Essays on the Economics of Self-Control and Lifestyle Brands

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Abstract

Chapter 1 addresses the role of self-set, non-binding goals as a source of internal motivation to attenuate the self-control problem of a hyperbolic discounter. Agents have linear reference-dependent preferences and endogenously set a goal that serves as the reference point. They face an infinite horizon, optimal stopping problem in continuous time. I show that goal-setting attenuates the hyperbolic agent's tendency to stop too early, but too much reference dependence leads an agent to wait longer than the first-best. Extending the model to social comparisons, I find that comparison to increasingly patient peers induces increasingly patient behavior. Nonetheless, every agent prefers to compare himself to a peer with the lowest degree of self-control possible.

Chapter 2 extends the framework developed in Chapter 1 to address the role of goal bracketing to improve a hyperbolic discounter's self-control. When setting non-binding goals in a sequential stopping problem, he also decides how and when to evaluate himself against such goals. He can bracket broadly by setting an aggregate goal for the entire project, or narrowly by setting incremental goals for individual stages. If the agent is sufficiently loss averse and ex-ante uncertainty is high, he will choose to bracket broadly; otherwise, he brackets narrowly despite the disutility from frequent goal evaluation. Whether he stops earlier or later than the first-best depends on the level of ex-ante uncertainty.

Chapter 3 offers an information-based account of the existence of lifestyle brands and analyzes firms' brand investment and pricing choices in a duopoly setting. If agents have uncertainty over their preferences but are aware that these are correlated with those of others, there exists an incentive to communicate and learn from others with similar tastes. When firms can offer branded goods as coordination mechanisms for their customers, they become associated with specific subgroups, forming lifestyle brands. In a duopoly setting, only one firm chooses to invest in a brand. Surpris-

ingly, although total surplus increases as a result of the provision of this mechanism, consumer surplus decreases. Consumers benefit from learning from one another, but all of this surplus (and more) is extracted by the firms through pricing.

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Contents

Chapter 1

Goal-Setting, Social Comparison, and Self-Control

1.1 Introduction

Personal development is a burgeoning, multi-billion dollar industry that focuses on self-improvement on a variety of levels, from career aspirations to lifestyle choices to spiritual well-being. In response to consumer demand, this market offers books, motivational speakers, workshops, and personal coaching, as well as innumerable weight-loss programs. In addition, institutional demand has resulted in consulting, employee training, and employee development programs whose purpose is to raise productivity in both individual and group settings at the workplace. A central tenet of the personal development industry is that goal-setting is a vital instrument for improving one's life in any aspect, including career, health, or lifestyle. For example, the "S.M.A.R.T. Goals" mnemonic is ubiquitous in project management, education, and self-help programs:[1]

1. Specific
2. Measurable
3. Attainable
4. Relevant
5. Timely.

This prescription is supported by extensive empirical evidence in psychology that *non-binding* goals tend to increase effort, attention, persistence (Klein, 1991; Latham and Locke, 1991; Locke and Latham, 2002), and that satisfaction is tied to achievement relative to such a goal, not just final outcomes (Mento et al., 1992).

Although much attention has been paid to the problem of present-biased preferences and intrapersonal conflict by individual decision makers, the role of self-set goals

[1]There exist many variations on this theme, which is generally attributed to Drucker (1954), a project management book.

in attenuating the self-control problem has been relatively unexplored by economists thus far.[2] In standard economic models, only binding goals can affect motivation and behavior. That is, motivation can be achieved through an explicit reward or punishment mechanism with external enforcement. For example, in a principle-agent model, the principal can motivate the agent's effort exertion by prescribing goals if the agent is evaluated according to those goals and compensated accordingly. In individual decision making, present-biased agents can enforce motivation on themselves by using binding precommitments or externally enforced contracts. Examples include alcoholics who take Antabuse to preclude drinking, gamblers who place themselves on casino "do not enter" lists, and shoppers who freeze their credit cards in blocks of ice.

However, the prevalence of non-binding goal-setting, and the success of the personal development industry, strongly suggests that less drastic measures may also serve as successful regulatory mechanisms. For example, individuals who wish to lose weight often set a specific target weight for themselves, and students motivate themselves to write research papers through self-imposed page targets and deadlines.[3] Individuals may also set goals for themselves in situations where there exists an option value of waiting due to the presence of uncertainty. A student will thus continue his education in order to achieve a target level of human capital or starting salary upon graduation. A person saving for retirement may set a target level of accumulated wealth upon retirement. In the marriage market, an individual may set a specified standard of quality for a prospective spouse in order to avoid settling too soon for an inadequate one.

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[2] Bénabou and Tirole (2004) develop a theory of self-enforcing personal rules, operating through self-reputation. Because agents have imperfect information about their own willpower and imperfect recall, they can achieve internal commitment out of fear of creating precedents and losing faith in themselves.

[3] Ariely and Wertenbroch (2002) conduct an experiment in which students set deadlines for themselves. However, these deadlines are a form of binding pre-commitment, since students were externally penalized when these deadlines were not met.

In this paper, I propose that a goal provides internal motivation by acting as the reference point for an agent who possesses reference-dependent preferences. I consider agents who derive utility from both standard consumption, arising from outcomes, and comparison utility, derived from the comparison of these outcomes to a self-set goal. At each point in time, the agent sets a goal based on his expectations about the outcome of future behavior, which his next "self" will inherit as a reference point in his utility function. In this way, the agent provides a degree of internal motivation that is desirable if he exhibits time inconsistency due to present-biased time preferences. Because expectations enter directly into utility through the goal choice, I consider the polar cases of sophistication and naivete about one's self-control problem in the manner of O'Donoghue and Rabin (1999), to study the impact of differing expectations on behavior and welfare. If agents are sophisticated, they can set only realistic objectives for himself, implying that the goal must ultimately coincide with the rational expectation of what he will actually do. If they are naive, they must set objectives that they *perceive* to be realistic.

The economic setting that I consider is the standard optimal stopping, or irreversible investment, problem in continuous time with an infinite horizon, in which there exists an option value of waiting due to uncertainty.[4] Present-biasedness then leads the agent to stop too early because he undervalues this option relative to his time-consistent counterpart. In the preceding examples, this means accumulating too little human capital, retiring with too little wealth, or settling for a mediocre spouse. However, I show that if the agent has reference dependent preferences, even of the simplest, *linear* kind, he can induce more patient behavior by setting a goal to be achieved at the stopping time, regardless of whether he is sophisticated or naive. Formally, I solve for the unique stationary Markov equilibrium of the intrapersonal game and show that greater reference dependence leads to later exercise of the option. The

[4]Dixit and Pindyck (1994) provide a thorough treatment of the irreversible investment problem in a variety of settings.

presence of a goal increases the agent's incentive to wait because he wishes to avoid incurring comparative disutility from falling short of his goal. In fact, for any degree of present-biasedness, there exists a level of reference dependence such that it allows a sophisticated agent to achieve the first best from an ex ante perspective. On the other hand, this implies that too much reference dependence causes the agent to wait *longer* than the first best. In particular, reference dependence always decreases the welfare of a time consistent agent because it causes him to wait longer than the first best, by distorting his incentives at the margin. In other words, goal setting *per se* can itself be a source of intrapersonal conflict. Another key result, which stands in contrast to previous literature and conjectures, is that goal-setting does not require any form of loss aversion or curvature in the comparison utility function to regulate behavior. Moreover, neither loss aversion nor ex-ante uncertainty over outcomes is required for goals to be detrimental to ex-ante welfare. Finally, in contrast to the "sophistication effect" (O'Donoghue and Rabin, 2001) that occurs in the absence of reference dependence, whereby the sophisticate's realistic pessimism makes him worse off than his naive counterpart, the naif can never achieve his first best, which is strictly worse than that of the sophisticate. Because the naif underestimates his present-biasedness, he persistently sets overly ambitious goals for himself and incurs disutility from falling short.

Next, I examine the optimal goal choice when it represents an individual's aspiration, rather than an expectation about what he can or will attain. When the goal is divorced from expectations, there exists a trade-off between consumption and comparison utility. I find that the agent must be sufficiently present-biased and exhibit diminishing sensitivity to gains in order to set a non-degenerate aspirational goal. In this case, an agent with a more severe self-control problem sets a higher aspirational goal. Otherwise, the incentive to maximize comparison utility dominates and he prefers to set the lowest aspirational goal possible.

Finally, I extend the model to social comparisons. In addition to, or instead of, engaging in individual goal-setting, an agent may also look to a peer or role model as a source of comparison. It has long been recognized that people derive utility from comparing their own outcomes, such as wealth, education, and consumption, against those of their peers (Veblen, 1953; Duesenberry, 1949; Frank, 1985). Furthermore, Bandura and Jourden (1991) find that social comparisons affect both individual goal-setting and the interpretation of personal outcomes. I assume that agents are fully aware of their own personal characteristics, but care about "keeping up with the Joneses." Since they are rational and fully informed about one another's characteristics, they must again hold correct expectations regarding everyone's outcomes. Considering first homogeneous peer groups, I show that measuring oneself against a peer with identical characteristics is no different from purely individual goal-setting. Turning next to a heterogeneous group where agents have differing amounts of self-control, I show that comparing oneself with *any* peer improves patience over having no goal at all, even if the peer has an even more severe self-control problem. An agent's patience increases with his peer's degree of self-control; it effectively sets a higher standard to be met, generating a stronger incentive to wait for a higher stopping value. At the same time, the agent with more self-control always exhibits strictly more patience than his more impulsive peer. Nonetheless, every agent prefers to compare himself to a peer with the lowest degree of self-control possible, regardless of the severity of his own self-control problem.

The paper proceeds as follows. Section 1.2 links this paper to related lines of research. Section 1.3 describes the model. Section 1.4 derives and characterizes the stationary Markov equilibrium and discusses the welfare implications of goal-setting when the agent is sophisticated. Section 1.5 derives the equilibrium when the agent is naive and discusses the welfare implications in comparison to the sophisticate. Section 1.6 considers the optimal choice of aspirational goals, when goals do not necessarily coincide with expectations. Section 1.7 extends the model to include social comparisons. Section 1.8 summarizes the results and discusses avenues for future research. Proofs are gathered in the Appendix.

1.2 Related Literature

This paper lies at the intersection of several lines of research. First, it links the work on reference dependence with that on self-control, which have each been studied quite separately, by considering the role of reference dependence preferences as a instrument to countervail a self-control problem arising from quasi-hyperbolic discounting. Secondly, it relates to the literatures on peer effects and social comparisons. Lastly, it analyzes behavior and welfare in the context of optimal stopping under uncertainty, where there exists an option value of waiting.

Goal-setting theory postulates that goals serve as a reference standard in a cognitive comparison process of self-evaluation and satisfaction (Latham and Locke, 1991; Locke and Latham, 2002; Bandura, 1989), while Latham and Locke (1991) argue that goal choice is a combination of what is desired and what is believed to be attainable. Heath et al. (1999) explicitly argue that a goal acts as the reference point in the value function of prospect theory (Kahneman and Tversky, 1979), while Loewenstein (2007) discusses goal-setting as a mechanism for self-control in the context of mountaineering, none of them provide a formal model with endogenous goals. In this paper, I assume that the value function is linear and show that, somewhat surprisingly, goal-setting can be effective without the assumptions of loss aversion or diminishing sensitivity. In parallel and independent work, Suvorov and van de Ven (2008) and Koch and Nafziger (2008) have also recently proposed a model of goals as reference points that serve to remedy self-control. While sharing a similar concept, the papers are quite complementary. Suvorov and van de Ven (2008) consider a three-period problem where costly effort on a task is required to receive a delayed benefit. They assume that a sophisticated, quasi-hyperbolic agent with loss aversion sets a goal regarding both effort and the task benefit, and they focus on the effects of uncertainty over effort and contingent self-rewards on self-control. They find that loss aversion is necessary to affect behavior, and that uncertainty over the cost of effort can lead to situations in which the optimal goals appear dysfunctional if reference dependence is not accounted for. The latter result regarding dysfunctional goals requires *both* loss aversion and uncertainty over outcomes. Likewise, Koch and Nafziger (2008) consider a three-period problem involving a task with immediate costly effort and a delayed benefit, with an extension to self-rewards in Koch and Nafziger (2009a). Their finding that only an agent with intermediate self-control problems will set a goal regarding the task outcome requires both loss aversion and outcome uncertainty. In contrast, I consider a continuous-time, optimal stopping problem with an infinite horizon, where the self-control problem arises purely from the tension between waiting and stopping now, since there is no intertemporal separation between realized costs and benefits. Assuming that the goal pertains to the final value and is evaluated only upon stopping, I find that goals attenuate impatience even in the absence of loss aversion, and can even cause an agent to wait too long. In contrast to related work on reference-dependent preferences, including Koch and Nafziger (2008, 2009), Suvorov and van de Ven (2008), and Kőszegi and Rabin (2006, 2009), neither loss aversion nor uncertainty over final payoffs are required for any results. I also study social comparisons, where agents evaluate themselves against others, and find that although the presence of any peer attenuates the self-control problem, every agent prefers to compare himself against the most impulsive peer possible.

Kőszegi and Rabin (2006) develop a model of reference dependent preferences that combines the neoclassical assumption of utility over final outcomes with comparative utility that exhibits the features of prospect theory (Kahneman and Tversky, 1979), where expectations determine the reference point. This framework has also been extended to a dynamic setting, where agents derive utility from the revision of beliefs as well as the comparison of outcomes to beliefs (Kőszegi and Rabin, 2009; Matthey and Dwenger, 2007; Matthey, 2008). In an experiment regarding the choice of effort exertion on a task, Abeler et al. (2009) find support for the theory that individuals are reference dependent and that rational expectations serve as the reference point. There also exists empirical evidence, though somewhat debated, that people use reference points in the form of income targets in their decisions to trade off labor and leisure (Camerer et al., 1997; Farber, 2005, 2008; Crawford and Meng, 2008; Fehr and Goette, 2007; Goette and Huffman, 2005).

The concept of intrapersonal conflict due to intertemporal differences in preferences within the self was first studied by Strotz (1956) and Schelling (1984), and more broadly developed by Ainslie (1992) and Laibson (1997). Subsequently, much attention has been devoted to exploring the circumstances under which present-biasedness can be attenuated or exacerbated. For example, O'Donoghue and Rabin (1999) consider how an individual's knowledge of his present-biasedness affects behavior, while Brocas and Carrillo (2001) examine the effects of competition versus complementarity in different agents' projects on their tendencies to rush or procrastinate. An individual can improve self-control through various self-imposed, binding commitments, including restricted information (Brocas and Carrillo, 2005; Carrillo, 2005), restricted choice sets (Gul and Pesendorfer, 2001), and externally enforced devices (Bisin and Hyndman, 2009). In contrast, Bénabou and Tirole (2004) develop a theory of internal regulation through self-enforcing personal rules based on a mechanism of self-reputation. The intrapersonal problem with time inconsistent preferences, which can be framed as a conflict among successive selves, certainly shares clear parallels with a principal-agent problem with moral hazard, a setting in which optimal compensation schemes have been studied (Ou-Yang, 2003; Kadan and Swinkels, 2008).

However, belief constraints and welfare interpretations can differ markedly between the two settings.

There also exists an extensive economics literature on social comparison and peer effects. When individuals care about their perceived status in the eyes of others, this desire can lead to conformity (Bernheim, 1994) or conspicuous consumption (Bagwell and Bernheim, 1996). Similarly, Austen-Smith and Fryer (2005) study the influence of cultural norms and expectations on racial differences in school performance. In finance, DeMarzo et al. (2008) consider the role of relative wealth concerns in the formation of financial bubbles. Battaglini et al. (2005) extend the framework developed by Bénabou and Tirole (2004) to endogenize peer group effects when individuals have incomplete information about the manageability of their self-control problem, but can learn more about it from observing others. Falk and Knell (2004) consider endogenous reference standards in a static, reduced-form social comparison model. Rayo and Becker (2007) propose an evolutionary model of how reference dependence and social comparison can arise as optimal mechanisms to maximize fitness.

