Advanced Economic Growth: Lecture 27: Structural Transformations and Market Failures in Development (Part 2)

Daron Acemoglu

MIT

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Introduction

- Development and structural change come with transformation of economy:
    - major social changes and greater coordination of economic activities.
- Now focus on multiple equilibria and credit market problems retarding economic development.
- Finally, some thoughts towards a unified framework.

Multiple Equilibria From Aggregate Demand Externalities and the Big Push I

- Version of Murphy, Shleifer and Vishny's (1989) model of "big push", which formalized ideas of Rosenstein-Rodan (1943), Hirschman and Nurske.
- Economic development viewed as a move from one (Pareto inefficient) equilibrium to another, more efficient equilibrium.
- Move requires coordination among different individuals and firms, thus a *big push.*
- Multiple equilibria, literally interpreted, are unlikely to be the root cause of persistently low levels of development.
- If there is indeed a Pareto improvement, it is unlikely coordination cannot be achieved for decades or even centuries.
- But forces leading to multiple equilibria highlight important economic mechanisms

Multiple Equilibria From Aggregate Demand Externalities and the Big Push II

- Also, dynamic versions of models of multiple equilibria can lead to multiple state states.
- Two-period economy, $t = 1$ and 2.
- Economy admits a representative household with preferences:

$$U = \frac{C(1)^{1-\theta} - 1}{1-\theta} + \beta \frac{C(2)^{1-\theta} - 1}{1-\theta}$$

- Representative household supplies labor inelastically and total labor supply is L.
- Resource constraint:

$$\begin{aligned} C(1) + I(1) &\leq Y(1) \\ C(2) &\leq Y(2), \end{aligned}$$

where $I(1)$ denotes investment in the first date.

Multiple Equilibria From Aggregate Demand Externalities and the Big Push III

- Households can borrow and lend, so budget constraint:

$$C(1) + \frac{C(2)}{R} \leq w(1) + \pi(1) + \frac{w(2) + \pi(2)}{R},$$

  where $\pi(t)$=profits, $w(t)$=wage rate, R=gross interest rate between 1 and 2.

- Individuals can borrow and lend, but in aggregate resource constraints hold so R determined to ensure this.
- Final good:

$$Y(t) = \left[\int_0^1 y(\nu, t)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

  where $y(\nu, t)$ is the output level of intermediate ν at date t, and $\varepsilon > 1$.

Multiple Equilibria From Aggregate Demand Externalities and the Big Push IV

- Production functions of intermediate goods:

$$y(\nu, 1) = l(\nu, 1)$$

and

$$y(\nu, 2) = \begin{cases} l(\nu, 2) & \text{with old technology} \\ \alpha l(\nu, 2) & \text{with new technology} \end{cases} \tag{1}$$

where $\alpha > 1$ and $l(\nu, t)$ denotes labor devoted to the production of intermediate good ν at time t.

- Labor market clearing:

$$\int_0^1 l(\nu, t)\, d\nu \leq L. \tag{2}$$

Multiple Equilibria From Aggregate Demand Externalities and the Big Push V

- At date 1, designated producer for each intermediate, but competitive fringe can also enter and produce each good as productively.
- At date 1, the designated producer can also invest in new technology, which costs F per firm.
- If this investment is undertaken, producer's productivity at date 2 will be higher by a factor α as indicated by (1).
- Fringe will not benefit from technological improvement, thus some degree of monopoly power.
- Profits from intermediate producers are naturally allocated to the representative household.
- Looking for a subgame perfect equilibrium.
- Focus on symmetric subgame perfect equilibria, SSPE.
- SSPE: allocation of labor, investment decisions, wages for both periods and an interest rate.

Multiple Equilibria From Aggregate Demand Externalities and the Big Push VI

- Since all goods are symmetric, first period labor market clearing:

$$l(\nu, 1) = L \text{ for all } \nu \in [0, 1]$$

- This implies that

$$Y(1) = L.$$

- At date 2, equilibrium will depend on how many firms have adopted the new technology.
- SSPE only consider two extremes: all firms adopt and no firm adopts.
- In either case, marginal productivity of all sectors are the same, so labor will be allocated equally:

$$l(\nu, 2) = L \text{ for all } \nu \in [0, 1].$$

Multiple Equilibria From Aggregate Demand Externalities and the Big Push VII

- Thus when technology is not adopted:

$$Y(2) = L$$

- When technology is adopted:

$$Y(2) = \alpha L.$$

- In the first date, designated producers have no monopoly power: charge marginal cost $w(1)$, and make zero profits.
- Since total output is equal to $Y(1) = L$, equilibrium wage rate is:

$$w(1) = 1.$$

Multiple Equilibria From Aggregate Demand Externalities and the Big Push VIII

- In the second date, if the technology is not adopted, the same situation repeats:

$$w(2) = 1$$

and thus no profits.

- In this case there is also no investment, so consumption at both dates is equal to L, thus

$$\hat{R} = \beta^{-1}. \tag{3}$$

- To see this recall that the standard Euler equation in this case is

$$C(1)^{-\theta} = R\beta C(2)^{-\theta}, \tag{4}$$

which can only be satisfied with $C(1) = C(2)$, if the gross interest rate is $\hat{R}$ as given in (3).

Multiple Equilibria From Aggregate Demand Externalities and the Big Push IX

- If designated producers have invested, they can produce α units of output with one unit of labor.
- Designated producers have some monopoly power, extent depends on the comparison of ε and α.
- Demand facing each producer. Solution to:

$$\max_{[y(\nu,2)]_{\nu\in[0,1]}} \left[\int_0^1 y(\nu,2)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 p(\nu,2)\, y(\nu,2)\, d\nu,$$

  where $p(\nu,2)$ is the price of intermediate ν at date 2.
- The first-order condition:

$$y(\nu,2)^{-1/\varepsilon}\, Y(2)^{1/\varepsilon} = p(\nu,2),$$

  or

$$y(\nu,2) = p(\nu,2)^{-\varepsilon}\, Y(2). \tag{5}$$

- Note demand for intermediate ν depends on the total amount of production, $Y(2)$.

