Multiple Selves and Endogenous Beliefs

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Anat Bracha

Dissertation Directors:
Professor Donald J. Brown
and
Professor Stephen Morris

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Abstract

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This dissertation consists of three essays. In the first essay I propose a new theoretical model of decision making under risk and uncertainty, titled "Affective Decision Making" (ADM). That is, I model the decision maker as having two inner processes: the "rational account" and the "mental account". The rational account chooses an action (insurance), given perceived risk (a perceived probability distribution). The mental account chooses perceived risk, given an action, to maximize expected utility net of mental costs. This captures the desire to hold the most favorable beliefs that one can justify. The agent's decision is a consequence of the interaction of the two accounts. This interaction is modeled as a simultaneous move intrapersonal game. The intrapersonal game is a potential game where the potential can be interpreted as the "composite" agent's utility function. The agent's choice is a pure strategy Nash equilibria of this game, reflecting consistency between the two accounts and the resolution of cognitive dissonance. The model gives rise to, generally, multiplicity of equilibria, which can be interpreted as framing (attentional) effects or uncertainty. For the insurance markets, the model allows for negative correlation between loss size and insurance level, and shows that the absolute risk aversion property can not be concluded from the data.

The second essay proves that given bounds on risk belief, the ADM model is testable.

That is, one can find a finite data set that refutes the model as well as one that is consistent with it. In addition, I show the existence of a mutual insurance equilibrium with affective agent and prove optimal sharing of *perceived* risk.

In the third essay, I consider a traditional screening model with a monopolistic firm facing two types of agents. The agents are Bayesian and observe a noisy private signal about their type. Consequently, the agents choose according to their perceived, rather than true, type. I show that the optimal endogenous information structure for the monopolist is always one of the corner solutions: either completely informed or completely uninformed agents.

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To my family,

Shlomo, Jaffa, and Michal Bracha

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Introduction

Information is at the core of economic analysis, as it determines the actions and strategies of economic agents. In many circumstances the relevant information is the individual's type; e.g., her tastes, abilities and risk level. A standard assumption in the literature is that the individual perfectly knows her type. This assumption has an intuitive appeal. However, in reality it is frequently violated. People find it difficult to determine their value of a product or to calculate their risk of being injured in a car accident; instead, agents use perceptions. In forming such perceptions, psychologists have recorded systematic biases and the influence of affective considerations. Affective considerations stand for the role of emotional motivation in determining one's belief.

The formation of individual perceptions and their economic consequences, in particular risk perception and insurance decision, are at the heart of my dissertation, as described below.

Chapter 1, Affective Decision Making in Insurance Markets, is a theoretical decision-making model that incorporates affective considerations into the expected utility paradigm. That is, the decision maker is modeled as an agent with two inner processes labeled the "rational account" and the "mental account". The rational account coincides with the process specified by the traditional economic model. Namely, it maximizes expected utility by choosing an action (insurance) for a given risk perception. The mental account is a process where risk perception is formed. Risk perception is determined by maximizing expected utility net of mental costs. In other words, the mental account chooses the risk perception which optimally justifies her insurance decision. Affective motivation - the desire to feel good about her insurance decision - is modeled using the expected utility term. This also captures the dependency of risk perception on outcomes, as supported in numerous psychological studies. The distinction between two thinking processes is supported by research in psychology and is known as the dual processes theory.

In order to make a decision, the two accounts interact; I model this interaction as a simultaneous-move intrapersonal game, where choice is a pure strategy Nash equilibrium of the game. The pure strategy Nash equilibrium reflects consistency of the two accounts once a choice is made. However, one can think of an adjustment process which converges to such a choice, i.e., a process of reaching an agreement. During this process the two accounts are constantly in disagreement, that is, in cognitive dissonance. Note that in this model both risk perception and insurance level are determined by choice. This gives rise to, generally, a multiplicity of equilibria and shows that almost surely the agent's decision differs from the traditional expected utility model. It also shows that one can not conclude the absolute risk aversion property of the utility function from the data and allows for a negative correlation between loss size and insurance level.

The multiplicity of equilibria in affective decision making (ADM), as described above, reflects several risk perceptions that the agent may hold. Since risk perception is not fixed in the ADM model, it is conceivable that this model can rationalize every finite data set. This conjecture is addressed and proven false in chapter 2, Consistency and Refutability of Affective Choice. In this chapter I formulate the ADM model in the state preference framework. Every insurance level and insurance premium can be translated into consumption and price vectors. In the case of two states of the world, multiplicity of equilibria is manifested as a finite number of discrete demand points on the budget line. The multiplicity of equilibria is due to a change of the utility function with risk perceptions. Using revealed preference arguments, I prove that the ADM model is testable in insurance markets. Testability means that there exists a finite data set consistent with the ADM model, and a finite data set that refutes the ADM model. I extend this model of analysis to equilibrium models of mutual insurance and a class of Knightian uncertainty models. Moreover, testability of the ADM model implies that there exists an axiomatization of affective choice in insurance markets. To see this, note that a finite family of Afriat's polynomial inequalities characterizes the solution of the ADM model. Applying the Tarski-Seidenberg theorem to this finite family of polynomial inequalities, I show that there exists an equivalent set of polynomial inequalities on the observables that characterize choice. Characterizing affective choice by a set of polynomial inequalities on the observables constitutes an implicit axiomatization of affective choice. For the case of two observations I derive conditions on the data that are necessary and sufficient for the data to be rationalized by a mutual insurance model with affective agents.

In addition, I extend the notion of Pareto optimality to ADM models of mutual insurance. Recall that the choice of an affective agent is a pair of perceived risk and an insurance level. Therefore, I define a strong preference relation: choice A is strongly preferred to B if the rational account prefers the insurance level at A considering both corresponding risk perceptions. Using the definition of strong preference, I prove that the mutual insurance equilibrium with affective agents is "affective Pareto optimal". That is, there does not exist another allocation that one agent strongly prefers to the equilibrium allocation, and the other is at least as well off as before. As a result, this is a version of Arrow's first welfare Theorem and it implies optimal sharing of perceived risk.

Chapter 3, A Perceived Screening Model with Bayesian Agents in Monopolistic Markets (PSM), considers the interaction among players where agents are Bayesian and have imperfect information about their own type. That is, each agent observes a noisy signal about her type and then computes her type assessment (posterior belief) using the conditional probability of the signal and the known distribution of types in the population. The monopolistic firm is assumed to control the conditional probability of the agent's private signal. This can be viewed as advertising by the firm in an attempt to influence agents' information. The essay examines the incentives a monopolistic firm has, in a market with Bayesian agents, to distort (or not) the agents' information about their type. I show that there are incentives for the firm to distort the agents' information, even if agents use Bayesian updating. Two cases are considered: a price discrimination model and a model of insurance markets. In the price discrimination model, the firm prefers perfectly informed or completely uninformed agents, but in the insurance model the company always prefers completely uninformed agents.

Chapter 1

Affective Decision Making in Insurance Markets

"...The mechanisms for behavior beyond drives and instincts use, I believe, both the upstairs and the downstairs: the neocortex becomes engaged *along with* the older brain core, and rationality results from their concerned activity.." Damasio (1994)

1.1 Introduction

Recent developments in the literature on economic decision making under risk and uncertainty is a result of a mutually enriching discourse between formal axiomatic models and empirical evidence. The axiomatization of expected utility with objective probabilities by von Neumann and Morgenstern (1944) served as the basis of analysis. However, ample empirical and experimental studies challenge its descriptive adequacy (e.g., Allais, 1953; Grether and Plott, 1979). This, in turn, has engendered other decision theories, such as prospect theory (Kahneman and Tversky, 1979), expected utility with rank dependent probability weight, and mixed fanning[1](for review see Camerer, 1995; Camerer, Loewenstein and Rabin, 2004). In order to accommodate the observable behavior, the new theories depart from the expected utility paradigm by introducing a probability weighting function[2] or a different shape of the utility function. The notion of a probability weighting function captures the well recorded phenomenon that probability weight, as judged by the agent, is different from the objective probability (Kahneman and Tversky, 1979, 2000; Camerer, Loewenstein and Rabin, 2004). In spite of the above differences, the new theories share the expected utility-type representation, i.e., a unitary process in which the agent maximizes some weighted average of future utility. In calculating the weighted average of future utility, probability weights are taken as given and independent of the size of outcomes.[3]

Although the separability of weights and outcomes has an intuitive appeal, it is often violated. Ample empirical evidence suggests that in the mind of the agent probability judgment and the size of outcomes are not independent, e.g., Edwards (1955, 1961) and Irwin (1953). In fact, the numerous records of optimism bias are evidence of such dependency. Optimism bias is the tendency of people to believe good outcomes are more likely, and bad outcomes are less likely, than they actually are (Weinstein, 1980; Armor and Taylor, 2002). This is a widespread phenomenon; optimism bias was shown to be correlated with normal mental health (Taylor and Brown, 1988), and to be present in various domains, one of which is risk perception (e.g., Slovic, 2000).

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[1]The mixed fanning hypothesis argues that indifference curves fan out for less favorable lotteries, while they fan in for more favorable ones.

[2]Weight assigned to a given state of the world according to a given function of the true probability.

[3]The probability weight in the rank dependent expected utility (RDEU) is determined by its ranking in the distribution of possible outcomes (see Lopes, 1995; Camerer, 1995). As long as the ranking is maintained, the weight function according to RDEU is unchanged.

Dependency of probability judgment and outcomes leads to another questionable feature of current decision making models: a fixed weighting function. Introspection leads one to conclude that we do not have a fixed weighting function, i.e., a fixed probability judgment. This is related to the previous point that probability weight and the size of outcomes are not independent; if weights assigned to future events are not independent of the size of outcomes then probability weights are endogenous to the problem and therefore not fixed.

Affective decision making (ADM) is a static model of decision making where the process of forming probability weight (hereafter termed perceived probability) is endogenous and explicitly modeled. The model extends expected utility theory in a new way, and is illustrated by focusing on the insurance markets.

For the insurance markets, the relevant probability distribution is personal risk. Risk is defined as the agent's probability distribution over future states of the world. As mentioned, the agent uses perceived, rather than objective risk, and she is optimistically biased. According to the psychology literature, one of the mechanisms underlying optimism bias is *motivated reasoning,* where the agent's beliefs are motivated by emotional goals (e.g., Kunda, 1990). Motivated reasoning suggests that the agent has preference over beliefs, which is a source of many psychological biases, including optimism bias. Preference over beliefs, and the biases resulting from motivated reasoning have been incorporated in some economic models. It has been shown that psychological biases have an interesting and important impact on information acquisition and optimal actions (e.g., Akerlof and Dickens, 1982; Rabin and Schrag, 1999; Koszĕgi, 2000; Bénabou and Tirole, 2001; Bodner and Prelec, 2001; Yariv, 2002; Eyster, 2002; Brunnermeier and Parker, 2002; Caplin and Leahy, 2004). Similarly, the model in this essay explores the impact of optimism bias on insurance decision, however takes a different modeling approach.

More specifically, ADM models the agent as having two inner accounts– the *rational account* and the *mental account.* The rational account coincides with the standard expected utility model. That is, for a given risk perception (perceived probability distribution), it maximizes expected utility with respect to insurance level. The mental account is where risk perception is formed. In accordance with the psychology research on motivated reasoning, the agent acts as if she *chooses*[4] her risk perception. In particular, the agent selects an optimal risk perception to balance two contradicting forces: (1) affective motivation and (2) a taste for accuracy. Affective motivation is the desire to hold favorable personal risk perception (optimism) for the given insurance level and is modeled using the expected utility term. That is, the mental account, like the rational account, wishes to maximize expected utility albeit doing so with respect to risk perception. Although the mental account desires favorable risk beliefs, it does come at a cost. In particular, the agent bears a mental cost for holding beliefs other than her *base rate.* Base rate is the belief that minimizes the mental cost function and can be thought of as the agent's probability reference point, or best assessment.[5] That is, a taste for accuracy[6].

Note that the mental process maps payoffs, which are determined by the rational account's insurance choice, into optimal risk perception. This, in turn, changes the optimal

[4]Choice stands for a result of personal motives or goals and does not mean a deliberate, fully conscious act.

[5]One can normalize the mental cost at the base rate to be zero.

[6]The term accuracy is taken from Kunda (1990). There, accuracy goal is the desire to come to the correct conclusion. In this case, the base rate can be thought of as the "correct" risk belief for the agent.

insurance level for the rational account. In order to reach a decision, the two accounts interact and aim for consistency. This interaction is modeled as a simultaneous move *intrapersonal* game. Consistency of the two accounts is characterized by the pure strategy Nash equilibria of the game and these are the candidates for choice.

The distinction between the two processes is based on research in psychology and neuroscience (Epstein, 1994; Wilson and Dunn, 2004; Camerer, Loewenstein and Prelec, 2004; Damasio, 1994). In psychology the distinction is between two systems, while in neuroscience decision making is the result of an interaction between various modules of the brain. These modules are often coarsely classified into two systems (upstairs, downstairs; new and old). In contrast to a common view that the two systems are competing, the model in this essay views them as complements. This is a result of the nature of the problem at hand and is consistent with recent developments in neuroscience (Damasio, 1994; LeDoux, 2000).

The principal results derived in the ADM model are: (1) affective considerations generally lead to multiple equilibria. This implies that people with the same information can have different beliefs about their risk and take different actions, (2) affective motivation can lead an outside observer to records both optimism and pessimism; the key is that it is always optimism in the mind of the agent, (3) the set of Nash equilibria consists of two types of equilibria, one that allows for negative correlation between objective risk and insurance and the other type captures an *enhancement effect.* That is, the presence of the mental account enhances the choice of the rational account. The possibility of negative correlation between insurance and objective risk is consistent with stylized facts in the life insurance market and suggests that educating the public about its higher-than-perceived-risk can lead to a counter intuitive result of less insurance. For the insurance markets the model suggests that (4) in the presence of affective considerations, the absolute risk aversion property of the utility function cannot be concluded from the data, and (5) the insurance premium affects risk perceptions. Also, I show that this model makes a natural distinction between choice and report tasks, and its framework, once applied to other contexts, is consistent with stylized facts outside the insurance markets. Finally note that multiplicity of equilibria means that there exists a set of possible perceived probabilities, which reflects uncertainty. However, the rational account is a model of risk. Therefore, ADM can be viewed as a bridge between risk and uncertainty.

The remainder of this essay is organized as follows: section 1.2 summarizes the psychology and economic literature and argues that the process underlying risk perception choice is as described above. Section 1.3 gives the setting of the rational and mental accounts. Section 1.4 proves the existence of a pure strategy Nash equilibrium for the intrapersonal game and provides comparative statics to examine the impact of changes in income, insurance premium and base rate on the individual's insurance decision. Section 1.5 discusses related economic studies and implications of the model, in and outside the context of insurance. For example, it discusses the relationship between absolute risk aversion, income and insurance choice, as well as the cautious optimism phenomenon. Section 1.6 concludes.

1.2 Related Literature

1.2.1 Risk Perception

In order to characterize the thinking process of forming risk perception, one has to turn to the psychology literature. Since this essay is concerned with the choice behavior of a single decision maker, it is important to understand the forces underlying perceived *personal* risk.[7] Casual observation shows that often people exhibit the "this is not going to happen to me" phenomenon. More scientifically put, most people, according to psychological experiments, believe that they are less likely than the average to be injured in a car accident or be involved in other bad experiences, but more likely than others to experience a positive event such as living longer, having a healthy life, and being successfully employed (Weinstein, 1980; Armor and Taylor, 2002). In other words, on average, people tend to be unrealistically optimistic, and this is correlated with mental health (Taylor and Brown, 1988).

Psychologists explain optimistic predictions as a way to achieve emotional and motivational goals such as good mood and self-confidence. The literature on motivated cognition (e.g., Kunda, 1990) suggests that confirmatory bias, optimism, cognitive dissonance, self-esteem and many other well-recorded psychological phenomena are due to individuals extracting utility from beliefs per se. That is, people have preferences over beliefs: people *want* to believe they are better than average, less likely to be ill, unemployed or unhappy. This suggests that people act as if they *choose* their beliefs. It is interesting to note that

[7] The psychology literature suggests a distinction between perceived *personal* risk and perceived *societal* risk. These two are fundamentally different. One implication is, as Tyler and Cook's (1984) research indicates, that factors influencing perceived societal risk do not necessarily influence perceived personal risk. More specifically they show that mass media information about risk will influence risk judgment on societal level but not on a personal level.

optimism bias is not exclusive to cases where an outcome is correlated with personal abilities; experiments show that subjects' prediction of what will occur is highly correlated with what they would like to see happen, rather than with what is objectively likely to happen (Irwin, 1953). This is true even in events that are totally random (Taylor and Brown, 1988 and references within).

In the context of risk perception, motivated reasoning means that individuals are choosing their personal risk perception and the desire for holding favorable beliefs guides them in that process, i.e., in choosing risk perceptions individuals are optimistically biased. Indeed research on risk perception finds optimism bias to be one of the sources in forming risk perception (Slovic et al, 1982). However, if motivation were the only driving force in the process of forming beliefs in general, and risk perception in particular, then people would generally hold arbitrary beliefs, which is not the case. Kunda (1990) argues that motivated cognition is restricted by personal experience, prior belief, knowledge or, in general, reality. In other words, individuals hold the most favorable risk perceptions that they can justify, i.e., that are reasonable.

In experiments regarding probability judgment, people are shown to use heuristics such as anchoring[8] and adjustment (Tversky and Kahneman, 1974; Shiller, 2000), representativeness, and availability (Gilovich, Griffin and Kahneman, 2002) to form their beliefs. These heuristics are related to motivated reasoning where agents are argued to balance motivation and accuracy. For example, consider the anchoring an adjustment heuristic. This heuristic was demonstrated in experiments using an exogenous anchor, to emphasize the thinking

[8] Anchoring is the tendency of people to stick to an anchor rate in an ambiguous situation.

process the decision maker use. However, in reality anchors are not exogenous, but rather, are endogenous. Therefore, it is not obvious what anchors decision makers use in reality. Motivated reasoning gives a possible answer – agents are anchored to motivated beliefs and adjust to reality as they perceive it (base rate).[9] In fact, Kunda (1990) acknowledges that anchoring and adjustment might not be distinguishable from motivated reasoning. Representativeness and availability can be seen as the cognitive strategy that the agent employs in order to justify her motivated beliefs. That is, the mind conveniently makes representative evidence, that is in support of the motivated beliefs, available.

1.2.2 Dual Processes

After reviewing the psychology literature with its implications for personal perception and, in particular, perceived personal risk, one needs to think about how the latter leads to choosing insurance. In other words, one should link together probability judgment and choice behavior. This brings us to the distinction often made in psychology, and supported by neuroscience, between two systems of reasoning.

The modern, most influential, mapping of the human mind into multiple processes was made by Freud. Freud suggests a distinction between two processes: the unconscious (primary process), and the conscious (secondary process). His hypothesis is that the primary process is symbolic and associative while the secondary process is a rational-thinking process. Throughout the years, many scholars in various fields of psychology have made similar distinctions between two processes, albeit distinguishing the two by different traits:

[9] Although experiments indicate that subjects are anchored to motivated beliefs and then adjust to the provided base rate, in reality the anchor rate is endogenous. Hence, one could argue that agents are anchored to the base rate and then adjust towards their motivational goals.

a verbal and nonverbal process, logical and prototypical systems, explicit and implicit, analytical and intuitive information processing and more (Wilson and Dunn, 2004; Epstein, 1994 and references within). Generally speaking, then, the psychology literature agrees on the distinction Freud proposes between two processes and agrees that one is a more deliberate, rational reasoning process and the other one is more intuitive and emotionally based. In contrast to Freud, psychologists now believe that the intuitive mode is not the source that undermines individual's attempt at rational thinking, but merely a different type of reasoning system. Epstein (1994), recognizing the similarities between the various dual process theories, developed a general theory of personality, the Cognitive-Experimental Self Theory (CEST), to encompass all such dual process models by making a distinction between the rational system – which is a deliberate, effortful, abstract system – and the experimental system – which is intuitive and emotionally driven. Recently, Kahneman (2003) introduced this distinction to economists, and argued that it should be incorporated in economic decision making. Moreover, advances in neuroscience research support the hypothesis of a modular brain (e.g., Damasio, 1994; LeDoux, 2000; Camerer, Loewenstein and Prelec, 2004 and reference within). The hypothesis of a modular brain has already been incorporated into economic decision making models. Thaler and Shefrin (1981) study the problem of self-control in the principal-many agents framework. More recent examples, drawing on new insights from neuroscience, are Bernheim and Rangel's (2004) model of addiction using two modes of decision processes, and Benhabib and Bisin's (2004) model of consumption and self-control.

Forming risk perception, as discussed above, involves emotional motives such as feeling good about oneself or one's future. Moreover, by introspection, forming risk perception is generally an unconscious process, i.e., agents do not have access to it. Therefore, it seems natural that the task of forming risk perception is generally an intuitive one performed by the experimental system in CEST and which I label the mental account, defined in section 1.3.2. Indeed, Wilson and Dunn (2004) argue that a source of self-knowledge failure is the inaccessibility of the mind to mental processes that involve perception, self esteem and alike. Thus, they agree that many self-perceptions are formed in implicit mental processes, which they argue, are generally unconscious. In contrast, choosing optimal action, such as insurance, is a deliberate task which demands logical effort, and it is therefore labeled the rational account. I follow the distinction psychologists often make between the rational and mental accounts and hypothesize that each is partially responsible for the insurance decision. The insurance decision is modeled as an outcome of the interaction between the two accounts, in accordance with the psychology literature. However, in contrast to the psychology literature where the two systems are often taken as rivals, this study views the two processes as complements, as does the recent research in cognitive neuroscience (Damasio 1994; LeDoux, 2000). The reason for this difference is that one can decompose the decision making into two main components; one is determined by the rational account and the other is determined by the mental account. In order to reach a decision, each process uses all available information including that supplied by the other process.

1.2.3 Preferences Over Beliefs in Economics

Some of the biases that are said to be explained by motivated cognition have been recognized in economic studies and shown to play an important role in information acquisition, strategic interactions and consumption choices. To mention a few, Akerlof and Dickens (1982) examine the consequences of cognitive dissonance for workers in a hazardous profession, and show that cognitive dissonance might prevent workers from purchasing available safety equipment. Koszĕgi (2000) shows that ego utility (capturing an agent's utility from positive beliefs about herself) can lead to suboptimal actions. That is, ego utility leads the agent to draw information strategically, generating distorted beliefs and hence suboptimal actions. Bénabou and Tirole (2001) study the value agents place on self-confidence, modeled as a probability distribution over possible individual abilities. They show that above a certain threshold of self confidence, agents will not acquire information. That will lead to non informative, self-handicapping activities. Similarly, Bodner and Prelec (2001) propose a model in which the agent's utility is composed of instrumental and dispositional elements. The agent's ability is not fully known; therefore she extracts information on her own ability by interpreting her actions (consumption choice). They show that the disposition element influence consumption: with a "face-value" interpretation the agent fails to realize the motivational goals in her actions thus reduces her consumption to supports overly positive view about the self; while with "true" interpretation the agent realizes the motivational goals in her actions and this can lead her to abstain from consumption altogether.

In a strategic analysis, Caplin and Leahy (2004) examine a principal agent model where the agent's utility is influenced by her probability beliefs; in particular, the agent is potentially anxious. That is, the agent's belief utility is influenced by both the probability belief and its distance from the uniform distribution (no information). They show that if it were only for anxiety, the principal (expert) who is concerned with the agent's (patient) private utility, will supply information to the patient only if her anxiety is reduced with early resolution of uncertainty, but not otherwise.

In intertemporal setting, Brunnermeier and Parker (2002) (BP) consider an agent who, in order to maximize expected total well-being, chooses at time zero probability beliefs for all subsequent periods. In all future periods (time one onward) the agent makes investment decision only, which is shown to depart from the standard intertemporal expected utility model. That is, if the agent, under the standard model, invests in an asset, then BP's agent will invest even more or alternatively will choose to sell it short.

Another source for anomalous behavior is consistency bias[10]. Yariv (2002) shows that a taste for consistency between beliefs in different stages can lead to either overconfidence or underconfidence, and presents conditions under which actions are persistent. Moreover, Yariv characterizes cases where a taste for consistency leads agents to prefer less information.

All the above studies use some notion of belief utility; the axiomatic foundation for this is provided by Caplin and Leahy (2001) and Yariv (2001).

In sum, psychology research suggests that people have preference over, and extract utility from, beliefs. This motivates agents to choose favorable beliefs, which are balanced against accuracy goals. This insight is the common feature of the papers cited above, i.e., agents choose beliefs in a strategic manner, resolving a trade-off between a standard instrumental payoff and some psychologically based payoff. The current essay formulates this trade-off by introducing an *internal game* between two accounts. The mental account chooses optimal beliefs (for a given action) to maximize *mental profit* from beliefs. The

[10] Eyster (2002) also considers a model with taste for consistency. He captures this by introducing a regret function that is a function of past and present actions.

rational account chooses optimal action (for a given belief) to maximize expected utility. Choice is an equilibrium outcome determined by the two accounts.

In addition, the ADM model embodies some existing concepts in the literature. To see that, note that the process of reaching a decision in our model can be viewed as a dynamic process, in which the two accounts are myopic and play in turns. In this process the two account constantly disagree, i.e., the agent is subject to cognitive dissonance. Choice, therefore, is a *resolution* of cognitive dissonance. That is, in what constitutes choice, actions and beliefs are consistent.

In particular, the choice of action reveals the beliefs for which it is optimal, which is similar in spirit to the face-value interpretation in Bonder and Prelec. In fact, by using the notion of intrapersonal equilibrium, the ADM model encompasses both sophistication levels; choice of action reveals the *optimal* probability belief for which it is optimal[11], however, note that in the process of reaching a choice (and true interpretation) the agent in our model fails to realize the motivational goal embodied in her *beliefs*.

1.3 The Model

Consider an agent who is facing two possible future states of the world, $s \in \{s_1, s_2\}$ with an associated wealth level of $w \in \{w_1, w_2\}$. I make the following assumptions:

Assumption 1 *Without loss of generality assume* $w_1 < w_2$

and denote the income shock as $z \equiv w_2 - w_1$.

[11] That is, in equilibrium the face-value and true interpretation coincide. To see that note that the mental account in the ADM model is the analogy to the dispositional element in Bodner and Prelec. Thus, the dispositional element is fully revealed by choice of action (i.e., equilibrium).

Assumption 2 *The agent has a strictly increasing, strictly concave, smooth utility function of wealth,* $U(w)$*, with* $\lim_{w\to-\infty} U'(w) = \infty$, $\lim_{w\to\infty} U'(w) = 0$.[12]

Unlike the standard insurance models, the agent is assumed *not* to know her risk level, rather she forms a risk perception and works with that. Risk perception is defined as the perceived probability $\beta \in [0,1]$ of being in state s_1. Note that since there are only two states of the world, β captures the perceived probability distribution over future states of the world.

As argued above, the agent has a preference over beliefs, which in this context is risk perception. Acknowledging that the agent has a preference over risk perception, leads one to believe that the agent *chooses* her perception of risk. The forces underlying this choice and the selection process itself will be discussed in section 1.3.2. In general, I distinguish between two processes: the rational account is presented in section 1.3.1 and the mental account presented in 1.3.2.

In order to avoid her (perceived) risk the agent can purchase insurance $I \in (-\infty, \infty)$ to smooth her wealth across states of the world. The insurance premium rate is $\gamma \in [0,1]$ and it is fixed for all levels of insurance purchase.

1.3.1 Rational Account

The rational account chooses insurance for a *given perceived risk* β to maximize expected utility. Thus, the rational account is the standard expected utility model using perceived rather than objective probabilities. More specifically, the rational account maximizes the

[12] All qualitative results are maintained for the case of $\lim_{w\to 0} U'(w) = \infty$, $\lim_{w\to\infty} U'(w) = 0$.

following objective function:

$$\max_{I} \{\beta U(w_1 + (1-\gamma)I) + (1-\beta)U(w_2 - \gamma I)\}$$

Where $U(\cdot)$ is the agent's utility of wealth and γ is the insurance premium. The optimal insurance level, I^*, satisfies the first order condition of this problem:

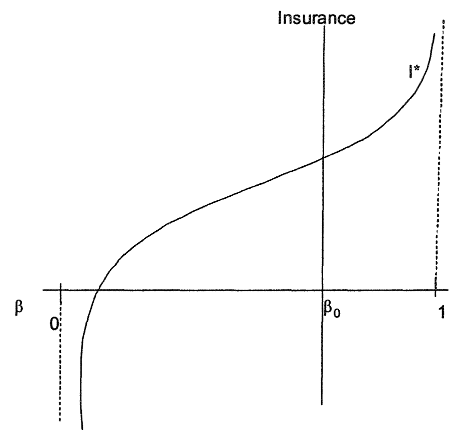
$$\beta U^{'}(w_1 + (1-\gamma)I^*)(1-\gamma) - (1-\beta)U^{'}(w_2 - \gamma I^*)\gamma = 0 \qquad (1.1)$$

Given a fixed wealth, income shock size and insurance premium, I^* is a function of risk perception β and will be denoted as $I^*(\beta)$. Since perceived risk is determined by the mental account, as discussed in section 1.3.2 below, *one can think of $I^*(\beta)$ as the best response of the rational account for any given strategy of the mental account.*

A straight-forward conclusion from the first order condition is that the best response $I^*(\beta)$ is a strictly increasing function of β and can be more than, less than, or exactly equal to full insurance. For stating the condition on the model's parameters which determine that, recall the definition of z as the income shock size ($z \equiv w_2 - w_1$). The lemma below summarizes the result.