Finally, the real options approach to investment under uncertainty was pioneered by Brennan and Schwartz (1985) and McDonald and Siegel (1986), and has been built upon extensively in economics and finance (Dixit, 1993; Dixit and Pindyck, 1994). More recently, Grenadier and Wang (2007) extend this framework to model the investment decisions of hyperbolic entrepreneurs, while Miao (2008) studies agents who possess Gul and Pesendorfer's (2001) temptation utility.

1.3 The Model

I first describe the economic environment, followed by the agent's preferences, which may include both hyperbolic discounting and reference dependence. I focus on an optimal stopping problem, where the self-control problem arises purely from the tension between waiting and stopping today. This framework applies directly to many economic situations, such as those described above - the student pursuing his education, the person saving a nest egg for retirement, and the person searching for a spouse. For example, a student who is deciding how long to remain in school will incur job search costs when he stops, and will generally obtain a better job outcome from staying in school longer.

1.3.1 Optimal Stopping

I consider the standard, continuous-time optimal stopping problem, in which an infinitely lived agent has a non-tradeable option to invest in a project.[5] The problem can also be interpreted as a project termination decision - the agent currently holds a project that has a fixed cost of disinvesting and an uncertain payoff or resale value.

At any time t, the agent knows the current value of the project's payoff $x_t \in [0, \infty)$ and decides whether to stop or to wait. In the latter case, the project's payoff evolves as a geometric Brownian motion:

$$dx_t = \mu x_t dt + \sigma x_t dz, \tag{1.1}$$

where z is a standard Wiener process, μ the average growth rate of x_t, and σ its standard deviation per unit time. At the stopping time $\bar{t}$, the project yields the lump-sum terminal payoff $x_{\bar{t}}$. The cost of stopping at any time is $I > 0$, and is incurred only at the stopping time.[6] Without loss of generality, there is no interim

[5]Thus, it has the same structure as an American stock option, where an investor holds an option that does not expire and decides when to strike.

[6]Here, there is no intertemporal separation of the costs and benefit. The agent's self-control problem arises purely from the tension between the option value of waiting for an uncertain period of time and stopping today at a known project value. In contrast, Brocas and Carrillo (2005) and Miao (2008) study irreversible consumption in discrete time models where costs are delayed until after consumption. Separating the costs and benefits of stopping in such a manner certainly exacerbates the self-control problem, but is not necessary to produce intrapersonal conflict.

flow payoff, nor any direct cost incurred prior to stopping.[7] Due to the stochastic nature of the payoff process, there exists an option value of waiting, in the hope that a higher project value will be realized at a later date.

1.3.2 Time Preferences

The agent may have present-biased preferences, creating a self-control problem. I model this present-biasedness by following Harris and Laibson (2004), who formulate a continuous-time version of quasi-hyperbolic preferences. At any time s, an agent's preferences are divided into a "present," which lasts from time s to time $s+\tau_s$, and a "future," which arrives at time $s+\tau_s$ and persists forever. The length of the present, τ_s, is stochastic and distributed exponentially with parameter $\lambda \in [0, \infty)$. While the agent knows the distribution governing the length of the present, he is unaware of when the future will arrive ex ante.[8] When the future for this self s arrives at time $s+\tau_s$, he is replaced by a new self who takes control of decision-making. Likewise, the preferences of this self $s+\tau_s$ are divided into a "present" of length $\tau_{s+\tau_s}$ and a "future" that arrives at time $(s+\tau_s)+\tau_{s+\tau_s}$ and persists forever. Hence, when each self's "future" arrives, it "dies" and is replaced by a new self.

[7] A more realistic setting might include a constant flow payoff $y \in (\underline{y}, \infty)$, where $\underline{y} < 0$ is the minimal flow payoff such that the agent stops immediately for any $x_t \geq 0$. For example, a student might incur some small positive or negative flow payoff from going to school. Given the other assumptions, the inclusion of a constant flow payoff has no qualitative effect on the results, so I assume $y = 0$ for simplicity of exposition. Likewise, incorporating a stochastic flow payoff that follows a known process with known current value leads to the same qualitative results.

[8] The assumption of a stochastic arrival time of the future allows us to obtain a stationary solution to the stopping problem, but is not necessary to obtain qualitative results. One can obtain stationarity by imposing a deterministic arrival time instead, though comparative statics with respect to differing time preferences would be less general.

Each self s has a stochastic discount function $D_s(t)$:

$$D_s(t) = \begin{cases} e^{-\rho(t-s)} & \text{if} \quad t \in [s, s+\tau_s) \\ \beta e^{-\rho(t-s)} & \text{if} \quad t \in [s+\tau_s, \infty). \end{cases} \tag{1.2}$$

where $\beta \in [0, 1]$ and $\rho > 0$.[9] To ensure that the agent never finds it optimal to wait forever in the optimal stopping problem, let $\rho > \mu$. The function $D_s(t)$ decays exponentially at the rate ρ throughout, but drops discontinuously at time $s + \tau_s$ to a fraction β of its prior level. Note that in this continuous time version of quasi-hyperbolic preferences, there are two parameters that determine the degree to which an agent's behavior deviates from that of a time-consistent individual. First, the parameter β retains the same role it plays in the discrete-time version, measuring how much the future is valued relative to the present. Second, the parameter λ determines the arrival rate of the future, and thus how often preferences change. In particular, when $\lambda \to \infty$ and $\beta < 1$, the agent discretely discounts all moments beyond the current instant. Harris and Laibson (2004) describe this limit case as "instantaneous gratification." When $\beta = 1$ or $\lambda = 0$, conversely, the preferences described by Equation (1.2) are equivalent to those of an exponential discounter with discount rate ρ.

1.3.3 Goals

The agent's preferences are assumed to be reference-dependent: his utility is composed of both standard consumption utility, which is based on absolute levels, and

[9]To see how the stochastic discount function $D_s(t)$ described by (1.2) is analogous to the discrete-time, quasi-hyperbolic version, note that it can be rewritten in the the following form, where $\rho = -\ln \delta$ and $\delta \in (0, 1]$:

$$D_s(t) = \begin{cases} \delta^{t-s} & \text{if} \quad t \in [s, s+\tau_s) \\ \beta \delta^{t-s} & \text{if} \quad t \in [s+\tau_s, \infty). \end{cases}$$

of comparison utility, which is concerned with gains and losses relative to a reference point, which here corresponds to his goal. In the optimal-stopping context with zero flow payoffs, the agent's consumption utility upon stopping at time $\bar{t}$ is simply his net terminal payoff, $x_{\bar{t}} - I$. As in Kőszegi and Rabin (2006; 2009), the agent's comparison utility is closely related to his consumption utility, and is derived by comparing his net terminal payoff at time $\bar{t}$ against his goal at that time, $r_{\bar{t}}$. A key difference, however, is that comparison utility here is simply a linear function, given by

$$\eta(x_{\bar{t}} - I - r_{\bar{t}}), \tag{1.3}$$

where $\eta \geq 0$.[10] Linear comparison utility implies that the agent exhibits *neither loss aversion nor diminishing sensitivity to gains and losses.* Although the absence of these two features deviates from Kahneman and Tversky's (1979) value function, it demonstrates that *neither of these features is necessary in order for goals to affect behavior in a meaningful way.* Indeed, one of the main points of the paper is that goal-setting and its effects can be understood completely independently of loss aversion.

The parameter η can be interpreted as the degree to which the agent cares about, or pays attention to, the difference between his outcome and his goal. It can also be seen as a measure of salience or "goal commitment," which is broadly defined in psychology as the degree to which a person is determined to achieve a goal, since concern for and attention to a goal clearly contribute to this trait. A central concept of goal-setting theory is that goal commitment is a necessary condition for a high goal to lead to high performance; that is, goal difficulty has little effect on behavior if commitment is not present (Locke and Latham, 2002). The absence of goal commitment corresponds to the absence of reference dependent preferences ($\eta = 0$), when the

[10] Alternatively, he could compare his gross terminal payoff $x_{\bar{t}}$ against his goal for the gross terminal payoff at that time, or separately compare the terminal payoff and cost using this comparison utility function, and the results would be unchanged.

existence of a goal has no effect on utility, and consequently, his behavior.[11] Here, I treat η as a fixed parameter and analyze the agent's subsequent behavior and welfare. The demand for personal development services and products can be interpreted as an attempt by individuals to improve self-regulation and welfare by changing η. For example, one such self-help book is suggestively titled "The Magic Lamp: Goal Setting for People Who Hate Setting Goals" (Ellis, 1998) and purports to help individuals improve their degree of goal commitment.[12] This interpretation is not inconsistent with this assumption, insofar as an agent must determine how he would fare for any given η, and thus the value of attempting to change his initial η. In this vein, I later consider the agent's preferences over η from an ex-ante perspective, given his degree of present-biasedness.

I assume that the agent only incurs comparison utility at the time at which he stops and receives the net terminal payoff. That is, although he is always aware that he will incur comparison utility at the moment of stopping, he does *not* directly experience it while waiting. This assumption accords with the notion from mental accounting that individuals do not necessarily "feel" gains and losses until they have been realized (Thaler, 1999).[13]

[11] Alternatively, we can interpret $\eta = 0$ as the case in which the agent has no goal or an ill-defined goal. If there is no basis against which to make a comparison, it seems natural to believe that an agent cannot incur comparison utility in this case. Likewise, if the point of comparison is poorly defined, then deriving comparison utility is a difficult exercise to accomplish. Such an interpretation accords with results in the psychology literature regarding the ineffectiveness of vague goals on motivation and effort (Latham and Locke, 1991; Mento et al., 1992).

[12] The inside flap declares, "**The Magic Lamp** is the first goal-setting guide for people who hate setting goals. Goals can take you anywhere you want to go, but they rarely give you the inspiration you need to get there . . . **The Magic Lamp** transforms the process of setting goals from a dull routine into an exciting adventure because it's the first book to combine the methods of goal setting with the magic of making your wishes come true." Despite the extravagance of the claim, it suggests both that goal commitment is necessary in conjunction with expectations and that individuals may be able to change their existing degree of goal commitment with some effort.

[13] For example, the disposition effect, where stockholders are reluctant to sell losing stocks, and hence realize losses relative to their original buying prices, is consistent with this idea (Odean, 1998; Barberis and Xiong, 2008).

For simplicity, overall utility is taken to be additively separable in its two components. The agent's total utility at the stopping time is thus

$$x_{\bar{t}} - I + \eta(x_{\bar{t}} - I - r_{\bar{t}}). \tag{1.4}$$

At any time s, the goal r_s is taken as given by self s and cannot be changed during his entire "lifetime," having been set by his previous self. Similarly, the goal that self $s + \tau_s$ inherits, denoted $r_{s+\tau_s}$, is set by self s, where τ_s, the lifespan of self s, is stochastically determined and a priori unknown to self s.

The assumption that the agent cannot change an inherited goal implies that such a goal can provide a degree of internal motivation to his (present-biased) future selves. Such "goal stickiness" is in fact necessary for it to matter at all: if the agent could simultaneously make the stopping decision and set a goal for himself for the *current* period, this goal would have no effect on his current behavior, since his present bias implies that he has no desire to behave more patiently in the current period. He would also have no means by which to influence future behavior. Thus, a goal would have no power to attenuate present-biased behavior, since the agent would have no way to impose ex-ante preferences for time-consistent behavior on himself.[14]

1.3.4 Expectations

In setting the goal, each self forms an expectation of his immediate "descendant"'s net terminal payoff if he does not stop himself. His descendant inherits this expectation as a given and compares his own net terminal payoff against this inherited goal if he

[14]Alternatively, we could assume that the agent can change an inherited goal at some cost. Thus imposing zero or infinite costs corresponds to no or maximal "goal stickiness," respectively. While relaxing "goal stickiness" would certainly weaken the effects of a goal, clearly the qualitative findings and comparative statics would still hold as long as it exists, without offering additional insights.

stops.[15] I will consider the two polar cases of sophistication and naivete, in the manner of O'Donoghue and Rabin (1999), to study the impact of differing expectations on behavior and welfare. In contrast to the case of hyperbolic discounting without reference dependent preferences, holding incorrect expectations directly affects the agent's comparison utility through his goal choice. However, I assume that his goals must be consistent with the outcome he expects to achieve. Because he observes the current project payoff perfectly, he holds no uncertainty over the anticipated outcome upon stopping ex ante, though such beliefs may or may not be correct. If the agent is sophisticated and correctly anticipates his actions, each self must have rational expectations about goal achievement. That is, the agent cannot consistently fool himself about what he can or cannot achieve - he must set goals that are realistic.[16] Likewise, if he is naive, so his beliefs regarding future behavior are incorrect, he must set objectives that he *perceives* to be realistic. Note, however, that these assumptions do *not* necessarily imply that each self must have the same goal. Each self cannot change the goal that he inherits, but is free to choose a different one for his future self if he so desires, as long as that he perceives, whether accurately or not, that it is realistic.

Because each self inherits his goal from a previous one, it is necessary to specify the source of the agent's goal when he is first able to stop the project. I assume that there exists a "self 0," an ex-ante self, who learns that the stopping opportunity will present itself in future and forms an expectation of how he will behave once the

[15]This formulation is consistent with Bandura's (1989) theory that goals serve as both targets to strive for and standards by which outcomes are evaluated, as well as empirical evidence that the degree of self-satisfaction varies depending on goal level. That is, two individuals who attain the same outcome will be unsatisfied or satisfied depending on whether their goals were higher or lower than that outcome, respectively (Mento et al., 1992).

[16]Based on the results of lab and field experiments, Latham and Locke (1991) conclude that goal choice is an integration of what one wants and what one believes is possible, suggesting that goals must be, and are, realistic to the agent. Carrillo and Dewatripont (2008) also discuss the tension between foresight and the credibility of promises in intrapersonal games, arguing that agents cannot simultaneously anticipate future behavior and fool future selves.

option becomes available for exercise.

Given the above time preferences and utility, at any time s the agent chooses the stopping rule that determines a (random) stopping time $\bar{t}$ to maximize the expected present value of his overall utility:

$$\max_{\bar{t}} E_s\{D_s(\bar{t})[x_{\bar{t}} - I + \eta(x_{\bar{t}} - I - r_{\bar{t}})]\}, \tag{1.5}$$

where E_s denotes the conditional expectation at time s and $D_s(\bar{t})$ is given by Equation (1.2). Because the project value is uncertain, there exists a benefit to waiting, in the hopes of realizing a higher project value. When the agent has hyperbolic time preferences, he is prone to stopping too early (Grenadier and Wang, 2007) because he undervalues the future, and thus the option value of waiting. The question of interest is to what extent reference dependent preferences and setting goals can attenuate this problem.

1.4 Sophistication

When the agent is quasi-hyperbolic and sophisticated, in that he is fully aware of his present-biasedness, the problem takes on the nature of a dynamic game between successive selves, and there could exist equilibria in which each self chooses a different stopping strategy. Here, I will focus on the most natural equilibrium, namely a stationary Markov equilibrium in which each self employs the same threshold strategy.

Since the geometric Brownian motion is continuous, the project value cannot jump discontinuously from one moment in time to the next. This implies that the sophisticated agent has no uncertainty over the net terminal payoff from stopping: if every self uses the same threshold $\overline{x}$, then $x_{\bar{t}} = \overline{x}$, so the expected net terminal payoff is $\overline{x} - I$. The source of ex ante uncertainty is the *timing* of this stopping - that is, when (and if) he will choose to stop. Furthermore, if a self-set goal must be realistic because the agent is sophisticated, every self will inherit, set, and meet the same goal in a stationary equilibrium, so that $r_{\bar{t}} = \overline{x} - I$.

