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Expectations and crises: theory, empirics and policy

A Dissertation
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Doctor of Philosophy

by

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Abstract

Expectations and crises: theory, empirics and policy

Bernardo de Vasconcellos Guimarães

2004

Expectations about what others will do are crucial in currency and financial crises: in many situations, if everybody is expected to attack a currency, it is optimal to attack it, but if everybody is expected to refrain from doing so, that is the optimal choice. A literature starting with Morris and Shin (1998) applies recent theoretical developments to such macroeconomic problems in order to get models with endogenous expectations and a unique equilibrium.

This dissertation contributes to this literature in theoretical, empirical and policy-oriented aspects: (i) it studies the dynamics of currency crises; (ii) it generates testable predictions on expectations and performs empirical work; (iii) it studies how risk aversion and wealth affect agents' decisions; and (iv) it studies the relation between an international lender of last resort and moral hazard.

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Chapter 1

Introduction

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This dissertation contributes to this literature in theoretical, empirical and policy-oriented aspects: (i) it studies the dynamics of currency crises; (ii) it generates testable predictions on expectations and performs empirical work; (iii) it studies how risk aversion and wealth affect agents' decisions; and (iv) it studies the relation between an international lender of last resort and moral hazard.

The first essay presents a dynamic model of self-fulfilling currency crises with asset market frictions. Agents face the trade-off between the risk of a devaluation and the positive interest rate differential. Expectations about what others will do play a key role and are endogenously determined. The model has a unique threshold equilibrium. Asset market frictions have important indirect effects: contrary to a non-friction environment, large currency depreciations may occur and an attack that quickly forces the government to abandon a pegged regime may take considerable time to be triggered. I analyze the effects on agents' behavior of: interest rates, frictions, macroeconomic prospects and government's commitment to the peg. The model generates testable predictions on agents' expectations.

Although expectations are a key factor in currency crises, research in the field has not aimed at generating testable implications on expectations and confronting them with data. The second essay takes this step. Using data on exchange rate options, it identifies the probability of a currency devaluation and its expected magnitude, and thus characterizes market participants' expectations in the period leading up to the end of the Brazilian pegged exchange rate regime (from January-1997 to January-1999). The probability of a currency devaluation is very volatile and mostly affected by the Asian and Russian crises. Such episodes also lead to exchange rate depreciations in other Latin American countries, which suggests a link between the probability of the end of the peg and the "shadow exchange rate". The expected magnitude of a devaluation, conditional on its occurrence, is stable and virtually unrelated to the risk of a crisis. Those findings are consistent with key implications of my dynamic model presented in the first essay.

The third essay (joint work with Stephen Morris) analyzes the effects of risk aversion, wealth and portfolio on the behavior of investors in a "global games" model of currency crises with continuous action choices. The model generates a rich set of striking theoretical predictions. For example, risk aversion makes currency crises significantly less likely; increased wealth makes crises more likely; and foreign direct investment (illiquid investments in the target currency) make crises more likely. While our analysis concerns currency crises, the modelling may be relevant to a wide array of macroeconomic issues. The analysis of risk and wealth is central to macro. Self-fulfilling beliefs and strategic complementarities play an important role in many macroeconomic settings. In the marriage of these two strands in this essay, risk, wealth and portfolio effects play a central role in determining how strategic complementarities translate into economic outcomes.

The fourth essay (joint work with Giancarlo Corsetti and Nouriel Roubini) presents an analytical framework to study how an international institution providing liquidity can help stabilize financial markets via coordination of agents' expectations, and influences the incentives faced by policy makers to undertake efficiency-enhancing reform. We show that the influence of such an institution is increasing in the size of its interventions and the precision of its information. More liquidity support and better information make agents more willing to roll over their debt and reduces the probability of a crisis. Different from the conventional view stressing debtor moral hazard, liquidity provision and good policies can be strategic complement: in some cases, domestic government will not undertake costly policies/reforms unless contingent liquidity assistance is provided.

Chapter 2

Dynamics of currency crises with asset market frictions

2.1   Introduction

Expectations play a key role in currency crises. The optimal choice for an agent deciding whether to attack a currency depends crucially on what he expects others to do. The first models exploring the role of expectations in crises presented multiple equilibria: sudden changes in expectations could determine the outcome of the game (see, for example, Obstfeld, 1996). Events completely disconnected from economic fundamentals (sunspots) could trigger a currency crisis. While this literature sheds light on important features of the problem, it does not help us to understand what drives agents' expectations, leaving most of the explanation to exogenous shifts in beliefs.

By applying the theory of global games (Carlsson and Van Damme, 1993) to currency crisis problems, Morris and Shin (1998) were able to link agents' expectations on others' actions to economic variables and find a unique rationalizable equilibrium in a model of self-fulfilling currency attacks. Morris and Shin (1998) provide a framework for analyzing what drives coordination among agents and bring many insights on how economic fundamentals can influence beliefs.

In Morris and Shin (1998), all agents' decisions are taken in a single period. So, their framework cannot capture some important dynamic features of the problem. For example, in reality, an agent may go long in a currency even if he foresees a crisis, provided he thinks it is very likely that he will

3

be able to escape *before the crisis comes*. Moreover, incentives for an agent to hold the currency increase if all the others are expected to do the same, but if the other players are expected to run, an agent's payoff is higher if he is able to get rid of the currency *before all others do it*. There is not only *coordination* but also *competition* among agents. Morris and Shin (1998) model captures the coordination issue but not the preemptive motivations of investors.[1]

This essay presents a dynamic model of currency crisis. Agents must decide about going long or not in one unit of money in the home country. There is a trade-off between the risk of a devaluation and the positive interest rate differential. The shadow exchange rate changes stochastically through time.[2] Government's ability to sustain the peg depends on how overvalued the currency is and on how many agents are long in the currency. Expectations on what others will do play a key role and are endogenously determined. The model may be seen as dynamic version of Morris and Shin (1998), with asset market frictions. Like Morris and Shin (1998), it focuses on the investors' problem. It does not attempt to explain why the government decided for a pegged exchange rate and what determines the 'shadow exchange rate'.

Asset market frictions are modelled in a stylized way: agents get the opportunity of changing position according to a Poisson process, as in Calvo (1983). This yields an interesting representation of the agent's problem. In the event of a run, the odds he will escape are the same as of any other agent's (due to the assumption of a Poisson process). The occurrence and size of a devaluation depend on others' decisions and on a stochastic shadow exchange rate. To make a decision, an agent tries to forecast what others will do and estimates his expected payoff of holding the currency.

Asset market frictions have important indirect effects in our model. Discrete currency devaluations may occur even when there is a 'secular deterioration of fundamentals' and no uncertainty on the economic variables.[3] An attack that would force the government to abandon the peg in a couple of weeks may take several months to be started. Agents may choose to face the risk of losing money with a devaluation because they profit with the higher domestic interest rates and may escape before the crisis comes. Indeed, many developing countries have been able to sustain their pegged regimes

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[1]Such essentially dynamic features of a currency crisis situation are also absent in the dynamic model of currency crisis in Morris and Shin (1999).

[2]The shadow exchange rate is what the exchange rate would be if the currency was floating.

[3]Discrete jumps in models a la Krugman (1979) have been obtained in the literature by introducing some kind of asset market imperfection. For example, Broner (2002) uses asymmetric information and learning to get it.

in spite of being vulnerable to currency attacks for quite a long time.

By modelling the macroeconomic environment and the asset market frictions in a stylized way and focusing on investor's decisions, this essay brings insights on the dynamics of currency crisis and analyzes the impacts on agent's behavior of some of the main instruments a government has to fight an attack: raising interest rates, increasing frictions, improving macroeconomic prospects and signalling strong commitment to the peg. Actually, some features of this model often show up in economic and policy analysis but are rarely incorporated in formal models. It would be desirable to incorporate this dynamic structure in a richer macroeconomic setup, although the weak links between the (shadow) exchange rate and the available economic models are not encouraging.

The model presented in this essay yields a distinguishable testable implication. It predicts that a speculative attack is triggered when the shadow exchange rate hits a threshold. Therefore, the probability of a devaluation varies according to the shadow exchange rate — it depends on how far the economy is from the threshold. The expected magnitude of a devaluation, conditional on its occurrence, is relatively stable, because agents know that when the speculative attack starts, the shadow exchange rate will be around the threshold. Completely different conclusions arise if a speculative attack is triggered by sunspots. The essay in chapter 3 verifies such predictions using data on Brazilian exchange rate options and finds empirical support for this model.

Although its motivation is applied, this essay is closely related to some theoretical work. Much of the structure of the model is borrowed from Burdzy, Frankel and Pauzner (2001) theoretical contribution and, especially, Frankel and Pauzner (2000) model of sectorial choice. Like in those papers, agents choose between 2 actions and get the opportunity to revise their decisions according to a Poisson process. Payoffs depend on two state variables: a random economic parameter that follows a Brownian motion and the fraction of agents that had chosen one of the actions. However, some assumptions made by Frankel and Pauzner (2000) do not hold in the context of this essay. In particular, they assume that agents' actions are strategic complements. In our model, although a higher willingness from the agents to go long in the currency reduces the likelihood of a crisis, it increases the magnitude of the devaluation, conditional on its occurrence. So, in some situations, an agent would go long in a currency if all others were *not* doing so but would stay out if all other agents were expected to go long in the currency. Although we cannot apply Frankel-Pauzner tools to obtain a unique rationalizable equilibrium, we show the existence of a unique *threshold equilibrium*

— a situation in which agents decide to invest or not according to a shadow-exchange-rate threshold.

In another related theoretical work, Abreu and Brunnemeier (2003) show how incomplete information can lead to bubbles and crashes. In their model, agents may decide to buy an overvalued asset although they know that the bubble will burst at some point in the future. While they focus on explaining asset market frictions, this essay tries to understand the dynamics of currency crisis in a world with frictions. Like in this model, Abreu and Brunnemeier (2003) also have elements of coordination and competition, preemption motivations are key in their work. The payoff structure in this essay is similar to Abreu and Brunnemeier (2003) but frictions are as in Calvo (1983) and Frankel and Pauzner (2000) for two main reasons: (i) it makes the analysis more tractable (qualitative predictions should be the same) and (ii) learning issues, while of independent interest, are not the heart of the story.

Some contributions to the study of the dynamics of currency crises have dealt with incomplete information and learning. In Broner (2002), there is a 'secular deterioration of fundamentals' and agents try to guess when the currency will be 'ripe for attack'. In Chamley (2003), agents try to learn whether the mass of speculators is enough to force the Central Bank to abandon the peg. This essay takes timing frictions as a starting point instead of attempting to explain what generates them. Although it may be common knowledge that an attack would be successful, agents may consider it is 'too early to leave' and refrain from attacking a currency for a while.

2.2 Setup

2.2.1 The exchange rate regime

The exchange rate at the home country is fixed at 1, but would be equal to $\exp(\theta)$, the *shadow exchange rate*, if the peg was abandoned. The parameter θ measures the amount of overvaluation of the home currency — a negative θ would indicate undervaluation.

There is a continuum of agents, with mass equal to 1, that choose either to go long or not in 1 unit of currency in the Home country — hereafter, we will denote agents' possible choices by *Long* and *Not*, respectively.[4] At any point in time, a fraction A of the agents is currently long in the currency. Foreign interest rate is normalized to 0 while Home country deposits pay interest $r > 0$.

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[4]Nothing substantial changes in the model if agents choose between going short or not, for example.

The government keeps the peg as long as it can (as in Krugman, 1979). There exists a function of A — $\tilde{\theta} : [0,1] \to R_+$ — such that the government is able to keep the peg whenever (θ, A) is at the left of the curve at figure 2.1. Pressure on the peg is higher when θ is higher (overvaluation is greater) and when A is lower (less people carry the currency). Following the literature (e.g., Morris and Shin, 1998), this essay assumes a positively inclined function $\tilde{\theta}$ ($\tilde{\theta}'(A) > 0$) instead of modelling the evolution of reserves and benefits of keeping the peg. In a reduced form way, this formulation captures the idea that higher values of A imply higher level of reserves and higher values of θ imply higher current account deficits and more economic distortions.
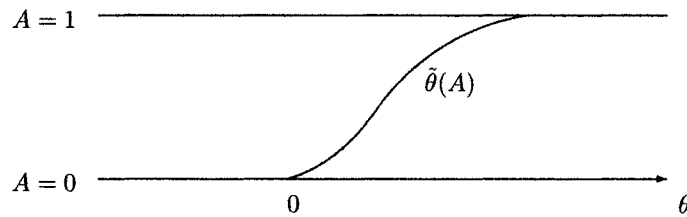
Figure 2.1: Government threshold for keeping the peg ($\tilde{\theta}(A)$)

The function $\tilde{\theta}$, as well as the current values of A and θ, are common knowledge. If $A = 0$, the peg cannot be even slightly overvalued, so $\tilde{\theta}(0) = 0$. Once the peg is abandoned, the exchange rate jumps to its shadow value (e^θ) and the game ends.

Particular attention will be paid to the case when $\tilde{\theta}(A) = \kappa A$. The higher κ, the higher is the willingness (and ability) of the government to keep the peg when the currency is overvalued.

The parameter θ follows a Brownian motion, which is a good approximation for the behavior of the shadow exchange rate in the short run.[5] So:

$$d\theta = \mu_\theta dt + \sigma_\theta dX$$

The state variables of the model are θ and A, so the current state of the economy is described by a point at figure 2.1.

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[5] A literature that started with Meese and Rogoff (1983) has shown that, in the short run, fundamental-based models cannot explain the exchange rate behavior better than a simple random walk. For a recent reference, see Bergin (2003).

2.2.2 Asset market frictions

Each investor gets the opportunity to change position according to an independent Poisson process with arrival rate δ, assumed to be greater than r.[6] So, if the currency is overvalued and, at a given moment, all agents decide they should take their money out of the country, all are ex-ante equally likely to succeed in running before the crisis comes. But ex-post, some will be caught by the devaluation while others will escape. As in Morris and Shin (1998), an agent's decision must take into account the risk of a devaluation, which depends on others' actions.

The frictions and the assumption that the currency will remain fixed until $\tilde{\theta}$ is reached give some time for the agents to run and compete against each other for the country's foreign reserves. Indeed, in reality, a peg is not abandoned at the day an attack starts. For example, the attack to the Brazilian *Real* started at the end of August-98 but the currency was only allowed to float in January-99. Such government behavior is justified by at least two reasons: (i) when an attack starts, a complicated interaction among the government, private investors and international organizations (e.g., the IMF) takes place and it is far from clear, at the beginning, that the attack will succeed in forcing the currency to float; (ii) even if the government knows the peg will have to be abandoned, it will be willing to use its reserves to avoid or reduce bankruptcies of un-hedged companies. Choosing ad-hoc those who will get such benefits may not be politically feasible, so letting the private sector deplete its stock of foreign currency may be the best strategy.

2.2.3 The agent's problem

When the opportunity of changing position comes, the agent decides between *Long* or *Not*. His decision holds until the next random signal comes. Choosing *Not* yields 0 up to the next opportunity of changing portfolio.

In general, a strategy could assign a decision for every history, that is, for every path of (θ, A). However, an agent's payoff of investing depends just on the current state and on others' strategies, so past values of (θ, A) are relevant only if agents condition their actions on past events in an arbitrary way. Future work may answer if it is possible to construct equilibria in which the path

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[6]Burdzy and Frankel (2002) extend Frankel and Pauzner (2000) model by allowing agents to raise their arrival rate (at a cost) and show that uniqueness of equilibrium still holds. It is reasonable to conjecture that the same would happen in the context of this essay.

of (θ, A) matters. This essay restricts our attention to strategies that depend only on the current state. So, a strategy for the agent yields a decision ($Long$ or Not) for every pair (θ, A), that is: $s : R^2 \rightarrow \{Long, Not\}$.

A process $\{x\} = \{\theta, A\}$ will denote a particular path of the state variables of the model, and $\{z\}$ will denote a particular realization of the Brownian motion. Suppose that all other agents are following a strategy around a threshold θ^* as shown in figure 2.2. The threshold θ^* defines 2 regions: the 'L' area, at its left, where agents choose $Long$ and the 'N' area, at its right, where agents choose Not. Let (θ_0, A_0) denote the current state of the economy. Call $\Delta t(z)$ the time it will take for the crisis and $\theta^{end}(z)$ the size of the devaluation.[7]
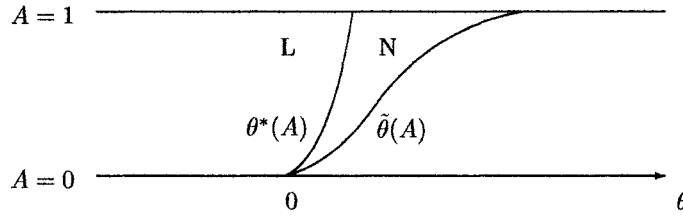
Figure 2.2: Investors's threshold for investing $(\theta^*(A))$

The difference between the expected payoffs of choosing $Long$ and Not equals the expected payoff of going long in the currency until the next opportunity of choosing.[8] Given z and θ^*, the only source of uncertainty is the realization of the Poisson process, and the expected payoff of choosing $Long$ is given by:

$$\pi(z; \theta_0, A_0, \theta^*) = \int_0^{\Delta t(z)} \delta e^{-(\delta-r)t} dt \; + \; e^{r\Delta t(z)-\theta^{end}} \int_{\Delta t(z)}^{\infty} \delta e^{-\delta t} dt - 1$$

The first term is what an agent gets if he receives a signal before time $\Delta t(z)$. The second term is the agent's return if he is caught by the devaluation.

Doing the algebra, we get:

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[7]The shadow exchange rate, θ, follows a Brownian motion so, eventually, the peg will be abandoned. Indeed, Obstfeld and Rogoff (1995) pointed that few countries were able to sustain their pegged regimes in the long run, and since that paper was published many developing countries were forced to let their currencies float.

[8]What happens after that is irrelevant because the present choice will have no influence in future decisions.

$$\pi(z; \theta_0, A_0, \theta^*) = \left(1 - e^{-(\delta-r)\Delta t(z)}\right) \frac{\delta}{\delta - r} + e^{-(\delta-r)\Delta t(z) - \theta^{end}(z)} - 1 \qquad (2.0)$$

Long is the optimal choice if $E\pi(\theta_0, A_0, \theta^*) \geq 0$ where:

$$E\pi(\theta_0, A_0, \theta^*) = \int_z \pi(z; \theta_0, A_0, \theta^*) f(z) dz \qquad (2.0)$$

2.2.4 Example

Next section shows that if $\sigma_\theta > 0$ the model exhibits a unique threshold equilibrium. The threshold $\theta^*(A)$ for specific parameters δ, r, κ, μ_θ and σ_θ can be found using numerical methods, described in the appendix. The task consists in finding a function $\theta^*(A)$ that solves (approximately) equation 2.2.3.

Figure 2.3 brings the threshold θ^* and an example of the path of $x = (\theta, A)$ for $\delta = 4$, $r = 0.03$, $\mu_\theta = 0$, $\sigma_\theta = 0.05$ and $\kappa = 1$.

The system starts at the point indicated by a circle ($x_0 = (0.095, 0.9)$). All agents choose *Not* and A drops to around 0.35 in just one week. But then, x crosses θ^*, gets to the 'L' area and everybody starts to go long in the currency again. Five months later, A is very close to 1 and x crosses the threshold θ^*. Two weeks later, $\tilde{\theta}$ is reached and the peg is gone. The exchange rate jumps to $e^{\theta^{end}}$, that is, to around 1.15. 85% of the agents are able to escape before the devaluation comes.

Ex-post, the payoff of a particular agent depends on luck: some will get a lot of interest on their money and pick *Not* right before the crisis, others will withdraw earlier, others will be caught by the devaluation, and so on. Ex-ante, every agent has the same expected payoff.

An agent that decided right before the threshold was crossed at the second time, at the point indicated by an 'x' in figure 2.3, chose *Long* for 2 reasons: first, in the case of a 'good' realization of z from then on, agents would keep choosing *Long* for a long time, as they had done for months and there would be no crisis. Second, in the case of a 'bad' realization of z, the agent could still get the signal and run before the crisis, as 85% of the agents did. In the two weeks preceding the devaluation, the interest earned was just 1.5%, much less than the depreciation of 15%, but good enough for encouraging agents deciding at the 'L' area to take the risk.
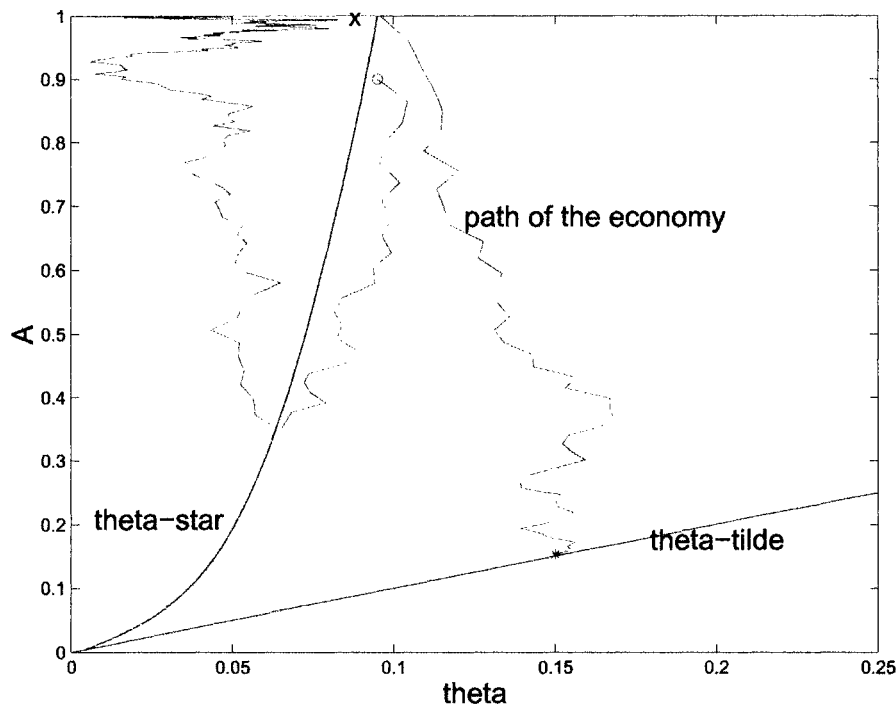
Figure 2.3: Example of the dynamics of the model

Sometimes, large speculative attacks follow small changes in economic variables. As Obstfeld and Rogoff (1995) point, "the speculative attack on the British pound in September 1992 would certainly have succeeded had it occurred in August — so why did speculators wait?".[9] In the model, a speculative attack would certainly succeed if it had started at the point indicated with an 'x' or even before. Speculators wait because it pays off to face the risk. In equilibrium, a crisis may be triggered by a small change in θ, if it pushes the shadow exchange rate beyond the threshold θ^*.

Countries usually resist for quite a long time before abandoning their pegs and many agents are able to escape from the crisis with little or no loss. Many times, the country wins the battle and the peg is not abandoned (in the example, that happened in the very first week, when A dropped from 0.9 to 0.35). Agents who decide not to withdraw their money in such cases make profits.

If herd behavior is understood as coping the others, there is no such a thing in this model. All agents are acting rationally based solely on the state variables (A and θ) and on what they expect others to do.

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[9]Page 86.

2.3 Equilibrium

This section discusses some technical properties of the proposed framework, compares it to the model in Frankel and Pauzner (2000), defines and shows existence and uniqueness of a *threshold* equilibrium.

2.3.1 Properties of the model

Frankel and Pauzner (2000) present a model in which agents choose between 2 actions and get the opportunity to revise their decisions according to a Poisson process. As in this essay, payoffs depend on two state variables: a random economic parameter that follows a Brownian motion and the fraction of agents that had choose one of the actions. They show that a unique strategy survives iterated deletion of strictly dominated strategies.

However, their results cannot be applied here because, in their framework, player's actions are *strategic complements*: the incentives to choose an action depend positively on the fraction of agents that are expected to choose it in the future.

It could seem that such property would hold in this model. It is true that the likelihood of a crisis depends negatively on the fraction of agents that will choose *Long*. However, the magnitude of the devaluation depends *positively* on the fraction of agents expected to go long in the currency in the future. The choice of *Not* may work as a 'discipline device', by preventing larger currency over-valuation.

Figure 2.4-a shows an example to illustrate why the property of strategic complementarities does not hold in this model.[10] Suppose that all other agents will choose *Not* . Then, an agent choosing *Long* at ($\theta = -0.05, A = 0$) has a positive expected payoff of around 0.0025, because the peg will be abandoned when θ hits 0, so there is no risk of losing money. On the other hand, if all other agents are choosing *long*, her expected payoff drops to around -0.0200. The graph at (a) shows an example of the path of θ when everybody picks *Long*, starting from the point marked with *: the possibility of getting a high devaluation makes going long in the currency a risky business.

The existence of *strategic complementarities* is key to the results of Frankel and Pauzner (2000) and also to the global games literature (see, for example, Morris and Shin (2002)).[11] If that property

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[10]The parameters used in the example are: $\delta = 1, r = 0.01, \mu_\theta = 0.1, \sigma_\theta = 0.1$ and $\tilde{\theta} = 0.25A^{\frac{1}{3}}$.

[11]Indeed, the argument for uniqueness in the global games model of Frankel, Morris and Pauzner (2003) and in the

holds, the worst possible scenario for a player that chose *Long* is everybody else choosing *Not*. Then, we can iteratively delete strictly dominated strategies and check if such procedure leads to a unique equilibrium.
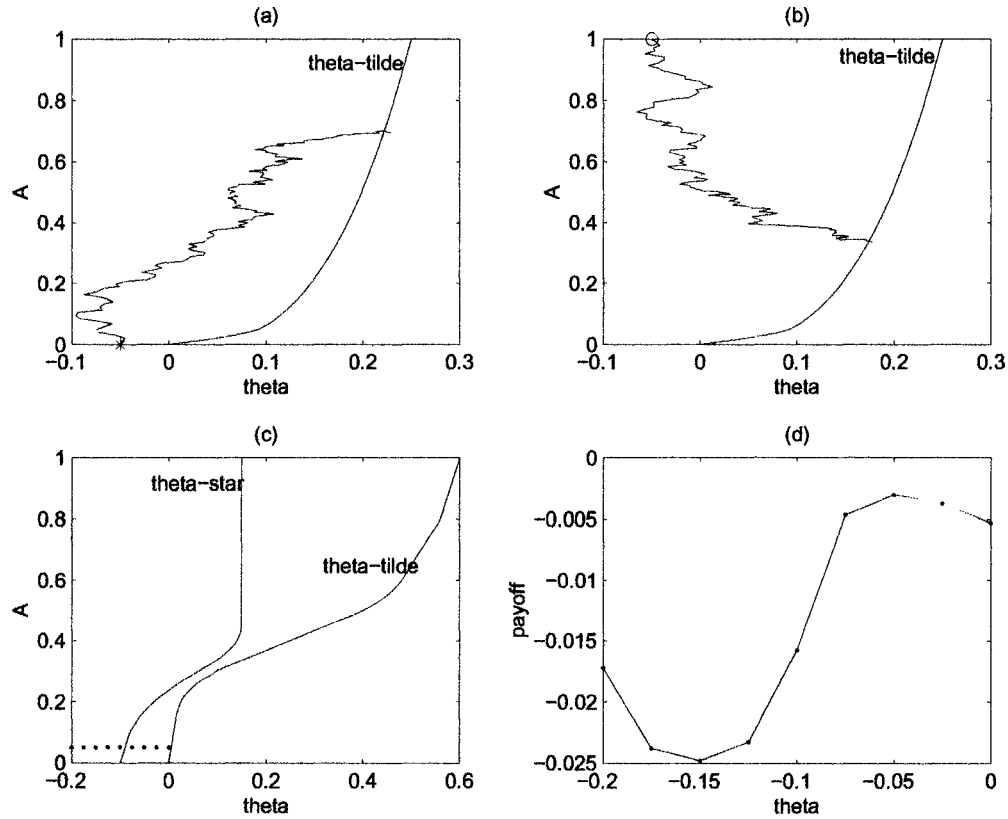
Figure 2.4: Counter examples for some usual assumptions

It is worth mentioning some other complicating features of this model. First, $E\pi$ is not always increasing in A. Suppose the same parameter values used in figure 2.4-a. Assume that everybody else will choose *Not* and $\theta = -0.05$. If $A = 0$, choosing *Long* yields a positive payoff, but if $A = 1$, the expected payoff of *Long* is around -0.0200. The graph at 2.4-b shows an example of the path of θ when everybody picks *Not*, starting from the point marked with a circle. The large devaluation would not happen if nobody was long in the currency at the beginning.

Second, it is also possible to build examples in which $E\pi$ is not monotonically decreasing in θ. This assumption sounds very natural: the higher is the shadow exchange rate, the less incentives an

dynamic model of Frankel and Pauzner (2000) are similar.

agent has to choose *Long*. That indeed holds *in equilibrium*, but it does not work for any threshold in any situation.

Graph 2.4-c shows an example of a situation in which payoff is sometimes *increasing* with respect to θ.[12] Graph 2.4-d shows the payoffs at points marked with a dot in graph 2.4-c. The key for this example are the high values of μ_θ and σ_θ. When the economy is close enough to $\tilde{\theta}$, in the 'N' area, the payoff is negative but small in absolute value because a small devaluation is very likely to come quickly. However, at those points further from 0, A and θ are likely to increase at the beginning and the devaluation is likely to be much greater and take not much time. So, choosing *Long* is worse in this case.

The last example may sound a bit contrived. Indeed, in equilibrium, payoffs are always decreasing in θ as we would expect. So, the failure of the such assumption may not have so interesting economic insights. However, it presents theoretical difficulties to prove the existence and uniqueness of equilibrium.

Although Frankel-Pauzner techniques cannot be applied here, we can still show that the model exhibits a unique *threshold equilibrium*, as we define now.

Last, if σ_θ is arbitrarily small or if the size of the devaluation is a known constant and θ changes just government's willingness/ability to keep the peg, there are strategic complementarities whenever $\theta \in (0,1)$, all 3 assumptions mentioned above hold and we can apply Frankel-Pauzner tools.

2.3.2 Existence and uniqueness

In a threshold equilibrium, choosing according to the threshold is the optimal decision for an agent given that all others are doing so. The next definition formalizes this concept.

Definition 1 *A Threshold equilibrium is characterized by a continuous function* $\theta^* : [0,1] \rightarrow (-\infty, \tilde{\theta}(1)]$. *An agent deciding at time t chooses (optimally)* Long *if* $\theta_t < \theta^*(A_t)$ *and* Not *if* $\theta_t > \theta^*(A_t)$.

The main general result of this essay is given by the following theorem:

Theorem 1 *If $\sigma_\theta \neq 0$, there exists a unique threshold equilibrium.*

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[12]The parameters are: $\delta = 1, r = .01, \mu_\theta = 0.2, \sigma_\theta = 0.1$, and θ^* and $\tilde{\theta}$ are shown in the figure.

The proof of the theorem is fully presented in the appendix. It is divided in 3 steps, that can be summarized as following:

1. There exists a function θ^* such that $\forall a, E\pi(\theta^*(a), a; \theta^*) = 0$.

2. If step 1 holds, *Long* is the optimal strategy at the 'L' area and *Not* is the optimal strategy at the 'N' area.

3. If steps 1 and 2 hold, there is a unique threshold equilibrium.

The proof of step 1 frames the problem in a way that allows it to apply the Schauder's fixed point theorem.[13] The existence of dominant regions and continuity of $E\pi(\theta^*(A), A; \theta^*)$ on θ^* are the 2 key factors of the proof.

The proof of step 2 starts by assuming that all agents are playing according to a threshold equilibrium around θ^*, such that $\forall a, E\pi(\theta^*(a), a; \theta^*) = 0$. At the 'L' area, players must have positive payoffs because when the economy gets to θ^*, they will be indifferent and, up to that moment, they will be profiting from the positive interest rates r. A key assumption for the argument is that the opportunity of changing position is a Poisson event.

At the 'N' area, the proof is trickier. It compares two processes: x', starting at $x'_0 = (\theta'_0, A)$, and x, starting at $x_0 = (\theta_0, A)$, such that $\theta_0 > \theta'_0 \geq \theta^*(A)$, as indicating at figure 2.5. The problem is that x' can get a lower payoff than x by achieving the 'L' area, going up in A and getting a higher devaluation. It is shown that although that can indeed happen for some realizations of the Brownian motion z, the *expected* payoff of x' is always higher than the *expected* payoff of x. In order to get a higher devaluation, x' needs to go to the 'L' area and, then, cross again the threshold θ^* to enter the 'N' area. The key to the argument is that when x' crosses the threshold, its expected payoff is zero, *as if it was crossing θ^* at any other point.*

As the argument above illustrates, the proof of step 2 relies on the expected payoff at any point of θ^* being equal to zero. That does not hold in the examples shown at figure 2.4-c-d.

The proof of step 3 supposes the existence of 2 threshold equilibria, characterized by $\underline{\theta}$ and $\bar{\theta}$, respectively, and finds a contradiction. Denote by a the point that maximizes the horizontal distance between the two curves — indicated at figure 2.6 — and consider a process x' starting at

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[13]Schauder's fixed point theorem extends Brouwer's theorem to more generic metric spaces.
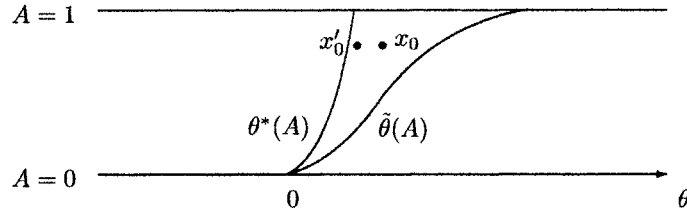
Figure 2.5: Proof of step 2

$x'_0 = (a, \underline{\theta}(a))$ when all agents follows a switching strategy around $\underline{\theta}(A)$ and a process x starting at $x_0 = (a, \bar{\theta}(a))$ when all agents follow a switching strategy around $\bar{\theta}(A)$.
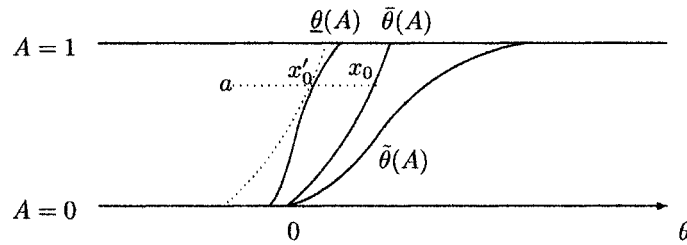
Figure 2.6: Proof of step 3

For any realization of the Brownian motion, at the first time x and x' are not at the same side of their own thresholds, the expected payoff of x is lower: either x' is over $\underline{\theta}$ (expected payoff is 0) and x is at its 'N' area (expected payoff is negative) or x' is at its 'L' area (expected payoff is positive) and x is over $\bar{\theta}$ (expected payoff is 0). If x and x' are always at the same side of the threshold until x gets to $\tilde{\theta}$, then, at that moment, x' will be at its 'N' area and will either get a smaller devaluation or hit $\underline{\theta}$ (which is the same as getting devaluation equal to 0).

The uniqueness result holds regardless of the sizes of σ_θ and δ.

Solutions for the general case are not available. Next, analytical results for a particular case and some numerical examples are presented.

2.4 Particular case: 'first generation' model

The so called 'first generation' models of currency crisis emphasized that a 'secular deterioration of fundamentals' would eventually lead to a speculative attack, a massive fall on reserves of a country and the abandonment of the peg. However, there would be no discrete jump in the exchange rate, as everybody would correctly foresee the crisis and arbitrage opportunities would be ruled out. Krugman (1979) and Flood and Garber (1984) are the main examples of such models. This situation corresponds to the case when the shadow exchange rate is increasing in time ($\mu_\theta > 0$) and there is no uncertainty ($\sigma_\theta = 0$). With asset market frictions, agents delay their decisions to attack the currency (pick *Not*) and try to 'beat the gun'. This behavior generates a discrete currency devaluation.

Existence and uniqueness of a threshold equilibrium come easier in this particular case. It is possible to find an expression for a curve such that expected payoff equals 0 at all its points provided everybody from that point on chooses *Not*. That is the threshold — it exists and is unique. As the time for a devaluation and its size are deterministic functions of the threshold and the current state, it is clear that *Long* is the optimal action at all points at the left of the threshold and *Not* is the optimal action at all points at its right. The appendix goes further on this issue.

To characterize the behavior of the system, we need to determine, for each A_0: $\theta^*(A_0)$ (Θ^* henceforth); the time that it takes for the peg to collapse since θ crosses the threshold (call it ΔT); the value of A when $\tilde{\theta}$ is reached (call it $\tilde{A}$); and the magnitude of the devaluation $\tilde{\theta}(\tilde{A})$ ($\tilde{\Theta}$ henceforth).

The time for a devaluation depends on the distance the parameter θ covers since agents start to choose *Not*:

$$\Delta T = \frac{\tilde{\Theta} - \Theta^*}{\mu_\theta} \tag{2.0}$$

As $\mu_\theta > 0$, an agent at $\theta^*(A_0)$ knows that everybody will choose *Not* from then on. So, the time for a devaluation is also related to the fraction of agents that is able to run before the devaluation:

$$\tilde{A} = A_0 \, e^{-\delta \Delta T} \tag{2.0}$$

As ΔT is known in equilibrium, the agents' optimization problem gets simpler: at θ^*, payoff is

given by equation 2.2.3, with the appropriate change on notation, that is:

$$E\pi = \left(1 - e^{-(\delta-r)\Delta T}\right)\frac{\delta}{\delta - r} + e^{(r-\delta)\Delta T - \tilde{\Theta}} - 1 = 0 \tag{2.0}$$

Solving for ΔT, we get:

$$\Delta T = \frac{\log\left(\frac{\delta}{r}\left(1 - e^{-\tilde{\Theta}}\right) + e^{-\tilde{\Theta}}\right)}{\delta - r} \tag{2.0}$$

Equations 2.4 and 2.4 yield:

$$\frac{\tilde{A}}{A_0} = \left(\frac{r}{re^{-\tilde{\Theta}} + \delta\left(1 - e^{-\tilde{\Theta}}\right)}\right)^{\frac{\delta}{\delta - r}} \tag{2.0}$$

The function $\tilde{\Theta} = \tilde{\theta}(\tilde{A})$ and equation 2.4 yield $\tilde{A}$ and we can easily back out all other parameters.

Note that even though there is no uncertainty on the path of θ, luck plays an important role in determining each agent's ex-post payoff.

Let $\psi = \frac{\delta}{r}$. Then, the equation 2.4 becomes:

$$\frac{\tilde{A}}{A_0} = \left(\psi\left(1 - e^{-\tilde{\Theta}}\right) + e^{-\tilde{\Theta}}\right)^{\frac{-\psi}{\psi - 1}} \tag{2.0}$$

Equation 2.4 shows that $\tilde{A}$ (and, consequently, the magnitude of the devaluation, $\tilde{\Theta}$) depends only on the initial state and on the ratio between δ and r. The next proposition presents some properties of the equilibrium.

Proposition 1 *Consider the model described above, when $\mu_\theta > 0$ and $\sigma_\theta = 0$. The magnitude of the devaluation ($\tilde{\Theta}$) and the fraction of agents that are not able to run before the crisis comes ($\tilde{A}$) are positively related to the interest rate differential (r) and negatively related to the arrival rate of the Poisson process (δ). Moreover, as δ goes to infinity, all endogenous variables ($\tilde{A}$, $\tilde{\Theta}$, ΔT and Θ^*) go to zero (but they do so at a slower rate than δ). That is:*

1. $\frac{d\tilde{A}}{d\delta} < 0$ (and so, $\frac{d\tilde{\Theta}}{d\delta} < 0$).

2. $\frac{d\tilde{A}}{dr} > 0$ (and so, $\frac{d\tilde{\Theta}}{dr} > 0$).

3. $\lim_{\delta\to\infty} \tilde{A} = \lim_{\delta\to\infty} \tilde{\Theta} = \lim_{\delta\to\infty} \Delta T = \lim_{\delta\to\infty} \Theta^* = 0$.

Corollary 1 *For any positive δ, $\tilde{A}$ and $\tilde{\Theta}$ are strictly positive.*

The higher is the interest rate differential, the higher are the incentives for an agent to go long in the currency and face the risk of not being able to run before the crowd. Moreover, higher frictions in the asset market (i. e. lower values of δ) imply a higher depreciation. One could think that if agents had less frequent opportunities of choosing (δ was lower), they would have an incentive to withdraw their money earlier, and so depreciation would come sooner. However, a lower δ also implies that *other agents* will take more time to change position, so it does not reduce the odds of escaping. The important effect of a reduction in δ is the increase in the time an attack takes (ΔT) and, therefore, in the interest got during this period. Loosely speaking, the benefit of going long in the currency is proportional to $1/\delta$ — as equation 2.4 shows, an increase in $1/\delta$ and an increase in r have exactly the same effect in $\tilde{A}$ and $\tilde{\Theta}$.

