

14.452 Economic Growth: Lectures 10 and 11, Endogenous Technological Change

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Introduction

- The key to understanding *technology* is that R&D and technology adoption are purposeful activities.
- This lecture, focus on technological change and R&D.
- The simplest models of endogenous technological change are those in which R&D expands the variety of inputs or machines used in production (Romer, 1990).
- Models with expanding input varieties:
 - research will lead to the creation of new varieties of inputs (machines) and a greater variety of inputs will increase the “division of labor”
 - *process innovation*.
- Alternative: *product innovation* (Grossman and Helpman (1991a,b)):
 - invention of new goods,
 - because of love-for-variety, “real” incomes increase

Key Insights

- Innovation as generating new blueprints or *ideas* for production.
- Three important features (Romer):
 - 1 Ideas and technologies *nonrival*—many firms can benefit from the same idea.
 - 2 Increasing returns to scale—constant returns to scale to capital, labor, material etc. and then ideas and blueprints are also produced.
 - 3 Costs of research and development paid as fixed costs upfront.
- We must consider models of *monopolistic competition*, where firms that innovate become monopolists and make profits.
- Throughout use the Dixit-Stiglitz constant elasticity structure.

The Lab Equipment Model with Input Varieties

- All that is required for research is investment in equipment or in laboratories
- That is, new machines and ideas are created using the final good.
 - rather than the employment of skilled or unskilled workers or scientists.
 - similar to Rebelo's *AK* economy.
 - useful benchmark, since it minimizes the extent of spillovers and externalities.

Demographics, Preferences, and Technology

- Infinite-horizon economy, continuous time.
- Representative household with preferences:

$$\int_0^{\infty} \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt. \quad (1)$$

- L = total (constant) population of workers. Labor supplied inelastically.
- Representative household owns a balanced portfolio of all the firms in the economy.

Demographics, Preferences, and Technology I

- Unique consumption good, produced with aggregate production function:

$$Y(t) = \frac{1}{1-\beta} \left[\int_0^{N(t)} x(\nu, t)^{1-\beta} d\nu \right] L^\beta, \quad (2)$$

where

- $N(t)$ = number of varieties of inputs (machines) at time t ,
- $x(\nu, t)$ = amount of input (machine) type ν used at time t .
- The x 's depreciate fully after use.
- They can be interpreted as generic inputs, intermediate goods, machines, or capital.
- Thus machines are *not* additional state variables.
- For given $N(t)$, which final good producers take as given, (2) exhibits constant returns to scale.

Demographics, Preferences, and Technology II

- Final good producers are competitive.
- The resource constraint of the economy at time t is

$$C(t) + X(t) + Z(t) \leq Y(t), \quad (3)$$

where $X(t)$ is investment on inputs at time t and $Z(t)$ is expenditure on R&D at time t .

- Once the blueprint of a particular input is invented, the research firm can create one unit of that machine at marginal cost equal to $\psi > 0$ units of the final good.

Innovation Possibilities Frontier and Patents I

- *Innovation possibilities frontier:*

$$\dot{N}(t) = \eta Z(t), \quad (4)$$

where $\eta > 0$, and the economy starts with some $N(0) > 0$.

- There is free entry into research: any individual or firm can spend one unit of the final good at time t in order to generate a flow rate η of the blueprints of new machines.
- The firm that discovers these blueprints receives a *fully-enforced perpetual patent* on this machine.
- There is no aggregate uncertainty in the innovation process.
 - There will be uncertainty at the level of the individual firm, but with many different research labs undertaking such expenditure, at the aggregate level, equation (4) holds deterministically.

Innovation Possibilities Frontier and Patents II

- A firm that invents a new machine variety v is the sole supplier of that type of machine, and sets a profit-maximizing price of $p^x(v, t)$ at time t to maximize profits.
- Since machines depreciate after use, $p^x(v, t)$ can also be interpreted as a “rental price” or the user cost of this machine.

The Final Good Sector

- Maximization by final the producers:

$$\begin{aligned} & \max_{[x(v,t)]_{v \in [0, N(t)]}, L} \frac{1}{1-\beta} \left[\int_0^{N(t)} x(v,t)^{1-\beta} dv \right] L^\beta & (5) \\ & - \int_0^{N(t)} p^x(v,t) x(v,t) dv - w(t) L. \end{aligned}$$

- Demand for machines:

$$x(v,t) = p^x(v,t)^{-1/\beta} L, \quad (6)$$

- Isoelastic demand for machines.
- Only depends on the user cost of the machine and on equilibrium labor supply but not on the interest rate, $r(t)$, the wage rate, $w(t)$, or the total measure of available machines, $N(t)$.

Profit Maximization by Technology Monopolists I

- Consider the problem of a monopolist owning the blueprint of a machine of type ν invented at time t .
- Since the representative household holds a balanced portfolio of all the firms, no uncertainty in dividends and each monopolist's objective is to maximize expected profits.
- The monopolist chooses an investment plan starting from time t to maximize the discounted value of profits:

$$V(\nu, t) = \int_t^\infty \exp \left[- \int_t^s r(s') ds' \right] \pi(\nu, s) ds \quad (7)$$

where

$$\pi(\nu, t) \equiv p^x(\nu, t)x(\nu, t) - \psi x(\nu, t)$$

denotes profits of the monopolist producing intermediate ν at time t , $x(\nu, t)$ and $p^x(\nu, t)$ are the profit-maximizing choices and $r(t)$ is the market interest rate at time t .

Profit Maximization by Technology Monopolists II

- For future reference, the discounted value of profits can also be written in the alternative Hamilton-Jacobi-Bellman form:

$$r(t) V(v, t) - \dot{V}(v, t) = \pi(v, t). \quad (8)$$

- This equation shows that the discounted value of profits may change because of two reasons:
 - 1 Profits change over time
 - 2 The market interest rate changes over time.