In order to construct the stationary Markov equilibrium, I solve the intrapersonal game backwards in the same manner as Grenadier and Wang (2007), but with the inclusion of a final goal-dependent payoff. Each self anticipates that his descendants will act according to a threshold that maximizes their own current benefit of waiting, so they will face a problem that is identical to his own. Constructing the stationary solution thus involves searching for a fixed point such that current and future selves stop at a common threshold.

Because each self controls the stopping decision in the present, and cares about - but cannot directly control - those of the future, two value functions are required to describe the intrapersonal problem. Suppose that all descendants inherit the goal $\hat{r}$. Then the Bellman equation for the continuation value function $v(x, \hat{r})$ is

$$v(x, \hat{r}) = \max\{x - I + \eta(x - I - \hat{r}), e^{-\rho dt} E[v(x + dx, \hat{r})]\}. \tag{1.6}$$

This continuation value function describes each self s's consideration (or internalization) of his future selves, following the random arrival of the future at time τ_s. That is, beyond time τ_s, he discounts any future utility flows exponentially. Hence, the continuation value function also describes how the current self would *prefer* his future selves to evaluate payoff streams - by discounting exponentially. That is, v describes the agent's preferences over the future from an ex ante perspective, including those of self 0. If the agent were time consistent ($\beta = 1$ or $\lambda = 0$), then all selves' preferences would coincide and he would choose the optimal strategy by maximizing v. Given the current project value x and his inherited goal $\hat{r}$, he would thus choose, as in (1.6), the maximum of the current net terminal payoff and the expected present discounted value of waiting for a higher realization of x, where this discounting occurs exponentially. In Section (1.4.1), I will describe the behavior of a special case, namely that of the "standard" agent who is neither present-biased nor reference-dependent, as a benchmark.

However, if the agent is present-biased ($\beta < 1$), he maximizes a different value function that overweights the present relative to the future. Denoting the goal inherited by the current self by $\overline{r}$, the Bellman equation that describes this current value function $w(x,r)$ is

$$w(x,\overline{r}) = \max\{x - I + \eta(x - I - \overline{r}), \\ (1 - e^{-\lambda dt})e^{-\rho dt}\beta E[v(x + dx, \hat{r})] + (e^{-\lambda dt})e^{-\rho dt}E[w(x + dx, \overline{r})]\}. \quad (1.7)$$

Given the current x and an inherited goal $\overline{r}$, and anticipating that his future selves will inherit $\hat{r}$ (with the knowledge that he sets $\hat{r}$ for his immediate descendant), the current self chooses the maximum of the current total utility from stopping and the expected present discounted value of waiting for a higher realization of x, where this discounting discontinuously drops by the factor β upon the random arrival of the future. A future self arrives in the next instant dt with probability $1 - e^{-\lambda dt}$, while the current self remains in control with probability $e^{-\lambda dt}$.[17]

[17] In an alternative analogy, consider a parent who derives utility from the payoff stream of his possible descendants. The continuation value v describes the parent's evaluation of the stream of his descendants' utilities - he discounts them exponentially. Thus he also prefers that every descendant evaluate the payoff streams from his entire family line, including his own, exponentially. When his child becomes the decision-maker, he discounts his own descendants' utilities exponentially, just as his parent did. But he also disproportionately underweights the stream of his descendants' utilities relative to his own by the factor β, in disagreement with his parent's wishes. Thus, w describes the child's evaluation of the payoff stream from his entire family line, including his own. Each parent sets a goal $\hat{r}$ for his child according to his beliefs about his child's preferences. While a sophisticated parent knows that his child will care less about his descendants than the parent prefers, a naive parent incorrectly believes that his child's preferences will be in complete agreement with his own.

1.4.1 Ex Ante Preferences

To construct the continuation value function $v(x, \hat{r})$, I first suppose that all future selves inherit goal $\hat{r}$ and employ a threshold $\hat{x}$ such that they wait if $x < \hat{x}$ and stop if $x \geq \hat{x}$. By continuity of the geometric Brownian motion, there is zero probability that the project value x_t can jump discontinuously from the "wait" region ($x < \hat{x}$) to the "stop" region ($x \geq \hat{x}$) from one moment to the next. Therefore, I can construct $v(x, \hat{r})$ by considering its behavior in the "wait" and "stop" regions separately, then joining them using appropriate boundary conditions.

By definition of the threshold strategy, the value of Equation (1.6) in the stop region ($x \geq \hat{x}$) is simply given by $x - I + \eta(x - I - \hat{r})$. In the wait region, standard results imply that $v(x, \hat{r})$ must obey the following linear differential equation:

$$\rho v(x, \hat{r}) = \mu x(\frac{\partial v}{\partial x}) + \frac{1}{2}\sigma^2 x^2(\frac{\partial^2 v}{\partial x^2}) \quad \text{if } x < \hat{x}. \tag{1.8}$$

By definition of the geometric Brownian x, $x = 0$ is an absorbing barrier. The continuation value function must also be continuous at the threshold $\hat{x}$ between the waiting and stopping regions. Therefore, the relevant boundary conditions for v are:[18]

$$\text{Boundary: } v(0, \hat{r}) = 0, \tag{1.9}$$

$$\text{Value Matching: } v(\hat{x}, \hat{r}) = \hat{x} - I + \eta(\hat{x} - I - \hat{r}). \tag{1.10}$$

Because there is no optimal decision embodied in the continuation value function $v(x, \hat{r})$, the smooth pasting condition does not apply to $v(x, \hat{r})$ if the agent is present-biased. The stopping decision is never made by future selves, only by *current* selves.

Combining Equation (1.8) with conditions (1.9) and (1.10), the solution for the

[18]The first boundary condition is obtained by noting that given any $\hat{r} \geq 0$, the agent never stops if $x = 0$, since he incurs negative overall utility. The fact that $\hat{r} \geq 0$ can be verified in equilibrium - thus, the boundary condition is $v(0, \hat{r}) = 0$. More generally, it would be sufficient that $v(0, \hat{r})$ is finite for any $\hat{r}$.

continuation value function is

$$v(x,\hat{r}) = \begin{cases} [(\hat{x}-I)+\eta(\hat{x}-I-\hat{r})](\frac{x}{\hat{x}})^{\gamma_1} & \text{if} \quad x < \hat{x} \\ x-I+\eta(x-I-\hat{r}) & \text{if} \quad x \geq \hat{x}, \end{cases} \tag{1.11}$$

where $\gamma_1 > 1$ is the positive root[19] of the quadratic equation

$$\frac{1}{2}\sigma^2\gamma_1^2 + (\mu - \frac{1}{2}\sigma^2)\gamma_1 - \rho = 0, \tag{1.12}$$

reflecting the fact that from an ex ante perspective, the agent discounts the future exponentially at the rate ρ.

Benchmark: The Standard Case

In the standard optimal-stopping problem, the agent is time-consistent ($\beta = 1$ or $\lambda = 0$) and is not reference dependent ($\eta = 0$). His optimal strategy is to use the fixed stopping threshold x^*: at any time s, he waits if $x_s < x^*$ and stops if $x_s \geq x^*$, where x^* is chosen to maximize the option value of waiting. Time consistency implies that the agent's preferences are fully described by the single value function v, so he chooses x^* to maximize v. Thus, the smooth pasting condition with respect to x must apply to the continuation value function v, so the marginal values of waiting and stopping must be equal at the optimal threshold. Since $\eta = 0$ here, the smooth pasting condition is given by:

$$\text{Smooth Pasting: } \frac{dv}{dx}(x^*) = 1. \tag{1.13}$$

[19]The negative root is ruled out by the boundary condition for $x = 0$. Writing out γ_1 explicitly, we have $\gamma_1 = -\frac{\mu}{\sigma^2} + \frac{1}{2} + \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}}$. To see that $\gamma_1 > 1$, note that $\sigma^2 > 0$ and the left-hand side of the quadratic is negative when evaluated at $\gamma_1 = 0$ and $\gamma_1 = 1$, implying that the negative root is strictly negative and the positive root is strictly greater than 1 if $\mu < \rho$.

Solving Equation (1.8) subject to (1.13) and the barrier absorption and value matching conditions given by (1.9) and (1.10) when $\eta = 0$ allows us to determine the optimal threshold x^*:

$$x^* = (\frac{\gamma_1}{\gamma_1 - 1})I, \tag{1.14}$$

where $\gamma_1 > 1$ is again the positive root of the quadratic equation $\frac{1}{2}\sigma^2\gamma_1^2 + (\mu - \frac{1}{2}\sigma^2)\gamma_1 - \rho = 0$. Unsurprisingly, this γ_1 is the same parameter value as that obtained in (1.12), because the continuation value is derived by exponentially discounting the future at the rate ρ in both cases.

The expression for x^* implies that the standard agent always waits for a project value that exceeds its direct cost ($x^* > I$). Due to the forgone possibility of higher realizations of x in the future, there exists an opportunity cost of stopping today. In equilibrium, the standard agent's value of the option to stop, denoted $v^*(x)$, is given by

$$v^*(x) = \begin{cases} (x^* - I)(\frac{x}{x^*})^{\gamma_1} & \text{if} \quad x < x^* \\ x - I & \text{if} \quad x \geq x^*. \end{cases} \tag{1.15}$$

1.4.2 Present-Biased Preferences

In contrast to the time-consistent case, the present-biased agent maximizes the current value function w, rather than the continuation value v. Proceeding with the derivation of $w(x, \overline{r})$ analogously, I first suppose that all current selves inherit goal $\overline{r}$ and employ a threshold $\overline{x}$ such that they wait if $x < \overline{x}$ and stop if $x \geq \overline{x}$. Again, I can construct $w(x, \overline{r})$ by characterizing it in the "wait" and "stop" regions separately, then joining the two regions through appropriate boundary conditions.

By definition of the threshold strategy, the value of w in the stop region ($x \geq \overline{x}$) is simply given by $x - I + \eta(x - I - \overline{r})$. In the wait region, standard results imply that $w(x, \overline{r})$ must obey the following linear differential equation:

$$\rho w(x, \overline{r}) = \lambda(\beta v(x, \hat{r}) - w(x, \overline{r})) + \mu x(\frac{\partial w}{\partial x}) + \frac{1}{2}\sigma^2 x^2(\frac{\partial^2 w}{\partial x^2}) \quad \text{if } x < \overline{x}. \tag{1.16}$$

Comparing Equation (1.16) to Equation (1.8), the additional term $\lambda(\beta v(x, \hat{r}) - w(x, \overline{r}))$ is the expected value of the change in the current value w that occurs through the stochastic arrival of a transition from the present to the future.

As with v, $x = 0$ is an absorbing barrier and w must clearly be continuous at the threshold $\overline{x}$ between the wait and stop regions. Finally, the smooth pasting condition must apply to w with respect to x, because the optimal threshold is chosen by the current self to maximize his current value function w. At this threshold $\overline{x}$, the current self must be unwilling to deviate from stopping, so the marginal value of waiting must equal that of stopping.[20] Thus, the relevant boundary conditions for w are

$$\text{Boundary: } w(0, \overline{r}) = 0, \tag{1.17}$$

$$\text{Value Matching: } w(\overline{x}, \overline{r}) = \overline{x} - I + \eta(\overline{x} - I - \overline{r}), \tag{1.18}$$

$$\text{Smooth Pasting: } \frac{\partial w}{\partial x}(\overline{x}, \overline{r}) = 1 + \eta. \tag{1.19}$$

Because the current self fully anticipates that his future selves will use threshold $\hat{x}$ given goal $\hat{r}$, we can substitute the continuation value function, given by Equation (1.11) into the differential Equation (1.16). Under the assumption that $x \leq \hat{x}$, which the fixed point condition will satisfy in a stationary equilibrium, it is the value of v in its wait region that applies to (1.16). Combining Equation (1.16) with (1.11) and conditions (1.17), (1.18), (1.19), we obtain the solution to the optimal threshold $\overline{x}$ as a function of current goal $\overline{r}$ and the conjectured future goals $\hat{r}$ and thresholds $\hat{x}$.

[20]Dixit (1993) provides a detailed treatment of the smooth pasting technique.

1.4.3 Stationary Equilibrium

In a stationary equilibrium, the sophisticated agent knows that all current and future selves employ the same threshold, which requires that $\overline{x} = \hat{x}$. To make clear the effect of goal-setting on the optimal stopping rule, I will first consider the sophisticate's stopping threshold given any fixed goal level (i.e., $r = \overline{r} = \hat{r}$), denoted $\overline{x}^{SE}$, provided that this goal is set ex ante and does not change during the stopping decision. Thus, this also describes a sophisticate's response to any externally set goal, which can differ from the agent's actual final payoff. Next, I derive the sophisticate's stopping rule when his goals are set internally, denoted $\overline{x}^{SI}$, and therefore are required to be met in equilibrium.

Exogenous Goals

Given that $r = \overline{r} = \hat{r}$ and imposing the fixed point condition that $\overline{x} = \hat{x} \equiv \overline{x}^{SE}$, the optimal threshold in response to an externally set goal $r \geq 0$, denoted $\overline{x}^{SE}$, is

$$\overline{x}^{SE} = (\frac{\overline{\gamma}}{\overline{\gamma}-1})I + r(\frac{\eta}{1+\eta})(\frac{\overline{\gamma}}{\overline{\gamma}-1}), \quad \text{with } \overline{\gamma} \equiv \beta\gamma_1 + (1-\beta)\gamma_2, \tag{1.20}$$

where γ_2 is the positive root[21] of the quadratic equation

$$\frac{1}{2}\sigma^2\gamma_2^2 + (\mu - \frac{1}{2}\sigma^2)\gamma_2 - (\rho + \lambda) = 0. \tag{1.21}$$

Note that the only difference between the quadratic equation for γ_2 and that of γ_1 is the presence of the parameter λ, which is the hazard rate for the arrival of the future. It is apparent that $\gamma_2 \geq \overline{\gamma} \geq \gamma_1$, with equality only if the future never arrives ($\lambda = 0$), i.e. preferences never change. That is, the parameter γ_2 reflects the fact that each self's expected "lifetime" shortens with λ, while the degree to which this

[21] As before, the negative root is ruled out by the boundary condition for $x = 0$. Writing out γ_2 explicitly, we have $\gamma_2 = -\frac{\mu}{\sigma^2} + \frac{1}{2} + \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + 2(\frac{\rho+\lambda}{\sigma^2})}$.

feature affects the stopping decision is determined by the degree of present-biasedness, measured by $1 - \beta$.

Equation (1.20) makes clear the effect of goal-setting on the optimal stopping rule. The first term is the agent's stopping threshold in the absence of reference dependence or a goal: if $\eta = 0$, then $\overline{x}^{SE} = (\frac{\overline{\gamma}}{\overline{\gamma}-1})I < x^*$.[22] A hyperbolic discounter without reference dependent preferences stops earlier than the standard agent, and this impatience is exacerbated as the present is more overweighted, i.e. as β decreases, and as preferences change more frequently, i.e. as λ increases. Recall that it is the combination of parameters β and λ that determines the degree of the self-control problem. In fact, (1.20) reveals that $\overline{\gamma} \equiv \beta\gamma_1 + (1-\beta)\gamma_2$ is a sufficient statistic for measuring the sophisticated agent's degree of impulsiveness, rather than β alone.[23] The second term in (1.20) is the effect of the goal on the stopping threshold. Because an agent is motivated to avoid settling for a lower project only if there exists a potential comparative penalty from falling short, the goal r only induces more patient behavior as long as $r > 0$, even in the case of instantaneous gratification. When $\beta < 1$ and $\lambda \to \infty$, the second term in (1.20) is strictly positive and is given by $r(\frac{\eta}{1+\eta})$. No potential penalty is imposed when $r = 0$, so behavior is unchanged by this goal and it is equivalent to having no goal.[24]

Proposition 1. *In a stationary equilibrium with exogenous goal r, the sophisticate's stopping threshold exhibits the following properties:*

1. *The threshold decreases with impulsiveness:* $\frac{\partial \overline{x}^{SE}}{\partial \overline{\gamma}} < 0$.