Lemma 1.1. *$I^*(\beta)$ is a continuous, strictly increasing function of β, and as $\beta \to 0$, $I^*(\beta) \to -\infty$ while as $\beta \to 1$, $I^*(\beta) \to \infty$. Moreover, if $\beta \gtreqless \gamma$ then the optimal insurance level is $I^*(\beta) \gtreqless z$.*

A possible illustration of $I^*(\beta)$ is presented in Figure 1.1 below:



(Figure 1.1)

1.3.2 Mental Account

The psychology literature dealing with motivated cognition (e.g., Kunda, 1990) suggests that individuals extract utility from beliefs per se. Acknowledging the effect of beliefs per se on well-being leads one to think that people *choose* their beliefs. Thus, I allow the agent to choose her beliefs and derive the impact of that choice on perception and insurance choice.

Bénabou and Tirole (2001) argue that people get a positive utility from favorable beliefs for consumption value and signaling value, among other reasons. Consumptions value simply means that people like to hold favorable beliefs about the self . Signaling value means that if one perceives herself as better than she really is, other people will tend to believe it as well. I model this by using an expected utility term. Note that the mental account takes the insurance level, and therefore the utility level, as given. Therefore, changing β, which is the weight assigned to the utility in state s_1, will change perceived expected utility. This also captures the dependency of perceived risk on payoffs. That is, the mental gain of changing β depends upon the utilities in both states of the world.

Notice that if expected utility were the only component in deciding on optimal perception of risk, then the agent would hold arbitrary values of β, ($\beta \to 0$) for values of $I < z$, and ($\beta \to 1$) for values of $I > z$. However, as argued in section 1.2, people's beliefs are not arbitrary and are a result of balancing motivation with a taste for accuracy. As discussed, balancing motivation and accuracy can be viewed as the phenomenon of anchoring and adjustment. I model this by introducing a mental cost function $f(\beta)$ that reaches a minimum at some point $\beta = \beta_0$. Henceforth the cost function will be denoted as $f(\beta; \beta_0)$. Note that the cost at β_0 can be normalized to zero, and therefore β_0 is labeled the belief reference point, or base rate; this can be thought of as the agent's best assessment[13].

There are at least two possible motivation for introducing a cost function. First, the cognitive strategies that the mental account employs to justify her beliefs, such as availability, become more costly as one is further away from her base rate. Second, the cost function can be viewed as capturing fear of being overly optimistic; the further away perception is from the agent's base rate the higher is her fear. Emotions such as fear then guide her in adjusting perception closer to her base rate. This is in accordance with the psychology literature and the affective neuroscience literature, where emotions are believed to influence decisions in general (Loewenstein et al, 2001; Damasio, 1994) and in particular negative emotions influence perceived risk (Johnson and Tversky, 1983; Shafir and LeBoeuf, 2002),

[13] This assessment need not be concious.

with fear increasing it (Lerner and Ketler, 2000; Lerner et al, 2003). Negative emotions such as regret and fear affect decisions because people will attempt to avoid or minimize those emotions (Bell, 1982; Mellers et al, 1998; Shafir and LeBoeuf, 2002).

Given the discussion above, the agent balances the desire of holding beliefs $\beta \neq \beta_0$ with her taste for accuracy. Hence, the optimal risk perception β^* solves the following

$$\max_{\beta} \{\beta U(w_1 + (1-\gamma)I) + (1-\beta)U(w_2 - \gamma I) - f(\beta;\beta_0)\}$$

The first order condition is:

$$U(w_1 + (1-\gamma)I) - U(w_2 - \gamma I) - \frac{\partial f(\beta^*;\beta_0)}{\partial \beta} = 0 \qquad (1.2)$$

Given a fixed income, and loss size β^* is a function of insurance level I and is indexed by β_0. Notice that the marginal mental gain[14] of infinitesimally changing beliefs, $U(w_1 + (1-\gamma)I) - U(w_2 - \gamma I)$[15], is influenced by the insurance level while the marginal cost of this, $\frac{\partial f(\beta^*;\beta_0)}{\partial \beta}$, is determined by β and β_0. $\beta^*(I;\beta_0)$ is the probability judgment, or belief, that balances these two forces. In other words, $\beta^*(I;\beta_0)$ is the perception that maximizes the mental well-being, for a given insurance level I, i.e., $\beta^*(I;\beta_0)$ *is the best mental response for a given strategy of the rational account.* It is interesting to note that the mental best response presents the risk perception one would expect the agent to report, for a given insurance level.

[14] Gain can be either positive or negative.

[15] For a low level of insurance, the difference in utilities in the two future states of the world is relatively large. Thus, changing beliefs will have a relatively large mental gain. As insurance increases, the same change in beliefs will have lower impact on the mental gain since the differences in the state contingent utilities is smaller.

To analyze the behavior of $\beta^*(I;\beta_0)$ I assume the following:

Assumption 3 *$f(\beta;\beta_0)$ is a continuous, smooth function of β and β_0,* it is strictly *convex in β* and *reaches a minimum at $\beta = \beta_0$.*

In words, the further away β is from β_0, the greater the psychological cost. This is, by definition, a taste for accuracy. I also assume that the cost function $f(\cdot)$ is submodular:

Assumption 4 *The mental cost function $f(\beta;\beta_0)$ is submodular, i.e., $\frac{\partial^2 f(\beta;\beta_0)}{\partial\beta\partial\beta_0} \leq 0$.*

This assumption implies that the marginal cost of holding a risk perception β is nonincreasing in β_0. This assumption is used in later sections.

Finally, experiments show that people attribute a special quality to certain situations, which correspond to extreme beliefs $\beta \in \{0,1\}$. Thus, people would not generally choose a belief $\beta \in \{0,1\}$ under conditions of risk and uncertainty. Hence, I assume the mental cost function is finite in $(\underline{\beta},\bar{\beta})$ with $0 < \underline{\beta} < \bar{\beta} < 1$ and approaches infinite at the endpoints. These arguments are summarized in Assumption 5 below:

Assumption 5 Mental costs are positive and $lim_{\beta\to\underline{\beta}} f(\beta;\beta_0) = lim_{\beta\to\bar{\beta}} f(\beta;\beta_0) = +\infty$.

Assumption 5 implies the following limiting behavior of the marginal mental cost: $lim_{\beta\to\underline{\beta}} \frac{\partial f(\beta;\beta_0)}{\partial\beta} = -\infty$, $lim_{\beta\to\bar{\beta}} \frac{\partial f(\beta;\beta_0)}{\partial\beta} = +\infty$.[16]

Using Assumption 3 and the first order condition, the following lemma is immediate.

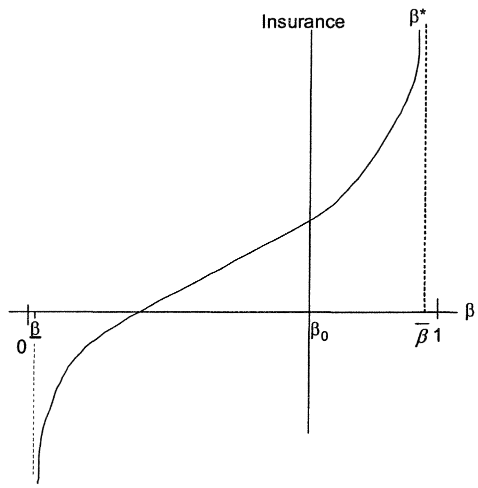
[16] Note that the function

$$f(\beta;\beta_0) = \ln\left(\frac{1}{(\beta-\underline{\beta})(\bar{\beta}-\beta)}\right) - \beta k$$

satisfies all of the above assumptions. β_0 is the minimum of this function.

Lemma 1.2. *$\beta^*(I;\beta_0)$ is a continuous and strictly increasing function of I. At full insurance, $\beta^*(I=z;\beta_0)=\beta_0$.*

Figure 1.2 is a possible illustration of $\beta^*(I;\beta_0)$:



(Figure 1.2)

The illustration of the best response locus above assumes it follows a particular shape: β^* is initially convex and then becomes concave in insurance I. This is not necessarily the case; in fact, since $\beta^*(I;\beta_0)$ balances marginal mental gain and cost, its shape with respect to I depends on the effect of a change in I on the marginal gain, $\frac{\partial[U(w_1+(1-\gamma)I)-U(w_2-\gamma I)]}{\partial I}$, and the rate at which this happens, $\frac{\partial\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right]}{\partial I}$, relative to the change in marginal cost as one changes belief β, $\frac{\partial f'(\beta^*;\beta_0)}{\partial\beta}$, and its speed of change $\frac{\partial^2 f'(\beta^*;\beta_0)}{\partial\beta^2}$. That is, the shape of $\beta^*(I;\beta_0)$ depends on the following condition:

Lemma 1.3. *$\beta^*(I;\beta_0)$ is concave in I iff*

$$\frac{\frac{\partial\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right]}{\partial I}}{\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right]^2} \leq \frac{\frac{\partial^2 f'(\beta^*;\beta_0)}{\partial\beta^2}}{\left[\frac{\partial f'(\beta^*;\beta_0)}{\partial\beta}\right]^2}$$

and it is convex otherwise

1.3.3 Both Accounts

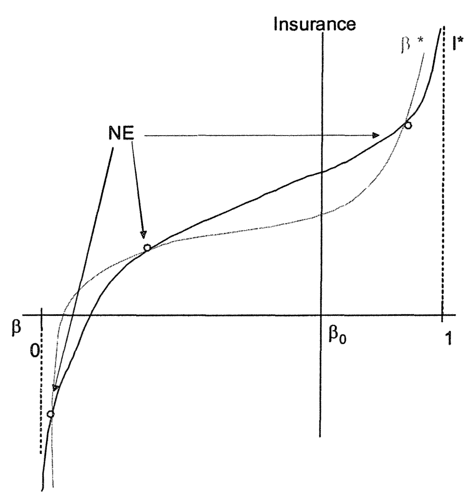
Although the two accounts are discussed separately, they are assumed to interact in the process of reaching a decision. A decision in this context is a pair consisting of risk perception and an insurance level. As previously argued, the rational account produces an insurance best response to a given risk perception β, while the mental account produces a belief best response to a given strategy of the rational account, i.e., insurance level I. The two accounts must equilibrate or the agent suffers the cost of cognitive dissonance. This situation is summarized in an intrapersonal static game between the two accounts, defined below.

Definition 1.1. *An intrapersonal game is a simultaneous move game of two players – the rational and the mental account. The strategy of the rational account is an insurance level, $I \in (-\infty,\infty)$, and the strategy of the mental account is a risk perception , $\beta \in (\underline{\beta},\overline{\beta})$. The rational account's payoff function is $g(\beta,I) \equiv \beta U(w_1+(1-\gamma)I)+(1-\beta)U(w_2-\gamma I)$, $g:(\underline{\beta},\overline{\beta})\times(-\infty,\infty)\rightarrow R$ and the mental account's payoff function is $\psi(\beta,I) \equiv g(\beta,I) - f(\beta;\beta_0)$, $\psi:(\underline{\beta},\overline{\beta})\times(-\infty,\infty)\rightarrow R$, where $f(\cdot)$ is the mental cost function of holding beliefs β and it reaches a minimum at β_0.*

Modeling the interaction of the two accounts as a simultaneous move game models a recent view in cognitive neuroscience research. Namely, both accounts mutually determine the performance of the task at hand. To quote Damasio(1994):

"...The mechanisms for behavior beyond drives and instincts use, I believe, both the upstairs and the downstairs: the neocortex becomes engaged *along with* the older brain core, and rationality results from their concerned activity.."

The pure strategy Nash equilibria of this game, if they exist, represent the mutual determination of behavior that resolve the conflict between the rational account and the mental account. Thus, the set of Nash equilibria are the natural candidates for the agent's choice. Given the best responses of the two accounts one can illustrate graphically the set of pure strategy Nash equilibria for this game. One possible illustration is presented in Figure 1.3 below:[17]



(Figure 1.3)

Notice that the implied risk perception at any Nash equilibrium is generally different from

[17]This is for a strictly concave utility functions such that $\beta^*(I)$ is convex in I for some range and then becomes concave (see Lemma 1.3 for exact conditions).

the self-reported risk perception one might record. To see this, note that if an agent is asked, in a report task, to indicate her perceived probabilities, she will answer by activating only her mental account. That is, the beliefs that makes her feel good about her situation. However, when engaged in a choice task, both accounts are activates and therefore her risk perception will be a part of a pure strategy Nash equilibrium of the intrapersonal game. This implies that the insurance choice need not be consistent with reported risk perception, a phenomenon recorded by Costa-Gomes and Weizsäcker (2003), albeit in a different context. This distinction is quite important, as will become clear in section 1.5.

1.4 Results

This section is divided into two parts. First, I prove the existence of pure strategy Nash equilibria for the intrapersonal game, present sufficient conditions for a unique equilibrium, and analyze the effect of affective considerations on insurance outcome. Second, I provide comparative statics to show the change in choice as one changes the parameters of the model.

1.4.1 Nash Equilibria

To study the pure strategy Nash equilibria of the intrapersonal game, it is useful to define two types of equilibria, as follows.

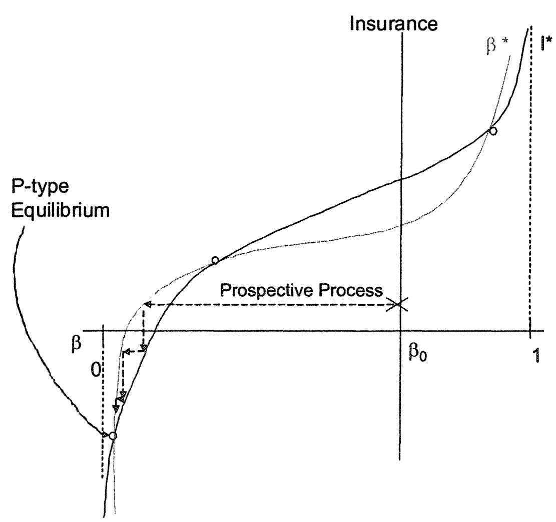
Definition 1.2. *Let* $I^* : (\underline{\beta}, \overline{\beta}) \to R$ *be the best response of the rational account.*[18] *Let* $\beta^* : R \to (\underline{\beta}, \overline{\beta})$ *be the best response of the mental account. Define a Prospective adjustment*

[18] $I^*(\beta)$ is defined over $(0, 1)$. However, since all Nash equilibria are within $\beta \in (\underline{\beta}, \overline{\beta})$, then we can restrict attention to this interval.

process (P) as sequential play where the mental and the rational accounts play in turns, $h = I^* \circ \beta^*$*, where* $h : R \to R$.

Definition 1.3. *A Nash equilibrium is of P-type if a prospective adjustment process P converges to it for all initial points in some neighborhood.*

That is, fix an insurance level close to I^{NE}, $\dot{h}(I) > 0$ for $I < I^{NE}$ and $\dot{h}(I) < 0$ for $I > I^{NE}$.[19] Graphically, a P-type equilibrium is an equilibrium point where the mental account's best response β^* crosses the rational account's best response I^* from below. For example:



(Figure 1.4)

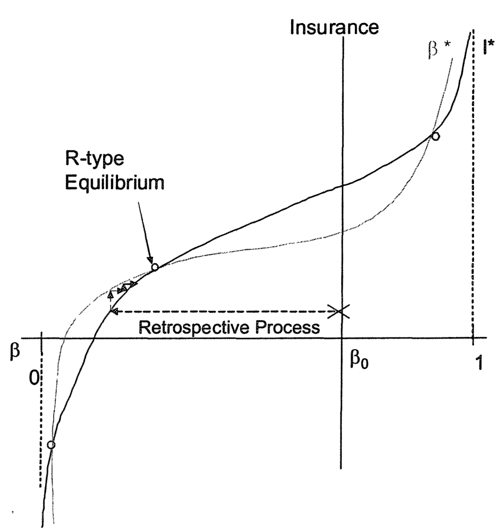
Definition 1.4. *Let* $I^{*-1} : R \to (\underline{\beta}, \overline{\beta})$ *be the belief that makes* I^* *the rational account's best response. Let* $\beta^{*-1} : (\underline{\beta}, \overline{\beta}) \to R$ *be the insurance level that makes* β^* *the mental account's*

[19] Note that $\dot{h}(I) = h(I) - I$. A Nash equiliria is where $\dot{h}(I) = 0$ and it is stable point of the function h iff $\frac{\partial \dot{h}}{\partial I} = \frac{\partial h(I)}{\partial I} - 1 < 0$ at the Nash equilibrium point. This is of course equivalent to $\dot{h}(I) > 0$ for $I < I^{NE}$ and $\dot{h}(I) < 0$ for $I > I^{NE}$.

best response. Define a Retrospective adjustment process (R) as sequential play where the rational and the mental accounts play in turns, $h^{-1} = I^{*-1} \circ \beta^{*-1}$, where $h^{-1} : (\underline{\beta}, \overline{\beta}) \to (\underline{\beta}, \overline{\beta})$.

Definition 1.5. *A Nash equilibrium is of R-type if a retrospective adjustment process R converges to it for all initial points in some neighborhood.*

That is, fix a belief close enough to β^{NE}, $\dot{h}^{-1}(\beta) > 0$ for $\beta < \beta^{NE}$ and $\dot{h}^{-1}(\beta) < 0$ for $\beta > \beta^{NE}$. Graphically, a R-type equilibrium is an equilibrium point where the mental account's best response β^* crosses the rational account's best response I^* from above. For Example:



(Figure 1.5)

Using the above definitions and excluding the case of tangency between the two accounts' best responses, the proposition below follows.

Proposition 1.1. *There exists a pure-strategy Nash equilibrium for the intrapersonal game.*

The set of Nash equilibria has an odd number of equilibria with a lowest and a highest arguments. If these equilibria are ordered with respect to the natural partial order on R^2, then they form a chain. Moreover, the Nash equilibria points alternate between being P-type and R-type. Under Assumption 5, the extreme equilibria are of P-type[20].

Proposition 1.1, using the argument in Milgrom and Roberts (1994) , assures us that an agent can achieve internal consistency between her rational and mental accounts. Moreover, there is an odd number of such equilibria and generally more than one. If there are multiple equilibria, then a chain guarantees a unique order of equilibria from (low risk perception, low insurance) to (high risk perception, high insurance)[21]. In addition, excluding the case of a unique equilibrium[22], there is at least one equilibrium of R-type. Since all equilibria are candidates for choice, the case of a unique equilibrium is of a special interest. For the sufficient condition for uniqueness, first I state the following proposition:

Proposition 1.2. *The intrapersonal game is a potential game, where the mental account's objective function is the potential function (potential) for the game. Moreover, since the potential is strictly concave in each variable (risk perception and insurance) its critical points are the pure strategy Nash equilibria of the game.*

Note that there is a natural interpretation for the potential: a "utility function" for the "composite" agent.

---

[20]Note that existence of pure strategy Nash equilibrium can be derived also for the case of a logarithmic utility function, where the agent's income in each state is non negative. The argument is in the appendix.

[21]For example, $u(w) = -\exp^{-0.01w}$, $f(\beta) = \frac{1}{8}\ln\frac{1}{(\bar{\beta}-\beta)(\beta-\underline{\beta})} - 0.2976\beta \Rightarrow \beta_0 = 0.66$, $\gamma = 0.66, w_1 = 0, w_2 = 50$. The solution set for this example is: $(\beta, I) = \{(0.19, -161.33), (0.66, 50), (0.78, 110.23)\}$.

[22]In which case, by the boundary conditions, it is of type P.

The properties of potential games make a clear sufficient condition for uniqueness: if the potential is strictly concave in both variables, and the strategy space is compact for all players[23], then it admits a unique critical point, i.e., a unique pure strategy Nash equilibrium. Proposition 1.3 gives the condition for the potential to be strictly concave.

Proposition 1.3. *The following is a sufficient condition for a unique pure strategy Nash equilibrium of the intrapersonal game:*

$$\frac{\partial^2 f(\beta;\beta_0)}{\partial\beta^2} > -\frac{\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right]^2}{[\beta U''(w_1+(1-\gamma)I)(1-\gamma)^2+(1-\beta)U''(w_2-\gamma I)\gamma^2]},$$
$$\forall (I,\beta) \in \left[I^*(\underline{\beta}'), I^*(\overline{\beta}')\right] \times \left[\underline{\beta}', \overline{\beta}'\right],$$

where $\underline{\beta}' \equiv \beta^*\left(I^*\left(\underline{\beta}\right)\right)$ *and similarly* $\overline{\beta}' \equiv \beta^*\left(I^*\left(\bar{\beta}\right)\right)$

From proposition 1.3 it is easy to see that if one considers a cost function $cf(\beta;\beta_0)$ instead of $f(\beta;\beta_0)$, then the above condition becomes

$$c\frac{\partial^2 f(\beta;\beta_0)}{\partial\beta^2} > -\frac{\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right]^2}{[\beta U''(w_1+(1-\gamma)I)(1-\gamma)^2+(1-\beta)U''(w_2-\gamma I)\gamma^2]}$$

Therefore, for large enough c there will be a unique equilibrium.

Note that the property of the potential being the objective function of one of the "players" gives rise to the following:

Proposition 1.4. *Consider a potential game where the potential is the objective function of player A. The set of pure strategy Nash equilibria in the sequential game, where player*

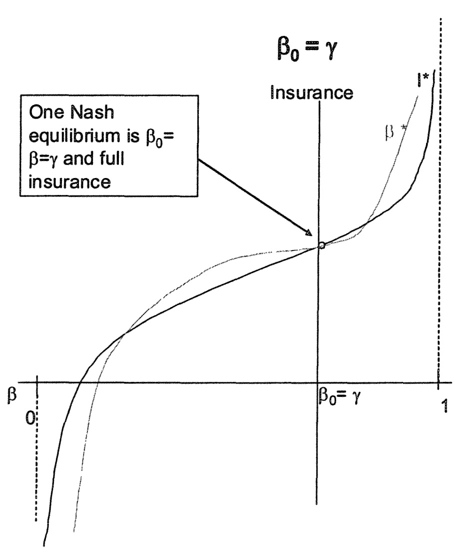
[23] In the proof of proposition 1.1 I show that one can restrict attention to a intrapersonal game where both players have a compact strategy space. This restricted game admits the same set of Nash equilibria as the original intrapersonal game.

A moves first, is a subset of the pure strategy Nash equilibria set of the simultaneous move game. Moreover, this subset is the set of global maximizers of the potential.

This proposition assure us that if the mental account moves first and the rational account moves second, then the set of the pure strategy Nash equilibria of the sequential intrapersonal game is a subset of the pure strategy Nash equilibria of the simultaneous intrapersonal game. Moreover it contains only P-type equilibria.

To better understand the effect of the mental account on choice, I split the discussion into three cases: $\gamma = \beta_0, \gamma > \beta_0$ and $\gamma < \beta_0$. The results are summarized in the following three propositions. Note that the graphical illustrations which accompany these propositions are assuming a specific shape of the β^* and I^* loci. However, the conclusions drawn are general and apply to all shapes of the mental and the rational best response functions.

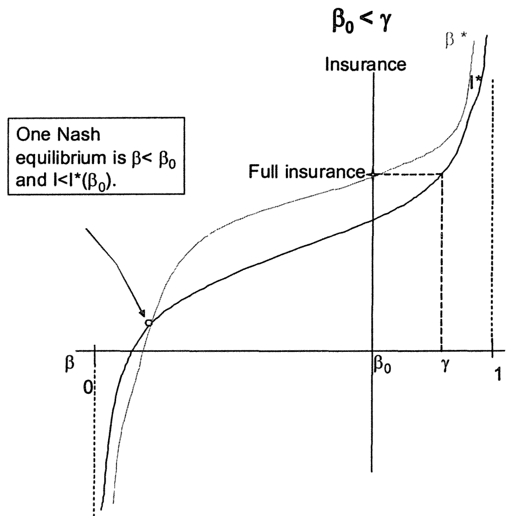
Proposition 1.5. *If $\gamma = \beta_0$, then there exists at least one Nash equilibrium with full insurance and $\beta = \beta_0 = \gamma$.*



(Figure 1.6)

Note that in this case the set of equilibria contains the expected utility model's outcome. To prove this note that at full insurance there is no mental gain for holding beliefs $\beta \neq \beta_0$ but there exists mental cost. Therefore at full insurance the mental account's best response is $\beta = \beta_0$. Given that $\gamma = \beta_0 = \beta$, the rational account's best response is full insurance. Consequently, full insurance and $\beta = \beta_0$ is a Nash equilibrium of this case.

Proposition 1.6. *If $\gamma > \beta_0$, then there exists at least one Nash equilibrium, with $\beta < \beta_0$ and $I < I^*(\beta_0)$.*

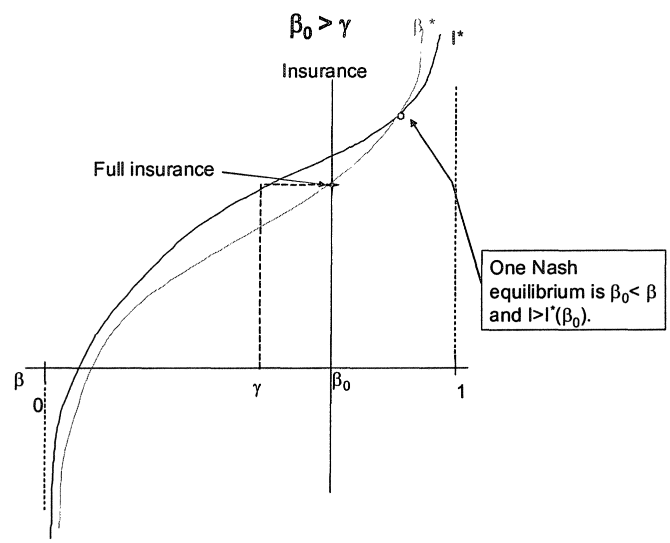


(Figure 1.7)

At this Nash equilibrium the agent buys less insurance relative to the expected utility model. To prove this, note that the insurance premium is higher than β_0. As a result, $I^*(\beta = \beta_0) < z$. Also, $\beta^* = \beta_0$ only at full insurance, where $I = z$. Therefore, at the point of $\beta = \beta_0$ the mental account's best response is above the rational account's best response. Since this relationship is reversed at the limit $\beta \to \underline{\beta}$ and both the mental and the rational best responses are increasing, the conclusion is that there exists a Nash equilibrium with

$\beta < \beta_0$ and less insurance than predicted by the expected utility model.

Proposition 1.7. *If $\gamma < \beta_0$, then there exists at least one Nash equilibrium, with $\beta_0 < \beta$ and $I > I^*(\beta_0)$.*



(Figure 1.8)

At this Nash equilibrium, the agent buys more insurance relative to the expected utility model. To see this, notice that because the insurance premium is lower than β_0, then $I^*(\beta = \beta_0) > z$, while only full insurance, $I = z$ will make $\beta^* = \beta_0$. Therefore, at the point of $\beta = \beta_0$ the mental account's best response is below the rational account's best response. Since this relationship is reversed at the limit $\beta \to \overline{\beta}$ and both the mental and the rational best responses are increasing , one can conclude that there exists a Nash equilibrium with $\beta > \beta_0$ and more insurance than predicted by the expected utility model.

Propositions 1.6 and 1.7 above illustrate the effect of affective considerations for P-type equilibria[24]: affective considerations cause the decision maker to enhance her rational

[24] This is the analysis for type P equilibria as by the above argument we capture the equilibria where

account decision. For example, suppose the base rate decreases such that $\gamma > \beta_0$; the rational account prescribes buying less than full insurance. The mental account, then, leads the decision maker to convince herself that she is at a lower risk, in order to feel better about her decision (motivational reasoning); this effect causes a further reduction in insurance purchase.

