Clarendon Lectures, Lecture 1

Directed Technical Change: Importance, Issues and Approaches

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Introduction

- New technologies not neutral towards different factors/groups.

  1. recent skill-biased technological changes
  2. many of the technologies of the Industrial Revolution and the early factory system appear to be biased against skilled artisans
  3. standard growth theory: long-run technological change must be labor augmenting.
  4. technological change in continental Europe during the 1980s and the 1990s appears to be capital biased.

Overview

- Lecture 1:
  - motivation and examples
  - summary of the approach here and contrast to alternatives
  - brief discussion of implications and applications
  - illustration with a simple example
  - outline of a simple dynamic general equilibrium approach.
- Lecture 2: Applications to pharmaceutical sector; implications for cross-country income and growth differences; the effect of international trade on innovation; study of the structure and assumptions of basic growth models.
- Lecture 3: General theory of directed technical change.

Understanding the Bias of Technology

- Important for the study of the nature of technology adoption and technological change.
- Also bias and direction of technological change key for **political economy** of technology—determines the distributional implications of technological change.
- Generally recognized that technological progress engine of modern economic growth.
  - But existing approaches focus almost exclusively on "aggregate technological change," ignoring the bias and direction of technology.
  - If technological progress and adoption are endogenous, so should their bias be.
  - Major implications for understanding **economic growth** and **cross-country income differences**.

Approach

- Key ingredient: technologies endogenous and partly driven by profits.
- John Stuart Mill:

  > “ The labor of Watt in contriving the steam-engine was as essential a part of production as that of the mechanics who build or the engineers who work the instrument; and was undergone, no less than theirs, in the prospect of a renumeration from the produces ”

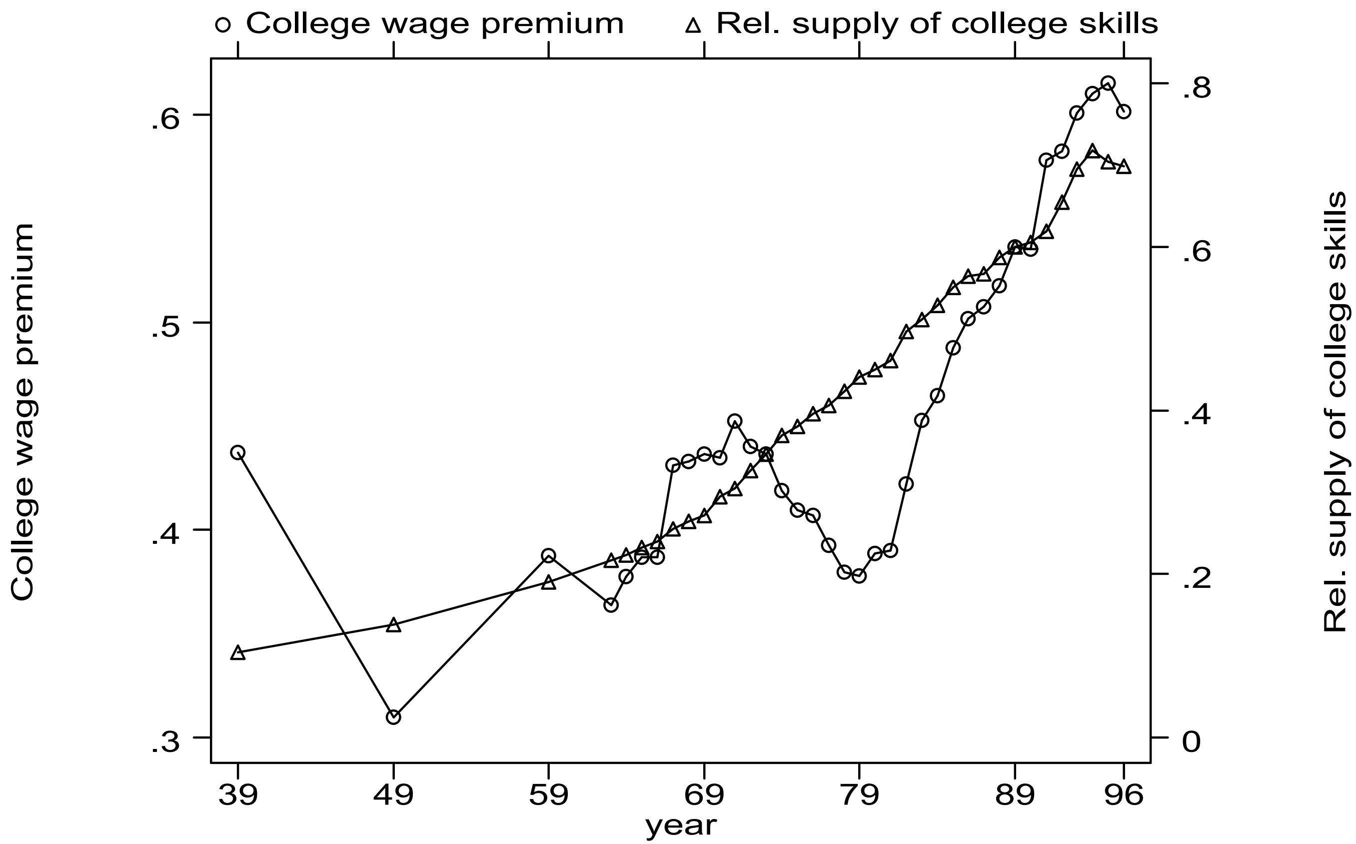
- Schmookler:

  > “ ...invention is largely an economic activity which, like other economic activities, is pursued for gain....”

- All the results in these lectures implications of this perspective.

Example 1: Recent Skill Biased Technological Change

- Evolution of skill premium and inequality in the US labor market.