2. *The threshold increases with goal level:* $\frac{\partial \overline{x}^{SE}}{\partial r} > 0$.

[22] If the agent is not reference dependent ($\eta = 0$), then Equation (1.20) is identical to the sophisticate's threshold obtained by Grenadier and Wang (2007), who study irreversible investment with naive and sophisticated hyperbolic agents.

[23] Clearly, $\overline{\gamma}$ is decreasing in β and increasing in λ, so comparative statics on each parameter separately yield the same qualitative results.

[24] This feature makes clear that η in itself does not simply act as a subsidy to reaching a higher net x.

3. *Responsiveness to a goal increases with goal commitment:* $\frac{\partial^2 \overline{x}^{SE}}{\partial r \partial \eta} > 0$.

4. *Responsiveness to a goal decreases with goal commitment:* $\frac{\partial^2 \overline{x}^{SE}}{\partial r \partial \overline{\gamma}} < 0$.

It is intuitive that the agent's threshold should decrease with his degree of impulsiveness irrespective of his degree of reference dependence, for two reasons. First, he undervalues the future himself. Second, being sophisticated, he anticipates that his future selves will undervalue their own futures as well, which decreases the value of waiting even further.

A second intuitive result is that, regardless of his degree of impulsiveness (including when $\beta = 1$ or $\lambda = 0$), the agent's threshold increases with the level of the goal if he is reference dependent ($\eta > 0$). Raising r increases the potential cost of settling for a lower project value. Thus, the goal induces more patient behavior by providing an additional incentive to wait for a higher realization of the project value. This result is consistent with experimental evidence that task performance increases with the goal difficulty, whether externally- or self-set.[25]

Consistent with empirical evidence on goal commitment (Klein et al., 1999), responsiveness to a goal is increasing in the agent's degree of reference dependence. An agent who cares more about falling short of his goal is more motivated to change his behavior to avoid such an outcome than one who cares less about this comparison.

Finally, an agent's responsiveness to a goal is decreasing in his degree of impulsiveness, illustrating the interaction between present-biasedness and reference dependence. A less present-biased agent values the future more than a more impulsive counterpart, so he has a stronger incentive to avoid incurring a comparative penalty assessed in the future. Because a more present-biased agent undervalues the future more, he not only exhibits more impatience in the absence of a goal, but is also less responsive to a given goal than a less impulsive counterpart.

[25] Locke and Latham (2002) even find a positive linear relationship between goal difficulty and performance.

Endogenous Goals

When the goal is internally set, rational expectations implies that each self correctly anticipates his descendant's threshold strategy, so that $\overline{r} = \overline{x} - I$ and $\hat{r} = \hat{x} - I$. But since $\overline{x} = \hat{x}$, this implies that $r = \overline{r} = \hat{r} = \overline{x} - I$. Each self correctly expects his descendants to use the same stopping rule as he does currently, so every self inherits and meets the same goal in equilibrium. That is, whether goals are externally or internally set, sophistication implies that $\overline{x} = \hat{x}$ and $\overline{r} = \hat{r}$. Moreover, a self-set goal must satisfy the additional condition that it coincides with the expected net terminal payoff, so that $r = \overline{x} - I$. Thus, the optimal threshold in a stationary equilibrium with internally set goals, denoted $\overline{x}^{SI}$, must satisfy $r = \overline{x}^{SI} - I$ and can be derived by imposing this additional condition on Equation (1.20), yielding

$$\overline{x}^{SI} = \left(\frac{\overline{\gamma}}{\overline{\gamma} - 1 - \eta}\right)I, \qquad \begin{array}{ll} \text{with} & \overline{\gamma} \equiv \beta\gamma_1 + (1-\beta)\gamma_2 \\ \text{and} & \eta < \overline{\gamma} - 1, \end{array} \tag{1.22}$$

and the equilibrium value functions w^{SI} and v^{SI}:

$$w^{SI}(x, r = \overline{x}^{SI} - I) = \begin{cases} \beta(\overline{x}^{SI} - I)(\frac{x}{\overline{x}^{SI}})^{\gamma_1} + (1-\beta)(\overline{x}^{SI} - I)(\frac{x}{\overline{x}^{SI}})^{\gamma_2} & \text{if} \quad x < \overline{x}^{SI} \\ x - I + \eta(x - \overline{x}^{SI}) & \text{if} \quad x \geq \overline{x}^{SI}, \end{cases} \tag{1.23}$$

$$v^{SI}(x, r = \overline{x}^{SI} - I) = \begin{cases} (\overline{x}^{SI} - I)(\frac{x}{\overline{x}^{SI}})^{\gamma_1} & \text{if} \quad x < \overline{x}^{SI} \\ x - I + \eta(x - \overline{x}^{SI}) & \text{if} \quad x \geq \overline{x}^{SI}. \end{cases} \tag{1.24}$$

The value of Equation (1.23) in its wait region is the expected present value of the option to stop, given the current value of the project's payoff, $x < \overline{x}^{SI}$, and the optimal threshold $\overline{x}^{SI}$. This is essentially the weighted average of two time-consistent option values, where the first, weighted by β, uses the discount rate ρ, which is reflected in γ_1. The second, weighted by $1 - \beta$, uses the discount rate $\rho + \lambda$, which is reflected in

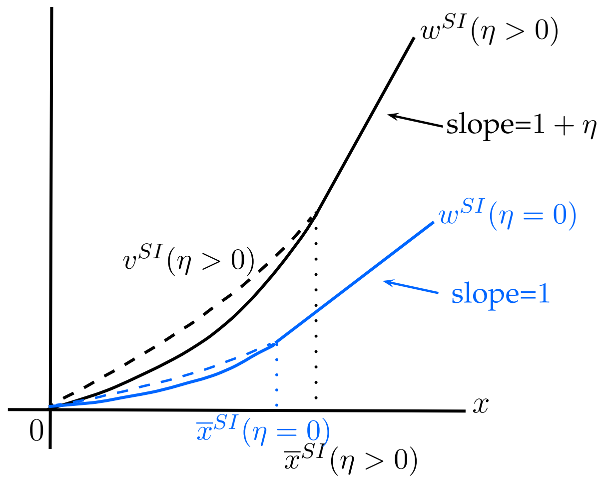

Figure 1.1: Option values with endogenous goals when $\beta < 1$ for varying degrees of η. The black lines represent the value functions when $\eta > 0$. The blue lines represent the value functions when $\eta = 0$. The solid lines depict the current value functions w, which are smooth and continuous everywhere. The dashed lines depict the continuation values v, which are kinked at the equilibrium thresholds $\overline{x}^{SI}$ and coincide with their respective current value functions w thereafter.

γ_2. The value of Equation (1.24) in its wait region is the expected present value of the option to stop, using only the discount rate ρ, reflected in γ_1. Thus, the equilibrium v also represents the option value that each current self would prefer his future selves to use from an ex ante perspective (or would like to commit them to), though he is resigned to the knowledge that they will maximize w rather than v.

Figure 1.1 depicts the equilibrium value functions when goals are self-set. Reference dependence increases the marginal value of waiting, increasing the slope of both the continuation and current value functions upon stopping, and thus the incentive to wait longer. The fact that v lies above its respective w reflects the fact that an exponential discounter values the option more than a present-biased agent. Finally, independently of the value of η, the slope of v is flatter than that of w at the equilibrium stopping threshold, reflecting the fact that the exponential discounter prefers to wait longer than the present-biased agent. Thus, it implies that the present-biased agent prefers that his future selves wait longer than they do in equilibrium.

Proposition 2. *By inducing more patient behavior, reference dependence attenuates impulsiveness in a stationary equilibrium with endogenous goals:* $\frac{\partial \overline{x}^{SI}}{\partial \eta} > 0$.

From Equation (1.22), it is clear that the equilibrium threshold increases with the degree of reference dependence for any given degree of impulsiveness if λ is finite. An agent with a high degree of reference dependence has a stronger incentive to meet his goal, since he puts more weight on the comparative disutility from falling short. It is only in the instantaneous gratification case that the agent, with infinite impatience and finite reference dependence, is unaffected by goal-setting. As noted in Equation (1.20), the agent *is* responsive to a goal even in the instantaneous gratification case, as long as $r > 0$. However, when the goal is self-set, his anticipation of extreme impatience makes him unable to set a realistic penalty to improve his patience, so he sets $r = 0$ in equilibrium and his behavior is unchanged by reference dependence. As shown in the following sections, this will not be the case when he is naive or when he is placed in a heterogeneous peer group. Note also that if $\beta = 1$ and $\eta = 0$, the threshold is equivalent to the equilibrium solution to the standard optimal stopping problem, where the agent is neither dynamically inconsistent nor reference-dependent.

1.4.4 Welfare

From an ex-ante perspective, the agent, no matter how severe his degree of present-biasedness or reference dependence, prefers that his future selves behave according to a time-consistent, optimal strategy. Therefore, I use the preferences of self 0, which determine the ex ante optimum, to evaluate the agent's welfare. Such an analysis allows us to evaluate the welfare consequences for a heterogeneous population of individuals, with varying degrees of present-biasedness and goal commitment. It also allows self 0 to determine the value of adjusting his degree of goal commitment η to improve future behavior, since this choice may be costly.

Sophistication implies that the agent always anticipates that he will meet his self-set goal exactly in equilibrium. He is thus aware that once he actually stops at some point in the future, his comparative utility will equal zero in equilibrium, so his overall utility will consist only of the net terminal payoff, just as in the $\eta = 0$ case. Therefore, even if he knows that he will be reference dependent, the ex ante self wants to maximize the expected present discounted value of his overall utility, which exactly equals the net terminal payoff alone, as if he were time consistent. That is, he wants to choose the stopping threshold that maximizes $v^{SI}(x)$. This problem is identical to that of an agent who is dynamically consistent ($\beta = 1$) and not reference dependent ($\eta = 0$). Thus, the first-best threshold for any sophisticated agent, no matter his degree of present-biasedness or reference dependence, coincides with the threshold that a standard agent chooses: $x^* = (\frac{\gamma_1}{\gamma_1 - 1})I$. This implies that for any degree of impulsiveness, there exists a degree of countervailing reference dependence that enables the agent to employ the first-best threshold strategy (i.e., $\overline{x}^{SI}(\eta^*) = x^*$). Unsurprisingly, this optimal reference dependence η^* is increasing in $\overline{\gamma}$ - as the degree of impulsiveness increases, the degree of reference dependence required to attenuate it increases.

Proposition 3. *For every $\overline{\gamma} \in (1, \infty)$, there exists an η^* such that the reference-dependent agent with $\eta = \eta^*$ achieves the first best, given by*

$$\eta^* = \frac{\overline{\gamma} - \gamma_1}{\gamma_1}.$$

This analysis makes clear the differential effect of increasing the degree of goal commitment η versus increasing the stopping cost I. While both have the same qualitative effect of increasing the equilibrium stopping threshold $\overline{x}^{SI}$, only the former improves welfare by closing the gap between equilibrium and first-best behavior if $\eta < \eta^*$. In contrast, increasing the stopping cost has no effect on the self-control problem, since both the equilibrium *and* first-best thresholds increase with I.

On the other hand, the degree of reference dependence can also be sufficiently high that the agent waits *longer* than is first-best. Consider now an agent who is dynamically consistent ($\overline{\gamma} = \gamma_1$, i.e. $\beta = 1$ or $\lambda = 0$) and reference dependent ($\eta > 0$). Although reference dependence does not distort final overall utility directly, it changes the *marginal* value of stopping. Because the agent has an incentive to avoid incurring comparative disutility, the marginal value of waiting at the first best threshold x^* exceeds the marginal value of stopping, so the agent waits longer. Although he achieves a payoff that exceeds x^*, its ex-ante discounted value is lower than the first best. Hence, reference dependence distorts his behavior *away from the first best* and causes him to be overly patient.[26] Since a time-consistent agent has no self-control problem, reference dependence offers no beneficial value, so $\eta^* = 0$ if $\overline{\gamma} = \gamma_1$. The same argument can clearly be applied to present-biased agents with reference dependence, if η overcompensates for the conflict in time preferences between current and future selves. Moreover, reference dependence can be so high that even an impulsive agent would be better off in the absence of goal-setting. That is, when $\eta > \overline{\eta}$, he would wait so long under goal-setting that he is actually *better off* if *he cannot set goals for himself* (i.e., $\eta = 0$) and behaves impatiently. Thus, goal-setting can itself be a source of intrapersonal conflict, since it can cause an agent to wait longer than is optimal from an ex ante perspective. Unsurprisingly, the level of reference dependence required to be detrimental to the agent's welfare is increasing in his degree of impulsiveness.

Corollary 1. *For every $\overline{\gamma} \in (1, \infty)$, there exists a range of η such that the sophisticated agent waits* longer *than the first best: $\eta^* < \eta < \overline{\gamma} - 1$. Moreover, he is strictly* worse off *under goal-setting if $\overline{\eta} < \eta < \overline{\gamma} - 1$, where $\overline{\eta} \geq \eta^*$ is increasing in $\overline{\gamma}$ and is*

[26] In contrast, Kőszegi and Rabin (2009) find that in the absence of uncertainty over final payoffs, reference dependence can lead to apparently impatient behavior, even without present-biased preferences. There, the reference point is a vector of plans (beliefs) over time, and the agent derives comparison utility in each period of a consumption-savings problem. If he values contemporaneous comparison utility more than future comparison utility, then he may overconsume relative to the ex-ante optimum.

defined by the following condition:

$$(\frac{\overline{\gamma}-1}{\overline{\gamma}-1-\overline{\eta}})^{\gamma_1-1}(\frac{1}{1+\overline{\eta}})-1=0.$$

In particular, a time-consistent agent stops at $\overline{x}^{SI}=(\frac{\gamma_1}{\gamma_1-1-\eta})I$ *and is strictly worse off if he is reference-dependent* ($\eta>0$)*.*

Although the sophisticate's self-set goal level is pinned down by rational expectations, he may further regulate his behavior by adjusting his degree of goal salience or commitment. Under the interpretation that demand for personal development products and services, such as self-help books and executive coaching, is akin to individuals' (costly) attempts to change η from some initial level, these welfare findings suggest that such behavior is quite rational and can, indeed, improve the sophisticate's welfare unambiguously if successful. Prescriptively, they imply that welfare improvement can be achieved purely through programs or services that educate individuals about goal-setting and the appropriate level of goal commitment. While much of this industry focuses on improving welfare by increasing goal commitment, there is evidence suggesting that the converse problem is recognized as well. In psychology, dysfunctional perfectionism is defined as "overdependence of self-evaluation on the determined pursuit of personally demanding, self-imposed, standards in at least one highly salient domain, despite adverse consequences" (Shafran et al., 2002). Consistent with the idea that η can be detrimentally high, Goldsmith (2008) discusses the prevalence of "goal obsession" among successful executives, many of whom sacrifice health or family life in the pursuit of their careers to their own regret.

1.5 Naivete

To highlight the impact of expectations on behavior and welfare, I now consider the case of naivete, where the agent mistakenly believes that he will be dynamically consistent in the future. Given the framework developed to study the sophisticate, the naive agent's strategy is straightforward to derive.