The choice of the timing of an attack depends on what an agent expects others to do. When δ is finite, there is a discrete jump in the exchange rate ($\tilde{\Theta} > 0$) and a positive number of agents ($\tilde{A} > 0$) lose money. An expected speculative attack following a 'secular deterioration of fundamentals' is not incompatible with a discrete devaluation, provided there are frictions in the asset markets.

Some examples help to illustrate the above relationships. Figure 2.7 shows the path to the devaluation when $\tilde{\theta}(A) = A$, $A_0 = 0.5$, $\mu = 0.01$/month, $r = 0.03$/month and $\delta = 2$ and 4, which means that the average time an agent waits to change position are 15 days and a week, respectively. Each dot in the figure is spaced by a period of 0.1 months. The currency is initially fixed at its shadow value.

As the bottom graph ($\delta = 4$) shows, although the exchange rate is always overvalued and initial A equals to 0.5, agents choose *Long* for the first 7.8 months. Then, θ hits the θ^* line and agents start to run. After just 0.62 months, the peg is abandoned. The exchange rate will be equal to $e^{0.0841} = 1.0877$, and 8.41% of the agents will be caught by the devaluation.

An agent that gets the chance to choose right before θ hits the threshold θ^* reasons that the likelihood of being able to run before the devaluation comes is high (around 91.6%) and the interest rate he gets during this period is worth the risk. Therefore, he picks *Long* and tries later to run faster than the crowd.

To get further intuition from the model, consider $\tilde{\theta}(A) = \kappa A, \kappa > 0$. Then, the above equation becomes:
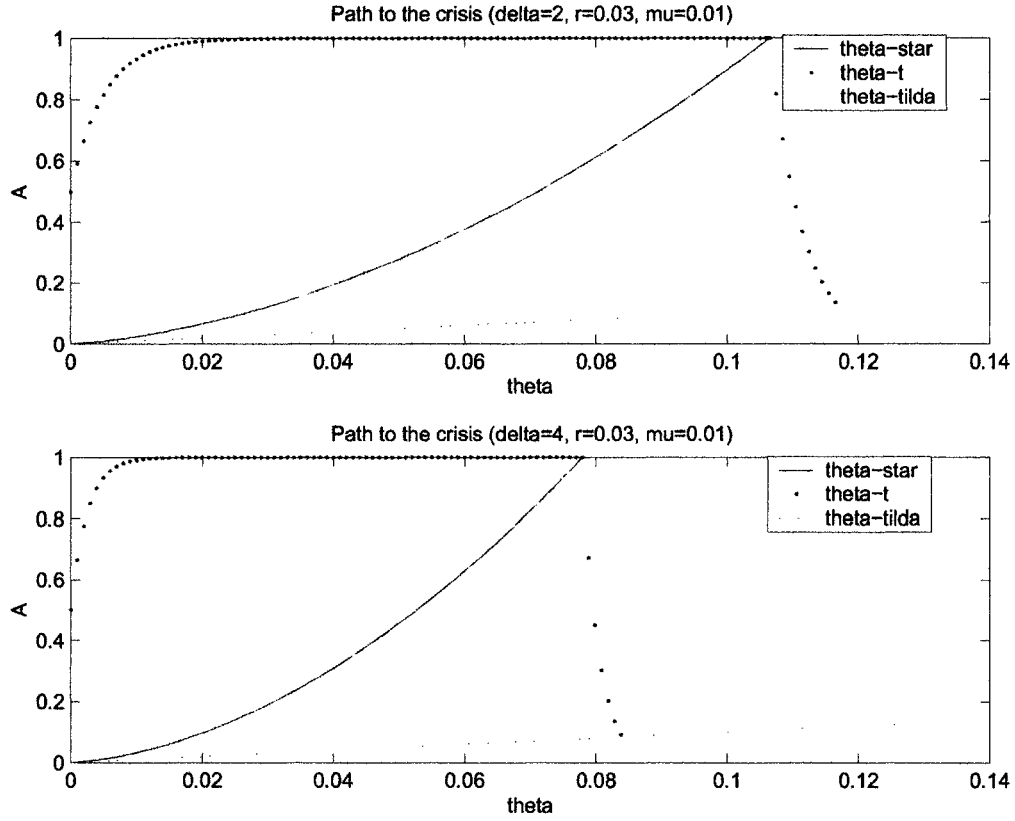
Figure 2.7: Path to the crisis

$$\frac{\tilde{A}}{A_0} = \left(\frac{r}{re^{-\kappa\tilde{A}} + \delta\left(1 - e^{-\kappa\tilde{A}}\right)}\right)^{\frac{\delta}{\delta-r}} \tag{2.0}$$

The parameter κ indicates how overvalued the currency needs to be to force the government to abandon the peg. For a given A, the higher κ, the higher is the difference between the exchange rate and its shadow value that forces the government to float the currency. High values of κ correspond to more commitment to a peg (or more ability to sustain it).

Implicitly differentiating equation 2.4 with respect to κ, we get that:

$$\frac{d\tilde{A}}{d\kappa} = -\frac{\tilde{A}^2\psi}{\left(e^{\kappa\tilde{A}} - 1\right)\psi + \psi\kappa\tilde{A} + 1} < 0$$

$$\frac{d\tilde{\Theta}}{d\kappa} = \frac{\left(\psi - (\psi - 1)e^{-\tilde{\Theta}}\right)\tilde{\Theta}}{\left(\psi - (\psi - 1)e^{-\tilde{\Theta}} + \psi\tilde{\Theta}e^{-\tilde{\Theta}}\right)\kappa} > 0$$

More ability/willingness to keep the peg (higher κ) implies that more people will be able to run before the peg is abandoned ($\tilde{A}$ will be lower). So, an agent attributes lower probability of being caught by the devaluation and, therefore, has more incentives to go long in the currency. Therefore, a higher value of κ imply a higher devaluation (higher $\tilde{\Theta}$) and more incentives to hold the currency (higher Θ^*).
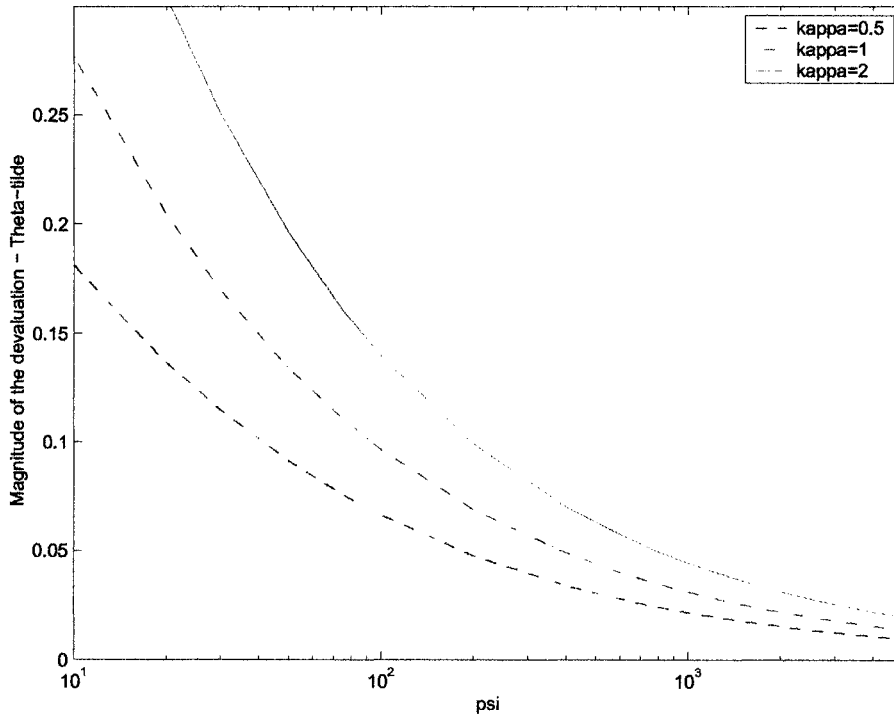
Figure 2.8: $\tilde{\Theta} \times \psi$

Figure 2.8 shows the magnitude of a devaluation ($\tilde{\Theta}$) as a function of ψ for some values of κ. We can see that $\tilde{\Theta}$ do not decay slowly: for $r = 1\%$ a month, $\delta = 20$ and $\kappa = 1$, (one signal for each business day, on average), the magnitude a devaluation is above 2%, which is the interest got in a couple of months, but a speculative attack takes less than 4 days to force the government to leave the peg.

The parameter μ_θ has no effect on any variable but Θ^*. It is clear from equation 2.4 that a

higher μ_θ leads to a lower Θ^*: agents willingness to choose *Long* depend negatively on the rate currency overvaluation is expected to increase. For very big values of μ_θ, the curve θ^* may depend negatively on A_0 for (at least) some values of A_0.

2.5 General case

If $\sigma_\theta \neq 0$, the threshold $\theta^*(A)$ for specific parameters δ, r, μ_θ, σ_θ and κ can be found using numerical methods. The task consists in finding a function $\theta^*(A)$ that solves (approximately) equation 2.2.3. The appendix comments on the procedure to find numerical solutions.

The numerical simulations confirm the results and insights of the particular case shown above. For empirically reasonable parameter values, the threshold $\theta^*(A)$ and, therefore, the expected size of a devaluation depend negatively on the arrival rate δ and positively on r and κ. The size of the devaluation approaches zero when δ approaches infinity.

As shown in the example at section 2, a large speculative attack may be triggered by a small change in θ, as long as it pushes the shadow exchange rate beyond the threshold θ^*.

2.5.1 Policy analysis

The model shows that the agents' willingness to go long in the currency, for a given level of θ and A, increases on: (i) frictions, (ii) interest rates, (iii) government's ability/willingness to keep the peg and (iv) the rate currency overvaluation tends to decrease. So, raising any of the above variables would reduce the probability of a crisis but would also *increase* the expected magnitude of a currency devaluation, conditional on its occurrence.[14]

The policy analysis, presented next, brings some useful insights but has no normative value as the model is silent about the costs and benefits of the peg. Moreover, the results refer to the impacts of policy variables on investors' decisions in a dynamic currency crisis game. Other effects are beyond the scope of this essay.

The parameter δ is related to the discussion of capital controls. Increasing controls may be thought as decreasing δ. In the model, 'throwing a bit of sand in the wheels of international

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[14]Although no counter-example was found for the above effects, there may be some set of parameter values to which one or more of those implications don't apply. However, the simulation results allow us to conjecture that such comparative statics hold for empirically plausible parameter values.

financial gears' increases the 'L' area, making a crisis less likely.[15] However, some critics of capital controls are skeptical about the ability of the government to influence δ — of course, the model has no say about that.

Raising interest rates (parameter r) could affect μ_θ, the shadow-exchange-rate trend. Disregarding such effect, the model shows a positive impact of interest rates on agent's willingness to choose *Long*. Such conclusion sounds natural but may not be obtained in a standard multiple equilibrium setting. Furman and Stiglitz (1998), for example, argue that the payoff of going long is of order of dt if the peg survives but is large and negative if the peg is abandoned. So, only unrealistic interest rates would be able to defend the currency. The missing piece in their analysis are the asset market frictions. Indeed, governments usually raise interest rates when a speculative attack starts in order to defend their pegs and, sometimes, they succeed (as Brazil in 1997, for example).

The parameter κ measures the degree of government's ability/willingness to keep the peg. The positive relation between κ and θ^* is behind the strong statements of commitment to the exchange rate regime made by Central Bankers and Finance Ministers. If agents believe that the peg will be abandoned in the event of any attack, they will have more incentives to attack the currency (choose *Not*).

The parameter μ_θ is the trend of the shadow exchange rate. A negative μ_θ can be thought as an expected reduction on currency overvaluation due to measures aiming at improving macroeconomic conditions in the long run. Those measures have a positive effect on the threshold θ^* but the numerical results show that such effects are not big.[16]

Figure 2.9 shows the equilibrium threshold for some given parameters — $\mu_\theta = 0$, $\delta = 4$, $r = 0.03$, $\kappa = 1$ and $\sigma_\theta = 0.02$ — as well as the curves θ^* for changes in individual parameters. We see that the effect of decreasing μ_θ from 0 to -0.01 is smaller than the impact of changing δ from 4 to 3, r from 0.03 to 0.04 or κ from 1 to 1.25. Note that having $\mu_\theta = -0.01$ implies that, in less than a year, the expected θ will be zero. In reality, macroeconomic reforms usually take a lot of time to

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[15]For an argument for throwing sand in the wheels of international finance, see Eichengreen, Tobin and Wyplosz (1995).

[16]One could argue that changes in the expected exchange rate should be reflected in the current exchange rate, as an undergraduate textbook would suggest. However, examples of abrupt currency appreciation in developing countries' are not the rule. Our negative trend in θ may be seen as representing improvements in the country's situation aimed by a government lacking credibility.
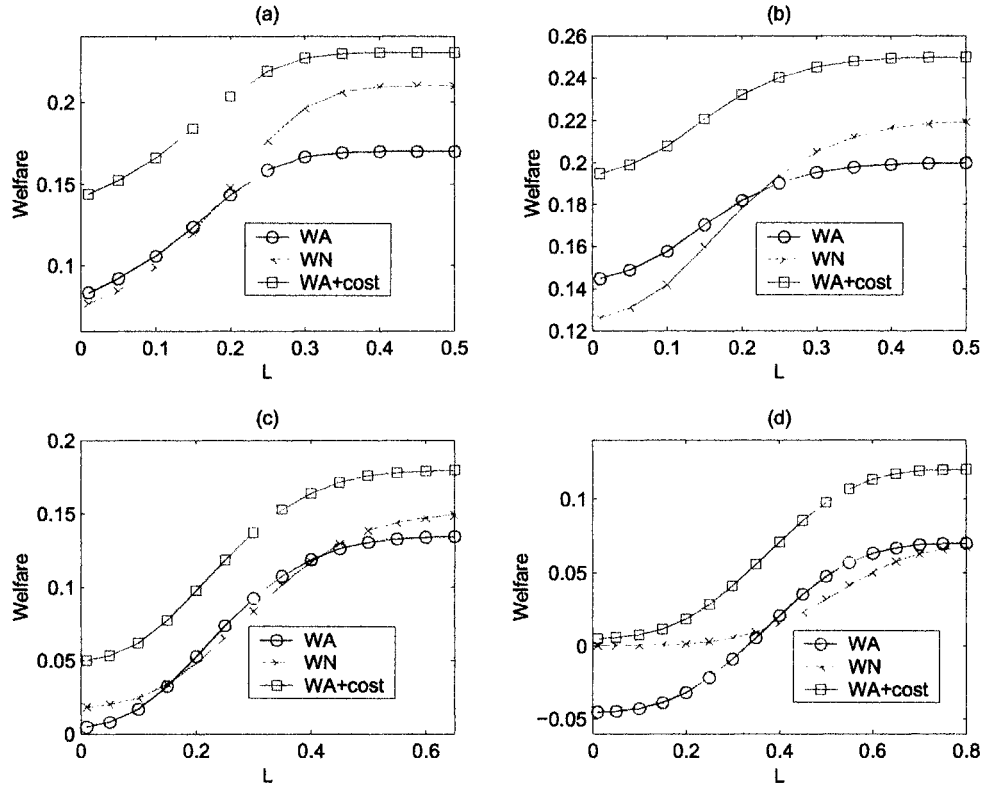
Figure 2.9: Effects of changing exogenous variables

yield such effects.

One could think that so large change in μ_θ would push the threshold far to the right because agents would attribute high probability for others choosing *Not* in the future. That could lead to a chain effect and have major effects on θ^*. That does not happen because, in the very short run, the stochastic component of the Brownian motion is more important than the trend and movements in A are even more dramatic. As an attack takes little time to deplete the country's reserves and force a devaluation, nothing but the near future matters in this game. The perspective of the economy in a year is not relevant for an agent deciding in the middle of the turmoil. That generates room for policies aiming at helping a country to survive a crisis conditional on improving macroeconomic conditions.

2.5.2 Effects of uncertainty on others' actions

Up to now, the analysis has highlighted the similarities between the general case and the particular case shown at section 2.4. This section shows a key difference between them.
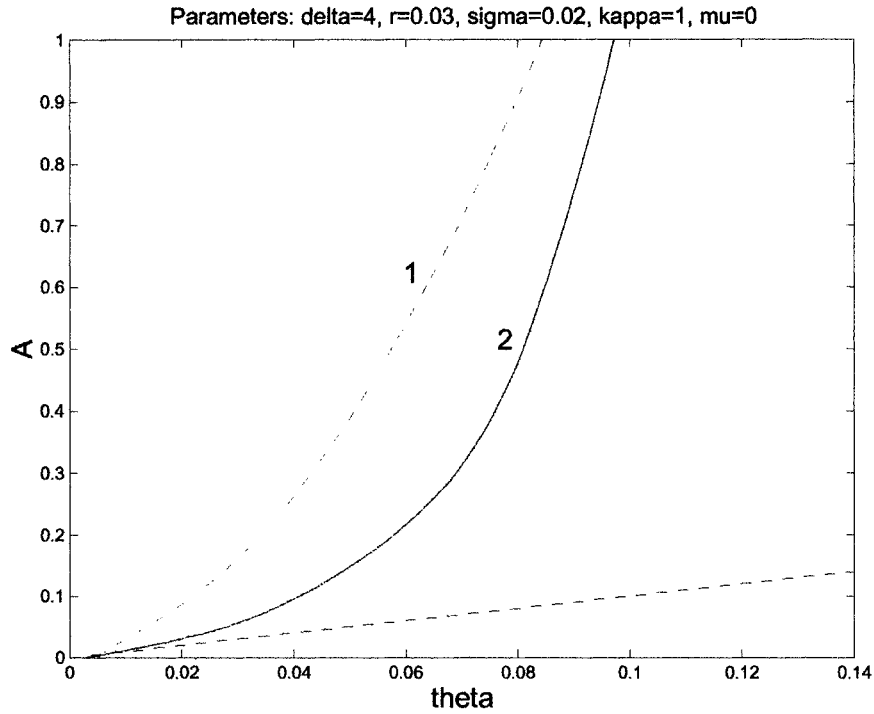
Figure 2.10: Effects of others' actions

Figure 2.10 shows the line $E\pi = 0$ conditional on every agent choosing *Not* from then on (curve 1) and the equilibrium threshold (curve 2).

The curve for $E\pi = 0$ when everybody is expected to choose *Not*, shown at figure 2.10, is almost identical to the threshold for the particular case described at section 2.4 when $\mu_\theta \to 0^+$. In that situation, agents also know that everybody will be choosing *Not* after the attack starts. The second order difference between both cases is virtually irrelevant.

The difference between the curves 1 and 2 can be seen as the effect of the possibility that others will choose *Long* later. For any positive μ_θ, the 'L' area is larger if there is some (not-too-big) degree of uncertainty on θ (as opposed to $\sigma_\theta = 0$) because that generates some possibility that others will choose *Long* in the future.

At the left of curve 1, the possibility of getting another signal before the crisis comes is enough for encouraging an agent to go long in the currency — as in the particular case of section 2.4. In the area between both curves, if an agent knew all others would pick *Not* from then on, he would also choose *Not*, but the possibility that others might choose *Long* in the future if θ gets low enough makes the crisis less likely and provides further stimulus for facing the risk of a devaluation. Needless to say, if all agents were expected to choose *Long*, the indifference curve ($E\pi = 0$) would be pushed far away to the right of the equilibrium threshold.

2.5.3 Probability and expected magnitude of a devaluation

When θ is far to the left of θ^*, the probability of a devaluation in the short run is small. However, the expected magnitude of a devaluation, conditional on its occurrence, will not depend much on how far the economy is from the threshold, because if there is a crisis in the future, θ will have crossed the barrier θ^*.

The following example illustrates this point: consider that the unit of time is a *month*, $r = 0.03$, $\delta = 4$, $\tilde{\theta}(A) = A$, $\sigma_\theta = 0.02$ and $\mu_\theta = 0$. Table 2.1 shows probabilities and expected sizes of a devaluation in the next 2 months, generated by Monte Carlo experiments, when the economy starts from $A = 0.7$ for different values of $\theta < \theta^*(0.7) = 0.0875$.

The probability of a devaluation varies a lot and increases as the shadow exchange rate gets closer to the threshold. Actually, it seems to be too sensitive to changes in θ. While in the model the probability of a devaluation in a month may get close to 100%, in reality it can't go beyond 10% or so, otherwise the interest rates paid by the country get too high. This drawback of the model gets further attention in the next section.

The expected magnitude of a devaluation, conditional on its occurrence, is virtually independent of θ.[17] It essentially depends on δ, r, σ_θ and μ_θ. The probability of a crisis also depends on the overvaluation of the currency (θ) and international reserves (A).

This prediction serves as the basis for the analysis developed in the next chapter.

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[17] Indeed, if $A = 1$ and $\theta < \theta^*(1)$, the expected size of a devaluation is independent of θ.

Table 2.1: Example: probability and expected jump size

θ	probability	mean(size)	std(size)
0.0400	0.0133	0.0995	0.0139
0.0450	0.0253	0.0996	0.0141
0.0500	0.0414	0.0984	0.0141
0.0550	0.0724	0.0980	0.0146
0.0600	0.1028	0.0993	0.0141
0.0650	0.1601	0.0989	0.0147
0.0700	0.2392	0.0990	0.0140
0.0750	0.3380	0.0980	0.0145
0.0800	0.4625	0.0974	0.0145
0.0850	0.6578	0.0953	0.0143

Chapter 3

Expectations and crises: an empirical evaluation

3.1 Introduction

Economists have been discussing the role played by *fundamentals* and *self-fulfilling beliefs* in currency crises for a long time. The weak links between changes in economic variables and speculative attacks in some recent episodes (e.g., the ERM crises in 1992-3 and the contagion of 1997-8) have stimulated the idea that bad fundamentals may be a pre-condition for a crisis, but its occurrence and timing are somewhat random events.

The so called second generation models of currency crisis formalize the above mentioned view.[1] This literature points out that, if fundamentals are not good enough, the optimal strategy for an agent in a currency crisis game depends on expectations: if everybody is expected to attack the currency, it is optimal to attack it, but if everybody is expected to refrain from doing so, then not attacking is the optimal choice. In a complete information setup, such models present multiple equilibria. The economic outcome may depend on *sunspots*. Sudden and exogenous shifts on expectations may trigger a crisis.

Although the second generation literature emphasizes the role of expectations in crises, it does not shed light on what drives agents' beliefs and, therefore, does not supply the tools for analyzing market

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[1]See, for example, Obstfeld (1996). Rangvid (2001) gives a survey of this literature.

behavior. Expectations are given exogenously in these models. In a path breaking contribution, Morris and Shin (1998) criticize the literature that relies on the existence of multiple equilibria showing that, when we remove some simplifying and unrealistic assumptions, we get a unique equilibrium in a model of self-fulfilling currency crises.[2] In the Morris-Shin model, expectations are *endogenously* determined by economic variables.

However, the empirical work on currency crises seems to be stuck in the dichotomy of 'fundamentals' versus 'multiple equilibria'. A large body of the literature tries to uncover which economic variables are relevant in explaining crises and whether fundamentals play an important role. The poor empirical fit of fundamental models is often taken as evidence of the existence of multiple equilibria — sudden changes of expectations would be driving the outcomes.[3] As Garber (1999) points out, the large deviations of observed variables from a model of fundamentals end up being taken as the empirical support for a theory that 'predicts' that large deviations may occur.[4]

If empirical work does not find clear links between economic fundamentals and crises, one may suspect that there is something missing. Raising the issue of expectations is legitimate, as there are models highlighting their role and some anecdotal evidence on their importance. A literature started by Morris and Shin (1998) has built models that aim at explaining what drives expectations.[5] This essay contrasts the predictions of the model presented in chapter 2 with empirical data.

The dynamic model of currency crises presented in chapter 2 predicts that a speculative attack is triggered when the shadow exchange rate hits a threshold. Therefore, the probability of a devaluation varies according to the shadow exchange rate — it depends on how far the economy is from the threshold. The expected magnitude of a devaluation, conditional on its occurrence, is relatively stable, because agents know that when the speculative attack starts, the shadow exchange rate will be around the threshold.[6]

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[2]As Morris and Shin (1998) show, multiplicity of equilibrium depends on assuming that all information is common knowledge in the economy and agents know others' actions in equilibrium.

[3]See, for example, Jeanne and Masson (2000) and Radelet and Sachs (1998).

[4]In one of the few explicit tests of sunspots, Jeanne (1997) executes a likelihood ratio test for the existence of multiple equilibria in the French Franc crisis. Some unexplainable shifts on expectations seem to be present, allowing him to conclude that jumps between different equilibria were playing a role. Although Jeanne (1997) does more than point to the weak links between fundamentals and crises, the empirical support for multiple equilibria in his paper is still the existence of mysterious changes that fundamentals do not seem to account for.

[5]See Morris and Shin (2002) for a survey of this literature.

[6]This implication is not at all trivial. If a currency crisis was triggered by a sunspot event, *independent of the*

The Asian crisis (in 1997) and the Russian crisis (in 1998) are said to have caused dramatic changes in expectations about pegged exchange rate regimes in countries with few direct economic links, like Brazil.[7] This essay characterizes expectations about the Brazilian *Real* in that period by estimating the probability and the expected magnitude of a currency devaluation. Then, it argues that key features of the empirical findings are consistent with the model presented in chapter 2.

The Brazilian crawling 'peg' was instituted in March-1995 as part of a plan intended to defeat inflation. Under the 'peg', the exchange rate could float inside a less-than-1%-wide mini-band. The mini-band was readjusted about 0.6% a month on a very regular basis, with each adjustment being distributed during the month in around 5 to 7 small increases. However, the large current account deficits suggested that the Brazilian *Real* was overvalued and that sustainability of the exchange rate regime was questionable. The peg survived the turmoil caused by the Korean crisis (in the end of 1997) and was finally abandoned in January-1999, five months after the Russian crisis (in August-1998).

The essay starts by presenting a method to identify the probability and the expected magnitude of a devaluation of Brazilian *Real* through the estimation of parameters of an asset pricing model, using data on options. In the model, the exchange rate follows a Brownian motion with a very small volatility, that may be interrupted at any time.[8] The interruption, a Poisson event, leads to a discrete jump in the exchange rate and to another Brownian motion, with a much higher volatility. Thus, the probability of a devaluation and its expected magnitude are key parameters of option prices. The proposed method is applied to the Brazilian pegged regime from January-1997 to January-1999, when the peg was abandoned. Exchange rate options between the Brazilian Real

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shadow exchange rate, then, in the 'multiple-equilibrium region' of the model, the probability of a devaluation would be independent of the shadow exchange rate, while the expected size of a jump in the short run, conditional on its occurrence, should approximately follow the shadow exchange rate.

[7]For example, Paul Krugman in January-1999 stated:"From my point of view, the power of contagion in [1997-8] settles a long-running dispute about currency crises in general: the dispute between 'fundamentalists' and 'self-fulfillers'. (...) I hereby capitulate. I cannot see any way to make sense of the contagion of 1997-8 without supposing the existence of multiple equilibria, with countries vulnerable to self-validating collapses in confidence, collapses that could be set off by events in faraway economies that somehow served as a trigger for self-fulfilling pessimism." Krugman (1999), page 35.

[8]Clearly, such a formulation does not correspond to a regime with a mini-band, but it serves as a good approximation of the Brazilian exchange rate path in the short run if the peg is kept.

and US Dollar were traded at São Paulo Futures Exchange (BM&F).

Among financial prices, options are the best sources of information on the expectations about a peg because their value at maturity is nonzero only if the exchange rate goes beyond a certain level (the strike price). So, if there is data on options of different strike classes, there is information about the probability density of the exchange rate at different points, and it is possible to identify two variables that characterize expectations: the probability of a change in regime and the expected magnitude of the devaluation conditional on its occurrence.

Extracting information about risks of discrete jumps in prices from data on options is not a novelty since Bates (1991) estimated Merton's (1976) model to test if the stock market crash of '87 had been expected. Merton's model was also used for assessing the credibility of Brazilian currency by Rocha and Moreira (1998, in Portuguese). None of the above papers was concerned about distinguishing between the probability of a jump and its expected magnitude. Campa, Chang and Refalo (2002) measured the credibility of the Brazilian exchange regime using non-parametric estimations. Guimarães and Silva (2002, in Portuguese) is an earlier attempt to pursue this essay's method of identifying the probability and expected magnitude of a devaluation, using a different asset pricing model (Merton, 1976).

The results in this essay show that the odds of a devaluation were very volatile and mostly driven by contagion from external crises, as the Asian and Russian crises triggered by far the greatest increases in the probability of the end of the peg. The conditional expected magnitude of a devaluation was very stable and was not affected at all by the Russian episode. Interestingly, the moments in which the probability of a crisis jumped up coincided with reasonably big depreciations of flexible exchange rates of countries in similar situations (like Mexico), suggesting a link between the probability of a crisis and the Brazilian 'shadow exchange rate'. It is also shown that the expected magnitude of a discrete jump was much below the observed depreciation of January-1999.

How are those findings related to the model predictions? Well, the empirical estimates show that the Asian and Russian crises caused large increases in the probability of a devaluation but little or no change in its expected magnitude. According to the model, those are just the expected consequences of a negative shock to the shadow exchange rate. Indeed, the Asian and Russian crises had a significant impact in a number of floating currencies of countries in a similar situation to Brazil (such as Mexico). This suggests that the Brazilian shadow exchange rate is likely to have

been affected by those episodes.

3.2 Empirical identification

This section presents a method to assess expectations about the Brazilian exchange rate regime by estimating the probability and expected magnitude of a devaluation of the *Real* from January-1997 to January-1999, when the peg was abandoned.

3.2.1 Intuition for identification

The exchange rate risk in a pegged regime depends on the probability that the peg will be abandoned and on the expected size of a consequent currency devaluation. The forward premium is roughly the product of those two variables and can be estimated through some (relatively) simple calculations. However, such approaches do not allow us to *identify* the probability of a devaluation and its expected magnitude: a forward premium of 3% a year may refer to an expected devaluation of 30% with probability 10% in a year, or an expected devaluation of 5% with probability 60% in a year, and so on.

Consider that the exchange rate in a given day in the future is described by a probability density function $f(y)$. In a risk neutral world, the future rate corresponds to the expectation of y:

$$F = \int_{-\infty}^{\infty} y f(y) dy$$

The price of a call option is:

$$c = \int_{X}^{\infty} (y - X) f(y) dy \qquad (3.0)$$

where X is the strike price. If there is data on options with different strike classes (X), there is information on different points of the distribution $f(y)$ — which is the key for identification. The asset pricing model described below imposes some structure on the exchange rate path and, therefore, parameterizes the probability distribution of the exchange rate. So, $f(y)$ and the price of an option (c) depend on a few parameters: the probability of a devaluation; its magnitude; and the volatility of the exchange rate before and after the peg is abandoned.

3.2.2 The asset pricing model

The model intends to capture the main features of the Brazilian crawling 'peg' in a simple way. Initially, the exchange rate follows a standard Black-Scholes diffusion process with a very small volatility:

$$\frac{dS}{S} = \mu_1 \cdot dt + \sigma_1 \cdot dX$$

This process may be interrupted at any time. The interruption, described by a Poisson event with hazard rate λ, leads to a discrete jump in the exchange rate and to a new diffusion process that is assumed to last forever.

The magnitude of the jump is a known constant equal to k, that is:

$$\frac{S^{after}}{S^{before}} = (1 + k)$$

The new diffusion process (the floating regime) is described by a Brownian motion with a drift and a much higher volatility:

$$\frac{dS}{S} = \mu_2 \cdot dt + \sigma_2 \cdot dX$$

It is easy to extend the model so as to incorporate a log-normal jump, and the formula for the price of an option is similar. However, as the standard deviation of the jump plays a role similar to σ_2, it is not possible to get significant estimates for both with the available data for options on Brazilian Real.[9]

Interest rates are assumed to be constant.[10] The formula presented below and the estimations in this essay consider risk-neutral agents.[11] A call option gives its owner the right to purchase one

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[9]A model with a log-normal jump and $\sigma_2 = 0$ yields similar results for λ and k, as discussed in appendix B.3.

[10]Empirical tests show that stochastic interest rates are not important for short term options. See, e.g., Bakshi et al (1997).

[11]The formula would also be valid for risk averse agents if the risk of a jump was diversifiable and uncorrelated with the market as in Merton (1976). In this case, it would be possible to get an instantaneous zero-beta portfolio and the price of an option would not depend on any individual's preferences. In particular, options would have the same value as in a risk-neutral world.

If the risk of a change in the exchange regime is systematic and cannot be diversified, there is no way to get a riskless portfolio and a price independent of agents' risk aversion. Then, using additional assumptions about individuals'

unit of foreign currency. As shown in the appendix B.1.1, the price of a European call with maturity

at time T is:

$$C^{mod} = e^{-\lambda T} BS \left(Se^{(-q-\lambda k)T}, T; X, r, \sigma_1^2 \right) + \tag{3.1}$$

$$\int_0^T \lambda e^{-\lambda t} BS \left(Se^{-qT-\lambda kt}(1+k), T; X, r, \frac{(\sigma_1^2 t + \sigma_2^2(T-t))}{T} \right) dt$$

where r is the domestic interest rate, q is the interest rate denominated in foreign currency, X

is the strike price, S is the spot exchange rate and $BS(S, T; X, r, \sigma^2)$ denotes the Black and Scholes

price of a call option. The first term of equation 3.1 represents the value of the option if the peg is

not abandoned at time T. The integrand of the second term is the option price given a devaluation

at time t (multiplied by its probability density function).

An example may be helpful for illustrating the identification issue. For some standard parameter

values[12], options with the same 'devaluation premium' ($\lambda.k$) but different λ's and k's, with strike

prices equal to 1020 and 1100, are priced as shown at table 3.1.

Table 3.1: Example for identification

λ	k	C(X=1020)	C(X=1100)	ratio
0.20	0.10	1.79	0.56	3.20
0.10	0.20	1.86	1.10	1.69

If $\lambda = 0.2$ and $k = 0.1$, an option with strike 1020 costs 3.2 options with strike 1100. This ratio

goes down to 1.7 if If $\lambda = 0.1$ and $k = 0.2$. That difference is the key for identifying the probability

and the expected size of a currency devaluation.

3.2.3 Data and Estimation

The observed price of a call option (C^{obs}) is assumed to be equal to the model price (C^{mod}) plus an

error term:

preferences and the correlation between their wealth and the underlying assets, it would be possible to get richer
theoretical models as in Bates (1991, 1996). However, the empirical results would depend on those assumptions. If
agents are risk-neutral, we can use just observable financial prices in the estimations.

[12]$S = 1000$, $\tau = 0.1$ year, $R = 0.2$/year, $B = 0.1$/year, $\sigma_1 = 0.01$/year and $\sigma_2 = 0.25$/year.

$$C^{obs} = C^{mod}(S, r, q, T; X, k, \lambda, \sigma_1, \sigma_2) + \epsilon \qquad (3.0)$$

The error term may be interpreted as measurement error in the dependent variable.

Estimating the parameters of equation 3.2.3 requires data on: domestic interest rates denominated in domestic and foreign currency; spot exchange rate; and option prices. Interest rate and exchange rate data are available from very liquid markets.[13] Unfortunately, the option market is much less liquid and there is no reliable information about the time each option was traded.[14] Thus, the price of the last trade for every option is used. The options are *European calls*,[15] the underlying asset is the US Dollar and the contracts are to be paid in Brazilian Real.

Options traded too close to maturity (less than 10 days) were discarded, as they contain little information about implicit distributions and their prices are not much greater than the bid-ask spread. In addition, transactions in at least 4 strike classes with the same maturity were required for each day. Finally, some questionable observations of a few far out-of-the-money option classes were excluded.[16] In the end, there were 3587 observations in the sample corresponding to 474 days and 25 months. 75% of the options in the sample were traded less than 45 days before maturity. So, the estimates are measures of expectations about the peg in the short run. Appendix B.2 gives more details on the data.

In the model, λ and k are constants, but in the estimations, they are allowed to vary over time. Although it is potentially inconsistent with the assumption of constant parameters used in deriving the model, some Monte Carlo experiments presented in appendix B.1.2 show that, for at least some diffusion processes of λ, such a procedure yields very reasonable estimates. That is not surprising, as prices of European calls do not depend on the particular paths of the hazard rate and magnitude of jump but on the probability distribution of the exchange rate at the maturity date. Indeed, the estimation of different λ's and k's is the standard procedure in the literature (see Bates (1991, 1996) and Jondeau and Rockinger (2000), for example).

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[13]All the data comes from contracts traded at São Paulo Futures Exchange (BM&F).

[14]In theory, options were traded at the exchange. In practice, options were traded over the counter and then registered at BM&F.

[15]European options can be exercised only at the maturity date, not before, so they are easier to price than *American* options.

[16]Campa *et al* (2002) and Guimarães and Silva (2002) also deleted those questionable observations.

Different assumptions were made about λ and k. In some cases, a parameter was constrained to be constant during each of the 25 months. On other occasions, one parameter was estimated for each of the 695 sets of options of a certain maturity traded in a given day.

The number of daily transactions of exchange rate options is small, so the available data refers to trades realized at potentially different times. Especially in nervous periods, this may introduce a large measurement error in the dependent variable.[17] As the measurement error depends on how much the exchange rate market fluctuates in a day, the variance of the error term may depend on the day an option was traded. A method for dealing with such heteroscedasticity is discussed in the next section and in appendix B.3.

The algorithms that solve for least square estimators of non linear models may stop at points that are local but not global minimizers. Appendix B.3 also shows that the results in this essay are very likely to correspond to the global minimum and comments on the robustness of the results.

3.3 Results

3.3.1 Description of the estimates

Figure 3.1 shows the results when λ and k are allowed to vary across days and maturity dates, assuming $\sigma_1 = 0.75\%/\text{year}$ and $\sigma_2 = 25.5\%/\text{year}$ — 25.5% is the standard deviation of the observed daily changes in the exchange rate from 1/19/99 to 12/31/99. Among the 695 (λ, k) estimated, 442 pairs have a t-statistic higher than 2 for both estimates. Figure 3.1 shows just the results for those 442 'significant' days. The estimates of λ higher than 0.17 are plotted as if they were equal to 0.17 (5 cases yield 'significant' estimates of λ between 0.25 and 0.40).[18] The vertical lines mark the periods in which the 'devaluation premium' is affected by the Asian and Russian crisis, in 1997 and 1998, respectively.[19]

Figure 3.2 presents the estimates when λ and k are constrained to be constant within each month. It also assumes $\sigma_1 = 0.75\%/\text{year}$ and $\sigma_2 = 25.5\%/\text{year}$.

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[17]See appendix B.2.

[18]Nothing substantial changes in the figures if the lower bound for t-statistics and the censorship limit is altered.

[19]Nouriel Roubini's website provides a detailed chronology of the Asian currency crisis and its global contagion: http://pages.stern.nyu.edu/ nroubini/asia/AsiaChronology1.html.

For a chronology of the Russian crisis and its contagion to Brazil, see Baig and Goldfajn (2001).