Relative Supply of College Skills and College Premium

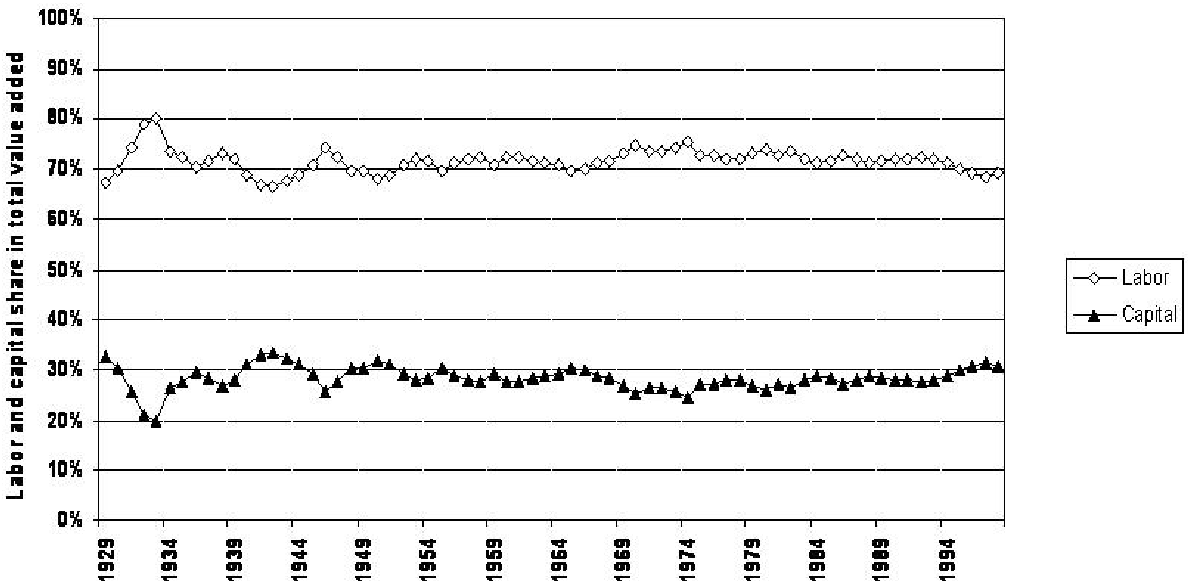
Example 1: Recent Skill Biased Technological Change (continued)

- Large simultaneous increases in relative supply and relative demand.
- Standard story: exogenous skill-biased technological change.
- But why is technological change skill biased?
  - It does not seem to have been so in the 19th century.

    "First in firearms, then in clocks, pumps, locks, mechanical reapers, typewriters, sewing machines, and eventually in engines and bicycles, interchangeable parts technology proved superior and replaced the skilled artisans working with chisel and file." (Mokyr 1990, p. 137)
  - New technologies can be used for scanners as well PCs.
- Should we think of skill bias as **endogenous**?

Example 2: Labor-Augmenting Technological Change

- Capital share in GDP approximately constant in the long run.
- In fact necessary for balanced growth.

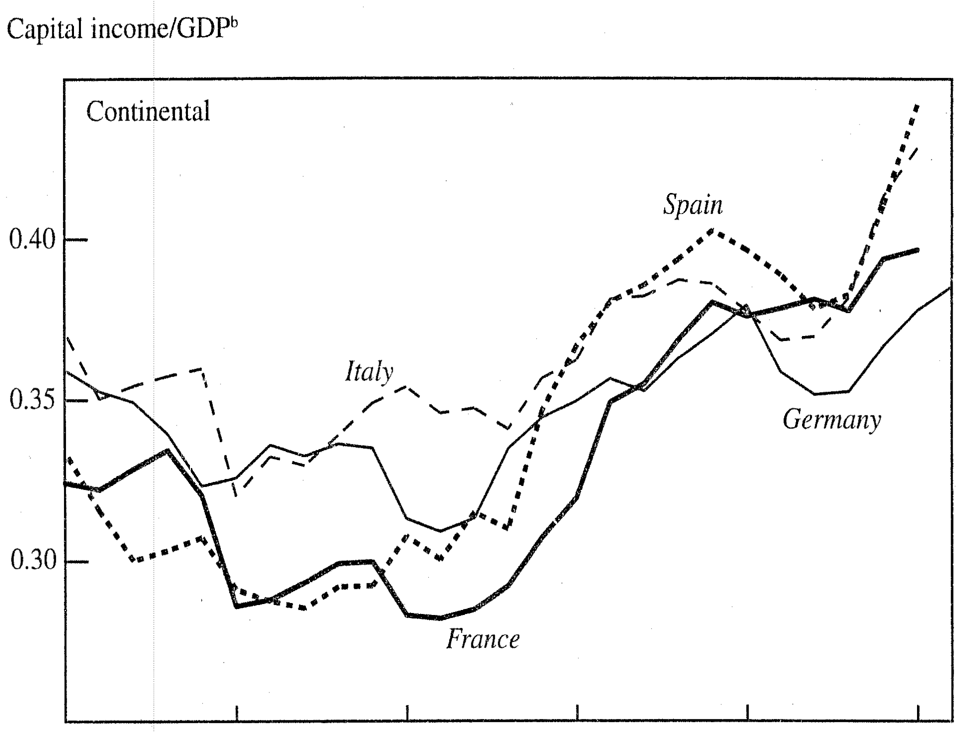


Example 2: Labor-Augmenting Technological Change (continued)

- Standard assumption in growth theory: all technological change labor-augmenting (Harrod-neutral).
- Otherwise, capital share not constant and there would be no balanced growth.
- But why should all technological change take this special form?

Example 3: Capital-Biased Technological Change in Continental Europe

- Large decline in capital share in continental European economies in the 1970s. Blanchard (1997): due to wage push.



Example 3: Capital-Biased Technological Change in Continental Europe (continued)

- However, followed by even a larger increase in capital share.
- Why?
- Possible answer: capital-biased technological change.

Framework

- To investigate these questions, we need a framework in which direction and bias of technological change is endogenous.
- Potentially responding to
  - prices
  - policies
  - market size
  - accumulated knowledge
- The focus of these lectures.

Applications and Implications

- In addition to shedding light on the above examples, new implications from the framework:

  1. Issues of inappropriate technology: why despite free flows of technology, less-developed countries may be "technologically disadvantaged"?
  2. New light on evolution of cross-country income differences and on the causes of the "lost decades" of the 1980s and 1990s.
  3. Will increase international trade affect the nature of technological change?
  4. Habakkuk hypothesis: does wage push create technological improvements?
  5. Why certain types of pharmaceutical innovations have been more likely?