Multiple Equilibria From Aggregate Demand Externalities and the Big Push X

- Imagine no fringe of competitive producers: each designated producer will act as an unconstrained monopolist and maximize

$$\pi(\nu, 2) = \left(p(\nu, 2) - \frac{w(2)}{\alpha}\right) y(\nu, 2).$$

- Substituting from (5):

$$\max_{p(\nu,2)} \pi(\nu, 2) = \left(p(\nu, 2) - \frac{w(2)}{\alpha}\right) p(\nu, 2)^{-\varepsilon} Y(2),$$

- First-order condition

$$p(\nu, 2)^{-\varepsilon} Y(2) - \varepsilon \left(p(\nu, 2) - \frac{w(2)}{\alpha}\right) p(\nu, 2)^{-\varepsilon-1} Y(2) = 0,$$

Multiple Equilibria From Aggregate Demand Externalities and the Big Push XI

- Thus,

$$p(\nu, 2) = \frac{\varepsilon}{\varepsilon - 1} \frac{w(2)}{\alpha}.$$

- Standard monopoly price formula of a markup related to demand elasticity over the marginal cost, $w(2)/\alpha$.
- But since competitive fringe can produce one unit using one unit of labor, the monopolist can only charge this price if $\varepsilon / ((\varepsilon - 1)\alpha) \leq 1$.
- Otherwise, if

$$\frac{\varepsilon}{\varepsilon - 1} \frac{1}{\alpha} > 1. \quad (6)$$

monopolist will be forced to charge a *limit price*:

$$p^* = w(2).$$

Multiple Equilibria From Aggregate Demand Externalities and the Big Push XII

- Thus each monopolist would make per unit profits equal to

$$w(2) - \frac{w(2)}{\alpha} = \frac{\alpha - 1}{\alpha} w(2).$$

- The profits of firms are then obtained from substituting from (5) as:

$$\pi(2) = \frac{\alpha - 1}{\alpha} w(2)^{1-\varepsilon} Y(2). \tag{7}$$

- Wage rate can be determined from income accounting.
- Total production will be equal to $Y(2) = \alpha L$, and this has to be distributed between profits and wages:

$$\frac{\alpha - 1}{\alpha} w(2)^{1-\varepsilon} \alpha L + w(2) L = \alpha L,$$

Multiple Equilibria From Aggregate Demand Externalities and the Big Push XIII

- Thus,

$$w(2) = 1,$$

- Increased marginal product does not translate into higher wages but to profits for firms.
- But since profits are redistributed to the agents, $C(2) = \alpha L$.
- With investment in the new technology at date 1, $C(1) = L - F$.
- Again interest rate has to adjust so that individuals are happy to consume these amounts:

$$(L - F)^{-\theta} = \tilde{R}\beta(\alpha L)^{-\theta}, \tag{8}$$

$$\tilde{R} = \beta^{-1}\left(\frac{\alpha L}{L - F}\right)^{\theta} > \hat{R}.$$

Multiple Equilibria From Aggregate Demand Externalities and the Big Push XIV

- Interest rate in this case is higher: individuals are being asked to forgo date 1 consumption for date 2 consumption.
- Also the greater is θ, the higher is $\tilde{R}$: there is less intertemporal substitution.
- Higher F, greater consumption sacrifice: higher interest rate.
- Key question: whether firms will find it profitable to undertake the investment at date 1.
- Possibility of multiplicity: answer will depend on whether other firms are undertaking the investment or not.
- Consider no other firm is undertaking investment (denote by N), and consider incentives of a single firm.
- Total output at date 2 is equal to L (firm considering investment is infinitesimal), market interest rate is $\hat{R}$.

Multiple Equilibria From Aggregate Demand Externalities and the Big Push XV

- From (7) and $w(2) = 1$, profits at date 2 are

$$\pi^N(2) = \frac{\alpha - 1}{\alpha} L.$$

- Thus net discounted profits at date 1 for the firm:

$$\begin{aligned} \Delta\pi^N &= -F + \frac{1}{\hat{R}} \frac{\alpha - 1}{\alpha} L \\ &= -F + \beta \frac{\alpha - 1}{\alpha} L. \end{aligned}$$

- Now case of all other firms investing (I). Profits at date 2:

$$\pi^I(2) = (\alpha - 1) L,$$

Multiple Equilibria From Aggregate Demand Externalities and the Big Push XVI

- Profit gain from investing:

$$\begin{aligned}\Delta\pi^I &= -F + \frac{1}{\tilde{R}}(\alpha - 1) L \\ &= -F + \beta\left(\frac{\alpha L}{L-F}\right)^{-\theta}(\alpha - 1) L.\end{aligned}$$

- Both no investment in the new technology and all firms investing in the new technology possible if:

$$\Delta\pi^N < 0 \text{ and } \Delta\pi^I > 0, \qquad (9)$$

- Possible as of the *aggregate demand externality* ensures that $\pi^I > \pi^N$:
  - other firms invest, produce more, more aggregate demand, and profits from having invested in new technology are higher.