There are two aspects of the set of Nash equilibria that are worth noting. First, note that the set of choice candidates is composed of Nash equilibria of the intrapersonal game which reflect the influence of insurance decision on both payoffs (thus utility) and probability judgment. This idea is similar to the moral hazard logic; in the moral hazard setting, agents are assumed to have two actions – insurance and, say, driving style. The argument is that agents will have incentives to change their driving style after buying insurance, leading to a change in their true risk. Thus insurance indirectly affect risk. In the current set-up, buying insurance *directly* affects the agent's *risk perception*. Therefore, the similarity is that insurance affects risk but the differences are that in the present model (1) insurance influences perceived risk, not true risk and (2) choice is derived from a Nash equilibrium implying that, in equilibrium, the insurance purchase will be consistent with risk perception. The second interesting point is that the set of Nash equilibria, representing the choice candidates, contains both P-type and R-type equilibria. Although economists typically focus attention only on P-type equilibria, the traditional result of full insurance and $\beta = \beta_0 = \gamma$ can be of R-type. In fact, below are the conditions that determine whether or not the expected utility outcome is stable under the prospective process. These conditions

---

$\dot{h}(t) > 0$ for $I < I^{NE}$ and $\dot{h}(t) < 0$ for $I > I^{NE}$.

depend on the relationship between the mental cost function and the utility function.

Proposition 1.8. *Consider the case $\beta_0 = \gamma$. Define $\overline{w} \equiv \beta_0 w_1 + (1 - \beta_0) w_2$ and let $r(\overline{w})$ be the measure of absolute risk aversion. The full insurance standard outcome is of P-type if and only if*

$$\frac{U'(\overline{w})}{r(\overline{w})} < \frac{\partial^2 f(\beta_0; \beta_0)}{\partial \beta^2} \beta_0 (1 - \beta_0)$$

This condition implies that there are cases with $\beta_0 = \gamma$, admitting the outcome of the expected utility model, where the equilibrium is of R-type.[25]

1.4.2 Comparative Statics

In this section I provide a comparative statics analysis for both P-type and R-type equilibria. As noted previously, if the intrapersonal game admits a unique equilibrium then this is a P-type equilibrium; hence, the analysis below is general, covering both the unique equilibrium case and the multiple equilibria case.

Changes in β_0

This part presents the changes in Nash equilibria, composed of (risk perception, insurance) as the agent's base rate, β_0, changes. For this analysis, I use the following lemma:

Lemma 1.4. *$\beta^*(I; \beta_0)$ is weakly increasing in β_0. Assuming a strictly submodular mental cost function $f(\beta; \beta_0)$, i.e. $\frac{\partial^2 f(\beta;\beta_0)}{\partial\beta\partial\beta_0} < 0$, then $\beta^*(I; \beta_0)$ is strictly increasing in β_0.*

Lemma 1.4 above suggests that as the base rate β_0 increases, indicating a higher chance of being in state s_1, the optimal belief for any given insurance level is nondecreasing. To see

[25] An example is $w_1 = 25, w_2 = 50$, $f(\beta; \beta_0 \approx 0.66) = \ln \frac{1}{(0.9-\beta)(\beta-0.1)} - 0.3\beta$, and a utility function of the form $-\exp^{-0.01w}$.

that, note that as the base rate β_0 increases, the marginal gain from holding any belief β is unchanged while the marginal cost is nonincreasing. If the marginal cost does not change, then the optimal belief for a given insurance level stays the same. If the marginal cost does change, it declines and leads to higher perception of risk at every given insurance level.

Using Lemma 1.4, one can deduce the effect of a change in the base rate β_0 on both types of Nash equilibria. This is summarized in proposition 1.9 below.

Proposition 1.9. *Consider a P (R)-type equilibrium of the intrapersonal game. This equilibrium is weakly increasing (decreasing) in β_0. Moreover, if $\beta^*(I;\beta_0)$ is strictly increasing in β_0, then it is strictly increasing (decreasing) in β_0.*

Proposition 1.9 suggests that as the base rate increases, the Nash equilibrium one considers might or might not increase, depending if it is equilibrium of P-type or R-type. If it is a retrospective equilibrium then, as β_0 increases, it will consist of insurance level and risk perception which are less than or equal to their previous level. This result is counter intuitive, as one would expect an increase in the base rate to generate a higher Nash equilibrium with higher insurance and higher perception of risk just like the case for a P-type equilibrium. However, this intuition, in the retrospective case, captures only one aspect: the change in β^*. Ceteris Paribus, the optimal belief is higher for a positive incremental change in β_0. However, the intrapersonal game is composed of both rational and mental considerations. If the rational and the mental account reinforce each other approaching the local Nash equilibrium, as is the case of P-type equilibria, then one would maintain the intuitive result. By definition, in some neighborhood of the R-type equilibrium, the insurance decision feeds beliefs in the opposite direction of the local Nash equilibrium. Therefore, for the retrospective equilibrium, an increase in the base rate results in a decrease in insurance choice *because* β^* increases with β_0. This result could have policy implications implying that manipulating the base rate upwards can cause the agent to choose less insurance and hold a more favorable risk perception!

Changes in income and shock size

This part studies the influence of a change in traditional insurance parameters, such as income and shock size, on the decision of an agent who is subject to both rational and mental considerations.

Proposition 1.10. *(i) Suppose that the income w_2 increases while the shock size, z, stays constant. Then, the change in choice is given by the following tables, distinguishing between choice due to P-type or R-type equilibrium as well as distinguishing between cases of a utility function with decreasing absolute risk aversion (DARA), constant absolute risk aversion (CARA) and increasing absolute risk aversion (IARA).*

$P-type$

	$DARA$	$CARA$	$IARA$
$I < z$	?	$NE \uparrow$	$NE \uparrow$
$I = z$	*unchanged*	*unchanged*	*unchanged*
$I > z$	?	$NE \downarrow$	$NE \downarrow$

$R-type$

	$DARA$	$CARA$	$IARA$
$I < z$	?	$NE \downarrow$	$NE \downarrow$
$I = z$	*unchanged*	*unchanged*	*unchanged*
$I > z$	?	$NE \uparrow$	$NE \uparrow$

where ? means that all kind of changes are possible.

(ii) Suppose that the income shock size, z increases, while w_2 stays constant. Then, the impact of this change on the Nash equilibria (of any type) is not clear. In particular, it is possible that an increase in the income shock size z will lead to a reduced insurance purchase.

Since choice is according to a Nash equilibrium and is composed of (risk perception, insurance), increase in equilibrium (or choice) means that both insurance and risk perception increases. Bearing this in mind, proposition 1.10 states that if the utility function exhibits CARA, the initial choice consists of less than full insurance and follows a P-type equilibrium, then as income increases choice will increase, leading to more insurance purchase and higher risk perception. However, in contrast to the expected utility model, the prediction in the ADM model is not constant; for instance if initially choice consists of more than full insurance, is due to a P-type equilibrium and the utility function has the CARA property, then higher income leads to purchasing less insurance. Moreover, part (ii) of proposition 1.10 implies that, ceteris paribus, an agent facing higher possible loss will not necessarily purchase more insurance. This is in contrast to the expected utility model, which predicts more insurance purchase for such a change.

Changes in insurance premium

This section examines the influence of a change in insurance premium on insurance decision. Generally speaking, my results differ from the comparative statics of the expected utility model. A summary of the results is presented below.

Proposition 1.11. *Suppose the insurance premium increases. Then, the impact of this change on the Nash equilibria is as follows:*

P − type

	DARA	CARA	IARA
$I > z$	?	?	?
$I = z$	$NE \downarrow$	$NE \downarrow$	$NE \downarrow$
$0 < I < z$	$NE \downarrow$	$NE \downarrow$	?
$I < 0$	?	?	?

R − type

	DARA	CARA	IARA
$I > z$	?	?	?
$I = z$	$NE \uparrow$	$NE \uparrow$	$NE \uparrow$
$0 < I < z$	$NE \uparrow$	$NE \uparrow$	?
$I < 0$	?	?	?

where ? means that all kind of changes are possible.

Proposition 1.11's results suggest that there are cases where insurance companies can increase the insurance premium without a diminution in demand. This depends on the agent's initial insurance level, and the type of equilibrium manifested in her choice.

1.5 Discussion

This section discusses (1) the relationship of affective decision making with the existing literature, (2) the implications of the model on risk perception and optimism, (3) provides a possible explanation for various stylized facts in the insurance markets, and (4) experimental studies in psychology that this model can indirectly explain. Explaining phenomena outside the insurance context suggests that the framework of dual processes in decision making is more general and can, with some adjustment, be applied to decision making in other contexts.

1.5.1 Related literature

I would like to return to three particular papers that are closely connected to the current essay, and discuss in more detail the crucial differences with the ADM model. Of the related literature, the closest model to ADM is the optimal expectations paper by Brunnermeier and Parker (2002) (BP). Optimal expectations is an intertemporal decision making model in which the agent chooses probability beliefs to maximize expected total well-being. In that sense, both models have the agent choosing her perceived probability to maximize a similar objective function. Indeed, this objective function can be viewed as an analogue to the mental objective function appearing in the current essay. In calculating this objective function, BP assume that the mental cost of distorting probability beliefs equals the expected utility loss, measured with respect to the true probability distribution, due to suboptimal actions. In contrast, the ADM does not assume an explicit functional form. Hence, ADM allows for other cost functions that may be more realistic.

In addition, the BP agent plays a dynamic sequential game between what I label, for the sake of comparison, the "BP mental account" (BPMA) and the "BP rational account" (BPRA). The BPMA moves first by choosing perceived probabilities for all subsequent periods; the BPRA moves in each of the following periods by choosing optimal actions for the given perceived probability. It is assumed that BPMA has a perfect foresight, perfectly anticipating how the future will unfold. Therefore, the BP construction is, in effect, a choice of one variable – probability belief. Moreover, the BP model can be viewed as a choice of an optimal prior, after which the belief dynamics are Bayesian. In contrast, the ADM is a static model where the mental and rational accounts move simultaneously. Indeed, as was shown in the text, one can define a dynamic adjustment process which converges to a Nash equilibrium of the ADM model. However, in this dynamic process the two accounts are myopic; they play in turns, and belief dynamics are not Bayesian. Since the ADM model is simultaneous, ADM is, in effect, a decision of two variables: probability beliefs and action. The two cannot be reduced to a problem in one variable as probability belief and actions are mutually determined. Nonetheless, if one want to think of a sequential ADM model, where the mental account moves first, similar to the BP construction, then the set of pure strategy Nash equilibria in the sequential ADM model is a subset of the simultaneous move ADM model (see proposition 1.4). The BP model does not have this property; in fact, if we take a simultaneous move game with the mental cost as in BPMA, then the outcome is a unique equilibrium with risk belief equal the insurance premium and the agent always fully insure.

The affective decision making framework makes the distinction between two types of processes. A distinction between two processes has already been incorporated into economic models; examples are Bernheim and Rangel's (2004) (BR) model of addiction, and Benhabib and Bisin's (2004) model of self-control. These models and others, model the two systems, or decision *modes*, to be mutually exclusive. In contrast, the two processes in ADM are simultaneously active, and mutually determine the individual's choice. Hence, ADM is essentially a model of one decision making *mode*, that is composed of two inner processes.

This difference is a result of the different questions that the papers are addressing. In the model of addiction and the model of self-control, both processes decide on *action*, while in this essay one is choosing action and the other is forming perceptions; both are necessary for decision making.

1.5.2 General Discussion

The ADM model shows that allowing motivational (affective) reasoning to interact with rational considerations gives rise to possibly multiple equilibria with different probability judgments and actions. Multiple equilibria are consistent with the casual observations that different people hold different beliefs despite being exposed to the same events or objective information. Hence, this model suggests that affective motivations may be one of the reasons for this.

Note that the adjustment processes, either prospective or retrospective, can be viewed as a selection mechanism. If the agent indeed uses either one of these mechanism then choice will be path dependent – the observed choice depends on whether the agent first chooses belief or insurance. This is in accordance with lab experiments where choices are shown to depend in part on the attribute subjects were induced to focus on. In the insurance context, a prospective process implies that manipulating subjects to think about their risk first will, generally, lead to lower insurance purchase, relative to the case where subjects are induced to think about insurance first.[26] However, as the mental cost of distorting one's beliefs are getting larger, framing (attentional) effects get arbitrarily small.[27]

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[26]Unless, of course, there is a unique equilibrium, or that there exists one P-type equilibrium between the two post manipulation points.

[27]Think of a cost function $cf(\beta, \beta_0)$ instead of $f(\beta, \beta_0)$. As c increases above a certain threshold, we will

Finally, an insight that arises from this model is that it is possible for the experimenter to record both pessimism and optimism (relative to β_0). However, recording pessimism is possible only *because* the action taken (insurance) can change a state from being a 'bad' state to being a 'good' state. Thus, if the available action can not change the bad state into a good state, one would record only optimism and people will purchase insurance which is less than optimal. This might hold also in other markets similar to insurance, such as pharmaceutical drugs consumption. This discussion leads one to think about issues of control. A negative relationship between sense of control and perceived risk is a well documented phenomenon. This seems to contradict the ADM model[28]; unless one makes the distinction between two sources of control sensation – available actions versus other factors such as familiarity. Accepting such a distinction, the ADM model suggests that the ability to take effective actions can lead an experimenter to record pessimism, while the other factors (which may or may not be familiarity) leads to record optimism.

1.5.3 Risk and Uncertainty

The reader might wonder at this point whether the ADM model is a model of risk or uncertainty. Recall the distinction between risk and Knightian uncertainty; in both, the agent is faced with a future which outcome is not certain. Risk is the case where the agent knows the probability distribution over future events. Uncertainty, is the case where the agent is *uncertain* about this probability distribution; the uncertainty is captured by having a *set* of possible probability distributions over future events. Having these definitions in

have a unique equilibrium. That is, no framing (attentional) effects. Note that as $c \to \infty$, equilibrium gets arbitrary close to the outcome of the expected utility model.

[28] Since ADM implies that the availability of insurance can lead to records of pessimism.

mind, it is clear that the *rational account* is a model of risk; the rational account takes the mental account's probability judgment and acts as if it is the true probability distribution. However, the agent's choice is a model of uncertainty since she does not know a priori the perceived probability: multiple equilibria mean there is a set of perceived probabilities which the agent can believe. The linkage between risk and uncertainty is the mental account.

1.5.4 Insurance Market

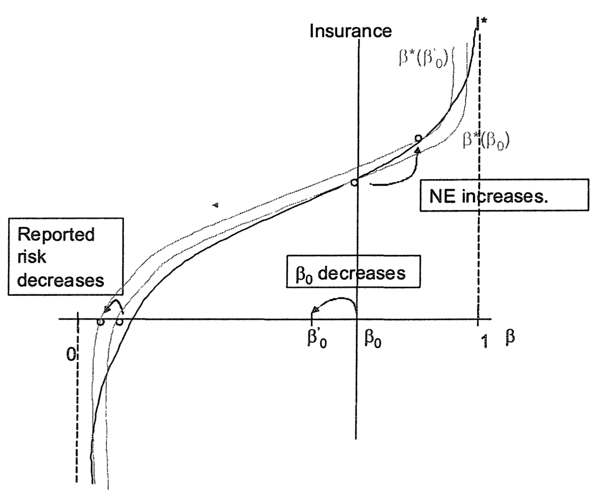
Income, insurance choice and the DARA hypothesis

Eisenhauer (1997) studies the life insurance market and argues that this market is not actuarially fair. Yet he finds that life insurance purchases increase with wealth. Controlling for household size and age[29], Eisenhauer concludes that the findings reject the DARA hypothesis. Indeed one can find other explanations for this finding, such that the data is still consistent with the DARA hypothesis. For example, wealthy individuals might have bequest motivations or they might have a different utility function altogether. The ADM model suggests another explanation; if the agent is subject to affective motivations then it is possible to have a utility function exhibiting DARA and to observe greater insurance purchase with an increase in income (see proposition 1.10). In fact, ADM implies that in the presence of motivated reasoning the absolute risk aversion property can not be extracted from observing changes in insurance.

[29] Indirectly through age-adjusted mortality probability.

Risk measure and insurance choice

Cawley and Philipson (1999) find that relatively risky individuals, using both self-reported and actual risk measures, are less likely to purchase life insurance, and once purchasing insurance the high-risk individuals, using self-reported risk measures, purchase less insurance. They find these results after controlling for wealth and proxis for bequest motives such as number of grandchildren, number of children, age of youngest child, average age of children, number of siblings, age of spouse and marital status. The ADM model is consistent with such results. To see this, note that the model captures objective risk with β_0; by proposition 1.9 I know that decreasing β_0, i.e., lower objective risk, the R-type equilibria increase, leading to higher insurance. As Lemma 1.4 shows, for a given insurance level, $\beta^*(I;\beta_0)$ moves always in the direction of β_0; Hence, for any given insurance level, lower reported risk will be associated with higher insurance. The figure below illustrates this point:



(Figure 1.9)

1.5.5 Other phenomena

Psychology Research: Cautious Optimism

Isen et al (1988) and Nygren et al (1996) examine the influence of positive affect on decision rule in risky situations. Both papers find a phenomenon labeled "cautious optimism". To illustrate this, consider the experiment in Nygren et al. In that experiment, participants are asked to make both numerical evaluations of verbal probabilities in three-outcome gambles and actual betting decisions for similar gambles. The participants who were induced with positive affect overestimated the probabilities of winning relative to losing, for the same phrases (optimism). However, when asked to gamble, these participants were much less likely to gamble relative to controls (cautious). The intrapersonal game is consistent with this phenomenon. Note that overestimated probabilities of winning in a report task is analogous to a shift in the mental best response, and therefore explaining cautious optimism is similar to explaining higher insurance purchase by low risk individuals (self reported measure). The key is that the intrapersonal game model makes the distinction between report and choice tasks. When engaged in a report task, participants will respond according to the mental best response. However, when engaged in a choice task, choice is according to a Nash equilibrium. The Nash equilibrium can be of R-type, leading individuals with lower perceived risk to choose more conservatively, analogous to higher insurance.

Note that Nygren, Isen et al (1996) conclude that ..."these findings suggest that positive affect can promote an overt shift from a decision rule focusing primarily on probabilities to one focusing on utilities or outcome values, especially for losses." Translating this: positive affect promotes a shift from an adjustment process where the mental account moves first to one where the rational account moves first. Indeed, such a shift, in the presence of multiple Nash equilibria, means choosing more insurance or more cautiously. However, in this model, such a shift cannot explain a higher reported win perception among the positive affect participants relative to controls.

1.6 Conclusions

Empirical evidence from the life insurance market as well as numerous experimental evidence leads one to reevaluate economic decision theory. One of the main findings in the psychology literature, as well as in recent economic studies, is that people have preferences over, and extract utility from, beliefs. Taking this point seriously and accepting the dual processes theory, I define a game – the intrapersonal game – in which the two accounts within the self, the rational and mental accounts, play simultaneously. The choice one observes is one of the Nash equilibria of this game.

Adding the mental account to the rational account can be viewed as adding a layer to, or enriching, the standard expected utility model. If the agent is not subject to affective considerations, then we are reduced to the classical model. However, if the agent's decision involves some mental gain and cost then the ADM model departs from the expected utility model. In other words, the ADM model suggests that the failure of the traditional expected utility model to explain the data is in part due to systematic mental biases.

Taking the intrapersonal game to the insurance context allows us to examine the insurance market and conduct comparative statics, given that people prefer to think the best outcome is more likely, or in other words, they are optimistically biased. Many interesting dynamics arise from this framework; in particular it suggests a possible explanation for Cawley and Philipson's (1999) finding in the life insurance market that high-risk individuals purchase less insurance. The ADM model can explain this since negative correlation between risk (actual and reported) and insurance decision is possible. In fact, this has interesting policy implications: educating the public to realize its higher-than-perceived risk can lead to an opposite reaction - lower insurance purchase!

Furthermore, this model is consistent (see proposition 1.10) with empirical results showing that risk aversion, as deduced from insurance decision, increases with wealth (Eisenhauer, 1997). However, since the ADM model suggests that the observed data is due to the interaction between the mental and rational account, then one cannot conclude the risk aversion characteristics of the utility function from the data. For example, it is possible to have a DARA utility function and still have insurance choice increasing with wealth. Like the effect of an increase in income, an increase in insurance premium can lead either to higher or lower insurance purchase. This will depend on the utility function, the equilibrium type of the initial choice, as well as the ratio between initial insurance level and the shock size. Thus, insurance companies might actually have an incentive to decrease their insurance premium.

Chapter 2

Consistency and Refutability of Affective Choice

2.1 Introduction

Assessment of probabilities is the driving force in decision making under conditions of risk and uncertainty. Modern axiomatization of risky choice assume that the probability distribution the agent holds is unique and fixed, albeit objective or subjective (von Neumann and Morgenstern,1944; Savage, 1954). But it has been shown in the psychology literature that agent's subjective probabilities are neither fixed nor unique and are subject to heuristics and biases (Tversky and Kahneman, 1974). In fact, psychologists refer to subjective probabilities as probability *judgments*. This is in accordance with Knight (1921) who suggests that an economic agent's choice is better described as made under conditions of uncertainty. That is, the agent does not have a single (subjective) probability distribution in mind, but rather a set of possible probability distributions.

Bewley (1986) suggested a formulation of decision making where agents have a set of probability distributions, his model is termed Knightian uncertainty; chapter 1 formulates affective decision making (ADM) in insurance markets, where an agent's choice is a pure strategy Nash equilibrium of an intrapersonal game between the agent's "rational" and "emotional" self. The rational self chooses the optimal level of insurance given a perceived probability distribution, and the emotional self chooses perceived probability distribution given insurance level. Choice of the affective agent is a pair of insurance level and a perceived probability distribution that constitute a Nash equilibrium in the intrapersonal game. In general there may be several Nash equilibria. Thus, in both Bewley's model and the ADM model the agent has a set of possible (perceived) probabilities; However, in the Knigthian uncertainty model the agent chooses to move away from the agent's status-quo if and only if the expected utility of the new bundle under all possible probability distribution exceeds the expected utility of the status-quo; but in the ADM model the perceived probability distribution and action (insurance level) mutually determine choice. Note that the freedom of distinguishing between objective and perceived probabilities in chapter 1 allows ADM to support all data sets that can be rationalized by expected utility theory as well as data sets which can not (see Figure 2.2 in the text).

It is conceivable that both Bewley's Knightian uncertainty and the ADM model are not refutable. That is, every finite set of observations on agents' choices in insurance markets can be "rationalized" by both models. Is this true? Simply put, are these models testable? This is the question addressed in this chapter.

This chapter shows that the ADM model is testable under both conditions of "complete" information, where risk perception is observable, and conditions of "incomplete" information where risk perception is unobservable. In order to prove testability of the ADM model for the complete information case I use Afriat's inequalities to characterize affective choice and provide an example which is not consistent with this system of inequalities. For the case of incomplete information, I formulate affective choice in the state preference model and use the notion of random utility functions as presented in Brown and Calsamiglia (2004) to show that the model is refutable. This proves that affective decision making is a meaningful theory in the sense that it is testable, even if perceived probabilities are unobservable. It also implies the testability of a class of the Knightian uncertainty models. To see this note that in Bewley's knightian uncertainty model the agent holds a *set* of possible probabilities. Consequently, if we define a class of Affective Knightian uncertainty models where the set of probability distributions is the convex hull of the perceived probabilities, supported by affective choice, then it follows from the previous results that this class of models is also testable. Despite the close relationship between the Affective Knightian model, it is possible to distinguish between the two. A discussion and an example are provided in section 2.3.3.

As mentioned above, in order to show testability for the case of complete information, I characterize affective choice by a system of Afriat's inequalities. Note, that this system of inequalities holds and characterizes affective choice also for the case of unobserved perceived probabilities. By invoking the Tarski-Seidenberg theorem (in Caviness and Johnson (eds.), 1998), together with the testability result, I prove existence of an implicit axiomatization of affective choice. That is, according to the Tarski-Seidenberg theorem one can reduce the Afriat's inequalities, which is a finite set of polynomial inequalities, to an equivalent set of inequalities on the parameters using quantifier elimination. Thus, testability of the model implies that equilibrium is characterized by a set of inequalities on the observables, i.e., there exists an implicit axiomatization of ADM. For the case of complete information, this axiomatization includes conditions on insurance choice, risk perception and prices; for the case of incomplete information, given bounds on risk belief, the axiomatization involves conditions on insurance choices and prices *only*. This is a surprising result since it states that even if one has information only on the bounds of perceived probabilities, she can still find a set of axioms on insurance choices and premium that are generating the ADM model.

To prove testability I established an isomorphism between the affective choice model and the state preference model, and explicitly derive axioms for the case of two observation. In addition, using the state preference model, I show existence and prove testability of an exchange economy of mutual insurance with affective agents. In equilibrium, agents can (and generally will) hold different perceived probabilities, which translate into different *perceived price ratios*. The prices one observes in equilibrium are the same for all agents, but not their perceptions.

A natural question arises: is this mutual insurance market Pareto optimal? For the expected utility model we have Arrow's (1971) result that the mutual insurance general equilibrium is indeed Pareto optimal, hence optimal risk sharing. However, with affective agent this is not clear, since perceived price ratios are not the same across agents and each agent has two inner selves. Consequently, in order to discuss Pareto optimality, I define a strong preference relation, where an insurance level x is said to be strongly preferred to y if it is preferred by the rational self under both the equilibrium risk perceptions at insurance level x and y. Given this definition, one can show that the equilibrium observed is "affective"

Pareto optimal; that is, there is no other feasible allocation which all agents strongly prefer. In other words, there is an efficient *perceived risk* allocation. Thus, I state a welfare theorem analogous to Arrow's for *perceived* rather than objective risk, and I label it the first affective welfare theorem; one should emphasize once more: with affective agents such allocations need not, and generally will not, be the point of optimal objective risk sharing. Following the first affective welfare theorem, I show that any affective Pareto optimal allocation can be supported by a general equilibrium with affective agents and zero sum transfers. This result is labeled the second affective welfare theorem.

The remainder of this essay is organized as follows: section 2.2 discuss the relationship between risk and uncertainty in the ADM model; section 2.3 proves testability of ADM under conditions of complete information, i.e., perceived probabilities are observable, and conditions of incomplete information, i.e., perceived probabilities are not observable. This section also formulates ADM in the state-preference model. Section 2.4 presents the explicit axioms for the case of two observations under incomplete information conditions; section 2.5 analyzes an equilibrium in an exchange economy with affective agents, and shows that this model is testable. All proofs of theorems appear in the Appendix.

2.2 Affective Choice

Affective choice consists of both insurance level and perceived probability, and is the outcome of the ADM model presented in chapter 1. That is, affective choice is a pure strategy Nash equilibrium of the intrapersonal game between the rational and the mental accounts. Proposition 1.1 proves that generally the intrapersonal game has a multiplicity of equilibria; this means that the agent's perceived probability changes across equilibria and depends upon the insurance level chosen. Consequently, one may wonder if the theory has too many degrees of freedom, or differently put, is the theory testable?

2.2.1 Risk and Uncertainty

Affective choice, as presented in chapter 1, encompasses both risk and uncertainty. That is, the rational account is a model of risk – the rational account is choosing *as if* the perceived probability is the true (albeit personal) risk. However, the multiplicity of equilibria, i.e., multiple possible perceived risks that the agent may hold in equilibrium, represents *uncertainty.* The agent does not know a priori which perceived probability she should focus on. Thus, constructing the convex hull of the possible perceived probabilities, generates a set of perceived probabilities. This set of perceived probabilities reflects uncertainty as modeled by Bewley (1986) and inspired by Knight (1921).

Consequently, the question can one test affective decision making has implications for testing Knightian uncertainty models.

2.3 An Implicit Axiomatization

2.3.1 Full Information

In this section I develop conditions that characterize a solution to the ADM model and thus allow one to test whether the observed data conform with the theory.

Consider N observations on insurance premia, insurance levels, and perceived probability distributions: (γ_i, I_i, β_i) for $i = 1, 2, ...N$. To conclude that the model can not rationalize *all* data sets, one must find a set of observations which rejects the model.