Previous Literature

- Early interest in the study of technological bias spurred by Hicks:

  > "A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind—**directed to economizing the use of a factor which has become relatively expensive**." (pp. 124-5)

- Implicit implication, as a factor becomes more abundant, and thus cheaper, technology should become less biased towards it.
- An important element of the 1960s induced-innovation literature, e.g., Kennedy (1964), Samuelson (1965), Drandakis and Phelps (1965), Ahmad (1966) and David (1975).
- However, models without microfoundations (in fact, not full optimization)
- Consequently, the most important insights missed and some of the conclusions incorrect.

Outline of Argument and Main Results

- Let us define relative bias as the effect of technology on the relative marginal products (relative prices) of two factors, say L and Z.
- Suppose that there are only two factors and only factor-augmenting technologies

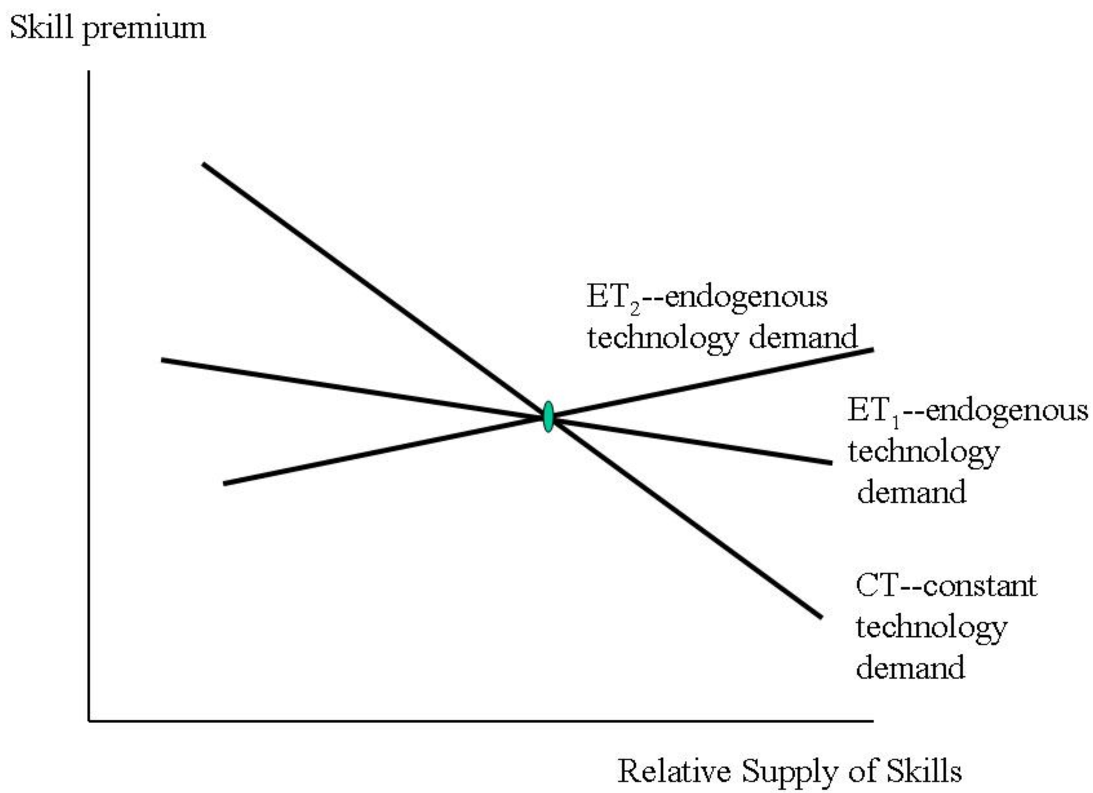
  Result I (Weak Bias Theorem): When the supply of factor Z increases, technology becomes more biased towards factor Z.

  Result II (Strong Bias Theorem): If the elasticity of substitution between the two factors is sufficiently high, an increase in the supply of factor Z makes technology sufficiently biased towards factor Z that the (relative) demand curve for Z becomes upward-sloping.

  Result III (Factor Augmenting Theorem): If the elasticity of substitution between two factors, Z and L, is greater than 1 [less than 1], then an increase in the supply of factor Z should lead to Z-augmenting [L-augmenting] technological change.

Heuristic Illustration

- Short-run (constant technology) relative demand downward sloping.
- Long-run (endogenous technology) relative demand more elastic or upward-sloping.



Intuition

- Key to both the weak and the strong bias theorems is: general equilibrium.
- Greater supply of factor Z implies greater **market size** for technologies working with factor Z.
- This encourages development of further technologies for this factor.
- Counteracting this is the **price effect**
- Goods produced and services supplied factor Z are cheaper and thus there should be less technologies for this factor.
- But the market size effect always dominates the price effect.

Relation to the LeChatelier Principle

- Samuelson's LeChatelier Principle: long-run demand curves more elastic than short-run demand curves
- Because other factors (such as capital) adjust "in the long run".
- Certain degree of similarity: "technology" adjusts in the background and "endogenous technology" $\approx$ "long run"
- Major difference: endogenous bias is a general equilibrium phenomenon, not a result of firm optimization.
- In the LeChatelier Principle, all demand curves are downward-sloping.

Contrast to Previous Literature

- The previous literature mostly about the **price effect**, e.g., Hicks's quote.
- Therefore, the opposite conclusions.
- But different conclusions not due to differences in modeling assumptions.
- Previous literature lacked micro-founded models of technology choice.
- Because once technology is endogenized, there are increasing returns to all factors, technology + usual factors of production.
- Thus with competitive markets, one can only analyze technology choices (aggregate or directed) by making arbitrary assumptions.
- General equilibrium with noncompetitive elements is key to the modern analysis of endogenous bias.

Implication 1: Skill Bias of Technology in the Long Run

- Weak bias theorem $\Rightarrow$ increases in the supply of skills should be associated with skill-biased technological change.

- Strong bias theorem $\Rightarrow$ increases in the supply of skills can lead to an increase in the skill premium.

- Weak bias theorem $\Rightarrow$ increases in the supply of unskilled labor in 19th-century British cities should be associated with the development of unskilled-biased technologies.

Implication 2: Labor-Augmenting Technological Change

- Elasticity of substitution between capital and labor generally less than 1.
- Factor-augmenting theorem $\Rightarrow$ capital-deepening should be associated with technological change that tends to be labor augmenting.
- Will it be purely labor augmenting (Harrod neutral)?
- Topic for Lecture 2.