Characterization of Equilibrium I

- An allocation in this economy is defined by time paths of:
 - consumption levels, aggregate spending on machines, and aggregate R&D expenditure $[C(t), X(t), Z(t)]_{t=0}^{\infty}$,
 - available machine types, $[N(t)]_{t=0}^{\infty}$,
 - prices and quantities of each machine and the net present discounted value of profits from that machine, $[p^x(v, t), x(v, t), V(v, t)]_{v \in N(t), t=0}^{\infty}$, and
 - interest rates and wage rates, $[r(t), w(t)]_{t=0}^{\infty}$.
- An equilibrium is an allocation in which
 - all research firms choose $[p^x(v, t), x(v, t)]_{v \in [0, N(t)], t=0}^{\infty}$ to maximize profits,
 - $[N(t)]_{t=0}^{\infty}$ is determined by free entry,
 - $[r(t), w(t)]_{t=0}^{\infty}$ are consistent with market clearing, and
 - $[C(t), X(t), Z(t)]_{t=0}^{\infty}$ are consistent with consumer optimization.

Characterization of Equilibrium II

- Since (6) defines isoelastic demands, the solution to the maximization problem of any monopolist $\nu \in [0, N(t)]$ involves setting the same price in every period:

$$p^x(\nu, t) = \frac{\psi}{1 - \beta} \text{ for all } \nu \text{ and } t. \quad (9)$$

- Normalize $\psi \equiv (1 - \beta)$, so that

$$p^x(\nu, t) = p^x = 1 \text{ for all } \nu \text{ and } t.$$

- Profit-maximization also implies that each monopolist rents out the same quantity of machines in every period, equal to

$$x(\nu, t) = L \text{ for all } \nu \text{ and } t. \quad (10)$$

Characterization of Equilibrium III

- Monopoly profits:

$$\pi(v, t) = \beta L \text{ for all } v \text{ and } t. \quad (11)$$

- Substituting (6) and the machine prices into (2) yields:

$$Y(t) = \frac{1}{1 - \beta} N(t) L. \quad (12)$$

- Even though the aggregate production function exhibits constant returns to scale from the viewpoint of final good firms (which take $N(t)$ as given), there are *increasing returns to scale* for the entire economy;
- An increase in $N(t)$ raises the productivity of labor and when $N(t)$ increases at a constant rate so will output per capita.

Characterization of Equilibrium IV

- Equilibrium wages:

$$w(t) = \frac{\beta}{1-\beta} N(t). \quad (13)$$

- Free entry

$$\begin{aligned} \eta V(v, t) &\leq 1, \quad Z(v, t) \geq 0 \quad \text{and} \\ (\eta V(v, t) - 1) Z(v, t) &= 0, \quad \text{for all } v \text{ and } t, \end{aligned} \quad (14)$$

where $V(v, t)$ is given by (7).

- For relevant parameter values with positive entry and economic growth:

$$\eta V(v, t) = 1.$$

Characterization of Equilibrium V

- Finally, the representative household's problem is standard and implies the usual Euler equation:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho) \quad (15)$$

and the transversality condition

$$\lim_{t \rightarrow \infty} \left[\exp \left(- \int_0^t r(s) ds \right) N(t) V(t) \right] = 0. \quad (16)$$

Equilibrium and Balanced Growth Path I

- We can now define an equilibrium more formally as time paths
 - $[C(t), X(t), Z(t), N(t)]_{t=0}^{\infty}$, such that (3), (??), (15), (16) and (14) are satisfied;
 - $[p^x(v, t), x(v, t)]_{v \in N(t), t=0}^{\infty}$ that satisfy (9) and (10),
 - $[r(t), w(t)]_{t=0}^{\infty}$ such that (13) and (15) hold.
- We define a *balanced growth path (BGP)* as an equilibrium path where $C(t)$, $X(t)$, $Z(t)$ and $N(t)$ grow at a constant rate. Such an equilibrium can alternatively be referred to as a “steady state”, since it is a steady state in transformed variables.

Balanced Growth Path I

- A balanced growth path (BGP) requires that consumption grows at a constant rate, say g_C . This is only possible from (15) if

$$r(t) = r^* \text{ for all } t$$

- Since profits at each date are given by (11) and since the interest rate is constant, $\dot{V}(t) = 0$ and

$$V^* = \frac{\beta L}{r^*}. \quad (17)$$

Balanced Growth Path II

- Let us next suppose that the (free entry) condition (14) holds as an equality, in which case we also have

$$\frac{\eta\beta L}{r^*} = 1$$

This equation pins down the steady-state interest rate, r^* , as:

$$r^* = \eta\beta L$$

- The consumer Euler equation, (15), then implies that the rate of growth of consumption must be given by

$$g_C^* = \frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r^* - \rho). \quad (18)$$

Balanced Growth Path III

- Note the current-value Hamiltonian for the consumer's maximization problem is concave, thus this condition, together with the transversality condition, characterizes the optimal consumption plans of the consumer.
- In BGP, consumption grows at the same rate as total output

$$g^* = g_C^*.$$

Therefore, given r^* , the long-run growth rate of the economy is:

$$g^* = \frac{1}{\theta} (\eta\beta L - \rho) \quad (19)$$

- Suppose that

$$\eta\beta L > \rho \text{ and } (1 - \theta)\eta\beta L < \rho, \quad (20)$$

which will ensure that $g^* > 0$ and that the transversality condition is satisfied.

Balanced Growth Path IV

Proposition Suppose that condition (20) holds. Then, in the above-described lab equipment expanding input variety model, there exists a unique balanced growth path in which technology, output and consumption all grow at the same rate, g^* , given by (19)..

- An important feature of this class models is the presence of the *scale effect*: the larger is L , the greater is the growth rate.