Multiple Equilibria From Aggregate Demand Externalities and the Big Push XVII

- Counteracting this effect: interest rate is also higher when all firms invest.
- Existence of multiple equilibria requires interest rate effect not to be too strong.
- Extreme case where preferences are linear, i.e., $\theta = 0$:

$$\Delta\pi^I = -F + \beta(\alpha - 1)L > \Delta\pi^N = -F + \beta\frac{\alpha - 1}{\alpha}L,$$

  so (9) is certainly possible.
- General condition for the existence of multiple equilibria :

$$\beta\left(\frac{\alpha L}{L - F}\right)^{-\theta}(\alpha - 1)L > F > \beta\frac{\alpha - 1}{\alpha}L. \qquad (10)$$

Multiple Equilibria From Aggregate Demand Externalities and the Big Push XVIII

- Whenever both equilibria exist, the equilibrium with investment Pareto dominates the one without investment:
  - (10): households are better-off with the upward sloping consumption profile giving them higher consumption at date 2.
- *Aggregate demand externalities:* investing is profitable only when there is sufficient demand at date 2, which occurs when all firms invest in the new technology.
- Investment decision:
  - positive (pecuniary) externality, but each firm does not realize the full increase in the social product,
  - monopoly markup implies marginal increases in output create first-order gain for consumers.
  - monopolist does not internalize this first-order gain, so demand linkages become aggregate demand externalities.

Multiple Equilibria From Aggregate Demand Externalities and the Big Push XIX

- Interpretation of Murphy, Shleifer and Vishny:
  - equilibrium with no investment as a "development trap," where economy remains in "underdevelopment"
  - equilibrium with investment corresponding to "industrialization": societies that can *coordinate* will industrialize
- Shortcomings:
  1. Industrialization is a dynamic process, but the model is static.
  2. Multiple equilibria: difficult to imagine a society remaining unable to coordinate
  3. More likely aggregate demand externalities (or other forces leading to multiple equilibria) are more important as sources of persistence or as mechanisms generating multiple steady states.

Inequality, Credit Market Imperfections and Human Capital

- Distribution of income and the organization of financial markets affect human capital investments.
- Show the possibility of multiple steady states, and more substantive questions related to the role of inequality and credit markets in the process of development.
- Focus on human capital investments, but inequality and credit market problems influences also occupational choices and other aspects of the organization of production.

A Simple Case With No Borrowing I

- Continuum 1 of dynasties.
- Each individual lives for two periods, childhood and adulthood, and gets an offspring in his adulthood.
- Consumption only at the end of adulthood.
- Preferences:

$$(1-\delta)\log c_i(t) + \delta \log e_i(t+1)$$

  where c is consumption at the end of the individual's life, and e is the educational spending on the offspring
- Budget constraint:

$$c_i(t) + e_i(t+1) \leq w_i(t),$$

- Preferences here have the "warm glow" type altruism: parents do not care about utility of their offspring, but about what they bequeath (education).

A Simple Case With No Borrowing II

- Preferences are logarithmic: constant saving rate, in terms of educational investments.
- Labor market is competitive, wage income is a linear function of individual's human capital:

$$w_i(t) = Ah_i(t)$$

- Human capital of the offspring of individual i of generation t:

$$h_i(t+1) = \begin{cases} e_i(t)^{\gamma} & \text{if } e_i(t) \geq 1 \\ \bar{h} & \text{if } e_i(t) < 1 \end{cases}, \tag{11}$$

  where $\gamma \in (0,1)$ and $\bar{h} \in (0,1)$.
- Key feature to generate multiple equilibria or multiple steady states: a *nonconvexity* in the technology of human capital accumulation.
- Each individual choose the spending on education that maximizes its own utility.

A Simple Case With No Borrowing III

- Implies "saving rate":

$$e_i(t) = \delta w_i(t) = \delta A h_i(t). \tag{12}$$

- One unappealing feature (not crucial for results): parents derive utility from educational spending so spend even when $e_i(t) < 1$.
- Assume that

$$\delta A > 1 > \delta A \bar{h}. \tag{13}$$

- Dynamics of human capital for dynasty i:
- If $h_i(0) < (\delta A)^{-1}$: dynasty that starts with $h_i(0) < (\delta A)^{-1}$ will never reach a human capital level greater than $\bar{h}$.
  - (12) implies that $e_i(t) < 1$, so the offspring will have $h_i(1) = \bar{h}$.
  - Given (13), $h_i(1) = \bar{h} < (\delta A)^{-1}$, and repeating this argument, we have $h_i(t) = \bar{h} < (\delta A)^{-1}$ for all t.

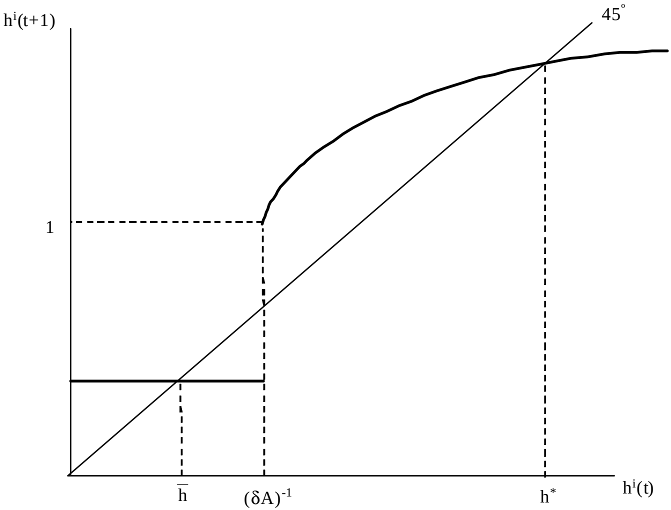

Figure: Dynamics of human capital with nonconvexities and no borrowing.

A Simple Case With No Borrowing IV

- $h_i(0) \in ((\delta A)^{-1}, h^*)$.
  - (13): $h_i(1) = (\delta A h_i(0))^\gamma > 1$, so gradually accumulate more and more and ultimately reach "steady state" $h^* = (\delta A h^*)^\gamma$ or

$$h^* = (\delta A)^{\frac{\gamma}{1-\gamma}} > 1.$$

- If $h_i(0) > h^*$: decumulate human capital
- Two steady-state levels of human capital for individuals, $\bar{h}$ and $h^* > \bar{h}$: dynasties with $h_i(0) < (\delta A)^{-1}$ will tend to $\bar{h}$, while those with $h_i(0) > (\delta A)^{-1}$ will tend to h^*.
- Simple dynamics:
  - Human capital of a single individual contains all information for dynamics of entire economy.
  - Reason is no prices determined in equilbrium.
  - "Markovian": summarized by a Markov process without any general equilibrium interactions.