Implication 3: Capital-Biased Technological Change in Europe

- An increase in wage push $\Rightarrow$ reduction in employment and reduction in the share of capital in national income (elasticity of substitution between capital and labor < 1).
- Short-run response.
- Weak bias theorem $\Rightarrow$ long-run response will be capital-biased technological change.

Application 1: A Theory of Cross-Country Income Differences

- Most new technologies invented and developed in OECD countries.
- Many of these technologies can then be used by LDCs.
- Factor proportions, in particular, supply of skills, different between the OECD and the LDCs.
- Directed technical change $\Rightarrow$ OECD technologies "optimized" for their factor proportions.
- In particular, these technologies will use and rely on skilled labor, engineers, managers, etc.
- When imported, these technologies will be **inappropriate** to the conditions in the LDCs and thus less productive.
- "Lost decades" potentially related to frontier technologies becoming more skill biased and less appropriate for many LDCs (Lecture 2).

Application 2: International Trade and Technology

- Globalization will affect product and factor prices.
- Price effect $\Rightarrow$ impact on direction of technological change
- Globalization makes labor-intensive products cheaper $\Rightarrow$ skill-biased and capital-biased technological change.
- Consequently: globalization $\Rightarrow$ greater inequality among nations.

Application 3: Demographics and Innovation

- With the baby boom and the baby bust, significant demographic changes $\Rightarrow$ changes in the markets for different types of goods.
- Market size effect $\Rightarrow$ impact on the types of technologies that are developed.
- Example: the pharmaceutical industry (Lecture 2).

Simplified Model

- To provide the main ideas, consider a static model with a simple constant elasticity of substitution production function.
- Generalization to a dynamic model of technology choice and growth (see below).
- More general theorems on the direction of technological change: Lecture 3.

Environment

- Take the aggregate production function as

$$F(Z,L,A_Z,A_L)=\left[\gamma_Z\left(A_Z Z\right)^{\frac{\sigma-1}{\sigma}}+\gamma_L\left(A_L L\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

- Here A_Z and A_L are two separate factor-augmenting technology terms.
- $\gamma_Z, \gamma_L \in (0,1)$
- $\sigma \in (0,\infty)$ is the elasticity of substitution between the two factors.
- When $\sigma=\infty$, the production function is linear. When $\sigma=1$, the production function is Cobb-Douglas. When $\sigma=0$, there is no substitution between the two factors, and the production function is Leontief.
- Let the aggregate supplies of the two factors be $\bar{Z}$ and $\bar{L}$.

Basic Definitions

- A_Z is Z-augmenting and A_L is L-augmenting.
- But A_Z [A_L] **not** Z-biased [L-biased].
- Relative wage equation: relative marginal product of Z given by:

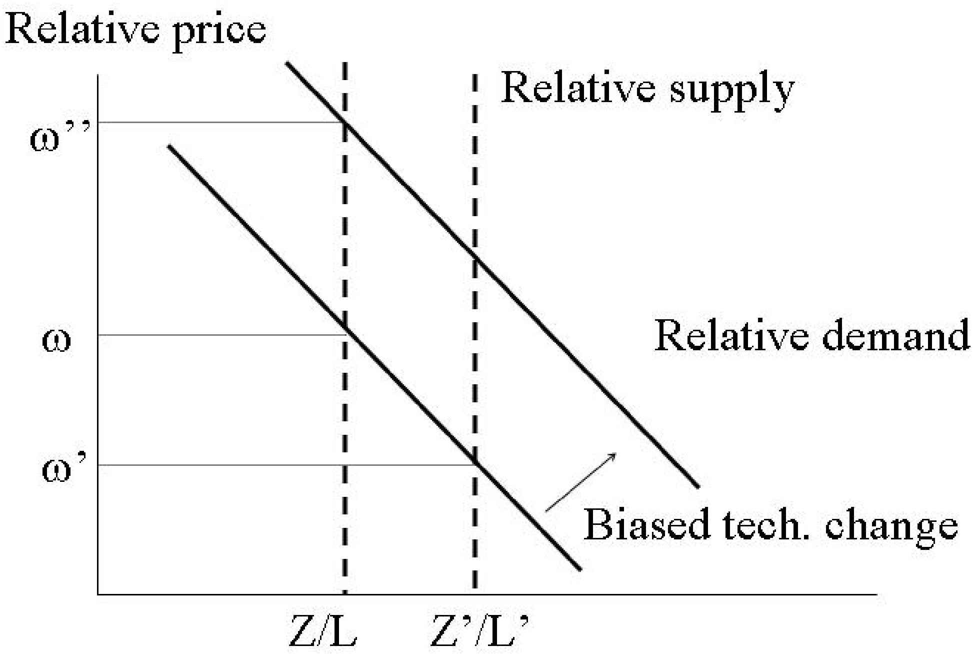
$$\frac{w_Z}{w_L} = \frac{\gamma}{1-\gamma}\left(\frac{A_Z}{A_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{\bar{Z}}{\bar{L}}\right)^{-\frac{1}{\sigma}},$$

  where $\gamma \equiv \gamma_Z/\gamma_L$

- Naturally decreasing in the relative supply of Z, $\bar{Z}/\bar{L}$.
- Also, clearly the measure of relative bias towards Z is $(A_Z/A_L)^{(\sigma-1)/\sigma}$.
- Implication: When $\sigma > 1$, factor-augmenting and skill-biased technologies are proportional.
- When $\sigma < 1$, they are **inversely proportional**.

Biased Technical Change

- Technical change biased in favor of factor Z:



Innovation Possibilities Frontier

- Innovation possibilities frontier determines how new technologies can be introduced ("produced").
- Equivalent to costs of producing new technologies.
- Suppose that the costs of producing new technologies in terms of the final good are

$$\eta_Z A_Z^{1+\delta}$$

$$\eta_L A_L^{1+\delta}$$

  with $\delta > 0$

- Suppose also that new technologies are produced by "a monopolist" and sold to final good producers (details below).