Figure 3.1: Estimation of 1 λ and 1 k for each day, with $\sigma_1 = 0.75\%, \sigma_2 = 25.5\%$.
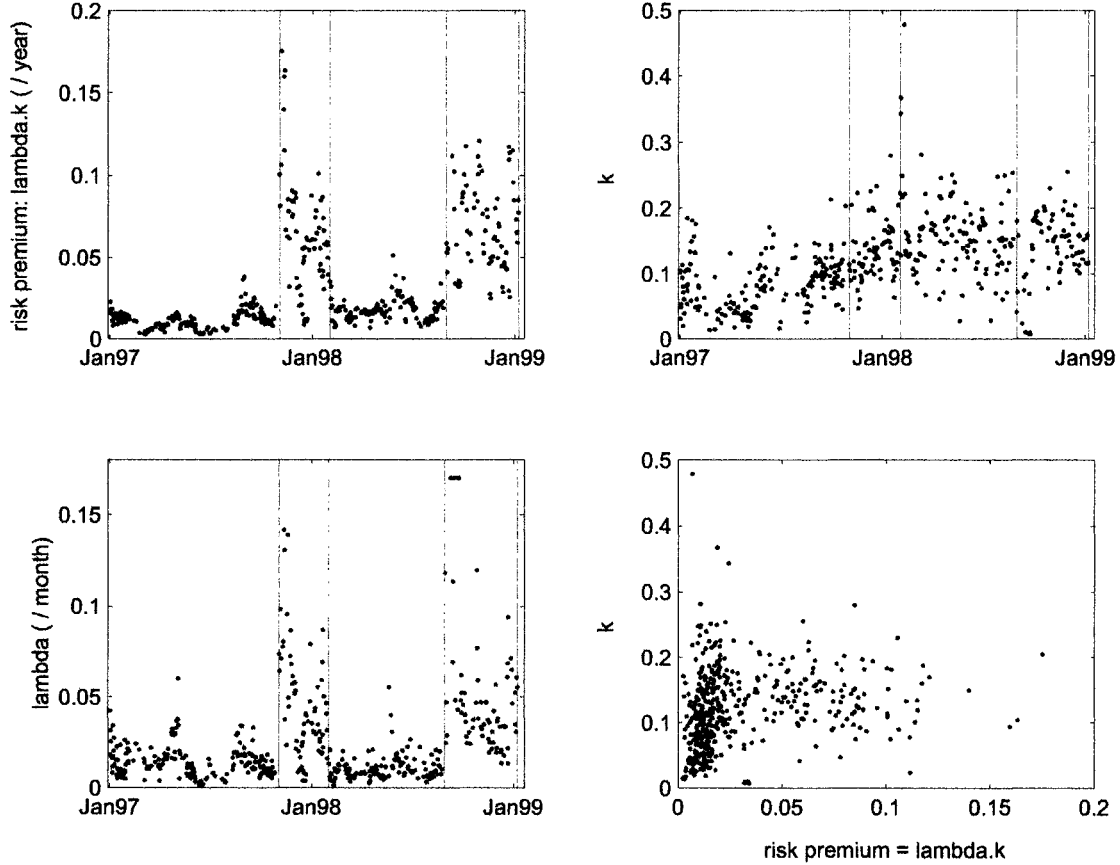
The probability of a crisis seems to change dramatically at certain days while the expected magnitude is very slow-moving. Figure 3.3 shows the estimates if k is constrained to be constant within a month and λ is constrained to be constant within a day — which allows us to capture sudden changes in the probability of a devaluation and get more accurate estimates of k. It also includes the estimation of σ_1 and σ_2. Their estimates are very reasonable: $\hat{\sigma}_1 = 0.74\%(0.02\%)$ and $\hat{\sigma}_2 = 28.46\%(1.17\%)$, standard deviations in parentheses.

The fit of the model is very good. In the case shown at figure 3.3, $R_c^2 = 0.9860$ and $\bar{R}_c^2 = 0.9825$.[20]

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[20]The uncentered $R^2 = 0.9919$ and $\bar{R}^2 = 0.9899$. If we estimate one volatility for each of the 695 'days' (695 parameters), using the standard Black-Scholes models, the RSS is 8609. Estimating the proposed asset pricing model with 722 parameters (allowing for 695 λ's, 25 k's, 1 σ_1 and 1 σ_2), we get a RSS equal to 384, 4.5% percent of what is obtained by using the Black and Scholes model. Considering that our dependent variable is very noisy, that is a

Figure 3.2: Estimation of 1 λ and 1 k each month, with $\sigma_1 = 0.75\%, \sigma_2 = 25.5\%$.
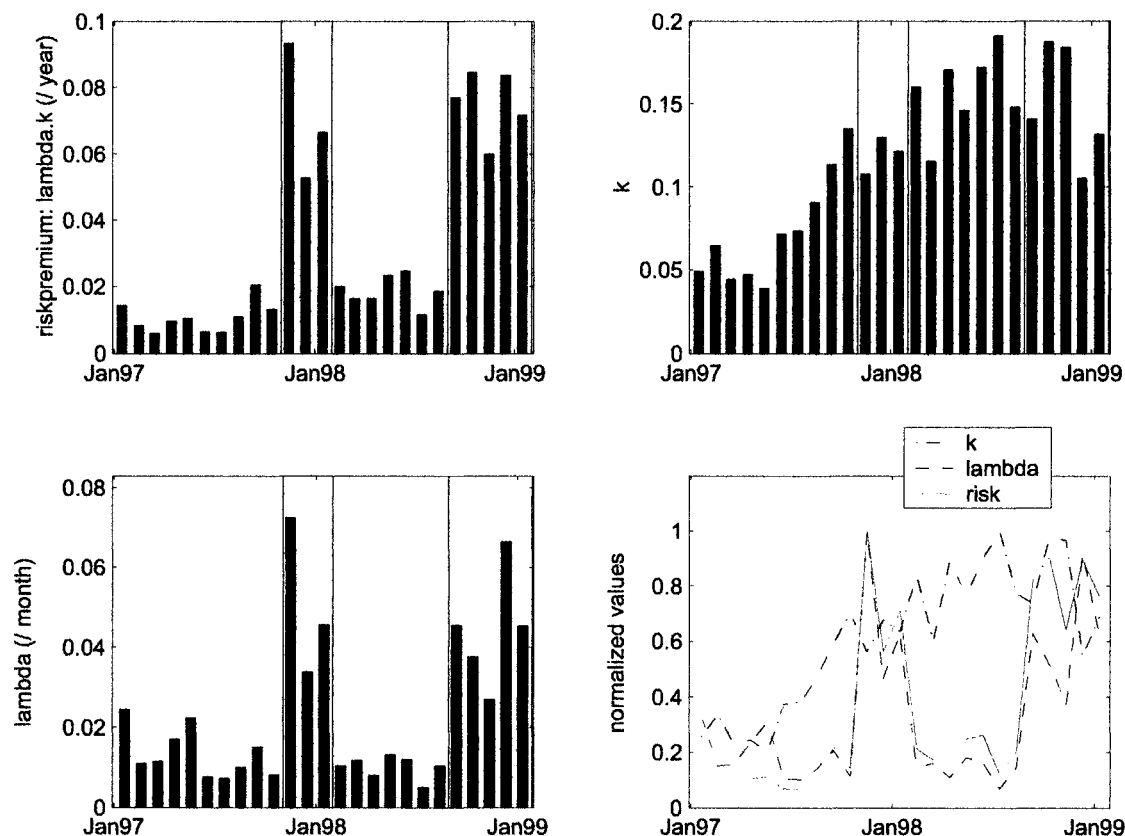
All 25 parameters k, shown at table 3.2, are significant at the 1% level of confidence.[21] Among the 695 estimators of λ, 85% of the t-statistics are greater than 2 and 92% are greater than 1.5. Joint tests of λ and k are always significant, clearly showing that the possibility of a currency devaluation was crucial for pricing options.

The errors are well behaved except for the high kurtosis coefficient, around 19. The asymmetry coefficient (−0.75) also seems too far from zero, but after excluding the residuals larger than 3 standard deviations (in absolute value), the asymmetry coefficient goes down to −0.028. The high kurtosis is due to the particularly large measurement errors in the dependent variable on days with

remarkable fit.

[21]Standard deviations were calculated dividing the residual squared sum by the number of degrees of freedom, 2865, not by the size of sample, 3587.

Figure 3.3: Estimation of 1 λ a day, 1 k for each month, 1 σ_1 and 1 σ_2.
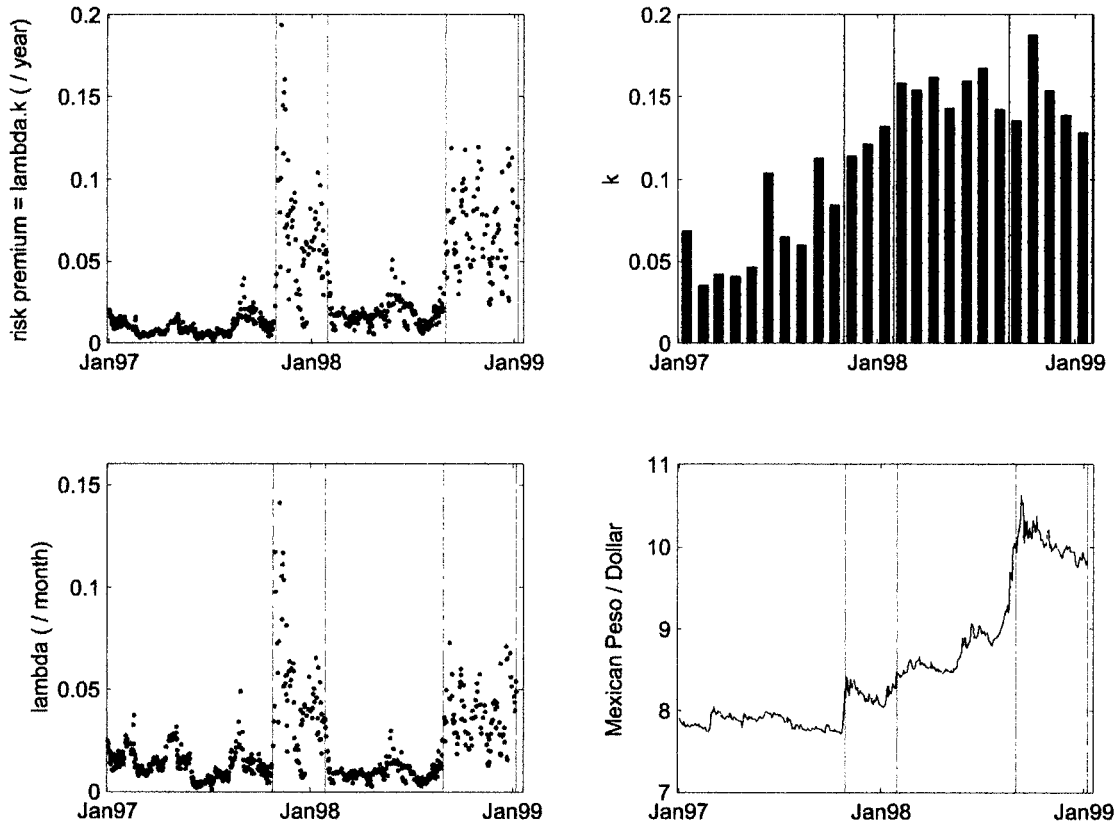
a high variation of prices and the fact that observations are not synchronized. For example, the 6 observations on 10/31/1997 present an average squared residual of 3.67, which is 34 times the mean squared residual of the full sample.[22] Such a high degree of heteroscedasticity suggests the use of FGLS to estimate the model. This was done, and the procedure is described in appendix B.3 — the NLLS and FGLS estimates are hardly distinguishable though.

3.3.2 The path of λ and k

The estimates shown at figures 3.1, 3.2 and 3.3 describe expectations about the Brazilian pegged regime from January-1997 to January-1999. The pictures show clearly that fluctuations in the

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[22]See appendix B.2.

'devaluation premium' are mostly explained by movements in λ. As shown in the bottom-right graph of figure 3.1, k seems to be almost uncorrelated with the 'devaluation premium'.

In the end of October-1997, the Korean crisis strongly affected the credibility of the *Real*. The probability of a devaluation reached its peak in November-1997 and almost vanished in February-1998. In 1998, λ stayed low until August, when Russia defaulted its debt. Then, λ rose sharply again and remained around 5%/month until January-1999, when the rupture effectively occurred.

The results suggest that k rose when the Korean crisis came and kept increasing until February-1998, while the risk of a devaluation was decreasing. Since then, k remained virtually unchanged, around 15%, until the end of the pegged regime. In 1998, virtually all changes in the 'devaluation premium' were due to movements in λ.

In sum, the Asian and Russian crises strongly affected the probability of a devaluation but had little effect on its expected magnitude.

Interestingly, the greatest jumps in Mexican exchange rate and in the probability of a devaluation in Brazil occurred at exactly the same moments, as figure 3.3 shows. Like Brazil, Mexico had large current account deficits by that time, but its currency was floating. Like Brazil, Mexico had few direct links with Russia or Korea. Therefore, it is reasonable to expect that the Brazilian 'shadow exchange rate' and the Mexican Peso would respond to the Asian and Russian crises in similar ways: if the Brazilian currency had been floating by then, it probably would have been depreciated.[23]

Table 3.3 summarizes our findings.

The probabilities of a crisis in a month were almost always below 10% and remained around 5% from September-1998 until January-1999 (see figure 3.3). Interestingly, λ did not increase sharply right before the regime break.[24] Indeed, the Brazilian government entered into an arrangement with the IMF at the end of 1998, interest rates were decreasing (from 2.93%/month in October-1998 to 2.38%/month in December-1998) and most macroeconomic reports were pointing to an increase in the credibility of the currency by December-1998.[25]

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[23]Actually, all the main Latin American currencies that were floating in that period were negatively affected by the crises of 1997-8. The *Chilean Peso*, despite the good economic performance of Chile, was hit by the Asian crisis. The *Colombian Peso* lost 10% of its value in the month following the Russian default (its average monthly devaluation in the period was 2%).

[24]There are estimates for λ until 01/08/99, 3 business days before the jump. Options get more expensive 1 or 2 days before the devaluation, but not by much.

[25]For example, the economic analysis bulletin of IPEA, a respected Brazilian economic research institute, states in

The 'devaluation premium' estimates shown in figures 3.1 and 3.3 are virtually the same. Indeed, estimating the 'devaluation premium' is not difficult, the tough issue is identifying probability and magnitude of a devaluation.

The results also show that agents underestimated the size of the jump: while the expected depreciation is never greater than 20%, the observed devaluation was as high as 60%. Before showing the theoretical model, some comments on the discrepancy between the expected and the observed devaluation are warranted. Also, one could argue that government interventions in the market to defend the exchange rate regime may have influenced our results. Next, we deal with both issues.

3.3.3   Expected and observed jump size

On 02/01/99, the first maturity day of options after the devaluation, the exchange rate was at 1.983 R$/US$, 63.7% higher than 3 weeks before. So, either the estimates of k are wrong or the expected magnitude of the jump was severely below its observed value. This section provides a 'back-of-the-envelope' estimation of the expected magnitude of a devaluation, using exchange rate options, and some facts suggesting that the size of the devaluation of the Brazilian *Real* in January-1999 was higher than agents expected.

There is a simple way to 'trade' the expected magnitude of the devaluation using options, that would generate profit opportunities if k was mispriced: pick options of the same maturity with 2 different strike prices that are worth zero if no change in regime occurs (strike prices need to be high enough).[26] Selling one and buying the other in the inverse proportion of their prices, we get a zero-value lottery. The ex-post value of the lottery is a function of the size of the devaluation and equals 0 if the peg is not abandoned.

Figure 3.4 shows the ex-post value of buying 875 call options with strike price of 1300 (they cost 6.00) and selling 600 call options with strike price of 1250 (they cost 8.75).[27] The break-even of

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December-1998 that '(...) the pressure on the exchange rate got milder, and now a speculative attack is not likely to occur', IPEA (1998, in Portuguese), page 6.

[26] Any option with strike price higher than the top of the projected mini-band is worth zero if the mini-band regime is kept.

[27] Figure 3.4 shows a 'median' example, using data of 12/28/1998, when US$1000 were worth BR$1208.40 in the spot exchange rate market. An option gives its holder the right to buy US$1000 at the strike price.

such operation occurs with a jump of 16.6%.

Figure 3.4: Ex-post value of the lottery
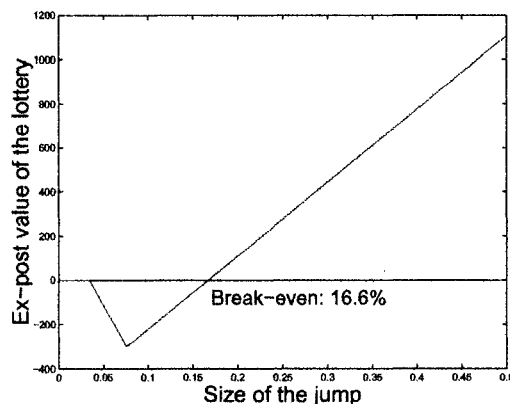
The estimates of k around 15% are compatible with the ex-post value of such lotteries. We are left to believe that the expected devaluation was below its observed value.

Actually, this belief is confirmed by the exchange rate path right after the devaluation. Table 3.4 shows the spot exchange rates in January-99 — Future rates display the same pattern. On January 13th, the end of the pegged regime was announced and the Central Bank tried to impose a new upper bound of fluctuation, at R\$1.32/US\$.[28] Two days later, the new-born band was abandoned and the exchange rate started to float. On the 15th, even though Brazilian *Real* had lost this first battle, the US Dollar was still just 21% more expensive than before the jump. The spot rate would go up gradually and increase every single day until the end of the month.

The behavior of the exchange rate in the very short run after the devaluation is interesting: there seems to be a clear and very strong upward trend for the price of the US Dollar, suggesting that either bad news for Brazilian economy was arriving every single day or that the market took a couple of weeks to update its more optimistic prior. A look at the newspapers of January-1999 favors the latter explanation.[29] It seems that financial prices were reflecting too-optimistic views of

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[28]That would mean a devaluation of around 9%.

[29]For example, the magazine Epoca published on 01/18/99, when the devaluation was already above 20%, brought Finance Minister Pedro Malan arguing that Brazilian currency overvaluation was slightly *lower than 10%* — he cited studies from institutions such as Morgan, Lloyds, IMF and Goldman Sachs that confirmed his opinion. He dismissed the estimations of an overvaluation of "20%, 25%, 30% and *even* 40%" as based on some "simplistic calculations". At that day, 40% sounded like a bad joke. Reality proved to be different: the real exchange rate kept depreciating

Brazilian currency.

3.3.4   Did government interventions bias results?

When extracting information on expectations about the Brazilian exchange rate regime from option prices, we are assuming that those prices reflect market beliefs. However, during the most critical periods of the pegged regime, Brazil lost a considerable fraction of its international reserves, that is, the Central Bank sold large amounts of foreign currency in the spot market. Moreover, during those periods, the government increased its stock of dollar-indexed debt. One could argue that such governmental interventions pushed down the forward premium and agents were not increasing their short positions due to risk considerations.

Now, assume that such an effect was big enough to substantially influence the forward premium. As the example plot at figure 3.4 shows, agents can trade the size of the jump without changing their bets on the risk of a devaluation. So, if agents believed $\lambda.k$ was 'too cheap' but didn't want to increase their short positions, the estimates of λ during the most nervous periods would be downward biased, but that should not affect our estimates of k because agents could make bets on the expected size of the jump without increasing their exposure to the risk of no-devaluation.

3.4   Empirical results and theoretical predictions

The dynamic model of currency crises presented in this essay matches the key empirical findings. It correctly predicts that the probability of a devaluation should be volatile while its expected magnitude should be more stable. Assuming that the shadow value of the Brazilian currency was pushed down when the crises in Asia and Russia occurred, as happened with the floating Mexican peso, the model also explains how those events in faraway economies lead to a jump in the probability of a devaluation with virtually no impact in its expected magnitude. A key result of the model is that agents expect a crisis to occur when overvaluation gets high enough.

However, some empirical results differ from the implications of the model. In particular, the expected size of a devaluation increases in 1997 while the model predicts it should not vary; and the probability of a crisis does not go up at the last moments of the pegged regime, although the model

and, since then, the *Real* has never been valued as much as it was on 01/18/99.

predicts it would be pushed up to 100%. Sections 3.4.1 and 3.4.2 comment on these two issues.

Note that if the occurrence of a currency crisis was completely disconnected from the shadow exchange rate as in the benchmark 'sunspot model' presented above, k would be roughly the currency overvaluation while λ would be given by factors outside of the model. The only prediction of that model, the fact that k should vary with θ, seems not to be confirmed by the data.

The impacts of foreign crises on the credibility of Brazilian *Real* or on the value of *Mexican Peso* go far beyond what we would be able to explain based solely on domestic fundamentals. A large literature studies what causes such strong contagion. Some recent papers (e.g., Morris and Shin, 2002b and Bacchetta and Van Wincoop, 2003) show that events that are public information may have an amplified effect on the exchange rate. External crises are public information: when Russia defaults, that is virtually common knowledge among all of the relevant players in Brazilian financial markets. That may be an important factor behind the large contagious effects of foreign crises.

3.4.1 The expected size of a devaluation in 1997

The expected magnitude of a devaluation goes up at the end of 1997 — when the Asian crisis is getting less severe. Why? Before the Asian crisis, the Brazilian pegged regime had not experienced any troubles of such magnitude and one could argue that the energetic defense of the peg by the government, that included keeping very high interest rates for some months despite the already weak state of the economy, worked as a signal to the private sector and changed agents' assessment of the government's commitment to the exchange rate regime. As shown in chapter 2 a stronger commitment to the peg makes agents less willing to attack the currency and, therefore, the crisis happens only for a higher θ. So, if the government's behavior at the end of 1997 was interpreted by market participants as a signal of a strong commitment to the peg, the expected magnitude of a jump should have increased and the probability of a crisis should have decreased — which indeed happened.

3.4.2 The path of the probability at the end

Despite its empirical success, the model presented here misses some important features of reality. While the estimates show that the probability of a devaluation stays around 5% during the last 5 months of the pegged regime, the model predicts that, right before the peg is abandoned, the

probability of a crisis is close to 100%.

The exact timing of the crisis seems to be random to market participants. Such timing inde-terminacy is usually seen as a sign of sunspots. However, the facts don't look like the multiple equilibrium story in which a large unexplainable shift of agents' behavior forces the government to abandon the peg. By far the largest losses of foreign reserves occurred in the first month following the Russian default: they fell from around US$67 billion at the end of August-1998 to US$45 billion at the end of September-1998. At the end of January-1999, when the peg was abandoned, foreign reserves totalled US$35.5 billion. Moreover, interest rates were declining since October-1998. There is some uncertainty about the government's decision to abandon the peg that is not captured by the model presented in this essay.

The analysis so far has considered that agents are deciding whether they hold *one unit* of domestic or foreign currency. The next chapter abstracts from the dynamic issues studied at chapters 2 and 3 and proposes a framework in which agents have a much richer set of options. The analysis brings insights on the effects of risk and wealth in currency crises.

Table 3.2: Parameter k

Month	k	std
Jan-97	0.0689	(0.0082)
Feb-97	0.0355	(0.0088)
Mar-97	0.0423	(0.0120)
Apr-97	0.0410	(0.0074)
May-97	0.0465	(0.0099)
Jun-97	0.1041	(0.0290)
Jul-97	0.0654	(0.0220)
Aug-97	0.0603	(0.0136)
Sep-97	0.1131	(0.0121)
Oct-97	0.0845	(0.0061)
Nov-97	0.1143	(0.0033)
Dec-97	0.1216	(0.0052)
Jan-98	0.1322	(0.0056)
Feb-98	0.1582	(0.0155)
Mar-98	0.1541	(0.0187)
Apr-98	0.1620	(0.0189)
May-98	0.1432	(0.0150)
Jun-98	0.1597	(0.0145)
Jul-98	0.1673	(0.0256)
Aug-98	0.1422	(0.0163)
Sep-98	0.1355	(0.0057)
Oct-98	0.1878	(0.0064)
Nov-98	0.1536	(0.0062)
Dec-98	0.1389	(0.0048)
Jan-99	0.1284	(0.0104)

Table 3.3: Summary of empirical findings

Probability of a devaluation	Expected size of a devaluation
- High volatility	- Low volatility
- High correlation with risk of a devaluation	- Low correlation with risk of a devaluation
- Strong reaction to foreign crises	- Little reaction to foreign crises
- Main jumps coincide with highest depreciations of Mexican Peso	

Table 3.4: Spot exchange rate right in January-99

Day	Exchange Rate	Jump
1/11/99	1.211	
1/12/99	1.211	
1/13/99	1.319	8.9%
1/14/99	1.319	8.9%
1/15/99	1.466	21.0%
1/18/99	1.538	27.0%
1/19/99	1.558	28.6%
1/20/99	1.574	29.9%
1/21/99	1.660	37.0%
1/22/99	1.705	40.7%
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Chapter 4

Risk and Wealth in a model of currency attacks

This essay is co-authored with Stephen Morris

4.1 Introduction

Currency crises are often self-fulfilling. Agents have an incentive to sell a currency short when they anticipate that others will short the currency and to go long in the currency when expecting others to do so. Many authors have developed coordination game models of currency crises.[1] In a complete information setting, such models yield multiple equilibria. Removing the assumption of common knowledge of fundamentals and knowledge of others' actions in equilibrium, Morris and Shin (1998) developed a "global games" model of currency crises with a unique prediction of when a currency crisis would occur. That work assumed that agents were risk neutral and were making a binary decision whether to attack or not. In this essay, we ask how risk aversion, wealth and portfolio composition of agents affect the likelihood of an attack. We derive a rich set of theoretical predictions.

We first assume that there is a continuum of homogenous agents, characterized by the degree of relative risk aversion, the composition of their portfolio of dollar and peso-denominated assets

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[1] Obstfeld (1996).

and their propensity to consume in dollar and peso denominated goods. They earn an interest rate premium from holding pesos. However, there is a possibility that the exchange rate will be devalued by a known amount. Each investor will choose an optimal portfolio given his beliefs about the likelihood of devaluation. Devaluation occurs if the aggregate net sales of pesos exceeds some stochastic threshold (the "fundamentals"). Each agent has a different noisy signal of fundamentals. We derive a closed-form solution for the unique equilibrium of this model. The equilibrium is characterized by the critical threshold where the currency is devalued. We examine how the critical threshold varies as parameters change.

The critical threshold will always be in the range where, if there was complete information, there would an equilibrium with devaluation and an equilibrium without devaluation. The threshold implicitly determines what we call the "sunspot" probability: the probability of devaluation conditional on fundamentals being in the multiple equilibrium range of the complete information model. The sunspot probability is a measure of the likelihood of a self-fulfilling attack when fundamentals do not require it. By showing how the sunspot probability depends on the parameters of the model, we develop comparative static predictions that would not arise in a complete information model.

Some key findings are:

1. **Risk Aversion.** Under the "one way bet" assumption - i.e., the interest rate differential from holding pesos is much smaller than the capital loss from holding pesos in the event of a devaluation - risk aversion has a very large effect. If agents are risk neutral, the sunspot probability is close to 1, but if agents' constant relative risk aversion is greater than 1, the sunspot probability is less than $\frac{1}{2}$ and, in the limiting case of infinite risk aversion, is close to 0. The intuition is that for a risk averse agent, increased returns to devaluation reduce the size of the long position he must take to hedge against the possibility of devaluation.

2. **Wealth.** The probability of a crisis is *increasing* in wealth (the opposite of the conventional wisdom underlying some contagion stories). Here it is key that agents can both short and go long in the peso. Increased wealth allows agents to short the currency more, increasing the likelihood of a successful attack. If there are incomplete markets and agents cannot short the currency, we regain the conventional result that increased wealth reduces the likelihood of crises.

3. **Portfolio.** If agents' wealth is shifted from dollar denominated assets to peso denominated assets, then a risk averse marginal agent who is uncertain whether a devaluation will occur or not will have a hedging incentive to go long in dollars. Thus foreign direct investment may, *ceteris paribus*, increase the likelihood of a currency crisis.

4. **Ownership.** Home residents consume more peso denominated goods than foreign residents. In the case of relative risk aversion coefficient greater than 1, if agents become foreign rather than domestic, the likelihood of an attack increases. Although the returns to attacking are higher for domestic residents, this leads to a lower probability of crisis because of risk aversion.

5. **Devaluation Size.** For reasonable levels of risk aversion, increasing the size of devaluation may increase or decrease the likelihood of crisis.

6. **Interest Rate Defense.** For sufficiently high risk aversion, devaluation size and interest rate differential, increasing the interest rate differential may increase the likelihood of attack. However, for reasonable parameter values, an interest rate defense will reduce the likelihood of crisis.

A common theme to many of the comparative static results is that when relative risk aversion is above 1, intuitive comparative statics may be reversed. As risk aversion increases, consumption in the two states (devaluation, no devaluation) become complements rather than substitutes. Thus income effects come to dominate substitution effects.

None of these results would arise in a complete information model, where risk aversion and ownership have no effect at all on the set of equilibria. Possibly one could derive related comparative statics in a complete information model with symmetric uncertainty. However, to make risk aversion matter in such a model, a large amount of uncertainty about fundamentals would be required. Our results continue to hold even when uncertainty about fundamentals is arbitrarily small.

Our results thus imply a rich set of empirical predictions from a global games model of currency crises that would probably be hard to replicate with other models that do not build on agents' strategic uncertainty in equilibrium. Unfortunately, the type of data required to directly test the predictions are surely not available. We leave the task of confronting our model to data to future work.

Our analysis focusses on the representative agent with known characteristics. However, we show that our analysis extends to an arbitrary distributions of characteristics. The devaluation threshold is *linear* in the distribution of characteristics in the population.

The analysis builds an approach to modelling currency crises due to Morris and Shin (1998), building on the global games analysis of Carlsson and van Damme (1993). Morris and Shin (1998) and other applied papers using these techniques (surveyed in Morris and Shin (2003)) make heavy use of the assumption that each player faces a binary choice (to attack the currency or not). Frankel, Morris and Pauzner (2003) showed how existence and uniqueness results can be extended to global games with many actions. The model in this essay is a tractable example of a global game with many actions where closed form solutions can be obtained. A theoretical contribution is that it identifies another setting where there is "noise independent selection": the threshold as noise becomes small does not depend on the shape of the noise. By allowing for a continuum of actions, we are able to endogenize the amount of "hot money" available in currency attacks and endogenize whether attacking or defending the currency is the riskier action. These have been arbitrary modelling choices in existing models.

This essay is related to an important work on contagion by Goldstein and Pauzner (2001). They model the idea that catastrophic losses in Russia, say, may reduce the wealth of investors. If those same investors are also investing in Brazil, and those investors have decreasing absolute risk aversion, then they will reduce their risky exposure to Brazil, thus generating a wealth contagion mechanism. Goldstein and Pauzner emphasize that risk aversion has a large impact on the unique equilibrium even though there may be an arbitrarily small amount of uncertainty about fundamentals.[2] The same mechanism underlies our results. We are able to allow for continuous rather than binary investment choices and extend this type of comparative statics to the range of other issues discussed above. Our results show how the Goldstein-Pauzner model — and the underlying intuition about contagion — rely on a (perhaps empirically plausible) incomplete markets modelling assumption that people who lost money in Russia were unable to short the Brazilian real. In a complete markets

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[2]Calvo and Mendoza (2000) modelled this type of contagion using an informational story. Kyle and Xiong (2001), like Goldstein-Pauzner, modelled a wealth effect version of the contagion story, but the mechanism is different, relying on a significant amount of uncertainty in equilibrium. These papers also rely on explicit or implicit assumptions that "attacking" (selling pesos) rather than "defending" (buying pesos) is the safe action.

model, their loss of wealth in Russia should reduce their ability to short the real, and under the one way bet assumption and plausible risk aversion, this would actually decrease the likelihood of a Brazilian crisis.

We describe and solve our basic representative agent model in section 4.2. Comparative statics with respect to risk aversion, wealth, portfolios, ownership, devaluation size and interest rate differentials are analyzed in section 4.3. In section 4.4, we highlight the key modelling assumptions that allow us to get a closed form solution and also noise independence, noting how this result implies that our representative agent analysis immediately extends to a heterogeneous agent world in a linear way; we also examine the robustness to various assumptions: the form of asymmetric information about fundamentals, the known size of the potential devaluation, the constant relative risk aversion assumption and the size of agents.

4.2 Basic Model

4.2.1 Setup

A continuum of agents (measure 1) will realize wealth w_D denominated in dollars and wealth w_P denominated in pesos next period. Each agent must decide his net demand for dollars today, y, with $-y$ being the dollar value of the agent's net demand for pesos.[3] Dollar investments earn an interest rate normalized to zero. Pesos investments earn an interest rate of r. The initial peso/dollar exchange rate is fixed at e_0, but there is a possibility that the exchange rate will be devalued next period. Thus the exchange rate next period (e_1) will be either e_0 or $E > e_0$.[4] Thus the agent's final period wealth (denominated in dollars) is given by

$$\widetilde{w}(y,e_1) = w_D + y + \left(\frac{w_P}{e_1} - y\frac{e_0}{e_1}(1+r)\right)$$
$$= w_D + \frac{w_P}{e_1} + y\left(1 - \frac{e_0}{e_1}(1+r)\right).$$

The agent may consume both foreign goods (x_D, denominated in dollars) and domestic goods (x_P, denominated in pesos). The agent's von-Neumann Morgenstern utility function over foreign and

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[3]For simplicity, we assume that the agent has zero liquid assets in the current period. If the agent had positive liquid assets, we could simply add their current dollar value to w_D, and our analysis would be unchanged.

[4]The assumption that the size of a potential devaluation is known is discussed in section 4.4.3.

domestic goods is Cobb-Douglas,

$$u\left(x_D, x_P\right) = x_D^{\alpha} x_P^{1-\alpha},$$

with $\alpha \in [0,1]$. Letting q_D and q_P be the constant prices of dollar and peso denominated goods, respectively, indirect vNM utility is

$$\left(\frac{\alpha \widetilde{w}\left(y, e_1\right)}{q_D}\right)^{\alpha} \left(\frac{(1-\alpha) e_1 \widetilde{w}\left(y, e_1\right)}{q_P}\right)^{1-\alpha}$$
$$= \left(\frac{\alpha}{q_D}\right)^{\alpha} \left(\frac{1-\alpha}{q_P}\right)^{1-\alpha} e_1^{1-\alpha} \widetilde{w}\left(y, e_1\right).$$

Dividing through by the constant

$$\left(\frac{\alpha}{q_D}\right)^{\alpha} \left(\frac{1-\alpha}{q_P}\right)^{1-\alpha} e_0^{1-\alpha},$$

we have normalized indirect vNM utility

$$v\left(y, e_1\right) = \left(\frac{e_1}{e_0}\right)^{1-\alpha} \widetilde{w}\left(y, e_1\right).$$

We will assume that the net return to attacking the currency by buying a dollar (and going short in pesos to do so) if there is a devaluation is positive, so

$$v_A = \frac{dv\left(y, E\right)}{dy} = \left(1 - (1+r)\frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{(1-\alpha)} > 0;$$

and the net return to defending the currency by selling a dollar (and purchasing pesos) if there is no devaluation is

$$v_D = -\frac{dv\left(y, e_0\right)}{dy} = r > 0.$$

We will often want to make the "one way bet" assumption[5] that

$$v_A > v_D$$

or

$$\left(1 - (1+r)\frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{(1-\alpha)} > r.$$

Writing

$$t = \frac{v_D}{v_A + v_D} = \frac{r}{\left(1 - (1+r)\frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{(1-\alpha)} + r},$$

---

[5]Betting in favor of a devaluation is often seen as a one-way bet because the opportunity cost of taking a temporary short position in the currency is small relative to the potential gains from devaluation.

the one way bet assumption is equivalent to the requirement that

$$t < \frac{1}{2}.$$

The agent has constant relative risk aversion ρ over his vNM index. So he chooses y to maximize the expected value of

$$\frac{1}{1-\rho} \left(\left(\frac{e_1}{e_0} \right)^{1-\alpha} \widetilde{w}(y, e_1) \right)^{1-\rho}$$

The agent's optimal portfolio choice will thus depend of the probability he attaches to devaluation. We assume that devaluation occurs if the aggregate net demand for dollars exceeds a stochastic threshold θ.[6] We assume that θ is uniformly distributed on the real line and that agents don't know θ but each agent i observes a signal x_i, $x_i = \theta + \varepsilon_i$, where the ε_i are distributed in the population according to probability density function f.[7]

The Inada condition implies non-negative wealth next period; now $\widetilde{w}(y, E) \geq 0$ implies that

$$y > \underline{y} = -\frac{w_D + \frac{w_P}{E}}{1 - (1+r)\frac{e_0}{E}};\qquad(4.-4)$$

and $\widetilde{w}(y, e_0) \geq 0$ implies

$$y < \overline{y} = \frac{w_D + \frac{w_P}{e_0}}{r}.\qquad(4.-4)$$

We will assume that the agent must choose $y \in \left[\underline{\theta}, \overline{\theta}\right]$. This implies that if there was complete information, there would be a tripartite division of fundamentals. If $\theta < \underline{\theta}$, there must be devaluation. If $\underline{\theta} \leq \theta \leq \overline{\theta}$, there are multiple equilibria. If $\theta > \overline{\theta}$, the peg must be maintained.

Most of our analysis will concern the "complete markets" model, with no limits on an agent's ability to go long or short in dollars and pesos, so that $\left[\underline{\theta}, \overline{\theta}\right] = \left[\underline{y}, \overline{y}\right]$. Note that $\underline{y}$ and $\overline{y}$ depend on some parameters of the model. In other cases, we will look at various "incomplete markets" scenarios, where there are exogenous limits on the position the agent can take. In this case, we will

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[6]This assumption should be understood as a reduced form description of an optimizing decision by the government whether to abandon the peg. Morris and Shin (1998) had a slightly more detailed modelling of government behavior - the government pays an exogenous reputational cost of abandoning the peg - that would give the same results in this setting.

[7]The assumption of a uniform prior is standard in the global games and used for convenience. As discussed in section 4.4.3, the results continue to hold for any prior if the noise is small.

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4.2.3 The "Sunspot Probability" with complete markets

It is both interesting and analytically convenient to introduce the variable

$$\widehat{\theta} = \frac{\theta^* - \theta}{\overline{\theta} - \underline{\theta}}.$$

Since $[\underline{\theta}, \overline{\theta}]$ is the range of fundamentals where there are multiple equilibria, we have that $\widehat{\theta}$ can be interpreted as the probability of a "bad sunspot". If we interpreted the data using a complete information model, the probability of a bad sunspot would be the proportion of times that a self-fulfilling run occurred when this was consistent with fundamentals but not required by fundamentals.

In the special case of complete markets, we can provide a very simple closed form characterization of $\widehat{\theta}$ which we will use extensively in our analysis. The first order conditions for (4.2.2) imply that

$$\pi \left(\widetilde{w}\left(y, e_0\right) \right)^{-\rho} v_D = (1 - \pi) \left(\left(\frac{E}{e_0} \right)^{1-\alpha} \widetilde{w}\left(y, E\right) \right)^{-\rho} v_A.$$

Thus

$$\left(\frac{\left(\frac{E}{e_0} \right)^{1-\alpha} \widetilde{w}\left(y, E\right)}{\widetilde{w}\left(y, e_0\right)} \right)^{-\rho} = \left(\frac{\pi}{1-\pi} \right) \left(\frac{t}{1-t} \right),$$

$$\frac{\widetilde{w}\left(y, E\right)}{\widetilde{w}\left(y, e_0\right)} = \left(\frac{1-\pi}{\pi} \right)^{\frac{1}{\rho}} \left(\frac{1-t}{t} \right)^{\frac{1}{\rho}} \left(\frac{e_0}{E} \right)^{1-\alpha},$$

and

$$y^* \left(\pi \right) = \frac{\left(\frac{1-\pi}{\pi} \right)^{\frac{1}{\rho}} \left(\frac{1-t}{t} \right)^{\frac{1}{\rho}} \left(\frac{e_0}{E} \right)^{1-\alpha} \left(w_D + \frac{w_P}{e_0} \right) - \left(w_D + \frac{w_P}{E} \right)}{\left(\frac{1-\pi}{\pi} \right)^{\frac{1}{\rho}} \left(\frac{1-t}{t} \right)^{\frac{1}{\rho}} \left(\frac{e_0}{E} \right)^{1-\alpha} r + 1 - \frac{e_0}{E} \left(1 + r \right)}.$$

But observe that

$$\underline{y} = y^* \left(1 \right) \text{ and } \overline{y} = y^* \left(0 \right),$$

so

$$
\begin{aligned}
\widehat{y} \left(\pi \right) &= \frac{y^* \left(\pi \right) - \underline{y}}{\overline{y} - \underline{y}} \\
&= \frac{y^* \left(\pi \right) - y^* \left(1 \right)}{y^* \left(0 \right) - y^* \left(1 \right)} \\
&= \frac{\frac{\left(\frac{1-\pi}{\pi} \right)^{\frac{1}{\rho}} \left(\frac{1-t}{t} \right)^{\frac{1}{\rho}} \left(\frac{e_0}{E} \right)^{1-\alpha} \left(w_D + \frac{w_P}{e_0} \right) - \left(w_D + \frac{w_P}{E} \right)}{\left(\frac{1-\pi}{\pi} \right)^{\frac{1}{\rho}} \left(\frac{1-t}{t} \right)^{\frac{1}{\rho}} \left(\frac{e_0}{E} \right)^{1-\alpha} r + 1 - \frac{e_0}{E} \left(1 + r \right)} + \frac{\left(w_D + \frac{w_P}{E} \right)}{1 - \frac{e_0}{E} \left(1 + r \right)}}{\frac{w_D + \frac{w_P}{e_0}}{r} + \frac{\left(w_D + \frac{w_P}{E} \right)}{1 - \frac{e_0}{E} \left(1 + r \right)}} \\
&= \frac{1}{1 + \frac{1 - \frac{e_0}{E} \left(1 + r \right)}{r} \left(\frac{\pi}{1-\pi} \right)^{\frac{1}{\rho}} \left(\frac{t}{1-t} \right)^{\frac{1}{\rho}} \left(\frac{E}{e_0} \right)^{(1-\alpha)}} \\
&= \frac{1}{1 + \left(\frac{\pi}{1-\pi} \right)^{\frac{1}{\rho}} \left(\frac{1-t}{t} \right)^{1-\frac{1}{\rho}}}.
\end{aligned}
$$

Thus

$$
\begin{aligned}
\widehat{\theta} &= \frac{\int_{\pi=0}^{1} y^{*}\left(\pi\right) d\pi - \underline{y}}{\overline{y} - \underline{y}} \\[2ex]
&= \int_{\pi=0}^{1} \widehat{y}\left(\pi\right) d\pi \\[2ex]
&= \int_{\pi=0}^{1} \frac{1}{1 + \left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}} \left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}} d\pi
\end{aligned}
\qquad (4.\text{-}13)
$$

A convenient feature of this expression is that it depends only on the determinants of t (r, $\frac{E}{e_0}$ and α) and risk aversion ρ, and not on portfolio variables (w_D and w_P).