Recall that the ADM model assumes two selves; The rational account objective function is

$$\max_{I} \sum_{s\in\{1,2\}} \beta_s u(w_s(I,\gamma)) \tag{2.1}$$

The mental account objective function is

$$\max_{\beta} \sum_{s\in\{1,2\}} \beta_s u(w_s(I,\gamma)) - f(\beta,\beta_0) \tag{2.2}$$

Proposition 2.1. *Take any two observations* (γ_1, I_1, β_1), (γ_2, I_2, β_2). *The following conditions are equivalent:*

(1) There exist locally nonsatiated, continuous, monotonic, concave utility function $u(\cdot)$, *and locally nonsatiated, continuous, monotonic, convex cost function* $f(\cdot)$ *that rationalizes the data;*

(2) There exist numbers $\left(u_s((I_i,\gamma_i)),\ f(\beta_i) \geq 0, \frac{\partial u(w_s(I_i,\gamma_i))}{\partial w} \geq 0, \frac{\partial f(\beta_i)}{\partial \beta}\right)$ *for* $i = 1,2$ *that satisfy the Afriat's inequalities* $\forall i,j \in 1,2;\ \forall r,s \in \{1,2\}$:

$$u(w_s(I_i,\gamma_i)) \leq u\left(w_r(I_j,\gamma_j)\right) + \frac{\partial u\left(w_r(I_j,\gamma_j)\right)}{\partial w}\left[w_s(I_i,\gamma_i) - w_r\left(I_j,\gamma_j\right)\right] \tag{a}$$

$$\sum_{s\in\{1,2\}} \beta_s^i \frac{\partial u(w_s(I_i,\gamma_i))}{\partial w}\frac{\partial w_s(I_i,\gamma_i)}{\partial I} = 0 \tag{b}$$

$$\begin{aligned}\sum_{s\in\{1,2\}} \beta_s^i u(w_s(I_j,\gamma_j)) - f(\beta_i) \quad \leq \quad & \sum_{s\in\{1,2\}} \beta_s^j u(w_s(I_j,\gamma_j)) - f(\beta_j) \\ & + \left(\left[u\left(w_1(I_j,\gamma_j)\right) - u\left(w_2(I_j,\gamma_j)\right)\right] - \frac{\partial f(\beta_i)}{\partial \beta}\right)\left[\beta_j - \beta_i\right]\end{aligned} \tag{c}$$

$$\left[u(w_1(I_i,\gamma_i)) - u(w_2(I_i,\gamma_i))\right] - \frac{\partial f(\beta_i)}{\partial \beta} = 0 \tag{d}$$

Note that conditions (a) and (c) are the concavity condition on the utility function $u_s((I_i, \gamma_i))$ and the mental account's objective function, respectively. Condition (b) and (d) are the first order conditions of the rational and the mental account, respectively.

Definition 2.1. *a model is said to be testable if there exists a data set which is it rationalizes, as well as a data set which refutes the model.*

Proposition 2.2. *The model is testable*

By the Tarski-Seidenberg theorem one can reduce the Afriat's inequalities, which is a finite set of polynomial inequalities over parameters and unknowns, to an equivalent set of inequalities on the parameters. Thus, testability of our model implies that equilibrium is characterized by a set of inequalities on the observables, i.e., there exists an implicit axiomatization of ADM.

However, since risk perception is unobservable and difficult to elicit, this implicit axiomatization is not useful in practice. Fortunately, in the following sections I show that the model is testable even when perceived probabilities are not observable. This also implies that a certain class of the Knightian models, proposed by Bewley, are testable. In order to prove this (somewhat) counter intuitive result, I first formulate affective choice in the state preference framework. Note that the set of Afriat's inequalities must hold, even if perceived probabilities are not observable. Thus, proving that the model with unobservable perceived probabilities is testable means that there is an implicit axiomatization in this case as well.

2.3.2 Affective Choice and State-Preference Model

Note that the *rational* account choice can be formulated in the state-preference model, shown below:

$$\max_{c_1,c_2\in R^2} \quad \beta_1 u(c_1) + (1-\beta_1)\, u(c_2) \tag{2.3}$$

$$s.t. \qquad p_1c_1 + p_2c_2 = y \tag{2.4}$$

$$\begin{aligned} where\ c_s &\triangleq w_s + k_s I, \quad k_s \triangleq \begin{cases} (1-\gamma) & \text{for } s=1 \\ -\gamma & \text{for } s=2 \end{cases} \\ y &\triangleq p_1\overline{c_1} + p_2\overline{c_2} \end{aligned}$$

where $\overline{c_1}, \overline{c_2}$ are the agent's endowment at state s_1, s_2, respectively. Note that $y \triangleq p_1\overline{c_1} + p_2\overline{c_2}$ implies that $\frac{p_1}{p_2} = \frac{\gamma}{(1-\gamma)}$; thus I will set, without loss of generality, $p_1 = \gamma$, $p_2 = (1-\gamma)$. Since income level and insurance premium are given, choosing consumption levels c_1, c_2 is equivalent to selecting an insurance level I.

The first order conditions of the state preference model are:

$$\beta_1 \frac{\partial u(c_1)}{\partial c} = \lambda\gamma \tag{2.5}$$

$$(1-\beta_1)\frac{\partial u(c_2)}{\partial c} = \lambda(1-\gamma) \tag{2.6}$$

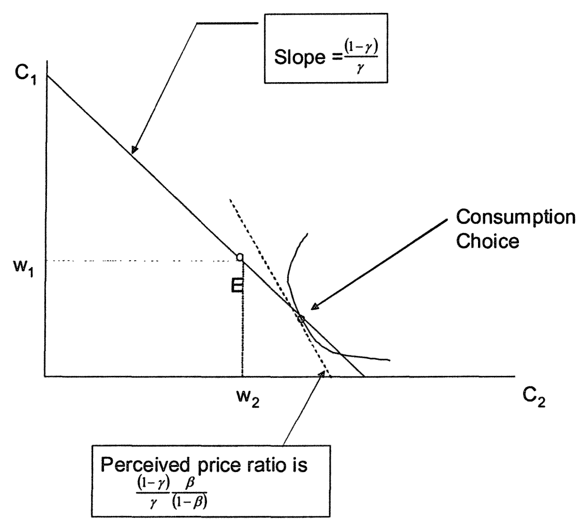
This can be written as

$$\frac{u^{'}(c_2)}{u^{'}(c_1)} = \frac{(1-\gamma)}{\gamma}\frac{\beta_1}{(1-\beta_1)} \tag{2.7}$$

Thus, every solution of the intrapersonal game can be translated into a point on the budget line in the (c_1, c_2) plane; this point is observable and the slope of the budget line is the ratio $\frac{(1-\gamma)}{\gamma}$. However, the agent's marginal rate of substitution equals, what I call the perceived price ratio, $\frac{(1-\gamma)}{\gamma}\frac{\beta_1}{(1-\beta_1)}$, which is determined by the perceived probabilities. These perceived probabilities must satisfy the first order condition of the mental account:

$$u(c_1) - u(c_2) = f'(\beta_1) \tag{2.8}$$

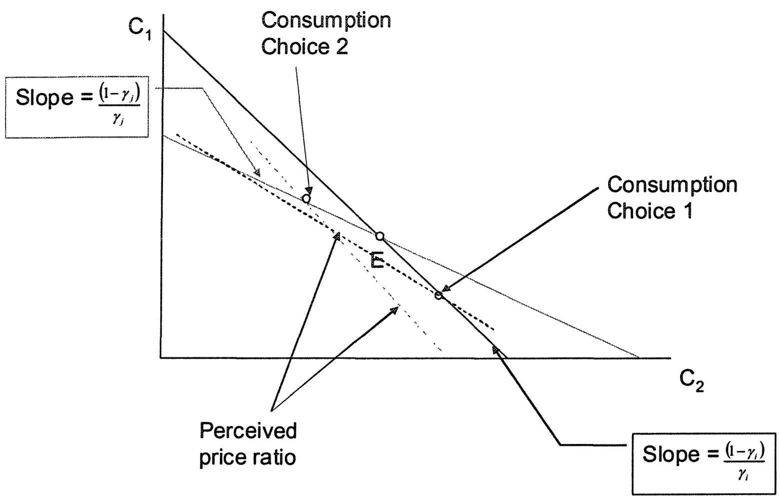
Thus, Figure 2.1 below is an illustration of a possible choice:



(Figure 2.1)

Definition 2.2. *An affective choice is a consumption vector* (c_1, c_2), *and price* $0 < \gamma < 1$, *such that the agent is maximizing utility subject to her budget constraint and is satisfying her mental account's first order condition.*

As argued, the distinction between objective and perceived probabilities allows the ADM model to support data sets which can not be rationalized by expected utility theory. This is shown in an example below:



(Figure 2.2)

Note that in the illustration above with objective probabilities the two observations violate WARP. Therefore, this agent is not an expected utility maximizer, i.e., these observations refute the expected utility paradigm. However, if this agent is an affective agent, then these observations need not violate WARP. To see that recall that with ADM perceived probabilities generally differ from the objective probabilities. Therefore, the agent's perceived price ratio might be very different than the one we observe. Indeed, as shown above, it well can be the case that with price perceptions the data does not violate WARP; hence I can find a utility function and a mental cost function such that the agent acts as if she is an affective expected utility maximizer. That is the ADM model can rationalize the data.

2.3.3 Unobservable Perceived Probabilities

In the case of unobservable perceived probabilities, the reader might be skeptical regarding the testability property of the ADM model. After all, if probabilities are allowed to vary, then one could always rationalize the observed insurance level. However, this is not true if perceived probabilities have known bound as is supported in experiments (Kahneman and Tversky, 1979 and reference within) and can be concluded with report task (see Chapter 1).

Recall our assumption that perceived probabilities are bounded between $\beta_{\min} < \beta < \beta_{\max}$ such that $0 < \beta_{\min} < \beta_{\max} < 1$. Let (γ_i, I_i) for $i = 1, ..., T$ be a finite number of observation of insurance premium and level, respectively. Using the relationship between affective choice and the state preference model, the proposition below follows:

Proposition 2.3. *The model is testable.*

The proof uses the fact that the marginal utility and the perceived probabilities are multiplicatively separable. Similar to the case of random utility functions, analyzed by Brown and Calsamiglia (2004), this allows one to disentangle the marginal utility of a Bernoulli agent from the unobservable component; in particular, this enables us to present the observed choice as a choice of a Bernoulli agent with *perceived price ratio*, as illustrated in the previous section. Having established the observed data as a choice of a Bernoulli agent, one can use revealed preference arguments to refute the model. The only difficulty is that the perceived price ratio is now unobservable. Fortunately, if the perceived probabilities are bounded (as I assume and is supported by experiments), then perceived price ratio is bounded as well, and the model is refutable[1]!.

As discussed above, testability implies in our framework that there exists a system of inequalities on the parameters only, i.e., observed insurance choices and prices which characterizes equilibrium (choice). In other words, given bounds on risk belief, this proves the existence of an implicit axiomatization for ADM involving insurance choices and prices *only.*

Note the similarity to Bewley (1986) knightian uncertainty model; there, the agent has a set of possible probabilities and she will choose a new consumption point over her status-quo if and only if the new point is preferred under all probability distributions in the set. Consequently, if one defines a class of Knightian uncertainty models where the set of probability distribution is the convex hull of the perceived probabilities, supported by affective choice, then this class of Knightian uncertainty models, labeled affective Knightian uncertainty, is also testable. To see why, consider two observations, A and B, that refutes the ADM model. These observations violate WARP for some perceived probabilities in the set. That is, no matter what is the initial choice, either A or B, the agent always prefers the status quo to the alternative. For example, if we take one of the observations, say A, to be the status quo, then the agent will never want to change her choice to B. Thus, observing the two choices A and B is a contradiction – refuting the affective Knightian model.

A natural question is how can one distinguish between the ADM model and the Knightian model? The example below makes it clear:

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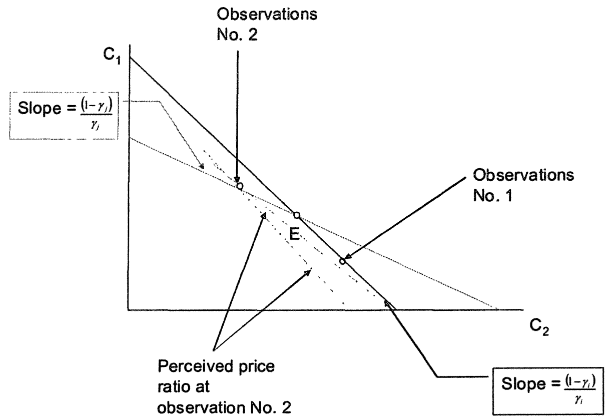
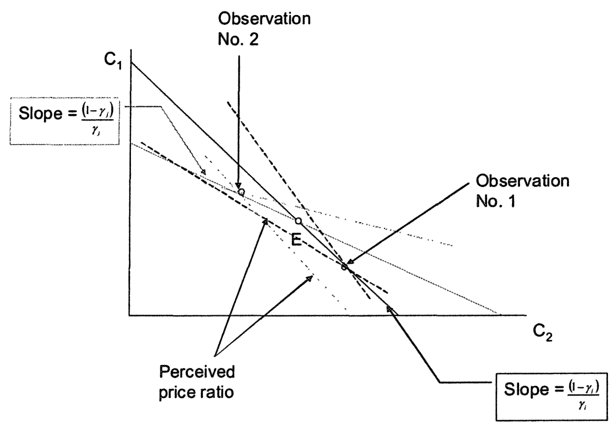
[1] Brown and Calsamiglia (2004) show that if probabilities are not bounded in their model, then it is not refutable. This is also true for the ADM model.



(*a*) (Figure 2.3)



(*b*)

Figure 2.3(a) and Figure 2.3(b) are examples where the two observations violated WARP for the standard expected utility model. Figure 2.3(a) is an example where, considering perceived probabilities, the agent does not violate WARP and thus is consistent with the ADM model. Moreover this is an instance where the agent is also consistent with the affective Knightian model. To see this, suppose the initial status quo is observation number

2, under all perceived probabilities the agent prefers observations number 1[2]. Note that in this example, the agent is also consistent with the ADM as with these perceived probabilities, the agent does not violates WARP. Figure 2.3(b) is an instance where the agent is not consistent with the expected utility paradigm, but is consistent with the ADM model. To see that note that there exist perceived probabilities such that the agent does not violate WARP. However, this example is not consistent with the affective Knightian model. To see that, suppose the agent's status quo is observation number 1. she will never change her choice to observation number 2 since there are perceived probabilities under which the status quo is preferred to observation number 2. A similar argument holds if observation number 2 is the status quo. Therefore, the two observations are not consistent with the affective Knightian uncertainty model.

In the next section I derive explicit axioms for the case of two observations and incomplete information.

2.4 Explicit Axioms

Consider two choices $(\gamma, x_1, x_2, \beta)$, $(\hat{\gamma}, \hat{x}_1, \hat{x}_2, \hat{\beta})$ of some affective agent, where $\gamma, \hat{\gamma}$ are prices, $(x_1, x_2), (\hat{x}_1, \hat{x}_2)$ are consumption bundles and $\beta, \hat{\beta}$ are the associated risk beliefs. However, for an outside observer the data consist of the consumption choices and prices only, i.e., (γ, x_1, x_2), $(\hat{\gamma}, \hat{x}_1, \hat{x}_2)$. By the discussion above, if the ADM rationalizes the data, i.e., these are choices of an affective agent, then the observables must satisfy the rational

[2] Note that this is enough to conclude that the agent is consistent with the Knightian uncertainty model, unless one knows that the agent started with observation number 1. In that case, one needs to show that for all perceived probabiliites at observation 1, the agent prefers observation number 2. This is the same argument as presented in the text. This was omitted for ease of exposition.

and the mental accounts' first order condition, as well as the Afriat's inequalities. For the case of two observations these conditions sum up to a total of 18 inequalities that can be reduced to the following:

$$\left[\beta - \hat{\beta}\right]\left[u_1 - u_2 - \hat{u}_1 + \hat{u}_2\right] \geq 0 \tag{2.9}$$

and

$$x_i > x_j \Rightarrow u_i^{'} < u_j^{'} \tag{2.10}$$

where

$$\beta = \frac{u_2^{'}\gamma}{u_1^{'}(1-\gamma) + u_2^{'}\gamma}; \hat{\beta} = \frac{\hat{u}_2^{'}\hat{\gamma}}{u_1^{'}(1-\hat{\gamma}) + \hat{u}_2^{'}\hat{\gamma}}$$

In other words, if one can find numbers $u_1, u_2, \hat{u}_1, \hat{u}_2$, $u_1^{'} \geq 0, u_2^{'} \geq 0, \hat{u}_1^{'} \geq 0, \hat{u}_2^{'} \geq 0$ $f^{'}(\beta)$ that satisfy the above conditions, then she can find a concave utility function and a convex mental cost function such that the observed choices constitute a pure strategy Nash equilibrium in the intrapersonal game of this affective agent. That is, affective payoff functions that rationalize the data. Using equation 2.9 and 2.10 the following proposition emerges:

Proposition 2.4. *Consider two observations* (γ, x_1, x_2), $(\hat{\gamma}, \hat{x}_1, \hat{x}_2)$ *where* $\gamma, \hat{\gamma}$ *are prices and* $(x_1, x_2), (\hat{x}_1, \hat{x}_2)$ *are consumption bundles. For any bounds on risk beliefs* $\underline{\beta}, \bar{\beta}$ *such that* $0 < \underline{\beta} < \bar{\beta} < 1$ *and numbers* $u_1^{'} \geq 0, u_2^{'} \geq 0, \hat{u}_1^{'} \geq 0, \hat{u}_2^{'} \geq 0$ *that satisfy condition 2.10 the data is rationalized by ADM if and only if:*

(i) $[u_1 - u_2 - \hat{u}_1 + \hat{u}_2] \geq 0$ *and* $\frac{\gamma}{(1-\gamma)}\frac{1-\bar{\beta}}{\bar{\beta}} < \frac{\hat{\gamma}}{(1-\hat{\gamma})}\frac{1-\underline{\beta}}{\underline{\beta}}$;

(ii) $[u_1 - u_2 - \hat{u}_1 + \hat{u}_2] \leq 0$ *and* $\frac{\hat{\gamma}}{(1-\hat{\gamma})}\frac{1-\bar{\beta}}{\bar{\beta}} < \frac{\gamma}{(1-\gamma)}\frac{1-\underline{\beta}}{\underline{\beta}}$;

(iii) the sign of $[u_1 - u_2 - \hat{u}_1 + \hat{u}_2]$ *is indeterminate.*

An example for two observations that can not be rationalized by the model is $x_1 = 10, x_2 = 5, \hat{x}_1 = 5, \hat{x}_2 = 10, \gamma = 0.4, \gamma = 0.4, \hat{\gamma} = 0.9, \underline{\beta} = 0.1, \bar{\beta} = 0.95$

Note that each part of the proposition corresponds to eight consumptions ranking. These ranking are provided in the proof.

2.5 Mutual Insurance with Affective Agents

The relationship established in the section 2.3.2 between affective choice and state-preference model, can be used to establish the existence of an equilibrium in a general equilibrium model of mutual insurance with affective agents. For this argument, I assume that agents consume strictly positive levels of consumption in all states. Agents with logarithmic utility functions are examples. Chapter 1 guarantees that the intrapersonal game for an agent with a logarithmic utility function admits pure strategy Nash equilibria; therefore, I shall focus on this class of affective agents. However, the results in this section apply also for affective agents each with a strictly concave utility function whose intrapersonal game have a Nash equilibrium with positive consumption in both states.

Consider two affective agents, i, j, each with a logarithmic utility function of wealth $u(\cdot)$ who are facing two possible future states of the world s_1, s_2, with an associated income level of w_1^k, w_2^k, respectively where $k \in \{i, j\}$. Without loss of generality, assume $w_1^k < w_2^k$ and denote the income shock as $z^k \equiv w_2^k - w_1^k$. Each agent can insure against the future income shock, z^k, by purchasing or selling mutual insurance $I^k \in (-\infty, +\infty)$ at some premium rate $0 < \gamma < 1$. Since the agents are affective agents, their choice is a pure-strategy Nash equilibrium of each agent's intrapersonal game, and it is a vector of insurance level and perceived probability. The intrapersonal game and its Nash equilibrium are as discussed in chapter 1 above.

As section 2.3.2 shows, the affective choice can be represented in a state-preference model. More specifically, agent i solves the following:

$$\max_{c_1, c_2} \quad \beta_i u(c_1) + (1 - \beta_i)\, u(c_2) \tag{2.11}$$

$$s.t. \qquad \gamma c_1 + (1 - \gamma)\, c_2 = y \tag{2.12}$$

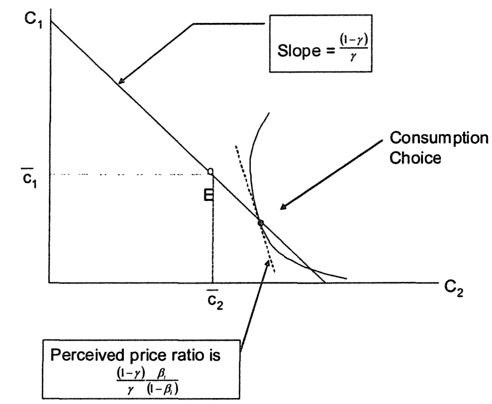
where the vector (c_1, c_2) is the consumption of agent i in state (s_1, s_2), respectively; β_i is the probability of state s_1 to occur, as perceived by agent i in equilibrium, and $y \triangleq \gamma w_1^1 + (1 - \gamma)\, w_2^1 \triangleq \gamma \bar{c}_1 + (1 - \gamma)\, \bar{c}_2$[3]. Since perceived probability β_i is optimal in equilibrium, it must satisfy the first order condition of the mental account (see chapter 1), as presented below:

$$u\,(c_1) - u\,(c_2) = f'\,(\beta_i) \tag{2.13}$$

where $f\,(\cdot)$ is agent $i's$ mental cost of holding beliefs β.

[3] $(\bar{c}_1, \bar{c}_2)$ are the agent's endowment in state (s_1, s_2), respectively.

The following is an illustration of individual $i's$ choice:



(Figure 2.4)

Similarly, agent j solves:

$$\max_{d_1,d_2} \quad \beta_j u(d_1) + \left(1 - \beta_j\right) u(d_2) \tag{2.14}$$

$$s.t. \qquad \gamma d_1 + (1 - \gamma)\, d_2 = I \tag{2.15}$$

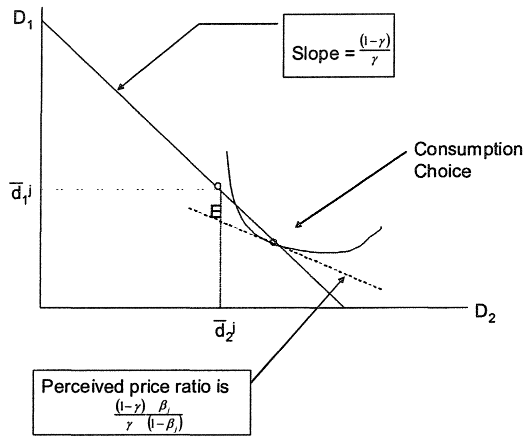
where the vector (d_1, d_2) is the consumption of agent j in state (s_1, s_2), respectively, and $I \triangleq \gamma w_1^2 + (1 - \gamma)\, w_2^2 \triangleq \gamma \bar{d}_1 + (1 - \gamma)\, \bar{d}_2$[4]; β_j is the probability of state s_1 to occur, as perceived by agent j in equilibrium. The perceived probability β_j must satisfy:

$$u\,(d_1) - u\,(d_2) = g'\left(\beta_j\right) \tag{2.16}$$

where $g\,(\cdot)$ is agent $j's$ mental cost of holding beliefs β. The following is an illustration of

[4] $(\bar{d}_1, \bar{d}_2)$ are the agent's endowment in state (s_1, s_2), respectively.

individual $j's$ choice:



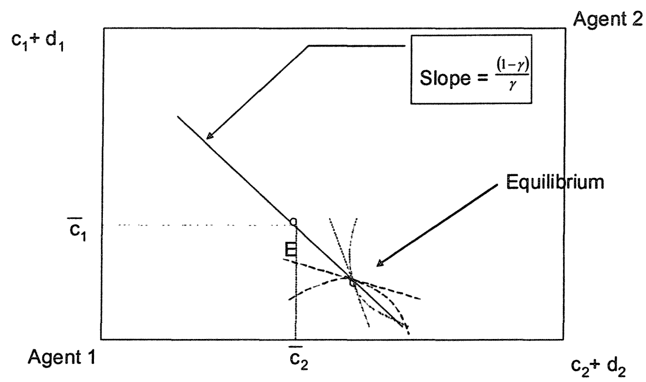
(Figure 2.5)

Definition 2.3. *A mutual insurance equilibrium is a consumption vector* (c_1, c_2, d_1, d_2) *and price* $0 < \gamma < 1$*, such that each agent is maximizing utility subject to her budget constraint, satisfying her mental account's first order condition and all markets clear.*

Since perceived probabilities are allowed to vary and are part of the solution, it is not clear a priori that a solution exists given that the demand correspondences are not convex-valued. Fortunately, the following assure us that one can construct an example of existence, hence the model is consistent.

Proposition 2.5. *There is an example where mutual insurance equilibrium with affective agents exists.*

The following figure is an example of such an equilibrium:



(Figure 2.6)

And the following is a numeric example that constitutes a mutual insurance equilibrium:

$$\begin{aligned}
Utility\ Function &: u_1(x) = u_2(x) = \ln(x) \\
\text{Pr}\,ices &: \gamma = 0.5 \\
Allocation &: (c_1, c_2) = (8, 12), (d_1, d_2) = (16, 10) \\
Mental\ Cost\ for\ agent\ 1 &: f(\beta_1) = \left[\log(\frac{1}{(\beta - \underline{\beta})(\bar{\beta} - \beta)}) + 0.9278\beta\right] \Rightarrow \beta_1^* = 0.4 \\
Mental\ Cost\ for\ agent\ 2 &: g(\beta_2) = \left[\log(\frac{1}{(\beta - \underline{\beta})(\bar{\beta} - \beta)}) - 3.101\beta\right] \Rightarrow \beta_2^* = 0.8 \\
Endowment, agent\ 1 &: \bar{c}_1 = 8 - \varepsilon, \bar{c}_2 = 12 + \varepsilon \\
Endowment, agent\ 2 &: \bar{d}_1 = 16 + \varepsilon, \bar{d}_2 = 10 - \varepsilon
\end{aligned}$$

As in the previous section, the agents' perceived probabilities vary since they are making affective choices. This raises the question if a mutual insurance equilibrium with affective agents can ever be refuted. Is everything possible? The answer is no, and it is summarized in the following proposition.

Proposition 2.6. *Mutual insurance with affective agents is testable.*

To prove proposition 2.6, one only needs to show that there exist an example where the model is refuted. Using the fact that affective decision making imposes bounds on perceived price ratios, I modify the example in Brown and Matzkin (1996) to provide the necessary example.

An important question is the Pareto optimality of a mutual insurance equilibrium with affective agents. For the expected utility model, we have the first welfare theorem which guarantees that the mutual insurance equilibrium with expected utility maximizers is Pareto optimal. However, with affective agent this is not obvious. In order to discuss Pareto optimality, I shall use the preference relation and Pareto optimality defined below:

Definition 2.4. *Let A =(I_A, β_A) and B=(I_B, β_B) be two possible affective choices for an agent i. A is* strongly preferred *to B, $A \succ_i B$ if the rational account prefers the insurance level I_A to I_B under both beliefs β_A and β_B.*

Definition 2.5. *An allocation is said to be "affective Pareto optimal" if there is no other allocation which is strongly preferred to it by both agents and is feasible.*[5]

There are two important points to notice; First, an allocation is feasible in our case if it

[5]Note that if the rational account prefers $x \succ y$ under both equilibrium perceptions β_x, β_y, then the mental account is better off at (x, β_x). To see that note that $x \succ y$ implies $\sum_s \beta_{sy} u(x_s) > \sum_s \beta_{sy} u(y_s)$. Thus, $\sum_s \beta_{sy} u(x_s) - f(\beta_y) > \sum_s \beta_{sy} u(y_s). -f(\beta_y)$. However we know that $\sum_s \beta_{sx} u(x_s) - f(\beta_x) \geq \sum_s \beta_{sy} u(x_s) - f(\beta_y)$ by definition.

is both feasible in the standard sense *and* if it is a Nash equilibrium in each of the agents' intrapersonal games. Second, for the expected utility model the strong preference relation reduces to the familiar one, as well as the definition of affective Pareto optimality.

Using the definition of strong preference relation and affective Pareto optimality, the following is true:

Proposition 2.7. *The mutual insurance general equilibrium with affective agents is affective Pareto optimal.*

The implications of this proposition is that a mutual insurance equilibrium is Pareto optimal as perceived by the affective agents. That is, there is an efficient *perceived risk* allocation in the sense that there is no other allocation which is feasible and both affective agents would like to move to. Thus, I label it the first affective welfare theorem. Arrow (1971) pointed out that, under certain hypotheses, the allocation of risk bearing is optimal under competitive securities markets. Proposition 2.7 above indicates that in the case of affective agents, Arrow's result still hold for *perceived* risk; this allocation need not, and generally will not, be the point of optimal objective risk sharing. Hence, Arrow's result holds for uncertainty as well as risk.

This is not all; below is a second affective welfare theorem:

Proposition 2.8. *For any affective optimal allocation, one can find a mutual insurance equilibrium with zero-sum transfers that supports it.*

The proof is omitted since it is similar to the proof of the standard second welfare theorem in exchange economies. However, the idea is that if one fixes the perceived probabilities at the Nash optimal allocation, then she is back to the standard model. Thus, the better than sets are convex and there exists a separating hyperplane. Now one simply needs to construct an endowment point such that the separating hyperplane is the budget line connecting this new endowment point and the equilibrium considered. This can be achieved with zero sum transfers.

Chapter 3

A Perceived Screening Model with Bayesian Agents in Monopolistic Markets

3.1 Introduction

Economic models with heterogeneous agents commonly assume that agents perfectly know their type. In reality, however, this is not usually the case. Take for example consumers and insurees. As consumers, we often find it difficult to evaluate the utility from consuming a product. As insurees, we find it difficult to determine the probability of having an accident.