Sketch of Equilibrium

- Monopolist's problem:

$$\max_{A_Z, A_L} \left[\gamma \left(A_Z \bar{Z}\right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left(A_L \bar{L}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \eta_Z A_Z^{1+\delta} - \eta_L A_L^{1+\delta}.$$

- Taking the ratio of the first-order conditions:

$$\left(\frac{A_Z}{A_L}\right)^* = \eta^{-\frac{\sigma}{1+\sigma\delta}} \gamma^{\frac{\sigma}{1+\sigma\delta}} \left(\frac{\bar{Z}}{\bar{L}}\right)^{\frac{\sigma-1}{1+\sigma\delta}}.$$

where $\eta \equiv \eta_Z / \eta_L$

Illustration of the Factor Augmenting Theorem

- Factor Augmenting Theorem:
- The effect of relative supplies on factor augmenting technology.

$$\frac{\partial \ln (A_Z/A_L)^*}{\partial \ln (Z/L)} = \frac{\sigma - 1}{1 + \sigma\delta}.$$

- An increase in $Z/L \Rightarrow Z$-augmenting technological change only if $\sigma > 1$.
- If $\sigma < 1$, the converse result applies.

Illustration of the Weak Bias Theorem

- Weak Bias Theorem:
- Change in relative wage only due to induced technology:

$$\Delta\left(w_Z/w_L\right) \equiv \frac{\partial \ln\left(w_Z/w_L\right)}{\partial \ln\left(A_Z/A_L\right)} \frac{\partial \ln\left(A_Z/A_L\right)^*}{\partial \ln\left(Z/L\right)} \geq 0.$$

- From relative wage equation: that

$$\frac{\partial \ln\left(w_Z/w_L\right)}{\partial \ln\left(A_Z/A_L\right)} = \frac{\sigma - 1}{\sigma}$$

- Therefore

$$\Delta\left(w_Z/w_L\right) = \frac{\sigma - 1}{\sigma} \times \frac{\sigma - 1}{1 + \sigma\delta} = \frac{\left(\sigma - 1\right)^2}{\left(1 + \sigma\delta\right)\sigma} \geq 0,$$

which is always true.

- Notice the contrast between the two cases: $\sigma > 1$ and $\sigma < 1$.

Illustration the Strong Bias Theorem

- Are relative demand curves upward sloping?

$$\frac{\partial \ln (w_Z/w_L)}{\partial \ln (Z/L)} + \frac{\partial \ln (w_Z/w_L)}{\partial \ln (A_Z^*/A_L^*)} \frac{\partial \ln (A_Z/A_L)^*}{\partial \ln (Z/L)} > 0.$$

- From relative wage equation:

$$\frac{\partial \ln (w_Z/w_L)}{\partial \ln (Z/L)} = -\frac{1}{\sigma}$$

and

$$\frac{\partial \ln (w_Z/w_L)}{\partial \ln (A_Z/A_L)} \frac{\partial \ln (A_Z/A_L)^*}{\partial \ln (Z/L)} = \frac{\sigma - 1}{\sigma} \times \frac{\sigma - 1}{1 + \sigma\delta} = \frac{(\sigma - 1)^2}{(1 + \sigma\delta)\,\sigma},$$

- Therefore, the condition for strong bias is

$$-\frac{1}{\sigma} + \frac{(\sigma - 1)^2}{(1 + \sigma\delta)\,\sigma} = \frac{\sigma - 2 - \delta}{1 + \sigma\delta} > 0.$$

- This will hold when $\sigma > 2 + \delta$

Lessons from the Simple Example

- This simple example illustrative, but also "reduced form".
- A dynamic general equilibrium is necessary to study these results in a more satisfactory manner.
- Natural candidate: endogenous growth models.
- These results generalize to the standard growth setup.
- In fact, they are substantially more general than suggested by this example (and shown by the growth models): Lecture 3.
- But let us start with a standard growth model.

Directed Technical Change and Growth: Environment and Preferences

- Expanding varieties model (extension of Romer, 1990)
- Lab equipment specification: only final good used for research.
- Useful to demonstrate that results do not depend on knowledge spillover is externalities.
- Constant supply of factors (e.g., skilled and unskilled workers) Z and L.
- Representative household with the standard CRRA preferences:

$$\int_0^\infty \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1 - \theta} dt,$$

- No loss of generality in this context in assuming existence of a representative household.

Production Structure

- Competitive final good sector represented by an aggregate production function:

$$Y(t) = \left[\gamma_L Y_L(t)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma_Z Y_Z(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

  where intermediate good $Y_L(t)$ is L-intensive, $Y_Z(t)$ is Z-intensive.

- Here ε is the elasticity of substitution between the two intermediates.

Production Structure (continued)

- Intermediate goods produced competitively with:

$$Y_L(t) = \frac{1}{1-\beta} \left(\int_0^{N_L(t)} x_L(\nu, t)^{1-\beta} d\nu \right) L^\beta \tag{1}$$

$$Y_Z(t) = \frac{1}{1-\beta} \left(\int_0^{N_Z(t)} x_Z(\nu, t)^{1-\beta} d\nu \right) Z^\beta, \tag{2}$$

  where machines $x_L(\nu, t)$ and $x_Z(\nu, t)$ are assumed to depreciate after use.

- $N_Z =$ range of intermediates that can be used with factor Z.

- $N_L =$ range of intermediates that can be used with factor L.

- (1) and (2) use different types of machines—different ranges $[0, N_L(t)]$ and $[0, N_H(t)]$. That these intermediates are entirely distinct for simplicity.

- Loosely speaking $N_L \approx A_L$ and $N_Z \approx A_Z$ in terms of the example above.

Resource Constraint

- The resource constraint for the economy (and budget constraint for the representative household):

$$C(t) + X(t) + R(t) \leq Y(t),$$

where

$$R(t) = R_L(t) + R_Z(t)$$

is research expenditure on L-augmenting and Z-augmenting machine varieties.

Innovation Possibilities Frontier

- New machine varieties created by research expenditure:

$$\dot{N}_L(t) = \eta_L R_L(t) \text{ and } \dot{N}_Z(t) = \eta_Z R_Z(t),$$

- All machines are supplied by monopolists that have a fully-enforced perpetual patent, at prices $p_L^x(\nu, t)$ for $\nu \in [0, N_L(t)]$ and $p_H^x(\nu, t)$ for $\nu \in [0, N_H(t)]$.

- Once invented, each machine can be produced at the fixed marginal cost ψ in terms of the final good.

- Normalize to $\psi \equiv 1 - \beta$ without loss of any generality.