4.3 Comparative Statics

For the complete markets model, we obtained a very convenient characterization of the unique equilibrium in the previous section. For equation (4.-13), we know how the sunspot probability $\widehat{\theta}$ depends on risk aversion ρ and the payoff parameter, t, which in turn depends on the interest rate differential r, the devalued exchange rate E and the preference parameter α. The actual threshold is then given by

$$
\theta^{*} = \left(1 - \widehat{\theta}\right) \underline{y} + \widehat{\theta}\overline{y},
$$

where $\underline{y}$ and $\overline{y}$ are given by (4.2.1) and (4.2.1); $\underline{y}$ and $\overline{y}$ depend on wealth and portfolio composition (w_D and w_P), r and E.

Thus we analyze the effect of risk aversion (ρ) and ownership (α) exclusively by looking at their effect on $\widehat{\theta}$ ($\underline{y}$ and $\overline{y}$ are independent of ρ and α); we then analyze the effect of wealth and portfolio composition (w_D and w_P) exclusively by looking at the their effect on $\underline{y}$ and $\overline{y}$ ($\widehat{\theta}$ is independent of w_D and w_P); finally, when analyzing the effect of r and E, we must take both kinds of effects into account.

As well as analyzing our benchmark complete markets scenario, we also look at a number of incomplete markets scenarios, to see how very different comparative statics conclusions may result under reasonable market restrictions.

4.3.1 Risk Aversion

Complete Markets

We are interested in comparative statics with respect to ρ. Here $\widehat{\theta}$ depends on ρ, but $\underline{y}$ and $\overline{y}$ are independent of ρ. When $\rho \to 0$,

$$\widehat{y}(\pi) \to \begin{cases} 0 & \text{if} \quad \pi > 1 - t \\ 1 & \text{if} \quad \pi < 1 - t \end{cases}$$

An almost risk-neutral agent will bet virtually all his future consumption unless his signal is arbitrarily close to $1 - t$. We get:

$$\widehat{\theta} \to 1 - t$$

$$\theta^* \to t\underline{y} + (1 - t)\overline{y}$$

In applications, t is often considered to be close to 0. That implies $\widehat{\theta}$ close to 1 in the risk neutral case — conditional on θ being in the multiple equilibrium region, the probability of a "bad sunspot" is very high. For this reason, it has been said that global-games currency-crisis models tend to select the 'bad' equilibrium.[9] But with risk aversion, $\widehat{\theta}$ drops dramatically.

When $\rho = 1$ (log utility), we get that:

$$\widehat{y}(\pi) = 1 - \pi$$

With logarithmic utility, the proportion invested in dollars is equal to the probability of a devaluation and does not depend on any other thing — prices are irrelevant. Then,

$$\widehat{\theta} = \frac{1}{2}$$

$$\theta^* = \frac{1}{2}\underline{y} + \frac{1}{2}\overline{y}$$

---

[9]See, for example, Chamley (2003).

Note the dramatic impact of risk aversion on $\hat{\theta}$. For example: if $t = 0.05$, the probability of a "bad sunspot" is 95% in the risk neutral case but equals only 50% if agents have logarithmic utility function.

When $\rho \to \infty$:

$$\hat{y}(\pi) \to t$$

$$\hat{\theta} \to t$$

When there is very little uncertainty (i.e., f puts most probability close to 0) and ρ is large, agents will be almost always choosing either $\underline{y}$ or $\overline{y}$, but $\hat{\theta}$ will be close to t anyway. Now

$$\theta^* \to (1-t)\underline{y} + t\overline{y} = \frac{w_P \left(\frac{1}{e_0} - \frac{1}{E} \right)}{\left(1 - (1+r)\frac{e_0}{E} \right) \left(\frac{E}{e_0} \right)^{(1-\alpha)} + r}.$$

Note that when $w_P = 0$ and ρ tends to ∞, θ^* tends to 0. With no hedging demand because of peso exposure, risk averse agents take zero positions.

Figure 4.1 shows $\hat{\theta}$ as a function of $\log(\rho)$ and t. Under the one way bet assumption ($t < \frac{1}{2}$), risk aversion reduces $\hat{\theta}$ and makes investors less willing to attack the currency. The opposite holds when $t > \frac{1}{2}$. The sunspot probability ($\hat{\theta}$) equals $\frac{1}{2}$ whenever $t = \frac{1}{2}$ or $\rho = 1$.[10]

Agents' expectations on others' actions play a crucial role in determining the outcome of the game. Risk aversion influences other agents' positions, which determines θ^*. What drives the results is the impact of risk aversion on what *all others* will do — not on what *a single individual* will do. The impact of a tiny fraction of agents with different levels of risk aversion on θ^* is negligible, as we show at section 4.4.2.

Figure 4.2-a cuts figure 4.1 at some given values of ρ. We can see that the impact of t on the sunspot probability depends crucially on risk aversion. Interestingly, for $\rho > 1$, $\hat{\theta}$ is *increasing* in t: with complete markets, for empirically plausible levels of risk aversion, a higher cost of attacking the currency leads to a higher probability of a "bad sunspot". This result may sound counter-intuitive at first. The intuition is that although the gains from a successful currency attack are decreasing

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[10]Analytically, we are able to show that for $t < \frac{1}{2}$, $\hat{\theta}$ is decreasing in ρ for $\rho \geq 1$ — we couldn't prove it for $\rho < 1$ although we believe it also holds in this case. Analogously, for $t > \frac{1}{2}$, we can show that $\hat{\theta}$ is increasing in ρ only for $\rho \leq 1$.

Figure 4.1: $\hat{\theta}$ as a function of t and $\log(\rho)$
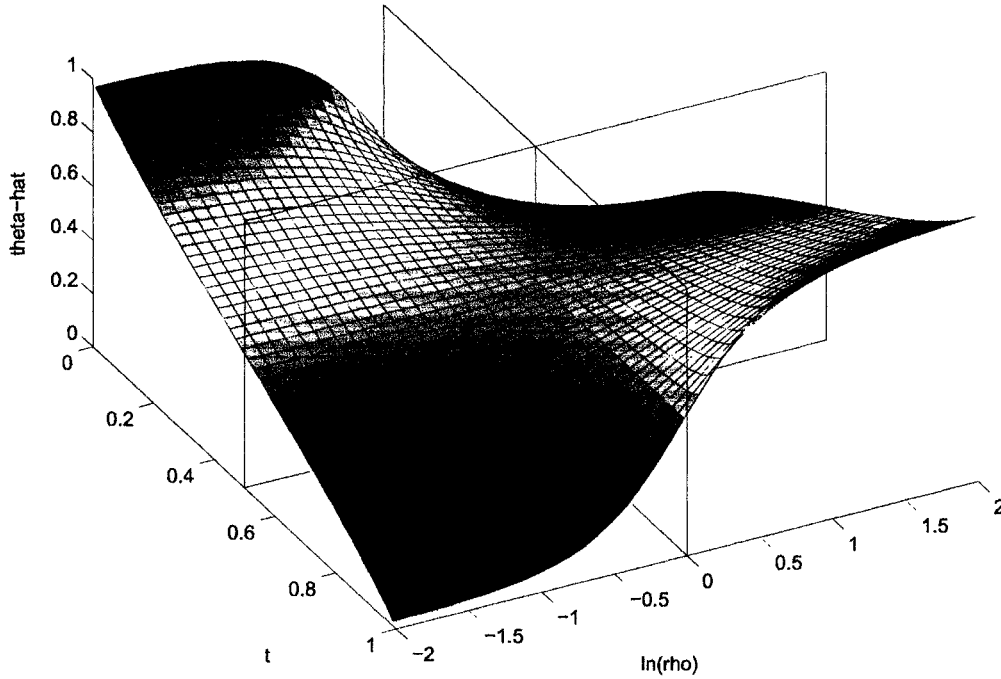
in t, the incentives for attacking are not increasing in the gains from attacking: depending on the level of risk aversion, hedging motivations may dominate the prospects of higher gains. Moreover, factors that influence t may also affect $\underline{y}$ and $\overline{y}$, so the overall effects of prices (E and r) on θ^* are not totally captured by figures 4.1 and 4.2-a.

Figure 4.2-b cuts figure 4.1 at some given values of t and shows that $\hat{\theta}$ reacts strongly to changes in risk aversion for low values of ρ. If t is small, the sunspot probability for empirically plausible degrees of risk aversion is completely different from the risk neutral case.

In sum, when agents are free to short any amount of dollars and pesos, risk aversion has huge impacts on $\hat{\theta}$ (and thus θ^*). Next, we check what happens when agents positions are restricted and compare results.

Figure 4.2: Effects of t and ρ in $\hat{\theta}$
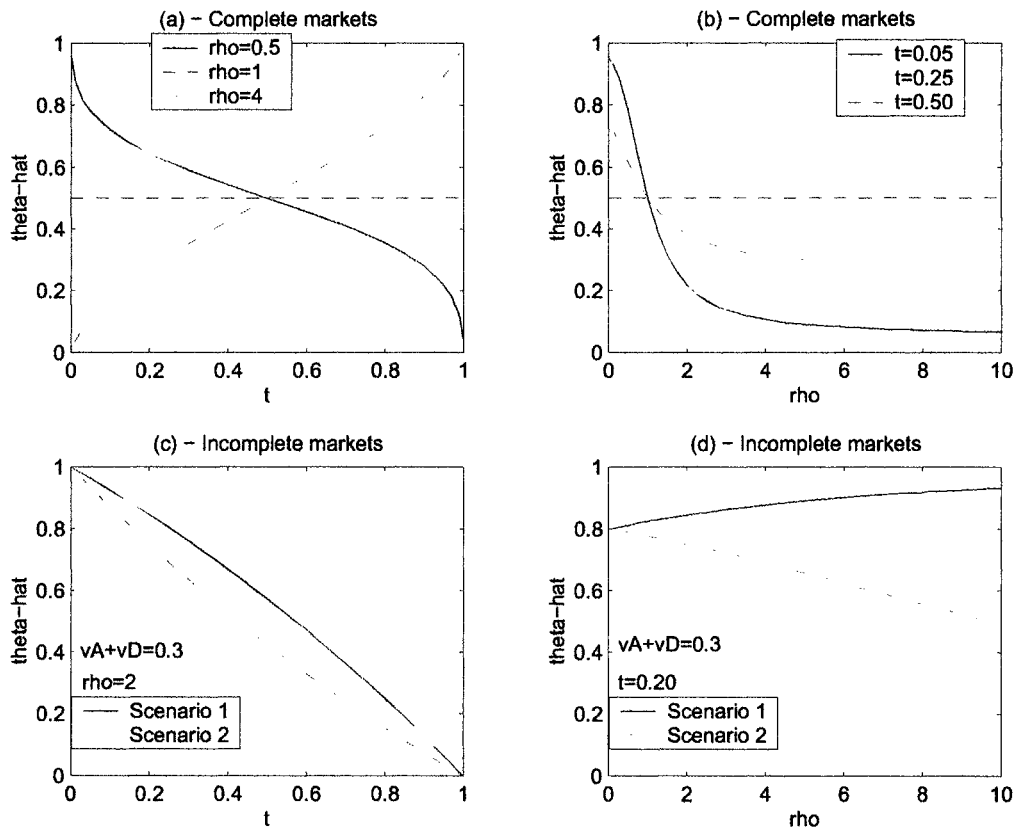
Incomplete Markets

With complete markets, we can sign the effect of risk aversion on both $\hat{\theta}$ and θ^*. In the complete markets analysis, it is endogenous whether attacking the currency (high y) or defending the currency (low y) is the riskier action.

Some of our intuition about the effect of risk aversion on currency crises comes from situations where we know that either defending or attacking is riskier for the investor. This intuition depends on some incompleteness of markets. Next, we present scenarios where risk aversion will unambiguously increase the probability of attacks or unambiguously reduce it, independent of the one-way bet assumption (i.e., the size of t).

INCOMPLETE MARKETS SCENARIO 1.

Foreign investors with all their wealth in dollars cannot go short in either currency. Thus, $w_D > 0$, $w_P = 0$, $\alpha = 1$, $\underline{\theta} = -w_D$ and $\bar{\theta} = 0$ (if $y^* = -w_D$, all his wealth is invested in pesos and if $y^* = 0$, all his wealth is invested in dollars). In this case, $\hat{\theta}$ is given by:

$$\hat{\theta} = \frac{\theta^* + w_D}{w_D}$$

For such an investor, the safe action is to hold his wealth in dollars (i.e., to attack the currency) and the risky action is to hold pesos (i.e., to defend the currency). In particular, this investor's problem is:

$$y^*(\pi) = \operatorname*{arg\,max}_{y \in [-w_D, 0]} \left[\pi \left(w_D - ry \right)^{1-\rho} + (1-\pi) \left(w_D + y \left(1 - \frac{e_0}{E}(1+r) \right) \right)^{1-\rho} \right]. \qquad (4.\text{-}13)$$

For any $\rho \in (0, \infty)$, if $\pi < 1 - t$, we will have that the investor would like to short pesos and go long in dollars, but cannot, so $y^*(\pi) = 0$; if $\pi > 1 - t$, the investor will hold less than his whole portfolio in dollars, so $y^*(\pi) < 0$.

Now if $\rho \to 0$, we will have

$$y^*(\pi) \to \begin{cases} -w_D & \text{if } \pi > 1 - t \\ 0 & \text{if } \pi < 1 - t \end{cases}, \; \hat{\theta} \to 1 - t \text{ and } \theta^* \to -tw_D$$

As $\rho \to \infty$, we will have

$$y^*(\pi) \to 0, \hat{\theta} = 1 \text{ and } \theta^* = 0.$$

Thus risk aversion increases the probability of attacks (independent of t), because attacking is the safe action by assumption.

INCOMPLETE MARKETS SCENARIO 2. A domestic investor with all wealth in pesos who cannot go short in either currency. Thus $w_D = 0$, $w_P > 0$, $\alpha = 0$, $\underline{\theta} = 0$ and $\bar{\theta} = \frac{w_P}{e_0}$ (if $y^* = \frac{w_P}{e_0}$ all his wealth is invested in dollars). In this case, $\hat{\theta}$ is given by:

$$\hat{\theta} = \frac{\theta^* e_0}{w_P}$$

For this investor, the safe action is to hold his wealth in pesos (i.e., to defend the currency) and the risky action is to hold dollars. The investor's problem is

$$y^*(\pi) = \underset{y \in \left[0, \frac{w_P}{e_0}\right]}{\arg\max} \left[\pi \left(\frac{w_P}{e_0} - ry\right)^{1-\rho} + (1-\pi)\left(\frac{w_P}{e_0} + y\left(\frac{E}{e_0} - (1+r)\right)\right)^{1-\rho}\right]. \qquad (4.-13)$$

For any $\rho \in (0, \infty)$, if $\pi > 1 - t$, the investor would like to short dollars and go long in pesos, but cannot, so $y^*(\pi) = 0$; if $\pi < 1 - t$, the investor will hold a positive amount of dollars, $y^*(\pi) > 0$.

Now if $\rho \to 0$, we will have

$$y^*(\pi) \to \begin{cases} 0 & \text{if } \pi > 1 - t \\ \frac{w_P}{e_0} & \text{if } \pi < 1 - t \end{cases}, \hat{\theta} \to 1 - t \text{ and } \theta^* \to (1-t)\frac{w_P}{e_0}$$

As $\rho \to \infty$, we will have

$$y^*(\pi) \to 0, \hat{\theta} = 0 \text{ and } \theta^* = 0.$$

Thus risk aversion reduces the probability of attacks (independent of t), because defending is the safe action by assumption.

If we included both the investors of scenario 1 and the investors of scenario 2, then the heterogeneous agent argument of section 4.4.2 shows that threshold would move linearly between the two results as a function of the proportion of investors of both types.

Figures 4.2-c and 4.2-d show numerical results for $\hat{\theta}$ in both scenarios. As shown above, when $\rho \to 0$, $\hat{\theta}$ approaches $(1 - t)$ — which is the result in a model with risk neutral agents. The effect of risk aversion in the sunspot probability depends on which is the risky action and its sign is independent of t.

The impacts of risk aversion with short-selling constraints (figures 4.2-c and 4.2-d), although not at all negligible, are not as huge as in the complete market case. When agents are free to take any position they want, they may end up shorting large amount of dollars or pesos and, therefore, facing the risk of getting almost no consumption if their bet goes wrong. Therefore, impacts of risk aversion on their decisions are potentially big. On the other hand, if investors' positions are limited, so are the effects of risk aversion.

With incomplete markets, when $v_A + v_D$ is small, there is little risk and ρ has not much impact on agent's decision. With complete markets, $\hat{\theta}$ does not depend on $v_A + v_D$ because investors choose the amount of risk they will face.

4.3.2 Ownership

We are interested in comparative statics with respect to the parameter α. We focus on the case of complete markets. Our interpretation is that a high α corresponds to foreign investors (who will use terminal wealth to purchase dollar denominated goods) and a low α corresponds to domestic investors. Here $\hat{\theta}$ in independent depends on α, but $\underline{y}$ and $\bar{y}$ are independent of α.

All the impact of α on the threshold goes through t. As:

$$\frac{d\hat{\theta}}{d\left(\frac{1-t}{t}\right)} = -\int_0^1 \frac{\left(\frac{\rho-1}{\rho}\right)\left(\frac{1-t}{t}\right)^{-\frac{1}{\rho}}}{\left(1 + \left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{\rho-1}{\rho}}\right)^2}d\pi$$

$\frac{d\hat{\theta}}{d\left(\frac{1-t}{t}\right)}$ is positive for $\rho < 1$ and negative for $\rho > 1$. Also, we have that:

$$\frac{d\left(\frac{1-t}{t}\right)}{d\alpha} = -\frac{1}{r}\left(1 - (1+r)\frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{1-\alpha}\ln\left(\frac{E}{e_0}\right) < 0$$

So, the effect of α on θ^* depends on ρ, as shown at table 4.1.

This result may sound counter intuitive — shouldn't a higher α imply a lower threshold? It is true that a higher α implies a higher t — i.e., a higher cost of attacking. A higher t turns investors

ρ	< 1	$= 1$	> 1
$\frac{d\hat{\theta}}{d\alpha}$	< 0	$= 0$	> 0

Table 4.1: $\frac{d\hat{\theta}}{d\alpha}$

less inclined to attack the currency if they are not very risk averse but also turn them more interested in holding dollars if $\rho > 1$ by increasing demand for hedging. When we have log utility, α does not impact the threshold: hedging motivations are just enough to offset the prospects of higher gains, as pointed by figures 4.1 and 4.2-a and the discussion at section 4.3.1.

4.3.3 Wealth

Complete Markets

It is often said that a negative wealth shock may threaten a currency peg because investors are forced to withdraw their money. For example, when Russia defaulted its debt in 1998, Brazil experienced a large capital outflow.

We first examine how a wealth shock could effect the coordination of agents and thus change θ^* even with complete markets. It is important to note that with complete markets, a decrease in wealth will decrease the size of the position that both attacking and defending agents can take (consistent with the Inada condition). We want to find out which effect is more important. For simplicity, we focus on the case where $w_P = 0$, $w_D > 0$ and $\alpha = 1$, so that all wealth belongs to foreigners.[11] We are interested in comparative statics with respect to wealth, w_D. Here the sunspot probability $\hat{\theta}$ in independent of wealth, but $\underline{y}$ and $\bar{y}$ depend on wealth.

We have:

---

[11]The result also holds if all money belong to domestic residents.

$$\theta^* = \hat{\theta}\bar{y} + \left(1 - \hat{\theta}\right)\underline{y}$$

$$= w_D \left[\frac{\hat{\theta}}{r} - \frac{1 - \hat{\theta}}{1 - \frac{c_0}{E}(1+r)}\right]$$

$$\frac{d\theta^*}{dw_D} = \frac{\hat{\theta}}{r} - \frac{1 - \hat{\theta}}{1 - \frac{c_0}{E}(1+r)}$$

$$= \left(\frac{1-\hat{\theta}}{r}\right)\left(\frac{\hat{\theta}}{1-\hat{\theta}} - \frac{t}{1-t}\right)$$

If $t < \frac{1}{2}$, $\hat{\theta} > t$ for any ρ. So

$$\frac{d\theta^*}{dw_D} > 0.$$

Thus increased wealth reduces the probability of a successful currency attack (while the sunspot probability remains unchanged). Note that θ^* is linear in w_D. Our result, with complete markets, is totally different from the usual intuition. If $t < \frac{1}{2}$, most agents are selling *Pesos* short. As the utility function is concave, the amount of risk one agent chooses to face depends positively on his wealth. So, a negative wealth shock reduces the munition an agent is willing to use to attack the currency.

As we show now, with incomplete markets, this result may not hold.

Incomplete Markets

Consider again scenario 1 above. The investor will choose:

$$y^* = \begin{cases} -w_D & \text{if } y^{foc} \le -w_D \\ y^{foc} & \text{if } y^{foc} \in (-w_D, 0) \\ 0 & \text{if } y^{foc} \ge 0 \end{cases}$$

where

$$y^{foc} = w_D \frac{1 - \left(\frac{\pi \ r}{(1-\pi)v_A}\right)^{-\rho}}{r + v_A\left(\frac{\pi \ r}{(1-\pi)v_A}\right)^{-\rho}}.$$

As in the complete markets case, y^* is given by a linear function of w_D. However, in the present example, the investor will always choose a negative y^*. Thus, increasing wealth will always increase the size of his position in absolute value, increasing the region of a successful currency attack.

This result is similar to those obtained in the contagion model by Goldstein and Pauzner (2001).[12] Short selling constraints are implicitly assumed in their essay and agents decide to invest or not one unit of money in the country. We show here that their conclusion still holds when agents have a continuum of actions but depends on positive investments in the country — agents need to be investing, not attacking.

4.3.4 Portfolio

Complete Markets

We are interested in comparative statics as we shift wealth between w_D and w_P. Again, $\widehat{\theta}$ is independent of such portfolio reallocations, but $\underline{y}$ and $\bar{y}$ depend on the portfolio. Suppose that we increase w_D by e_0 units and reduce w_P by 1 unit. That is a change in the initial allocation of portfolio without changing agents' wealth. Then, we have:

$$\frac{1}{e_0}\frac{d\theta^*}{dw_D} - \frac{d\theta^*}{dw_P} = \frac{E\left(1-\widehat{\theta}\right)}{E - e_0(1+r)}\left(\frac{e_0 - E}{e_0.E}\right) < 0$$

We could also increase w_D by E units and reduce w_P by 1 unit. We still get:

$$\frac{1}{E}\frac{d\theta^*}{dw_D} - \frac{d\theta^*}{dw_P} = \frac{\widehat{\theta}}{r}\left(\frac{e_1 - E}{e_1.E}\right) < 0$$

An increase in the share of peso assets creates incentives for agents to *reduce* y_D for they are risk averse and search for hedge. This result says that investors who are exposed to exchange rate risk (say, through foreign direct investment), will be more likely to attach the currency at the margin because of hedging motives.

Figure 4.3 shows that the quantitative importance of portfolio effects, with complete markets, is small.[13] The threshold θ^* is very sensitive to ρ but seems to be almost unaffected by changes in portfolio allocation.

Next, we investigate whether portfolio effects still occur in the presence of short selling constraints and if their magnitude is higher in that case.

---

[12]Their result is in a more general model: in particular, they allow for any decreasing absolute risk aversion utility function (not just constant relative risk aversion); and they allow for general strategic complementarities in investment in each country.

[13]Parameters in this example: $E = 1.2$, $e_0 = 1$, $r = 0.03$ and $\alpha = 0.5$.

Figure 4.3: Portfolio effects
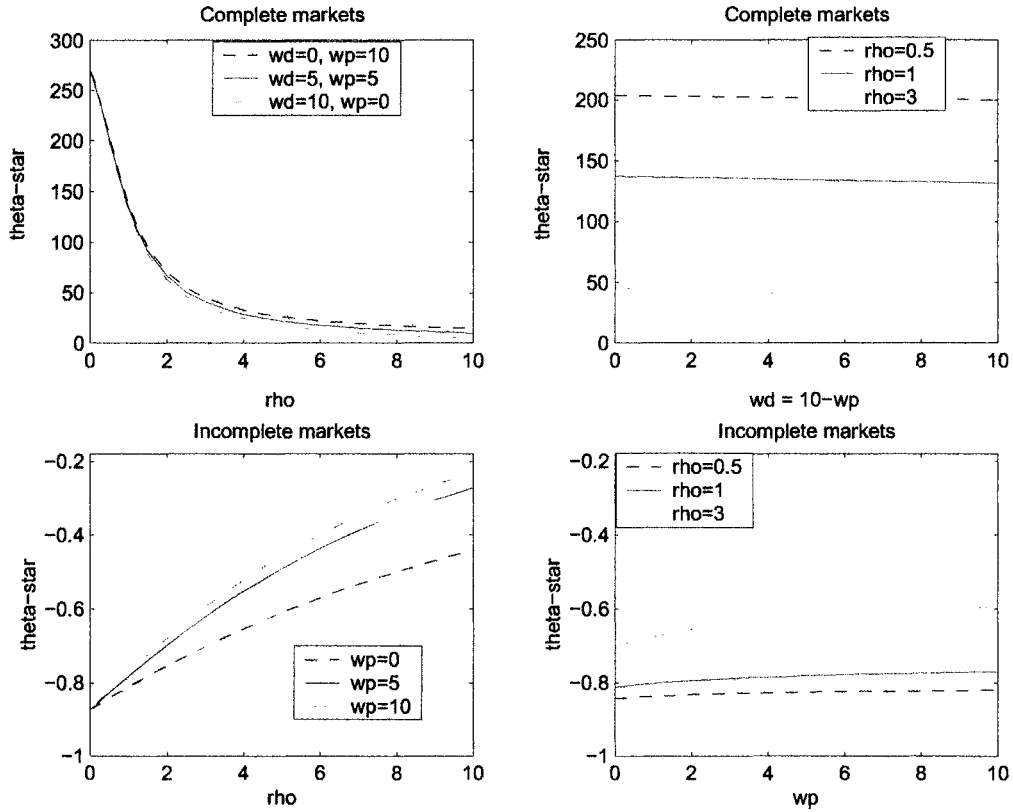
Incomplete Markets

With complete markets, we are allowing the investor to borrow against his peso wealth. A more interesting scenario might be when investors' peso wealth is illiquid and short sales are restricted. Next, we study an example in which agents can sell their dollar wealth but not their peso wealth and can't go short in either currency.[14]

INCOMPLETE MARKETS SCENARIO 3. A foreign investor has all his liquid wealth in dollars but may have illiquid investments in pesos. Thus $w_D > 0$, $w_P > 0$, $\alpha = 1$, $\underline{\theta} = -w_D$ and $\overline{\theta} = 0$. In this case,

---

[14]Allowing agents to short some amount of either currency (say, at most twice their dollar wealth) has little impact on our results.

$$y^* \left(\pi \right) = \underset{y \in [-w_D, 0]}{\arg\max} \left[\pi \left(w_D + \frac{w_P}{e_0} - ry \right)^{1-\rho} + (1 - \pi) \left(w_D + \frac{w_P}{E} + y \left(1 - \frac{e_0}{E} \left(1 + r \right) \right) \right)^{1-\rho} \right].$$

Figure 4.3 shows that, with incomplete markets, higher illiquid investment in pesos also implies a higher threshold θ^* and, therefore, a larger set of states in which devaluation occurs.[15] Such effect is increasing in ρ and vanishes as $\rho \to 0$.

As in the complete market case, risk aversion is much more important than portfolio allocation in determining θ^*.

4.3.5   The Size of Devaluation

We are interested in comparative statics with respect to the parameter E. An increase in E represents an increase in the size of the devaluation, if it occurs.

Observe first that an increase in E increases $\underline{y}$, since agents must then take a smaller peso position and thus a smaller (negative) dollar position; $\overline{y}$ does not depend on E. Thus an increase in E shifts the complete information multiple equilibria range unambiguously in the direction of more attacks.

An increase in E decreases t. As we have already observed at section 4.3.1, $\widehat{\theta}$ is decreasing in t if $\rho < 1$ but increasing in t if $\rho > 1$. Thus for empirically plausible levels of risk aversion, a higher value of E leads to a *higher* probability of a bad sunspot. The intuition is simply that a higher E leads to an increase in the gains from a successful currency attack and, for $\rho > 1$, hedging motivations are the dominant forces.

Combining the two effects, we know that for $\rho \leq 1$, we must have θ^* increasing in E. But for $\rho > 1$, the combined effect may go either way for reasonable values of the parameters. Figure 4.4-a shows θ^* as function of E for different levels of risk aversion.[16] We can see that for $\rho \leq 1$, θ^* is increasing in E: higher rewards for a successful attack make it more likely. For $\rho = 3$, however, θ^* is decreasing in E for sufficiently high values of E (i.e., for sufficiently low values of t). Figure 4.4-b plots the same function θ^* using different scale and shows clearly that, for $\rho = 3$, an increase E may turn a devaluation less likely.

---

[15]Parameters in this example: $E = 1.2$, $e_0 = 1$, $r = 0.03$ and $w_D = 5$.

[16]Parameters of this example: $w_D = 1$, $w_P = 1$, $r = 0.03$, $e_0 = 1$, $\alpha = 0.5$. The non-monotonicity of θ^* as function of E holds for empirically plausible values of parameters regardless of the values of α, w_D and w_P.

Figure 4.4: Effects of E and r
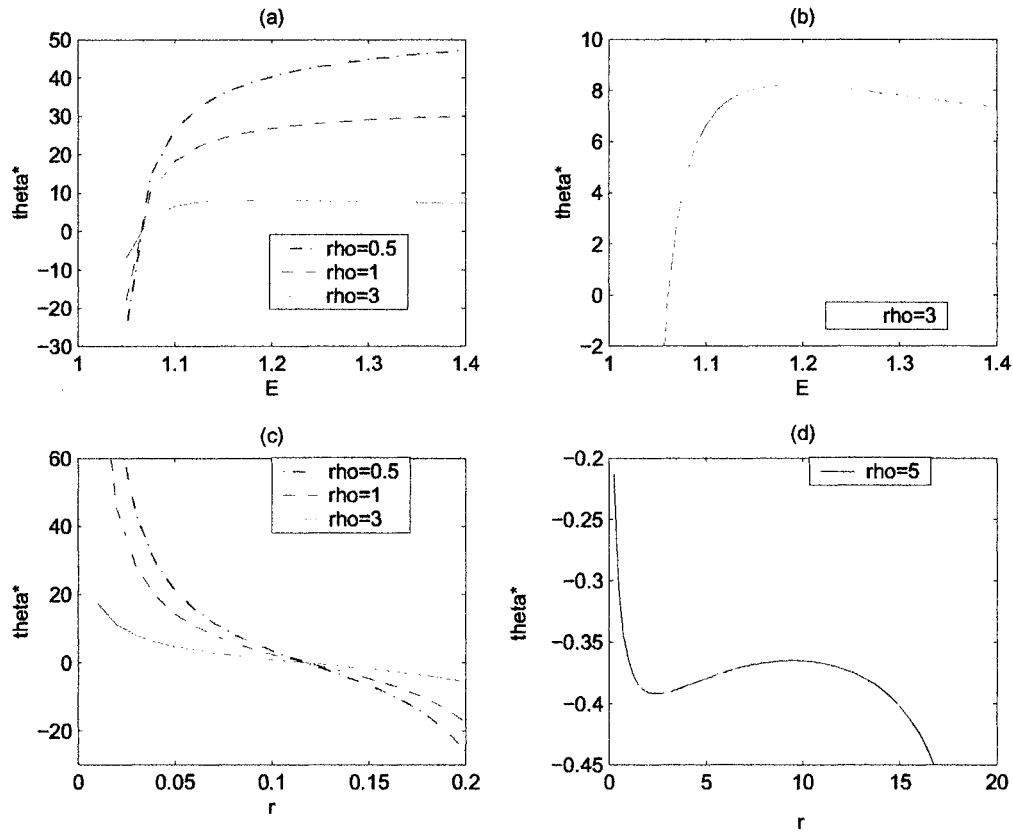
4.3.6 Interest Rate Defense

We are interested in comparative statics with respect to the parameter r. How does an increase in r affect the likelihood of currency crises?[17] Again, $\widehat{\theta}$ depends on r (through t), but so do $\underline{y}$ and $\overline{y}$.

Observe first that an increase in r reduces $\underline{y}$, since the reduced return to attacking the currency allows investors to take a larger peso position and thus a larger (negative) dollar position; an increase in r also reduces $\overline{y}$, since the largest dollar position consistent with being able to pay the interest differential on the short peso position is reduced. Thus an increase in r shifts the complete information multiple equilibria range unambiguously in the direction of less attacks.

An increase in r increases t. As we have already observed, this increases $\widehat{\theta}$ if $\rho < 1$ but decreases $\widehat{\theta}$

---

[17]In our model, the choice of r does not serve a signal of private information of the central bank. In reality, the interest rate may be a signal. See Angeletos, Hellwig and Pavan (2003) for a global games model with interest rate signalling by the central bank.

if $\rho > 1$. Again, for empirically plausible levels of risk aversion, an interest rate defense paradoxically increases the probability of a bad sunspot — a higher interest rate means that the full hedging demand for dollars is increased.

Which effect wins out overall? For $\rho \leq 1$, we must have θ^* decreasing in r. We find that this comparative static is maintained for $\rho > 1$ for reasonable values of the parameters. Figure 4.4-c shows θ^* for different levels of risk aversion.[18] Regardless of the value of ρ, an increase in r turns a devaluation less likely.

However, it is possible to construct examples where θ^* is increasing in r. Figure 4.4-d shows an example of such curious behavior of θ^*. The parameters in such example ($e_0 = 1$, $E = 20$ and $r \in (200\%, 900\%)$) are too unrealistic in the context of currency crisis.[19] However, in other applications in which the difference between agent's utility in the 2 possible states is too big, we may find this perverse effect of an interest rate defense.[20]

4.4 Model Robustness

4.4.1 A General Noise Independence Result for Two State Global Games

Consider a game played by I continua of agents, where each agent with index i chooses an action $a \in [\underline{\theta}, \overline{\theta}]$. Let λ_i be the mass of agents with index i. Let $u_i : A \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ be agent i's utility function, so that

$$u_i\left(a, \overline{a}, \theta\right)$$

is agent i's utility if he chooses action a, the average action in the population is given by $\overline{a}$ and the state is θ. Assume that the payoff function takes the following special form:

$$u_i\left(a, \overline{a}, \theta\right) = \begin{cases} \underline{v}_i\left(a\right), & \text{if } \theta \geq \overline{a} \\ \overline{v}_i\left(a\right), & \text{if } \theta < \overline{a} \end{cases}$$

---

[18]Parameters of this example: $w_D = 1$, $w_P = 1$, $E = 1.25$, $e_0 = 1$, $\alpha = 0.5$. We could not find an example of θ^* increasing in r for any reasonable values of E, e_0 and r.

[19]Other parameters are: $w_P = 1$, $w_D = 1$, $\alpha = 0.5$.

[20]One could extend our model to the case of debt crises. If there is risk of total default, could an interest rate defense make matters worse?

Further assume that $a' > a$ implies

$$\overline{v}_i\left(a'\right) - \overline{v}_i\left(a\right) > \underline{v}_i\left(a'\right) - \underline{v}_i\left(a\right). \tag{4.-17}$$

Let

$$\widetilde{a}_i\left(\pi\right) = \arg\max_a \left(1 - \pi\right)\overline{v}_i\left(a\right) + \pi\underline{v}_i\left(a\right).$$

Assumption (4.4.1) ensures that $\widetilde{a}_i\left(\pi\right)$ is weakly decreasing in π. Let $\widetilde{a}_i\left(0\right) = \lim_{\pi\to 0} \widetilde{a}_i\left(\pi\right)$ and $\widetilde{a}_i\left(1\right) = \lim_{\pi\to 0} \widetilde{a}_i\left(\pi\right)$.

We maintain the information assumptions: thus we assume that θ is uniformly distributed on the real line and that each class of agents doesn't know θ but observe a signal x, $x = \theta + \varepsilon$, where the ε are distributed in the population according to probability density function f.

These assumptions ensure that key supermodularity and limit dominance properties of Frankel, Morris and Pauzner (2003) are satisfied. As a consequence, there will be an essentially unique equilibrium where each type of agent has a non-increasing strategy $s_i : \mathbb{R} \to A$. Corresponding to this strategy profile $s = (s_i)_{i=1}^{I}$, there is an average action

$$\widehat{a}\left(\theta\right) = \sum_{i=1}^{I}\lambda_i \int_{\varepsilon = -\infty}^{\infty} s_i\left(\theta + \varepsilon\right) d\varepsilon.$$

This is non-increasing in θ. Thus there will be a unique θ^* solving $\theta = \widehat{a}\left(\theta\right)$. Now recall that an agent who observes signal x attaches probability $F\left(x - \theta^*\right)$ to $\theta \geq \theta^*$. Thus $s_i\left(x\right) = \widetilde{a}_i\left(F\left(x - \theta^*\right)\right)$. Thus

$$\begin{aligned}
\theta^* &= \sum_{i=1}^{I}\lambda_i \int_{\varepsilon = -\infty}^{\infty} \widetilde{a}_i\left(F\left(\left(\theta^* + \varepsilon\right) - \theta^*\right)\right) f\left(\varepsilon\right) d\varepsilon \\
&= \sum_{i=1}^{I}\lambda_i \int_{\varepsilon = -\infty}^{\infty} \widetilde{a}_i\left(F\left(\varepsilon\right)\right) f\left(\varepsilon\right) d\varepsilon \\
&= \sum_{i=1}^{I}\lambda_i \int_{\pi = 0}^{1} \widetilde{a}_i\left(\pi\right) d\pi.
\end{aligned}$$

Thus we have a closed form characterization of the unique equilibrium: an agent of type i observing signal x chooses action $\widetilde{a}_i\left(F\left(x - \theta^*\right)\right)$, where

$$\theta^* = \sum_{i=1}^{I}\lambda_i \int_{\pi = 0}^{1} \widetilde{a}_i\left(\pi\right) d\pi. \tag{4.-20}$$

Frankel, Morris and Pauzner (2003) established the existence of a unique equilibrium in a global game with a uniform prior.[21] However, some games satisfy a "noise independence property," where the structure of equilibrium is independent of the shape of the noise. In particular, the limiting behavior of the unique equilibrium as the noise goes to zero is independent of the distribution of the noise. Such a noise independence property holds in symmetric games with a continuum of agents and binary actions, and Morris and Shin (2003) argue how the simple "Laplacian" characterization of the unique equilibrium in this case is useful in applications. Frankel et al. (2003) and Morris and Ui (2002) give other examples of games where the noise independence property holds. However, the noise independence property does not always hold, as shown by an example in Frankel et al. (2003) and the currency crisis application of Corsetti, Dasgupta, Morris and Shin (2003). The above argument identifies another sufficient condition for noise independent selection. It relies on the fact that there even though there are a continuum of actions, others' actions and the state θ only enter each agent's utility via a binary classification. This is a fairly restrictive condition. However, it allows for arbitrary action sets and arbitrary asymmetry (consistent with monotonicity properties) among agents' payoff functions. The sufficient condition could clearly be weakened a little bit - for example, the binary classification might depend on an increasing aggregate statistic of all agents' actions (rather than a linear function). With this general argument, we immediately see how we could introduce heterogeneous agents into the earlier model and the only impact on the results would threshold θ^* would be a weighted sum of the thresholds that would have arisen in a homogenous agent model, with the weights of each type equal to their proportion in the population.