Under such circumstances, consumers use type *assessments* based on information from friends, experts, and ads. This accords with the marketing literature which views ads as supplying the consumer with information (e.g., Lushbough 1974, Ehrlich and Fisher 1982, Soberman 2001 and reference within), which is either truthful or not. It is also in agreement with the casual observation that non-informative ads project an image or elicit an emotion to generate consumption. Emotion eliciting ads can be viewed as a mechanism to distort potential buyers' information. Support for this can be found in behavioral and psychological research which claims that emotional states affect information processing (e.g., Mellers, Schwartz and Cooke 1998), type perception (e.g., Johnson and Tversky 1983), and choice (e.g., Lerner and Keltner 2000). Additional support in the marketing literature shows that there is a correlation between perceived quality and advertising expenditure (e.g., Moorthy and Zhao 2000).

Acknowledging the ability of ads to influence type perception implies that the information structure is in part endogenously determined by the firm. Therefore, it is of economic interest to find this firm's optimal consumer perceptions. This essay examines the incentive a monopolist has to influence consumers' type perceptions in the specific class of screening models.

To capture the endogenous information structure, generated by type perception, this essay departs from the traditional screening models (e.g., Stiglitz's 1977 and for a review of these models, Salanié 1999), by relaxing the assumption of perfectly informed agents. Alternatively, it is assumed that the agents only observe a noisy signal of their type. The monopolist is assumed to influence agents' perception by setting the accuracy of the agents' signal in advance.

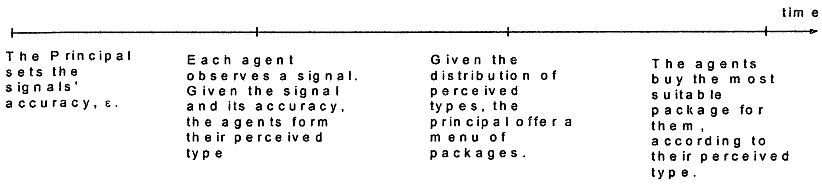
Agents are assumed to process their signal rationally using Bayes' rule. There are two reasons for this: (1) to minimize the departure from classic models and (2) to show that a monopolist can have incentives to distort an agent's information even in a rational setting. One can think of this as a benchmark for future research incorporating systematic biases in perceptions.

In a related paper, Lewis and Sappington (1994) analyze the endogenous information structure in a monopolistic price-discrimination adverse-selection model, assuming continuous types (product valuation) and fixed marginal cost. They also present an example with two discrete types, high and low. Their set up is similar to the one presented in this essay, in the sense that the monopolist can determine the accuracy of the agents' signals in advance. However, their analysis of the discrete case has strong assumptions. In particular, they assume linear belief updating over two *equally* likely valuations and each consumer's demand is restricted to either zero or one unit of the good. The unit demand assumption is particularly strong as it excludes second degree price discrimination. Excluding second degree price discrimination means a market break down for the low value types in the case of perfect information. In contrast, this essay analyzes a more general setting of monopolistic price discrimination with continuous demand, convex costs and Bayesian updating over all possible binary distributions of the two valuations, high and low. The analysis indicates, similar to Lewis and Sappington, that there will be a corner solution: either agents have no information or full information. However, in contrast to Lewis and Sappington, providing full information does not necessarily yield a break down of the market for low types. This depends on the distribution of, and difference between, high and low valuations. Moreover, the current set up allows a careful examination of the forces that generate the corner solutions. Consequently, in this essay I provide specific conditions on the parameters that determine which corner solutions will prevail and who participates in the market. In addition, general sufficient conditions on the profit function that ensure corner solutions are provided. This essay also analyzes the optimal endogenous information structure in the context of insurance. In this case, there is only one solution – no information.

The rest of the essay is organized as follows. Section 3.2 presents and analyzes a monopolistic price discrimination screening example that departs from the classic model by incorporating perceived types, rather than true types. I call this the *perceived price discrimination screening model.* Section 3.3 generalizes the model of section 3.2 and provides sufficient conditions for the existence of a corner solution. Section 3.4 presents a monopolistic *perceived insurance screening model* and discusses both the optimal information structure for the firm and the robustness of the solution. Section 3.5 concludes and suggests possible extensions of this research.

3.2 Perceived Price Discrimination Screening Model

Consider a monopolist price discrimination screening model with imperfectly informed consumers. The consumers privately observe a noisy signal about their type and rationally process it to form a type assessment, which I define as a consumer's perceived type. The principal is assumed to be uninformed, but is able to influence type perception by selecting the agent's private signal accuracy. Selecting signal accuracy is done prior to the agent observing her signal. More specifically, the timing of the model is as follows:



(Figure 3.1)

3.2.1 Model

Consider a monopolistic price discrimination screening model. Suppose there are two possible types of consumers (product valuation), low and high, which I denote as θ_1, θ_2 respectively. Without loss of generality, assume $\theta_2 > \theta_1$ and $\{\theta_1, \theta_2\}$ are distributed with probability $\{p_1, 1 - p_1\}$, respectively. The principal is uninformed but knows the distributions of types, while each agent i observes a noisy signal $s_i \in \{s_1, s_2\}$ about her type θ_i[1]. The signal s_i is generated according to the following conditional probability:

$$\Pr(s_1|\theta_1) = \Pr(s_2|\theta_2) = (1-\varepsilon),\ \Pr(s_1|\theta_2) = \Pr(s_2|\theta_1) = \varepsilon$$

Each agent i, after observing s_i, forms her perceived type according to Bayes' rule:

$$\hat{\theta}_i = E\left(\theta_i|s_i\right) = \begin{cases} \hat{\theta}_1 = \dfrac{p_1(1-\varepsilon)\theta_1 + (1-p_1)\varepsilon\theta_2}{p_1(1-\varepsilon) + (1-p_1)\varepsilon}, & if\ s_i = s_1 \\ \hat{\theta}_2 = \dfrac{p_1\varepsilon\theta_1 + (1-p_1)(1-\varepsilon)\theta_2}{p_1\varepsilon + (1-p_1)(1-\varepsilon)}, & if\ s_i = s_2 \end{cases} \tag{3.1}$$

The distribution of perceived types $\hat{\theta}_i \in \{\ \hat{\theta}_1,\ \hat{\theta}_2\}$, $\hat{\theta}_1 \leq \hat{\theta}_2$, is $\{\hat{p}_1, 1 - \hat{p}_1\}$, respectively,

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[1] In contrast to the standard asymmetric information model, where each insuree has perfect information of θ.

where $\hat{p}_1$ denotes the probability of observing low signal and is defined as:

$$\hat{p}_1 \triangleq p(s_1) \triangleq p_1(1-\varepsilon) + (1-p_1)\varepsilon$$

The uninformed principal designs a menu which consists of a package of quantity, $\hat{q}_k$, and transfer, $\hat{t}_k$, for each perceived type k. The monopolist is assumed to have some control over perceived types and the distribution of perceived types, through ε. Each agent observes a private signal and knows ε, which she then use to form an assessment and select a package to maximize her (subjective) expected utility. Agent i's utility function is quasilinear:

$$u_i(q) = \hat{\theta}_i q - t_i$$

The monopolist's cost function is assumed quadratic:

$$c(q) = \frac{1}{2}q^2$$

3.2.2 The Solution

The solution is the optimal information structure for the monopolist, captured by ε. This is the signal accuracy which maximizes the monopolist's profits. Due to the timing of the model, this solution is obtained using backwards induction. First, the monopolist solves the perceived screening problem and then its implied profits. Second, the monopolist chooses the information structure which yields the highest possible profits.

To obtain the backwards induction solution, it will be proved useful to analyze the principal's profits function in a symmetric information world, where the principal observes the same signal as the agents.

Definition: $\Pi^S(\hat{\theta}, q) = \hat{\theta} q - \frac{1}{2} q^2$ is the symmetric information profit.

Assuming symmetric information one can disregard incentive compatibility constraints and therefore is assured the principal is always selling to both perceived types[2]. For any type $\hat{\theta}_k$, the profits maximizing package is $q = \hat{\theta}_k$ and consequently the profits are:

$$\pi_k^{sym} = \frac{1}{2} \hat{\theta}_k^2$$

If there are two perceived types in the population, then the total profits are:

$$\Pi^{sym} = \hat{p}_1 \pi_1^{sym} + (1 - \hat{p}_1) \pi_2^{sym} = \frac{1}{2} \left[\hat{p}_1 \hat{\theta}_1^2 + (1 - \hat{p}_1) \hat{\theta}_2^2 \right]$$

Proposition 3.1. *As ε increases, from 0 to $\frac{1}{2}$, Π^{sym} is decreasing and approach a minimum at $\hat{\theta}_1 = \hat{\theta}_2 = \bar{\theta} \triangleq p_1 \theta_1 + (1 - p_1) \theta_2$.*

Proposition 3.1 implies that in a symmetric information world, where both the company and the agent observe the same signals about the agent's type, the firm is better off with informed agent, i.e. $\varepsilon^* = 0$. Note the equivalence to the case in which a firm has information advantage about the agent's type. The first proposition, means then, that the firm is better off conveying its information to the agents.

Proposition 3.1 allows us to analyze the case of asymmetric information. To cover all possible cases of asymmetric information, I shall make the following distinction between two asymmetric information settings: (1) $\theta_1 = 0, \theta_2 > 0$ and (2) $\theta_1 > 0, \theta_2 = \lambda \theta_1$ where

[2]Except for the case of $\hat{\theta}_k = 0$. In such a case, the profit formula still hold, but the firm does not sell to type $\hat{\theta}_k$.

$\lambda > 1$.

Case 1: $\theta_1 = 0, \theta_2 > 0$

The optimal information structure when $\theta_1 = 0, \theta_2 > 0$ follows immediately from proposition 1 and it is stated in proposition 3.2 below:

Proposition 3.2. *The optimal information structure in the case of asymmetric information where* $\theta_1 = 0, \theta_2 > 0$ *is* $\varepsilon^* = 0$.

Proposition 3.2 implies that if there exists a group of consumers who would not purchase the good when they perfectly knew their type, then it is better for the firm to forgo this sector by perfectly informing them about their type. This is equivalent to offering only a package for the high type, while educating the high type agents about the product value for them. An example can be found in the fine wine industry. The wineries participating in this market produce and sell only to a specific high class market and it uses certain tasters to convey the quality of the wine.

Case 2: $\theta_1 > 0, \theta_2 = \lambda\theta_1$ where $\lambda > 1$.

This case is more general and together with the previous one concludes all cases. To find the optimal information structure (ε) in this case, below is the perceived adverse selection model for some value of ε :

$$\max_{t_1,t_2,q_1,q_2} \hat{p}_1 \left[\hat{t}_1 - c(\hat{q}_1)\right] + (1-\hat{p}_1)\left[\hat{t}_2 - c(\hat{q}_2)\right]$$

$s.t.$

$$(IR_1)\ \hat{\theta}_1\hat{q}_1 - \hat{t}_1 \geqslant 0$$

$$(IR_2)\ \hat{\theta}_2\hat{q}_2 - \hat{t}_2 \geqslant 0$$

$$(IC_1)\ \hat{\theta}_1\hat{q}_1 - \hat{t}_1 \geqslant\ \hat{\theta}_1\hat{q}_2 - \hat{t}_2$$

$$(IC_2)\ \hat{\theta}_2\hat{q}_2 - \hat{t}_2 \geqslant\ \hat{\theta}_2\hat{q}_1 - \hat{t}_1$$

$$\hat{t}_k \geq 0, \hat{q}_k \geq 0\ \forall k \in \{1,2\},$$

The two (IR) constraints are the individual rationality constraints which guarantee that each perceived type accepts her designated package. The two (IC) constraints are the incentive compatibility constraints and they assure us that each perceived type prefers her designated package to the alternative package.

Notice that this maximization problem is different from the classic screening model *only* in using *perceived types*, instead of true types. Due to this similarity, it is straightforward to see that only the low-perceived-type rationality constraint (IR_1) and the high-perceived-type incentive constraint (IC_2) are binding in equilibrium.

The solution of the perceived model, assuming quadratic cost function, is similar to the classic one:

$$\hat{q}_1^* = \hat{\theta}_1 - \frac{1-\hat{p}_1}{\hat{p}_1}\left(\hat{\theta}_2 - \hat{\theta}_1\right)$$
$$\hat{q}_2^* = \hat{\theta}_2$$

I need to verify that at the optimum $\hat{q}_1^* \geq 0$. $\hat{q}_1^*$ is non-negative if and only if:

$$\varepsilon^2\left(p_1 + 3\lambda p_1 - \lambda - 2p_1^2 - 2\lambda p_1^2\right) + \varepsilon\left(p_1 + p_1^2 - 3\lambda p_1 + 3\lambda p_1^2\right) - \left(p_1 - \lambda p_1 + \lambda p_1^2\right) < 0 \tag{3.2}$$

Condition (3.2) can not be solved for ε in a closed form. However, numerically solving condition (3.2) yields the following two sub-cases:

1. $\exists$ pairs (λ, p_1) such that condition (3.2) is satisfied $\forall \varepsilon \in \left[0, \frac{1}{2}\right]$. Therefore, the principal is selling to both perceived types, whatever is the value of ε.

2. $\exists$ pairs (λ, p_1) such that condition (3.2) is satisfied $\forall \varepsilon \in \left[\bar{\varepsilon}, \frac{1}{2}\right]$, where $0 < \bar{\varepsilon} < \frac{1}{2}$ and not otherwise. Thus, if $\varepsilon \in [0, \bar{\varepsilon}]$ the principal is selling only to the high-perceived type; if $\varepsilon \in \left[\bar{\varepsilon}, \frac{1}{2}\right]$ the principal is selling to both perceived types.

Case 2.1: consider a pair (λ, p_1) such that condition (3.2) is satisfied $\forall \varepsilon \in \left[0, \frac{1}{2}\right]$.

In this sub-case, the profits one should analyze are the profits of selling to both types:

$$\Pi^{Asy} = \frac{1}{2}\{\hat{\theta}_1^2 + \frac{(1-\hat{P}_1)}{\hat{P}_1}(\hat{\theta}_2 - \hat{\theta}_1)^2\} \tag{3.3}$$

Taking first order derivative, $\frac{\partial \Pi^{Asy}}{\partial \varepsilon}$, and numerically solving it reveals that $\frac{\partial \Pi^{Asy}}{\partial \varepsilon}$ increases with ε. Thus proposition 3.3 immediately follows:

Proposition 3.3. *The optimal information structure is at the extremes. i.e., either* $\varepsilon^* = 0$ *or* $\varepsilon^* = \frac{1}{2}$. *And the exact solution is given by the following condition:*

$$\text{if} \begin{cases} 1 \leq \lambda < \frac{(1-p_1)(1+p_1+p_1^2)}{1-p_1^3-2p_1(1-p_1)} \Rightarrow \varepsilon^* = \frac{1}{2} \\ \lambda = \frac{(1-p_1)(1+p_1+p_1^2)}{1-p_1^3-2p_1(1-p_1)} \quad \Rightarrow \varepsilon^* = \{0, \frac{1}{2}\} \\ Otherwise \quad \Rightarrow \varepsilon^* = 0 \text{ and selling to both} \end{cases}$$

To explain the above results, note that there are two contradicting effects at play. The first is the "symmetric information profits" effect, which is decreasing with ε as shown in proposition 3.1. The second effect is the "asymmetric information costs" which is a sum of the information rent and costs of distorting the low-type contract. These costs increase with information asymmetry (as ε declines) and therefore tend towards no information, $\varepsilon = \frac{1}{2}$. When λ is small enough, the "asymmetric information costs" effect dominates, and the solution is no information. However, when λ is large enough, the "symmetric information profits" effect dominates and the result is full information. If λ is exactly equal the threshold level of $\frac{(1-p_1)(1+p_1+p_1^2)}{1-p_1^3-2p_1(1-p_1)}$ then the firm is indifferent between the informational separating and pooling equilibria. This means that the reduction in symmetric profits with $\varepsilon^* = \frac{1}{2}$ is exactly offset by the decline in asymmetric information costs. Since the company is indifferent, while the consumers are better off with $\varepsilon^* = 0$, then one can conclude that $\varepsilon^* = 0$ is welfare-superior. In such a case, taxing the firm for distorting information can lead to the welfare-superior outcome, $\varepsilon^* = 0$.

Proposition 3.3 suggests that there are cases where the monopolist prefers privately informed agents, even though it does not observe this information. An example of such a case can be found in the dental industry. Dental labs use a few patent products that are produced monopolistically. These monopolists usually allow dental labs to try out their unique products, as well as conduct educational seminars to market them. This essentially allows agents to receive private information which the principal does not observe. Proposition 3.3 suggests an explanation for this: if the product value varies a lot across labs, then the producer is better off with perfectly informed labs.

Case 2.2: consider a pair (λ, p_1) such that condition (3.2) is satisfied only for $\varepsilon \in \left[\bar{\varepsilon}, \frac{1}{2}\right]$, for some $0 < \bar{\varepsilon} < \frac{1}{2}$. This implies that for values $\varepsilon \in [0, \bar{\varepsilon}]$ the principal sells only to high-perceived type and for values $\varepsilon \in \left[\bar{\varepsilon}, \frac{1}{2}\right]$ the principal sells to both perceived types.

In order to solve this case, the analysis is divided into three steps: the first step analyzes the monopolist's profit when it sells to both high and low perceived types for all ε. The second step examines the monopolist's profits when it sells only to the high perceived type for all values of ε. The third step incorporates the two previous separate analyses and accounts for a regime change at some threshold $\bar{\varepsilon}$.

Case 2.1 already studied the case of selling to both perceived types. To complete the second stage of the current analysis, one needs to examine the profits from selling exclusively to the high perceived type. When the monopolist is selling only to the high perceived type[3], its profits are:

$$\Pi^{High} = \begin{cases} \frac{1}{2}(1-\hat{p}_1)\hat{\theta}_2^2, & \text{if } \varepsilon \neq \frac{1}{2} \\ \frac{1}{2}\bar{\theta}^2, & \text{if } \varepsilon = \frac{1}{2} \end{cases} \tag{3.4}$$

Where $\bar{\theta} \triangleq p_1\theta_1 + (1-p_1)\theta_2$.

[3] This happens in cases where the optimal quantity is $\hat{q}_1^* = \hat{\theta}_1 - \frac{1-\hat{p}_1}{\hat{p}_1}\left(\hat{\theta}_2 - \hat{\theta}_1\right) < 0$ which is ruled out.

The first order derivative, $\frac{\partial \Pi^{High}}{\partial \varepsilon} \gtreqless 0 \Leftrightarrow \lambda \lesseqgtr \frac{p_1 + p_1(1-\varepsilon) - 2p_1^2(1-\varepsilon)}{(1-p_1)(1-\varepsilon) + 2p_1\varepsilon(1-p_1)}$. Notice that $\frac{p_1 + p_1(1-\varepsilon) - 2p_1^2(1-\varepsilon)}{(1-p_1)(1-\varepsilon) + 2p_1\varepsilon(1-p_1)}$ is increasing with ε, thus proposition 3.4 immediately follows:

Proposition 3.4. *The optimal information structure, selling only to the high perceived type, is at the extremes. i.e., either $\varepsilon^* = 0$ or $\varepsilon^* = \frac{1}{2}$. And the exact solution is given by the following condition:*

$$\text{if} \begin{cases} 1 \leq \lambda < 1 + \frac{1}{\sqrt{1-p_1}} & \Rightarrow \varepsilon^* = \frac{1}{2} \\ \lambda = 1 + \frac{1}{\sqrt{1-p_1}} & \Rightarrow \varepsilon^* = \left\{0, \frac{1}{2}\right\} \\ \textit{Otherwise} & \Rightarrow \varepsilon^* = 0 \textit{ and selling to the high type only} \end{cases}$$

Again, this proposition reflects two contradicting forces that underline the change in profits as ε changes. As ε increases, symmetric information profit declines while *market participation* increases. Similar to the previous case, the solution is either one of the two corner solutions. The exact solution is determined by the level of λ – the ratio of the private value for the high and low type. In the special case of λ=1 + $\frac{1}{\sqrt{1-p_1}}$, both equilibria are possible and are equivalent from the firm's point of view.

Combining proposition 3.3 and 3.4, proposition 3.5 below follows:

Proposition 3.5. *The optimal information structure is either $\varepsilon = 0$ or $\varepsilon = \frac{1}{2}$, regardless of the profit regime.*

Recall that proposition 3.3 states the conditions that determine which extreme information structure will prevail in the case of selling to both perceived types for all $\varepsilon \in \left[0, \frac{1}{2}\right]$. Proposition 3.4 states the conditions that determine which extreme information structure will prevail in the case of selling only to the high perceived type for all $\varepsilon \in \left[0, \frac{1}{2}\right]$. Incorporating the conditions from these two propositions to take account for the two regimes yields proposition 3.6:

Proposition 3.6. *The information structure in the presence of two profit regimes is determined by the following condition:*

$$if \begin{cases} 1 \leq \lambda < 1 + \frac{1}{\sqrt{1-p_1}} \Rightarrow \varepsilon^* = \frac{1}{2} \\ \lambda = 1 + \frac{1}{\sqrt{1-p_1}} \qquad \Rightarrow \varepsilon^* = \{0, \frac{1}{2}\} \\ Otherwise \quad \Rightarrow \varepsilon^* = 0 \text{ and selling to the high type only} \end{cases}$$

Notice that the binding conditions are the ones resulting from the regime of selling only to the high perceived type. As mentioned above, these conditions determine when the decline in symmetric profit is outweigthed by capturing larger fraction of the market.

3.2.3 Discussion

Adding costs

The above analysis assumes that firm's information-related activity is costless. A natural question to ask is whether the above results are robust to adding information-related costs. To answer that, note that the driving forces behind the qualitative results are a decline in symmetric information profits as well as a reduction in asymmetric information costs as ε increases. Together, these forces generate asymmetric information profit which is convex in ε. Clearly, any cost function that leaves the structure of the problem unchanged in ε will preserve the qualitative results of a corner solution[4]. Examples for such cost functions are fixed costs, fixed regardless of ε, and costs that increase with information accuracy.

[4] Profits must remain non-negative for some values of $\varepsilon \in [0, 1]$ or otherwise the firm will shut down.

Market Structure

Since the analysis is conducted for the special case of a monopolist firm, an interesting question is whether different market structures will change the above results. One implication of a non-monopolistic market, in the case of endogenous information structure, is that each company's profits depend, among other things, on the information supplied to the agents by the other companies.

Suppose that the agents' type perception depends only on the highest accuracy available. Consider a competitive market and let η be the best accuracy supplied by one of the firms. Given η, one can find the perceived types in the population and their distribution. At this point, the model is the same as the traditional screening model only working with perceived, rather than true, types. For the traditional competitive case, a pure strategy equilibrium does not always exist. However, if such an equilibrium exists then each firm's profit is zero in equilibrium.

Now consider the endogenous information structure case. Suppose there is a set of $\eta^* \in [0, \frac{1}{2}]$ for which there exist equilibrium in pure strategies. Define η^* as the informational equilibria.

Proposition 3.7. *In contrast to the traditional case, pooling equilibria in informational competitive screening model exist.*

To see that, consider the case of $\eta^* = \frac{1}{2}$. In this case, there is only one perceived type and therefore an equilibrium exists. Since all firms expect to gain zero economic profit under all other informational structure (if they exist), no firm has a profitable deviation and this is an informational equilibrium.

Corollary 3.1. *An informational equilibrium always exists.*

Indeed competition introduces the possibility of other non-corner solutions, as opposed to the monopoly case. However, as argued above, one corner solution of no information (informational pooling) is always possible and it could well be the unique equilibrium.

An example of a competitive market with possible asymmetric information, in which firms sell only to one average type can be found in the vitamin industry. Casual observation indicates that vitamins tend to sell in "one dosage fits all" and its ads are usually positive emotion eliciting. According to proposition 3.7, this equilibrium is a possible equilibrium of a competitive perceived price-discrimination market and could be unique.

Acquiring information

One can wonder what would happen if agents could engage in activities to improve their information. Consider the case where the optimal solution is $\varepsilon = \frac{1}{2}$. The amount the agents as a group would be willing to pay, in order to improve their information, is up to the change in informational costs moving from $\varepsilon = 0$ to $\varepsilon = \frac{1}{2}$. Note that the change in profits for the firm, moving from $\varepsilon = 0$ to $\varepsilon = \frac{1}{2}$ is:

$$\begin{array}{rcl} \triangle\pi & = & \triangle(symmetric \text{ information } profit) - \triangle(\text{informational costs}) \\ & & \qquad \downarrow \quad \text{as } \varepsilon \longrightarrow \frac{1}{2} \qquad\qquad\qquad\qquad \downarrow \quad \text{as } \varepsilon \longrightarrow \frac{1}{2} \end{array}$$

Given that the equilibrium is at $\varepsilon = \frac{1}{2}$, one can conclude that comparing the profit at $\varepsilon = 0$ and $\varepsilon = \frac{1}{2}$, the gap in profit is less than the change in informational costs. Thus, if consumers are able to pay the firm for better information it is possible to have perfect information at all times.

3.3 Generalization

The example of the previous section suggested that in the monopolistic two types price discrimination screening model, the monopolist would either choose no information or full information, in either case, a corner solution. In general, however, this result may be sensitive to the exact functional form assumed. In this section I provide sufficient conditions on the value and cost functions which guarantee a corner solution.

Consider the following adverse selection model. Again, there are two possible types, $\{\theta_1, \theta_2\}$, with a probability $(p_1, 1-p_1)$ respectively. Without loss of generality assume $\theta_2 > \theta_1$ and $1 > p_1 > 0$. Each individual observes a signal $s_i \in \{s_1, s_2\}$ about her type with conditional probability

$$\begin{aligned} \Pr(s_i|\theta_i) &= (1-\varepsilon) \\ \Pr(s_j|\theta_1) &= \varepsilon \end{aligned}$$

After observing the signal, each agent forms her perceived type:

$$\begin{aligned} \hat{\theta}_1 &= \frac{p_1(1-\varepsilon)\theta_1 + (1-p_1)\varepsilon\theta_2}{p_1(1-\varepsilon) + (1-p_1)\varepsilon}, \quad if\ s_i = s_1 \\ \hat{\theta}_2 &= \frac{p_1\varepsilon\theta_1 + (1-p_1)(1-\varepsilon)\theta_2}{p_1\varepsilon + (1-p_1)(1-\varepsilon)}, \quad if\ s_i = s_2 \end{aligned}$$

The proportion of perceived type $\hat{\theta}_1$ is $\hat{p}_1 = p_1(1-\varepsilon) + (1-p_1)\varepsilon$ and the proportion of perceived type $\hat{\theta}_2$ is $(1-\hat{p}_1)$. Notice that $(1-\hat{p}_1)$ is strictly positive. Each agent has a product valuation of $g(\hat{\theta},\hat{q}) \triangleq E\left(v(\theta,q)\,|s\right)$, which is assumed to be strictly concave in q, twice differentiable with respect to both θ and q and satisfies the following properties:

$$\frac{\partial g(\hat{\theta},\hat{q})}{\partial\hat{\theta}} > 0,\ \frac{\partial g(\hat{\theta},\hat{q})}{\partial\hat{q}} > 0,\ \frac{\partial^2 g(\hat{\theta},\hat{q})}{\partial\hat{q}^2} < 0,\ \frac{\partial^2 g(\hat{\theta},\hat{q})}{\partial\hat{\theta}\partial\hat{q}} > 0,\ \frac{\partial}{\partial\hat{\theta}}\left[\frac{\partial^2 g(\hat{\theta},\hat{q})}{\partial\hat{q}^2}\right] < 0$$

Again, i's payoff is then determined by a quasilinear utility function

$$V_i(\hat{\theta}_i,\hat{q}_i,\hat{t}_i) = g(\hat{\theta}_i,\hat{q}_i) - \hat{t}_i$$

where $\hat{t}_i$ is the monetary transfer from the agent to the principal.