Prices

- Normalize the price of the final good at every instant to 1, which is equivalent to setting the ideal price index of the two intermediates equal to one, i.e.,

$$\left[\gamma_L^{\varepsilon}\left(p_L\left(t\right)\right)^{1-\varepsilon}+\gamma_Z^{\varepsilon}\left(p_Z\left(t\right)\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}=1 \text{ for all } t,$$

  where $p_L\left(t\right)$ is the price index of Y_L at time t and $p_Z\left(t\right)$ is the price of Y_H.

- Relative prices across states determined by the interest rate, $r(t)$.

- Denote factor prices by $w_L\left(t\right)$ and $w_Z\left(t\right)$.

Equilibrium

- Equilibrium defined in the usual manner (as time path of prices and allocations).
- Maximization problem of producers in the two sectors:

$$\max_{L(t),[x_L(\nu,t)]_{\nu\in[0,N_L(t)]}} p_L(t)\,Y_L(t) - w_L(t)\,L(t)$$
$$-\int_0^{N_L(t)} p_L^x(\nu,t)\,x_L(\nu,t)\,d\nu,$$

and

$$\max_{Z(t),[x_Z(\nu,t)]_{\nu\in[0,N_Z(t)]}} p_Z(t)\,Y_H(t) - w_Z(t)\,Z(t)$$
$$-\int_0^{N_Z(t)} p_Z^x(\nu,t)\,x_Z(\nu,t)\,d\nu.$$

Characterization of Equilibrium

- Thus, demand for machines in the two sectors:

$$x_L(\nu, t) = \left[\frac{p_L(t)}{p_L^x(\nu, t)}\right]^{1/\beta} L \quad \text{for all } \nu \in [0, N_L(t)] \text{ and all } t,$$

and

$$x_Z(\nu, t) = \left[\frac{p_Z(t)}{p_Z^x(\nu, t)}\right]^{1/\beta} Z \quad \text{for all } \nu \in [0, N_Z(t)] \text{ and all } t.$$

- Maximization by technology monopolists implies a constant markup over marginal cost $1-\beta$:

$$p_L^x(\nu, t) = p_Z^x(\nu, t) = 1 \text{ for all } \nu \text{ and } t.$$

Characterization of Equilibrium (continued)

- Equilibrium demands:

$$x_L(\nu, t) = p_L(t)^{1/\beta} L \quad \text{for all } \nu \text{ and all } t,$$

and

$$x_Z(\nu, t) = p_Z(t)^{1/\beta} Z \quad \text{for all } \nu \text{ and all } t.$$

- Therefore, profits are also independent of the machine type:

$$\pi_L(t) = \beta p_L(t)^{1/\beta} L \text{ and } \pi_Z(t) = \beta p_Z(t)^{1/\beta} Z. \tag{3}$$

Characterization of Equilibrium (continued)

- Combining machine demands with (1) and (2), the derived production functions for the two intermediate goods are:

$$Y_L(t) = \frac{1}{1-\beta} p_L(t)^{\frac{1-\beta}{\beta}} N_L(t) L,$$

and

$$Y_Z(t) = \frac{1}{1-\beta} p_Z(t)^{\frac{1-\beta}{\beta}} N_Z(t) Z.$$

Equilibrium Relative Prices

- From first-order conditions of final the producers:

$$
\begin{aligned}
p(t) &\equiv \frac{p_Z(t)}{p_L(t)} = \gamma \left(\frac{Y_Z(t)}{Y_L(t)} \right)^{-\frac{1}{\varepsilon}} \\
&= \gamma \left(p(t)^{\frac{1-\beta}{\beta}} \frac{N_Z(t)\, Z}{N_L(t)\, L} \right)^{-\frac{1}{\varepsilon}} \\
&= \gamma^{\frac{\varepsilon\beta}{\sigma}} \left(\frac{N_Z(t)\, Z}{N_L(t)\, L} \right)^{-\frac{\beta}{\sigma}},
\end{aligned}
$$

where $\gamma \equiv \gamma_Z / \gamma_L$ and

$$
\begin{aligned}
\sigma &\equiv \varepsilon - (\varepsilon - 1)(1 - \beta) \\
&= 1 + (\varepsilon - 1)\beta.
\end{aligned}
$$

- What is σ?

Equilibrium Factor Prices

- Relative factor prices:

$$\begin{aligned} \omega(t) &\equiv \frac{w_Z(t)}{w_L(t)} \\ &= p(t)^{1/\beta} \frac{N_Z(t)}{N_L(t)} \\ &= \gamma^{\frac{\varepsilon}{\sigma}} \left(\frac{N_Z(t)}{N_L(t)} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Z}{L} \right)^{-\frac{1}{\sigma}}. \end{aligned}$$

- Thus σ is the (derived) elasticity of substitution between the two factors, since it is exactly equal to

$$\sigma = -\left(\frac{d \log \omega(t)}{d \log (Z/L)} \right)^{-1}.$$

Free Entry and Optimization

- Free entry conditions:

$$\eta_L V_L(t) \leq 1 \text{ and } \eta_L V_L(t) = 1 \text{ if } R_L(t) > 0.$$

  and

$$\eta_Z V_Z(t) \leq 1 \text{ and } \eta_Z V_Z(t) = 1 \text{ if } R_Z(t) > 0.$$

  where V_L and V_Z are the values of the monopolists selling machines of different types.

- Consumer Euler equation:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}\left(r(t) - \rho\right),$$

  and transversality condition:

$$\lim_{t\to\infty}\left[\exp\left(-\int_0^t r(s)\,ds\right)\left(N_L(t)V_L(t) + N_Z(t)V_Z(t)\right)\right] = 0.$$

Balance Growth Path (BGP)

- Let us focus on BGP, where consumption grows at a constant rate.
- Therefore, interest rate constant at some level r^* from the consumer Euler equation.
- This implies

$$V_L = \frac{\beta p_L^{1/\beta} L}{r^*} \text{ and } V_Z = \frac{\beta p_Z^{1/\beta} Z}{r^*},$$

- Taking the ratio of these two expressions, we obtain

$$\frac{V_Z}{V_L} = \left(\frac{p_Z}{p_L}\right)^{\frac{1}{\beta}} \frac{Z}{L}.$$

- **Market size** and **price** effects.