A Simple Case With No Borrowing V

- Key implication: poverty traps due to the nonconvexities created by the credit market problems.
- Contrast two identical economies, but starting out with different distributions of income.
- Consider economy with two groups starting at income levels h_1 and $h_2 > h_1$ such that $(\delta A)^{-1} < h_2$.
- If inequality (poverty) is high so that $h_1 < (\delta A)^{-1}$, a significant fraction of the population will never accumulate much human capital.
- If inequality is limited, $h_1 > (\delta A)^{-1}$, all agents will accumulate human capital, eventually reaching h^*.

A Simple Case With No Borrowing VI

- Parallel between the multiplicity of steady states here and the multiple equilibria, but also differences
  - Multiple equilibria in a static model: nothing determines which equilibrium the economy will be in.
  - Can at best appeal to "*expectations*," or informally to the role of "*history*," but this ismisleading.
    1. Static model, so discussion of an economy "that has been in the low equilibrium for a while" is not meaningful.
    2. Even if the model were turned to a dynamic one by repeating it, history of being in one equilibrium will have no effect on multiple equilibria at the next instant.
  - Thus models with multiple equilibria have indeterminacy that are both theoretically awkward and empirically difficult to map to reality.
  - Multiple steady states avoids these thorny issues: equilibrium is *unique*, initial conditions determine where the dynamical system will end up
  - No issue of indeterminacy or expectations, and multiple steady states can be useful for thinking of development traps.

A Simple Case With No Borrowing VII

- Distribution of income affects which individuals will invest and influences the long-run income level.
- Sometimes interpreted as implying that an unequal distribution of income will lead to lower output (and growth).
- But not a general result and no specific predictions about relationship between inequality and growth.
- E.g., now starting with $h_1 < h_2 < (\delta A)^{-1}$, neither group will accumulate but redistributing from 1 to 2 so that $h_2 > (\delta A)^{-1}$ would increase human capital accumulation.
- General feature: in models with nonconvexities, *no unambiguous general* results about whether greater inequality is good or bad for accumulation and growth.
- Depends on whether greater inequality pushes more people below or above the critical thresholds.

Human Capital Investments with Imperfect Credit Markets I

- Simplified version Galor and Zeira, 1993.
- Each individual still lives for two periods.
- In youth, he can either work or acquire education.
- Utility function of each individual is

$$(1-\delta)\log c_i(t) + \delta \log b_i(t),$$

- Budget constraint is

$$c_i(t) + b_i(t) \leq y_i(t),$$

- Preferences still "warm glow" form, but now depends on monetary bequest rather than level of education expenditures.
- Logarithmic formulation once again ensure constant saving rate δ.

Human Capital Investments with Imperfect Credit Markets II

- Education: binary outcome, and educated (skilled) workers earn wage w_s while uneducated workers earn w_u.
- Expenditure to become skilled is h, and not earn the unskilled wage w_u during the first period.
- Binary education: introduces the nonconvexity.
- Imperfect capital markets: some amount of monitoring required for loans to be paid back.
- Cost of monitoring: wedge between the borrowing and the lending rates.
- Linear savings technology, which fixes lending rate at some constant r, but borrowing rate is $i > r$.
- Also assume:

$$w_s - (1 + r) h > w_u (2 + r) \tag{14}$$

Human Capital Investments with Imperfect Credit Markets III

- Implies investment in human capital is profitable when financed at the lending rate r.
- Consider an individual with wealth x.
  - If $x \geq h$, assumption (14) implies that individual will invest in education.
  - If $x < h$, then whether it is profitable to invest in education will depend on wealth of individual and borrowing interest rate, i.
- Utility of this agent (with $x < h$), when he invests in education:

$$
\begin{aligned}
U_s(x) &= \log\left(w_s + (1+i)(x-h)\right) + \log (1-\delta)^{1-\delta} \delta^{\delta} \\
b_s(x) &= \delta \left(w_s + (1+i)(x-h)\right),
\end{aligned}
$$

Human Capital Investments with Imperfect Credit Markets IV

- When he chooses not to invest:

$$\begin{aligned} U_u(x) &= \log\left(\left(1+r\right)\left(w_u+x\right)+w_u\right)+\log\left(1-\delta\right)^{1-\delta}\delta^{\delta} \\ b_u(x) &= \delta\left(\left(1+r\right)\left(w_u+x\right)+w_u\right). \end{aligned}$$

- Individual likes to invest in education if and only if:

$$x \geq f \equiv \frac{\left(2+r\right)w_u+\left(1+i\right)h-w_s}{i-r}$$

- Equilibrium correspondence describing equilibrium dynamics is

$$x\left(t+1\right)=\begin{cases} b_u=\delta\left(\left(1+r\right)\left(w_u+x\left(t\right)\right)+w_u\right) & \text{if } x\left(t\right)<f \\ b_s=\delta\left(w_s+\left(1+i\right)\left(x\left(t\right)-h\right)\right) & \text{if } h>x\left(t\right)\geq f \\ b_n=\delta\left(w_s+\left(1+r\right)\left(x\left(t\right)-h\right)\right) & \text{if } x\left(t\right)\geq h \end{cases} \tag{15}$$

Human Capital Investments with Imperfect Credit Markets V

- Equilibrium dynamics: (15) describes both the behavior of the wealth of each individual and the behavior of the wealth distribution in the economy ("Markovian").
- Define x^* as the intersection of the equilibrium curve (15) with the 45 degree line, when the equilibrium correspondence is steeper than the 45 degree line.
- Such an intersection will exist when the borrowing interest rate, i, is large enough.