The principal maximizes the asymmetric profits for each set of perceived types:

$$\begin{aligned}
\max_{\hat{q}_1,\hat{q}_2} \Pi^A &= \hat{p}_1(\hat{t}_1 - c(\hat{q}_1)) + (1-\hat{p}_1)(\hat{t}_2 - c(\hat{q}_2)) \\
S.t.& \\
(IR_i)\ \ g(\hat{\theta}_i,\hat{q}_i) - \hat{t}_i &\geq 0\ \forall i \\
(IC_i)\ \ g(\hat{\theta}_i,\hat{q}_i) - \hat{t}_i &\geq g(\hat{\theta}_i,\hat{q}_j) - \hat{t}_j\ \forall i,j
\end{aligned}$$

In equilibrium, only (IR_1) and (IC_2) are binding. $\Rightarrow$

$$\begin{aligned}
\hat{t}_1 &= g(\hat{\theta}_1,\hat{q}_1) \\
\hat{t}_2 &= g(\hat{\theta}_2,\hat{q}_2) - g(\hat{\theta}_2,\hat{q}_1) + \hat{t}_1 \\
\hat{t}_2 &= g(\hat{\theta}_2,\hat{q}_2) - g(\hat{\theta}_2,\hat{q}_1) + g(\hat{\theta}_1,\hat{q}_1)
\end{aligned}$$

Hence, the monopolist's profit maximizing problem is:

$$\begin{aligned}\max_{\hat{q}_1,\hat{q}_2} \Pi^A &= \hat{p}_1(g(\hat{\theta}_1,\hat{q}_1) - c(\hat{q}_1)) + (1-\hat{p}_1)(g(\hat{\theta}_2,\hat{q}_2) - g(\hat{\theta}_2,\hat{q}_1) + g(\hat{\theta}_1,\hat{q}_1) - c(\hat{q}_2)) \\ &= \hat{p}_1(g(\hat{\theta}_1,\hat{q}_1) - c(\hat{q}_1)) + (1-\hat{p}_1)(g(\hat{\theta}_2,\hat{q}_2) - c(\hat{q}_2)) - (1-\hat{p}_1)(g(\hat{\theta}_2,\hat{q}_1) - g(\hat{\theta}_1,\hat{q}_1))\end{aligned}$$

I define symmetric information profit $\Pi^S(\theta, q) = g(\theta, q)q - c(q)$. Solving the monopolist's general problem using this definition yields the following proposition.

Proposition 3.8. *The optimal information structure is a corner solution of either $\varepsilon^* = 0$ or $\varepsilon^* = \frac{1}{2}$ if the following sufficient condition is satisfied.*

$$\begin{aligned}\frac{\partial^2 \Pi^S(\hat{\theta}_1,\hat{q}_1^*)}{\partial \varepsilon^2} &> 2(1-2p_1)\left[\frac{\partial \Pi^S(\hat{\theta}_2,\hat{q}_2^*)}{\partial \varepsilon} - \frac{\partial \Pi^S(\hat{\theta}_2,\hat{q}_1^*)}{\partial \varepsilon}\right] \\ &\quad -(1-\hat{p}_1)\left[\frac{\partial^2 \Pi^S(\hat{\theta}_2,\hat{q}_2^*)}{\partial \varepsilon^2} - \frac{\partial^2 \Pi^S(\hat{\theta}_2,\hat{q}_1^*)}{\partial \varepsilon^2}\right]\end{aligned}$$

3.4 Perceived Insurance Screening Model

Consider a monopolistic insurance adverse-selection model with an uninformed principal and a population of insurees. There are two possible states of the world, a good state and a bad state. The wealth in the good state is denoted by w_G, and in the bad w_B, where $w_B < w_G$. Each insuree i has a risk type θ_i, which is her true probability to be in a bad state. There are two possible types, $\theta_i \in \{\theta_1, \theta_2\}$. Without loss of generality, let $\theta_1 < \theta_2$ with probability $\{p_1, 1-p_1\}$, respectively and $0 < p_1 < 1$. The insurance company knows only the distributions of types, while each agent i observes a noisy signal $s_i \in \{s_1, s_2\}$ on her true type θ_i. The signal s_i is generated according to the following conditional probability:

$$\Pr(s_1|\theta_1) = \Pr(s_2|\theta_2) = (1-\varepsilon),\ \Pr(s_1|\theta_2) = \Pr(s_2|\theta_1) = \varepsilon$$

Each insuree i, after observing s_i and using ε, forms her perceived type according to Bayes' rule:

$$\hat{\theta}_i = \hat{\theta}(s_i) = \begin{cases} \hat{\theta}_1 = \dfrac{p_1(1-\varepsilon)\theta_1 + (1-p_1)\varepsilon\theta_2}{p_1(1-\varepsilon)+(1-p_1)\varepsilon}, & if\ s_i = s_1 \\ \hat{\theta}_2 = \dfrac{p_1\varepsilon\theta_1 + (1-p_1)(1-\varepsilon)\theta_2}{p_1\varepsilon + (1-p_1)(1-\varepsilon)}, & if\ s_i = s_2 \end{cases}$$

The distribution of perceived types $\hat{\theta}_i \in \{\ \hat{\theta}_1,\ \hat{\theta}_2\}$ with $\hat{\theta}_1 \leq \hat{\theta}_2$, is $\{\hat{p}_1, 1-\hat{p}_1\}$, respectively, where $\hat{p}_1$ denotes the probability of observing the low signal and is defined as:

$$\hat{p}_1 \triangleq p(s_1) \triangleq p_1(1-\varepsilon) + (1-p_1)\varepsilon$$

The uninformed insurance company designs an insurance menu which consists of monetary reimbursement I_k and premium $\hat{t}_k$ packages for each perceived type k. The firm is assumed to have some control over type perception by selecting ε.

After observing her signal and ε, each agent forms her perceived type. According to her perceived type, she selects a package to maximize her (subjective) expected utility, which is:

$$Eu_i(I_k, t_k) = \hat{\theta}_i u(w_B + I_k - t_k) + (1-\hat{\theta}_i)u(w_G - t_k)$$

where $u(\cdot)$ is assumed to be strictly concave.

In the symmetric information case, the company's profit from selling insurance to perceived type $\hat{\theta}_i$ is:

$$\begin{aligned}\Pi^{Sym} &= \hat{\theta}_i w_B + (1-\hat{\theta}_i) w_G - u^{-1}\left(Eu_i(0,0)\right) \\ &= \hat{\theta}_i w_B + (1-\hat{\theta}_i) w_G - u^{-1}\left(\hat{\theta}_i u(w_B) + (1-\hat{\theta}_i) u(w_G)\right)\end{aligned}$$

Analyzing the symmetric information profits as defined above yields the following propositions.

Proposition 3.9. *As ε increases, from 0 to $\frac{1}{2}$, Π^{sym} is increasing and approaches a maximum at $\hat{\theta}_1 = \hat{\theta}_2 = \bar{\theta} \triangleq p_1\theta_1 + (1-p_1)\,\theta_2$.*

Proposition 3.10. *Asymmetric information profits, Π^{Asym}, reaches a maximum at $\varepsilon = \frac{1}{2}$*

In contrast to the price discrimination problem, the symmetric information profits in the insurance problem are concave with respect to θ. This implies that the symmetric information profits increases with ε. Consequently, $\varepsilon^* = \frac{1}{2}$ is the unique solution and insurees get no information in equilibrium.

3.4.1 Discussion

Adding costs

As in the price discrimination example, the study of the insurance example assumes that information related activities are costless. It is interesting to investigate whether or not adding such costs will change the above results. The driving forces behind these qualitative results are the increase in symmetric information profits and the reduction in asymmetric information costs as ε increases. Therefore, adding costs such that the structure of the problem in θ stays unchanged [5] maintains the qualitative results. Again, examples of such cost functions are fixed costs (unchanged with ε) and costs that increase with information accuracy.

Market Structure

The analysis of changing the market structure for the insurance example is also very similar to the one presented in section 3.2.3. Again, there will be either multiple informational equilibria (possibly a continuum) or a unique informational pooling equilibria. In contrast to the traditional insurance case, an informational pooling equilibrium is possible in both cases.

Acquiring information

As discussed in section 3.2.3, if the consumers were to pay the firm for better information, they would pay up to the change in information costs moving from $\varepsilon = 0$ to $\varepsilon = \frac{1}{2}$. However, in the insurance case at $\varepsilon = \frac{1}{2}$, the firm enjoys both the reduction in information costs and an increase in symmetric information. Therefore, even if consumers are allowed to pay the firm for better information, perfect information equilibria is not possible and the previous solution remains.

3.5 Conclusions

The examples in this essay show that inspite of assuming Bayesian agents, introducing imperfect information into monopolistic screening models generates incentives for a monop-

[5] Profits must remain non-negative for some values of $\varepsilon \in [0, 1]$ or otherwise the firm will shut down.

olist to distort the information that agents hold. The analysis reveals that the incentive for distorting information exist because information costs decrease as information becomes more symmetric.

In the insurance adverse selection example, this incentive is complemented by an increase in the symmetric information profits as agents' information is reduced. The unique solution is therefore no information. Allowing the insurees to pay for better information will not change the solution. Having a competitive market can have other informational equilibria, but will always contain an informational pooling equilibrium.

In the price discrimination case, the firm's symmetric information profit declines as agents have less information. Therefore, the firm balances two contradicting forces: the change in information costs tends toward no information while the change in symmetric information profit tends towards full information. This leads to one of two possible corner solutions: (1) either full information or (2) no information. The exact solution depends on the relationship between the private values of the two types of consumers. Allowing for the consumers to purchase better information can lead to a unique full information equilibrium. In a competitive market setting, an informational pooling equilibrium always exists. However, it may or may not be unique.

The difference between the two examples is due to a property of the symmetric information profit function. In the price discrimination setting, the symmetric profit is convex with respect to ε while in the insurance model it is concave. The concavity/convexity property is directly generated from the monopolist's degrees of freedom in designing a symmetric contract. In the price discrimination example, the firm tailors the symmetric information quantity and transfer for each perceived type. In the insurance symmetric case, once an agent forms her type perception, the cost of supplying insurance is fixed and thus the optimal insurance is uniquely determined (for each utility function). Note that in both price discrimination and insurance models information distortion could prevail when informational activity costs and different market structure are introduced.

Natural extensions of this essay include looking for incentives a firm has to influence agents' information when agents are assumed to be systematically biased.

Appendix

Proof of Lemma 1.1. Recall that I^* satisfies condition (1.1), denoted below as (RA). By the Implicit Function Theorem and assumption 2, we get:

$$\frac{\partial I^*}{\partial \beta} = -\frac{\partial RA/\partial \beta}{\partial RA/\partial I^*} = -\frac{U'(w_1 + (1-\gamma)I^*)(1-\gamma) + U'(w_2 - \gamma I^*)\gamma}{\beta U''(w_1 + (1-\gamma)I^*)(1-\gamma)^2 + (1-\beta)U''(w_2 - \gamma I^*)\gamma^2} > 0$$

Note that by assumption 2 $U''(\cdot) < 0$, and for all values of $I \in (-\infty, \infty)$, $U'(\cdot) > 0$. In addition, by assumption 2 and condition (1.1) as $\beta \to 0$, $I^* \to -\infty$, as $\beta \to 1$, $I^* \to \infty$. It is straight-forward to check that if $\beta \gtreqless \gamma$ then the optimal insurance level is $I^*(\beta) \gtreqless z$. $\square$

Proof of Lemma 1.2. Recall that β^* satisfies condition (1.2), denoted below as (M). By Assumptions 2, 3, and the Implicit Function Theorem:

$$\frac{\partial \beta^*}{\partial I} = -\frac{\partial M/\partial I}{\partial M/\partial \beta^*} = -\frac{U'(w_1 + (1-\gamma)I)(1-\gamma) + U'(w_2 - \gamma I)\gamma}{-\frac{\partial^2 f(\beta^*;\beta_0)}{\partial \beta^2}} > 0$$

By condition (1.2) if $I = z$, $f'(\cdot) = 0 \Rightarrow \beta = \beta_0$. $\square$

Proof of Lemma 1.3. The proof of Lemma 1.2 explicitly gives $\frac{\partial \beta^*}{\partial I}$. Taking its derivative we obtain:

$$\frac{\partial^2 \beta^*}{\partial I^2} = \frac{\partial \left[\frac{U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma}{\frac{\partial^2 f(\beta^*;\beta_0)}{\partial \beta^2}} \right]}{\partial I}$$

Recall that β^* is a function of I.

Taking the derivatives, and rearranging implies that $\frac{\partial^2 \beta^*}{\partial I^2} \gtreqless 0 \Leftrightarrow$

$$\frac{\frac{\partial\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right]}{\partial I}}{\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right]^2} \gtreqless \frac{\frac{\partial^2 f'(\beta^*;\beta_0)}{\partial \beta^2}}{\left[\frac{\partial f'(\beta^*;\beta_0)}{\partial \beta}\right]^2}$$

□

Proof of Proposition 1.1. By Assumption 5 all Nash equilibria constitute of beliefs $\beta \in (\underline{\beta}, \overline{\beta})$, where $0 < \underline{\beta} < \overline{\beta} < 1$. Moreover, we know that all pure strategy Nash equilibria of this game will have insurance level in the interval $\left[I^*(\underline{\beta}), I^*(\overline{\beta})\right]$. That, in turns, implies that all Nash equilibria will have perceived probabilities in the interval $\left[\beta^*\left(I^*(\underline{\beta})\right), \beta^*\left(I^*(\overline{\beta})\right)\right]$ where $0 < \underline{\beta} < \beta^*\left(I^*(\underline{\beta})\right) < \beta^*\left(I^*(\overline{\beta})\right) < \overline{\beta} < 1$. Define $\beta^*\left(I^*(\underline{\beta})\right) \equiv \underline{\beta}'$, $\beta^*\left(I^*(\overline{\beta})\right) \equiv \overline{\beta}'$; since all the Nash equilibria of the intrapersonal game for $\beta \in (\underline{\beta}, \overline{\beta})$ are within $\beta \in \left[\underline{\beta}', \overline{\beta}'\right]$ we can focus on the latter probability space.

Definition A1 Define $\tilde{I}(\beta; \beta_0)$ to be the insurance level such that $\beta^*(\tilde{I}; \beta_0) = \beta$ for some $\beta \in \left[\underline{\beta}', \overline{\beta}'\right]$. That is, $\tilde{I}(\beta; \beta_0)$ is the inverse function β^{*-1}

Definition A2 Define $\Pi(\beta; \beta_0) = I^*(\beta) - \tilde{I}(\beta; \beta_0)$, $\Pi : \left[\underline{\beta}', \overline{\beta}'\right] \to R$

Existence —We know that $\tilde{I}(\underline{\beta}'; \beta_0) < I^*(\underline{\beta}') < I^*(\overline{\beta}') < \tilde{I}(\overline{\beta}'; \beta_0)$, since $\tilde{I}(\underline{\beta}'; \beta_0) = I^*(\underline{\beta})$ and $\tilde{I}(\overline{\beta}'; \beta_0) = I^*(\overline{\beta})$, $\underline{\beta} < \underline{\beta}' < \overline{\beta}' < \overline{\beta}$ and $I^*(\cdot)$ is strictly increasing in β. Consequently $\Pi(\underline{\beta}'; \beta_0) > 0$ and $\Pi(\overline{\beta}'; \beta_0) < 0$. Since both $\tilde{I}(\beta; \beta_0)$ and $I^*(\beta)$ are continuous in β, then $\Pi(\beta;\beta_0)$ is continuous in β $\forall\beta_0 \in (\underline{\beta},\overline{\beta})$. Fix some $\beta_0 \in (\underline{\beta},\overline{\beta})$. Having a continuous function Π which is $\Pi(\underline{\beta}';\beta_0) > 0$ and $\Pi(\overline{\beta}';\beta_0) < 0$ guarantees a solution $\Pi(\beta;\beta_0) = 0$ in the $(\underline{\beta}',\overline{\beta}')$ interval, which is a pure strategy Nash equilibrium of the intrapersonal game as it is a point of intersection of the two best responses.

Lowest and highest argument — By the boundary conditions $\Pi(\underline{\beta}';\beta_0) > 0$ and $\Pi(\overline{\beta}';\beta_0) < 0$, $\beta_L(\beta_0) \equiv \inf\{\beta|\Pi(\beta;\beta_0) \leq 0\}$, $\beta_H(\beta_0) \equiv \sup\{\beta|\Pi(\beta;\beta_0) \geq 0\}$ exist. One needs to show that $\beta_L(\beta_0), \beta_H(\beta_0)$ are solutions. Consider $\beta_L(\beta_0)$. By definition of β_L, the $\limsup$ of $\Pi(\beta;\beta_0)$ as $\beta \uparrow \beta_L$ is nonnegative. Thus, $\Pi(\beta_L(\beta_0),\beta_0) \geq 0$. If $\beta_L(\beta_0) = \overline{\beta}'$, then $\Pi(\overline{\beta}';\beta_0) < 0$ — contradiction. If $\beta_L(\beta_0) < \overline{\beta}'$, and $\Pi(\beta_L(\beta_0);\beta_0) > 0$ then continuity implies that there is some $\varepsilon > 0$ such that $\Pi(\beta_L(\beta_0) + \varepsilon;\beta_0) > 0$ $\forall\beta \in [\beta_L(\beta_0), \beta_L(\beta_0) + \varepsilon]$ which is a contradiction to the definition of $\beta_L(\beta_0)$. Therefore, the conclusion is that $\Pi(\beta_L(\beta_0);\beta_0) = 0$. The case of $\beta_H(\beta_0)$ can be proved to be a solution by similar arguments.

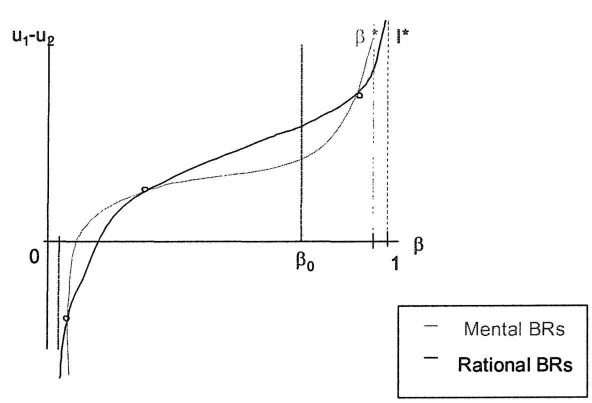
Odd number of equilibria — Suppose there are two equilibria points $\beta_1 < \beta_2$. By the boundaries conditions, in the neighborhood of $\beta_1(\beta_0)$ $\Pi(\beta_1-\varepsilon;\beta_0) > 0$ and $\Pi(\beta_1+\varepsilon;\beta_0) < 0$. However, since there is only one more equilibrium point and Π is continuous then it must be that in the neighborhood of $\beta_2(\beta_0)$, $\Pi(\beta_2 - \varepsilon;\beta_0) < 0$ and $\Pi(\beta_2 + \varepsilon;\beta_0) > 0$ which is a contradiction to the boundaries conditions. This argument can be repeated for any case of even number of equilibrium points.

A chain — Note that the set of equilibrium points is defined as $\beta^{NE} \equiv \{\beta|\Pi(\beta;\beta_0) = 0\}$. Thus, the equilibrium points are beliefs $\beta \in \left[\underline{\beta}',\overline{\beta}'\right]$ which form a chain by definition. Since $I^*(\beta)$ is an increasing function, then it follows that the Nash equilibria of intrapersonal game which are vectors (I,β) form a chain.

Equilibrium Type — Recall the definition of the prospective adjustment process: $h = I^* \circ \beta^*$, where β^* is the mental BR and I^* is the rational BR. Given an insurance point I, β^* moves first and I^* second such that $h : R \rightarrow R$. A Nash equilibrium is of P-type iff the prospective adjustment process converges to it. In other words, take a NE $\left(I^{NE}, \beta^{NE}\right)$. In the neighborhood of this equilibrium point it must be the case that for $I < I^{NE}$, $\dot{h} > 0$ and $I > I^{NE}$, $\dot{h} < 0$. This is equivalent to requiring that the slope of $\frac{\partial \tilde{I}(\beta^{NE};\beta_0)}{\partial \beta} > \frac{\partial I^*(\beta^{NE})}{\partial \beta}$ which is equivalent to $\Pi(\beta^{NE} - \varepsilon; \beta_0) > 0$ and $\Pi(\beta^{NE} + \varepsilon; \beta_0) < 0$. By definition, then, β_L and β_H are of P-type. A Nash equilibrium is of R-type iff the retrospective adjustment process converges to it. In other words, take a NE $\left(I^{NE}, \beta^{NE}\right)$. In the neighborhood of this equilibrium point it must be the case that for $\beta < \beta^{NE}$, $\dot{h}^{-1} > 0$ and $\beta > \beta^{NE}$, $\dot{h}^{-1} < 0$. This is equivalent to requiring that the slope of $\frac{\partial \tilde{I}(\beta^{NE};\beta_0)}{\partial \beta} < \frac{\partial I^*(\beta^{NE})}{\partial \beta}$ which is equivalent to $\Pi(\beta^{NE} - \varepsilon; \beta_0) < 0$ and $\Pi(\beta^{NE} + \varepsilon; \beta_0) > 0$. By continuity of Π, then, the NE alternate from being of P-type to R-type.

Note that the existence, lowest and highest Nash equilibria and the chain results can be proved by defining a restricted intrapersonal game where the insurance pure-strategy space is restricted between $[I^*(\underline{\beta}), I^*(\overline{\beta})]$ and the perceived probabilities are restricted to $\beta \in \left[\underline{\beta}', \overline{\beta}'\right]$ such that the equilibria points of the intrapersonal game are not altered. The restricted game can be shown to be a supermodular game and thus these results follow from the properties of this class of games (see Topkis, 1998).

For the case of a logaritmic utility function. Plot the rational and mental accounts' best responses on the $(u_1 - u_2) \times\beta$ space.



(Figure A1)

This reveals the same geometry as in the original problem. Therefore, one can construct an analogeous argument for this proposition and prove existence for the case of logaritmic utility function. □

Proof of Proposition1.2. Denote the rational account's payoff function as (R) and the mental account's payoff function as (M). A necessary and sufficient condition for the intrapersonal game to have a potential (Monderer and Shapley, 1996) is

$$\frac{\partial^2 R}{\partial\beta\partial I} = \frac{\partial^2 M}{\partial\beta\partial I}$$

This is clearly satisfied in the ADM model. To find the potential, we need to find a function

$P(\beta, I)$ such that (Monderer and Shapley, 1996):

$$\frac{\partial P}{\partial \beta} = \frac{\partial M}{\partial \beta}$$
$$\frac{\partial P}{\partial I} = \frac{\partial R}{\partial I}$$

Since $\frac{\partial M}{\partial I} = \frac{\partial R}{\partial I}$ we see that (M) can serve as a potential.The critical points of the potential are

$$\frac{\partial P}{\partial \beta} = \frac{\partial M}{\partial \beta} = 0$$
$$\frac{\partial P}{\partial I} = \frac{\partial R}{\partial I} = 0$$

Since the potential is strictly concave in each variable, then we know that at each critical point each account is maximizing its objective function, given the strategy of the other account. Therefore, the critical points of the potential are the pure strategy Nash equilibria of the intrapersonal game, and all pure strategy Nash equilibria are critical points of the potential. □

Proof of Proposition 1.3. Notice that the mental account's objective function

$$\beta U(w_1 + (1-\gamma)I) + (1-\beta)U(w_2 - \gamma I) - f(\beta;\beta_0) \tag{P}$$

is the potential function of our game. That is, $\frac{\partial P}{\partial I} = 0$ is condition (1.1), and $\frac{\partial P}{\partial \beta} = 0$ is condition (1.2). Therefore the maximization of (P) with respect to a pair (I, β) gives rise to a pure strategy Nash equilibria of the game. Since all Nash equilibria of the intrapersonal game are within $\beta \in \left[\underline{\beta}', \overline{\beta}'\right]$ and $I \in \left[I^*(\underline{\beta}'), I^*(\overline{\beta}')\right]$ (see Proof of Proposition 1.1), then one can restrict attention to this restricted intrapersonal game where both players' strategy space are compact. By Neyman (1997), a potential game with a strictly concave, smooth potential function, where all players have a compact and convex strategy space, has a unique pure strategy Nash equilibrium. That is, the Hessian is negative definite, which is given by the condition below.

$$\frac{\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right]^2}{-\left[\beta U''(w_1+(1-\gamma)I)(1-\gamma)^2+(1-\beta)U''(w_2-\gamma I)\gamma^2\right]}<\frac{\partial^2 f(\beta;\beta_0)}{\partial\beta^2}$$

$\forall\,(I,\beta)\in\left[I^*(\underline{\beta}'),I^*(\overline{\beta}')\right]\times\left[\underline{\beta}',\overline{\beta}'\right]$ □

Proof of 1.4. Consider a normal form potential game with $N=2$ players $j\in\{1,2\}$, who have payoff function u_1,u_2, and action set X_1,X_2, respectively. The potential function is denoted by (P). The pure strategy Nash equilibria are $\{x_1,x_2\}\in X_1\times X_2$ such that $\frac{\partial u_j}{\partial x_j}=\frac{\partial P}{\partial x_j}=0$ for $j\in\{1,2\}$. That is, a critical point of the potential function. Suppose, wlog, that the potential (P) is the objective function of player 1, i.e., $u_1=(P)$. Consider a sequential play where player 1 moves first, and player 2 second. Clearly player 1 wants to maximize her objective function, that is (P), taking into account that in period 2 players 2 will maximize her payoffs as well. Therefore, the pure strategy Nash equilibria of the sequential game are $\{x_1,x_2^*(x_1)\}\in X_1\times X_2$ such that $\frac{\partial u_j}{\partial x_j}=\frac{\partial P}{\partial x_j}=0$ for $j\in\{1,2\}$. That is, a global maxima of the potential (P). Clearly, this is a subset of the set of the normal-form pure strategy Nash equilibria. This argument is true for any arbitrary N. □

Proof of Proposition 1.8. As in the proof of Proposition 1.1 a P-type equilibrium is where at (β^{NE},I^{NE}) $\frac{\partial\tilde{I}(\beta^{NE};\beta_0)}{\partial\beta}>\frac{\partial I^*(\beta^{NE})}{\partial\beta}$ or $\frac{\partial I^*(\beta^{NE})}{\partial\beta}\frac{\partial\beta^*(I^{NE})}{\partial I}<1$.

$\frac{\partial I^*(\beta^{NE})}{\partial\beta}\frac{\partial\beta^*(I^{NE})}{\partial I}=$

$$= \frac{U'(w_1+(1-\gamma)I^{NE})(1-\gamma)+U'(w_2-\gamma I^{NE})\gamma}{\beta^{NE}U''(w_1+(1-\gamma)I^{NE})(1-\gamma)^2+(1-\beta^{NE})U''(w_2-\gamma I^{NE})\gamma^2} \frac{U'(w_1+(1-\gamma)I^{NE})(1-\gamma)+U'(w_2-\gamma I^{NE})\gamma}{-\frac{\partial^2 f(\beta^{NE};\beta_0)}{\partial\beta^2}}$$

At $I^{NE} = z$ and $\beta^{NE} = \beta_0 = \gamma$, define $\overline{w} \equiv \beta_0 w_1 + (1-\beta_0)\, w_2$:

$$\begin{aligned}\frac{\partial I^*(\beta)}{\partial \beta}\frac{\partial \beta^*(I)}{\partial I} &= -\frac{U'(\overline{w})^2}{\beta_0(1-\beta_0)U''(\overline{w})\frac{\partial^2 f(\beta_0;\beta_0)}{\partial\beta^2}} \\ &= \frac{U'(\overline{w})}{\beta_0(1-\beta_0)r(\overline{w})\frac{\partial^2 f(\beta_0;\beta_0)}{\partial\beta^2}} < 1\end{aligned}$$

Rearranging:

$$\frac{U'(\overline{w})}{r(\overline{w})} < \frac{\partial^2 f(\beta_0;\beta_0)}{\partial\beta^2}\beta_0(1-\beta_0)$$

□

Proof of Lemma 1.4. This proof is using the properties of the mental account's best response and the submodularity of the mental cost function (assumption 4).