Balanced Growth Path (continued)

- Free entry: $\eta_L V_L = \eta_Z V_Z$.
- Equilibrium free entry condition:

$$\left(\frac{p_Z}{p_L}\right)^{-\frac{1}{\beta}} = \eta \frac{Z}{L}$$

where $\eta \equiv \eta_Z / \eta_L$.

- BGP ratio of relative technologies:

$$\left(\frac{N_Z}{N_L}\right)^* = \eta^\sigma \gamma^\varepsilon \left(\frac{Z}{L}\right)^{\sigma-1}, \quad (4)$$

- Direction of technology endogenous: determined by the innovation possibilities frontier and the relative supply of the two factors.
- Immediate implication: Factor Augmenting Theorem.

Summary of BGP

Proposition: Suppose that

$$\beta\left[\gamma_Z^{\varepsilon}\left(\eta_Z Z\right)^{\sigma-1}+\gamma_L^{\varepsilon}\left(\eta_L L\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} > \rho$$

$$\text{and } (1-\theta)\,\beta\left[\gamma_Z^{\varepsilon}\left(\eta_Z Z\right)^{\sigma-1}+\gamma_L^{\varepsilon}\left(\eta_L L\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} < \rho.$$

Then there exists a unique BGP equilibrium in which the relative technologies are given by (4), and consumption and output grow at the rate

$$g^{*}=\frac{1}{\theta}\left(\beta\left[\gamma_Z^{\varepsilon}\left(\eta_Z Z\right)^{\sigma-1}+\gamma_L^{\varepsilon}\left(\eta_L L\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}-\rho\right).$$

Moreover, this unique BGP is globally stable starting with any initial values $N_Z(0)$ and $N_L(0)$.

Weak and Strong Bias Theorems

Proposition: In the equilibrium there is always **weak equilibrium bias** in the sense that an increase in the supply of Z induces technological change biased towards Z.

- Identical intuition to the example above.
- In addition, substituting for $(N_Z/N_L)^*$ into the relative wage equation, we obtain that endogenous-technology relative wages are given by

$$\omega^* \equiv \left(\frac{w_H}{w_L}\right)^* = \eta^{\sigma-1}\gamma^{\varepsilon}\left(\frac{H}{L}\right)^{\sigma-2}.$$

- Therefore:

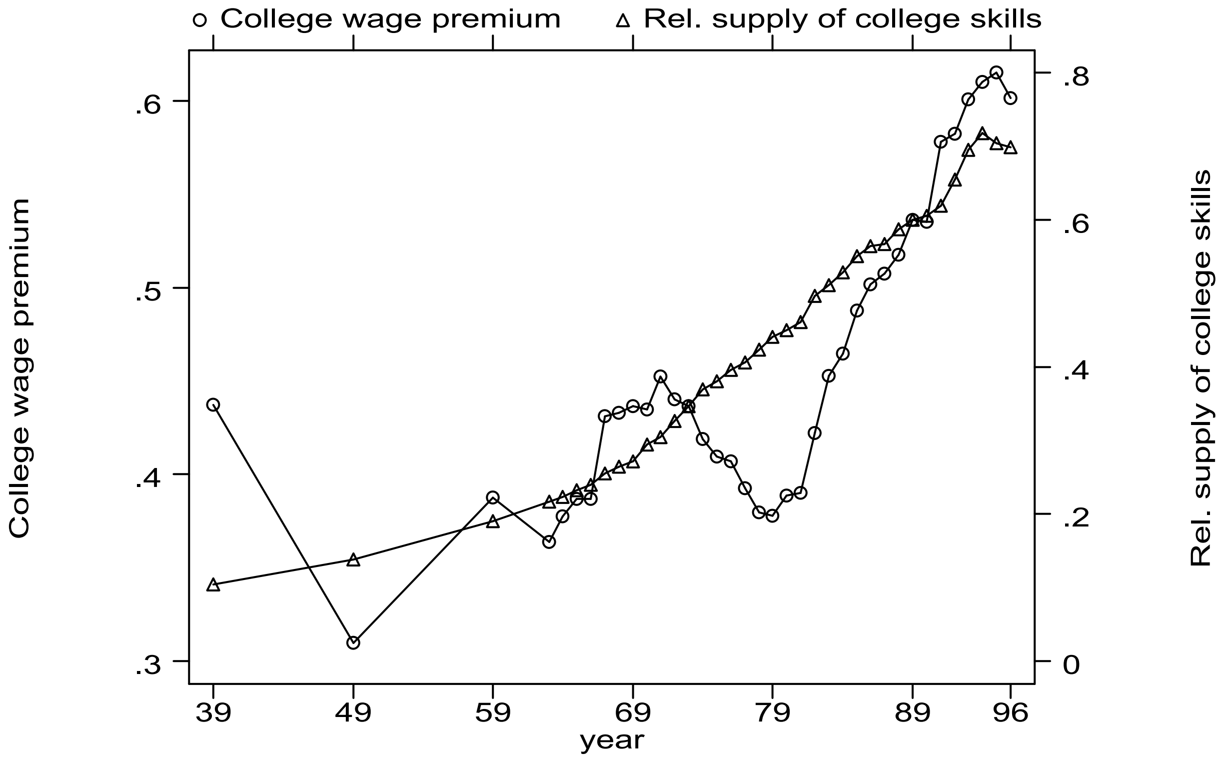
Proposition: In the equilibrium there is **strong equilibrium bias** in the sense that the endogenous-technology relative demand curve is upward-sloping if $\sigma > 2$.

Back to Skill Bias

- **Weak Bias Theorem** can account for:
  1. Secular skill bias over the past hundred years or so in response to the steady increase in the supply of skills .
  2. Acceleration in skill bias over the past 30 years .
  3. Technologies biased against skilled workers during the early phases of the Industrial Revolution.
- **Strong Bias Theorem** can account for: increase in skill premium in the face of acceleration in supply.