4.4.2 Heterogeneous Agents

For notational convenience, we will analyze the complete markets model with a finite set of possible types. However, the incomplete markets models will generalize in a similar way and we could deal with a continuum of types essentially by replacing summations with integrals.

An agent of type i is characterized by a coefficient of constant relative risk aversion ρ_i, dollar-denominated wealth w_{Di}, peso-denominated wealth w_{Pi}, and preference parameter α_i. If this agent

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[21]The result is actually proved for an arbitrary prior and small noise. However, a step in the argument involves showing uniqueness for a uniform prior and arbitrary noise.

assigned probability π to the peg being maintained, this agent demand for dollars would be

$$
y_i^*(\pi) = \underset{y \in [\underline{\theta}, \bar{\theta}]}{\arg\max} \left[\begin{array}{l} \pi \left(w_{Di} + \frac{w_{Pi}}{e_0} - yr \right)^{1-\rho_i} \\ + (1-\pi) \left(\left(\frac{E}{e_0} \right)^{1-\alpha_i} \left(w_{Di} + \frac{w_{Pi}}{E} + y \left(1 - \frac{e_0}{E}(1+r) \right) \right) \right)^{1-\rho_i} \end{array} \right]. \tag{4.-20}
$$

If there was a homogenous continuum of agents of type i, we know that the critical threshold would be

$$
\theta_i^* = \int\limits_{\pi=0}^{1} y_i^*(\pi) \, d\pi.
$$

But if there was a heterogeneous population, with proportion λ_i of type i, then the argument of the previous section and equation (4.4.1) imply that:

$$
\begin{aligned}
\theta^* &= \sum_{i=1}^{I} \lambda_i \theta_i^* \\
&= \sum_{i=1}^{I} \lambda_i \int\limits_{\pi=0}^{1} y_i^*(\pi) \, d\pi.
\end{aligned}
$$

4.4.3 Assumptions

At this point, it is useful to review the role of some of the assumptions made in our analysis.

The Uniform Prior Assumption

We made the convenient assumption that θ was uniformly distributed on the real line. This is a standard simplifying assumption in the global games literature (see Morris and Shin (2003)). If we bounded the support of the noise distribution f, we could have had θ uniform on a bounded interval, with no change in the analysis. In addition, if θ were drawn from a smooth, but non-uniform, prior and we let the variance of the noise shrink to zero (i.e., the support f shrinks to zero), then the limiting equilibrium threshold is equal to threshold identified under the uniform prior assumption. Intuitively, if the noise is small, variation in the density of the prior becomes irrelevant. Thus our results should be understood as applying if either uncertainty is small or uncertainty is large but there is not too much prior or public information about θ.

The Known Devaluation Assumption

A crucial assumption in our model is that the exchange rate at period 1, conditional on the occurrence of a devaluation, is common knowledge and constant (equal to E). The size of the devaluation

is independent of θ, which represents the ability of the government to defend the peg. A more realistic assumption would be that if a devaluation occurred in state θ, then the new exchange rate would be $e_1(\theta)$, where e_1 is a decreasing function of θ.[22] Both our noise independence property and our ability to get a closed form solution would go away in this model. In particular, we heavily exploited the fact that we were always evaluating two state gambles. In general, there would be a complicated interaction between the binary uncertainty about whether will be a devaluation or not, and the richer uncertainty about the size of the devaluation.

However, there is some hope of extending our results if there was a small amount of uncertainty about θ. In this case, agents' uncertainty about the size of the devaluation would go away even as strategic uncertainty about others' actions remained (this is the key insight of the global games approach). Thus if we restricted the noise to have finite support and let the support shrink to zero, then the existing analysis might apply.

If there is no uncertainty about the size of the devaluation, a bit of algebra on the results of section 4.2.3 shows that θ^* is the unique value of θ solving:

$$
\theta = \frac{w_D + \frac{w_P}{e_0}}{r} \left(\int_{\pi=0}^{1} \left[1 + \left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}} \left(\frac{\left(1-(1+r)\frac{e_0}{E}\right)}{r}\right)^{1-\frac{1}{\rho}} \right]^{-1} d\pi \right)
$$
$$
+ \frac{w_D + \frac{w_P}{E}}{1-(1+r)\frac{e_0}{E}} \left(1 - \int_{\pi=0}^{1} \left[1 + \left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}} \left(\frac{\left(1-(1+r)\frac{e_0}{E}\right)}{r}\right)^{1-\frac{1}{\rho}} \right]^{-1} d\pi \right)
$$

If agents' demand for dollars was increasing in E (implying that the right hand side is increasing in E), then with uncertainty about the size of a devaluation, our candidate solution would be the unique value of θ solving:

$$
\theta = \frac{w_D + \frac{w_P}{e_0}}{r} \left(\int_{\pi=0}^{1} \left[1 + \left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}} \left(\frac{\left(1-(1+r)\frac{e_0}{e_1(\theta)}\right)}{r}\right)^{1-\frac{1}{\rho}} \right]^{-1} d\pi \right) \qquad (4.-23)
$$
$$
+ \frac{w_D + \frac{w_P}{e_1(\theta)}}{1-(1+r)\frac{e_0}{e_1(\theta)}} \left(1 - \int_{\pi=0}^{1} \left[1 + \left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}} \left(\frac{\left(1-(1+r)\frac{e_0}{e_1(\theta)}\right)}{r}\right)^{1-\frac{1}{\rho}} \right]^{-1} d\pi \right)
$$

---

[22]This is essentially the model analyzed in Morris and Shin (1998). In that risk neutral incomplete markets model, the uniqueness result continues to hold.

A unique solution will exist since the left hand side is increasing in θ and the right hand side is decreasing in θ.

Our results at section 4.3.5 imply that, if $\rho \leq 1$, agents' demand for dollars is increasing in E and, therefore, the right hand side of equation (4.-23) is increasing in E. In this case, we have a unique equilibrium, as shown at figure 4.5-a. The decreasing function is the (exogenous) relation between fundamentals (θ) and the exchange rate (the inverse of $e_1(\theta)$). The increasing function is what θ^* would be if E was a known constant. The intersection of both curves gives the unique equilibrium.[23]

Figure 4.5: Equilibria

$(a) - rho=1$

$(b) - rho=3$
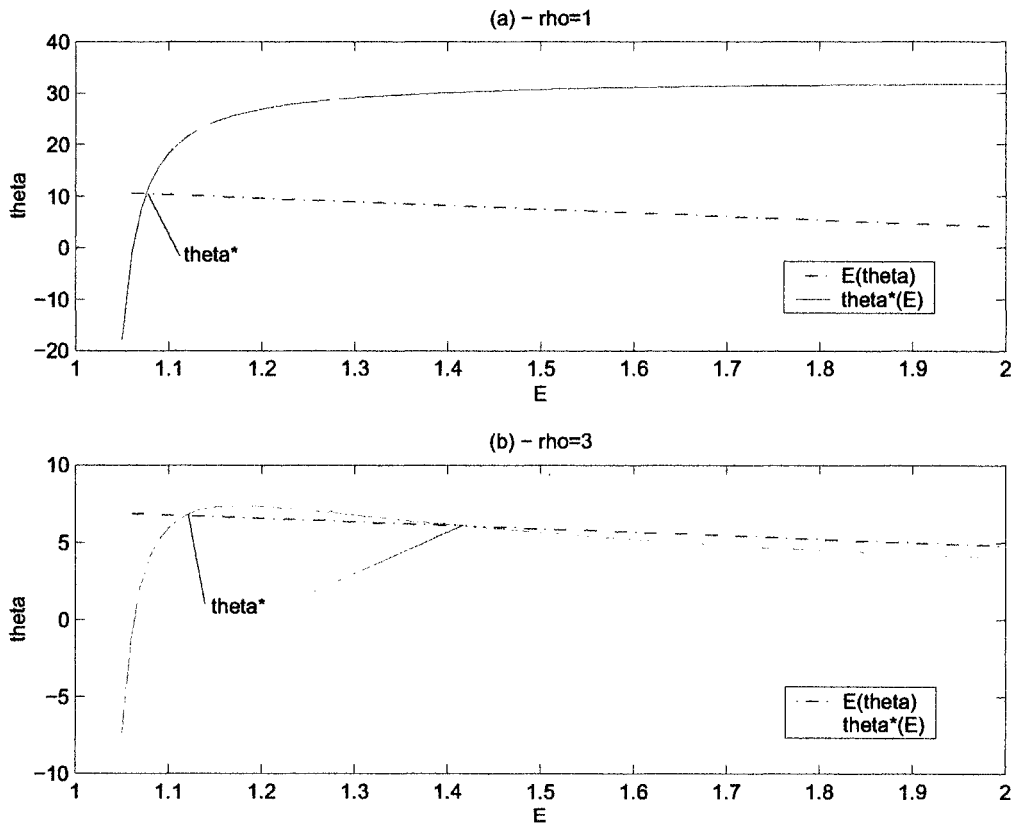
However, for $\rho > 1$, agents' demand for dollars may be decreasing in E, so there might be multiple solutions to equation (4.-23) and thus multiple equilibria in the game. Figure 4.5-b shows an example for $\rho = 3$ in which the (exogenous) relation between fundamentals and the exchange rate

---

[23]Parameters used for the graphs at 4.5-a and 4.5-b: $w_D = 1$, $w_P = 1$, $e_0 = 1$, $r = 0.03$ and $\alpha = 0$.

crosses the function $\theta^*(E)$ more than once. In this case, we would require additional restrictions —

e.g., upper bounds on the slope of $e_1(.)$ — to restore uniqueness.

The Constant Relative Risk Aversion Assumption

We assumed throughout that agents had constant relative risk aversion. This allowed simple solutions. Suppose instead that the agent has utility function $U\left(\cdot\right)$ over his vNM index. We report how some of our results would vary with more general utility functions. Clearly, we would have

$$y^*\left(\pi\right) = \underset{y \in \left[\underline{\theta}, \overline{\theta}\right]}{\arg\max} \left[\pi U\left(\widetilde{w}\left(y, e_0\right)\right) + \left(1 - \pi\right) U\left(\left(\frac{E}{e_0}\right)^{1-\alpha} \widetilde{w}\left(y, E\right)\right)\right]. \qquad (4.\text{-}24)$$

and

$$\theta^* = \int\limits_{\pi=0}^{1} y^*\left(\pi\right) d\pi.$$

Hyperbolic Absolute Risk Aversion Consider the three parameter family of "hyperbolic absolute risk aversion" utility functions, with

$$U\left(w\right) = \zeta \left(\eta + \frac{w}{\rho}\right)^{1-\rho}.$$

The special case where $\eta = 0$ corresponds to constant relative risk aversion coefficient ρ and the special case with $\eta > 0$ and $\rho \rightarrow \infty$ corresponds to constant absolute risk aversion coefficient $\frac{1}{\eta}$.[24] In this case, the Inada conditions no longer hold (for $\eta \neq 0$) so the comparison between the incomplete information game and the complete information game is harder to interpret. However, limiting optimal demands are

$$y^*\left(1\right) = -\frac{w_D + \frac{w_P}{E} + \eta\rho}{1 - \frac{e_0}{E}\left(1 + r\right)} \text{ and } y^*\left(0\right) = \frac{w_D + \frac{w_P}{e_0} + \eta\rho}{r}$$

The first order conditions imply the same formula for

$$\widehat{y}\left(\pi\right) = \frac{y^*\left(\pi\right) - y^*\left(1\right)}{y^*\left(0\right) - y^*\left(1\right)} = \frac{1}{1 + \left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}}.$$

Thus we have a general formula for the hyperbolic absolute risk aversion case:

$$\theta^* = \left(1 - \widehat{\theta}\right)\left(-\frac{w_D + \frac{w_P}{E} + \eta\rho}{1 - \frac{e_0}{E}\left(1 + r\right)}\right) + \widehat{\theta}\left(\frac{w_D + \frac{w_P}{e_0} + \eta\rho}{r}\right)$$

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[24]For derivations of these results and more on this class of utility functions, see Gollier (2001); these utility functions are defined only when $w > -\eta\rho$; concavity of U requires that we have $\frac{\zeta(1-\rho)}{\rho} > 0$.

$$\text{where } \widehat{\theta} = \int_{\pi=0}^{1} \frac{1}{1 + \left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}} \left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}} d\pi.$$

In the special case where $w_P = 0$, this simplifies to

$$\begin{aligned}
\theta^* &= (w_D + \eta\rho) \left(\left(1 - \widehat{\theta}\right) \left(-\frac{1}{1 - \frac{e_0}{E}(1+r)}\right) + \widehat{\theta}\left(\frac{1}{r}\right) \right) \\
&= \frac{w_D + \eta\rho}{r + 1 - \frac{e_0}{E}(1+r)} \left(\left(1 - \widehat{\theta}\right) \left(-\frac{1}{1-t}\right) + \widehat{\theta}\left(\frac{1}{t}\right) \right)
\end{aligned}$$

If $\alpha = 1$, one can show that as $\rho \to \infty$,

$$\begin{aligned}
\rho\left(\widehat{\theta} - t\right) &= \rho \left(\int_{\pi=0}^{1} \frac{1}{1 + \left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}} \left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}} d\pi - t \right) \\
&\to -t(1-t)\ln\frac{t}{1-t}.
\end{aligned}$$

Thus as $\rho \to \infty$,

$$\begin{aligned}
\theta^* &\to \frac{w_D + \eta\rho}{r + 1 - \frac{e_0}{E}(1+r)} \left(\begin{array}{c} \left(1 - t + \frac{t(1-t)}{\rho}\ln\frac{t}{1-t}\right)\left(-\frac{1}{1-t}\right) \\ + \left(t - \frac{t(1-t)}{\rho}\ln\frac{t}{1-t}\right)\left(\frac{1}{t}\right) \end{array} \right) \\
&= \frac{w_D + \eta\rho}{\left(r + 1 - \frac{e_0}{E}(1+r)\right)\rho} \ln\frac{t}{1-t} \left(t(1-t)\left(-\frac{1}{1-t}\right) - t(1-t)\left(\frac{1}{t}\right) \right) \\
&= -\frac{\eta}{r + 1 - \frac{e_0}{E}(1+r)} \ln\frac{t}{1-t}.
\end{aligned}$$

Thus under the one way bet assumption with constant absolute risk aversion case, we have that θ^* is increasing in the coefficient of constant absolute risk aversion, going from $-\infty$ as the absolute risk aversion coefficient $\left(\frac{1}{\eta}\right)$ tends to 0 to 0 as the coefficient goes to ∞.

The $t = \frac{1}{2}$ case We mention one case where we can solve for a general utility function $U(\cdot)$ that provides some useful intuition for our earlier results. If $\alpha = 1$, $w_P = 0$ and $t = \frac{1}{2}$ (i.e., $r = 1 - \frac{e_0}{E}(1+r)$), then, by symmetry, $y^*(\pi) = -y^*(1 - \pi)$. This implies that $\widehat{\theta} = \frac{1}{2}$ and $\theta^* = 0$.

Decreasing Absolute Risk Aversion Most of arguments do not generalize to an arbitrary decreasing absolute risk aversion utility function. However, when we performed comparative statics of wealth in the incomplete markets case earlier, we were only using the DARA property of our CRRA utility function (as in Goldstein and Pauzner (2001)).

The Infinitessimal Agent Assumption

We assumed a continuum of agents. With a finite set of agents, each agent would anticipate his own impact on whether a crisis would occur. The direction of this effect would vary across the scenarios we considered. If agents were risk neutral, $\alpha = 1$ and $w_P = 0$, then each agent has a private interest in having a successful attack. So attacks would be more likely with large traders (this is the case analyzed by Corsetti, Dasgupta, Morris and Shin (2003)). But if $w_P > 0$ (e.g., the agent has foreign direct investment in the country) then an agent has a lot to lose from devaluation. In a continuum model, this does not influence his best response. But with large traders, this would make attacks less likely.

Chapters 2, 3 and 4 have focused on currency crises. The last essay of this dissertation, presented next, focuses on financial crises and, more specifically, on liquidity runs. The analytical framework shown at chapter 5 brings insights on how an international institution providing liquidity can help stabilize financial markets via coordination of agents' expectations, and how it influences the incentives faced by policy makers to undertake efficiency-enhancing reform.

Chapter 5

IMF, catalytic finance and moral hazard

This essay is co-authored with Giancarlo Corsetti and Nouriel Roubini

5.1 Introduction

In the last decade many emerging market economies have experienced currency, debt, financial and banking crises: Mexico, Thailand, Indonesia, Korea, Russia, Brazil, Ecuador, Turkey, Argentina and Uruguay, to name the main ones. At different times, each of these countries faced massive reversal of capital flows, and experienced a large drop in asset prices and economic activity. Even if current account deficits were sharply reduced via domestic policy adjustment and painful economic contraction, external financing gaps remained large because of strong capital outflows and the unwillingness of investors to rollover short-term claims on the country (including its government, its banks and its residents).

Crisis resolution has thus involved, in addition to domestic policy adjustment, some combination of official financing (or 'bailouts') by International Financial Institutions and other official creditors, and private financing in the form of 'bailins' of private investors (the latter is also referred to as private sector involvement (or PSI) in crisis resolution).[1] Indeed, the issue of 'bailouts' versus

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[1] Bail-ins can take various forms in a spectrum going from very coercive to very soft forms of PSI: at one extreme are

'bailins' – or private sector involvement in crisis resolution – is the most controversial question in the debate on the reform of the international financial architecture.

In this debate, an important view holds that international currency and financial crises are primarily driven by liquidity runs and panics, and could therefore be avoided via the provision of sufficient international liquidity to countries threatened by a crisis. According to this view, the global financial architecture should be reformed by creating an international lender of last resort. Not only would such an institution increase efficiency ex-post by eliminating liquidation costs and default in the event of a run: by severing the link between illiquidity and insolvency, it would also prevent crises from occurring in the first place (see Sachs (1995) and Fischer (2001)). The opposing view doubts that international illiquidity is the main factor driving crises. When crises can also be attributed to fundamental shocks and policy mismanagement, liquidity support may turn into a subsidy to insolvent countries, thus generating debtor and creditors moral hazard (see the Meltzer Commission Report (2001)). Accordingly, IMF interventions should be limited in frequency and size so as to reduce moral hazard distortions, even if limited support would not prevent liquidity runs.

The official IMF/G7 position is somewhere between the two extreme views described above: provided a crisis is closer to illiquidity than to insolvency, a *partial* bailout granted conditional on policy adjustment by the debtor country can galvanize investors' confidence and therefore stop destructive runs — i.e., can have a "catalytic effect".[2] If the "catalytic" approach is successful, official resources do not need to be unlimited (i.e., so large as to fill in any potential financing gap), since some official liquidity provision and policy adjustment will convince private investors to rollover their positions (rather than run) while restoring market access by the debtor country.[3] But

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defaults on external (and domestic) claims (as in the case of Ecuador, Argentina, Russia); somewhere between extremes are debt suspension and standstills, semi-coercive bonded debt exchange offers, semi-coercive rollover agreements (as in the case of Ukraine, Pakistan, Uruguay Korea, Indonesia, Thailand); on the softer corner of the PSI spectrum are semi-voluntary rollover agreements and other mild forms of PSI (Brazil in 1999, Turkey in 2001) or outright bailouts with little PSI (Mexico in 1995, Turkey in 2002-2003).

[2]Liquidity support is effective both directly and indirectly. *Directly*, it reduces liquidation costs against speculative withdrawal of credit as liquidity provision reduces the amount of illiquid investments that need to be liquidated. *Indirectly*, it reduces the number of speculators willing to attack a country for each realization of the economic fundamentals. In other words, the presence of the IMF means that, over some crucial range of fundamentals, private investors are more likely to rollover their positions — this is the essence of 'catalytic finance.'

[3]See Cottarelli and Giannini (2002) and Mody and Saravia (2003) for an analysis of the IMF's catalytic approach and an assessment of its success.

can partial "catalytic" bailouts be ever successful or, as argued by many, only full bailouts or full bailins can be effective in preventing destructive runs?[4]

This essay contributes to the current debate on these issues by providing a theoretical model of financial crises and the main policy trade-offs in the design of liquidity provision by an international financial institution. In our model, a crisis can be generated both by fundamental shocks and by self-fulfilling panics, whereas liquidity provision affects the optimal behavior of the government in the debtor country (possibly generating moral hazard distortions). Our study draws on the theoretical model by Corsetti, Dasgupta, Morris and Shin (2002) and the policy analysis by Corsetti, Pesenti and Roubini (2002), on the role of large speculative traders in a currency crisis. Consistent with these contributions, we model the official creditor (the IMF or ILOLR) as a *large player* in the world economy, with a well-defined objective function and financial resources. In our model, the strategies of the official creditor, international speculators and domestic governments are all endogenously determined in equilibrium.

There are two major areas in which our model contributes to the debate on the reform of the international financial architecture: the effectiveness of catalytic finance and the trade-off between liquidity support and moral hazard distortions. As regards the *first area*, our analysis lends support to the hypothesis that catalytic liquidity provision by an official institution can work to prevent a destructive run — although in our model the success of partial bailouts is realistically limited to cases in which macroeconomic fundamentals are not too weak. In reality, the IMF does not have infinite resources and cannot close by itself the possibly very large external financing gaps generated by speculative runs, i.e., the IMF cannot rule out debt defaults due to illiquidity runs. According to our results, however, even when relatively small, contingent liquidity support lowers the likelihood

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[4]On these controversial questions, there is a wide range of opinions, but little analytical and formal work. The G7 doctrine and framework for PSI policy has evolved over time as a reform of the international financial architecture was started after the Asian and global crisis of 1997-98. The current tentative but fragile G7/IMF consensus approach can be summarized as follows. Based on a case-by-case discretional assessment of each crisis, the IMF should finance troubled countries with large packages when the crisis is closer to illiquidity and country policy adjustment can ensure solvency. The IMF should limit its financial support, and proceed with debt restructuring/reduction when a country is close to insolvency and unable to adopt adjustment policies. A combination of limited official financing, bailins (such as debt reprofiling, restretching or restructuring) and policy adjustment is appropriate in cases between the two extremes — whenever problems are more severe than illiquidity but not as severe as in insolvency. Clearly, the crucial issue is to determine when large catalytic finance is appropriate.

of a crisis by enlarging the range of economic fundamentals at which international investors find it optimal to rollover their credit to the country. This 'catalytic effect' is stronger, the larger the size of IMF funds, and the more accurate the IMF information. But our results also make clear that catalytic finance cannot and will not be effective when the fundamental turns out to be very weak ex post: as more and more agents receive bad signals about the state of the economy, massive withdrawals will cause a crisis regardless of whether the IMF intervenes.

Our result runs counter to the hypothesis, first suggested by Krugman and King and then formalized by Zettelmeyer (1999) Jeanne and Wyplosz (2000), that IMF bailouts can be effective only when there are enough resources to fill financing gaps of any possible size. These authors base their view on models with multiple equilibria, in which partial bailouts cannot rule out the possibility of self-fulfilling runs, i.e., partial IMF interventions are not an effective coordination mechanism of private investors. In such a framework, liquidity support is effective only insofar as it is large enough to prevent a run and eliminate all liquidation costs in the presence of a run.[5]

As regards the *second area*, contrary to the widespread view linking provision of liquidity to moral hazard distortions, we show that under some circumstances liquidity assistance can actually make a government willing to implement efficiency-enhancing but costly reforms. More specifically, the conventional view on debtor moral hazard is that, by insulating the macroeconomic outcome from ruinous speculative runs, liquidity assistance gives the government an incentive to avoid the costs of implementing good policies. But this is not the only possible effect of an ILOLR. It is also plausible that some governments may be discouraged from implementing good but costly policies *because* speculative runs jeopardize the chances of their success. In this case, liquidity support that reduces liquidation costs in the event of a run can actually motivate the government to implement socially desirable policies. Our model show that liquidity support can have either effect depending on circumstances — very large bailouts induce moral hazard distortions, but moderately large support

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[5]Models drawing on the traditional bank run literature prescribe that the IMF should have very deep pockets. Usually, in the analysis underlying such a view, the cost of a crisis is independent of the size of the financial gap, i.e. the difference between short term obligations and the liquid financial resources available to the country. In other words, by falling either one cent or one billion dollars short of obligations, the country pays the same large cost. More general and realistic models would allow for partial liquidation of long term investment (selling one bit of it may provide the required resources without incurring a macroeconomic crisis). See appendix A of this essay for further discussion.

can be a complementary to good policy behavior.

Building on the main insights from the literature on global games,[6] this essay contributes to our understanding of how and why catalytic finance can work. Our analysis is related to a vast and fast-growing literature on the merits of bailouts vs. bailins as a crisis resolution strategy and the arguments in favor of an ILOLR. We contribute to this literature in a number of dimensions.

First, we model the role of official financial institutions as large players whose behavior is endogenously derived in equilibrium. Relative to global games and the literature on the ILOLR building on them (see Morris and Shin (2002) but also the closed-economy model by Goldstein and Pauzner (2002) and Rochet and Vives (2002)) much of our new analytical insight stems exactly from this feature of our model. In specifying the preferences of its shareholders or principals, we model a 'conservative' IMF, in the sense that it seeks to lend to illiquid countries, but not to insolvent countries. Consistently, in our equilibrium the IMF is more likely to provide liquidity support when the crisis is caused by a liquidity run, as opposed to crises that are closer to the case of insolvency.

Second, in our framework domestic expected GNP is a natural measure for national welfare — which may differ from the objective function of the domestic government because of the (political) costs of implementing reforms and adjustment policies. We can therefore analyze the impact on the welfare of domestic citizens and the government of alternative intervention strategies by the IMF.

Third, we develop a model where a crisis may be anywhere in the spectrum going from pure illiquidity to insolvency. Most studies of ILOLR builds on Diamond and Dybvig (1983) — D&D henceforth — interpreting crises as a switch across instantaneous (rational-expectations) equilibria, but ignoring or downplaying macroeconomic shocks or any other risk of fundamental insolvency. Relative to this literature, we present a more realistic specification of an open economy where fundamentals, in additions to speculation, can cause debt crises.

Fourth, in our global games model the probability of a crisis and coordination among agents are endogenous, and the equilibrium is unique. We can therefore study the equilibrium implications

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[6]Specifically, our framework draws on the literature on global games as developed by Carlsson and van Damme (1993) and Morris and Shin (2003). As is well known, in global games the state of the economy and speculative activity is not common knowledge among agents. With asymmetric information, there will be some heterogeneity in speculative positions even if everybody follows the same optimal strategy in equilibrium. Moreover, the precision of information need not be the same across individuals. Arguably, global games provide a particularly attractive framework to analyze the coordination problem in financial markets.

of varying the size of IMF's support, the precision of its information and other parameters of the model without relying on arbitrary assumptions on the likelihood of a speculative attack. This is in sharp contrast with multiple equilibrium models.

As already mentioned, the conventional wisdom is that official finance exacerbates the moral hazard problem: the novel result from our comparative static analysis is that, in some circumstances, the existence of official liquidity assistance can give a debtor country the right incentive to implement policy adjustment. In addition, the framework of global games allows us to assess the role of IMF information precision in strengthening the IMF's influence on private investors' strategies and government behavior. In general, a better-informed IMF reduces the aggressiveness of private speculators, and therefore lowers the likelihood of a crisis.

While the model of catalytic finance shapes the traditional, official view of liquidity provision by the IMF, until very recently there was no theoretical analysis of it. This essay and Morris and Shin (2002) are, to our knowledge, the first contributions to fill such gap in the literature (see also the closed-economy model by Rochet and Vives (2002)).

Some recent literature has contributed to our understanding of the policy trade-offs between liquidity and moral hazard. Haldane *et al* (2002) present a model that allows for fundamentals-driven runs, and assess the arguments in favor debt standstills, relative to official finance, as crisis resolution mechanisms. These authors discuss the implications of moral hazard but do not develop a model of the tradeoff between these objectives and the optimal intervention policy. Gale and Vives (2002) study the role of dollarization in overcoming moral hazard distortions deriving from domestic (but not international) bailout mechanisms (such as central bank injection of liquidity in a banking system subject to a run). Allen and Gale (2000a) introduce moral hazard distortions in a model of fundamental bank runs, but do not consider analytically the role of an international lender of last resort. Rochet and Vives (2002) study domestic lending of last resort as a solution to bank runs in a global game model.[7] They find that liquidity and solvency regulation can solve the creditor coordination problem that leads to runs but that their cost is too high in terms of foregone returns. Thus, emergency liquidity support is optimal in addition to such regulations. However, they do not model the lender of last resort as a player — as it is done here. Therefore, they do not analyze the tradeoff between bailouts and moral hazard, and the role of a large official creditor (IMF) that is

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[7]See also Goldstein and Pauzner (2002).

central to our study. The closest analysis to our model is that of Morris and Shin (2002) who, in a parallel contribution, also consider formally catalytic IMF finance — although they do not model the IMF as a 'large player'.[8]

The core policy tradeoff in creating an ILOLR is between liquidity provision and moral hazard distortions. For instance, in analyzing the IMF role in the new international financial architecture, many authors have stressed the need to complement provision of liquidity with punishment mechanisms that reduce the incentive to default strategically (as in Dooley and Verma (2000)) or tune down policy efforts to ensure solvency (as in Kumar, Masson and Miller (2000)). Specifically, Dooley and Verma (2000) strongly argued against reducing the costs of renegotiating debt. To the extent that this leads an opportunistic sovereign to use debt suspensions/standstill/defaults too often, the flow of capital to emerging markets would drop to socially inefficient levels in equilibrium (see also Gai, Hayes and Shin (2001) for a refinement of the argument).

In a closed economy setup, moral hazard distortions can be reduced via incentive-compatible deposit insurance, capital adequacy regulation and overall supervision and regulation of the banking sector. In case financial distress occurs, the central bank and/or the authorities in charge of regulation and supervision have the power to seize the banks, change their management, restructure and merge them with other banks or even liquidate them. But in an international context, there is no international authority with comparable powers: debtors with sovereign immunity cannot be seized, merged or closed down. Moral hazard distortions deriving from the existence of an ILOLR are thus potentially exacerbated.[9]

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[8]Several recent studies (Cottarelli and Giannini (2001), Mody and Saravia (2003) and Roubini and Setser (2003)) have provided useful empirical evidence on catalytic finance. According to these studies, and consistent with the main results of our analysis, catalytic finance is more likely to be sucessful when fundamentals are not bad and/or the amount of policy adjustment required to achieve debt sustainability is feasible and credible.

[9]A number of recent contributions have discussed several dimensions of this issue. Kumar, Masson and Miller (2000) focus on the maturity structure of external debt: short-term debt imposes discipline on the debtor, but only at the cost of raising the probability of self-fulfilling runs. In this model, moral hazard distortions imply that full insurance by international institutions is never optimal. Rather, liquidity lending should be made conditional on policy/effort changes that can be effectively monitored by the IMF.

The accuracy of the IMF information is an important issue in assessing whether liquidity provision is appropriate or not and whether it exacerbates moral hazard distortions. Gai, Hayes and Shin (2001) model the IMF role in reducing the costs of disorderly adjustment following debt servicing difficulties in the presence of debtor moral hazard. International liquidity support is more likely to be beneficial if the IMF can make an accurate assessment of the

5.2   The model

Consider a small open economy with a three-period horizon — periods are denoted 0, 1 (or interim) and 2. The economy is populated by a continuum of agents of mass 1, each endowed with E units of resources. These agents can borrow up to D from a continuum of international fund managers also of mass 1, willing to lend to the country only short term. Moreover, there exists one international financial institution, the International Monetary Fund (IMF), which may provide the country with international liquidity up to L. For simplicity, all international lending and borrowing by domestic agents takes place at the same international interest rate r^*, which is normalized to zero.

Domestic agents invest in domestic projects which yield a stochastic rate of return equal to R in period 2, or to $R/(1+\kappa)$ if projects are discontinued and liquidated early in the interim period. The expected return from these projects in period 2 is well above the international interest rate, i.e., $E_0 R > 1 + r^* = 1$. Yet, investment is illiquid, in the sense that projects can be discontinued in period 1 at the cost $\kappa > 0$ per unit of investment.[10]

The sequence of decisions can be summarized as follows. *In period 0*, agents in the economy invest their own endowment and the borrowed resources $E + D$ in the domestic risky technology I and in an international liquid asset M.

country's policy efforts. If the IMF is not very effective in distinguishing between a crisis due to bad luck and a crisis due to opportunistic default, lowering the ex-post cost of a crisis will raise the incentive to default strategically. In equilibrium a larger role for the IMF can reduce social welfare because it leads to a sizeable drop in the amount of international lending. As argued by Ghosal and Miller (2002), if official creditors cannot detect the nature of the crisis (insolvency versus opportunistic/moral hazard-driven default), there is a case for "constructive ambiguity", i.e. there should be some uncertainty about the provision of official liquidity support as a way to reduce moral hazard distortions. In our model, we formally analyze the role of the relative informational advantage of an official creditor relative to the private ones.

[10]While our model analyzes speculative portfolio positions given prices, a more general model should also derive risk premia in equilibrium. The well know difficulty in this step is that market prices reveal information, and therefore reduce the importance of agents' private signal. Interestingly, however, the empirical evidence on the IMF catalytic effect on asset prices is consistent with many of our results. For instance, Mody and Saravia (2003) find the IMF programs improve market access and the frequency of bond issuance, and lower spreads. IMF programs seem to contain the negative effect on spread of high export variability. Finally, the IMF influence is larger for intermediate level of the fundamentals, while the catalytic role of the IMF increases with the size of its programs. The link between these empirical findings and our theoretical conclusions are apparent (see Mody and Saravia (2003) for a review of the empirical literature).

In the interim period, fund managers decide whether to rollover their loans or withdraw. Denoting with x the fraction of managers who decide to withdraw, xD measures the short-term liquidity need of the country. To meet short-term obligations, domestic agents can use their stock of liquid resources, and can liquidate some fraction z of the long term investment I, getting $zRI/(1+\kappa)$. In addition, if the IMF decides to intervenes, the country can obtain funds up to L.

Let Λ denote total international liquidity available to the country, including both the predetermined component M and the contingent component L. Clearly, the country will incur some liquidation costs when $xD > \Lambda$ (i.e., z will be such that $xD - \Lambda = zRI/(1+\kappa)$); it will default when $xD > \Lambda + RI/(1+\kappa)$ (i.e., domestic agents will not be able to meet their short-term obligations despite complete liquidation of long-term investment). When the country defaults, we assume that all lenders will be paid pro-rata, up to exhausting the resources available to the country.

In the last period, the country total resources consist of $R(1-z)I$ (corresponding to GDP), plus any money left over from the previous period, i.e., $\max\{\Lambda - xD, 0\}$. Its liabilities consists of private debt $(1-x)D$ plus any outstanding IMF loan L. As for the case of default in the interim period, we assume that lenders are treated symmetrically and paid pro-rata also when the country defaults in period 2 (the case in which IMF loans have priority over private loans is discussed in one of the extensions of the model presented in Section 7).

Note that the difference (if any) between total resources and debt obligations is the country GNP, available to domestic consumption:

$$Y = \max\left\{R(1-z)I + (\Lambda - xD)_+ - (1-x)D - L_+, 0\right\} \qquad (5.1)$$
$$= \max\left\{RI\left[1 - \frac{z\kappa}{1+\kappa}\right] + M - D, 0\right\}$$

whereas we make use of the notation convention $(\Lambda - xD)_+ = \max\{\Lambda - xD, 0\}$.[11] Note that GNP and domestic consumption are zero in the event of default. In what follows, we take GNP as a

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[11]To derive the expression in the second line (5.1) note that the term $(\Lambda - xD)_+$ is zero when liquidation costs are positive. Adding and subtracting xD we get

$$Y = R(1-z) + xD - D - L_+$$

whereas accounting for liquidation costs we have

$$xD - D - L_+ = M - D + \frac{RI\kappa}{1+\kappa}.$$

measure of national welfare.

5.2.1 Payoffs and information

In this subsection we describe the payoffs and information set of fund managers and the IMF. The objective function of the government will be introduced later on, in the subsection on moral-hazard distortions.

As in Rochet and Vives (2002), fund managers face a structure of payoffs that depend on taking the "right decision". When the country does not default, rolling over loans in period 1 is the right thing to do, and yields a benefit that is higher than withdrawing — the difference in utility between rolling over debt and withdrawing is equal to a positive constant b. When the country defaults, managers who do not withdraw in the interim period make a mistake and therefore pay a cost. The difference in utility between rolling over loans and withdrawing is negative, and set equal to $-c$.

Different from previous literature, we model an international institution providing liquidity, the International Monetary Fund (IMF) as an additional player that is large in the world economy. In specifying the IMF objective function, we want to capture the idea that the IMF is concerned with the inefficiency costs associated with early liquidation, but cannot provide subsidized loans or grants to a country with bad fundamentals. The payoff of the managing board of the IMF is isomorphic to that of private fund managers: if the country ends up not defaulting, lending L is the right thing to do. By providing liquidity rather than denying it, the IMF gets a benefit B. If the country defaults, instead, the IMF loses money when lending. Relative to doing nothing, the benefit from providing liquidity is negative and equal to $-C$.

Note that, in the above specification of payoffs, the utility for funds' managers and the IMF is independent of the extent of default. Our analysis thus abstracts from distributional issues between the country and the creditors, as well as between private creditors and the IMF, that arise in debt crises.

As regards the stochastic process driving the fundamental, we assume that the rate of return R is distributed normally with mean R_j and variance $1/\rho$. The mean R_j — with $j = A, N$ — depends on the "effort" of the government, as analyzed later on in the essay. In period 0, the distribution of R (but the value of its mean) is common knowledge in the economy; R is realized in the interim period.

In the interim period, international fund managers do not know the true R but each of them receives a private signal $\widetilde{s}_i$ such that

$$\widetilde{s}_i = R + \varepsilon_i \tag{5.0}$$

whereas individual noise is normally distributed with precision α and its cumulative distribution function is denoted by $G(.)$.

By the same token, the management of the IMF also ignore the true R, but receive a signal $\widetilde{S}$ such that

$$\widetilde{S} = R + \eta \tag{5.0}$$

where η is also normally distributed, with precision β and its cumulative distribution function is denoted by $H(.)$.

Now, note that the posteriors of both funds managers and the IMF will depend on public information (the prior distribution of R), private signals *and* on probability assigned to the event 'government took action A' (call it p_A). The posterior s for a fund manager that gets signal $\widetilde{s}_i$ is equal to:

$$s = p_A \left(\frac{R_A \rho + \widetilde{s}_i.\alpha}{\rho + \alpha} \right) + (1 - p_A) \left(\frac{R_N \rho + \widetilde{s}_i \alpha}{\rho + \alpha} \right) \tag{5.0}$$

Analogously, the posterior of the IMF is:

$$S = p_A \left(\frac{R_A \rho + \widetilde{S}.\beta}{\rho + \beta} \right) + (1 - p_A) \left(\frac{R_N \rho + \widetilde{S} \alpha}{\rho + \beta} \right) \tag{5.0}$$

The interaction between private and public signals in coordination games is the focus of recent literature including Hellwig (2002) and Morris and Shin (2002b). Encompassing the main results of these essays in the context of our model would complicate the analysis considerably, without necessarily adding essential insights. To keep our work focused, we abstract from the above issue altogether. In Sections 4 and 5 below, we will proceed as in Corsetti, Dasgupta, Morris and Shin (2003) by assuming a very uninformative public signal ($\rho \to 0$). In section 6, instead, we will focus on government behavior, affecting the mean of the distribution of R. Hence, we will set ρ equal to a finite value, and consider the limiting case in which private information is arbitrarily precise, although precision is not necessarily identical for funds' managers and the IMF. In either cases — $\rho \to 0$ (for α and β finite) or α, $\beta \to \infty$ (for ρ finite) —,

$$\lim_{\substack{\rho \\ \alpha \to 0}} s_i = \widetilde{s}_i \tag{5.0}$$

$$\lim_{\frac{\varrho}{\beta} \to \infty} S = \widetilde{S} \tag{5.0}$$

so that we can disregard public information in building our equilibrium.[12]

5.2.2 Solvency and liquidity

To illustrate the logic of the model, suppose that no early withdrawal of funds could ever occur (debt is effectively long-term), so that $x = 0$. In this case, the country is solvent if the cash flow from investment is at least equal to its net debt[13]

$$RI \geq D - M. \tag{5.0}$$

Thus, the minimum rate of return at which the country is solvent conditional on no liquidity drain in the interim period (the break-even rate) is

$$R_s = \frac{D - M}{I}. \tag{5.0}$$

In the presence of liquidity runs, a return on investment as high as R_s may no longer be sufficient for the country to avoid default. Specifically, if the IMF has not lent to the country in the interim period, the country will be solvent in period 2 if and only if:

$$R(1 - z)I = RI - (1 + \kappa)\,[xD - M]_+ \geq (1 - x)\,D. \tag{5.0}$$

Denoting by $\bar{R}$ the minimum rate of return at which the country is solvent conditional on no IMF intervention, we can write:

$$\bar{R} = R_s + \kappa \frac{[xD - M]_+}{I} \geq R_s. \tag{5.0}$$

With early liquidation of investment (i.e., when $xD - M > 0$), the break-even rate must increase above R_s, as the failure of international investors to roll over their debt results in wasteful liquidation costs and hence ex-post efficiency losses.