Because the naif holds incorrect beliefs about his future behavior, it is the *perceived*, rather than actual, behavior of future selves that influences his stopping decision. Here, let $\hat{x}$ be the threshold that the naif perceives future selves will employ, such that they wait if $x < \hat{x}$ and stop otherwise, and let $\hat{r}$ be their goal. Both the sophisticate and naif have identical *evaluations* of their future behavior, discounting it exponentially. Thus, the naif's (perceived) continuation value v is still given by Equation (1.11). However, the sophisticate and naif differ drastically in their *beliefs* over their future behavior. The key difference is that the naive agent believes that given a goal $\hat{r}$, future selves will choose $\hat{x}$ such that

$$\hat{x} = (\frac{\gamma_1}{\gamma_1 - 1})I + \hat{r}(\frac{\eta}{1+\eta})(\frac{\gamma_1}{\gamma_1 - 1}). \tag{1.25}$$

The naive agent believes that future selves will be exponential discounters, so he thinks they will choose a stopping rule that maximizes v, rather than w. Thus, the naif derives (1.25) by combining the smooth pasting condition with respect to x with Equation (1.11), in addition to the absorbing barrier (1.9) and value matching (1.10) conditions.[27]

Let $\overline{x}$ be the threshold used by the current self and $\overline{r}$ be the goal that he inherits. Again, because the naif's preferences are identical to those of the sophisticate, the wait region of his current value function w is still given by Equation (1.16).

1.5.1 Equilibrium

Since the naif's beliefs about future selves' behavior are incorrect, current and *perceived* future selves employ different stopping thresholds, so a fixed point condition

[27]Equivalently, Equation (1.25) can be derived from Equation (1.20) by evaluating $\overline{x}^{SE}$ when $\beta = 1$ and the goal is some $\hat{r}$.

does not apply, in contrast with the sophisticate's case. Rather, $\hat{x}$ is given by Equation (1.25). As before, I will first consider the naif's response to any fixed goal level (i.e., $r = \overline{r} = \hat{r}$), denoted $\overline{x}^{NE}$, before describing his stopping threshold when goals are self-set, denoted $\overline{x}^{NI}$.

Exogenous Goals

Let the naif's goal be set ex ante and unchanged during the stopping decision, so $r = \overline{r} = \hat{r}$. Assuming that $\overline{x}^{NE} \equiv \overline{x} \leq \hat{x}$ (and verifying that this holds in equilibrium), we can combine (1.16) with the (perceived) continuation value function (1.15) in its wait region and perceived stopping rule (1.25), and boundary conditions (1.17), (1.18), (1.19) to obtain an implicit function for the naive agent's threshold $\overline{x}^{NE}$ when the goal is exogenous and fixed:

$$\overline{x}^{NE} = \frac{1}{(\gamma_2 - 1)(1+\eta)}[\beta(\gamma_2-\gamma_1)(\hat{x}^{NE}-I+\eta(\hat{x}^{NE}-I-r))(\frac{\overline{x}^{NE}}{\hat{x}^{NE}})^{\gamma_1}+\gamma_2(1+\eta)I+\gamma_2\eta r], \tag{1.26}$$

where $\hat{x}^{NE} = (\frac{\gamma_1}{\gamma_1-1})I + r(\frac{\eta}{1+\eta})(\frac{\gamma_1}{\gamma_1-1})$. Unsurprisingly, the comparative statics with respect to $\overline{x}^{NE}$ are analogous to those described in Proposition (1). Since the intuition is virtually identical, the details are provided in Appendix A.3.

Endogenous Goals

If the naive agent sets goals for himself, I assume that he must believe they are realistic, implying that $\hat{r} = \hat{x} - I$. But since each self, including self 0, persistently misperceives future preferences, the current self inherits the same goal that he will pass on to his descendant (i.e., $\overline{r} = \hat{r}$). Thus, whether goals are externally or internally set, the condition that $\overline{r} = \hat{r}$ must hold. Moreover, a self-set goal must satisfy the additional condition that it is perceived to be realistic, $\hat{r} = \hat{x} - I$. Imposing this condition on Equation (1.25) and letting $\overline{r} = \hat{r}$ yields the perceived threshold employed by future selves when the goal is endogenous, denoted $\hat{x}^{NI}$:

$$\hat{x}^{NI} = (\frac{\gamma_1}{\gamma_1 - 1 - \eta})I. \tag{1.27}$$

Thus, $\hat{x}^{NI}$ is precisely the threshold employed by a time-consistent agent whose goals are endogenously set and who is both time-consistent ($\beta = 1$ or $\lambda = 0$).[28] Likewise, the naif's equilibrium threshold when the goal is endogenous, denoted $\overline{x}^{NI}$, is derived by imposing $r = \hat{x}^{NI} - I$ on Equation (1.26):

$$\overline{x}^{NI} = \frac{1}{(\gamma_2 - 1)(1+\eta)}[\beta(\gamma_2 - \gamma_1)(\hat{x}^{NI} - I)(\frac{\overline{x}^{NI}}{\hat{x}^{NI}})^{\gamma_1} + \gamma_2(\eta\hat{x}^{NI} + I)]. \tag{1.28}$$

His current value function w^{NI} is given by

$$w^{NI}(x, \overline{r} = \hat{x}^{NI} - I) = \begin{cases} \beta(\hat{x}^{NI} - I)(\frac{x}{\hat{x}^{NI}})^{\gamma_1} + \\ (1+\eta)(\frac{\gamma_1 - 1}{\gamma_2 - \gamma_1})(\hat{x}^{NI} - \overline{x}^{NI})(\frac{x}{\overline{x}^{NI}})^{\gamma_2} & \text{if} \quad x < \overline{x}^{NI} \\ x - I + \eta(x - \hat{x}^{NI}) & \text{if} \quad x \geq \overline{x}^{NI}. \end{cases}$$

Proposition 4. *The naive agent stops after the sophisticated agent, but falls short of his goal*[29]*:* $\overline{x}^{SI} < \overline{x}^{NI} < \hat{x}^{NI}$.

The naive agent incorrectly believes that he will be more patient in the future, so he sets higher goals accordingly. But when faced with the stopping decision in the present, he undervalues the future more than he had expected, and stops earlier. Thus, he falls short of his overly ambitious goal. The fact that the naive agent stops after his sophisticated counterpart can be attributed to two factors. The first effect, the "sophistication effect" (O'Donoghue and Rabin, 1999), occurs irrespective

[28]Note that it also corresponds to (1.22) when $\beta = 1$ or $\lambda = 0$.

[29]As shown in Appendix (A.3), this result also holds for exogenous $r \geq 0$: $\overline{x}^{SE} < \overline{x}^{NE} < \hat{x}$, where $\hat{x}$ is given by (1.25). The intuition is essentially identical. When $r = \hat{x} - I$, the naif's over-optimism leads him to set his goal higher than the sophisticate's, making the gap between their stopping thresholds even larger.

of reference dependence. Both the sophisticate and the naif overweight the present in the same manner. But because the sophisticate correctly foresees that he will also undervalue the option to wait in the future, he undervalues the present option to wait even further. In contrast, the naif is more optimistic about his future behavior, so he values the present option relatively more than the sophisticate.[30] The second effect is caused by the interaction of reference dependence and expectations. The naif's optimism leads him to set higher goals for himself than the sophisticate, who realistically tempers his expectations. Since he incurs a higher potential penalty upon stopping, the naif has a larger incentive to wait for a higher project value.

1.5.2 Welfare

In the absence of reference dependence, the naive agent is clearly better off than his sophisticated counterpart from an ex-ante perspective. When $\eta = 0$, there is no comparison utility and the ex ante self wants to maximize the expected present discounted value of his net terminal payoff alone. Therefore, the first best is to employ the threshold $x^* = (\frac{\gamma_1}{\gamma_1 - 1})I$. Since $\overline{x}^{SI}(\eta = 0) < \overline{x}^{NI}(\eta = 0) < x^*$, the naive agent is unambiguously better off due to the "sophistication effect," whereby the sophisticate's realistic pessimism leads to a relatively detrimental outcome (O'Donoghue and Rabin, 1999).

However, the naif's welfare is less rosy when he has reference dependent preferences, because he incurs direct comparative disutility from falling short of his optimistic expectations. From an ex ante perspective, the naif's *true* continuation value when goals are self-set is given by $\tilde{v}^{NI}(x)$:

$$\tilde{v}^{NI}(x, r = \hat{x}^{NI} - I) = \begin{cases} [\overline{x}^{NI} - I + \eta(\overline{x}^{NI} - \hat{x}^{NI})](\frac{x}{\overline{x}^{NI}})^{\gamma_1} & \text{if} \quad x < \overline{x}^{NI} \\ x - I + \eta(x - \hat{x}^{NI}) & \text{if} \quad x \geq \overline{x}^{NI}, \end{cases} \tag{1.29}$$

[30] This is the same result obtained by Grenadier and Wang (2007), whose model corresponds to $\eta = 0$ here.

where $\hat{x}^{NI}$ and $\overline{x}^{NI}$ are defined by Equations (1.27) and (1.28), respectively. Since his unanticipated impulsiveness leads him to fall short of his overly optimistic goal at any point in time, he incurs disutility $\eta(\overline{x}^{NI} - \hat{x}^{NI})$ upon stopping, in addition to receiving the net terminal payoff $\overline{x}^{NI} - I$.

Because the naive agent incurs comparative disutility that his sophisticated counterpart does not, the welfare comparison between naivete and sophistication is not as clear-cut as in the absence of reference-dependence, and there are circumstances in which the naif is unambiguously *worse off* than the sophisticate. For example, consider the case when reference dependence is quite high: let $\eta \geq \eta^*$, so $\overline{x}^{SI} \geq x^*$. Since the naif always waits longer than his sophisticated counterpart, the ex-ante expected present discounted value of his net terminal payoff is strictly less than that of the sophisticate. Furthermore, the naif incurs comparative disutility from falling short of his overly optimistic goal while the sophisticate incurs no comparative disutility. Thus, the naif is worse off than the sophisticate in both components of overall utility when $\eta \geq \eta^*$.

Proposition 5. *When goals are endogenous, let $\tilde{v}^{N^*}(x)$ denote the (true) first-best option value that the naif can attain, and let $v^{S^*}(x)$ denote the first-best option value that the sophisticate can attain. When $\eta > 0$, the first best that the naif can achieve is strictly lower than that of the sophisticate: $\tilde{v}^{N^*}(x) < v^{S^*}(x)$.*

In the previous section, I have shown that the sophisticate's maximum attainable option value is achieved when $\eta = \eta^*$, since he maximizes the expected present discounted value of his consumption utility (i.e., the net terminal payoff) and incurs zero comparison utility by stopping at $\overline{x}^{SI} = x^*$. In contrast, the naif persistently sets his goal to be $\hat{x}^{NI} - I$, and incurs comparative disutility from falling short. Given that he sets this goal, the threshold that would maximize his overall utility, but which *no value of η can attain* due to his over-optimism, is the one that allows him to meet that goal. If this were possible, he would incur no comparative disutility and the option value would only consist of the discounted value of consumption utility from stopping at $\hat{x}^{NI}$. But since $\hat{x}^{NI} > x^*$, the expected present discounted value of stopping at $\hat{x}^{NI}$ is strictly less than that of stopping at x^* for any $\eta > 0$. Thus, in contrast to O'Donoghue and Rabin's (1999) finding that the "sophistication effect" makes sophisticates worse off than naifs in the absence of reference-dependence, the first-best that the naif can achieve is *strictly lower* than that of the sophisticate when goals are self-set.

In contrast to the welfare findings regarding the sophisticate, larger intervention would be required to make naifs as well off as sophisticates. In essence, Proposition 5 implies that the naif's ignorance regarding his limitations is a handicap that cannot be overcome by changing η alone. Beyond education about goal-setting and goal commitment, the naif would need to recognize his limitations in some way, which is arguably more difficult to achieve. In the absence of such learning, more direct intervention through externally set goals is one possible solution to this problem if an outside party is aware of the naif's impulsiveness. Clinical psychologists have long recognized that individuals may persistently fall short of self-set goals while simultaneously maintaining the belief that they can attain them. Hamacheck (1978) describes neurotic perfectionists as individuals "whose efforts - even their best ones - never seem quite good enough, at least in their own eyes. It always seems to these persons that they could - and should - do better . . . " In reality, individuals are likely to lie in between the two extremes of sophistication and naivete. That is, they may be aware of their impulsiveness but underestimate its magnitude, as in O'Donoghue and Rabin's (2001) concept of "partial naivete." The prescription that a "S.M.A.R.T. Goal" should be attainable, and associated advice about how to determine whether a goal is attainable, such as examining past achievement experience, is consistent with recognition that individuals may misperceive their limitations to some degree.

1.6 Aspirational Goals

Thus far, I have made the assumption that the agent's goal must coincide with the outcome he expects to achieve. Because there is no uncertainty over the realized outcome, regardless of whether *beliefs* are correct or not, it is unclear how the agent could simultaneously anticipate a particular outcome with certainty, knowingly set a goal that differs from that expectation, and also derive utility, whether positive or negative, from comparing the realized outcome to a goal that he knew to be unrealistic.

An alternative interpretation is that the goal represents an individual's aspiration, rather than an expectation about what he can or will attain. In this case, the goal may arguably be divorced from expectations without logical conflict. Under this interpretation, he can strategically set goals for future selves, even if he is aware that they are unrealistic ex ante. I assume that the feasible set of goals is given by $r \in [-I, \infty)$.[31] Since a goal can only be set for a future self, it is clear that he seeks to maximize his total utility, i.e. the sum of consumption and comparison utility, from an ex-ante perspective. Thus, the optimal aspirational goal r^* is chosen to maximize the continuation value function $v(x, \hat{r})$ described by Equation (1.11) in its wait region, where the agent anticipates future selves' behavior in response to r^*. In this intrapersonal problem, the agent's optimal goal choice involves a trade-off between the expected present values of consumption and comparison utility. Choosing a more ambitious goal increases his material payoff by providing a stronger incentive to wait for a higher project payoff, but reduces his comparison utility, while choosing a less ambitious goal leads to the opposite effect. Here, the agent does have an incentive to choose goals that may be unrealistic in order to improve his material outcome, and pessimistic goals may be beneficial.[32] The question of interest is what

[31] Since the project payoff process is bounded below by zero, the lowest net terminal payoff the agent can receive is $-I$, if he stops the process when it is zero.

[32] In a somewhat similar vein, Brunnermeier and Parker (2005) consider the optimal

goal optimizes ex-ante welfare when both consumption and comparison utility are accounted for, and under what circumstances the optimal goal is realistic. I focus on the case of sophistication, where the agent is aware of and would like to attenuate his self-control problem.[33] Thus, he anticipates that future selves employ threshold $\overline{x}^{SE}(r^*)$, described by Equation (1.20).

Consider an agent who has no self-control problem (i.e., $\overline{\gamma} = \gamma_1$). Then any positive goal induces him to wait longer than is first-best to maximize expected consumption utility. Thus, he is clearly better off setting a goal which is lower than the outcome he will actually achieve. Even in the more interesting case when the agent has a self-control problem ($\overline{\gamma} > \gamma_1$) and can benefit from additional motivation, however, choosing the lowest possible goal is optimal from an ex ante perspective for *any* degree of present-biasedness, $\overline{\gamma} \in (\gamma_1, \infty)$. For each of the two additively separable components of v, namely the expected discounted values of consumption and comparison utility, the two forces affecting each are the utility incurred upon stopping and the time value of waiting for it. As the agent's goal decreases, the decrease in consumption utility reduces the value function, but time discounting counteracts this reduction, since achieving a lower terminal payoff does not require waiting as long, on average. On the other hand, as the goal decreases, both the increase in comparison utility and time discounting positively affect the value function, since he realizes higher comparison utility and stops earlier due to the weaker force on self-discipline. Consequently, a decrease in the goal raises (expected) discounted comparison utility more than it reduces discounted consumption utility, so that choosing the lowest possible goal is globally optimal when his comparison utility function is linear.