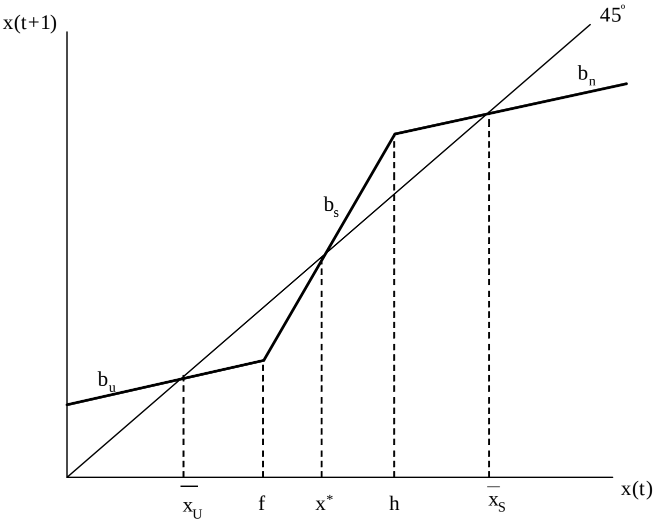

Figure: Multiple steady-state equilibria in the Galor and Zeira model.

Human Capital Investments with Imperfect Credit Markets VI

- All individuals with $x(t) < x^*$ converge to the wealth level $\bar{x}_U$, while all those with $x(t) > x^*$ converge to the greater wealth level $\bar{x}_S$.
- "Poverty trap," attracts agents with low initial wealth.
- Distribution of income again has a potentially first-order effect, but it is straightforward to construct examples where an increase inequality can lead to either worse or better outcomes.
- Implications of financial development: i smaller given r.
  - More agents will escape the poverty trap, and poverty trap may not exist

Human Capital Investments with Imperfect Credit Markets VII

- Shortcomings.
  - Partial equilibrium model:
    - ★ Multiple steady states here may not be robust to addition of noise in income dynamics—long-run equilibrium then corresponds to a stationary distribution of human capital levels.
    - ★ Models in which prices determined in general equilibrium affect wealth (income) dynamics generate more robust multiplicity of steady states.
  - Focus on human capital investments:
    - ★ Some, e.g. Banerjee and Newman (1994), believe effect of income inequality on occupational choices is potentially more important.

Heterogeneity, Stratification and the Dynamics of Inequality I

- More general (Benabou (1996a)): study dynamics of inequality and its costs for efficiency of production resulting from its effect on human capital.
- Aggregate output in the economy at time t:

$$Y(t) = H(t),$$

- $H(t)$ is an aggregate of the human capital of all the individuals in the society.
- Normalizing total population to 1 and denoting the distribution of human capital at time t by $\mu_t(h)$:

$$H(t) \equiv \left(\int_0^\infty h^{\frac{\sigma-1}{\sigma}} d\mu_t(h) \right)^{\frac{\sigma}{\sigma-1}}, \tag{16}$$

Heterogeneity, Stratification and the Dynamics of Inequality II

- σ=degree of complementarity or substitutability in the human capital of different individuals.
  - $\sigma \to \infty$: perfect substitutes and $H(t)$ is simply equal to the mean of the distribution.
  - $\sigma \in (0, \infty)$: complementarity between the human capital levels of different individuals.
- Effect of heterogeneity of human capital on aggregate productivity, for given mean level, is most severe when σ is close to 0.
- But formulation is general enough to allow for the case in which greater inequality is productivity-enhancing.
  - Defined for $\sigma < 0$ as well: in this case, greater inequality for a given mean level increases $H(t)$ and productivity.
  - Extreme case $\sigma \to -\infty$, $H(t) = \max_i \{h_i(t)\}$.
- Focus on potential costs of inequality on human capital: $\sigma \geq 0$.

Heterogeneity, Stratification and the Dynamics of Inequality III

- Then, mean preserving spread of the human capital distribution μ will lead to a lower level of $H(t)$
- Human capital of an individual from dynasty i at time $t+1$:

$$h_i(t+1) = \xi_i(t) B (h_i(t))^{\alpha} (N_i(t))^{\beta} (H(t))^{\gamma}, \qquad (17)$$

- B is a positive constant, $h_i(t)$ human capital of parent, $\xi_i(t)$ random shock, and $N_i(t)$ "average" human capital in the neighborhood.
- Assume neighborhood human capital is also a constant elasticity of substitution aggregator,with an elasticity ε:

$$N_i(t) \equiv \left(\int_0^{\infty} h^{\frac{\varepsilon-1}{\varepsilon}} d\mu_t^i(h) \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

- $\mu_t^i(h)$ denotes the distribution of human capital in the neighborhood of individual i at time t.

Heterogeneity, Stratification and the Dynamics of Inequality IV

- $\varepsilon \in (0, \infty)$: mean preserving spread of neighborhood human capital will reduce the human capital of all the offsprings.
- Plausible if presence of some low human capital children will slow down learning by those with higher potential (one "bad apple" will spoil the pack).
- Suggests segregation of high and low human capital parents might be beneficial for human capital accumulation.
- Multiplicative structure in (17): tractable evolution of human capital if initial distribution of human capital and the $\xi(t)$s are log normal.

Heterogeneity, Stratification and the Dynamics of Inequality V

- Assume:

$$\begin{aligned} \ln h_i(0) &\sim \mathcal{N}\left(m_0, \Delta_0^2\right) \\ \ln \xi_i(t) &\sim \mathcal{N}\left(-\frac{\omega^2}{2}, \omega^2\right), \end{aligned} \tag{18}$$

  where $\mathcal{N}$ denotes the normal distribution.
- The draws of $\xi_i(t)$ are independent across time and across individuals.
- Distribution of $\ln \xi$ is assumed to have mean $-\omega^2/2$ so that ξ has a mean equal to 1 (that is independent of its variance).