Transitional Dynamics I

- There are no transitional dynamics in this model.
- Substituting for profits in the value function for each monopolist, this gives

$$r(t) V(v, t) - \dot{V}(v, t) = \beta L.$$

- The key observation is that positive growth at any point implies that $\eta V(v, t) = 1$ for all t . In other words, if $\eta V(v, t') = 1$ for some t' , then $\eta V(v, t) = 1$ for all t .
- Now differentiating $\eta V(v, t) = 1$ with respect to time yields $\dot{V}(v, t) = 0$, which is only consistent with $r(t) = r^*$ for all t , thus

$$r(t) = \eta\beta L \text{ for all } t.$$

Transitional Dynamics II

Proposition Suppose that condition (20) holds. In the above-described lab equipment expanding input-variety model, with initial technology stock $N(0) > 0$, there is a unique equilibrium path in which technology, output and consumption always grow at the rate g^* as in (19).

- While the microfoundations here are very different from the neoclassical AK economy, the mathematical structure is very similar to the AK model (as most clearly illustrated by the derived equation for output, (12)).
- Consequently, as in the AK model, the economy always grows at a constant rate.
- But the economics is very different.

Social Planner Problem I

- Monopolistic competition implies that the competitive equilibrium is not necessarily Pareto optimal. The model exhibits a version of the *aggregate demand externalities*:
 - ① There is a markup over the marginal cost of production of inputs.
 - ② The number of inputs produced at any point in time may not be optimal.
- The first inefficiency is familiar from models of static monopoly, while the second emerges from the fact that in this economy the set of traded (Arrow-Debreu) commodities is endogenously determined.
- This relates to the issue of endogenously incomplete markets (there is no way to purchase an input that is not supplied in equilibrium).

Social Planner Problem II

- Given $N(t)$, the social planner will choose

$$\max_{[x(v,t)]_{v \in [0, N(t)]}, L} \frac{1}{1-\beta} \left[\int_0^{N(t)} x(v,t)^{1-\beta} dv \right] L^\beta - \int_0^{N(t)} \psi x(v,t) dv,$$

- Differs from the equilibrium profit maximization problem, (5), because the marginal cost of machine creation, ψ , is used as the cost of machines rather than the monopoly price, and the cost of labor is not subtracted.
- Recalling that $\psi \equiv 1 - \beta$, the solution to this program involves

$$x^S(v,t) = (1 - \beta)^{-1/\beta} L,$$

Social Planner Problem III

- The *net* output level (after investment costs are subtracted) is

$$\begin{aligned} Y^S(t) &= \frac{(1-\beta)^{-(1-\beta)/\beta}}{1-\beta} N^S(t) L \\ &= (1-\beta)^{-1/\beta} N^S(t) L, \end{aligned}$$

- Therefore, the maximization problem of the social planner can be written as

$$\max \int_0^{\infty} \frac{C(t)^{1-\theta} - 1}{1-\theta} \exp(-\rho t) dt$$

subject to

$$\dot{N}(t) = \eta (1-\beta)^{-1/\beta} \beta N(t) L - \eta C(t).$$

where $(1-\beta)^{-1/\beta} \beta N^S(t) L$ is net output.

Social Planner Problem IV

- In this problem, $N(t)$ is the state variable, and $C(t)$ is the control variable. The current-value Hamiltonian is:

$$\hat{H}(N, C, \mu) = \frac{C(t)^{1-\theta} - 1}{1-\theta} + \mu(t) \left[\eta(1-\beta)^{-1/\beta} \beta N(t) L - \eta C(t) \right].$$

- The conditions for a candidate Pareto optimal allocation are:

$$\begin{aligned} \hat{H}_C(N, C, \mu) &= C(t)^{-\theta} - \eta\mu(t) = 0 \\ \hat{H}_N(N, C, \mu) &= \mu(t) \eta(1-\beta)^{-1/\beta} \beta L \\ &= \rho\mu(t) - \dot{\mu}(t) \\ \lim_{t \rightarrow \infty} [\exp(-\rho t) \mu(t) N(t)] &= 0. \end{aligned}$$

Social Planner Problem V

- It can be verified easily that the current-value Hamiltonian of the social planner is (strictly) concave, thus these conditions are also sufficient for an optimal solution.
- Combining these conditions:

$$\frac{\dot{C}^S(t)}{C^S(t)} = \frac{1}{\theta} \left(\eta (1 - \beta)^{-1/\beta} \beta L - \rho \right). \quad (21)$$

Comparison of Equilibrium and Pareto Optimum

- The comparison to the growth rate in the decentralized equilibrium, (19), boils down to that of

$$(1 - \beta)^{-1/\beta} \beta \text{ to } \beta,$$

- The socially-planned economy will always grow faster than the decentralized economy the former is always greater since $(1 - \beta)^{-1/\beta} > 1$ by virtue of the fact that $\beta \in (0, 1)$.

Comparison

Proposition In the above-described expanding input variety model, the decentralized equilibrium is always Pareto suboptimal. Starting with any $N(0) > 0$, the Pareto optimal allocation involves a constant growth rate

$$g^S = \frac{1}{\theta} \left(\eta (1 - \beta)^{-1/\beta} \beta L - \rho \right),$$

which is strictly greater than the equilibrium growth rate g^* given in (19).

Comparison

- Why is the equilibrium growing more slowly than the optimum allocation?
- Because the social planner values innovation more
- The social planner is able to use the machines more intensively after innovation, *pecuniary externality* resulting from the monopoly markups.
- Other models of endogenous technological progress we will study in this lecture incorporate technological spillovers and thus generate inefficiencies both because of the pecuniary externality isolated here and because of the standard technological spillovers.

Policies

- What kind of policies can increase equilibrium growth rate?
- ① *Subsidies to Research*: the government can increase the growth rate of the economy, and this can be a Pareto improvement if taxation is not distortionary and there can be appropriate redistribution of resources so that all parties benefit.
- ② *Subsidies to Capital Inputs*: inefficiencies also arise from the fact that the decentralized economy is not using as many units of the machines/capital inputs (because of the monopoly markup); so subsidies to capital inputs given to final good producers would also increase the growth rate.
- But note, the same policies can also be used to distort allocations.
- When we look at a the cross-section of countries, taxes on research and capital inputs more common than subsidies.