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choice of subjective beliefs when the agent faces a trade-off between material outcomes and belief-based utility. However, in their setting, there is no incentive to distort beliefs in order to change actions. Rather, the only motivation for belief distortion is the benefit for anticipatory utility flows, so any belief distortion decreases material benefits and pessimistic beliefs can only hurt.

[33]Since the naif believes that he has no self-control problem, he sees no instrumental benefit to setting a positive goal.

Proposition 6. *When the agent's comparison function is linear, his ex-ante welfare is monotonically decreasing in his goal r. Thus, the aspirational goal r^* that maximizes ex-ante welfare is the lowest possible.*

The finding that no matter how severe his self-control problem, an agent prefers to set the lowest possible goal is reliant on both the asymmetric effect of time discounting, which is due to the trade-off between consumption and comparison utility, and the linearity of comparison utility, which implies that the marginal benefit from exceeding a given goal is invariant to the gap between goal and outcome. However, it is arguably more realistic to assume that the agent exhibits diminishing sensitivity to gains, so that the marginal benefit from exceeding his goal is decreasing as the gap between low aspirations and realized outcomes increases. Consider a comparison utility function given by $\psi(x - I - r)$, where $\psi(\cdot)$ exhibits the following properties:

A1. $\psi(y)$ is continuous and thrice differentiable for all y.

A2. $\psi(0) = 0$.

A3. $\psi(y)$ is increasing everywhere: $\psi'(y) \geq 0, \ \forall y \in \mathbb{R}$.

A4. $\psi(y)$ is concave: $\psi''(y) \leq 0, \ \forall y \in \mathbb{R}$.

A5. $\psi(y)''' \geq 0, \ \forall y \in \mathbb{R}$.

Clearly, the case studied thus far, in which the comparison utility function is linear, corresponds to the case in which $\psi'(y) = \eta$ and $\psi''(y) = 0$ for all $y \in \mathbb{R}$. The first three assumptions are standard and intuitive features of comparison utility. The fourth implies that when $\psi''(y) < 0$ everywhere, the agent is diminishingly sensitive to gains and increasingly sensitive to losses. While the former is a feature of Kahneman and Tversky's (1979) value function, the latter property is less commonly assumed. Given that the effects of time discounting in the optimal stopping problem bias the agent toward setting goals that are *lower* than anticipated (and realized) outcomes, the key region of interest is that of *gains*, and its feature is consistent with previous work.[34] When $\psi''(y) < 0$ everywhere, comparison utility is asymmetric across gains and losses, and the agent exhibits loss aversion. We also can obtain an exact mapping to the above comparison utility function $\psi(\cdot)$ through an alternative model. A formally equivalent specification is that the comparison utility function is linear, but the agent faces convex costs, incurred upon stopping, of setting a goal that deviates from his actual net terminal payoff, whether that goal is below or above his realized outcome.[35] The fifth assumption is a regularity condition.

When $\psi(\cdot)$ is non-linear, we must derive the optimal stopping threshold given a goal r. As in the linear case, the optimal threshold is determined by joining the waiting and stopping regions of the value function. It is only the stopping region that now changes, since the comparison of an outcome against the goal upon stopping differs. Then the optimal stopping threshold $\overline{x}^{NL}$ is given by the following implicit function:

$$0 = (\overline{\gamma} - 1)\overline{x}^{NL} - \overline{\gamma} I + \overline{\gamma}\psi(\overline{x}^{NL} - I - r) - \overline{x}^{NL}\psi'(\overline{x}^{NL} - I - r), \qquad (1.30)$$

where we can verify that, unsurprisingly, $\overline{x}^{NL}$ is increasing in r and decreasing in $\overline{\gamma}$ given assumptions A1-A4. As in the linear case, the optimal aspirational goal r^* is chosen to maximize the continuation value function $v(x, \hat{r})$ in its wait region, where the comparison utility is now evaluated non-linearly according to $\psi(\cdot)$ and $\overline{x}^{NL}(\hat{r})$ is

[34] Moreover, the assumption of increasing sensitivity to losses can be interpreted as a regularity condition, to ensure that the continuation value function can be single-peaked, but is not strictly necessary to obtain a positive optimal goal.

[35] Costs that rise more quickly with the gap between the goal and the expected payoff thus correspond to a $\psi(\cdot)$ that is more concave.

given by Equation (1.30):

$$r^* = \operatorname*{argmax}_{\hat{r}} v(x, \hat{r}) = \operatorname*{argmax}_{\hat{r}} \ [\overline{x}^{NL} - I + \psi(\overline{x}^{NL} - I - \hat{r})](\frac{x}{\overline{x}^{NL}})^{\gamma_1}. \tag{1.31}$$

An optimal,non-degenerate aspirational goal (i.e., $r^* > -I$) must satisfy the first-order condition, which is equivalent to finding the r^* such that

$$\begin{aligned} &\overline{x}^{NL}\psi''(\overline{x}^{NL} - I - r^*)[-\overline{\gamma} + \gamma_1(1 + \psi'(\overline{x}^{NL} - I - r^*)] \\ &- \overline{\gamma}(\gamma_1 - 1)\psi'(\overline{x}^{NL} - I - r^*)(1 + \psi'(\overline{x}^{NL} - I - r^*)) = 0, \end{aligned} \tag{1.32}$$

where $\overline{x}^{NL}(r^*)$ is given by (1.30). The second term of (1.32) is non-positive for all values of r^* by property A3. Since $\psi(\cdot)$ is concave, the first term is only positive if $\overline{\gamma} > \gamma_1(1 + \psi'(\overline{x}^{NL} - I - r^*))$, where the right-hand side is greater than γ_1. Unsurprisingly, setting a non-generate goal is only desirable if it is needed to correct impulsiveness, which must be sufficiently high to counteract the marginal benefit of setting a lower goal.[36] Moreover, the first term of (1.32) must be sufficiently positive for the equality to hold, which implies that $\psi''(\cdot)$ must be sufficiently negative. The incentive to set a very low goal diminishes when the marginal benefit of exceeding it decreases sufficiently quickly.[37] Thus, comparison utility must diminish sufficiently quickly in gains for the agent to prefer setting a non-degenerate aspirational goal. If either of these conditions is not satisfied at a given goal r, then the left-hand side of (1.32) and the optimal aspirational goal is the lowest possible. Note that when $\psi(\cdot)$ is linear, Equation (1.32) cannot hold and left-hand side is negative, implying that the optimal aspirational goal is the lowest possible, as stated in Proposition (6).

[36]Note that this condition has a clear analog to the case in which goals are expectations over outcomes. As described in Proposition 3, a given degree of reference dependence is only welfare improving if impulsiveness is sufficiently severe relative to marginal comparison utility.

[37]As shown in the proof in the Appendix, the second-order condition is satisfied as long as an (positive) upper bound on $\psi'''(\cdot)$ is satisfied at r^*.

Proposition 7. *For any $r^* > -I$, the following two conditions must be satisfied for r^* to be an optimal aspirational goal:*

1. *The agent is sufficiently impulsive: $\overline{\gamma} > \gamma_1[1 + \psi'(\overline{x}^{NL} - I - r^*)]$.*

2. *Sensitivity to gains is diminishing sufficiently fast: $\psi''(\overline{x}^{NL} - I - r^*)$ is sufficiently negative that Equation (1.32) holds,*

where $\overline{x}^{NL}$ is the stopping threshold given r^* and $\psi(\cdot)$.

Given these findings, it is evident that setting an optimal aspirational goal that is realistic ($r^* = \overline{x}^{NL} - I$) is simply a special case in which the agent sets an aspiration goal that coincides with his outcome. But it is only ex-ante optimal when the conditions described in Proposition (7) at the r^* such that $\overline{x}^{NL}(r^*) - I - r^* = 0$. In particular, the marginal benefit of setting a goal slightly lower than is achievable must be diminishing sufficiently quickly at the origin, implying that he must be sufficiently loss averse over small stakes in order to set a realistic goal.

Corollary 2. *Given any degree of impulsiveness, there exists a unique aspirational goal $\tilde{r}$ that is realistic. That is, for any $\overline{\gamma}$ and $\psi(\cdot)$, there exists a unique $\tilde{r}$ such that $\tilde{r} = \overline{x}^{NL} - I$, given by*

$$\tilde{r} = \left(\frac{1 + \psi(0)}{\overline{\gamma} - 1 - \psi(0)}\right)I. \tag{1.33}$$

However, $\tilde{r}$ is only ex-ante optimal if the conditions of Proposition (7) are satisfied.

As an agent's impulsiveness becomes more severe, the motivational benefit of setting a more ambitious aspirational goal increases. On the other hand, undervaluing the future implies that his responsiveness to a given goal declines as well, weakening the benefits of setting a higher goal relative to the loss in comparison utility. Nonetheless, the former effect dominates the latter when the comparison utility function satisfies A1 and A3-A5. Thus, more impulsive agents set higher aspiration goals for themselves in order to improve their patience, despite the ensuing loss in comparison utility. Although their aspirational goals are higher than those of less impulsive agents, they still stop earlier.

Proposition 8. *More impulsive agents set higher aspirational goals:* $\frac{\partial r^*}{\partial \overline{\gamma}} > 0$. *Nonetheless, more impulsive agents stop earlier: if* $\overline{\gamma}_i < \overline{\gamma}_j$, *then* $\overline{x}_i^{NL} > \overline{x}_j^{NL}$.

If the agent can set an aspirational goal that differs from his expectations regarding his material outcome, then the question of interest is what his optimal goal choice is, given the trade-off between improving his material outcome and decreasing his expected comparison utility. I find that when his comparison utility function is linear, he prefers to set the lowest goal possible. He will only choose a non-degenerate goal if he is sufficiently impulsive and his sensitivity to gains diminishes sufficiently quickly. Because the motivational benefit of setting a more ambitious goal increases with impulsiveness, more impulsive agents set higher aspirational goals, though they still stop earlier than their less impulsive counterparts.

1.7 Social Comparison

Thus far, I have assumed that an agent is solely engaged in self-comparison; his point of reference is his own expected net terminal payoff. But in addition to, or rather than, engaging in self-comparison, he may look to a role model or peer(s) as the basis of comparison. Many previous theories of social comparison have considered their costly and wasteful effects. For example, Frank (1985) argues that the pursuit of status results in a positional arms race that decreases societal welfare, because individuals are engaged in costly signaling, most notably through conspicuous consumption, in a zero-sum game. Here, I explore whether social comparison can serve the functional purpose of attenuating the self-control problem by providing individuals with reference points that allow them to set goals. More generally, I examine the differences in behavior and welfare when an agent engages in self versus social comparisons, focusing on the case of sophistication. Previously, Battaglini et al. (2005) have shown how the presence of a peer can ameliorate or exacerbate one's impulsive behavior, since his behavior can act as either "good news" or "bad news" about one's own ability to resist temptation. In their model, because agents have incomplete information about their own and their peers' vulnerability to temptation but are aware that they are correlated, observing a peer's behavior affects one's decisions by offering information about one's own degree of self-control. On the other hand, I consider the effects of social comparison under full information and instead emphasize how they can be motivated to exercise patience because they derive utility from comparing their own outcomes to others'.

Consider two agents, i and j, who compare themselves against one another. When the agent's goal is no longer a matter of pure self-comparison, the natural extension of sophistication is that the agent's goal must be derived from correct expectations about his own and his peer's outcomes, given full information about one another's characteristics.[38] Suppose that each agent's goal, or reference point, is a convex combination of his own expected net terminal payoff and his peer's[39]:

$$r_i = \alpha_i(\overline{x}_i - I) + (1-\alpha_i)(\overline{x}_j - I), \text{ where } a_i \in [0,1]$$
$$r_j = \alpha_j(\overline{x}_j - I) + (1-\alpha_j)(\overline{x}_i - I), \text{ where } a_j \in [0,1].$$

If $\alpha_i = 1$, agent i evaluates himself exclusively against his own expected net terminal payoff, so this case is equivalent to the preceding analysis of an individual agent. If

[38] As in the case of self-comparison, the model could also be extended to the case in which the agent persistently misperceives his peer's "ability," with similar insights.

[39] Due to linearity of the reference point and of the comparison utility function, this formulation is equivalent to assuming that an agent separately compares his net terminal payoff against its own expectation and against the expectation of his peer's net terminal payoff, and that his overall comparison utility is formed by taking a convex combination, dictated by his parameter α, of these two comparisons.

$\alpha_i = 0$, agent i evaluates himself exclusively against his expectation of his peer's (i.e., agent j's) net terminal payoff.[40] I also allow for differences in agents' degrees of impulsiveness (β_k, λ_k), reference dependence (η_k), and the characteristics of their projects (μ_k, σ_k), where $k = i, j$. Note that if the agents in a peer group have differing net terminal payoffs in equilibrium, they will incur non-zero comparative utility. Contrary to the individual case, an agent k can thus *fall short of or exceed his goal* if that goal is influenced by the behavior of another ($\alpha_k > 0$). For this reason, a welfare analysis of social comparison from the ex ante perspective must include both consumption and comparison utility. In this context, I explore the effects of homogeneous and heterogeneous peer groups on stopping thresholds and welfare.

1.7.1 Homogeneous Agents

Consider first the simplest case, in which agents i and j are identical in all respects: $\mu_i = \mu_j, \sigma_i = \sigma_j$, $\lambda_i = \lambda_j$, $\beta_i = \beta_j$, $\eta_i = \eta_j$, and $\alpha_i = \alpha_j$. For clarity of exposition, I drop the subscripts on all matching parameters.

Recall that Equation (1.20) describes an agent's optimal threshold in a stationary equilibrium with an arbitrary goal level. Therefore, I can immediately derive the optimal thresholds in a homogeneous peer group by substituting the appropriate goals. Since each agent has complete information about both his own and his peer's preferences, the optimal thresholds must satisfy the following system of equations

[40] Although the expectation of a peer's net terminal payoff and that peer's actual material outcome coincide, I assume that an agent's goal is the expectation, so that his comparison utility is incurred when he himself stops. If the comparison were made once both agents had stopped, then comparison utility could be incurred after one's stopping time. For example, if two identical agents face separate payoff processes with identical parameters, they will use the same thresholds but stop at different times. Since the stopping is irreversible and the expectation of his peer's outcome coincides with its realization, the assumption that the comparison is assessed upon stopping against an expectation seems reasonable.

simultaneously in equilibrium:

$$\overline{x}_i = (\frac{\overline{\gamma}}{\overline{\gamma}-1})I + r_i(\frac{\eta}{1+\eta})(\frac{\overline{\gamma}}{\overline{\gamma}-1}) \tag{1.34}$$

$$\overline{x}_j = (\frac{\overline{\gamma}}{\overline{\gamma}-1})I + r_j(\frac{\eta}{1+\eta})(\frac{\overline{\gamma}}{\overline{\gamma}-1}), \tag{1.35}$$

where $r_i = \alpha(\overline{x}_i - I) + (1-\alpha)(\overline{x}_j - I)$ and $r_j = \alpha(\overline{x}_j - I) + (1-\alpha)(\overline{x}_i - I)$.

Proposition 9. *For the sophisticated agent, belonging to a homogeneous peer group is equivalent to pure self-comparison. In equilibrium,*

$$\overline{x}_i = \overline{x}_j = (\frac{\overline{\gamma}}{\overline{\gamma}-1-\eta})I, \quad \textit{where } \eta < \overline{\gamma} - 1.$$

Unsurprisingly, the degree to which agents in a homogeneous group uses a peer's net terminal payoff rather than his own as a component of his own goal is irrelevant: $\frac{\partial \overline{x}_i}{\partial \alpha} = 0$. Intuitively, there is no difference between comparing oneself against one's own expected outcome and comparing oneself against the expected outcome of an identical twin. Hence, all of the preceding welfare analysis for the individual case applies for agents in a homogeneous peer group. Nonetheless, having an identical peer certainly induces patience relative to having no goal at all ($\eta = 0$), and is welfare improving for both parties if the degree of reference dependence is not too high.