Heterogeneity, Stratification and the Dynamics of Inequality VI

- Thus distribution of human capital within every generation will remain log normal:

$$\ln h_i(t) \sim \mathcal{N}\left(m_t, \Delta_t^2\right), \tag{19}$$

  for some endogenous mean m_t and variance Δ_t, which will depend on parameters and the organization of society.
- Analysis of output and inequality dynamics boils down to characterizing the law of motion of m_t and Δ_t.
- Two alternative organizations: full segregation and full mixing.
- Full segregation: each parent is in a neighborhood with identical parents.
  - Because the neighborhood human capital is the same as the parent's human capital, (17) becomes

$$h_i(t+1) = \xi_i(t) B (h_i(t))^{\alpha+\beta} (H(t))^{\gamma}, \tag{20}$$

Heterogeneity, Stratification and the Dynamics of Inequality VII

- Full mixing: each neighborhood is a mirror image of the entire society.
  - Thus for all neighborhoods $N^i(t) = N(t) \equiv \left(\int_0^\infty h^{\frac{\varepsilon-1}{\varepsilon}} d\mu_t(h)\right)^{\frac{\varepsilon}{\varepsilon-1}}$, where μ_t refers to the aggregate distribution.
  - Accumulation equation:

$$h_i(t+1) = \xi_i(t) B(h_i(t))^\alpha N(t)^\beta H(t)^\gamma. \tag{21}$$

- Intuition above: segregation might be preferable.
- But not entirely accurate:
  - lack of segregation may reduce long-run inequality leading to better economic outcomes.

Heterogeneity, Stratification and the Dynamics of Inequality VIII

- With full segregation:

$$
\begin{aligned}
m_{t+1} &= \ln B - \frac{\omega^2}{2} + (\alpha + \beta + \gamma)\, m_t + \gamma \left(\frac{\sigma - 1}{\sigma} \right) \frac{\Delta_t^2}{2} \qquad (22) \\
\Delta_{t+1}^2 &= (\alpha + \beta)^2\, \Delta_t^2 + \omega^2
\end{aligned}
$$

- With full integration:

$$
\begin{aligned}
\hat{m}_{t+1} &= \ln B - \frac{\omega^2}{2} + (\alpha + \beta + \gamma)\, \hat{m}_t + \left[\begin{array}{c} \gamma \left(\frac{\sigma - 1}{\sigma} \right) \\ + \beta \left(\frac{\varepsilon - 1}{\varepsilon} \right) \end{array} \right] \frac{\hat{\Delta}_t^2}{2} \qquad (23) \\
\hat{\Delta}_{t+1}^2 &= \alpha^2 \hat{\Delta}_t^2 + \omega^2,
\end{aligned}
$$

- $\hat{m}_t$ and $\hat{\Delta}_t^2$ refer to the values of the mean in the variance of the distribution under full integration.

Heterogeneity, Stratification and the Dynamics of Inequality IX

- Note there will be persistence in the distribution of human capital (autoregressive nature of the behavior of m_t):
  - human capital of offsprings reflects that of parents (either through direct effect or through neighborhood and aggregate spillovers).
- Dispersion of the parents' human capital affects the mean of the distribution.
  - when $\sigma < 1$ or when $\varepsilon < 1$, so degree of complementarity in the aggregate or the neighborhood spillovers is high, greater dispersion reduces the mean of the distribution of human capital.

Heterogeneity, Stratification and the Dynamics of Inequality X

- Behavior of the variance of the distribution:
  - With full segregation, costs of heterogeneity resulting from neighborhood spillovers are avoided.
  - But variance of log human capital is more persistent than under full integration.
  - In paticular, when $\varepsilon < 1$, starting with the same m_t and Δ_t:

$$\hat{m}_{t+1} < m_{t+1} \text{ and } \hat{\Delta}^2_{t+1} < \Delta^2_{t+1},$$

  - Thus human capital in the next period is higher under segregation.
  - But inequality is also higher and from (16) inequality has efficiency costs.
- To determine which effect dominates, first find the long-run level of inequality under segregation and integration.

Heterogeneity, Stratification and the Dynamics of Inequality XI

- Equations (22) and (23) imply these variances are given by:

$$\Delta_\infty^2 = \frac{\omega^2}{1-(\alpha+\beta)^2} > \hat{\Delta}_\infty^2 = \frac{\omega^2}{1-\alpha^2},$$

- i.e., greater inequality of human capital and income with segregation of neighborhoods.
- Mean of the two distributions will also be different: suppose $\alpha+\beta+\gamma<1$, so steady state distribution exists under both full segregation and full integration.

Heterogeneity, Stratification and the Dynamics of Inequality XII

- Then:

$$m_\infty = \frac{1}{1-(\alpha+\beta+\gamma)}\left[\ln B - \frac{\omega^2}{2} + \gamma\left(\frac{\sigma-1}{\sigma}\right)\frac{\omega^2}{2\left(1-(\alpha+\beta)^2\right)}\right],$$

and

$$\hat{m}_\infty = \frac{1}{1-(\alpha+\beta+\gamma)}\left[\begin{array}{c}\ln B - \frac{s^2}{2} \\ +\left[\gamma\left(\frac{\sigma-1}{\sigma}\right)+\beta\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]\frac{s^2}{2(1-\alpha^2)}\end{array}\right].$$

- Mean level of human capital in the long run may be higher or lower under full integration or full segregation.