Recall that the choice of risk perception satisfies equation (1.2), denoted below as (M). Therefore, if the mental cost function $f(\beta;\beta_0)$ is submodular in β_0, i.e., $\frac{\partial^2 f(\beta;\beta_0)}{\partial\beta\partial\beta_0} \leq 0$, and strictly convex in β, then:

$$\frac{\partial \beta^*}{\partial \beta_0} = -\frac{\frac{\partial M}{\partial \beta_0}}{\frac{\partial M}{\partial \beta^*}} = -\frac{-\frac{\partial^2 f(\beta^*;\beta_0)}{\partial\beta\partial\beta_0}}{-\frac{\partial^2 f(\beta^*;\beta_0)}{\partial\beta^2}} = \frac{-\frac{\partial^2 f(\beta^*;\beta_0)}{\partial\beta\partial\beta_0}}{\frac{\partial^2 f(\beta^*;\beta_0)}{\partial\beta^2}} \geq 0$$

Furthermore, if $\frac{\partial^2 f(\beta;\beta_0)}{\partial\beta\partial\beta_0} < 0$ then

$$\frac{\partial \beta^*}{\partial \beta_0} = -\frac{\frac{\partial M}{\partial \beta_0}}{\frac{\partial M}{\partial \beta^*}} = -\frac{-\frac{\partial^2 f(\beta^*;\beta_0)}{\partial\beta\partial\beta_0}}{-\frac{\partial^2 f(\beta^*;\beta_0)}{\partial\beta^2}} = \frac{-\frac{\partial^2 f(\beta^*;\beta_0)}{\partial\beta\partial\beta_0}}{\frac{\partial^2 f(\beta^*;\beta_0)}{\partial\beta^2}} > 0$$

□

Proof of Proposition 1.9. From Lemma 1.4, $\beta^*(I;\beta_0)$ is weakly increasing in β_0. Therefore

$\tilde{I}(\beta;\beta_0)$,the inverse function, is weakly decreasing in β_0, leading $\Pi(\beta;\beta_0) = I^*(\beta) - \tilde{I}(\beta;\beta_0)$ to be weakly increasing in β_0. Using Lemma 1 in Milgrom and Roberts [1994], provided below, one can conclude that the P-type Nash equilibria are weakly increasing in β_0. To see this, for every P-type equilibrium one can find a local game that admits the same boundary conditions as the entire game and the equilibrium we consider is one of the game's extreme points. For the local game the following define the extreme equilibria $\beta_L(\beta_0) = \inf\{\beta|\Pi(\beta) \leq 0\}$ and $\beta_H(\beta_0) = \sup\{\beta|\Pi(\beta) \geq 0\}$. By Milgrom and Roberts these are weakly increasing in β_0. Suppose $\beta^*(I;\beta_0)$ is strictly increasing in β_0, then $\tilde{I}(\beta;\beta_0)$ is strictly decreasing in β_0. Then $\Pi(\beta;\beta_0) = I^*(\beta) - \tilde{I}(\beta;\beta_0)$ is strictly increasing in β_0 meaning that there are no β such that $\Pi(\beta;\beta_0) = \Pi(\beta;\beta_0') = 0$. Using Lemma 1, that means that the extreme Nash equilibria are strictly increasing in β_0. Note that for an equilibrium of R-type one can define a local game where it is one of the extreme equilibria. However, such a local game have the opposite boundary conditions and therefore the results are exactly the opposite.

Lemma 1 [Milgrom and Roberts, 1994]: *Let $X \subset R$ and let $f, g : X \to R$. Suppose that for all $x \in X$, $g(x) \leq f(x)$. Then* $\inf\{x|g(x) \leq 0\} \leq \inf\{x|f(x) \leq 0\}$ *and* $\sup\{x|g(x) \geq 0\} \leq \sup\{x|f(x) \geq 0\}$ □

Proof of Proposition 1.10. Recall that I^* satisfies condition (1.1) denoted below as (RA). By the Implicit Function Theorem, $\frac{\partial I^*}{\partial w_2} = -\frac{\partial RA/\partial w_2}{\partial RA/\partial I^*}$

$$= -\frac{\left[U''(w_2-z+(1-\gamma)I^*)U'(w_2-\gamma I^*)-U'(w_2-z+(1-\gamma)I^*)U''(w_2-\gamma I^*)\right]\left[U'(w_2-\gamma I^*)\right]^2}{\left[U'(w_2-\gamma I^*)\right]^2\left[U''(w_2-z+(1-\gamma)I^*)U'(w_2-\gamma I^*)(1-\gamma)+U'(w_2-z+(1-\gamma)I^*)U''(w_2-\gamma I^*)\gamma\right]}$$

$$\frac{\partial I^*}{\partial w_2} \gtreqless 0 \Leftrightarrow \frac{U''(w_2-z+(1-\gamma)I^*)}{U'(w_2-z+(1-\gamma)I^*)} \gtreqless \frac{U''(w_2-\gamma I^*)}{U'(w_2-\gamma I^*)}$$

Using definition of absolute risk aversion $r(x) = -\frac{U''(x)}{U'(x)}$

$\frac{\partial I^*}{\partial w_2} \gtreqless 0 \Leftrightarrow r\left(w_2 - z + (1-\gamma)I^*\right) \lesseqgtr r\left(w_2 - \gamma I^*\right)$

Recall that β^* satisfies condition (1.2), denoted below as (M). By the Implicit Function Theorem and assumption 3:

$$\frac{\partial \beta^*}{\partial w_2} = -\frac{\partial M/\partial w_2}{\partial M/\partial \beta^*} = -\frac{U'(w_2-z+(1-\gamma)I)-U'(w_2-\gamma I)}{-\frac{\partial^2 f(\beta^*;\beta_0)}{\partial \beta^2}} \gtreqless 0$$

$$\Rightarrow \frac{\partial \beta^*}{\partial w_2} \gtreqless 0 \Leftrightarrow w_2 - z + (1-\gamma)I \lesseqgtr w_2 - \gamma I \Leftrightarrow I \lesseqgtr z$$

Thus:

	$DARA$	$CARA$	$IARA$
$I<z$	$\frac{\partial I^*}{\partial w_2}<0, \frac{\partial \beta^*}{\partial w_2}>0$	$\frac{\partial I^*}{\partial w_2}=0, \frac{\partial \beta^*}{\partial w_2}>0$	$\frac{\partial I^*}{\partial w_2}>0, \frac{\partial \beta^*}{\partial w_2}>0$
$I=z$	$\frac{\partial I^*}{\partial w_2}=0, \frac{\partial \beta^*}{\partial w_2}=0$	$\frac{\partial I^*}{\partial w_2}=0, \frac{\partial \beta^*}{\partial w_2}=0$	$\frac{\partial I^*}{\partial w_2}=0, \frac{\partial \beta^*}{\partial w_2}=0$
$I>z$	$\frac{\partial I^*}{\partial w_2}>0, \frac{\partial \beta^*}{\partial w_2}<0$	$\frac{\partial I^*}{\partial w_2}=0, \frac{\partial \beta^*}{\partial w_2}<0$	$\frac{\partial I^*}{\partial w_2}<0, \frac{\partial \beta^*}{\partial w_2}<0$

Recall $\Pi(\beta;\beta_0) = I^*(\beta) - \tilde{I}(\beta;\beta_0)$. $\Pi(\underline{\beta}';\beta_0) > 0$ and $\Pi(\overline{\beta}';\beta_0) < 0$ and equilibria of this game is where $\Pi(\beta;\beta_0) = 0$.

	$DARA$	$CARA$	$IARA$
$I<z$	$\Delta\Pi \gtreqless 0$	$\Delta\Pi > 0$	$\Delta\Pi > 0$
$I=z$	$\Delta\Pi = 0$	$\Delta\Pi = 0$	$\Delta\Pi = 0$
$I>z$	$\Delta\Pi \gtreqless 0$	$\Delta\Pi < 0$	$\Delta\Pi < 0$

Using Lemma 1 in Milgrom and Roberts (1994) one can conclude the following for any equilibria of P-type (see proof of Proposition 1.9 above for Lemma 1 and an argument why this holds for any P-type equilibrium) :

	$DARA$	$CARA$	$IARA$
$I<z$	?	$NE\uparrow$	$NE\uparrow$
$I=z$	*unchanged*	*unchanged*	*unchanged*
$I>z$	?	$NE\downarrow$	$NE\downarrow$

Note that if the NE is of R-type then the result is exactly the opposite.

For the second part of the Proposition:

An increase in the shock size z will increase I^* and will decrease β^* as is shown below:

By the Implicit Function Theorem, $\frac{\partial I^*}{\partial z} = -\frac{\partial RA/\partial z}{\partial RA/\partial I^*} =$

$$-\frac{\left[-U''(w_2-z+(1-\gamma)I^*)\right]\left[U'(w_2-\gamma I^*)\right]^2}{\left[U'(w_2-\gamma I^*)\right]\left[U''(w_2-z+(1-\gamma)I^*)U'(w_2-\gamma I^*)(1-\gamma)+U'(w_2-z+(1-\gamma)I^*)U''(w_2-\gamma I^*)\gamma\right]}$$

$$\Rightarrow \frac{\partial I^*}{\partial z} > 0$$

By the Implicit Function Theorem and assumption 3,

$$\frac{\partial \beta^*}{\partial z} = -\frac{\partial M/\partial z}{\partial M/\partial \beta^*} = -\frac{-U'(w_2-z+(1-\gamma)I)}{-\frac{\partial^2 f(\beta^*;\beta_0)}{\partial \beta^2}} < 0$$

Consequently, as z increases both $\tilde{I}(\beta;\beta_0)$ and $I^*(\beta)$ increase and the change in $\Pi(\beta;\beta_0)$ is unclear. Therefore, it is not clear how NE changes with z. □

Proof of Proposition 1.11. Recall that I^* satisfies condition (1.1), and we can rewrite it as $G \equiv \frac{U'(w_1+(1-\gamma)I^*)}{U'(w_2-\gamma I^*)} - \frac{\gamma}{(1-\gamma)}\frac{(1-\beta)}{\beta} = \frac{U'(w_2-z+(1-\gamma)I^*)}{U'(w_2-\gamma I^*)} - \frac{\gamma}{(1-\gamma)}\frac{(1-\beta)}{\beta} = 0$. By the Implicit Function Theorem $\frac{\partial I^*}{\partial \gamma} = -\frac{\partial G/\partial \gamma}{\partial G/\partial I^*}$

$$= \frac{I^*\left[U'(w_2-z+(1-\gamma)I^*)U''(w_2-\gamma I^*)-U''(w_2-z+(1-\gamma)I^*)U'(w_2-\gamma I^*)\right]-\frac{1}{(1-\gamma)^2}\frac{(1-\beta)}{\beta}\left[U'(w_2-\gamma I^*)\right]^2}{-\left[U''(w_2-z+(1-\gamma)I^*)U'(w_2-\gamma I^*)(1-\gamma)+U'(w_2-z+(1-\gamma)I^*)U''(w_2-\gamma I^*)\gamma\right]}$$

Rearranging and using $\frac{U'(w_1+(1-\gamma)I^*)}{U'(w_2-\gamma I^*)} = \frac{\gamma}{(1-\gamma)}\frac{(1-\beta)}{\beta} \Rightarrow$

$$\frac{\partial I^*}{\partial \gamma} \gtreqless 0 \Leftrightarrow I^*\gamma(1-\gamma)\left[\frac{1}{r(w_2-z+(1-\gamma)I^*)} - \frac{1}{r(w_2-\gamma I^*)}\right] \gtreqless 1$$

Recall that I^* satisfies condition (1.2), denoted below as (M). By the Implicit Function Theorem and assumption 3, $\frac{\partial \beta^*}{\partial \gamma} = -\frac{\partial M/\partial \gamma}{\partial M/\partial \beta^*}$

$$= -\frac{U'(w_2-z+(1-\gamma)I)(-I)+U'(w_2-\gamma I)I}{-\frac{\partial^2 f(\beta^*;\beta_0)}{\partial \beta^2}} = -\frac{\left[U'(w_2-\gamma I)-U'(w_2-z+(1-\gamma)I)\right]I}{-\frac{\partial^2 f(\beta^*;\beta_0)}{\partial \beta^2}}$$

$$\Rightarrow \frac{\partial \beta^*}{\partial \gamma} \gtreqless 0 \Leftrightarrow \left[U'(w_2-\gamma I) - U'(w_2-z+(1-\gamma)I)\right] I \gtreqless 0$$

$$\Leftrightarrow \begin{cases} if\ I > 0 & \frac{\partial \beta^*}{\partial \gamma} \gtreqless 0 \Leftrightarrow I \gtreqless z \\ if\ I < 0 & \frac{\partial \beta^*}{\partial \gamma} > 0 \end{cases}$$

Thus:

	$DARA$	$CARA$	$IARA$
$I<0$	$\frac{\partial I^*}{\partial\gamma}?, \frac{\partial\beta^*}{\partial\gamma}>0$	$\frac{\partial I^*}{\partial\gamma}<0, \frac{\partial\beta^*}{\partial\gamma}>0$	$\frac{\partial I^*}{\partial\gamma}<0, \frac{\partial\beta^*}{\partial\gamma}>0$
$0<I<z$	$\frac{\partial I^*}{\partial\gamma}<0, \frac{\partial\beta^*}{\partial\gamma}<0$	$\frac{\partial I^*}{\partial\gamma}<0, \frac{\partial\beta^*}{\partial\gamma}<0$	$\frac{\partial I^*}{\partial\gamma}?, \frac{\partial\beta^*}{\partial\gamma}<0$
$I=z$	$\frac{\partial I^*}{\partial\gamma}<0, \frac{\partial\beta^*}{\partial\gamma}=0$	$\frac{\partial I^*}{\partial\gamma}<0, \frac{\partial\beta^*}{\partial\gamma}=0$	$\frac{\partial I^*}{\partial\gamma}<0, \frac{\partial\beta^*}{\partial\gamma}=0$
$I>z$	$\frac{\partial I^*}{\partial\gamma}?, \frac{\partial\beta^*}{\partial\gamma}>0$	$\frac{\partial I^*}{\partial\gamma}<0, \frac{\partial\beta^*}{\partial\gamma}>0$	$\frac{\partial I^*}{\partial\gamma}<0, \frac{\partial\beta^*}{\partial\gamma}>0$

Recall $\Pi(\beta;\beta_0) = I^*(\beta) - \tilde{I}(\beta;\beta_0)$. $\Pi(\underline{\beta}';\beta_0) > 0$ and $\Pi(\overline{\beta}';\beta_0) < 0$ and equilibria of this game is where $\Pi(\beta;\beta_0) = 0$.

	$DARA$	$CARA$	$IARA$
$I<0$	$\Delta\Pi \gtreqless 0$	$\Delta\Pi \gtreqless 0$	$\Delta\Pi \gtreqless 0$
$0<I<z$	$\Delta\Pi<0$	$\Delta\Pi<0$	$\Delta\Pi \gtreqless 0$
$I=z$	$\Delta\Pi<0$	$\Delta\Pi<0$	$\Delta\Pi<0$
$I>z$	$\Delta\Pi \gtreqless 0$	$\Delta\Pi \gtreqless 0$	$\Delta\Pi \gtreqless 0$

Using Lemma 1 in Milgrom and Roberts (1994) one can conclude the following for any equilibria of P-type (see proof of Proposition 1.9 above for Lemma 1 and an argument why this holds for any P-type equilibrium) :

	$DARA$	$CARA$	$IARA$
$I<0$	?	?	?
$0<I<z$	$NE\downarrow$	$NE\downarrow$	?
$I=z$	$NE\downarrow$	$NE\downarrow$	$NE\downarrow$
$I>z$	?	?	?

Note that if the NE is of R-type, then the result is exactly the opposite . □

Proof of Proposition 2.1. If there exists a nonsatiated differentiable, monotone, concave function that rationalizes the data, then the data satisfies Afriat's inequalities[6] the concavity conditions – conditions (a), (c), and the first order conditions – conditions (b), (d). By Varian (1983) and Chiappori-Rochet (1987), if we have a data set that satisfies the concavity and the first order conditions, then we can find a nonsatiated, differentiable, monotone, concave function that rationalizes the data. The differentiability assumption can be weakened to continuity; since we assumed a continuous concave function, it has subgradient at every point and thus conditions $(a)-(d)$ are satisfied. Consequently, if we have a data generated by the ADM model, then it must satisfy conditions $(a)-(d)$. Moreover, if we have a data set satisfying conditions $(a)-(d)$, one can find a nonsatiated, continuous,monotone,concave utility function $u(\cdot)$ and nonsatiated, continuous, monotone, convex cost function $f(\cdot)$ that rationalize the data. □

Proof of Proposition 2.2. Existence:

Theorem 1.1 assures that for a given insurance premium γ_i and income levels (w_1, w_2) there exist a solution (I_i, β_i).

Refutable:

One needs to show that some observations (γ_i, I_i, β_i) can not be rationalized by the model. Consider the following two observations $(\gamma_1, I_1, \beta_1) = (0.5, 80, 0.2); (\gamma_2, I_2, \beta_2) = (0.4, 110, 0.1)$ and suppose that $(w_1, w_2) = (0, 100)$. Equation (d) of Afriat's inequalities is in this case:

[6]See Afriat (1967) . Two new proofs of Afriat's theorem were recently suggested by Fostel, Scarf and Todd (2003).

$$\begin{aligned}
\left[u\left(0.5*80\right)-u\left(100-0.5*80\right)\right]\left[0.1-0.2\right] &\leq f\left(0.1\right)-f\left(0.2\right) && \text{(A1)}\\
\left[u\left(0.6*110\right)-u\left(100-0.4*110\right)\right]\left[0.2-0.1\right] &\leq f\left(0.2\right)-f\left(0.1\right) && \text{(A2)}\\
\left[u\left(40\right)-u\left(60\right)\right]\left[0.1-0.2\right] &\leq \left[u\left(66\right)-u\left(56\right)\right]\left[0.1-0.2\right] && \text{(A3)}\\
\text{negative} &\geq \text{positive}
\end{aligned}$$

which is impossible . □

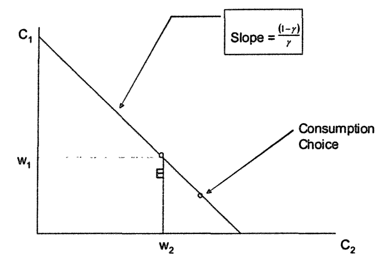
Proof of Proposition 2.3. The proof has two parts: an existence proof where the inequalities have a solution, and an example where the inequalities do not have a solution.

Existence:

Proposition 1.1 assures that for a given insurance premium γ_i and income levels (w_1, w_2) there exist a solution (I_i, β_i) . Given (w_1, w_2) ,γ_i and I_i we have a corresponding consumption levels (c_1, c_2). With (c_1, c_2) and the endowment point, prices are determined and therefore, we establish existence.

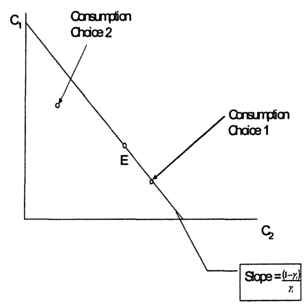
Refutable:

One needs to show that some observations (I_i, γ_i) can not be rationalized by the model. Take the state preference model to represent the rational account choice; observing a consumption choice (which is equivalent to observing (I_i, γ_i)) can be illustrated on a graph:



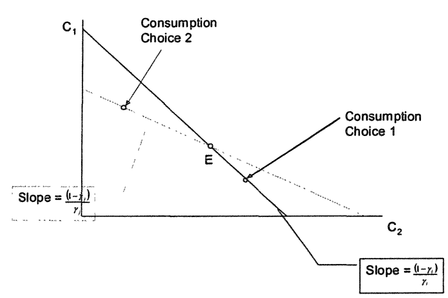
(Figure A2)

observing another choice pair (I_j, γ_j) can be illustrated as follows:



(Figure A3)

Since the endowment point E must lie on the budget line, as well as the consumption choice, then with a single observation point one can find the budget line. This is illustrated below:



(Figure A4)

This is clearly a case that can not be rationalized by an expected utility maximizer agent with constant probabilities, objective or subjective. This can not be rationalized by our model either. To see this note that our model solves:

$$\frac{p_2}{p_1} = \frac{(1-\gamma)}{\gamma} = \frac{(1-\beta_1)}{\beta_1}\frac{u'(c_2)}{u'(c_1)} \tag{A4}$$

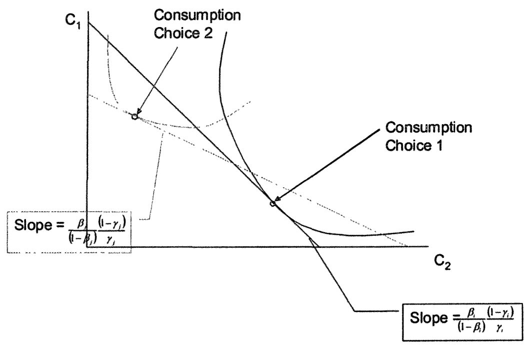
This is similar to the random utility case, taking β_1 to be the random element; Brown and Calsamiglia (2004) show that a random utility model is refutable if and only if the random error is bounded. We shall use their method for our purposes. For that, note that the rational account can be represented as a Bernoulli expected utility maximizer such that

$$u'(c_1) = \frac{\lambda p_1}{\beta_1} \tag{A5}$$

$$u'(c_2) = \frac{\lambda p_2}{(1-\beta_1)} \tag{A6}$$

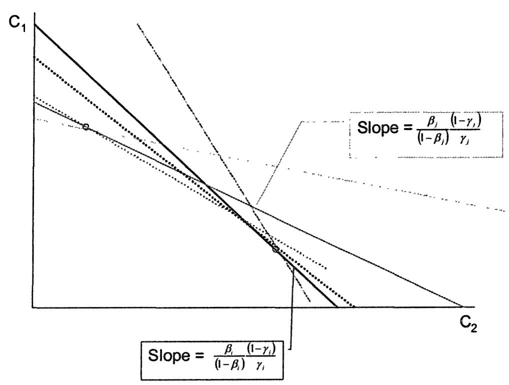
$$\begin{aligned} \frac{u'(c_2)}{u'(c_1)} &= \frac{\lambda p_2 \beta_1}{(1-\beta_1)\lambda p_1} && \text{(A7)} \\ &= \frac{\beta_1}{(1-\beta_1)}\frac{p_2}{p_1} && \text{(A8)} \\ &= \frac{\beta_1}{(1-\beta_1)}\frac{(1-\gamma)}{\gamma} && \text{(A9)} \end{aligned}$$

Thus, the random effect can be "transferred" to the price line, allowing one to illustrated the previous choice as a choice of a Bernoulli agent, as shown in the text and in the following figure:



(Figure A5)

As mentioned, the marginal utilities of the Bernoulli agent is independent of perceived probabilities; instead, the perceived probabilities are embodied in the (perceived) prices the agent is facing. Thus, we need to show that for any set of perceived prices, we contradict the Weak Axiom of Revealed Preference (WARP). See the example illustrated in the Figure below:



(Figure A6)

As mentioned, the model imposes bounds on the perceived probabilities which, in turn, bound the perceived prices and therefore the model can be refuted. Note that only the upper bound on perceived probabilities for choice j, and the lower bound on the perceived probabilities for choice i are binding. $\square$

Proof of Proposition 2.4. Consider two observations (γ, x_1, x_2), $(\hat{\gamma}, \hat{x}_1, \hat{x}_2)$ with associated risk belief $\beta, \hat{\beta}$, respectively; if these observations are choices of some affective agent then we must find numbers $u_1, u_2, \hat{u}_1, \hat{u}_2$, $u_1^{'} \geq 0, u_2^{'} \geq 0, \hat{u}_1^{'} \geq 0, \hat{u}_2^{'} \geq 0$ $f^{'}(\beta)$ that satisfy the rational and the mental accounts' first order conditions, as follows:

$$\beta u_1^{'}(1-\gamma) - (1-\beta)\, u_2^{'}\gamma = 0 \tag{A10}$$

$$\hat{\beta}\hat{u}_1^{'}(1-\hat{\gamma}) - \left(1-\hat{\beta}\right)\hat{u}_2^{'}\hat{\gamma} = 0 \tag{A11}$$

$$u_1 - u_2 - f^{'}(\beta) = 0 \tag{A12}$$

$$\hat{u}_1 - \hat{u}_2 - f^{'}\left(\hat{\beta}\right) = 0 \tag{A13}$$

where $u_i = u(x_i)$, and $f(\cdot)$ is the mental cost function. The observed behavior should also satisfy the Afriat's inequalities:

$$u_1 \leq u_2 + u_2^{'}(x_1 - x_2) \tag{A14}$$

$$u_2 \leq u_1 + u_1^{'}(x_2 - x_1) \tag{A15}$$

$$\hat{u}_1 \leq \hat{u}_2 + \hat{u}_2^{'}(\hat{x}_1 - \hat{x}_2) \tag{A16}$$

$$\hat{u}_2 \leq \hat{u}_1 + \hat{u}_1^{'}(\hat{x}_2 - \hat{x}_1) \tag{A17}$$

$$u_1 \leq \hat{u}_1 + \hat{u}_1^{'}(x_1 - \hat{x}_1) \tag{A18}$$

$$u_1 \leq \hat{u}_2 + \hat{u}_2^{'}(x_1 - \hat{x}_2) \tag{A19}$$

$$u_2 \leq \hat{u}_1 + \hat{u}_1^{'}(x_2 - \hat{x}_1) \tag{A20}$$

$$u_2 \leq \hat{u}_2 + \hat{u}_2^{'}(x_2 - \hat{x}_2) \tag{A21}$$

$$\hat{u}_1 \leq u_1 + u_1^{'}(\hat{x}_1 - x_1) \tag{A22}$$

$$\hat{u}_1 \leq u_2 + u_2^{'}(\hat{x}_1 - x_2) \tag{A23}$$

$$\hat{u}_2 \leq u_1 + u_1^{'}(\hat{x}_2 - x_1) \tag{A24}$$

$$\hat{u}_2 \leq u_2 + u_2^{'}(\hat{x}_2 - x_2) \tag{A25}$$

$$f(\beta) \geq f\left(\hat{\beta}\right) + f^{'}\left(\hat{\beta}\right)(\beta - \hat{\beta}) \tag{A26}$$

$$f\left(\hat{\beta}\right) \geq f(\beta) + f^{'}(\beta)(\hat{\beta} - \beta) \tag{A27}$$

From equation A12, A13 and A26, A27 we have

$$\left(\beta - \hat{\beta}\right)[u_1 - u_2 - \hat{u}_1 + \hat{u}_2] \geq 0 \tag{A28}$$

From A10, A11 we have:

$$\beta = \frac{u_2^{'}\gamma}{u_2^{'}\gamma + u_1^{'}(1-\gamma)}; \hat{\beta} = \frac{\hat{u}_2^{'}\hat{\gamma}}{\hat{u}_2^{'}\hat{\gamma} + \hat{u}_1^{'}(1-\hat{\gamma})} \tag{A29}$$

Conditions A14 to A25 can be reduced to

$$\begin{aligned}
(x_1 - x_2)\left(u_2^{'} - u_1^{'}\right) &\geq 0 \\
(\hat{x}_1 - \hat{x}_2)\left(\hat{u}_2^{'} - \hat{u}_1^{'}\right) &\geq 0 \\
(\hat{x}_1 - x_1)\left(u_1^{'} - \hat{u}_1^{'}\right) &\geq 0 \\
(x_1 - \hat{x}_2)\left(\hat{u}_2^{'} - u_1^{'}\right) &\geq 0 \\
(\hat{x}_1 - x_2)\left(u_2^{'} - \hat{u}_1^{'}\right) &\geq 0 \\
(\hat{x}_2 - x_2)\left(u_2^{'} - \hat{u}_2^{'}\right) &\geq 0
\end{aligned}$$

which can be summaraized as :

$$x_i > x_j \Rightarrow u_i^{'} < u_j^{'} \tag{A30}$$

Therefore, we can restrict attention to conditions $A28, A29$ and $A30$.Now we shall split the observations into three groups of eight rankings:

Group (i)

$$\begin{aligned}
x_1 &> \hat{x}_1 > \hat{x}_2 > x_2 \\
x_1 &> x_2 > \hat{x}_2 > \hat{x}_1 \\
x_1 &> \hat{x}_2 > \hat{x}_1 > x_2 \\
x_1 &> \hat{x}_2 > x_2 > \hat{x}_1 \\
\hat{x}_2 &> \hat{x}_1 > x_1 > x_2 \\
\hat{x}_2 &> x_1 > \hat{x}_1 > x_2 \\
\hat{x}_2 &> x_1 > x_2 > \hat{x}_1 \\
\hat{x}_2 &> x_2 > x_1 > \hat{x}_1
\end{aligned}$$

If there are numbers $u_1^{'} \geq 0, u_2^{'} \geq 0, \hat{u}_1^{'} \geq 0, \hat{u}_2^{'} \geq 0$ that satisfy condition $A30$, then all members of group (i) must satisfy:

$$\frac{u_1^{'}}{u_2^{'}} < \frac{\hat{u}_1^{'}}{\hat{u}_2^{'}}$$

By assumption of the ADM model, each risk belief β is between some bounds $\{\underline{\beta}, \bar{\beta}\}$ where $0 < \underline{\beta} < \bar{\beta} < 1$. Using condition $A29$ we know

$$\begin{aligned}
\underline{\beta} &< \beta = \frac{u_2^{'}\gamma}{u_2^{'}\gamma + u_1^{'}(1-\gamma)} < \bar{\beta} \\
&\Rightarrow \\
\frac{\gamma}{1-\gamma}\frac{1-\bar{\beta}}{\bar{\beta}} &< \frac{u_1^{'}}{u_2^{'}} < \frac{\gamma}{1-\gamma}\frac{1-\underline{\beta}}{\underline{\beta}}
\end{aligned}$$

Similarly,

$$\underline{\beta} \;<\; \hat{\beta} = \frac{\hat{u}_2'\hat{\gamma}}{\hat{u}_2'\bar{\gamma} + \hat{u}_1'(1-\hat{\gamma})} < \bar{\beta}$$

$$\Rightarrow$$

$$\frac{\hat{\gamma}}{1-\hat{\gamma}}\frac{1-\bar{\beta}}{\bar{\beta}} \;<\; \frac{\hat{u}_1'}{\hat{u}_2'} < \frac{\hat{\gamma}}{1-\hat{\gamma}}\frac{1-\underline{\beta}}{\underline{\beta}}$$

if

$$\frac{\hat{\gamma}}{1-\hat{\gamma}}\frac{1-\underline{\beta}}{\underline{\beta}} < \frac{\gamma}{1-\gamma}\frac{1-\bar{\beta}}{\bar{\beta}}$$

then it could not be that

$$\frac{u_1'}{u_2'} < \frac{\hat{u}_1'}{\hat{u}_2'}$$

contradiction.