The Effects of Competition I

- Recall that the monopoly price is:

$$p^x = \frac{\psi}{1 - \beta}.$$

- Imagine, instead, that a fringe of competitive firms can copy the innovation of any monopolist.
 - But instead of a marginal cost ψ , the fringe has marginal cost of $\gamma\psi$ with $\gamma > 1$.
- If $\gamma > 1/(1 - \beta)$, no threat from the fringe.
- If $\gamma < 1/(1 - \beta)$, the fringe would forced the monopolist to set a “*limit price*”,

$$p^x = \gamma\psi. \tag{22}$$

The Effects of Competition II

- Why? If $p^x > \gamma\psi$, the fringe could undercut the price of the monopolist, take over to market and make positive profits. If $p^x < \gamma\psi$, the monopolist could increase price and make more profits. Thus, there is a unique equilibrium price given by (22).
- Profits under the limit price:

$$\text{profits per unit} = (\gamma - 1)\psi = (\gamma - 1)(1 - \beta) < \beta,$$

- Therefore, growth with competition:

$$\hat{g} = \frac{1}{\theta} \left(\eta \gamma^{-1/\beta} (\gamma - 1) (1 - \beta)^{-(1-\beta)/\beta} L - \rho \right) < g^*.$$

Growth with Knowledge Spillovers I

- In the lab equipment model, growth resulted from the use of final output for R&D. This is similar to the endogenous growth model of Rebelo (1991), since the accumulation equation is linear in accumulable factors. In equilibrium, output took a linear form in the stock of knowledge (new machines), thus a AN form instead of Rebelo's AK form.
- An alternative is to have “scarce factors” used in R&D: we have scientists as the key creators of R&D.
- With this alternative, there cannot be endogenous growth unless there are knowledge spillovers from past R&D, making the scarce factors used in R&D more and more productive over time.

Innovation Possibilities Frontier I

- Innovation possibilities frontier in this case:

$$\dot{N}(t) = \eta N(t) L_R(t) \quad (23)$$

where $L_R(t)$ is labor allocated to R&D at time t .

- The term $N(t)$ on the right-hand side captures spillovers from the stock of existing ideas.
- Notice that (23) imposes that these spillovers are proportional or linear. This linearity will be the source of endogenous growth in the current model.
- In (23), $L_R(t)$ comes out of the regular labor force. The cost of workers to the research sector is given by the wage rate in final good sector.

Characterization of Equilibrium I

- Most of equilibrium characterization very similar.
- Labor market clearing:

$$L_R(t) + L_E(t) \leq L.$$

- Aggregate output of the economy:

$$Y(t) = \frac{1}{1-\beta} N(t) L_E(t), \quad (24)$$

and profits of monopolists from selling their machines is

$$\pi(t) = \beta L_E(t). \quad (25)$$

- The net present discounted value of a monopolist (for a blueprint ν) is still given by $V(\nu, t)$ as in (7) or (8), with the flow profits given by (25).

Characterization of Equilibrium II

- The free entry condition is no longer the same. Instead, (23) implies:

$$\eta N(t) V(v, t) = w(t), \quad (26)$$

where $N(t)$ is on the left-hand side because it parameterizes the productivity of an R&D worker, while the flow cost of undertaking research is hiring workers for R&D, thus is equal to the wage rate $w(t)$.

- The equilibrium wage rate must be the same as before:

$$w(t) = \beta N(t) / (1 - \beta)$$

- Balanced growth again requires that the interest rate must be constant at some level r^* .

Characterization of Equilibrium III

- Using these observations together with the free entry condition, we obtain:

$$\eta N(t) \frac{\beta L_E(t)}{r^*} = \frac{\beta}{1 - \beta} N(t). \quad (27)$$

Hence the BGP equilibrium interest rate must be

$$r^* = (1 - \beta) \eta L_E^*,$$

where $L_E^* = L - L_R^*$. The fact that the number of workers in production must be constant in BGP follows from (27).

- Now using the Euler equation of the representative household, (15), for all t :

$$\begin{aligned} \frac{\dot{C}(t)}{C(t)} &= \frac{1}{\theta} ((1 - \beta) \eta L_E^* - \rho) \\ &\equiv g^*. \end{aligned} \quad (28)$$

Characterization of Equilibrium IV

- To complete the characterization of the BGP equilibrium, we need to determine L_E^* . In BGP, (23) implies that the rate of technological progress satisfies

$$\frac{\dot{N}(t)}{N(t)} = \eta L_R^* = \eta (L - L_E^*)$$

This implies that the BGP level of employment is

$$L_E^* = \frac{\theta\eta L + \rho}{(1 - \beta)\eta + \theta\eta}. \quad (29)$$

Summary of Equilibrium in the Model with Knowledge Spillovers

Proposition Consider the above-described expanding input-variety model with knowledge spillovers and suppose that

$$(1 - \theta)(1 - \beta)\eta L_E^* < \rho < (1 - \beta)\eta L_E^*, \quad (30)$$

where L_E^* is the number of workers employed in production in BGP, given by (29). Then there exists a unique balanced growth path in which technology, output and consumption grow at the same rate, $g^* > 0$, given by (28) starting from any initial level of technology stock $N(0) > 0$.

- As in the lab equipment model, the equilibrium allocation is Pareto suboptimal.

Growth without Scale Effects: Motivation

- The models so far feature a scale effect.
- A larger population $L \implies$ higher interest rate and a higher growth rate.
- Potentially problematic for three reasons:
 - 1 Larger countries do not necessarily grow faster.
 - 2 The population of most nations has not been constant. If we have population growth as in the standard neoclassical growth model, e.g., $L(t) = \exp(nt) L(0)$, these models would not feature balanced growth, rather, the growth rate of the economy would be increasing over time.
 - 3 In the data, the total amount of resources devoted to R&D appears to increase steadily, but there is no associated increase in the aggregate growth rate.

Knowledge Spillovers Model with two Differences

- Differences:

- Population growth at exponential rate n , $\dot{L}(t) = nL(t)$.