Heterogeneity, Stratification and the Dynamics of Inequality XIII

- Using the production function, taking logs on both sides of (16) and using log normality:

$$\ln Y(t) = \ln H(t) = m_t + \left(\frac{\sigma - 1}{\sigma}\right)\frac{\Delta_t^2}{2},$$

- Thus long-run income levels under full segregation and full integration are:

$$\begin{aligned} \ln Y(\infty) &= m_\infty + \left(\frac{\sigma - 1}{\sigma}\right)\frac{\Delta_\infty^2}{2} \\ \ln \hat{Y}(\infty) &= \hat{m}_\infty + \left(\frac{\sigma - 1}{\sigma}\right)\frac{\hat{\Delta}_\infty^2}{2}. \end{aligned}$$

Depending on parameters long-run income levels may be higher or lower under full segregation and full integration.

Heterogeneity, Stratification and the Dynamics of Inequality XIV

- Richer framework and highlights various different costs arising from income inequality.
- Tractability: attractive for study of political economy decisions, such as a voting over education budgets, and education reform (Benabou (1996a,b)).
- But costs of inequality are introduced in a reduced-form way.
  - E.g., why there could not be segregation in production: high human capital produce with other high human capital individuals preventing costs of inequality?
    - ★ Acemoglu (1997b): individuals with different levels of human capital are matched with firms via a imperfect matching technology.
    - ★ Technology-based justifications for (16) can also be provided.

Towards a Unified Theory of Development and Growth? I

- Have emphasized the transformation of the economy and the society over the process of development or potential reasons for why such a transformation might be halted.
- Transformation:
  - structure of production changing, process of industrialization getting underway, greater fraction of the population migrating from rural areas to cities, financial markets becoming more developed, mortality and fertility rates changing via health improvements and the demographic transition, and the extent of inefficiencies and market failures becoming less pronounced over time.
- In many instances this driving force is self-reinforced by the structural transformation that it causes.
- In all of the models, economic development is associated with capital deepening.

Towards a Unified Theory of Development and Growth? II

- Thus we can also approximate the growth process with an increase in the capital-labor ratio of the economy, $k(t)$.
- Not necessarily mean that capital accumulation is the engine of economic growth:
  - technological change is often at the root and capital deepening may be the result of technological change.
  - crucial variable capturing stage of development might be the distance to the world technology frontier.
  - certain aspects of the technological change as endogenous, especially when link between development and changes in the extent of market failures is highlighted.
- But an increase in capital-labor ratio will take place along the equilibrium path: use as proxy for the stage of development.

Towards a Unified Theory of Development and Growth? III

- Take the capital-labor ratio as the proxy for the stage of development and use the Solow model to represent the dynamics of the capital-labor ratio:
  - Caveat: careful not to confuse increasing the capital-labor ratio with ensuring economic development.
- Can we then construct a unified model: single force drives the process of development and the structural transformations spurred by this force contribute to the evolution of this driving force?
  - An attempt to pack many different aspects of development into a single model will lead to a framework that is complicated and involved.
  - Economic growth and development literatures have not made great progress towards such unified model.
- Instead, provide a very reduced-form canonical model of development and structural change:
  - bring out the common features of the models we have seen in a very stylized and reduced-form manner.

Towards a Unified Theory of Development and Growth? IV

- Continuous-time economy.
- Output per capita:

$$y(t) = f(k(t), x(t)), \tag{24}$$

- $x(t)$ is some "social variable," such as financial development, urbanization, structure of production, the structure of the family etc.
- f =twice continuously differentiable and also increasing in concave in k.
- Convention: think of an increase in x as corresponding to structural change, so f is increasing in x, $f_x \geq 0$.
- Reduced-form model of social change:

$$\dot{x}(t) = g(k(t), x(t)), \tag{25}$$

- g is also assumed to be twice continuously differentiable, increasing in k, that is, $g_k > 0$.

Towards a Unified Theory of Development and Growth? V

- Mean reversion type reasoning suggests that g_x should be negative, $g_x < 0$.
- Capital accumulates according to the most basic Solow growth model:

$$\dot{k}(t) = sf(k(t), x(t)) - \delta k(t), \tag{26}$$

- No population growth no technological change for simplicity.
- Differential equations (25) and (26) provide a simple reduced-form representation of structural change driven by economic growth (capital accumulation).
- First consider the case in which $f_x(k, x) \equiv 0$ so that the social variable x has no effect on productivity.

Towards a Unified Theory of Development and Growth? VI

- Dynamics in this case are shown in Figure:
  - Thick vertical line corresponds to the locus for $\dot{k}(t)/k(t) = 0$, i.e., the zero of the differential equation (26).
  - This locus is a vertical line: only a single value of $k(t)$, k^*, is consistent with steady state.
  - Upward sloping line: (25), locus of the values of k and x such that $\dot{x}(t)/x(t) = 0$.
  - Upward sloping, since g is increasing in k and decreasing in x.
  - Laws of motion represented by the arrows follow from (25) and (26).
- Dynamical system is globally stable: starting with any $k(0) > 0$ and $x(0) > 0$, economy will travel towards unique steady state (k^*, x^*).
- Dynamics of a less-developed economy, that starts with a low $k(0)$ and a low $x(0)$:
  - gradual capital deepening and a corresponding increase in $x(t)$ towards x^*.

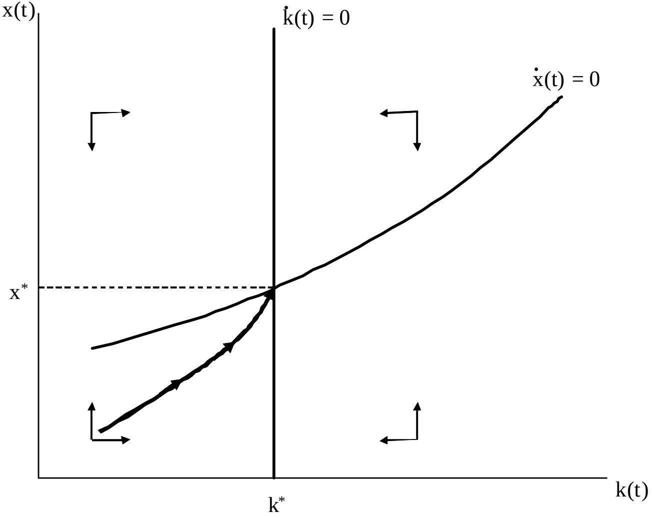

Figure: Capital accumulation and structural transformation without any effect of the "social variable" x on productivity.