Conversely, if the IMF intervenes in the first period, ex-post efficiency losses will be contained, and the solvency threshold for the rate of return conditional on a given x will be lower. Namely,

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[12]See Hellwig (2002) (Theorem 1) among others.

[13]Note that the following is true whether or not the IMF lends to the country — if it does so, the country will increase its gross stock of international safe assets in period 1, and use the additional reserves to pay back the IMF in period 2.

the country will be solvent if

$$R(1 - z)I = RI - (1 + \kappa)[xD - M - L]_+ \geq (1 - x)D + L. \tag{5.0}$$

Denoting by $\bar{R}_L$ the relevant threshold for default, we have

$$\bar{R}_L = R_s + \kappa \frac{[xD - M - L]_+}{I} \geq R_s. \tag{5.0}$$

IMF interventions increase the country GNP to the extent that they reduce early liquidation. It is worth noting that there are two ways in which the IMF can reduce early liquidation: *directly*, as the IMF provides liquidity against fund withdrawals, and *indirectly*, as the presence of the IMF may reduce the fund managers' willingness to withdraw for any given fundamental (lowering x for any given realization of R). This latter effect is at the heart of our analysis in the following sections.[14]

5.3 Speculative runs and liquidity provision in equilibrium

We now turn to the characterization of the equilibrium in our three-period economy for given government policies (i.e., for a given distribution of the fundamental R). According to our specification, in the interim period the IMF and the fund managers take their decisions independently and simultaneously. In effect, we envision a world in which the contingent fund L initially committed by the IMF may not be available ex post, and this is understood by fund managers, who correctly compute the likelihood of IMF interventions. As mentioned above, the idea here is that the IMF will refuse to lend if, according to its information, there is no prospect to recover its loans L fully — so that contingent financial assistance would turn into a subsidy. A different timing of decisions — with the IMF moving prior to private lenders — is discussed in section 7.

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[14]Default in the interim period is also possible. For this to happen, it must be the case that the speculative attack in the interim period exceeds all liquidity resources plus the liquidation value of domestic investment

$$xD \geq \frac{RI}{1 + k} + M + L_+$$

The minimum rate of return at which early liquidation x leads to early default is

$$\begin{aligned} \bar{R}_{ED} &= (1 + \kappa)\left[\frac{[xD - M]_+}{I} - \frac{L_+}{I}\right] \\ &= (1 + \kappa)R_s\left[\frac{[xD - M]_+}{D - M} - \frac{L_+}{D - M}\right] \end{aligned}$$

where ED stands for early (period 1) default.

At the heart of our model lies the coordination problem faced by fund managers in the interim period. Fund managers are uncertain about the information reaching all other managers and the IMF, and therefore face strategic uncertainty about their actions. But the expected payoff of each fund manager from rolling over a loan to the country depends positively on the fraction $(1 - x)$ of managers not withdrawing in the interim period, as well as on the IMF willingness to provide liquidity. The IMF expected payoff from providing liquidity, in turn, depends positively on the fraction of agents who roll over their debt. Clearly, the decision by the fund managers and the IMF are strategic complements.

As in Corsetti, Dasgupta, Morris and Shin [2003] — hereafter CDMS — in our model there is a unique equilibrium[15] in which agents employ trigger strategies: a fund manager will withdraw in period 1 if and only if her private signal on the rate of return of the risky investment is below some critical value $\tilde{s}^*$, identical for all managers. Analogously, the IMF will intervene in support of a country in distress if and only if its own private signal is above some critical value $\tilde{S}^*$. Using the argument in CDMS , it can be shown that a focus on trigger strategies is without loss of generality, as there is no other equilibrium in other strategies. The proof is omitted, since it can be derived from the appendix A of the CDMS paper.

The equilibrium is characterized by *four critical thresholds*. The first two thresholds are critical values for the fundamental R, below which the country always defaults — one conditional on no IMF intervention, $\bar{R}$, the other conditional on IMF intervention, $\bar{R}_L$. The other two are the thresholds $\tilde{s}^*$ and $\tilde{S}^*$ for the private signal reaching the funds managers and the IMF, discussed above. In this and the next section we will assume that the public signal is arbitrarily uninformative — i.e., $\rho \to 0$, so that posteriors will coincide with private signals (see (5.2.1) and (5.2.1)). We will therefore express signals and thresholds of individual managers and the IMF in terms of these agents' posterior, denoted without tilde (i.e., s_i, S, s^* and S^*).

Let us first derive the equations determining $\bar{R}$ and $\bar{R}_L$. If funds managers follow a trigger strategy with threshold s^*, the proportion of fund managers who receive a signal such that their

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[15]The equilibrium is familiar to readers of the global-game literature. It is a Bayes Nash equilibrium in which, conditional on a player signal, the action prescribed by this player's strategy maximize his conditional expected payoff when all other players follow their equilibrium strategy.

posterior is below s^* and hence withdraw in the first period crucially depends on the realization R:

$$x = prob\,(s_i \leq s^* \mid R) \equiv G(s^* - R).$$ (5.0)

Using our definition of the threshold for failure $\bar{R}$, if the IMF does not intervene, there will be a crisis for any R such that $R \leq \bar{R}$. Then, at $R = \bar{R}$ the mass of international managers that withdraw is just enough for causing the country to fail. This mass is $x = G(s^* - \bar{R})$. Using (5.2.2), the first equilibrium condition — defining $\bar{R}$ — is therefore:

$$\bar{R} = R_s \left[1 + \kappa \frac{[G(s^* - \bar{R}) \cdot D - M]}{D - M} \right].$$ (5.0)

If the IMF intervenes, there will be a crisis for any R such that $R \leq \bar{R}_L$. As above, at $\bar{R}_L$ the critical mass of speculator to cause debt liquidity-related problems is $x = G(s^* - \bar{R}_L)$. From (5.2.2), the threshold for failure conditional on IMF intervention $\bar{R}_L$ is:

$$\bar{R}_L = R_s \left[1 + \kappa \frac{[G(s^* - \bar{R}_L) \cdot D - M - L]}{D - M} \right].$$ (5.0)

This is the second equilibrium condition — defining $\bar{R}_L$. At the thresholds $\bar{R}$ and $\bar{R}_L$, xD must be greater than M conditional on no IMF intervention, and greater than $M + L$ otherwise. So, in equilibrium

$$\left[G(s^* - \bar{R}) \cdot D - M\right] > 0 \quad \text{and} \quad \left[G(s^* - \bar{R}_L) \cdot D - M - L\right] > 0.$$

Equations (5.3) and (5.3) imply $\bar{R}_L < \bar{R}$. [16]

We now turn to the equations determining the triggers s^* and S^*, starting from the latter. Upon receiving the signal $\widetilde{S}$, the IMF assigns probability $H(\bar{R}_L - S)$ to the failure of the country despite its intervention. The IMF expected payoff (denoted $\mathcal{W}_{IMF}$) is therefore

$$\mathcal{W}_{IMF} = B \cdot \left(1 - H(\bar{R}_L - S)\right) - C \cdot H(\bar{R}_L - S)$$

---

[16]From (5.3) and (5.3) we have:

$$
\begin{aligned}
s^* &= \bar{R} + G^{-1} \left[\left(\frac{\bar{R}}{R_s} - 1\right) \frac{D - M}{\kappa D} + \frac{M}{D} \right] = \\
&= \bar{R}_L + G^{-1} \left[\left(\frac{\bar{R}_L}{R_s} - 1\right) \frac{D - M}{\kappa D} + \frac{M + L}{D} \right].
\end{aligned}
$$

Taking differences:

$$\bar{R} - \bar{R}_L = G^{-1} \left[\left(\frac{\bar{R}_L}{R_s} - 1\right) \frac{D - M}{\kappa D} + \frac{M + L}{D} \right] - G^{-1} \left[\left(\frac{\bar{R}}{R_s} - 1\right) \frac{D - M}{\kappa D} + \frac{M}{D} \right].$$

Suppose $\bar{R} \leq \bar{R}_L$. Then the LHS of the above equation would be less or equal to zero, while the RHS is positive. So it must be the case that $\bar{R} > \bar{R}_L$.

which is decreasing in S. The optimal strategy consists of lending to the country if and only if this expected payoff is non-negative, that is, if and only if $S \geq S^*$ where S^* is defined by

$$S^* = \bar{R}_L - H^{-1}\left(\frac{B}{B+C}\right).$$ (5.0)

The investor's problem is more complex, as discussed in CDMS. Whether or not the IMF intervenes, the country will default for $R < \bar{R}_L$. So, a fund manager receiving signal $\tilde{s}$ will assign probability $G(\bar{R}_L - s)$ to the event 'default regardless of the IMF's action'. However, for R comprised between $\bar{R}_L$ and $\bar{R}$, the country will default only if the IMF fails to intervene. So, the managers' expected payoff (denoted $\mathcal{W}_{FM}$) from rolling over their fund in period 1 includes a term accounting for the conditional probability that the IMF fails to provide liquidity to the country, $H(S^* - R)$:

$$\mathcal{W}_{FM} = b\left[1 - \left(G(\bar{R}_L - s) + \int_{\bar{R}_L}^{\bar{R}} g(R - s) \cdot H(S^* - R)dR\right)\right]$$ (5.1)
$$-c\left(G(\bar{R}_L - s) + \int_{\bar{R}_L}^{\bar{R}} g(R - s) \cdot H(S^* - R)dR\right)$$

where g is the probability density function. The optimal trigger s^* for funds' managers is implicitly defined by the zero-profit condition (in expected terms) below:

$$\frac{b}{b+c} = G(\bar{R}_L - s^*) + \int_{\bar{R}_L}^{\bar{R}} g(R - s^*) \cdot H(S^* - R)dR.$$ (5.0)

The appendix A2 shows that there is a unique value s^* that solves this equation.

The four equations (5.3), (5.3), (5.3) and (5.3) in four endogenous variables ($\bar{R}$, $\bar{R}_L$, S^* and s^*) completely characterize the equilibrium. Note that in our equilibrium the country will always default when the realization of the fundamentals is worse than $\bar{R}_L$, it will never default when R is above $\bar{R}$ (when $R \geq \bar{R}$, whether or not the IMF intervenes, the fundamentals are good enough for the country to withstand any speculative run in the interim period). But for R comprised between $\bar{R}$ and $\bar{R}_L$, default may or may not occur, depending on the IMF. Analytical solutions for the general case are not available, but after identifying the relevant questions we want to address, we can resort to numerical simulations and derive some analytical results.

Figure 5.1: Partition of Fundamental R

default	no default conditional on IMF intervention	no default
$\bar{R}_L$		$\bar{R}$
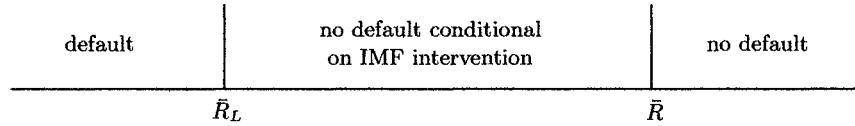
5.4 The effect of IMF lending on the likelihood and severity of debt crises

A distinctive feature of our global-game model is that crises have both a fundamental component and a speculative component. Not only must the rate of return be low enough for a speculative withdrawal to cause a solvency crises: withdrawals are more likely when the fundamentals are weak. The presence of an institutional lender of liquidity – even if with limited resources – affects the strategy of the fund managers. By changing the likelihood of speculative withdrawals, its presence can therefore influence the macroeconomic performance of the country.

In this section, we analyze the effects of IMF lending on the likelihood and severity of debt crises. More specifically, we can articulate our analysis addressing the following four questions:[17]

1. Does a larger availability of resources to the IMF increase the 'confidence' of the fund managers in the country — as captured by their willingness to roll over their loans for a relatively worse signal on the state of fundamentals?

2. To what extent does IMF lending affect the likelihood of a crisis?

3. Does the precision of the information of the IMF relative to the market matter? In other words, is the impact of IMF lending stronger as its information becomes more accurate?

4. To what extent IMF lending creates moral hazard, in the sense that because of liquidity support governments and/or corporations do not take (costly) steps to reduce vulnerability to crises?

We discuss the first three questions in this section. The last question on moral hazard — where

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[17]CDMS analyze questions related to the first three in our list in the context of a study focused on the role of large speculative players in currency crises.

our work yields the most novel result — will be analyzed in detail in the next section. Throughout our analysis, we will constrain L such that $L < D - M$. Obviously, when L becomes large enough to cover all possible withdrawals, liquidity is no more a concern — the break even rate is R_s.

5.4.1   Size of interventions

As regards question 1 and 2 above, we summarize our comparative static exercise by means of the following proposition:

Proposition 2 *All thresholds ($\bar{R}_L$, $\bar{R}$, s^* and S^*) are decreasing in L.*

Proof: see appendix.

To see how this proposition answers to question 1, note that if a larger L lower s^*, funds managers are now willing to rollover their loans for weaker private signals about fundamentals — hence they are less aggressive in their trading. A larger IMF raises the proportion of investors who are willing to roll over their debt at *any* level of the fundamental. Moreover, since the rate of return is normally distributed, if $\bar{R}$, $\bar{R}_L$ and S^* are all decreasing in L, the ex-ante probability of a crisis also falls with L. Then, the answer to question 2 is that bigger IMF interventions indeed lower the likelihood of a crisis. Observe that a lower S^* increases the probability of the IMF intervention for each level of the fundamentals. Expected GNP correspondingly increases. Figures 5.2 and 5.3 illustrate such effects.

Figure 5.2: A larger L makes investors less agressive
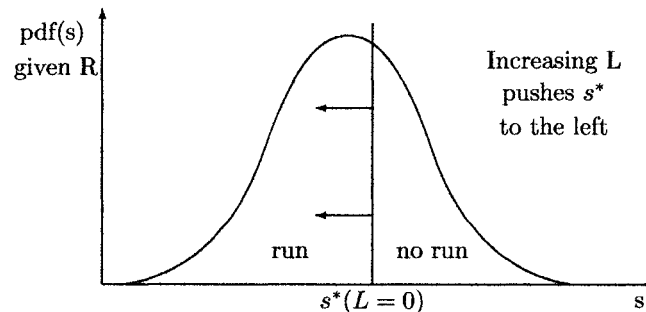
These results lend theoretical support to the notion that an international lender of last resort increases the country's expected GNP not only through the direct effects of liquidity provision

Figure 5.3: A larger L makes a crisis less likely
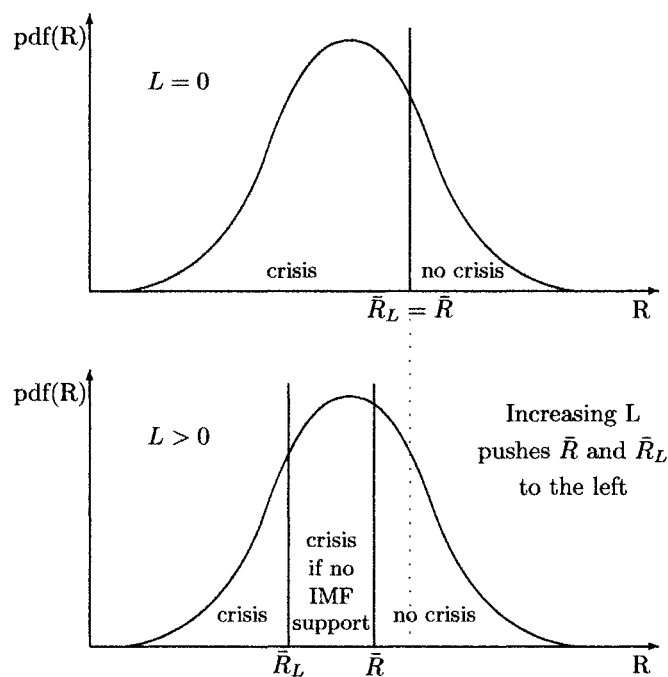
(interventions obviously reduce costly liquidation of existing capital). There is also an indirect effect on the coordination problem faced by fund managers: the possibility of interventions of size L lowers the threshold at which private managers refuse to roll over their debt, to an extent that increases with the size of contingent interventions. It follows that an international lender can avoid some early liquidation even if it does not act ex post.

To enhance the comparison between our analysis and the literature (especially contributions stressing multiple equilibria and self-fulfilling runs), it is useful to look at the equilibrium in our model when the precision of signals becomes arbitrarily large. When the errors ε_i go to zero, all private signals are arbitrarily close to the true fundamental R. Yet, signals are not common knowledge and agents still face strategic uncertainty about each other actions (i.e., they do not 'know' each other action in equilibrium). But with $\alpha, \beta \to \infty$, except in a measure-0 set in which the fundamental happens to be arbitrarily close to the threshold $\bar{R}_L$, either everybody withdraws early and the IMF does not intervene or nobody withdraws early. In this limiting case, there is no heterogeneity in managers' action, and there will be (almost) no provision of liquidity in equilibrium.

Thus, the prediction of our model is observationally equivalent to the model with common knowledge after Diamond and Dybvig [1983] — for a comparison, Appendix A.1 develops a D&D version of our model.

With $\alpha, \beta \to \infty$, *all* the benefit of a lender of last resort come through the coordination effect (as the IMF almost never intervenes saving liquidation costs). To coordinate markets, however, the IMF need not have 'deep pockets'. A marginal increase in the size of conditional interventions L lowers the threshold s^* chosen by all agents in equilibrium (at which x endogenously drops from 1 to 0).

5.4.2 The precision of IMF information

Above we have characterized the equilibrium when private signals become arbitrarily precise. Question 3 raises the interesting issue regarding the role, if any, of the *relative* precision of the information of the IMF. This is a central issue in the analysis of the influence of large players in currency crises by CDMS, as these players are usually believed to act on superior information. In our context, the main interest is in the equilibrium effect of improving the quality of IMF information.

What happens when the IMF private information becomes more accurate? The following proposition synthesize our result.

Proposition 3 *An increase in the IMF information precision decreases all thresholds.*

Proof: see appendix[18]

Ceteris paribus, a higher precision of information by the IMF increases the willingness by fund managers to roll-over their loans to the country, and reduces the probability of default. Intuitively, if the IMF has the ability to estimate the state of the country fundamentals arbitrarily well, funds' managers need not worry about idiosyncratic noise in the IMF intervention decisions. Provided that the IMF's objective function is common knowledge, private investors understand its strategy (lending to possibly illiquid but not to insolvent countries). At the margin, increasing the accuracy of IMF information makes them more willing to lend, because they will be confident that the IMF

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[18]CDMS show an analogous result for the limiting case when all players have arbitrarily accurate information. Our proposition generalize their result.

assessment of the fundamentals will not be far away from their own assessment — they can therefore expect the IMF to intervene when they believe that the state of the economy grant intervention.

5.4.3 A remark on portfolio managers' incentives

In our model, the strategies of all agents are endogenous in equilibrium. The parameters describing investors' payoffs affect more than the fund managers' own investment thresholds: because of their influence on the market coordination problem, these parameters also affect the equilibrium strategy by the IMF.

In our analysis, we focus on IMF's incentives and strategy. Yet our framework can also shed light on the general equilibrium effect of corporate governance and managers' behavior. By way of example, the next proposition establishes a link between the structure of incentives faced by the funds' managers, and the likelihood of IMF interventions. Recall that in our model b is the net gain in utility when a manager rolls over debt and the country is solvent, relative to withdrawing in the first period. The same net utility is negative and equal to $-c$ when the country defaults. Formally:

Proposition 4 *All thresholds ($\bar{R}_L$, $\bar{R}$, s^* and S^*) are decreasing in b and increasing in c.*

Proof: see appendix.

Intuitively, a weaker reward to successful long-term investment makes fund managers more wary about rolling over their credit. This in turn leads the IMF to be more cautious in providing liquidity. The likelihood of debt default correspondingly increases in equilibrium.

This proposition touches upon a topic extensively discussed by the literature, regarding the reason why rewards to long-term investment strategies may be perceived as weak by managers. For instance, funds' performance may be assessed against industry-wide benchmarks, so that individual managers may be reluctant to take positions at odds with those benchmarks, even if these position may have good risk-adjusted payoffs in the long term. Although our example is admittedly stylized, it shows the potential importance of general equilibrium analysis of these issues.

5.5   Liquidity and moral hazard

In the previous section we have shown that the ex-ante GNP of the country — our measure of national welfare — is increasing in the size of the IMF for any given distribution of the fundamental.

Yet, moral-hazard considerations may invalidate such conclusion, since liquidity assistance by the IMF could reduce the incentive for the government to implement costly policies that enhance the likelihood of good macroeconomic outcomes.

We now develop our framework and assume that the government can take a costly action improving the expected value of R without affecting the variance of the distribution. The government decides its level of effort in period 0, when international investors lend D to the country and the IMF states the size of its contingent intervention L. The action by the government is not observed at any point (and the IMF cannot make the provision of liquidity conditional on it). .

For simplicity, we will initially assume that the government can take one single action A (say, a policy reform and fiscal adjustment) that raises $E_0 R$ from R_N to R_A (let $\Delta R = R_A - R_N$). The welfare cost of undertaking action A is Ψ. This cost falls on the government only, and is motivated by exogenous considerations, say, electoral costs of reforms and fiscal adjustment. The government welfare function is

$$W = \mathcal{U} - \Psi = E_0 Y - \Psi \quad (5.0)$$

where $\mathcal{U}$ is the utility of the domestic representative agent. Note that W does not coincide with social welfare $\mathcal{U}$, that is measured by expected GNP only. At this end of this section, we will show that our main results carry over to a more general setup.

5.5.1 Liquidity provision and government behavior

It is convenient to focus our analysis on the limiting case when private signals become arbitrarily precise. An important reason is that, as the government affects the mean of the prior distribution, we need to relax the assumption of an uninformative public signal and conduct our analysis by setting a strictly positive ρ. With arbitrarily precise private information, we can do so without unnecessarily complicating the analysis. A second reason is that, as we have shown in the previous section, the case of arbitrarily precise private information brings the results of our model more closely into line with the predictions of models after Diamond-Dybvig, and therefore makes it easier to stress core differences between the two. Namely, with $\alpha \to \infty$, all agents will take the same action in equilibrium for almost all realizations of R (except when R happens to be arbitrarily close to $\bar{R}_L$), so that in equilibrium there will be no heterogeneity (but the equilibrium is unique) and no

partial liquidation (except in a measure-zero set). Thus, the utility of the government conditional on its action simplifies to:

$$\lim_{\alpha \to \infty} \mathcal{W}(A) = \int_{\bar{R}_L}^{\infty} [R \cdot I + M - D] \, f(R \mid R_A) \, dR - \Psi \qquad (5.1)$$

$$\lim_{\alpha \to \infty} \mathcal{W}(N) = \int_{\bar{R}_L}^{\infty} [R \cdot I + M - D] \, f(R \mid R_N) \, dR$$

Notably, the integrand in the above expressions does not depend on the liquidation cost κ — the set of realizations of R at which funds' withdrawals in period 1 lead to partial liquidation has measure zero. But the above expression is not independent of κ: in fact the lower extreme of integration (i.e., the threshold $\bar{R}_L$) crucially depends on this cost.

Taking the difference in government welfare with and without the costly action we obtain:

$$\lim_{\alpha \to \infty} \mathcal{W}(A) - \mathcal{W}(N) \equiv \Delta \mathcal{W} = I \cdot \Delta R \cdot \left(1 - F\left(\bar{R}_L \mid R_N\right)\right) \qquad (5.1)$$

$$+ \int_{\bar{R}_L}^{\bar{R}_L + \Delta R} [R \cdot I + M - D] \cdot f(R \mid R_A) \, dR - \Psi$$

In deciding whether to undertake the action A, the government compares the utility costs of a reform Ψ with the gains in expected GNP that come both in terms of higher average realization of R (first term on the RHS), and in terms of lower expected liquidation costs (second term on the RHS) because of the drop in the probability of a run on debt.

As the size of the IMF impacts the limits of integration, depending on parameter values there may be some critical L at which the government switches policy. The question is therefore how the net gain from the action A, $\Delta \mathcal{W}$, vary with the size of the IMF, L. The answer is stated by the following proposition.

Proposition 5 $\Delta \mathcal{W}$ *is decreasing in* L *if and only if* $\bar{R}_L < \frac{R_A + R_N}{2}$.

Proof: using our proposition (2), we know that for a given distribution of the fundamental, $\bar{R}_L$ is decreasing in L. We can therefore study the response of $\Delta \mathcal{W}$ to changes in $\bar{R}_L$, rather than in L. We have:

$$\frac{d(\Delta \mathcal{W})}{d\bar{R}_L} = (\bar{R}_L I + M - D) \left[f\left(\bar{R}_L \mid R_N\right) - f\left(\bar{R}_L \mid R_A\right)\right] \qquad (5.0)$$

The first term in brackets is non-negative (because $(\bar{R}_L I + M - D) = (\bar{R}_L - R_s)I$ and $\bar{R}_L \geq R_s$) but the second term can have either sign. As $R_A > R_N$, we have that

$$f\left(\bar{R}_L \mid R_N\right) > f\left(\bar{R}_L \mid R_A\right) \Leftrightarrow \bar{R}_L < \frac{R_A + R_N}{2} \qquad (5.0)$$

which is the condition for a positive $\frac{d(\Delta W)}{dR_L}$. $\square$

Suppose that $\bar{R}_L$ is lower than both R_N and R_A — implying that the probability of a crisis is less than 50 percent irrespective of government behavior. In this case, the difference on the RHS is positive: a *decrease* in $\bar{R}_L$, corresponding to a more abundant liquidity provision L, reduces the extra-utility a government gets for taking the costly action A.

This case is illustrated by Figure 5.4-a.[19] In equilibrium, the position of $\bar{R}_L$ in this figure is such that the density at $\bar{R}_L$ is higher conditional on R_N than conditional on R_A. A decrease in $\bar{R}_L$ will therefore *reduce* the gain in expected GNP from 'good' government behavior.
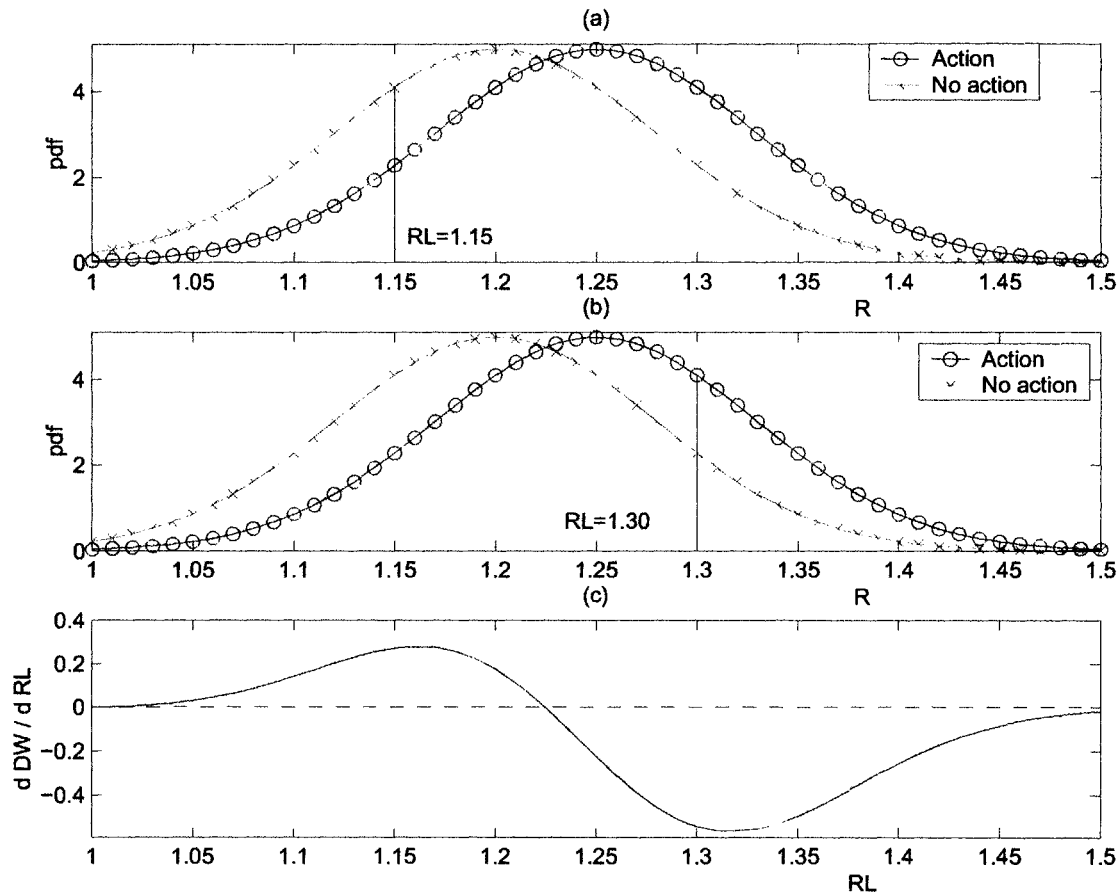

Figure 5.4: Government's decision: ΔW and $\bar{R}_L$

[19]Parameters employed: $R_A = 1.25$, $R_N = 1.20$, $\sigma_R = 0.08$ and $\bar{R}_L = 1.15$.

The equilibrium in Figure 5.4-a is consistent with the commonly held view of moral hazard distortions from liquidity provision. The argument underlying the traditional view is that the cost Ψ is high while the chances of a favorable macroeconomic outcome are good despite no government effort. In this case, additional liquidity provision is more likely to be helpful if the government does not take the costly action, so it further reduces the incentive for good behavior. However, our proposition make it clear that what is drawn in figure 5.4-a is not the only possible scenario.

5.5.2 Liquidity provision as an incentive for the government to 'do the right thing'

We are now ready to state a key result of our analysis. Suppose that the country fundamentals are relatively weak, in the sense that the ex-ante probability of a crisis is more than 50 percent even if the government chooses the costly action A. Then, according to proposition 5, ΔW will be *increasing* in L.

Intuitively, if — at some given L — the probability of a failure is relatively high, the government has little incentive to bear the costs of improving the macro outcome: the chance that a good outcome will materialize is low whether or not it exerts any effort. In this case, additional liquidity provision is more likely to be helpful if the government takes the costly action, so it *increases* the incentives for good behavior. By reducing the likelihood of runs and their costs in terms of forgone output, larger support by an international lender of last resort improves the trade off between the cost of government effort and the related improvement in the country's GNP. Whether the government chooses the action A in equilibrium will then depend on whether the net utility gain from action A exceeds its cost. To illustrate this case, in Figure 5.4-b the equilibrium $\bar{R}_L$ is drawn to the right of R_A. Clearly, a decrease in $\bar{R}_L$ raises the gains in expected GNP from the government action A.

Figure 5.4-c, instead, shows how the derivative of ΔW with respect to $\bar{R}_L$ varies with the level $\bar{R}_L$.[20] In looking at this figure, recall that $\bar{R}_L$ is monotonically decreasing in the size of IMF intervention L — so what the graph shows is the marginal effect of L on government incentives at different level of liquidity assistance. If $\bar{R}_L$ is small (i.e., L is large), the difference $\left[f\left(\bar{R}_L \mid R_N \right) - f\left(\bar{R}_L \mid R_A \right) \right]$ is positive and so is $\frac{d(\Delta W)}{d\bar{R}_L}$. For intermediate values of $\bar{R}_L$ (and L),

---

[20]Parameters employed: $R_A = 1.25$, $R_N = 1.20$, $\sigma_R = 0.08$, $I = 1$, $M = 0.2$, $D = 1.2$.

the ex-ante probability of a crisis gets higher. When $\bar{R}_L$ is large enough, $f\left(\bar{R}_L \mid R_N\right) < f\left(\bar{R}_L \mid R_A\right)$ and $\frac{d(\Delta W)}{d\bar{R}_L}$ is negative. In this case, by lowering $\bar{R}_L$, additional liquidity provision (larger L) actually strengthens the incentive for the government to take the costly action.

Relative to the traditional view, global-game models point to a different and intriguing possibility, one of *strategic complementarity* between the actions by the IMF and the domestic government (see the discussion of a similar result in Morris and Shin (2002)). When the ex-ante probability of a crisis is high, the payoff to the government from action A is increasing in L. Note that also the payoffs of the IMF is increasing in the action A undertaken by the government.

In closing this section, we note that our conclusion remains unchanged when government welfare depends on GDP, rather than GNP — this is equivalent to assume that the amount paid to foreigners is independent of the realization of R, perhaps because there are other resources in the economy in addition to the payoffs of domestic investment I. By discussing this case, we stress that our results are not driven by the assumption of "limited liability" for the economy as a whole. Even if the government cares about GDP, a marginal increase in the size of the IMF would still reduce $\bar{R}_L$, producing *marginal saving on liquidation costs*. Its effect on the incentives to take the costly action A depends on the likelihood that it will benefit the government in either situation (conditional on choosing R_A or R_N). The intuition is exactly the same as when government cares about GNP — we present some calculation in the appendix C.2.5 to shed further light on this issue.

5.5.3 Policy tradeoffs and the optimal size of L: numerical examples

The properties of our model can be illustrated by means of four numerical examples, all depicted in Figure 5.5. To draw this figure, we adopt the parameter values shown in table 1, and set $D = 1.2$ and $I = 1$. For each example, we plot $W(A)$, $W(N)$ and the expected GNP of the country against different values of L.

As shown above, the government chooses the costly action whenever $W(\mathcal{A}) > W(\mathcal{N})$. The country's GNP is therefore $W(\mathcal{N})$ if the action is not taken, and $W(A) + \Psi$ if the action is taken. Thus, the various graphs in figure 5.5 show for which values of L the government takes the costly action A, as well as the country expected GNP, as a function of L.

Figures 5.5-a and 5.5-b illustrate the case in which a large ILOLR unambiguously creates moral

Table 5.1: Value of the parameters in figure 5.5

figure	5.5-a	5.5-b	5.5-c	5.5-d
κ	.3	.25	.25	.25
M	.25	.20	.20	.20
Ψ	.06	.05	.045	.05
b	2	1	1	1
c	10	10	10	10
B	2	2	2	2
C	10	10	10	10
R_A	1.18	1.25	1.18	1.12
R_N	1.16	1.22	1.15	1.065
σ_R	.03	.05	.05	.05

hazard distortions. Comparing $W(A)$ with $W(N)$: the former exceeds the latter — i.e., governments prefer to take the costly action — only for relatively low values of L, between 0 and (approximately) 0.18. Liquidity provision in excess of this value creates a clear incentive for the government not to act. To trace the behavior of expected GNP for different levels of L, compare $W(A) + \Psi$ with $W(N)$. Increasing the size of the IMF contingent interventions above 0 at first raises expected GNP monotonically along $W(A) + \Psi$. At L around 0.18, however, the moral hazard distortion kicks in, determining a discrete drop in expected GNP and national welfare to $W(N)$. Conditional on R_N, providing more liquidity assistance has again a positive effect on expected GNP.

Thus, in a global sense, there could be different trade-offs between liquidity provision and moral hazard. In Figure 5.5-b, the country GNP is at a maximum when liquidity provision is just below the level at which the government would give up the costly action A. Globally, moral hazard distortions are more important than the costs of liquidity crises. Conversely, in Figure 5.5-a, the country GNP is highest for high values of L despite moral hazard distortions. Liquidity costs in this case are more important than the output costs due to moral hazard.

Figure 5.5-c illustrates the possibility of strategic complementarity between IMF lending and government policies towards solvency. In this figure the tradeoff between liquidity and moral hazard vary with L. For sufficiently low values of L, $W(A) < W(N)$ and the government does not
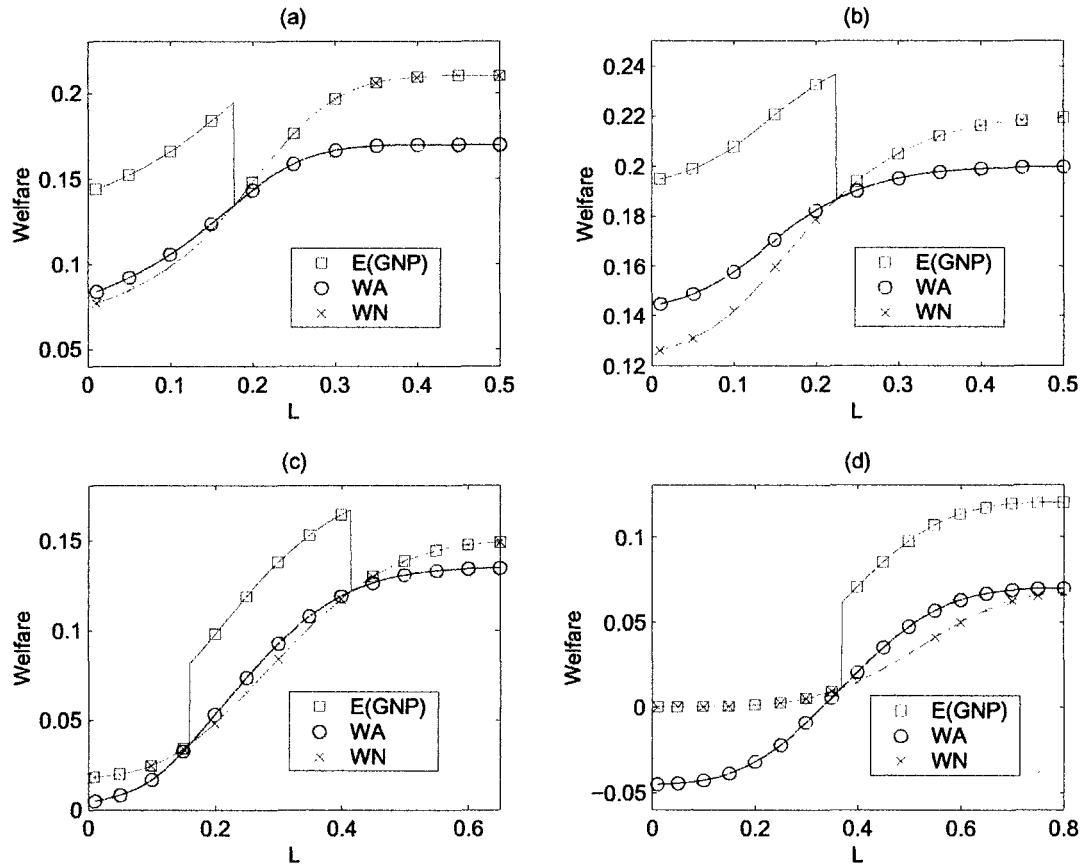
Figure 5.5: Policy tradeoffs

undertake any action because it is discouraged by bleak prospects of success. For intermediate level of liquidity support, however, the government welfare becomes higher conditional on undertaking the action A. Liquidity provision eventually becomes excessive. For levels of L in excess of 0.4, once again $W(A) < W(N)$: the government does not exert any effort, and the country's expected GNP falls. Note that, relative to Figure 5.5-b, the only parameter change consists of decreasing both R_A and R_N by a few percentage points — enough to worsen the macroeconomic outcome in such a way that, within some range of the fundamental, the government would not undertake any costly policy without liquidity assistance by the IMF.

Relative to Figure 5.5-c, in Figure 5.5-d we further reduce both R_A and R_N, while allowing for a larger difference ΔR. In figure 5.5-d, government welfare conditional on the costly action is actually

higher than $\mathcal{W}(N)$ if the IMF provides sufficiently large contingent funds. The country GNP is always higher conditional on A: globally, there is no trade off between liquidity provision and moral hazard.

These considerations may be useful as building blocks towards a normative study of the optimal size of IMF interventions. As apparent from figure 5.5, local governments like the highest possible level of liquidity assistance by the IMF. Once moral hazard considerations are taken into account, however, the level of liquidity assistance preferred by policymakers may not be the one that maximizes expected GNP and national welfare. Since the welfare cost Ψ does not fall on the country's citizens, these may prefer a low level to a large level of L. This is the case in the economy depicted by Figure 5.5-b.