Representative household, also growing at the rate n , with preferences:

$$\int_0^{\infty} \exp(-(\rho - n)t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt, \quad (31)$$

- R&D sector only admits limited knowledge spillovers and (23) is replaced by

$$\dot{N}(t) = \eta N(t)^{\phi} L_R(t) \quad (32)$$

where $\phi < 1$ and $L_R(t)$ is labor allocated to R&D activities at time t . Labor market clearing requires

$$L_E(t) + L_R(t) = L(t), \quad (33)$$

Growth without Scale Effects I

- Aggregate output and profits are given by (24) and (25) as in the previous section. An equilibrium is also defined similarly.
- Focus on the BGP. Free entry with equality:

$$\eta N(t)^\phi \frac{\beta L_E(t)}{r^* - n} = w(t). \quad (34)$$

- As before, the equilibrium wage is determined by the production side, (13), as

$$w(t) = \beta N(t) / (1 - \beta).$$

Thus,

$$\eta N(t)^{\phi-1} \frac{(1 - \beta) L_E(t)}{r^* - n} = 1.$$

Growth without Scale Effects II

- Differentiating this condition with respect to time, we obtain

$$(\phi - 1) \frac{\dot{N}(t)}{N(t)} + \frac{\dot{L}_E(t)}{L_E(t)} = 0.$$

- Since in BGP, the fraction of workers allocated to research is constant, we must have

$$\dot{L}_E(t) / L_E(t) = n$$

- Thus,

$$g_N^* \equiv \frac{\dot{N}(t)}{N(t)} = \frac{n}{1 - \phi}. \quad (35)$$

$$\begin{aligned} g_C^* &= g_N^* \\ &= \frac{n}{1 - \phi}. \end{aligned} \quad (36)$$

Summary of Equilibrium without Scale Effects

Proposition In the above-described expanding input-variety model with limited knowledge spillovers as given by (32), starting from any initial level of technology stock $N(0) > 0$, there exists a unique balanced growth path in which, technology and consumption per capita grow at the rate g_N^* as given by (35), and output grows at rate $g_N^* + n$.

- Sustained equilibrium growth of per capita income is possible with growing population.
- Instead of the linear (proportional) spillovers, only a limited amount of spillovers.
- Without population growth, these spillovers would affect the level of output, but not sufficient to sustain long-run growth.
- Population growth increases the market size for new technologies steadily and generates growth from these limited spillovers.

Discussion I

- “Growth without scale effects”?
- There are two senses in which there are still scale effects:
 - ① A faster rate of population growth translates into a higher equilibrium growth rate.
 - ② A larger population size leads to higher output per capita.
- Empirical evidence?
- “Semi-endogenous growth” models, because growth is determined only by population growth and technology, and does not respond to policies.
 - Extensions to allow for the impact of policy and growth possible (though under somewhat restrictive assumptions).

Schumpeterian Growth

- Alternative: quality improvements (over existing technologies or products).
 - Similar to vertical differentiation rather than horizontal differentiation.
- But more important difference is that now new technologies *replace* old ones.
- *Creative destruction*: when a higher-quality machine is invented it will replace (“destroy”) the previous vintage of machines.

Preferences and Technology I

- Continuous time.
- Representative household with standard CRRA preferences.
- Constant population L ; labor supplied inelastically.
- Resource constraint:

$$C(t) + X(t) + Z(t) \leq Y(t), \quad (37)$$

- Normalize the measure of inputs to 1, and denote each machine line by $\nu \in [0, 1]$.

Preferences and Technology II

- Engine of economic growth: *quality improvement*.
- $q(\nu, t)$ =quality of machine line ν at time t .
- “Quality ladder” for each machine type:

$$q(\nu, t) = \lambda^{n(\nu, t)} q(\nu, 0) \text{ for all } \nu \text{ and } t, \quad (38)$$

where:

- $\lambda > 1$
- $n(\nu, t)$ =innovations on this machine line between 0 and t .
- Production function of the final good:

$$Y(t) = \frac{1}{1-\beta} \left[\int_0^1 q(\nu, t) x(\nu, t | q)^{1-\beta} d\nu \right] L^\beta, \quad (39)$$

where $x(\nu, t | q)$ =quantity of machine of type ν quality q .

- Implicit assumption in (39): at any point in time only one quality of any machine is used.

Innovation Possibilities Frontier I

- Cumulative R&D process.
- $Z(\nu, t)$ units of the final good for research on machine line ν , quality $q(\nu, t)$ generate a flow rate

$$\eta Z(\nu, t) / q(\nu, t)$$

of innovation.

- Note one unit of R&D spending is proportionately less effective when applied to a more advanced machine.
- Free entry into research.
- The firm that makes an innovation has a perpetual patent.
- But other firms can undertake research based on the product invented by this firm.

Innovation Possibilities Frontier II

- Once a machine of quality $q(v, t)$ has been invented, any quantity can be produced at the marginal cost $\psi q(v, t)$.
- New entrants undertake the R&D and innovation:
 - The incumbent has weaker incentives to innovate, since it would be replacing its own machine, and thus destroying the profits that it is already making (*Arrow's replacement effect*).

Equilibrium

- Allocation: time paths of
 - consumption levels, aggregate spending on machines, and aggregate R&D expenditure $[C(t), X(t), Z(t)]_{t=0}^{\infty}$,
 - machine qualities $[q(v, t)]_{v \in [0,1], t=0}^{\infty}$,
 - prices and quantities of each machine and the net present discounted value of profits from that machine, $[p^x(v, t | q), x(v, t), V(v, t | q)]_{v \in [0,1], t=0}^{\infty}$, and
 - interest rates and wage rates, $[r(t), w(t)]_{t=0}^{\infty}$.

Equilibrium: Innovations Regimes

- Demand for machines similar to before:

$$x(\nu, t | q) = \left(\frac{q(\nu, t)}{p^x(\nu, t | q)} \right)^{1/\beta} L \quad \text{for all } \nu \in [0, 1] \text{ and all } t, \quad (40)$$

where $p^x(\nu, t | q)$ refers to the price of machine type ν of quality $q(\nu, t)$ at time t .