Now suppose that

$$\frac{\hat{\gamma}}{1-\hat{\gamma}}\frac{1-\underline{\beta}}{\underline{\beta}} > \frac{\gamma}{1-\gamma}\frac{1-\bar{\beta}}{\bar{\beta}}$$

then we can find numbers u_1', u_2' and $\hat{u}_1', \hat{u}_2'$ such that

$$\frac{u_1'}{u_2'} < \frac{\hat{u}_1'}{\hat{u}_2'}$$

This gurantees also that $u_1 - u_2 - \hat{u}_1 + \hat{u}_2 > 0$. Therefore, for the ADM model to be consistent with the data condition $A28$ must be satisfied. That is, we must have $\left(\beta - \hat{\beta}\right) \geq 0$.

$$\begin{aligned}
\beta &= \frac{u_2'\gamma}{u_2'\gamma + u_1'(1-\gamma)} \geq \hat{\beta} = \frac{\hat{u}_2'\hat{\gamma}}{\hat{u}_2'\bar{\gamma} + \hat{u}_1'(1-\hat{\gamma})} \\
\frac{u_2'\gamma}{u_2'\gamma + u_1'(1-\gamma)} &\geq \frac{\hat{u}_2'\hat{\gamma}}{\hat{u}_2'\bar{\gamma} + \hat{u}_1'(1-\hat{\gamma})} \\
u_2'\gamma\left[\hat{u}_2'\hat{\gamma} + \hat{u}_1'(1-\hat{\gamma})\right] &\geq \hat{u}_2'\hat{\gamma}\left[u_2'\gamma + u_1'(1-\gamma)\right] \\
u_2'\gamma\hat{u}_2'\hat{\gamma} + u_2'\gamma\hat{u}_1'(1-\hat{\gamma}) &\geq \hat{u}_2'\hat{\gamma}u_2'\gamma + \hat{u}_2'\hat{\gamma}u_1'(1-\gamma) \\
u_2'\gamma\hat{u}_1'(1-\hat{\gamma}) &\geq \hat{u}_2'\hat{\gamma}u_1'(1-\gamma) \\
\frac{u_1'}{u_2'}\frac{\hat{\gamma}}{(1-\hat{\gamma})} &\leq \frac{\gamma}{(1-\gamma)}\frac{\hat{u}_1'}{\hat{u}_2'}
\end{aligned}$$

This implies that one needs to find numbers u_1', u_2' and $\hat{u}_1', \hat{u}_2'$ such that

$$\frac{u_1'}{u_2'}\frac{\hat{\gamma}}{(1-\hat{\gamma})} \leq \frac{\gamma}{(1-\gamma)}\frac{\hat{u}_1'}{\hat{u}_2'}$$

If $\hat{\gamma} < \gamma$ this is obvious; if $\hat{\gamma} > \gamma$ then one need to find numbers u_1', u_2' and $\hat{u}_1', \hat{u}_2'$ such that

$$\begin{aligned}
\frac{u_1'}{u_2'} &\leq \frac{(1-\hat{\gamma})\gamma}{(1-\gamma)\hat{\gamma}}\frac{\hat{u}_1'}{\hat{u}_2'} \\
&= k\frac{\hat{u}_1'}{\hat{u}_2'}
\end{aligned}$$

which is trivial.

Group (ii)

$$x_1 > x_2 > \hat{x}_1 > \hat{x}_2$$
$$x_1 > \hat{x}_1 > x_2 > \hat{x}_2$$
$$x_2 > x_1 > \hat{x}_2 > \hat{x}_1$$
$$x_2 > \hat{x}_2 > x_1 > \hat{x}_1$$
$$\hat{x}_1 > \hat{x}_2 > x_1 > x_2$$
$$\hat{x}_1 > x_1 > \hat{x}_2 > x_2$$
$$\hat{x}_2 > \hat{x}_1 > x_2 > x_1$$
$$\hat{x}_2 > x_2 > \hat{x}_1 > x_1$$

Suppose the Afriat inequalities are satisfied, for members of group (ii) the sign of $[u_1 - u_2 - \hat{u}_1 + \hat{u}_2]$ is undertermined. Therefore one can always find a set of numbers $u_1, u_2, \hat{u}_1, \hat{u}_2,\ u_1^{'} \geq 0, u_2^{'} \geq 0, \hat{u}_1^{'} \geq 0, \hat{u}_2^{'} \geq 0\ f^{'}(\beta)$ that will satisfy conditions $A28, A29$ and $A30$. That is, the data is always rationalized by the ADM model.

Group (iii)

$$\begin{aligned}
x_2 &> x_1 > \hat{x}_1 > \hat{x}_2 \\
x_2 &> \hat{x}_1 > x_1 > \hat{x}_2 \\
x_2 &> \hat{x}_1 > \hat{x}_2 > x_1 \\
x_2 &> \hat{x}_2 > \hat{x}_1 > x_1 \\
\hat{x}_1 &> \hat{x}_2 > x_2 > x_1 \\
\hat{x}_1 &> x_2 > x_1 > \hat{x}_2 \\
\hat{x}_1 &> x_2 > \hat{x}_2 > x_1 \\
\hat{x}_1 &> x_1 > x_2 > \hat{x}_2
\end{aligned}$$

If condition A30 holds, then all observation that belongs to this group satisfies:

$$\frac{u_1^{'}}{u_2^{'}} > \frac{\hat{u}_1^{'}}{\hat{u}_2^{'}}$$

That implies that $u_1 - u_2 - \hat{u}_1 + \hat{u}_2 < 0$. Therefore, by similar arguments as used for group (i) one can conclude that this group can be rationalized iff $\frac{\hat{\gamma}}{(1-\hat{\gamma})}\frac{\underline{\beta}}{1-\underline{\beta}} < \frac{\gamma}{(1-\gamma)}\frac{\bar{\beta}}{1-\bar{\beta}}$ □

Proof of Proposition 2.5. Consider the case of two agents. Each agent solves the state preference choice problem, as shown above. Thus, each agent solves:

$$\frac{u_i'(c_2)}{u_i'(c_1)} = \frac{\beta_i}{(1-\beta_i)}\frac{(1-\gamma)}{\gamma} \tag{A31}$$

$$\frac{u_j'(d_2)}{u_j'(d_1)} = \frac{\beta_j}{(1-\beta_j)}\frac{(1-\gamma)}{\gamma} \tag{A32}$$

Both agents face the same prices and each agent's consumption bundle must be feasible, i.e.,

$$\gamma c_1 + (1-\gamma) c_2 = y_i \tag{A33}$$

$$\gamma d_1 + (1-\gamma) d_2 = y_j \tag{A34}$$

Each agent's perceived probabilities must satisfy the mental account restriction:

$$u(c_1) - u(c_2) = f'(\beta_i) \tag{A35}$$

$$u(d_1) - u(d_2) = g'(\beta_j) \tag{A36}$$

And market clears

$$c + d = \bar{c} + \bar{d} \tag{A37}$$

where $\bar{c}, \bar{d}$ are the endowment vectors.

Take logarithmic utility functions $u_i(\cdot), u_j(\cdot)$; choose cost functions $f(\cdot), g(\cdot)$ and y_i, y_j, γ. Compute Nash equilibrium for each agent and label it $(c_1^*, c_2^*, \beta_i^*), (d_1^*, d_2^*, \beta_j^*)$. These equilibria satisfies $\gamma c_1^* + (1-\gamma) c_2^* = y_i, \gamma d_1^* + (1-\gamma) d_2^* = y_j$. Construct an Edgeworth box of dimensions $c_1^* + d_1^* = E, c_2^* + d_2^* = F$. Now choose $\bar{c}, \bar{d}$ such that $\gamma \bar{c}_1 + (1-\gamma) \bar{c}_2 = y_i$,

$\gamma\bar{d}_1 + (1-\gamma)\,\bar{d}_2 = y_j$ and $\bar{c}_1 + \bar{d}_1 = E, \bar{c}_2 + \bar{d}_2 = F$. This is

$$\begin{aligned}
\bar{c}_1 &= c_1^* - \varepsilon \\
\bar{d}_1 &= d_1^* + \varepsilon \\
\bar{c}_2 &= c_2^* + \eta \\
\bar{d}_2 &= d_2^* - \eta
\end{aligned}$$

where

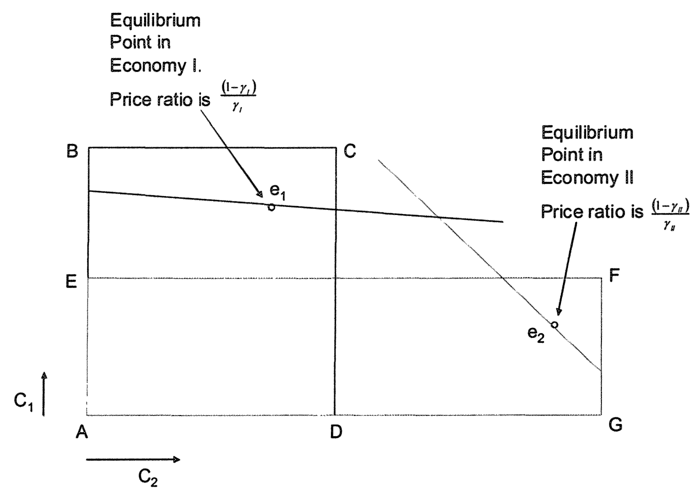
$$0 < \varepsilon < c_1^*$$

$$0 < \eta < d_2^*$$

$$\varepsilon = \frac{(1-\gamma)}{\gamma}\eta$$

□

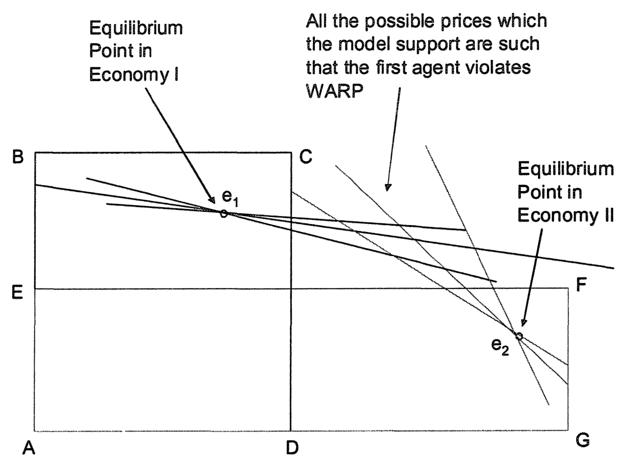
Proof of Proposition 2.6. As Proposition 2.5 shows one can find an example where a mutual insurance equilibrium with affective agents exists. Thus, to complete the proof, one need to show that the model is refutable. We use the example in Brown and Matzkin (1996). Take a logarithmic utility function $u(\cdot)$, and choose cost functions $g(\cdot), f(\cdot)$. Construct two exchange economies, their endowments and equilibrium price ratios. Solve for Nash equilibrium in each economy and construct Edgeworth boxes. Any allocation on the budget line in the Edgeworth box is a possible allocation in this economy, as shown below.



(Figure A7)

The first agent lives at the A vertex in both boxes and the second at vertex C in box (I) and F in box (II). The points e_1, e_2 are the equilibrium point between the two agents; e_1, e_2 are used to construct the boxes and together with prices define the budget line.

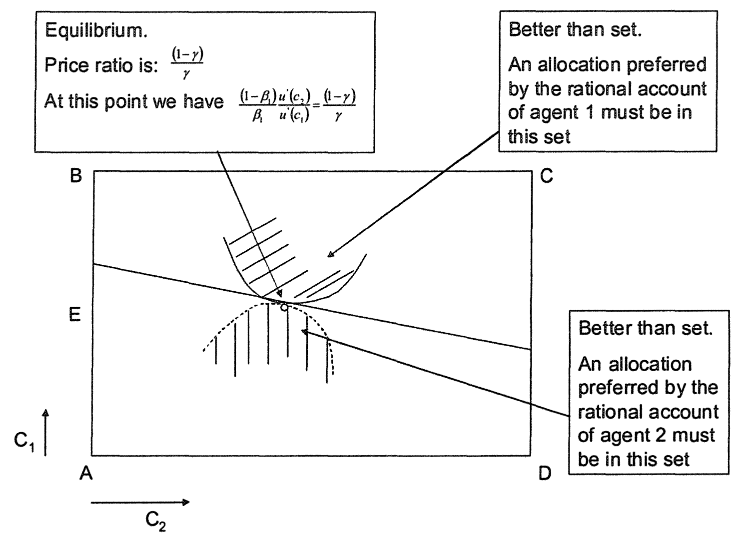
Given that we observe the prices and the endowment, the claim is that one can refute the model. To see that, note that the two budget lines intersect outside the box, which contradicts WARP for agent A. By Afriat's theorem, there is no solution to the inequalities that define solution in this economy. However, this is an example for refuting a pure exchange economy with non affective agents. Having affective agents, one can claim that the individual's perceived prices are different than $\frac{(1-\gamma)}{\gamma}$. However, recall that affective choice imposes bounds on the perceived probabilities, and thus price ratios, that the agents can hold. Thus, there is a bounded price range for agent A in each of the two Edgeworth boxes. If for all such perceived price ratios, the budget lines intersect outside the boxes, we have an example where the model is refutable. Such an example is illustrated below:



(Figure A8)

□

Proof of Proposition 2.7. The proof is by contradiction. Suppose there exists another allocation E which affective-Pareto dominates the affective mutual insurance general equilibrium M, i.e., $E \succ M$. Consider agent $i = 1$; $E \succ_1 M$, means that fixing beliefs and insurance premium at M to be β_1 and γ, respectively, she prefers E to M. Since all possible affective choices, given γ, are on the budget line, E must lie in the better than set, as illustrated in the graph below. Clearly, this must be true also for agent 2, which implies that such allocation E is not feasible in this exchange economy – contradiction. The following is an illustration of our argument:



(Figure A9)

□

Proof of Proposition 3.1. The proof is straight forward. Notice that the symmetric profit put of an individual θ_k is the strictly convex function $\frac{1}{2}\theta_k^2$. The symmetric profit from the entire population is a weighted average of two points on the function $\frac{1}{2}\theta_k^2$, evaluated at $\hat{\theta}_1, \hat{\theta}_2$. As ε increases, $\hat{\theta}_1$ and $\hat{\theta}_2$ converge, but the weighted point $\left[\hat{p}_1\hat{\theta}_1 + (1-\hat{p}_1)\hat{\theta}_2\right]$ does not change. Therefore, one gets that as $\varepsilon \longrightarrow \frac{1}{2}$, profits decreases. □

Proof of Proposition 3.2: Notice that the asymmetric information profits coincide exactly with the symmetric information profits at two points $\varepsilon = \left\{0, \frac{1}{2}\right\}$. For $\varepsilon \in \left(0, \frac{1}{2}\right)$, the profits in the asymmetric information case are always less than or equal to the symmetric information profits due to information rent, thus we can write $\Pi^{sym}(\varepsilon) \geq \Pi^{Asym}(\varepsilon)\ \forall\varepsilon$. Proposition 3.1 tells us that the highest Π^{sym} is at $\varepsilon = 0$ where $\Pi^{sym} = \Pi^{Asym}$, thus one can conclude that the optimal information structure for the principal is $\varepsilon^* = 0$. □

Proof of Proposition 3.3. Since the first order derivative $\dfrac{\partial \Pi^{Asy}}{\partial \varepsilon}$ increases with ε, then the optimal solution is a corner solution $\varepsilon^* = 0$ or $\varepsilon^* = \frac{1}{2}$. Comparing the profits at the two corner solution will determine the optimal solution, i.e.,

$$\begin{cases} \varepsilon^* = 0 \text{ if } \frac{1}{2}\{\theta_1^2 + \frac{(1-P_1)}{P_1}(\theta_2 - \theta_1)^2\} > \frac{1}{2}\bar{\theta}^2 = \frac{1}{2}(P_1\theta_1 + (1-P_1)\theta_2)^2 \\ \varepsilon^* = \left\{0, \frac{1}{2}\right\} \text{ if } \frac{1}{2}\{\theta_1^2 + \frac{(1-P_1)}{P_1}(\theta_2 - \theta_1)^2\} = \frac{1}{2}\bar{\theta}^2 = \frac{1}{2}(P_1\theta_1 + (1-P_1)\theta_2)^2 \\ \varepsilon^* = \frac{1}{2} \text{ if } \frac{1}{2}\{\theta_1^2 + \frac{(1-P_1)}{P_1}(\theta_2 - \theta_1)^2\} < \frac{1}{2}\bar{\theta}^2 = \frac{1}{2}(P_1\theta_1 + (1-P_1)\theta_2)^2 \end{cases}$$

Comparing:

$$\frac{1}{2}\left\{\theta_1^2 + \frac{(1-P_1)}{P_1}(\theta_2 - \theta_1)^2\right\} \gtreqless \frac{1}{2}\{P_1\theta_1 + (1-P_1)\theta_2\}^2$$

$$\left\{\theta_1^2 + \frac{(1-P_1)}{P_1}(\theta_2 - \theta_1)^2\right\} \gtreqless \{P_1\theta_1 + (1-P_1)\theta_2\}^2$$

Plug in $\theta_2 = \lambda\theta_1$

$$\left\{\theta_1^2 + \frac{(1-P_1)}{P_1}(\lambda - 1)^2\theta_1^2\right\} \gtreqless \{P_1\theta_1 + (1-P_1)\lambda\theta_1\}^2$$

$\Leftrightarrow$

$$\lambda^2\left\{2P_1 - 2P_1^2 + P_1^3 - 1)\right\} + \lambda\left\{2P_1^2 - 2P_1^3 + 2 - 2P_1\right\} + P_1^3 - 1 \lesseqgtr 0$$

Solve for λ :

$$\lambda = \frac{-\left[\left(P_1^2+1\right)(1-P_1)\right] \pm P_1\left[1-P_1\right]}{\left[2P_1(1-P_1)+P_1^3-1\right]}$$
$$= \begin{cases} \dfrac{\left[1-P_1\right]\left[P_1-1-P_1^2\right]}{\left[2P_1(1-P_1)+P_1^3-1\right]} > 0 \\ -\dfrac{\left[1-P_1\right]\left[P_1+1+P_1^2\right]}{\left[2P_1(1-P_1)+P_1^3-1\right]} > 0 \end{cases}$$

Notice that since the denominator is negative:

$$\lambda \leq \frac{2P_1-1-2P_1^2+P_1^3}{\left[2P_1(1-P_1)+P_1^3-1\right]} \text{ or}$$
$$\lambda \geq -\frac{\left[1-P_1\right]\left[P_1+1+P_1^2\right]}{\left[2P_1(1-P_1)+P_1^3-1\right]}$$

$$\lambda \leq 1 \text{ or}$$
$$\lambda \geq -\frac{\left[1-P_1\right]\left[P_1+1+P_1^2\right]}{\left[2P_1(1-P_1)+P_1^3-1\right]}$$

$$\Downarrow$$

$$\text{If} \begin{cases} 1 \leq \lambda < -\dfrac{\left[1-P_1\right]\left[P_1+1+P_1^2\right]}{\left[2P_1(1-P_1)+P_1^3-1\right]}, \ \varepsilon^* = \frac{1}{2} \\ \lambda = -\dfrac{\left[1-P_1\right]\left[P_1+1+P_1^2\right]}{\left[2P_1(1-P_1)+P_1^3-1\right]}, \ \varepsilon^* = \left\{0, \frac{1}{2}\right\} \\ \textit{Otherwise}, \ \varepsilon^* = 0 \end{cases}$$ □

Proof of Proposition 3.4: Since the first order derivative is $\dfrac{\partial \Pi^{High}}{\partial \varepsilon} \gtreqless 0 \Leftrightarrow$
$\lambda \lesseqgtr \dfrac{p_1+p_1(1-\varepsilon)-2p_1^2(1-\varepsilon)}{(1-p_1)(1-\varepsilon)+2p_1\varepsilon(1-p_1)} \triangleq c(\varepsilon,p_1)$

and $c(\varepsilon,p_1)$is increasing with ε, then there are 3 possibilities:

$$\begin{cases} \lambda < c(\varepsilon,p_1) \ \ldots\ldots\ldots\ldots\ldots\ldots \ \forall\varepsilon \in [0,\frac{1}{2}] \\ \lambda \gtreqless c(\varepsilon,p_1) \text{ if } \varepsilon \lesseqgtr \bar{\varepsilon} \ for \text{ some } \bar{\varepsilon} \in (0,\frac{1}{2}) \\ \lambda > c(\varepsilon,p_1) \ \ldots\ldots\ldots\ldots\ldots\ldots \ \forall\varepsilon \in [0,\frac{1}{2}] \end{cases} .$$

These cases indicate a corner solution. Thus, the condition for being in either corner solution is achieved by comparing the profits at the two extremes:

$$\frac{1}{2}\{1-P_1\}\theta_2^2 \gtreqless \frac{1}{2}[P_1\theta_1+(1-P_1)\theta_2]^2$$

Plug in $\theta_2 \equiv \lambda\theta_1$:

$$\lambda^2\left[P_1-P_1^2\right]-2\lambda P_1(1-P_1)-P_1^2 \gtreqless 0$$

Solve for λ :

$$\lambda=\frac{(1-P_1)\pm\sqrt[2]{(1-P_1)}}{(1-P_1)}=1\pm\frac{1}{\sqrt[2]{(1-P_1)}}$$

Since $1-\frac{1}{\sqrt[2]{(1-P_1)}}<1$, we can write$\pi(0)\leq\pi(\frac{1}{2})\Leftrightarrow 1\leq\lambda\leq 1+\frac{1}{\sqrt[2]{(1-P_1)}}$ □

Proof of Proposition 3.6. Notice that $\frac{(1-p_1)(1+p_1+p_1^2)}{1-p_1^2-2p_1(1-p_1)}<1+\frac{1}{\sqrt{1-p_1}}$ $\forall p_1\neq 1$. Suppose that λ is such that $\lambda<\frac{(1-p_1)(1+p_1+p_1^2)}{1-p_1^2-2p_1(1-p_1)}<1+\frac{1}{\sqrt{1-p_1}}$, then under both profits $\varepsilon=\frac{1}{2}$ is the optimal information structure. Suppose $\frac{(1-p_1)(1+p_1+p_1^2)}{1-p_1^2-2p_1(1-p_1)}<1+\frac{1}{\sqrt{1-p_1}}<\lambda$ then under both regimes the optimal information structure is $\varepsilon=0$, however in this case the monopolists would sell only to the high perceived type (both is not possible). Suppose $\frac{(1-p_1)(1+p_1+p_1^2)}{1-p_1^2-2p_1(1-p_1)}<\lambda<1+\frac{1}{\sqrt{1-p_1}}$, then the conditions that we got from the profits of selling to both tell us that the optimal information structure is $\varepsilon=0$. However, it is not possible to sell to both at $\varepsilon=0$, and if one considers the profits of selling only to the high perceived type, then the optimal information structure is $\varepsilon=\frac{1}{2}$! To conclude, in the presence of two profits regimes, the optimal information structure is determined as if the monopolist is to sell to the high perceived type only, i.e., by the following condition:

$$if \begin{cases} 1 \leq \lambda \leq 1 + \frac{1}{\sqrt{1-p_1}} & \Rightarrow \varepsilon^* = \frac{1}{2} \\ Otherwise & \Rightarrow \varepsilon^* = 0 \text{ and selling to the high perceived type only} \end{cases}$$

□

Proof of Proposition 3.8. We could write the asymmetric profit in terms of symmetric information profits, as follows:

$$\begin{aligned}
\Pi^A &= \hat{p}_1(g(\hat{\theta}_1, \hat{q}_1^*) - c(\hat{q}_1^*)) + (1-\hat{p}_1)(g(\hat{\theta}_2, \hat{q}_2^*) - g(\hat{\theta}_2, \hat{q}_1^*) + g(\hat{\theta}_1, \hat{q}_1^*) - c(\hat{q}_2^*)) \\
&= \hat{p}_1(g(\hat{\theta}_1, \hat{q}_1^*) - c(\hat{q}_1^*)) + (1-\hat{p}_1)(\Pi^S(\hat{\theta}_2) - g(\hat{\theta}_2, \hat{q}_1^*) + g(\hat{\theta}_1, \hat{q}_1^*)) \\
&= \hat{p}_1\Pi^S(\hat{\theta}_1, \hat{q}_1^*) + (1-\hat{p}_1)g(\hat{\theta}_1, \hat{q}_1^*) + (1-\hat{p}_1)(\Pi^S(\hat{\theta}_2, \hat{q}_2^*) - g(\hat{\theta}_2, \hat{q}_1^*)) \\
&= \hat{p}_1\Pi^S(\hat{\theta}_1, \hat{q}_1^*) + (1-\hat{p}_1)\left[g(\hat{\theta}_1, \hat{q}_1^*) - c(\hat{q}_1^*)\right] + (1-\hat{p}_1)\left[\Pi^S(\hat{\theta}_2, \hat{q}_2^*) - g(\hat{\theta}_2, \hat{q}_1^*) + c(\hat{q}_1^*)\right] \\
&= \hat{p}_1\Pi^S(\hat{\theta}_1, \hat{q}_1^*) + (1-\hat{p}_1)\Pi^S(\hat{\theta}_1, \hat{q}_1^*) + (1-\hat{p}_1)\left[\Pi^S(\hat{\theta}_2, \hat{q}_2^*) - \Pi^S(\hat{\theta}_2, \hat{q}_1^*)\right] \\
&= \Pi^S(\hat{\theta}_1, \hat{q}_1^*) + (1-\hat{p}_1)\left[\Pi^S(\hat{\theta}_2, \hat{q}_2^*) - \Pi^S(\hat{\theta}_2, \hat{q}_1^*)\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Pi^A}{\partial \varepsilon} &= \frac{\partial \Pi^S(\hat{\theta}_1, \hat{q}_1^*)}{\partial \varepsilon} - (1-2p_1)\left[\Pi^S(\hat{\theta}_2, \hat{q}_2^*) - \Pi^S(\hat{\theta}_2, \hat{q}_1^*)\right] \\
&\quad +(1-\hat{p}_1)\left[\frac{\partial \Pi^S(\hat{\theta}_2, \hat{q}_2^*)}{\partial \varepsilon} - \frac{\partial \Pi^S(\hat{\theta}_2, \hat{q}_1^*)}{\partial \varepsilon}\right]
\end{aligned}$$

A sufficient condition for a corner solution is $\dfrac{\partial^2 \Pi^A}{\partial \varepsilon^2} > 0 \Rightarrow$

$$\frac{\partial^2 \Pi^S(\hat{\theta}_1, \hat{q}_1^*)}{\partial \varepsilon^2} - 2(1-2p_1)\left[\frac{\partial \Pi^S(\hat{\theta}_2, \hat{q}_2^*)}{\partial \varepsilon} - \frac{\partial \Pi^S(\hat{\theta}_2, \hat{q}_1^*)}{\partial \varepsilon}\right] + (1-\hat{p}_1)\left[\frac{\partial^2 \Pi^S(\hat{\theta}_2, \hat{q}_2^*)}{\partial \varepsilon^2} - \frac{\partial^2 \Pi^S(\hat{\theta}_2, \hat{q}_1^*)}{\partial \varepsilon^2}\right] > 0$$

□

Proof of Proposition 3.9. Since w_B, w_G are given, once the perceived type is determined the cost are fixed at $u^{-1}\left(\hat{\theta}_i u(w_B) + (1-\hat{\theta}_i)u(w_G)\right)$. The symmetric profits will be the horizontal gap between $\hat{\theta}_i w_B + (1-\hat{\theta}_i)w_G$ and $u^{-1}\left(\hat{\theta}_i u(w_B) + (1-\hat{\theta}_i)u(w_G)\right)$. To find the symmetric profit as a function of θ, one needs to repeat this for all possible $\hat{\theta}_i$. Doing so yields strictly concave symmetric profit function in $\hat{\theta}_i$, which implies that a weighted average of two points on the graph, weighted by $\hat{p}_1$ is increasing as ε increases. To see that, note that the weighted point $\left[\hat{p}_1\hat{\theta}_1 + (1-\hat{p}_1)\hat{\theta}_2\right]$ does not change, but the two perceived types get closer. Since the symmetric profit is strictly concave in $\hat{\theta}_i$, as $\varepsilon \longrightarrow \frac{1}{2}$, profits increases. □

Proof of Proposition 3.10. note that the symmetric information profits are always at least as large as the asymmetric one. At the point $\varepsilon = \frac{1}{2}$, the symmetric information reach a maximum and the two profits coincide. Thus it is the maximum for asymmetric information profits as well. □

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