The level of L preferred by the IMF need not coincide with either the level of L preferred by national governments, or the level preferred by the country's citizens. In our specification, the structure of IMF preferences penalizes any loss of funds in case of national default, yet as a simplification the penalty from lending liquidity to crisis countries is exogenously given. Thus, for any given disutility from loosing its loans to the country — the main concern of the IMF is whether or not to limit L below its feasible level, as a way to mitigate moral-hazard distortions that could raise discretely the likelihood of a crisis.

5.5.4 Moral hazard with a continuous set of actions for the government

We conclude this section by reconsidering our analysis of moral hazard in a more general framework. Let ΔR denote policy effort, raising linearly the expected value of the fundamental, i.e., $E_0 R = R_0 + \Delta R$. Policy effort entails a utility cost $\Psi (\Delta R)^\nu / \nu$, affecting the government only. Thus, assuming that the noise in private signal is arbitrarily small ($\alpha \to \infty$), the policy problem is to maximize:

$$\lim_{\alpha \to \infty} \mathcal{W}(\Delta R) = \int_{\bar{R}_L}^{\infty} [R \cdot I + M - D] \ f(R \mid R_0 + \Delta R) \ dR - \frac{\Psi (\Delta R)^\nu}{\nu} \tag{5.1}$$

$$= \int_{\bar{R}_L - \Delta R}^{\infty} [(R + \Delta R) \cdot I + M - D] \ f(R \mid R_0) \ dR - \frac{\Psi (\Delta R)^\nu}{\nu}$$

Taking the derivative with respect to ΔR, we get:

$$\frac{dW(\Delta R)}{d\Delta R} = (\bar{R}_L I + M - D)f(\bar{R}_L - \Delta R \mid R_0) + \int_{\bar{R}_L - \Delta R}^{\infty} I \cdot f(R \mid R_0)\, dR - \Psi(\Delta R)^{\nu-1} (5.1)$$

$$= (\bar{R}_L - R_s)If(\bar{R}_L - \Delta R \mid R_0) + I(1 - F(\bar{R}_L - \Delta R \mid R_0)) - \Psi(\Delta R)^{\nu-1}$$

It is easy to show that, for $\nu > 1$ and reasonable values of Ψ, our results for the binary-action case still apply. Namely, when ex-ante odds of a crisis are high enough, the government chooses little or no policy effort. By reducing the ex-ante probability of a crisis, a larger L would then raise the government incentive to choose a higher effort ΔR. Conversely, when the ex-ante probability of a run is small, additional liquidity provision induces the government to reduce ΔR.
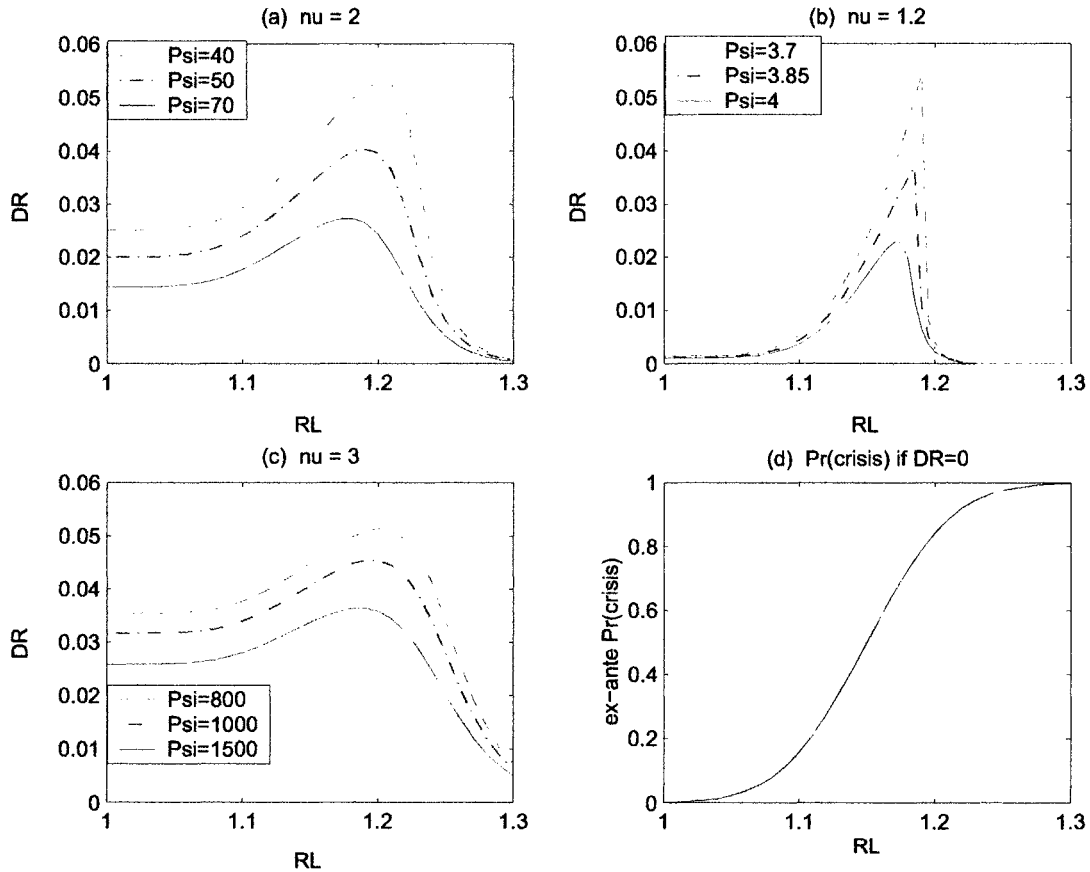
Figure 5.6: Continuous set of actions for the government

These results are illustrated by Figures 5.6-a,b,c, which plot the optimal effort level ΔR as a function of $\bar{R}_L$, for ν equal to 2, 1.2 and 3, respectively. Figure 5.6-d, instead, shows the ex-ante

odds of a crisis as a function of $\bar{R}_L$ conditional on $\Delta R = 0$.[21] The first three graphs appear quite

similar: effort (ΔR) is increasing in $\bar{R}_L$ up to a point (around 1.18 or 1.20, depending on parameters'

values), after which it is decreasing in $\bar{R}_L$. Note that the elasticity of ΔR falls with ν.

---

[21]Parameters used in the figures: $R_s = 1$, $R_0 = 1.15$, $\sigma_R = 0.05$, $I = 1$.

Chapter 6

Conclusion

This first essay of this dissertation analyzes a dynamic game of currency crises. To decide about going long in the currency, a rational agent must evaluate the probability that he'll be caught by the devaluation, forecasting what the others will do and knowing that the economic environment is continuously changing. In this framework, expectations about others' future actions are endogenous, agents' decisions are uniquely determined by the model, and payoffs are stochastic, regardless of whether there is a 'secular deterioration of fundamentals' or not.

The essay provides a new argument to establish existence and uniqueness of a threshold equilibrium. Analytical results for a particular case (with no uncertainty on the path of the shadow exchange rate) and numerical simulations yield insights about the effects on agents' behavior of: interest rates, frictions, macroeconomic prospects and government's willingness to keep the peg.

A large body of the literature of currency crises (as Krugman, 1999), has claimed that economic fundamentals cannot explain why events in Asia or Russia led to crises in countries with few direct links, like Brazil. Expectations would have played an important role. By estimating the probabilities and expected magnitudes of a currency devaluation, the second essay of this dissertation confirms Paul Krugman's view that the expectations with respect to the maintenance of Brazilian pegged regime were strongly affected by the Asian and Russian episodes. Then, it relates the empirical findings to the predictions of the dynamic model of currency crises presented in the first essay. In the model, agents expect others to attack the currency when the overvaluation gets high enough. Therefore, increases in the shadow exchange rate should affect the probability of a devaluation, but

not its expected magnitude — as the data suggests.

The third essay (written in co-authorship with Stephen Morris) builds a 'global-games' model of currency crisis in order to analyze the impact on agents behavior of issues related to risk and wealth. While the analysis here concerns currency crises, the modelling may be relevant to a wide array of macroeconomic issues. The analysis of risk and wealth is central to macro. Self-fulfilling beliefs and strategic complementarities play an important role in many macroeconomic settings. In the marriage of these two strands in this essay, risk, wealth and portfolio effects play a central role in determining how strategic complementarities translate into economic outcomes.

Under the naive, static, complete markets model of agents' portfolio choices, we were able to derive a number of striking predictions about the likelihood of currency crises. However, our conclusions were sensitive to the market assumptions: plausible sounding incomplete market restrictions can have a dramatic impact on comparative statics. Real currency markets reflect the transaction, hedging and speculative demands of many private traders, the policy interventions of central banks and the strategies of large institutions such as hedge funds that may be hard to explain and model as the aggregation of individual utility maximizing behavior. One message of this essay is that if currency crises are self-fulfilling, the motives and strategies of market participants may be important in a way they are not in models where an arbitrage condition (and not strategic considerations) pins down the equilibrium.

It is often argued that the provision of liquidity by the international institutions such as the IMF to countries experiencing balance of payment problems can have catalytic effects, i.e., it can reduce the scale of liquidity runs by inducing investors to roll over their financial claims to the country. Critics point out that official lending also causes moral hazard distortions: expecting to be bailed out by the IMF, debtor countries have weak incentives to implement good but costly policies, thus raising the probability of a crisis.

The fourth essay (written in co-authorship with Giancarlo Corsetti and Nouriel Roubini) presents an analytical framework to study the trade-off between official liquidity provision and debtor moral hazard. In our model international financial crises are caused by the interaction of bad fundamentals, self-fulfilling runs and policies by three classes of optimizing agents: international investors, the local government and the IMF. We show how an international financial institution helps prevent liquidity runs via coordination of agents' expectations, by raising the number of investors willing to lend to the

country for any given level of the fundamental. We show that the influence of such an institution is increasing in the size of its interventions and the precision of its information: more liquidity support and better information make agents more willing to roll over their debt and reduces the probability of a crisis.

Different from the conventional view stressing debtor moral hazard, we show that, in some situations, official lending may actually strengthen a government incentive to implement desirable but costly policies. By worsening the expected return on these policies, destructive liquidity runs may well discourage governments from undertaking them, unless they can count on contingent liquidity assistance.

Appendix A

Dynamic model

A.1 Proof of Theorem 1

Step 1 *There exists a continuous function $\theta^* : [0,1] \to (-\infty, \tilde{\theta}(1)]$, such that $E\pi(\theta^*(A), a; \theta^*) = 0$ for any $A \in [0,1]$.*

The payoff function is defined only at the left of $\tilde{\theta}$. For analytical convenience, we will define a function q which coincides with the expected payoff of investing at a point over the threshold θ^* if the payoff is defined at that point:

$$
q(\theta^*, A) = \begin{cases} E\pi(\theta^*(A), A; \theta^*) & \text{if } \theta^*(A) \leq \tilde{\theta}(A) \\ \lim_{\theta \to \tilde{\theta}(A)^-} E\pi(\theta, A; \theta^*) & \text{if } \theta^*(A) = \tilde{\theta}(A) \\ E\pi(\tilde{\theta}(A), A; \theta^*) - \left(\theta^*(A) - \tilde{\theta}(A)\right) & \text{if } \theta^*(A) > \tilde{\theta}(A) \end{cases}
$$

It will be shown the existence of a function $\theta^* : [0,1] \to [\underline{\theta}, \bar{\theta}]$ such that $q(\theta^*, A) = 0$ for all values of $A \in (0,1)$.

The proof starts by arguing that q is a continuous mapping:

Lemma 1 *The mapping q is continuous in θ. That is:*

$$
\lim_{\theta' \to \theta^*} q(\theta', A) = q(\theta^*, A)
$$

Sketch of the argument:
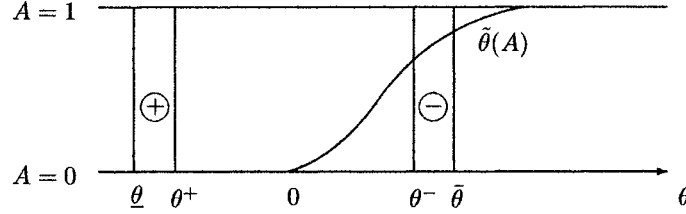
Figure A.1: Proof of step 1

$E\pi(\theta^*(A), A; \theta^*)$ and $E\pi(\theta'(A), A; \theta')$ may be different for two reasons: (i) as $\theta^*(A) \neq \theta'(A)$, the devaluation obtained for a given process z starting at those points will be different; (ii) as the curves are different, the path of A given a process z may depend on whether agents are playing according to $\theta^*(A)$ or to $\theta'(A)$.

It is easy to see that changes due to (i) satisfy continuity. For point (ii), note that as $\theta^*(A) \to \theta'(A)$, the 'L' and 'N' areas will be the same in both cases except for a measure 0 set.

Note also that:

Remark 1 *There exists a number θ^+ such that $\forall$ A and θ^*, if $\theta^*(A) < \theta^+$, $q(\theta^*, A) > 0$.*

Remark 2 *There exists a number θ^-, strictly smaller than $\tilde\theta(1)$, such that $\forall$ A and θ^*, if $\theta^*(A) > \theta^-$, $q(\theta^*, A) < 0$.*

Define:

$$\mathcal{F} = \left\{ f \mid f \text{ is continuous and } f : [0,1] \to [\underline{\theta}, \bar\theta] \right\} \text{ and}$$

$$y : \mathcal{F} \to \mathcal{F}, \quad y(\theta^*) = \theta^* + \alpha q(\theta^*) \text{ for some small } \alpha$$

where $\underline{\theta} < \theta^+$ and $\bar\theta > \theta^-$ as indicated in figure A.1. For small enough α, the image of the function $y(\theta^*)$ is always inside $[\underline{\theta}, \bar\theta]$ given the behavior of function q.

By lemma 1, y is a continuous mapping of $\mathcal{F}$. The set $\mathcal{F}$ is convex and compact. Thus, we can apply Schauder's fixed point Theorem.[1] So there exists a continuous θ^* such that:

---
[1]see, e.g., Smart (1974).

$$y(\theta^*) = \theta^* + \alpha q(\theta^*) = \theta^* \;\Rightarrow\; q(\theta^*) = 0$$

which yields the claim. Due to the behavior of the Brownian motion, as $\theta \to \tilde{\theta}(A)$, the probability of an 'instantaneous' devaluation approaches 1. Therefore, $\theta^*(A) < \tilde{\theta}(A)$ for all $A > 0$. $\square$

Step 2 *If step 1 holds,* Long *is the optimal choice at the 'L' area and* Not *is the optimal choice at the 'N' area. At θ^*, the agent is indifferent.*

The argument starts by emphasizing an important characteristic of the framework and showing some auxiliary results:

Remark 3 *Consider a process x starting at time 0. Suppose that at time $\bar{t}$ the economy is at (θ, A). Then the (conditional) expected payoff of x is:*

$$E\pi(x; \theta^*) = \left(1 - e^{-(\delta - r)\bar{t}}\right)\frac{\delta}{\delta - r} + e^{-(\delta - r)\bar{t}}\left[E\pi(\theta, A; \theta^*) + 1\right] - 1 \tag{A.0}$$

Equation 3 is similar to equation 2.2.3. The difference is that instead of having the value obtained at a terminal point, we have the expected payoff at a given state. This friendly formulation depends on the assumption of a Poisson process for δ.

Lemma 2shows that we can work with a modified form of equation 2.2.3:

Lemma 2 *The effects on expected payoff of hitting θ^* and hitting $\tilde{\theta}$ with devaluation equal to 0 are the same.*

That is, we can substitute equation 2.2.3 by the following equation:

$$\pi(z; \theta_0, A_0, \theta^*) = \left(1 - e^{-(\delta - r)\Delta t(z)}\right)\frac{\delta}{\delta - r} + e^{-(\delta - r)\Delta t(z) - \theta^{mod}(z)} - 1 \tag{A.0}$$

where $\Delta t(z)$ is the time it takes for reaching $\tilde{\theta}$ *or* θ^* for the first time, and θ^{mod} is defined below:

$$\theta^{mod} = \begin{cases} \theta^{end} & \text{if } \tilde{\theta} \text{ is reached before } \theta^* \\ 0 & \text{if } \theta^* \text{ is reached before } \tilde{\theta} \end{cases}$$

Proof:

Consider a process x starting at (θ, A) and a particular realization z of the Brownian motion. Suppose θ^* is reached before $\tilde{\theta}$. Call $\Delta t(z)$ the time it takes for the economy to get to θ^* and a_z the value of A when θ^* is reached. The payoff is:

$$
\begin{aligned}
\pi(z; \theta, A, \theta^*) &= \left(1 - e^{-(\delta-r)\Delta t(z)}\right) \frac{\delta}{\delta - r} + e^{-(\delta-r)\Delta t(z)} \left[E\pi(\theta^*(a_z), a_z; \theta^*) + 1\right] - 1 \\
&= \left(1 - e^{-(\delta-r)\Delta t(z)}\right) \frac{\delta}{\delta - r} + e^{-(\delta-r)\Delta t(z)} - 1 \\
&= \left(1 - e^{-(\delta-r)\Delta t(z)}\right) \frac{\delta}{\delta - r} + e^{-(\delta-r)\Delta t(z) - \theta^{mod}(z)} - 1
\end{aligned}
$$

if we define $\theta^{mod}(z) = 0$. The first line uses equation 3 and the second equality comes from expected payoff being equal to 0 at any point of θ^*. $\square$

Corollary 2 $E\pi(\theta, A; \theta^*) > e^{-\tilde{\theta}(A)}$

Corollary 2 comes from applying equation A.1, noting that $\theta^{mod} \leq \tilde{\theta}(A)$.

Now, we are ready to show Step 2, which is a consequence of 2 lemmas. Lemma 3 shows that the payoff at the 'L' area is always positive and decreasing in θ. Lemma 4 shows that the payoff at the 'N' area is always negative and decreasing in θ.

Lemma 3 *For all (θ, A) in the 'L' area:*

1. *$E\pi(\theta, A, \theta^*) > 0$,*

2. *$E\pi(\theta, A, \theta^*)$ is decreasing in θ.*

Proof:

Any process starting in the 'L' area will reach θ^* before reaching $\tilde{\theta}$. For any process z, call $t_1(z)$ the time it takes for reaching θ^*. Equation A.1 simplifies to:

$$
\pi(z; \theta_0, A_0, \theta^*) = \left(1 - e^{-(\delta-r)t_1(z)}\right) \frac{\delta}{\delta - r} + e^{-(\delta-r)t_1(z)}
$$

Using equation 2.2.3, we get:

$$
E\pi(\theta, A; \theta^*) = \int_z \frac{r}{\delta - r} \left(1 - e^{-(\delta-r)t_1(z)}\right) f(z) dz > 0 \tag{A.-3}
$$

which proves the first statement.

Now, consider two processes: x, starting at (θ, A) and x', starting at (θ', A), such that $\theta' > \theta > \theta^*(A)$. Call $t_1(z)$ and $t_1'(z)$, the time it takes for x and x' to reach the threshold θ^*, respectively. It is easy to see that for any z, $t_1(z) < t_1'(z)$. The difference of expected payoffs is given by:

$$
\begin{aligned}
\Delta\pi &= E\pi(\theta', A; \theta^*) - E\pi(\theta, A; \theta^*) \\
&= \int_z \frac{r}{\delta - r} \left(e^{-(\delta-r)t_1(z)} - e^{-(\delta-r)t_1'(z)} \right) f(z)dz \\
&> 0
\end{aligned}
$$

which completes the proof. $\square$

Lemma 4 *For all (θ, A) in the 'N' area:*

1. $E\pi(\theta, A, \theta^*) < 0$,

2. $E\pi(\theta, A, \theta^*)$ is decreasing in θ.

Proof:

Consider two processes: x, starting at (θ, A) and x', starting at (θ', A), such that $\theta^*(A) \geq \theta' > \theta$.

Consider also a process x'' that starts at starting at (θ', A) (with x') but moves in a different way: until x reaches either θ^* or $\tilde{\theta}$ for the first time, A'' always decreases as if the economy was at the 'N' area; and θ follows the Brownian motion z except that it never crosses the threshold θ^* to get to the 'L' area, as shown at figure A.2. After x hits θ^* or $\tilde{\theta}$, x'' behaves in the regular way (as x and x').

The proof of lemma 4 comes in 2 parts:

1. $E\pi(x'; \theta^*) > E\pi(x'', \theta^*)$

2. $E\pi(x''; \theta^*) > E\pi(x, \theta^*)$

First part: $E\pi(x'; \theta^*) > E\pi(x'', \theta^*)$

The argument goes as follows: the process x'' is automatically substituting states that yield a positive payoff by states that yield zero payoff. By equation 3, we know that the expected payoff of x', at any moment before x hits $\tilde{\theta}$, is a function of the expected payoff of its reachable states. Every
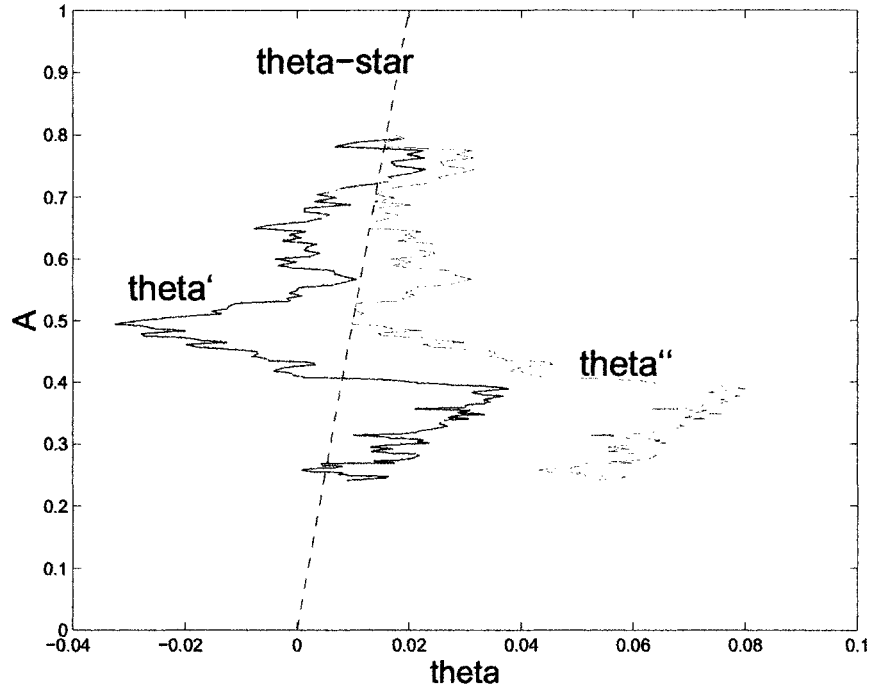
Figure A.2: Path of θ' and θ''

time we apply the 'x'' rule with respect to θ' and prevent the economy from reaching the 'L' area, we reduce the expected payoff of the process from that point on. The 'x'' rule with respect to A' has no effect on payoffs because the payoff at any point over θ^* is zero.

The strict inequality depends on the fact that x' will reach θ^* with positive probability.

Second part: $E\pi(x''; \theta^*) > E\pi(x', \theta^*)$

Suppose that x reaches θ^*. Then, from that point on, paths of x and x'' coincide.

Suppose that x gets to $\tilde{\theta}$ (without never hitting θ^*) at time t. From that point on, x'' follows the regular laws of motion of the game. From equation 3 and corollary 2, expected payoff of x'' in this case is greater than expected payoff of x.

The strict inequality depends on the fact that x' will reach θ^* with probability smaller than one.

Combining both parts, we get the claim. $\square$

Step 3 *If steps 1 and 2 hold, there is a unique threshold equilibrium.*

Proof: Suppose 2 equilibria, one with threshold $\underline{\theta}(A)$ and other with threshold $\bar{\theta}(A)$. Define the point a such that $a = \mathrm{argmax}\{|\bar{\theta}(A) - \underline{\theta}(A)|\}$ as shown in figure 2.6.

Suppose a process x' starting at $x'_0 = (a, \underline{\theta}(a))$ when all agents follows a switching strategy around $\underline{\theta}(A)$ and a process x starting at $x_0 = (a, \bar{\theta}(a))$ when all agents follow a switching strategy around $\bar{\theta}(A)$.

The argument goes as following: if both curves do not coincide, the expected payoff of x' is strictly greater that the expected payoff of x, which contradicts the fact that investors are indifferent at all points of those thresholds.

Let Γ be the set of realizations of the Brownian motion (z) such that x and x' are never at different sides of their own thresholds and Γ' its complement.

For any $z \in \Gamma$, we know that x will get to $\tilde{\theta}$ before x and the value of A will be the same for both processes (as x and x' were always at the same side of their own thresholds). By corollary 2 and equation 3, we get:

$$\int_{z \in \Gamma} \left[\pi(z; x'_0, \underline{\theta}) - \pi(z; x_0, \bar{\theta}) \right] f(z) dz \; > \; 0 \tag{A.-6}$$

For all $z \in \Gamma'$, call $t^1(z)$ the first moment in which x and x' are not at the same side of their own thresholds. It will happen when one process (x or x') hits its own threshold while the other has not reached it. As a maximizes the distance between curves, it must be true that $x'_{z,t^1(z)}$ will be at the 'L' area or over $\underline{\theta}$, which implies:

$$\int_{y \in \Omega} \pi(y; x'_{z,t^1(z)}, \underline{\theta}) f(y) dy > 0$$

while $x_{z,t^1(z)}$ will be at the 'N' area or over $\bar{\theta}(A)$ and so:

$$\int_{y \in \Omega} \pi(y; x_{z,n^1(z)}, \bar{\theta}) f(y) dy \; < 0$$

Combining both inequalities and considering there is no difference in payoffs for agents that got a signal before $n^1(z)$, we get:

$$\int_{z \in \Gamma'} \left[\pi(z; x_0', \underline{\theta}) - \pi(z; x_0, \bar{\theta}) \right] f(z) dz = \qquad \text{(A.-5)}$$

$$\int_{z \in \Gamma'} e^{-(\delta - r)t^1(z)} \left(\int_{y \in \Omega} \left[\pi(y; x_{z, t^1(z)}, \underline{\theta}) - \pi(y; x_{z, t^1(z)}, \bar{\theta}) \right] f(y) dy \right) f(z) dz > 0$$

Summing (A.1) and (A.-5), we get a contradiction. $\square$

A.2 Particular case: $\mu_\theta > 0$ and $\sigma_\theta = 0$

A.2.1 Existence and uniqueness

If $\mu_\theta > 0$, there exist an area in which *Long* is the optimal choice regardless what others do and another area in which *Not* is the optimal decision in any case. Thus, there cannot be any equilibrium in which people choose one of the actions regardless of the state of the economy.

As argued in section 2.4, if equation 2.4 defines a unique threshold, then it is easy to see that *Long* is the optimal choice at the left of θ^* and *Not* is the optimal choice at its right. To see that for any $A_0 \in [0, 1]$ there exists a unique $\tilde{A}$ satisfying equation 2.4, note that: (i) its left hand side is increasing in $\tilde{A}$ and its right hand side is decreasing in $\tilde{A}$ — as $\tilde{\theta}$ is an increasing funtion — and (ii) its right hand side assumes values inside the $(0, 1)$-interval and its left hand side equals 0 if $\tilde{A} = 0$ and 1 if $\tilde{A} = A_0$.

$$\frac{\tilde{A}}{A_0} = \left(\psi \left(1 - e^{-\tilde{\Theta}} \right) + e^{-\tilde{\Theta}} \right)^{\frac{-\psi}{\psi - 1}} \qquad \text{(equation 2.4)}$$

A.2.2 Proof of Proposition 1

Proof of Statements 1 and 2

It's clear that $\tilde{A}$ must lie in the interval $(0, 1)$. The agents that choose right before the devaluation must prefer *Not*, so $\tilde{A}$ must be smaller than 1. ΔT must be finite, so $\tilde{A}$ must be greater than 0.

To show that $\frac{d\tilde{A}}{d\delta} < 0$ and $\frac{d\tilde{A}}{dr} > 0$, we need to show that $\frac{d\tilde{A}}{d\psi} < 0$.

Taking logs of equation 2.4, we get:

$$\frac{\psi - 1}{\psi} \log \left(\frac{\tilde{A}}{A_0} \right) = -\log \left(e^{-\tilde{\Theta}} + \psi \left(1 - e^{-\tilde{\Theta}} \right) \right) \qquad \text{(A.-6)}$$

Differentiating equation A.2.2 with respect to ψ, we get:

$$\left[\frac{\psi - 1}{\tilde{A}\psi} + \frac{(\psi - 1)e^{-\tilde{\Theta}}\left(\frac{d\tilde{\theta}}{d\tilde{A}}\right)}{e^{-\tilde{\Theta}} + \psi\left(1 - e^{-\tilde{\Theta}}\right)}\right]\frac{d\tilde{A}}{d\psi} = -\frac{\log\left(\frac{\tilde{A}}{A_0}\right)}{\psi^2} - \frac{\left(1 - e^{-\tilde{\Theta}}\right)}{e^{-\tilde{\Theta}} + \psi\left(1 - e^{-\tilde{\Theta}}\right)} \tag{A.-5}$$

Substituting equation A.2.2 in A.-5, we get:

$$\left[\frac{\psi - 1}{\tilde{A}\psi} + \frac{(\psi - 1)\left(\frac{d\tilde{\theta}}{d\tilde{A}}\right)}{e^{-\tilde{\Theta}} + \psi\left(1 - e^{-\tilde{\Theta}}\right)}\right]\frac{d\tilde{A}}{d\psi} = \frac{\log\left(e^{-\tilde{\Theta}} + \psi\left(1 - e^{-\tilde{\Theta}}\right)\right)}{\psi(\psi - 1)} - \frac{\left(1 - e^{-\tilde{\Theta}}\right)}{e^{-\tilde{\Theta}} + \psi\left(1 - e^{-\tilde{\Theta}}\right)}$$

As $\left[e^{-\tilde{\Theta}} + \psi\left(1 - e^{-\tilde{\Theta}}\right)\right] > 1$,

$$log\left(e^{-\tilde{\Theta}} + \psi\left(1 - e^{-\tilde{\Theta}}\right)\right) < \left(e^{-\tilde{\Theta}} + \psi\left(1 - e^{-\tilde{\Theta}}\right) - 1\right)$$

and so:

$$\left[\frac{\psi - 1}{\tilde{A}\psi} + \frac{(\psi - 1)\left(\frac{d\tilde{\theta}}{d\tilde{A}}\right)}{e^{-\tilde{\Theta}} + \psi\left(1 - e^{-\tilde{\Theta}}\right)}\right]\frac{d\tilde{A}}{d\psi} < \frac{\left(e^{-\tilde{\Theta}} + \psi\left(1 - e^{-\tilde{\Theta}}\right) - 1\right)}{\psi(\psi - 1)} - \frac{\left(1 - e^{-\tilde{\Theta}}\right)}{e^{-\tilde{\Theta}} + \psi\left(1 - e^{-\tilde{\Theta}}\right)}$$

Simplifying,

$$\left[\frac{\psi - 1}{\tilde{A}\psi} + \frac{(\psi - 1)\left(\frac{d\tilde{\theta}}{d\tilde{A}}\right)}{e^{-\tilde{\Theta}} + \psi\left(1 - e^{-\tilde{\Theta}}\right)}\right]\frac{d\tilde{A}}{d\psi} < -\frac{\left(1 - e^{-\tilde{\Theta}}\right)(\psi - 1)e^{-\tilde{\Theta}}}{\psi\left(e^{-\tilde{\Theta}} + \psi\left(1 - e^{-\tilde{\Theta}}\right)\right)} < 0$$

which yields: $\frac{d\tilde{A}}{d\psi} < 0$. $\square$

Proof of Statement 3

Taking limit of equation 2.4, we get:

$$\lim_{\delta \to \infty} \Delta T = 0$$

Moreover, suppose $\delta \to \infty$ and $\tilde{\Theta} = \epsilon$, bounded away from 0. Then, by equation 2.4, $\tilde{A} \to 0$. But that implies $\tilde{\Theta} = 0$, a contradiction. Thus:

$$\lim_{\delta \to \infty} \tilde{\Theta} = 0 \; \Rightarrow \; \lim_{\delta \to \infty} \tilde{A} = 0$$

From equation 2.4, $\lim_{\delta \to \infty} \Theta^* = 0$. $\square$

A.3 Numerical estimations

An equilibrium in the model is characterized by a threshold θ^* such that the expected payoff of investing equals 0 at all points of the curve. The payoff of investing is stochastic and depends on two random variables: (i) the signal for deciding again and (ii) the motion of θ.

It is easy to handle the first issue: given the time it takes for a crisis to occur and the size of the devaluation, the expected payoff is given by equation 2.2.3. Dealing with the motion of θ is harder for two reasons: analytically, the analysis in this essay does not go further than equation 2.2.3, so we need to simulate paths of x to calculate the payoff at any point in the economy. Second, the motion of x around the threshold θ^* is quite complicated if z follows a Brownian motion.

In order to simulate the path of x, we need to approximate the Brownian motion by a random walk process, in which each period takes Δt units of time. However, close to the threshold θ^*, the motion of x depends on Δt, specially if σ_θ is small.[2] So, section A.3.1 makes explicit a discrete time version of the model. Of course, when $\Delta t \to 0$, it converges to the continuous time framework.

To get an approximation of the equilibrium threshold, we rely on a heuristic procedure to obtain a function θ^* in which the payoff in some points are close to zero and the other points are determined by a piecewise cubic hermite interpolating polynomial.

A.3.1 Discrete time version of the model

The model is exactly as before but in discrete time.

---

[2]For an intuition on the non-trivial behavior of the economy close to the threshold for small σ_θ, see Burdzy et al (2001).

One period lasts for Δt units of time. The parameter θ follows a *random walk*, so:

$$\Delta\theta \sim N(\mu_\theta.\Delta t, \sigma_\theta.\sqrt{\Delta t}).$$

Timing is as follows:

1. Period t starts and θ_t is observed.

2. Agents allowed to change their portfolio take their decisions. In a period, $(1 - e^{-\delta.\Delta t})$ agents move, but they do it sequentially. At the end of the period, A_t agents are investing.

3. If $\theta_t > \tilde{\theta}(A_t)$, the peg is abandoned and the game ends. Otherwise, period t ends and period $t + 1$ starts immediately. Interests are paid and consumed exactly when period t ends, right before θ_{t+1} is revealed.

Analogously to the continuous time case, a process $\{x\} = \{\theta, A\}$ will denote a particular path of the state variables of the model, and $\{z\}$ will denote a particular realization of the random walk process.

Suppose that all other agents are following a strategy around a threshold θ^* as shown in figure 2.2. Let z be a giving realization of the random walk process and (θ_0, A_0) the starting point at next period — all agents know where x will be at the end of the current period, so (θ_0, A_0) is the relevant starting point.

Let $n(z)$ be the number of periods it will take for the crisis and $\theta^{end}(z)$ the size of the devaluation. The payoff of choosing *Long* in this case is given by:

$$
\begin{aligned}
\pi(z; \theta_0, A_0, \theta^*) &= e^{-(\delta-r)n(z)\Delta t - \theta^{end}(z)} \\
&+ \left(1 - e^{-(\delta-r)n(z)\Delta t}\right) \frac{(1 - e^{-\delta\Delta t})e^{rt}}{1 - e^{-(\delta-r)\Delta t}} - 1
\end{aligned}
\tag{A.-10}
$$

Long is the optimal decision if $E\pi(\theta_0, A_0; \theta^*) \geq 0$ where:

$$
E\pi(\theta_0, A_0; \theta^*) = \int_z \pi(z; \theta_0, A_0, \theta^*) f(z) dz
\tag{A.-10}
$$

The threshold is the function θ^* such that $\forall A, E\pi(\theta_0, A_0; \theta^*) = 0$.

Appendix B

Empirical analysis

B.1   The asset pricing model

B.1.1   Formula for the price of a call

This section provides an intuitive explanation of equation 3.1, that is:

$$C^{mod} = e^{-\lambda T} BS\left(Se^{(-q-\lambda k)T}, T; X, r, \sigma_1^2\right) +$$
$$\int_0^T \lambda e^{-\lambda t} BS\left(Se^{-qT-\lambda kt}(1+k), T; X, r, \frac{\left(\sigma_1^2 t + \sigma_2^2(T-t)\right)}{T}\right) dt$$

where $BS\left(S, T; X, r, \sigma^2\right)$ denotes the Black-Scholes price of a European call option if the underlying asset follows a Brownian motion with a drift $\left(\frac{dS}{S} = \mu.dt + \sigma.dX\right)$, r is the interest rate, X is the strike price, S is the spot exchange rate and T is the time to maturity.

The price of an exchange rate option with the above characteristics is:

$$BS\left(S.e^{-qT}, T; X, r, \sigma^2\right) \tag{B.-2}$$

where q is the interest rate denominated in foreign currency.

The first term of equation 3.1 is the value of the option if there is no devaluation until time T. This happens with probability $e^{-\lambda T}$. Conditional on that, the value of a call option is given by:

$$BS\left(Se^{(-q-\lambda k)T}, T; X, r, \sigma_1^2\right)$$

which is equation B.1.1 with the spot exchange rate S multiplied by $e^{-\lambda kT}$. This term accounts for the devaluation premium — the instantaneous expected return on domestic currency equals its return conditional on no devaluation *minus* λk.

The probability density of a devaluation at time t is $\lambda e^{-\lambda t}$. Conditional on that, the value of a call option is:

$$BS\left(Se^{-qT-\lambda kt}(1+k), T; X, r, \frac{(\sigma_1^2 t + \sigma_2^2(T-t))}{T}\right)$$

The exchange rate in this case is distributed as if it followed a regular Brownian motion starting from $Se^{-qT-\lambda kt}(1+k)$ and with volatility $\frac{(\sigma_1^2 t + \sigma_2^2(T-t))}{T}$. The spot exchange rate needs to be corrected by the jump (multiplied by $(1+k)$) and by the devaluation premium up to time t (multiplied by $e^{-\lambda kt}$). The volatility is just a weighted average of the variances in the 2 regimes.

The second term of equation 3.1 integrates the products of prices and probability densities.

B.1.2 Theoretical option price if λ varies

Although the model assumes a fixed hazard rate λ, our estimations do not impose such constraint. So, how different would be theoretical option prices if λ was allowed to vary?

The answer may depend on the underlying process for λ. Monte Carlo simulations were used to approximate option prices for a particular case, when the hazard rate λ behaves according to the following equation:

$$d\log(\lambda) = \sigma_\lambda.dX$$

The table below shows the prices of options with 0.2 year to maturity for different σ_λ's but same expected λ after 0.1 year[1].

The lack of sensitivity to σ_λ is not due to little volatility. If $\sigma_\lambda = 0.5$ and $\lambda(t = 0) = 0.1975$, $E(\lambda|t = 0.10) = 0.20$ but the 95% confidence interval for $\lambda(t = 0.20)$ is wide: $[0.127, 0.306]$ — λ varies significantly in the 0.2-year period.

The results show that, at least for this particular case, changes in the standard deviation of the diffusion process for λ have no impact on option prices. This example seems to confirm our intuition

---

[1]Some simplifications were made to reduce computational cost of this exercise, so all prices are probably slightly overestimated. The parameters used were: $\sigma_1 = .01, \sigma_2 = .10; k = .20, S = 1000, X = 1100, \tau = .20, R = .22, B = .11$.

$E(\lambda\|t = 0.10)$	$\sigma_\lambda = 0$	$\sigma_\lambda = 0.5$
0.15	3.499 (0.008)	3.487 (0.010)
0.20	4.602 (0.009)	4.600 (0.008)
0.25	5.651 (0.009)	5.664 (0.009)

that the estimates of λ obtained in this work should be close to what agents perceived as an average hazard rate.

B.2   The Data

Table B.1 shows the data for the last week in October-1997 and the first week in November-1997. All information refers to contracts with maturity at the last day of November. The data contains 695 rows like the 10 presented in table B.1.