1.7.2 Heterogeneous Agents

A more interesting environment arises when agents are heterogeneous, since comparative utility may be non-zero in equilibrium. In particular, I focus on the case in which the agents differ in their degrees of impulsiveness, $(\beta_i, \lambda_i) \neq (\beta_j, \lambda_j)$, but are identical in all other characteristics $(\alpha, \eta, \mu, \sigma)$.[41] When peers have different degrees

[41] Allowing the other parameters to differ leads to similar comparative statics in the expected directions. I focus on heterogeneity in self-control alone in order to highlight

of self-control, how does the degree of equilibrium patience vary between them? If there is a difference in their threshold strategies, then both agents incur comparative utility, so an evaluation of ex ante welfare must include any utility or disutility incurred from the falling short of or exceeding a goal. Thus, the choice of a peer who will maximize ex ante welfare includes a trade-off between the expected discounted value of the two components of his overall utility, the material payoff and comparison utility.

For simplicity, suppose that each agent derives comparison utility exclusively from the expected outcome of his peer, $\alpha_i = \alpha_j = 0$, implying that $r_i = \overline{x}_j - I$ and $r_j = \overline{x}_i - I$ in equilibrium.[42] As before, I use $\overline{\gamma}$ as the measure of impulsiveness, where $\overline{\gamma}_k = \beta_k\gamma_1 + (1-\beta_k)\gamma_{2k}$ for $k = i, j$. Without loss of generality, let agent i have more patience than agent j: $\overline{\gamma}_i < \overline{\gamma}_j$. As above, we can use Equation (1.20) and substitute the relevant goals to obtain the optimal thresholds:

$$\overline{x}_i = (\frac{\overline{\gamma}_i}{\overline{\gamma}_i - 1})I + (\overline{x}_j - I)(\frac{\eta}{1+\eta})(\frac{\overline{\gamma}_i}{\overline{\gamma}_i - 1}) \quad (1.36)$$

$$\overline{x}_j = (\frac{\overline{\gamma}_j}{\overline{\gamma}_j - 1})I + (\overline{x}_i - I)(\frac{\eta}{1+\eta})(\frac{\overline{\gamma}_j}{\overline{\gamma}_j - 1}). \quad (1.37)$$

The key features of such an interpersonal equilibrium are illustrated in Figure 1.2, where the equilibrium thresholds lie at the intersection of the agents' optimal threshold functions and $\eta < \min\{\overline{\gamma}_i - 1, \overline{\gamma}_j - 1\}$.[43]

Proposition 10. *In a heterogeneous peer group:*

1. *Relative to having no goal ($\eta = 0$), having any peer increases an agent's stopping threshold, even if the peer is more impulsive:* $\frac{\partial \overline{x}_i}{\partial \eta} > 0$ *for all* $\overline{\gamma}_j \in (1, \infty)$.

the effect of heterogeneity in a situation that has a more conceptually interesting interpretation.

[42]The results can easily be extended the case when $\alpha_i \neq 0$ and $\alpha_j \neq 0$.

[43]This is a technical assumption to ensure that the equilibrium thresholds are bounded. It implies that if each agent were to engage in self-comparison (or were part of a homogeneous peer group), his threshold would be positive and well-defined.

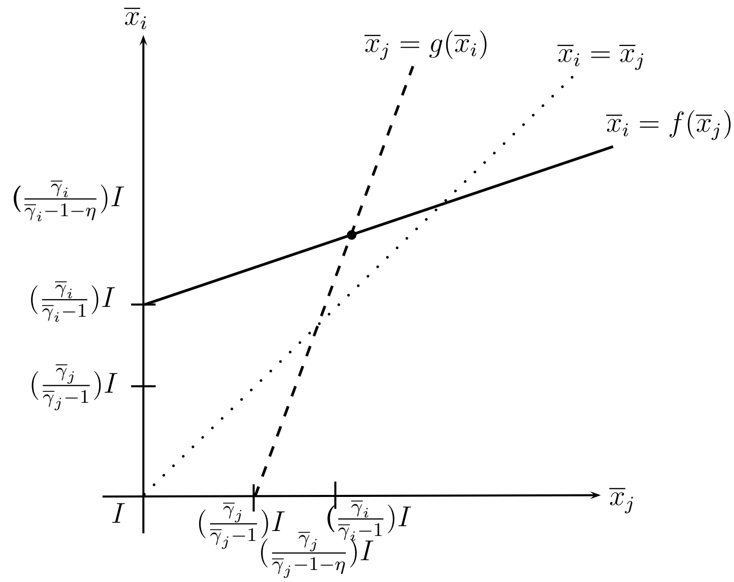

Figure 1.2: Interpersonal equilibrium with heterogeneous agents: $\overline{\gamma}_i < \overline{\gamma}_j$. The solid line depicts agent i's optimal threshold as a function of j's threshold. The dashed line depicts agent j's optimal threshold as a function of i's threshold. The dotted line represents the 45° line, where the agents' thresholds are equal.

2. *Each agent's stopping threshold is decreasing in his peer's degree of impulsiveness:* $\frac{\partial \overline{x}_i}{\partial \overline{\gamma}_j} < 0$ *and* $\frac{\partial \overline{x}_j}{\partial \overline{\gamma}_i} < 0$.

3. *The more patient agent always has a higher stopping threshold than his more impulsive peer: If* $\overline{\gamma}_i < \overline{\gamma}_j$, *then* $\overline{x}_i > \overline{x}_j$.

In the absence of a goal or peer, an agent k stops too early, choosing the stopping threshold $(\frac{\overline{\gamma}_k}{\overline{\gamma}_k-1})I$. A peer provides the agent with a goal that increases the potential cost of settling for a lower project value, since he incurs a comparative penalty if he falls short of his peer. The fact that the peer is relatively more impulsive merely implies that the goal will not be as high as it would be with a less impatient peer. Hence, the potential penalty and consequently, the agent's incentive to wait, increase with his peer's degree of self-control. This result is consistent with Bandura and Jourden's (1991) finding that individuals adjust their goals in response to their peers' performance. When their peers' performance is lower than their own, individuals adjust their goals downward and are content with a lower performance level because they are still outperforming their peers. In particular, as long as at least one member of the group exhibits some self-control (e.g., $\overline{\gamma}_i$ is finite), *both* members exhibit more patient behavior than they would in the absence of goal, even if the other has a preference for instantaneous gratification (i.e., $\overline{\gamma}_j \to \infty$). Note that the mitigation of j's preference for instantaneous gratification stands in contrast to the purely individual and homogeneous cases, when goal-setting could not improve his patience. When agent i has some self-control, however limited, he stops at $\overline{x}_i > I$ even in the absence of a goal. This sets a potential penalty that j wishes to avoid, mitigating his extreme impulsiveness; in turn, j's more patient behavior has a positive feedback effect on i's own behavior.[44] It is only when both agents have a preference for instantaneous gratification that social comparison does not improve patience, just as in the homogeneous peer group or in the individual setting, when j is unable to provide such a penalty for himself. Because the agent with more self-control is able to wait longer even in the absence of a goal and is more responsive to goals, he sets a higher stopping threshold than his more impulsive counterpart.

1.7.3 Optimal Peers

I now consider the choice of peer that maximizes ex ante welfare, maintaining the assumption that $r_i = \overline{x}_j - I$ and $r_j = \overline{x}_i - I$.[45] This decision involves a trade-off between the expected present values of consumption and comparison utility. Choosing a relatively less impulsive peer increases the material payoff but induces negative

[44] This feedback effect between heterogeneous peers certainly does not rely on $\eta_i = \eta_j$, and is proportional to the product $\eta_i \eta_j$.

[45] The optimal choice of peer groups for agents with self-control problems was also studied in Battaglini et al. (2005). As explained earlier, however, the observation of a peer's behavior affects an agent's behavior purely through its informational content. Thus, the optimal peer is determined by the quality of information inferred from his behavior. Here, the choice of peer weighs the instrumental effect on self-control against the value of relative performance or achievement.

comparison utility and increases the expected wait, while choosing a more impulsive peer leads to the opposite effect.

Proposition 11. *In a heterogeneous peer group, an agent's ex ante welfare* v_i *is monotonically increasing in his peer's degree of impulsiveness:* $\frac{\partial v_i}{\partial \overline{\gamma}_j} > 0$. *Thus, the partner* j^* *who maximizes agent* i*'s ex ante welfare is the most impulsive possible:* $\overline{\gamma}_j^* \to \infty$ *for all* $\overline{\gamma}_i \in (\gamma_1, \infty)$.

The finding that any agent, no matter his degree of impulsiveness, prefers to choose the most impulsive peer possible mirrors the result stated in Proposition 6, in which every agent with a linear comparison utility function sets the lowest aspirational goal possible.[46] This similarity is unsurprising, since choosing a peer whose expected outcome can differ from one's own is akin to choosing an aspirational goal that can differ from one's expected outcome. Both the agent who chooses a peer for comparison and the agent who chooses an aspirational goal face the same trade-off between consumption and comparison utility, and the same forces that affect an aspirational goal-setter, described in Section 1.6, apply to the agent who selects the optimal peer.

As in the case of intrapersonal comparison, there is the question of whether the marginal utility of a social comparison remains the same between any pair of peers. It seems reasonable that two peers with similar characteristics might choose to compare themselves against one another. But at some point, the gap between two agents may be too large to merit credible comparison. For example, it seems unlikely that an honors student derives significant utility from comparing himself to a remedial student. Diminishing sensitivity to comparative gains would be consistent with this phenomenon and would lead to a less extreme choice of the optimal peer, just as in

[46] In contrast, Battaglini et al. (2005) find that the optimal peer is always slightly weaker, $0 < \beta_j < \beta_i$, when agents infer information about their own degree of self-control by observing the behavior of others. There, having a much weaker partner is undesirable because it is "bad news" about one's own willpower, while a stronger partner provides relatively less information.

the case of aspirational goal-setting with diminishing sensitivity to gains, described in Proposition 7.

1.8 Conclusion

This paper addresses the role of self-set, non-binding goals as a source of internal motivation in an optimal stopping problem. When agents have linear reference-dependent preferences and endogenously set a goal regarding the expected outcome that serves as the reference point, they can attenuate the self-control problem, and sophisticates can even achieve the first best from an ex ante perspective. Too much reference dependence, on the other hand, leads an agent to wait longer than the first best, and is always detrimental in the absence of present-biased time preferences. In contrast to the "sophistication effect" that occurs in the absence of reference dependence, the first best that naifs can achieve is strictly worse than that of their sophisticated counterparts. The naif's over-optimism is detrimental because it leads him to set overly ambitious goals that he cannot achieve. Notably, none of the effects of goal-setting require any form of loss aversion or curvature in the comparison utility function, nor do they rely on ex-ante uncertainty over outcomes. These findings suggest that the demand for goods and services aimed at educating individuals about goal-setting and changing their goal commitment is, to some degree, a rational response to impulsiveness, and that the need for stringent external enforcement may be overestimated. But while sophisticated individuals may be able to improve their welfare purely through these means, naive individuals may require more extensive measures to correct for over-optimism.

When the goal represents an aspiration and can be divorced from expectations, there exists a trade-off between consumption and comparison utility. I find that the agent must be sufficiently present-biased and exhibit diminishing sensitivity to gains in order to set a non-degenerate aspirational goal. In this case,more impulsive agents set higher aspirational goals to counteract impatience. Otherwise, the incentive to maximize comparison utility dominates and he prefers to set the lowest aspirational goal possible.

I also extend the model to include social comparisons, where each agent compares his material outcome against the expectation of his peer's. When a peer group is homogeneous, engaging in social comparison is identical to purely self-centered comparison. In a heterogeneous group where agents have differing degrees of impulsiveness, comparison to *any* peer induces more patient behavior relative to the absence of a goal. Comparison to increasingly patient peers naturally induces increasingly patient behavior. Nonetheless, every agent prefers to compare himself to a peer with the lowest degree of self-control possible, regardless of the severity of his impulsiveness.

The current model limits attention to a single stopping decision. A natural extension, which is the subject of current work, is to consider the problem of goal-setting when the agent faces a multi-stage project. In this richer setting, the agent may also be able to "break down" or aggregate goals to improve his welfare. This decision corresponds to Read et al.'s (1999) discussion of "motivated bracketing," where an agent with a self-control problem chooses to frame a problem narrowly or broadly to accomplish a goal. I pursue this idea in Hsiaw (2009a) and show that in the presence of uncertainty over outcomes and loss aversion, the decision to engage in narrow or broad framing involves a trade-off between greater motivation and more expected comparative disutility due to outcome variance when goal evaluation is frequent. Similarly, the agent could perform the same exercise when he faces multiple projects. There, an important consideration will be the timing of the comparative evaluation if a goal pertains to multiple outcomes that are realized at different times.

While this paper considers situations in which goals are evaluated upon the endogenous stopping time, one can imagine situations in which the relevant goals are time-dependent. For example, a dieter might set a target weight to achieve by the end of the year, and feel bad about herself on December 31 if she has not reached it. In social comparisons, attaining a favorable outcome before a peer or rival can provide positive utility as well. In such settings, an agent might explicitly set time-contingent goals or engage in continual evaluation, relative to expectations about himself or to his peer.

Finally, the interaction of hyperbolic discounting and reference-dependent preferences has implications for a number of contexts beyond intrapersonal motivation. For example, contracting between a principal and agent may be underestimating the effect of external incentive schemes if they are reinforced by goal-setting. Likewise, the existence of peer effects through social comparison suggests that group or team settings may be beneficial to managers or educators who are interested in improving productivity. Further pursuit of these or related lines of inquiry would enrich our understanding of such effects and their interactions with standard mechanisms.

Chapter 2

Goal Bracketing and Self-Control

2.1 Introduction

When contemplating a series of potentially difficult choices to achieve a desired outcome, individuals often realize that they will face the temptation to abandon their efforts, despite the foreknowledge that persistence is beneficial. One commonly suggested method of improving perseverance is to take such choices "one step at a time." Most literally, an individual training for a marathon may mentally break down a planned 10,000 meter run into 1,000 meter increments, then focus on achieving each segment sequentially in order to avoid the temptation to give up early. Likewise, dieters often break down an overall weight target into intermediate targets, and long-term projects, like writing a book, can be segmented into intermediate components.

Neoclassical theory assumes that when making many choices, an agent maximizes utility by considering one choice in conjunction with the consequences of other choices. However, there exists extensive evidence that individuals generally do not make decisions on such a global basis. Rather than considering choices broadly, they tend to consider each choice in isolation - that is, they bracket narrowly - in the domains of both consumption (Heath and Soll, 1996) and risk (Tversky and Kahneman, 1981; Gneezy and Potters, 1997; Rabin and Weiszacker, 2009). Similarly, the concept of mental accounting posits that individuals allocate expenses into various categories, so that they essentially engage in narrow bracketing within each category (Thaler, 1999; Heath and Soll, 1996). Both literatures have suggested, but not formalized, the idea that these practices may be employed as a means of self-control.

