}(t) = n(t)$ and $L^{A}(t) = 1 - n(t)$.
- Manufacturing productivity, $X(t)$: evolves as a result of learning-by-doing externalities as in Romer's (1986) model
- Growth of $X(t)$ proportional to amount of current production in manufacturing:

$$\dot{X}(t) = \delta Y^{M}(t), \tag{74}$$

  where $\delta > 0$ measures extent of learning-by-doing effects and initial level of $X(0) > 0$ given.
- Learning-by-doing effects are external to individual firms.

Agricultural Productivity and Industrialization VI

- Can endogenize technology choices by introducing monopolistic competition and generate a market size affect and lead to an equation similar to (74).
- Each firm demands to equate value of marginal product to wage rate, $w(t)$.
- Price of agricultural goods is numeraire (i.e., normalize it to 1).
- Assume equilibrium is interior with both sectors being active.
- Then, equilibrium labor demand equations in two sectors:

$$w(t) = B^A G'(1 - n(t)) \text{ and } w(t) = p(t) X(t) F'(n(t))$$

where $p(t)$ is relative price of manufactured good.

- Market clearing:

$$B^A G'(1 - n(t)) = p(t) X(t) F'(n(t)). \tag{75}$$

Agricultural Productivity and Industrialization VII

- $\gamma^A > 0$ implies preferences are non-homothetic and income elasticity of demand for agricultural goods will be less than unity (for manufacturing goods greater than unity).
- Assume aggregate productivity is high enough to meet minimum agricultural consumption requirements:

$$B^A G(1) > \gamma^A > 0. \tag{76}$$

- Budget constraint of representative household:

$$c^A(t) + p(t)\, c^M(t) \leq w(t) + \pi(t)$$

where $\pi(t)$=profits per representative household, resulting from diminishing returns.

Agricultural Productivity and Industrialization VIII

- Equilibrium: sequence of consumption levels in two sectors and allocations of labor between two sectors at all dates, such that consumers maximize utility and firms maximize profits given prices, and goods and factor prices are such that all markets clear.
- Maximization of (71) implies:

$$c^A(t) = \gamma^A + \eta p(t) c^M(t). \tag{77}$$

- Economy is closed, production must equal consumption:

$$c^A(t) = Y^A(t) = B^A G(1 - n(t))$$

and

$$c^M(t) = Y^M(t) = X(t) F(n(t))$$

Agricultural Productivity and Industrialization IX

- Combining with (75) and (77):

$$\phi(n(t)) = \frac{\gamma^A}{B^A}, \tag{78}$$

where

$$\phi(n) \equiv G(1-n) - \eta G'(1-n) F(n) / F'(n),$$

is a strictly decreasing function.

- Moreover, $\phi(0) = G(1)$ and $\phi(1) < 0$.
- ϕ function can be interpreted as "excess demand" function for manufacturing over agriculture.
- Equilibrium has to satisfy (78). From Assumption (76) and properties of ϕ function, we can conclude that equilibrium condition (78) has a unique interior solution in which

$$n(t) = n^* \in (0, 1).$$

Agricultural Productivity and Industrialization X

- Thus structural change only in composition of output—fraction of labor force in agriculture remains $1 - n^*$.
- Using (78), unique equilibrium allocation of labor between two sectors satisfies

$$n^* = \phi^{-1}\left(\frac{\gamma^A}{B^A}\right). \tag{79}$$

- Key result: greater fraction of labor force allocated to manufacturing sector when agricultural productivity is higher.
  - Since ϕ is strictly decreasing, so is ϕ^{-1} and thus n^* is strictly increasing in B^A.
  - Cobb-Douglas production function and homothetic preferences would imply constant allocation of employment between sectors independent of their productivity.
  - But here preferences are non-homothetic: a certain amount of food production is necessary first.
  - When B^A is high a small fraction of labor generates this minimal level of food, thus more can be employed in manufacturing.

Agricultural Productivity and Industrialization XI

- Relationship between agricultural productivity and industrialization:
  - (74) implies that output in manufacturing grows at constant rate, $\delta F(n^*)$, also positively related to B^A in view (79).
- Constant shares of employment and no technological progress in agricultural sector imply:
  - Agricultural output is constant, all growth by manufacturing production.
  - Manufacturing and agricultural goods are imperfect substitutes: relative prices change and expenditure on agricultural goods increases.

Proposition In the above-described model, the combination of learning-by-doing and Engel's law generates a unique equilibrium in which the share of employment of manufacturing is constant at $n^* \equiv \phi^{-1}(\gamma^A / B^A)$, and manufacturing output and consumption grow at the rate $\delta F(n^*)$, which is increasing in agricultural productivity B^A.

Agricultural Productivity and Industrialization XII

- Higher agricultural productivity enables to allocate larger fraction of labor force to knowledge-producing sector, manufacturing.
- Impact of trade opening on industrialization?
  - Implications of closed and open economies are very different.
  - Higher agricultural productivity with international trade can lead to delayed industrialization or even to deindustrialization:
    - ★ specialization according to comparative advantage may have negative long-run consequences with sector-specific externalities.
  - But recall evidence for large externalities of this sort are not very strong.

Conclusions I

- Demand-side reasons for why growth can be non-balanced: Engel's law in basic neoclassical growth so that households spend a smaller fraction of their budget on agricultural goods as they become richer.
- Supply-side reasons for non-balanced growth:
  - Sectoral differences in capital intensity can lead to non-balanced growth.
  - Capital-intensive sectors tend to grow more rapidly as a result of an equi-proportionate increase in capital-labor ratio.
  - Combined with capital deepening at economy level, naturally leads to a pattern of non-balanced growth.
- Reconcile non-balanced growth at sectoral level with pattern of relatively balanced growth at aggregate.

Conclusions II

- Origins of industrialization:
  - agricultural productivity might have an important effect on timing of industrialization.
  - effect might depend on whether or not economy is open to international trade.
- But far from a satisfactory framework for understanding process of reallocation of capital and labor across sectors, how this changes at different stages of development, and how this remains consistent with relatively balanced aggregate growth and Kaldor facts.