Table B.1: A subset of the data

Day	X						F	S	τ	DI
	1113	1115	1120	1150	1170	1200				
10/27		2.25	2.20	1.40	1.50	1.30	1115.8	1102.7	32	97958
10/28		3.50	3.50	1.40	2.00	2.10	1116.9	1106.4	31	97841
10/29		3.00	4.50		2.00	2.20	1118.2	1102.4	30	97746
10/30	12.00	11.00	12.50	5.00	5.00	5.00	1125.8	1106.3	29	97473
10/31	11.00	7.00	11.00	4.00	4.50	3.30	1124.9	1103.1	28	97056
11/03		7.00	8.00	5.51	4.50	3.00	1121.6	1103.0	25	97123
11/04	5.50	5.50	6.40	3.51	3.49	2.00	1116.9	1104.1	24	97338
11/05	4.30	3.50	3.25	2.60	1.75	2.00	1118.1	1104.1	23	97402
11/06	8.00	7.00	6.00	4.30	2.70	3.00	1118.4	1106.9	22	97541
11/07	13.70	10.50	11.00	7.50	8.00	8.00	1123.5	1108.2	21	97392

The first column shows the trading day. Columns 2 to 7 show the prices of options with strike price shown at the first line of the table: for example, at 10/27, options that give its holder the

right to buy US$1000 for BR$1115 were traded at price BR$2.25. F denotes the future exchange rate: at 10/27, US$1000 at the last day of November were priced at BR$1115.80. S is the spot exchange rate: at 10/27, US$1000 cost BR$1102.70. τ in this table is just the number of days until maturity and DI is an interest rate derivative contract: at 10/27, BR$100,000 at the first day of December were worth BR$97,958. The information on future contracts of interest rate and exchange rate allows us to calculate interest rates denominated in domestic and foreign currency.

The peg was not abandoned in November-1997, so, at the maturity date of those options, the exchange rate was BR$1109 for US$1000 and all options shown at table B.1 were worth 0.

Option prices present huge daily variations, which suggests that large intra-day fluctuations may also occur. As the data refers to options traded in potentially different times, this may bring severe measurement error to the dependent variable of equation 3.2.3. In an extreme example, at 10/31/97, the price of a call with strike 1115 (Reais/US$1000) and maturity 12/01/97 is 7.00 and a call with strike 1120 and same maturity costs 11.00. The sum of the absolute measurement error is therefore greater than 4.00! There are plenty of examples like this, less dramatic though.

The price of a call option must be decreasing in the strike price (otherwise, there would be 'free lunch' in the market). Violation of such property is evidence of noise in the data for option prices, probably due to trades being realized at different times. As shown in table B.2, in 30% of the days in our sample (and in more than half of the days when there are 7 or more strike classes traded), the price of a call option is not strictly decreasing in the strike price.

Table B.2: Violation of monotonicity in the sample

points in a 'day'	4	5	6	7	8	9	total	%
number of 'days'	236	228	146	57	24	4	695	100.0
no (strict) monotonicity	19	44	42	20	15	3	143	20.6
no (weak) monotonicity	37	70	59	26	19	4	215	30.9

B.3 More on the estimation

This appendix starts presenting evidence that our results correspond to the global minimum of the residual squared sum. Then, it answers the question about robustness of the results: could small changes in the model or in the estimation procedure alter the main conclusions? It will be shown that the answer is no: all the conclusions of the essay survive a deeper scrutiny. Then, it is presented an attempt to deal with the measurement error in the dependent variable using FGLS. Al last, the computational work is briefly described.

B.3.1 Local × global minimum

This subsection argues that we have strong reasons to believe that the residual squared sum minimized in this essay is well approximated by a quasi-convex function and so, if there are other points of minimum, they are very close to the ones presented in this essay. It considers the case when λ is allowed to vary across days and maturities, k is constrained to be constant within a month and σ_1 and σ_2 don't change during the whole period.

Let i index months $(i = 1 \ldots m)$, j index days $(j(i) = 1 \ldots n(i))$ and l index every single option traded $(l = 1 \ldots m(j(i)))$. Note that:

$$Min_{\sigma_1,\sigma_2,k_1,\ldots,k_m,\lambda_1,\ldots,\lambda_N} \sum_{i=1}^{m} \sum_{j=1}^{n(i)} \sum_{l=1}^{x(n(i))} e^2_{l(j(i))}$$

$$=$$

$$Min_{\sigma_1,\sigma_2} \sum_{i=1}^{m} Min_{k_i} \sum_{j=1}^{n(i)} Min_{\lambda_{j(i)}} \sum_{l=1}^{x(j(i))} e^2_{l(j(i))}$$

That means, our optimization problem is equivalent to 3 nested minimizations. For checking the properties of the objective functions, it is easier to check the properties of the above 3 functions than to work with the original 722 variables.

We begin asking whether the residual squared sum (RSS) is likely to be convex or quasi-convex on λ. Taking the optimum values of σ_1, σ_2 and k as given, the RSS is calculated for values of λ in two grids:

1. $\lambda = [0.80, 0.82, \ldots, 1.18, 1.20] \times \lambda^*$

2. $\lambda = [0.980, 0.984, \ldots, 1.016, 1.020] \times \lambda^*$

where λ^* is the optimum value of λ.

It is checked if it is possible to reject: convexity; quasi-convexity; and if the point found is really a point of minimum. Taking every consecutive 3 points, convexity is rejected if the average value of the function in the extreme points is lower than its value in the middle point. Quasi-convexity is rejected if the function presents any decrease after any increase.

There are 695 λ's and, therefore, 695 tests performed. 692 were successful, 2 of the rejections appear only in the finer grid and the respective λ's are small, and the other occurs for λ smaller than 2%. The results lead to the conclusion that the residual squared sum is approximately convex in λ.

Then, it is investigated the behavior of RSS as a function of k, taking σ_1 and σ_2 as given and finding the optimal λ's for every k. So, for each k in the grids, several λ's need to be found. The tests are performed as above and the grids are:

1. $k = [0.2, 0.3, \ldots, 2.9, 3] \times k^*$

2. $k = [0.80, 0.82, \ldots, 1.18, 1.20] \times k^*$

3. $k = [0.980, 0.984, \ldots, 1.016, 1.020] \times k^*$

For all 25 k's, no failure were reported for grids 2 and 3. In the case of grid 1, there were 24 rejections of convexity, and no other failure. This result doesn't seem to a problem though. The shape of the right side of all RSS functions looks a bit like an upside-down bell, but such non-convexity, far from the minimum, doesn't matter for our purposes.

At last, the properties of the RSS with respect to σ_1 and σ_2 were also examined. The chosen grids were:

1. $\sigma_1 = [-0.2, -0.1, 0, 0.1, 0.2] + \sigma_1^*$

2. $\sigma_2 = [-0.6, -0.5, \ldots, 0.5, 0.6] + \sigma_2^*$

For each pair of (σ_1, σ_2), all k's and λ's were estimated and the RSS was calculated. In this case, it was possible to test for convexity and quasi-convexity in all directions, varying σ_1, σ_2 or both. No failure was detected.

B.3.2   Robustness

The main conclusions of the essay are not affected by changes in the estimation or small modifications in the model. In particular, k is very stable and virtually uncorrelated to the 'devaluation premium'; k is greater in 98 than in 97, but too small if compared to the observed jump; and λ, very volatile, is mostly affected by foreign crises. Moreover, the 'devaluation premium' measures always display the same pattern and the estimates of σ_1 are stable, only the results for σ_2 are not robust.

If a log-normal jump is included in the model, the formula for the price of a call changes slightly with the inclusion of the variance of the jump size, σ_j, in the integrand:

$$C^{mod} = e^{-\lambda T} BS\left(Se^{(-q-\lambda k)T}, T; X, r, \sigma_1^2\right) +$$
$$\int_0^T \lambda e^{-\lambda t} BS\left(Se^{-qT-\lambda kt}(1+k), T; X, r, \frac{(\sigma_1^2 t + \sigma_2^2(T-t) + \sigma_j^2)}{T}\right) dt$$

However, the available data doesn't allow us to identify σ_2 from σ_j. Both parameters capture the fact that the jump size is not known by the agents and none is specially successful in describing the shape of this uncertainty. So, in virtually all of the attempted estimations, one of them is not significantly different from zero. The results when σ_j is included in the model and $\sigma_2 = 0$ are available upon request. The main difference is that the estimates for k are, on average, 16% smaller. Estimating the model with either σ_2 or σ_j fixed at any reasonable value doesn't lead to any different conclusions.

In our estimates, each λ is related to a given day *and* a given maturity date. When we estimate just one λ per day, in spite of having options with different maturity dates in the sample, the estimate of σ_2 jumps to around 70%, while k's and λ's go down. The parameter σ_2 is in fact capturing an increase in the risk as the time to maturity goes up - which means that λ is not perceived as a constant parameter.[2] Even in this case, the main conclusions of this essay remain untouched.

B.3.3   FGLS estimation

It was pointed in the main text that asynchronous data could bring measurement error to our dependent variable. The variance of the error term is likely to depend in a large extent on the day

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[2]Indeed, the model suggests that λ should be increasing in time.

the option was traded because such measurement errors are stronger when there is higher intra-day fluctuations of prices.

One way to deal with that is to assume that the error term (ϵ_i) can be decomposed in a random error (μ_i) and a measurement error (ν_i), independent of the former:

$$\epsilon_i = \mu_i + \nu_i$$

such that:

$$var\,(\mu_i) = s_\mu$$

$$var\,(\nu_i) = s_\nu(t)$$

where i indexes options and t is an index for days.

The model is first estimated by non-linear least squares. Then, s_μ and $s_\nu(t)$ are found and the model is re-estimated by weighted least squares. This step is repeated until convergence. To get s_μ and $s_\nu(t)$, the residual squared sum is computed for each day, divided by s_μ and compared to the corresponding value of a $\chi^2_{1\%}$. Any excess is attributed to measurement error[3].

The obvious drawback of this approach is the assumption that big deviations from the model are due to bad data and not to a bad model. Although it is not a reliable way to test a model, it allows us to compare results and check robustness.

The estimate of σ_2 is smaller in this case ($\sigma_2 = 23.9\%$), and standard deviations of estimators are obviously smaller. The estimates of k and λ present no significative difference and are available upon request.

B.3.4 Details on the computational work

Matlab was the software used for estimations. Gauss-Newton was the optimization algorithm and user supplied gradients were found to reduce substantially the time for estimations. Numerical integrals were solved using adaptive Simpson quadrature.

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[3]So, s_μ is found as a fixed point.

Appendix C

IMF, catalytic finance and moral hazard

C.1   Multiple equilibrium benchmark

The coordination effect that is captured by global-game models cannot be accounted for by models of international lender of last resort after Diamond and Dybvig [1983]. This framework assumes common knowledge of the signal, and in equilibrium each agent conditions his choice on the speculative position taken by all other agents in the economy. Because of this strong assumption, multiple equilibria are possible for intermediate range of fundamentals, whereas a country defaults if a speculative run occurs, is solvent otherwise. In this range, a debt crisis occurs when each fund manager believes (i.e., knows) that all other fund managers will also refuse to roll over their debt. There is no heterogeneity in managers' action: either everybody attacks (i.e., $x = 1$), or nobody attacks (i.e., $x = 0$). In this appendix we study a version of our model without private information — in the tradition of the bank-run literature stressing multiple equilibria and self-fulfilling runs. Without private information, IMF interventions of limited size can lower, at the margin, the break-even rate of return conditional on the whole market denying credit (i.e., $x = 1$), but not the break-even rate conditional on nobody withdrawing their funds ($x = 0$). Most important, they are irrelevant in determining whether the market rollover or withdraw loans (i.e., whether $x = 0$ or $x = 1$) — see the analysis in Corsetti Pesenti and Roubini (2002) — unless the IMF has enough resources to bailout

the country completely.

Assume that in our model all investors and the IMF have complete information about R. We are interested in understanding the main features of this economy, and identify the values of the fundamental R corresponding to which the equilibrium is not unique. The equilibrium is characterized by the following partition of R. Suppose that L is small relative to the external financing gap of the country. If $R < R_s$, the country is insolvent and there is a unique equilibrium with $x = 1$ and a crisis. If $R > Max \left[R_s, R_s(1 + \kappa) - \kappa \frac{L}{I} \right]$ the country has enough liquidity to pay everybody and there is a unique equilibrium with $x = 0$ and no crisis. If $R_s < R < R_s(1 + \kappa) - \kappa \frac{L}{I}$, there are 2 equilibria, depending on whether investors attack the country or keep lending to it. Note that, when the IMF has sufficiently large resources relative to the next stock of short-term liability of the country, the equilibrium is unique: a crisis occurs if and only if $R < R_s$.

To see this, consider first an equilibrium with a crisis. In equilibrium, rational investors will all withdraw in the first period ($x = 1$) and the IMF does not intervene. It is easy to see that, in equilibrium, no investor will have an incentive to deviate and lend to the country. In our specification, a single investor of infinitesimal size makes no difference in the amount of resources that the country will have in period 2. So, an individual agent have an incentive to deviate if and only if R is sufficiently high that the country will be left with non-negative resources in the second period, independently of the run. But if the R is large enough, a crisis cannot be an equilibrium outcome, and nobody attacks.

Things are slightly different for the IMF, as this agent is not infinitesimal, i.e., its intervention makes a difference as regards the end-point resources of the country. Namely, the IMF has an incentive to deviate if the country has non-negative resources in the second period *conditional on its intervention*. If all agents withdraw ($x = 1$) and the IMF intervenes, equation (5.2.2) tells us that the country will have non-negative resources in the second period if $R > \bar{R}_{x=1}$, where $\bar{R}_{x=1}$ is given by:

$$\bar{R}_{x=1} I - (1 + \kappa)[D - M - L]_+ = L$$

Now there are two cases to consider, depending on the size of the IMF. If $L < D - M$, which means that the IMF endowment of liquidity in not large enough to fill the whole external financing gap,

we have:

$$\bar{R}_{x=1} = R_s(1+\kappa) - \kappa\frac{L}{I}$$

Conversely, if $L > D - M$, the IMF can solve the liquidity problem by itself, and we have

$$\bar{R}_{x=1} = R_s$$

Note that $\bar{R}_{x=1}$ depends on L. If L is limited, the IMF can contain the costs of a speculative run over some range of fundamentals. If L is not limited, instead, liquidity is not an issue, and a crisis only occurs if the country is insolvent.

Consider now a scenario without a crisis: rational agents do not withdraw $(x = 0)$ and the IMF intervenes. An equilibrium without crisis is consistent with rational expectations if the return on investment plus the money from the IMF is enough to pay all creditors and the IMF in the second period — in other words, there are no liquidation costs. Nobody has an incentive to deviate if $R > \bar{R}_{x=0}$, which is given by:

$$I\bar{R}_{x=0} + L = D - M + L$$

which yields:

$$\bar{R}_{x=0} = R_s$$

As there are no early withdrawals, the IMF is not saving any liquidation cost, so the equilibrium with no crisis exists if and only if the country is solvent. Note that $\bar{R}_{x=0}$ does not depend on L.

So, according to the multiple-equilibria framework, the IMF can solve the liquidity problem if and only if it has deep pockets. Otherwise, it has a limited impact on the range of fundamentals for which multiple equilibria are possible. Observe that, in equilibrium, either (a) all investors withdraw, the IMF does not intervene and there is a crisis, or (b) all investors roll over their debts, the IMF intervenes and there is no crisis.

In this model, there is no endogenous mechanism of equilibrium selection. Which equilibrium will investors choose? The solution usually adopted by the literature consists in attributing arbitrary probabilities ξ to a sunspot event selecting between equilibria. Note that this solution has many well-known conceptual problems: why would everybody pick the same action? what is the sunspot? But even abstracting from these particular issues, consider the goal of analyzing catalytic finance and moral hazard distortion within the framework sketched above. The sunspot probabilities ξ must

be known ex-ante. The equilibrium would depend crucially on ξ, but not on preference parameters b, c, B, C. Conditional on ξ, the IMF will have an effect only on the threshold $\bar{R}_{x=1}$, but not on the other thresholds. It will play no role in the equilibrium selection (unless this role is posited by assumption). All the above problems are avoided in our formulation.

C.2 Proofs

C.2.1 Uniqueness and existence of equilibrium

We have seen in the main text that the equilibrium value of s^* is determined by the following equation:

$$\frac{b}{b+c} = G(\bar{R}_L - s^*) + \int_{\bar{R}_L}^{\bar{R}} g(R - s^*) \cdot H(s^* - R)dR$$

We want to show that there is a unique value that solves this equation. Define $w = R - s^*$, $\bar{w} = \bar{R} - s^*$, and $\bar{w}_L = \bar{R}_L - s^*$ (where clearly $\bar{w} > \bar{w}_L$). Changing variables in equation (5.3) and using (5.3) we get:

$$G(\bar{w}_L) + \int_{\bar{w}_L}^{\bar{w}} g(w) \cdot H\left(\bar{w}_L - w - H^{-1}\left(\frac{B}{B+C}\right)\right) dw - \frac{b}{b+c} = 0 \qquad \text{(C.0)}$$

Key to our proof is that the RHS of this equation is monotonically increasing in $\bar{w}$ and $\bar{w}_L$, and both $\bar{w}$ and $\bar{w}_L$ in turn are monotonically increasing in s^*. To see why, note that increasing $\bar{w}_L$ is equivalent .... As regards $\bar{w}_L$, substituting (5.3) in the definition of this variable we can write

$$-\bar{w}_L - \frac{\kappa R_S \cdot D}{D - M} G(\bar{w}_L) - s^* + \text{constant} = 0$$

Differentiating

$$\frac{\partial \bar{w}_L}{\partial s^*} = \frac{1}{1 + \dfrac{\kappa R_S \cdot D}{D - M} g(\bar{w}_L)} > 0$$

By the same token

$$\frac{\partial \bar{w}}{\partial s^*} = \frac{1}{1 + \dfrac{\kappa R_S \cdot D}{D - M} g(\bar{w})} > 0$$

just as in CDMS. Thus, for sufficiently large s^* the LHS of (C.2.1) is positive, while it is negative for sufficiently small s^*. Since the LHS is continuous in s^*, there is a unique solution to (C.2.1). Once s^* is uniquely determined, S^* follows from (5.3).

C.2.2   Proof of proposition 2

This appendix proves proposition 1. Differentiating equations (5.3) and (5.3) and rearranging, we get:

$$\frac{ds^*}{dL} = \left(1 + \frac{1 - M/D}{R_s \cdot \kappa \cdot g(s^* - \bar{R})}\right) \cdot \frac{d\bar{R}}{dL} \tag{C.0}$$

$$\frac{ds^*}{dL} = \left(1 + \frac{1 - M/D}{R_s \cdot \kappa \cdot g(s^* - \bar{R}_L)}\right) \cdot \frac{d\bar{R}_L}{dL} + \frac{1}{g(s^* - \bar{R}_L)} \tag{C.0}$$

To ease notation, define ζ_1 and ζ_2 as follows

$$\zeta_1 = \left(1 + \frac{1 - M/D}{R_s \cdot \kappa \cdot g(s^* - \bar{R})}\right)^{-1}$$

$$\zeta_2 = \left(1 + \frac{1 - M/D}{R_s \cdot \kappa \cdot g(s^* - \bar{R}_L)}\right)^{-1}$$

Note that $\zeta_1, \zeta_2 \in (0, 1)$.

Now, define $w = R - s^*$, $\bar{w} = \bar{R} - s^*$, and $\bar{w}_L = \bar{R}_L - s^*$. Using (C.2.2) and (C.2.2) we have:

$$\frac{d\bar{w}}{dL} = -(1 - \zeta_1)\frac{ds^*}{dL} \tag{C.0}$$

$$\frac{d\bar{w}_L}{dL} = -(1 - \zeta_2)\frac{ds^*}{dL} - \frac{\zeta_2}{g(\bar{w}_L)} \tag{C.0}$$

Changing variables in equation (5.3) and using (5.3) we get:

$$\frac{b}{b+c} = G(\bar{w}_L) + \int_{\bar{w}_L}^{\bar{w}} g(w).H\left(\bar{w}_L - w - H^{-1}\left(\frac{B}{B+C}\right)\right) dw \tag{C.0}$$

Differentiating C.2.2 and rearranging terms:

$$\frac{d\bar{w}}{dL}\zeta_3 + \frac{d\bar{w}_L}{dL}\zeta_4 = 0$$

where:

$$\zeta_3 = g(\bar{w}) \, H\left(\bar{w}_L - \bar{w} - H^{-1}\left(\frac{B}{B+C}\right)\right) > 0$$

$$\zeta_4 = g(\bar{w}_L)\left(\frac{B}{B+C}\right) + \int_{\bar{w}_L}^{\bar{w}} g(w)\, h\left(\bar{w}_L - w - H^{-1}\left(\frac{B}{B+C}\right)\right) \, dw > 0$$

This yields:

$$\frac{ds^*}{dL} = -\frac{\zeta_2\zeta_4}{g(\bar{w}_L)\left[(1 - \zeta_1)\zeta_3 + (1 - \zeta_2)\zeta_4\right]} < 0$$

Using (C.2.2), (C.2.2) and (5.3) we get that:

$$\frac{d\bar{R}}{dL} < 0 \,, \ \frac{d\bar{R}_L}{dL} < 0 \text{ and } \frac{dS^*}{dL} < 0$$

which completes the proof.

C.2.3 Proof of Proposition 3

Let Φ be the standard normal distribution. Then, equation (5.3) can be written as:

$$\Phi\left(\sqrt{\beta+\rho}\left(S^* - \bar{R}_L\right)\right) = \frac{B}{B+C}$$

Differentiating with respect to the precision of IMF information (β), we get:

$$\phi\left(\sqrt{\beta+\rho}\left(S^* - \bar{R}_L\right)\right) \cdot \left[\sqrt{\beta+\rho}\left(\frac{dS^*}{d\beta} - \frac{d\bar{R}_L}{d\beta}\right) + \frac{S^* - \bar{R}_L}{2\sqrt{\beta+\rho}}\right]$$

Defining $w_S^* = S^* - s^*$, using $\bar{w}_L$ as defined above and rearranging, we obtain:

$$\sqrt{\beta+\rho}\,\frac{dw_S^*}{d\beta} = \sqrt{\beta+\rho}\,\frac{d\bar{w}_L}{d\beta} - \frac{(w_S^* - \bar{w}_L)}{2\sqrt{\beta+\rho}} \tag{C.0}$$

Moreover, as above:

$$\frac{d\bar{R}}{d\beta} = \zeta_1 \frac{ds^*}{d\beta}$$

$$\frac{d\bar{R}_L}{d\beta} = \zeta_2 \frac{ds^*}{d\beta}$$

So:

$$\frac{d\bar{w}}{d\beta} = -(1 - \zeta_1)\frac{ds^*}{d\beta} \tag{C.0}$$

$$\frac{d\bar{w}_L}{d\beta} = -(1 - \zeta_2)\frac{ds^*}{d\beta} \tag{C.0}$$

Differentiating (5.3), using (C.2.3), (C.2.3), (C.2.3) and rearranging, we get:

$$\frac{ds^*}{d\beta} = \frac{\int_{\bar{w}_L}^{\bar{w}} g(w)\, h\left(w_S^* - w\right)(\bar{w}_L - w)\, dw}{2\sqrt{\beta+\rho}\left[(1 - \zeta_1)\zeta_3 + (1 - \zeta_2)\zeta_5\right]} < 0$$

where

$$\zeta_5 = g(\bar{w}_L)\left(\frac{B}{B+C}\right) + \int_{\bar{w}_L}^{\bar{w}} g(w)\, h\left(\bar{w}_L - w - H^{-1}\left(\frac{B}{B+C}\right)\right)\sqrt{\beta+\rho}\,dw > 0$$

Finally, using (C.2.3), (C.2.3), (C.2.3), we obtain:

$$\frac{d\bar{R}_L}{d\beta},\ \frac{d\bar{R}}{d\beta},\ \frac{dS^*}{d\beta} < 0$$

which concludes our proof.

C.2.4 Proof of Proposition 4

Here, we show that $s*$, $S*$, $\bar{R}$ and $\bar{R}_L$ depend negatively on b and positively on c. As above, we get that:

$$\frac{d\bar{R}}{db} = \zeta_1 \frac{ds^*}{db} \tag{C.0}$$

$$\frac{d\bar{R}_L}{db} = \zeta_2 \frac{ds^*}{db} \tag{C.0}$$

Changing variables in the same way, we get that:

$$\frac{ds^*}{db} = -\frac{c}{(b+c)^2} \frac{1}{\zeta_3(1-\zeta_1) + \zeta_4(1-\zeta_2)} < 0$$

Using (C.2.4), (C.2.4) and (5.3) we get that:

$$\frac{d\bar{R}}{db} < 0 \ , \ \frac{d\bar{R}_L}{db} < 0 \text{ and } \frac{dS^*}{db} < 0$$

which completes the proof.

Analogously,

$$\frac{ds^*}{dc} = \frac{b}{(b+c)^2} \frac{1}{\zeta_3(1-\zeta_1) + \zeta_4(1-\zeta_2)} > 0$$

and also:

$$\frac{d\bar{R}}{dc} > 0 \ , \ \frac{d\bar{R}_L}{dc} > 0 \text{ and } \frac{dS^*}{dc} > 0$$

C.2.5 Liquidity provision with no 'limited liability'

Assume that the government cares about GDP — and that there are additional resources in the economy to pay international investors. We have:

$$\lim_{\alpha \to \infty} W(A) = \int_0^{\bar{R}_L} \left[\frac{R \cdot I}{1 + \kappa} + M - D \right] f(R \mid R_A) \, dR$$

$$+ \int_{\bar{R}_L}^{\infty} [R \cdot I + M - D] \ f(R \mid R_A) \, dR \ - \ \Psi$$

$$\lim_{\alpha \to \infty} W(N) = \int_0^{\bar{R}_L} \left[\frac{R \cdot I}{1 + \kappa} + M - D \right] f(R \mid R_N) \, dR$$

$$+ \int_{\bar{R}_L}^{\infty} [R \cdot I + M - D] \ f(R \mid R_N) \, dR$$

The difference in government's welfare is:

$$\Delta W = \int_0^{\bar{R}_L} \left[\frac{R \cdot I}{1 + \kappa} + M - D \right] [f(R \mid R_A) - f(R \mid R_N)] \, dR$$

$$+ \int_{\bar{R}_L}^{\infty} [R \cdot I + M - D] \ [f(R \mid R_A) - f(R \mid R_N)] \, dR \ - \ \Psi$$

The marginal effect of a change in $\bar{R}_L$ is:

$$\frac{d\left(\Delta W\right)}{d\bar{R}_L} = \bar{R}_L I \cdot \frac{\kappa}{1+\kappa}\left[f\left(\bar{R}_L \mid R_N\right) - f\left(\bar{R}_L \mid R_A\right)\right]$$

Therefore, the sign of the marginal effect of L in this case is the same as in equation (5.5.1).

Bibliography

[1] Abreu, Dilip and Markus Brunnermeier, 2003, "Bubbles and crashes", *Econometrica* 71, 173-204.

[2] Allen, Franklin and Douglas Gale, 2000, "Financial Contagion," *Journal of Political Economy* 108, 1-33.

[3] Allen, Franklin and Douglas Gale, 2000, "Optimal Currency Crises", unpublished mimeo.

[4] Allen, Franklin and Douglas Gale, 2001, "Comparative Financial Systems", MIT Press.

[5] Angeletos, George-Marios, Christian Hellwig and Alessandro Pavan, 2003, "Coordination and policy traps", mimeo.

[6] Bacchetta, Philippe and Eric Van Wincopp, 2003, "Can information heterogeneity explain the exchange rate determination puzzle?", mimeo.

[7] Baig, Taimur and Ilan Goldfajn, 2001, "The Russian default and the contagion to Brazil". In: Claessens, Stijn and Forbes, Kristin (eds.), *International Financial Contagion*, Kluwer.

[8] Bakshi, Gurdip, Charles Cao and Zhivu Chen, 1997, "Empirical performance of alternative option pricing models", *Journal of Finance* 52, 2003-2049.

[9] Bates, David S., 1991, "The crash of '87: Was it expected? The Evidence from Options Markets", *Journal of Finance* 46, 1009-1044.

[10] Bates, David S., 1996, "Jumps and stochastic volatility: Exchange rate processes implicit in Deutsche Mark options", *Review of Financial Studies* 9, 69-108.

[11] Bergin, Paul, 2003, "Putting the New Open Economy Macroeconomics to a test", *Journal of International Economics* 60, 3-34.

[12] Broner, Fernando, 2002, "On the timing of balance of payment crises: disaggregated information and interest rate policy", mimeo, University of Maryland.

[13] Bryant, John, 1980, "A Model of Reserves, Bank Runs, and Deposit Insurance", *Journal of Banking and Finance*, 4, pp.335-44.

[14] Bulow, Jeremy and Kenneth Rogoff, 1989, "Sovereign Debt: Is to Forgive to Forget?" *American Economic Review* 79, No.1.

[15] Burdzy, Krzysztof, David Frankel and Ady Pauzner, 2001, "Fast equilibrium selection by rational players living in a changing world", *Econometrica* 69, 163-189.

[16] Burdzy, Krzysztof and David Frankel, 2002, "Equilibrium selection in dynamic models with seasonal and mean-reverting shocks", mimeo.

[17] Calvo, Guillermo, 1983, "Staggered prices in a utility-maximizing framework", *Journal of Monetary Economics* 12, 383-398.

[18] Calvo, Guillermo and Enrique Mendoza, 2000, "Rational contagion and the globalization of securities markets", *Journal of International Economics* 51, 79-113.

[19] Campa, José M., P. H. Kevin Chang and James F. Refalo, 2002, "An options-based analysis of emerging market exchange rate expectations: Brazil's Real plan, 1994 - 1997", *Journal of Development Economics* 69, 227-253.

[20] Carlsson, Hans, and Eric van Damme, 1993, "Global Games and Equilibrium Selection," *Econometrica* 61, 989-1018.

[21] Chamley, Christophe, 2003, "Dynamic speculative attacks", *American Economic Review* 93, 603-621.

[22] Chang, Roberto and Andres Velasco, 1999, "Liquidity Crises in Emerging Markets: Theory and Policy", *NBER Macroeconomic Annual* 11-58, MIT Press.

[23] Cole, Harold and Patrick Kehoe, 1998, "Models of Sovereign Debt: Partial Versus General Reputations", *International Economic Review* 39, 55-70.

[24] Corsetti, Giancarlo, Amil Dasgupta, Stephen Morris and Hyun Song Shin, 2003, "Does one Soros make a difference? A theory of currency crises with large and small traders", *Review of Economic Studies* 71, 87-114.

[25] Corsetti Giancarlo, Paolo Pesenti and Nouriel Roubini, 2002, "The role of large traders in currency crises", in "Preventing currency crises in emerging markets" edited by S. Edwards and J. Frankel, University of Chicago Press.

[26] Cottarelli, Carlo and Curzio Giannini, 2002, "Bedfellows, Hostages, or Perfect Strangers? Global Capital Markets and the Catalytic Effect of IMF Crisis Lending", mimeo.

[27] Diamond, Douglas and Philip Dybvig, 1983, "Bank Runs, Deposit Insurance, and Liquidity", *Journal of Political Economy* 91, 401-419.

[28] Dooley, Michael and Sujata Verma, 2001, "Rescue Packages and Output Losses Following Crises," *NBER Working Paper* 8315.

[29] Eichengreen, Barry, 1999, "Toward a New International Financial Architecture Institute for International Economics", mimeo.

[30] Eichengreen, Barry, James Tobin, and Charles Wyplosz, 1995, "Two cases for sand in the wheels of international finance", *Economic Journal* 105, 162-172.

[31] Fischer, Stanley, 1999, "On the Need for an International Lender of Last Resort", unpublished mimeo (http://www.imf.org/external/np/speeches/1999/010399.htm).

[32] Flood, Robert and Peter Garber, 1984, "Collapsing exchange-rate regimes: some linear examples", *Journal of International Economics* 17, 1-13.

[33] Frankel, David, Stephen Morris, and Ady Pauzner, 2003, "Equilibrium selection in global games with strategic complementarities", *Journal of Economic Theory* 108, 1-44.

[34] Frankel, David and Ady Pauzner, 2000, "Resolving indeterminacy in dynamic settings: the role of shocks", *Quarterly Journal of Economics* 115, 283-304.

[35] Frankel, Jeffrey and Nouriel Roubini, 2001, "The Role of Industrial Country Policies in Emerging Market Crises," NBER Working Paper 8634.

[36] Furman, Jason and Joseph Stiglitz, 1998, "Economic crisis: evidence and insights from East Asia", *Brookings Papers on Economic Activity* 1998, n.2, 1-114.

[37] Gai, Prasanna, Simon Hayes and Hyun Song Shin, 2001, "Crisis costs and debtor discipline: the efficacy of public policy in sovereign debt crises", Bank of England Working Paper 136 (http://www.bankofengland.co.uk/workingpapers/wp136.pdf).

[38] Gale, Douglas and Xavier Vives, 2001, "Dollarization, Bailouts and the Stability of the Banking System", mimeo.

[39] Garber, Peter, 1999, "Comment on 'Balance sheets, the transfer problem, and financial crises' ", in *International finance and financial crises : essays in honor of Robert P. Flood, Jr.*, edited by Peter Isard, Assaf Razin and Andrew K. Rose.

[40] Ghosal, Sayantan and Marcus Miller, 2002, "Co-ordination Failure, Moral Hazard and Sovereign Bankruptcy Procedures," Bank Of England Conference on: The Role Of The Official And Private Sectors In Resolving International Financial Crises, Bank Of England, July.

[41] Goldstein, Itay and Ady Pauzner, 2001, "Contagion of self-fulfilling financial crises due to diversification of investment portfolios", mimeo.

[42] Goldstein Itay and Ady Pauzner, 2002, "Demand Deposit Contracts and The Probability of Bank Runs", mimeo

[43] Gollier, Christian, 2001, *The Economics of Risk and Time*, MIT Press.

[44] Gorton, Gary, 1988, "Banking Panics and Business Cycles", *Oxford Economic Papers* 40, 751-781.

[45] Guimarães, Bernardo and Marcos E. Silva, 2002, "A possibilidade de saltos no câmbio implícita nos preços das opções", *Revista Brasileira de Economia* 56, 397-428.

[46] Haldane, Andrew, Simon Hayes, Adrian Penalver, Victoria Saporta and Hyun Song Shin, 2002, "Binding-in the Private Sector", unpublished, Bank of England, August (http://www.bankofengland.co.uk/conferences/conf0207/bindinginps.pdf).

[47] Heinemann, Frank and Gerhard Illing, 2002, "Speculative attacks: unique equilibrium and transparency", *Journal of International Economics* 58, 429-450.

[48] Hellwig, Christian, 2002, "Public information, provate information and the multiplicity of equilibria in coordination games" *Journal of Economic Theory* 107, 191-222.

[49] IPEA, 1998, Carta de Conjuntura 85, december/98.

[50] Jacklin, Charles and Sudipto Bhattacharya, 1988, "Distinguishing Panics and Information-based Bank Runs: Welfare and Policy Implications," *Journal of Political Economy* 96, 568-592.

[51] Jeanne, Olivier, 1997, "Are currency crises self-fulfilling? A test", *Journal of International Economics* 43, 263-286.

[52] Jeanne, Olivier, 1999, "Sovereign Debt Crises and the Global Financial Architecture." Unpublished manuscript.

[53] Jeanne, Olivier, 2000, "Foreign Currency Debt and the Global Financial Architecture," *European Economic Review* 44, 719-727.

[54] Jeanne, Olivier and Paul Masson, 2000, "Currency crises, sunspots and Markov-switching regimes", *Journal of International Economics* 50, 327-350.

[55] Jeanne, Olivier and Charles Wyplosz, 2001, "The International Lender of Last Resort: How Large is Large Enough?", *NBER Working Paper* 8381.

[56] Jeanne, Olivier and Jeromin Zettelmeyer, 2001, "International Bailouts, Moral Hazard, and Conditionality," *Economic Policy* 16, issue No 33, October.

[57] Jondeau, Eric and Michael Rockinger, 2000, "Reading the smile: the message conveyed by methods which infer risk neutral densities", *Journal of International Money and Finance* 19, 885-915.

[58] Krueger, Anne, 2001a, "International Financial Architecture for 2002: A New Approach to Sovereign Debt Restructuring," IMF (http://www.imf.org/external/np/speeches/2001/112601.htm).

[59] Krueger, Anne, 2001b, "A New Approach to Sovereign Debt Restructuring," IMF, (http://www.imf.org/external/np/speeches/2001/122001.htm).

[60] Krueger, Anne, 2002, "New Approaches to Sovereign Debt Restructuring: An Update on Our Thinking," Conference on "Sovereign Debt Workouts: Hopes and Hazards" Institute for International Economics, Washington DC, April 1, 2002 (http://www.imf.org/external/np/speeches/2002/040102.htm).

[61] Krugman, Paul, 1979, "A model of balance-of-payments crises", *Journal of Money, Credit and Banking* 11, 311-325.

[62] Kumar, Manmohan, Paul Masson, and Marcus Miller, 2000, "Global Financial Crises: Institutions and Incentives," IMF Working Paper 00/105.

[63] Kyle, Albert and Wei Xiong, 2001, "Contagion as a Wealth Effect", *Journal of Finance* 56, 1401-1440.

[64] Meese, Richard and Kenneth Rogoff, 1983, "Empirical exchange rate models of the seventies: do they fit out of sample?", *Journal of International Economics* 14, 3-24.

[65] Meltzer Commission, 2000, Report of the International Financial Institutions Advisory Commission ("Meltzer Commission"), Washington D.C.

[66] Merton, Robert C., 1976, "Option pricing when underlying stock returns are discontinuous", *Journal of Financial Economics* 3, 125-144.

[67] Mody, Ashoka and Diego Saravia, "Catalyzing Capital Flows: Do IMF-Supported Programs Work as Commitment Devices?", 2003, mimeo.

[68] Morris, Stephen and Hyung Song Shin, 1998, "Unique equilibrium in a model of self-fulfilling currency attacks", *American Economic Review* 88, 587-597.

[69] Morris, Stephen and Hyung Song Shin, 1999, "A theory of the onset of currency attacks", in "Asian financial crisis: causes, contagion and consequences", Agenor, Vines and Weber, Eds.

[70] Morris, Stephen and Hyun Song Shin, 2002a, "Catalytic Finance", mimeo.

[71] Morris, Stephen and Hyung Song Shin, 2002b, "Social value of public information", *American Economic Review* 92.

[72] Morris, Stephen and Shin, Hyun S., 2003, "Global games: theory and applications", in *Advances in Economics and Econometrics (Proceedings of the Eighth World Congress of the Econometric Society)* (M. Dewatripont, L. Hansen and S. Turnovsky, Eds). Cambridge, England: Cambridge University Press.

[73] Morris, Stephen; and Ui, Takashi, 2002, "Generalized potentials and robust sets of equilibria," Cowles Foundation Discussion Paper #1394.

[74] Obstfeld, Maurice, 1996, "Models of currency crises with self-fulfilling features", *European Economic Review* 40, 1037-1047.

[75] Obstfeld, Maurice and Kenneth Rogoff, 1995, "The mirage of fixed exchange rates", *Journal of Economic Perspectives* 9, 73-96.

[76] Radelet, Steven and Jeffrey Sachs, 1998, "The East Asian financial crisis: diagnosis, remedies, prospects", *Brookings Paper on Economic Activity* 28, no. 1, 1-74.

[77] Rangvid, Jesper, 2001, "Second generation models of currency crises", *Journal of Economic Surveys* 15, 613-646.

[78] Revista Época, January 18th, 1999.

[79] Rocha, Katia M. C. and Ajax R. B. Moreira, 1998, "Credibilidade da política cambial e as opções cambiais", *Pesquisa e Planejamento Econômico* 28, 509-529.

[80] Rogoff, Kenneth, 1996, "The purchasing power parity puzzle", *Journal of Economic Literature* 34, 647-668.

[81] Rogoff, Ken and Jeromin Zettelmeyer, 2002, "Early Ideas on Sovereign Bankruptcy Reorganization: A Survey," IMF Working Paper 02/57.

[82] Rochet, Jean-Charles and Xavier Vives, 2002, "Coordination Failure and the lender of last resort: Was Bagehot right after all" mimeo.

[83] Sachs, Jeffrey, 1984, "Theoretical Issues in International Borrowing," Princeton Studies in International Finance 54, Princeton University Press.

[84] Sachs, Jeffrey, 1995, "Do We Need an International Lender of Last Resort," Frank D. Graham Lecture at Princeton University 8, April 20, 1995, unpublished manuscript (http://www.ksg.harvard.edu/cid/ciddirector/publicat.html#Working).

[85] Smart, D. R., 1974, "Fixed point theorems", Cambridge University Press.

[86] Tirole, Jean, 2002, "Financial Crises, Liquidity and the International Monetary System", Princeton University Press.

[87] Wallace, Neil, 1988, "Another Attempt to Explain an Illiquid Banking System: The Diamond and Dybvig Model with Sequential Servicing Taken Seriously", *Federal Reserve Bank of Minneapolis Quarterly Review* 12, No. 4, pp. 3-16.

[88] Zettelmeyer, Jeronim, 1999, "On the Short-Run Effectiveness of Official Crisis Lending", unpublished manuscript.