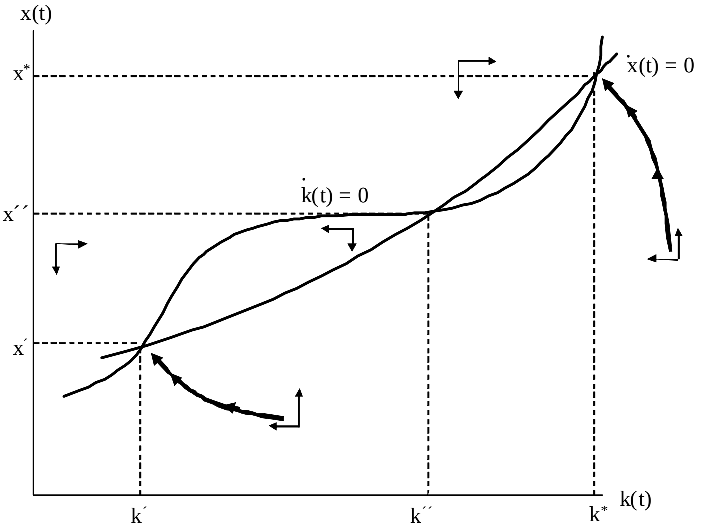

Figure: Capital accumulation and structural transformation with multiple steady states.

Towards a Unified Theory of Development and Growth? VII

- Case in which $f_x(k,x) > 0$:
  - Locus for $\dot{k}(t)/k(t) = 0$ also be upward sloping: $f_x > 0$ and the right-hand side of (26) is decreasing in k by standard arguments (by the strict concavity of $f(k,x)$ in k, $f(k,x)/k > f_k(k,x)$ for all k and x).
  - Steady state: intersection of the loci for $\dot{k}(t)/k(t) = 0$ and $\dot{x}(t)/x(t) = 0$.
  - Multiple steady states are possible as shown in Figure.
- Capture in reduced-form way potential multiple equilibria arising from aggregate demand externalities or from the interaction between non-convexities and imperfect credit markets:
  - low steady state (k', x'): social variable x is low and thus productivity is low, and this makes the economy settle into an equilibrium with a low k.
  - high steady state (k^*, x^*): the high level of x supports greater productivity and thus a greater k consistent with steady state.

Towards a Unified Theory of Development and Growth? VIII

- Both the low and the high steady states are typically locally stable.
- Starting from the neighborhood of one, economy will converge to the nearest steady state: importance of *historical factors*.
- Low steady state: "development trap", at least in part caused by lack of structural change (i.e., a low value of the social variable x).
    - Figure makes it clear that multiplicity requires the locus for $\dot{k}(t)/k(t) = 0$ to be relatively flat, at least over some range.
    - Equation (26): this will be the case when $f_x(k, x)$ is large, at least over some range.
    - Intuitively: multiple steady-state equilibria when the social variable x has a large effect on productivity.

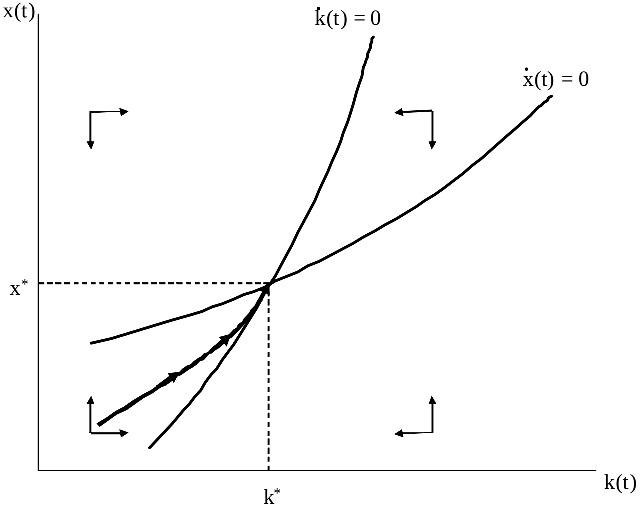

Figure: Capital accumulation and structural transformation when the "social variable" x affects but there exists a unique steady state.

Towards a Unified Theory of Development and Growth? IX

- More interesting: situation with same forces present, but a unique steady state:
  - If $f_x(k, x)$ is relatively small: locus $\dot{k}(t)/k(t) = 0$ will be everywhere steeper than locus $\dot{x}(t)/x(t) = 0$.
  - Unique steady state (k^*, x^*) and is globally stable (see Figure).
- Again less-developed economy starting with a low level of $k(0)$ and $x(0)$:
  - dynamics qualitatively similar to those in first Figure
  - economics is slightly different:
    - capital accumulation (deepening) leads to an increase in $x(t)$ as before, but now this structural change also improves productivity.
    - increase in productivity leads to faster capital accumulation and there is a *self-reinforcing* ("cumulative") process of development.
    - But since the effect of x on productivity is limited, this process ultimately takes us towards a unique steady state.

Conclusions

- Large number of models focusing on various aspects of the structural transformation accompanying economic development.
- No single framework unifying all these distinct aspects, even though there are many common themes
- Many of the topics are at the frontier of current research
- Also open way for a more constructive interaction between empirical development studies and the theories of economic development surveyed.
- Fruitful area for future research: combination of theoretical models of economic growth and development (that pay attention to market failures) with the rich empirical evidence on the incidence, characterization and costs of these market failures.