- Two regimes:
 - 1 innovation is “drastic” and each firm can charge the unconstrained monopoly price,
 - 2 limit prices have to be used.
- Assume drastic innovations regime: λ is sufficiently large

$$\lambda \geq \left(\frac{1}{1 - \beta} \right)^{\frac{1 - \beta}{\beta}}. \quad (41)$$

- Again normalize $\psi \equiv 1 - \beta$

Monopoly Profits

- Profit-maximizing monopoly:

$$p^x(v, t | q) = q(v, t). \quad (42)$$

- Combining with (40)

$$x(v, t | q) = L. \quad (43)$$

- Thus, flow profits of monopolist:

$$\pi(v, t | q) = \beta q(v, t) L.$$

Characterization of Equilibrium I

- Substituting (43) into (39):

$$Y(t) = \frac{1}{1-\beta} Q(t) L, \quad (44)$$

where

$$Q(t) = \int_0^1 q(v, t) dv \quad (45)$$

- Equilibrium wage rate:

$$w(t) = \frac{\beta}{1-\beta} Q(t). \quad (46)$$

Characterization of Equilibrium II

- Value function for monopolist of variety v of quality $q(v, t)$ at time t :

$$r(t) V(v, t | q) - \dot{V}(v, t | q) = \pi(v, t | q) - z(v, t | q) V(v, t | q), \quad (47)$$

where:

- $z(v, t | q)$ = rate at which new innovations occur in sector v at time t ,
 - $\pi(v, t | q)$ = flow of profits.
- Last term captures the essence of Schumpeterian growth:
 - when innovation occurs, the monopolist loses its monopoly position and is replaced by the producer of the higher-quality machine.
 - From then on, it receives zero profits, and thus has zero value.
 - Because of Arrow's replacement effect, an entrant undertakes the innovation, thus $z(v, t | q)$ is the flow rate at which the incumbent will be replaced.

Characterization of Equilibrium III

- Free entry:

$$\eta V(v, t \mid q) \leq \lambda^{-1} q(v, t) \quad (48)$$

and $\eta V(v, t \mid q) = \lambda^{-1} q(v, t)$ if $Z(v, t \mid q) > 0$.

- Note: Even though the $q(v, t)$'s are stochastic as long as the $Z(v, t \mid q)$'s, are nonstochastic, average quality $Q(t)$, and thus total output, $Y(t)$, and total spending on machines, $X(t)$, will be nonstochastic.
- Consumer maximization implies the Euler equation,

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho), \quad (49)$$

- Transversality condition:

$$\lim_{t \rightarrow \infty} \left[\exp\left(-\int_0^t r(s) ds\right) \int_0^1 V(v, t \mid q) dv \right] = 0 \quad (50)$$

for all q .

Definition of Equilibrium

- $V(v, t | q)$, is nonstochastic: either q is not the highest quality in this machine line and $V(v, t | q)$ is equal to 0, or it is given by (47).
- An equilibrium can then be represented as time paths of
 - $[C(t), X(t), Z(t)]_{t=0}^{\infty}$ that satisfy (37), (??), (50),
 - $[Q(t)]_{t=0}^{\infty}$ and $[V(v, t | q)]_{v \in [0,1], t=0}^{\infty}$ consistent with (45), (47) and (48),
 - $[p^x(v, t | q), x(v, t)]_{v \in [0,1], t=0}^{\infty}$ given by (42) and (43), and
 - $[r(t), w(t)]_{t=0}^{\infty}$ that are consistent with (46) and (49)
- Balanced Growth Path defined similarly to before (constant growth of output, constant interest rate).

Balanced Growth Path I

- In BGP, consumption grows at the constant rate g_C^* , that must be the same rate as output growth, g^* .
- From (49), $r(t) = r^*$ for all t .
- If there is positive growth in BGP, there must be research at least in some sectors.
- Since profits and R&D costs are proportional to quality, whenever the free entry condition (48) holds as equality for one machine type, it will hold as equality for all of them.
- Thus,

$$V(v, t | q) = \frac{q(v, t)}{\lambda\eta}. \quad (51)$$

- Moreover, if it holds between t and $t + \Delta t$, $\dot{V}(v, t | q) = 0$, because the right-hand side of equation (51) is constant over time— $q(v, t)$ refers to the quality of the machine supplied by the incumbent, which does not change.

Balanced Growth Path II

- Since R&D for each machine type has the same productivity, constant in BGP:

$$z(v, t) = z(t) = z^*$$

- Then (47) implies

$$V(v, t | q) = \frac{\beta q(v, t) L}{r^* + z^*}. \quad (52)$$

- Note the *effective discount rate* is $r^* + z^*$.
- Combining this with (51):

$$r^* + z^* = \lambda \eta \beta L. \quad (53)$$

- From the fact that $g_C^* = g^*$ and (49), $g^* = (r^* - \rho) / \theta$, or

$$r^* = \theta g^* + \rho. \quad (54)$$

Balanced Growth Path III

- To solve for the BGP equilibrium, we need a final equation relating g^* to z^* . From (44)

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{Q}(t)}{Q(t)}.$$

- Note that in an interval of time Δt , $z(t) \Delta t$ sectors experience one innovation, and this will increase their productivity by λ .
- The measure of sectors experiencing more than one innovation within this time interval is $o(\Delta t)$ —i.e., it is second-order in Δt , so that

$$\text{as } \Delta t \rightarrow 0, o(\Delta t)/\Delta t \rightarrow 0.$$

- Therefore, we have

$$Q(t + \Delta t) = \lambda Q(t) z(t) \Delta t + (1 - z(t) \Delta t) Q(t) + o(\Delta t).$$

Balanced Growth Path IV

- Now subtracting $Q(t)$ from both sides, dividing by Δt and taking the limit as $\Delta t \rightarrow 0$, we obtain

$$\dot{Q}(t) = (\lambda - 1) z(t) Q(t).$$

- Therefore,

$$g^* = (\lambda - 1) z^*. \quad (55)$$

- Now combining (53)-(55), we obtain:

$$g^* = \frac{\lambda \eta \beta L - \rho}{\theta + (\lambda - 1)^{-1}}. \quad (56)$$

Summary of Balanced Growth Path

Proposition Consider the model of Schumpeterian growth described above. Suppose that

$$\lambda\eta\beta L > \rho > (1 - \theta) \frac{\lambda\eta\beta L - \rho}{\theta + (\lambda - 1)^{-1}} . \quad (57)$$

Then, there exists a unique balanced growth path in which average quality of machines, output and consumption grow at rate g^* given by (56). The rate of innovation is $g^* / (\lambda - 1)$.