Dynamics of Skill Bias and Skill Premia

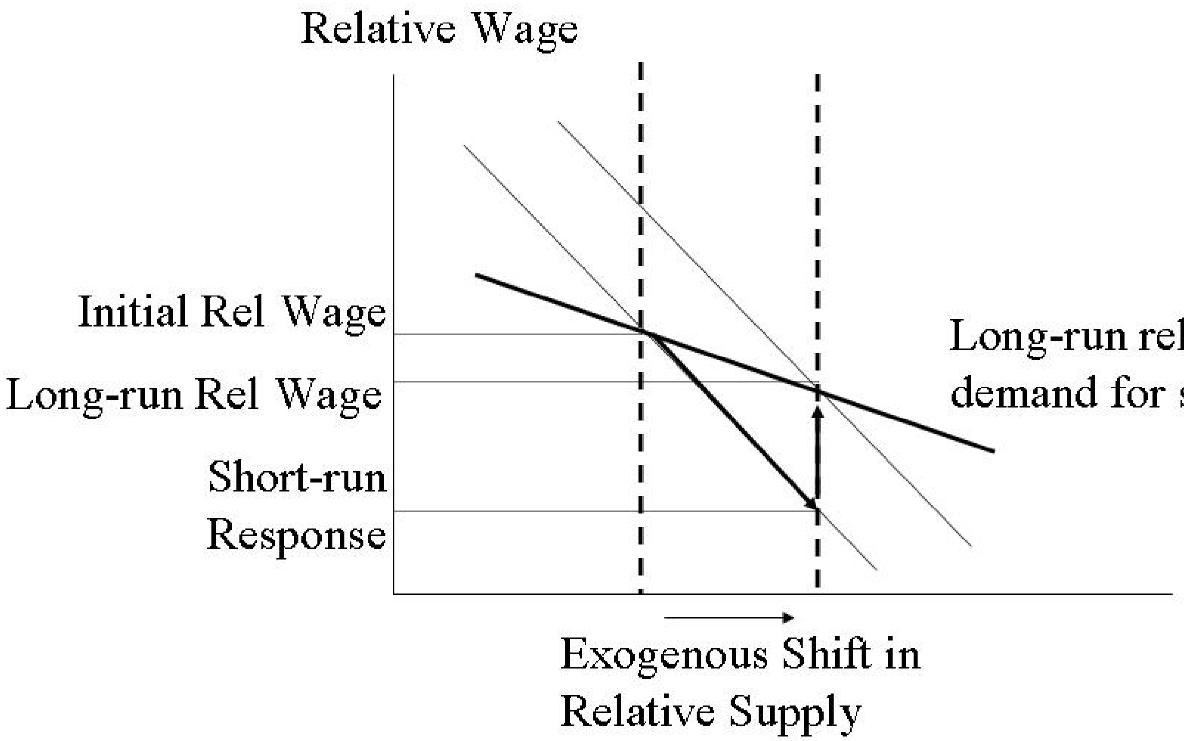
- Moreover, N_Z and N_L state variables thus changing slowly.
- So the model can account for an initial decline in skill premium followed by a larger increase.
- Recall the empirical pattern:



Relative Supply of College Skills and College Premium

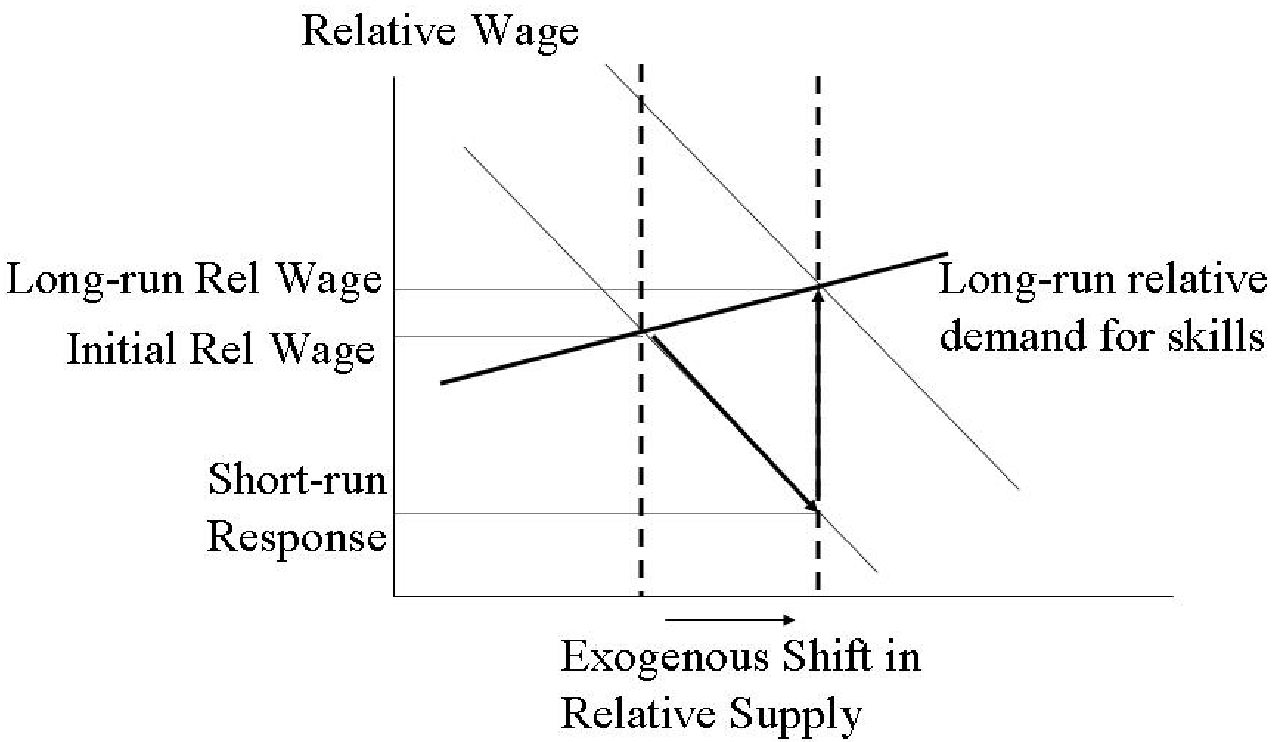
Dynamics of Skill Bias and Skill Premia (continued)

- From the theory there is always weak bias and slow adjustment of state variables:



Dynamics of Skill Bias and Skill Premia (continued)

- In the presence of strong bias:



Summary and Looking Ahead

- Importance of directed technical change.
- Relatively strong results on the equilibrium direction of technical change.
- Implications for the evolution of skill bias of technology.
- Lecture 2: implications for innovation, growth and cross-country income differences.
- Lecture 3: results presented here much more general than the illustrative examples.