- Also, as in the expanding input for IT model, there are no transitional dynamics.

Transitional Dynamics

Proposition In the model of Schumpeterian growth described above, starting with any average quality of machines $Q(0) > 0$, there are no transitional dynamics and the equilibrium path always involves constant growth at the rate g^* given by (56).

- Note only the average quality of machines, $Q(t)$, matters for the allocation of resources.
- Moreover, the incentives to undertake research are identical for two machine types ν and ν' , with different quality levels $q(\nu, t)$ and $q(\nu', t)$

Pareto Optimality in Schumpeterian Growth

- This equilibrium is typically Pareto suboptimal.
- But now distortions more complex than the expanding varieties model.
 - monopolists are not able to capture the entire social gain created by an innovation.
 - Business stealing effect.
- The equilibrium rate of innovation and growth can be too high or too low because of the business stealing effect.

Patents

- A very useful source of data on the quantity, quality and nature of innovation comes from patents data.
- A significant fraction of new innovations are patented to protect the property rights of the inventor.

USPTO defines a patent as:

A patent is a property right granted by the Government of the United States of America to an inventor to exclude others from making, using, offering for sale, or selling the invention throughout the United States or importing the invention into the United States for a limited time in exchange for public disclosure of the invention when the patent is granted.

What Can Be Patented

- To be patented, an invention must be:
 - Novel,
 - Nonobvious,
 - Adequately described or enabled (for one of ordinary skill in the art to make and use the invention), and
 - Claimed by the inventor in clear and definite terms.
- Utility patents are provided for a novel, nonobvious and useful:
 - Process,
 - Machine,
 - Article of manufacture, or
 - Composition of matter.
- The Patent Act of 1790 was the first federal patent statute of the United States, and set the length of a patent as 14 years. Since 1995, it is 20 years.

Some Examples: Watt's Steam Engine



A.D. 1769 N° 913.

Steam Engines, &c.

WATT'S SPECIFICATION.

TO ALL TO WHOM THESE PRESENTS SHALL COME, I, JAMES WATT, of Glasgow, in Scotland, Merchant, send greeting.

WHEREAS His most Excellent Majesty King George the Third, by His Letters Patent under the Great Seal of Great Britain, bearing date the Fifth day of January, in the ninth year of His said Majesty's reign, did give and grant unto me, the said James Watt, His special licence, full power, sole privilege and authority, that I, the said James Watt, my exors, adfiors, and assigns, should and lawfully might, during the term of years therein expressed, use, exercise, and vend, throughout that part of His Majesty's Kingdom of Great Britain called England, the Dominion of Wales, and Town of Berwick upon Tweed, and also in His Majesty's Colonies and Plantations abroad, my "NEW INVENTED METHOD OF LESSENING THE CONSUMPTION OF STEAM AND FUEL IN FIRE ENGINES;" in which said recited Letters Patent is contained a proviso obliging me, the said James Watt, by writing under my hand and seal, to cause a particular description of the nature of the said Invention to be inrolled in His Majesties High Court of Chancery within four calendar months after the date of the said recited Letters Patent, as in and by the said Letters Patent, and the Statute in that behalf made, relation being thereunto respectively had, may more at large appear.

NOW KNOW YE, that in compliance with the said proviso, and in pursuance of the said Statute, I, the said James Watt, do hereby declare that the

Some Examples: Watt's Steam Engine (continued)

A.D. 1769.—N^o 913.

3

Watt's Method of Lessening the Consumption of Steam & Fuel in Fire Engines.

weights are pressed, but not in the contrary. As the steam vessel moves round it is supplied with steam from the boiler, and that which has performed its office may either be discharged by means of condensers, or into the open air.

Sixthly, I intend in some cases to apply a degree of cold not capable of reducing the steam to water, but of contracting it considerably, so that the engines shall be worked by the alternate expansion and contraction of the steam.

Lastly, instead of using water to render the piston or other parts of the engines air and steam tight, I employ oils, wax, resinous bodies, fat of animals, quicksilver and other metals, in their fluid state.

In witness whereof, I have hereunto set my hand and seal, this Twenty-fifth day of April, in the year of our Lord One thousand seven hundred and sixty-nine.

JAMES WATT. (i.s.)

15 Sealed and delivered in the presence of

COLL. WILKIE.
GEO. JARDINE.
JOHN ROSSBUCK.

Be it remembered, that the said James Watt doth not intend that any thing in the fourth article shall be understood to extend to any engine where the water to be raised enters the steam vessell itself, or any vessell having an open communication with it.

JAMES WATT.

Witnesses,

25 COLL. WILKIE.
GEO. JARDINE.

AND BE IT REMEMBERED, that on the Twenty-fifth day of April, in the year of our Lord 1769, the aforesaid James Watt came before our said Lord the King in His Chancery, and acknowledged the Specification aforesaid, and all and every thing therein contained and specified, in form above written. And also the Specification aforesaid was stampt according to the tenor of the Statute made in the sixth year of the reign of the late King and Queen William and Mary of England, and so forth.

35 Inrolled the Twenty-ninth day of April, in the year of our Lord One thousand seven hundred and sixty-nine.

LONDON:

Printed by GEORGE EDWARD EYRE and WILLIAM SPOTTISWOODE,
Printers to the Queen's most Excellent Majesty. 1855.

WATT, ETC.

Some Examples: Apple's Touchscreen

<p>(12) United States Patent Ording</p> <hr/> <p>(54) DEVICE, METHOD, AND GRAPHICAL USER INTERFACE FOR LIST SCROLLING ON A TOUCH-SCREEN DISPLAY</p> <p>(75) Inventor: Bas Ording, San Francisco, CA (US)</p> <p>(73) Assignee: Apple Inc., Cupertino, CA (US)</p> <p>(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 763 days.</p> <p>This patent is subject to a terminal disclaimer.</p> <p>(21) Appl. No.: 12/270,807</p> <p>(22) Filed: Nov. 13, 2008</p> <p>(65) Prior Publication Data US 2009/0073194 A1 Mar. 19, 2009</p> <p>Related U.S. Application Data</p> <p>(63) Continuation of application No. 11/956,969, filed on Dec. 14, 2007, now Pat. No. 7,469,381.</p> <p>(60) Provisional application No. 60/937,993, filed on Jun. 29, 2007, provisional application No. 60/946,971, filed on Jun. 28, 2007, provisional application No. 60/945,858, filed on Jun. 22, 2007, provisional application No. 60/879,469, filed on Jan. 8, 2007, provisional application No. 60/883,801, filed on Jan. 7, 2007, provisional application No. 60/879,253, filed on Jan. 7, 2007.</p> <p>(51) Int. Cl. <i>G06F 3/00</i> (2006.01) <i>G06F 3/01</i> (2006.01)</p> <p>(52) U.S. Cl. 715/700; 715/786; 715/763; 715/784; 345/173; 345/156</p>	<p>(10) Patent No.: US 8,209,606 B2</p> <p>(45) Date of Patent: *Jun. 26, 2012</p> <p>(58) Field of Classification Search 345/173, 345/180, 156; 715/784, 763, 786, 700 See application file for complete search history.</p> <p>(56) References Cited</p> <p>U.S. PATENT DOCUMENTS 5,463,725 A 10/1995 Henckel et al. 395/155 (Continued)</p> <p>FOREIGN PATENT DOCUMENTS AU 2007283771 A1 4/2008 (Continued)</p> <p>OTHER PUBLICATIONS Examiner's Amendment dated Oct. 29, 2008, to related U.S. Appl. No. 11/956,969. (Continued)</p> <p><i>Primary Examiner</i> — Mark Rinehart <i>Assistant Examiner</i> — Tzun Vu (74) <i>Attorney, Agent, or Firm</i> — Morgan, Lewis & Bockius LLP</p> <p>(57) ABSTRACT In accordance with some embodiments, a computer-implemented method for use in conjunction with a touch screen display is disclosed. In the method, a movement of an object on or near the touch screen display is detected. In response to detecting the movement, a list of items displayed on the touch screen display is scrolled in a first direction. If a terminus of the list is reached while scrolling the list in the first direction while the object is still detected on or near the touch screen display, an area beyond the terminus of the list is displayed. In response to detecting that the object is no longer on or near the touch screen display, the list is scrolled in a second direction until the area beyond the terminus of the list is no longer displayed.</p> <p>21 Claims, 38 Drawing Sheets</p>
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Some Examples: Apple's Touchscreen (continued)

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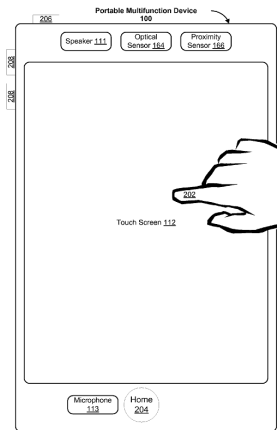


Figure 2

Some Examples: Apple's Touchscreen (continued)

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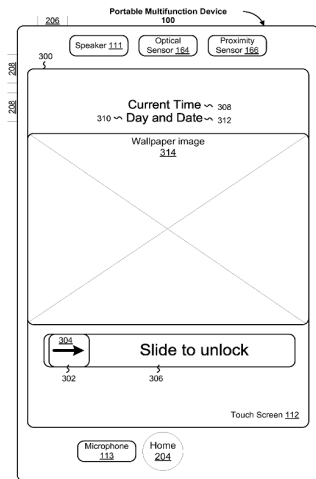


Figure 3

Patent Citations

- What makes patents a particularly useful source of data for measuring and modeling innovation is the data on patent citations.
- We know essentially the entire universe of patent citations.
- For example, between 1975 and 1990, a patent filed with the USPTO received about 8 cites (with a maximum of 631 cites) from other patents in the same time window. Only about 13-14% of this is self citation.

Patent Citations and Patent Value

- Considerable evidence suggests that patent value, and thus presumably patent quality, is correlated with patent citations, though there are many mitigating factors.
- For example:
 - Trajtenberg (1990): Individual patent specific social value for Computed Tomography Scanners related to citations
 - Hall, Jaffe and Trajtenberg (2005): Stock market value related to citations.
 - Bessen (2008): Patent renewals (decision to pay the annual renewal fee) related to citations

Patent Citations and Spillovers

- Another prima facie evidence in favor of the idea that innovation creates knowledge spillovers is that most patents “cite” other patents, indicating that they are “building” on them.
- However, this is not conclusive, since the citation may be done purely for bureaucratic reasons and after the fact (and in fact, many of the citations are added by patent examiners).
- If so, we would not know exactly how much “building on the shoulders of giants” there is.
 - Nevertheless, this would be an interesting source of data to exploit for this purpose.

Conclusions

- Knowledge spillovers are an important form of externality. Though they are not necessary for endogenous technological change, it is plausible that they are quite sizable.
- A variety of diverse evidence is consistent with the importance of these spillovers, but not always based on solid inference.
- Patent data and patent citation data can be used to investigate this question, as well as more generally as a very useful source of data in empirical work on innovation and technological change.
- Estimates of the spillovers that attempt to deal with major endogeneity issues and also spillovers taking place through product market competition suggest that knowledge spillovers are present and perhaps quite large.