In this paper, I therefore study the optimal level, frequency, and grouping of outcome-based goals, which act as milestones or targets. In the language of mental accounting, I study how an individual improves motivation by creating categories, where he places decisions into separate categories and evaluates himself against goals for each category. Since frequent goal evaluation can be expected to increase motivation, a natural question is why an individual might choose to set broader, aggregate goals that are evaluated less frequently and more holistically. Intuitively, the agent may dislike frequent evaluation if there is ex-ante uncertainty regarding how he will fare relative to his goal, since there is a greater likelihood that he incurs disutility from falling short at each milestone. Setting broader goals allows the agent to pool the risk from uncertain endeavors, minimizing the expected comparative disutility from evaluation.[1] For example, consider a writer who is completing a book. He enforces stronger motivation if he evaluates each chapter or section individually against his expectation of its quality upon its completion. But given ex-ante uncertainty about the quality of each chapter, he is more likely to disappoint himself at one of these stages, so he might instead choose to evaluate himself less frequently in order to avoid this. That is, he could instead evaluate the entire book against his expectation of overall quality, which he does only once it is completely written.

Formally, I consider the optimal bracketing of self-set goals, which I model as reference points for an agent with reference-dependent preferences. Such an agent derives utility from both standard consumption, arising from outcomes, and comparison utility, arising from the comparison of these outcomes to a self-set goal. At each point in time, his goal is based on his expectations about the outcome of future behavior, which his next "self" will inherit as a reference point in his utility function. In this way, he provides a degree of internal motivation that is desirable if he exhibits time inconsistency due to present-biased time preferences. I assume that the agent is loss averse, which leads him to prefer one-shot rather than gradual resolution of ex-ante uncertainty over outcomes. In addition, the agent determines whether to set incremental goals, evaluating "one step at a time," or to set aggregate goals, evaluating less frequently and more holistically. Finally, I assume that he is sophisticated,

[1]This intuition has been previously been explored in psychology, where more immediate goals are "proximal" and temporally distant goals are "distal." Kirschenbaum (1985, p. 503) states "Using very specific and proximal plans may lead to many failures to reach short-term subgoals. Failures to achieve such subgoals will occur frequently because people often fail to predict accurately fluctuations in situational demands . . . "

implying that his goals must ultimately coincide with the rational expectation of what he will actually do.

The economic setting I consider is a sequential stopping problem in continuous time with an infinite horizon, in which there exists an option value of waiting due to uncertainty. In the context of completing a book, a writer must complete the first chapter before continuing on to a second. Just as in a single optimal stopping problem, present-biasedness leads the agent to stop too early because he undervalues this option relative to his time-consistent counterpart.

I show that the extent to which an agent with a self-control problem chooses to bracket his decisions intertemporally is determined by a tradeoff between motivational power and expected disutility from ex-ante uncertainty over outcomes. Formally, I solve for the unique stationary Markov equilibrium of the intrapersonal game when the agent sets either incremental or aggregate goals, then determine the conditions under which each form of goal bracketing maximizes the value of the sequential stopping option ex-ante. Either form of goal bracketing attenuates the self-control problem, relative to having no goal. But setting an aggregate goal is less effective at improving patience than setting an incremental goal because aggregating outcome uncertainty across stages decreases the anticipated disutility incurred upon stopping. Thus the incentive to wait longer in order to compensate for such disutility is weaker. However, when the agent is sufficiently reference-dependent and loss averse and when ex-ante outcome uncertainty is sufficiently high, an aggregate goal is preferable to incremental goals for precisely this reason. Moreover, when the agent is present-biased, whether he stops earlier or later than the first-best is dependent on the level of ex-ante uncertainty. In particular, he stops earlier than is first-best when ex-ante uncertainty is sufficiently low, and later than is first-best when it is sufficiently high.

The paper proceeds as follows. Section 2.2 links this paper to related lines of research. Section 2.3 describes the model. Section 2.4 describes the general method of equilibrium construction. Section 2.5 characterizes the stationary Markov equilibrium when the agent sets incremental goals, while Section 2.6 considers the case of aggregate goals. Section 2.7 derives the conditions under which each form of goal bracketing maximizes the ex-ante value of the sequential stopping option. Section 2.8 summarizes the results and discusses avenues for future research. Proofs are gathered in the Appendix.

2.2 Related Literature

This paper connects several lines of research. First, it links the work on reference dependence with that on self-control, particularly due to hyperbolic discounting, through the concept of self-imposed, non-binding goals. Second, it relates to the literature on choice bracketing.

Psychologists have long posited that goals serve as a reference standard in a cognitive comparison process of self-evaluation and satisfaction (Latham and Locke, 1991; Locke and Latham, 2002; Bandura, 1989), while Heath et al. (1999) explicitly argue that a goal acts as the reference point in the prospect theory value function formulated by Kahneman and Tversky (1979). Goal-setting as a mechanism for self-control is discussed by Loewenstein (2007) with respect to mountaineering, and has been the subject of recent interest by economists. Hsiaw (2009b) addresses the role of self-set, non-binding goals to attenuate the self-control problem of a hyperbolic discounter in the context of an optimal stopping problem in continuous time. I show that even in the absence of loss aversion or diminishing sensitivity, outcome-based goal-setting can attenuate the hyperbolic agent's tendency to undervalue the option and stop too early, regardless of whether he is sophisticated or naive. However, too much reference dependence can itself be a source of intrapersonal conflict, causing an agent to wait longer than the first best. Suvorov and van de Ven (2008) and Koch and Nafziger

(2008) study a three-period problem where costly effort on a task is required to receive a delayed benefit, and a sophisticated agent sets a goal regarding both effort and task benefit. They study the interaction between uncertainty and loss aversion and find that the latter is necessary for goals, formulated as plans much like those specified by Kőszegi and Rabin (2009), to affect behavior.

Kőszegi and Rabin (2006) develop a model of reference dependent preferences in which an individual derives utility from both final outcomes and comparison to a reference point, which is endogenously determined by rational expectations. This framework has also been extended to a dynamic setting, where agents derive comparison utility from the revision of beliefs (Kőszegi and Rabin, 2009; Matthey and Dwenger, 2007; Matthey, 2008). Abeler et al. (2009) find support for the theory that expectations serve as the reference point in a task experiment. There also exists empirical evidence, though somewhat debated, that people use reference points in the form of income targets in their decisions to trade off labor and leisure (Camerer et al., 1997; Goette and Huffman, 2005; Farber, 2005, 2008; Crawford and Meng, 2008).

Much attention has been devoted to the means through which individuals can attenuate self-control problems that arise from intrapersonal conflict, particularly through external commitments. Binding restrictions that individuals can effectively self-impose include those on choice sets (Gul and Pesendorfer, 2001), information (Brocas and Carrillo, 2005; Carrillo, 2005), and asset liquidity (Laibson, 1997). In contrast, the study of internal regulation is less extensive, though growing. Bénabou and Tirole (2004) develop a theory of internal regulation through self-enforcing personal rules based on a mechanism of self-reputation, while the aforementioned papers study the use of non-binding goals (Hsiaw, 2009b; Suvorov and van de Ven, 2008; Koch and Nafziger, 2008). Suvorov and van de Ven (2008) and Koch and Nafziger (2009a) also extend the goal-setting framework to contingent self-rewards.

There exists extensive evidence that individuals generally do not make decisions on the global basis assumed in neoclassical theory. Rather, they tend to consider choices in isolation - that is, they bracket narrowly - in the domains of both consumption (Heath and Soll, 1996) and risk (Tversky and Kahneman, 1981; Gneezy and Potters, 1997; Rabin and Weiszacker, 2009). The concept of narrow bracketing, in conjunction with loss aversion, has been applied in behavioral finance to explain a variety of empirical puzzles, such as the equity premium puzzle (Benartzi and Thaler, 1995), low stock market participation (Barberis et al., 2006), and individual investors' portfolio choices (Odean, 1998). The concept of mental accounting posits that individuals create budgets by earmarking expenses into categories, so that they essentially engage in narrow bracketing within each category (Thaler, 1999; Heath and Soll, 1996).

While it is evident that individuals engage in narrow bracketing, the question of when and why they choose to do so has been relatively unexplored and has rarely been formalized. Consistent with the interpretation of bracketing as a result of cognitive limitations rather than an optimization problem (Read et al., 1999), most empirical work has inferred the degree of bracketing necessary to explain observed phenomena (Benartzi and Thaler, 1995; Odean, 1998), while existing theoretical work has studied behavior under the assumption of some form of narrow bracketing (Barberis et al., 2001, 2006). In contrast, Read et al. (1999) suggest the motivated use of narrow versus broad bracketing intertemporally, such as recovering alcoholics taking things "one day at a time" or dieters viewing a single candy as part of a larger health plan, as a means of self-control. Similarly, Thaler (1999) discusses how people may deliberately engage in mental accounting across purchase categories, by assigning "tempting" goods to categories with small budgets. The psychological literature includes studies of intertemporal bracketing in the form of short-term versus long-term goals, referred to as proximal versus distal goals, respectively, for repeated or lengthy tasks. These studies usually involve the comparison of subjects' performance when proximal versus distal goals are *assigned* by the experimenter, and tend to find a larger response to goals when they are proximal (Bandura and Simon, 1977; Locke and Latham, 2002). Recognizing that the terms "proximal" and "distal" are inherently relative, Kirschenbaum (1985) considers the circumstances under which proximal versus distal goals are preferable and pinpoints the central factor as a trade-off between motivation and uncertainty. In parallel and independent work, Koch and Nafziger (2009b) extend the three-period model developed in Koch and Nafziger (2008), involving a task with immediate costs and delayed benefits, to study the trade-off between risk pooling and motivation in an agent's bracketing decision as the probability of incurring a loss relative to the self-set goal varies. They find that when tasks are conducted sequentially, narrow bracketing is always optimal because the motivation provided by a broad goal is too weak to be preferable ex ante. When tasks are completed simultaneously, narrow bracketing is only optimal when the ex-ante probability of falling short of the goal and incurring a loss is sufficiently small. In contrast, I consider a continuous-time, sequential stopping problem with an infinite horizon, where the self-control problem arises purely from the tension between waiting and stopping now. Moreover, I consider the effect of mean-preserving spreads over the agent's expected outcome, in order to isolate the effect of uncertainty alone. That is, the ex-ante probability of falling short of the goal is fixed, but the magnitude of expected comparative utility varies with the degree of uncertainty. While goal evaluation can only occur at fixed points in time in the model constructed in Koch and Nafziger (2008), the sequential stopping problem studied here endogenizes the timing of goal evaluation given the bracketing choice, leading to a different intuition for the agent's behavior under broad bracketing. Consequently, I find that broad bracketing can be optimal if the agent's aversion to ex-ante uncertainty is sufficiently strong. In a different conception of proximal and distal goals, Suvorov and van de Ven (2008) find that the optimal use of proximal goals, which are modeled as non-contingent goals, versus distal goals, which are formulated as contingent goals, depends on the correlation of cost shocks involved in a task that takes place over two periods.

2.3 The Model

I first describe the economic environment, followed by the agent's preferences, which may include both hyperbolic discounting and reference dependence. I focus on a sequential optimal stopping problem, where the self-control problem arises purely from the tension between stopping today and waiting for a better outcome. A number of stopping problems can naturally and more realistically be reframed as sequential stages. For example, completing a book can involve writing one chapter after another, or completing a preliminary draft before submitting a polished manuscript. Other problems usually studied in the self-control literature, such as smoking, dieting, or exercising, can also involve multiple stages of (dis)investment.[2]

2.3.1 Sequential Stopping

I consider a sequential continuous-time stopping problem, in which an infinitely lived agent is engaged in a two-stage project. For example, consider an individual who is writing a book. First, he must complete the first chapter at some cost I_1. Completing it buys himself another option to write a second chapter at cost I_2. Thus he must complete the first stage in order to continue to the second.

Formally, the agent decides whether to stop or to wait, based on an observation of the current value of the first-stage payoff. In the latter case, the payoff of the first stage, $x_{1t} \in [0, \infty)$, evolves as a geometric Brownian motion:

$$dx_{1t} = \mu_1 x_{1t} dt + \sigma_1 x_{1t} dz_1, \tag{2.1}$$

[2]For example, training for a marathon can be broken up into stages of a training regimen. Many individuals who are trying to recover from an addiction participate in n-step programs.

where z_1 is a standard Wiener process, μ_1 the average growth rate of x_{1t}, and σ_1 its standard deviation per unit time. Completion of the first stage of the project at time $\bar{t}_1$ yields the lump-sum terminal payoff $x_{1\bar{t}_1}$. For the writer, x_{1t} can be interpreted as a measure of the chapter's quality. In addition, it yields the option to complete a second stage. The cost of completing the first stage at any time is $I_1 > 0$, and is incurred only at the stopping time.[3] In the case of writing a book, this terminal cost may be the cost of finalizing and printing the manuscript and sending or shopping it around to an editor. Without loss of generality, there is no interim flow payoff, nor any direct cost incurred prior to stopping.[4]

Conditional on completion of the first stage, the agent again decides whether to stop or to wait, based on an observation of the current value of the second-stage payoff. In the latter case, the payoff of the second stage, $x_{2t} \in [0, \infty)$, evolves as a geometric Brownian motion:

$$dx_{2t} = \mu_2 x_{2t} dt + \sigma_2 x_{2t} dz_2, \tag{2.2}$$

where z_2 is a standard Wiener process, μ_2 the average growth rate of x_{2t}, and σ_2 its standard deviation per unit time. The second-stage payoff process x_{2t} only starts upon completion of the first stage, at time $\bar{t}_1$, and evolves thereafter independently of x_{1t}, which terminates at time $\bar{t}_1$. The processes x_{1t} and x_{2t} are only linked at

[3]Here, there is no intertemporal separation of the costs and benefit. The agent's self-control problem arises purely from the tension between the option value of waiting for an uncertain period of time and stopping today at a known project value. In contrast, Brocas and Carrillo (2005) and Miao (2008) study irreversible consumption in discrete time models where costs are delayed until after consumption. Separating the costs and benefits of stopping in such a manner certainly exacerbates the self-control problem, but is not necessary to produce intrapersonal conflict.

[4]A more realistic setting might include a constant flow payoff $y \in (\underline{y}, \infty)$, where $\underline{y} < 0$ is the minimal flow payoff such that the agent stops immediately for any $x_{1t} \geq 0$. For example, a writer might incur some small positive or negative flow payoff from writing. Given the other assumptions, the inclusion of a constant flow payoff has no qualitative effect on the results, so I assume $y = 0$ for simplicity of exposition. Likewise, incorporating a stochastic flow payoff that follows a known process with known current value leads to the same qualitative results.

one point in time, $\bar{t}_1$. In particular, the initial value of x_{2t} is some proportion of the payoff value of the first stage process upon its termination, $x_{1\bar{t}}$. In a number of settings, including the one described above, it seems natural that the initial value of a payoff process depends on the payoff from the preceding stage. For example, the initial value of the second chapter's expected quality may be lower if the writer was not diligent on the first; equivalently, he expects to wait longer for the quality of the second chapter to reach a certain level if he has devoted less time to his first chapter than if he had meticulously honed his writing skills on it.

Completion of the second stage of the project at time $\bar{t}_2$ yields the lump-sum terminal payoff $x_{2\bar{t}_2}$. The cost of stopping the second stage at any time is $I_2 > I_1 > 0$, and is incurred only at the second-stage stopping time.[5] Thus, each stage of the project is a standard optimal stopping problem, where the agent can only complete the second stage by completing the first.[6] Note, however, that there is nothing to preclude the agent from completing both stages simultaneously if it is optimal to do so.

I assume that $\mu_1 = \mu_2 \equiv \mu$ and $\sigma_1 = \sigma_2 \equiv \sigma$, for simplicity. However, note that the processes x_{1t} and x_{2t} still evolve independently. None of the results rely on or require this simplification, which is made for ease of exposition.

2.3.2 Uncertainty

In each stage i where $i = 1, 2$, the agent observes the project payoff values imperfectly, and does not learn the true realizations of x_{it} until *after* he has made the stopping decision. In the context of the above example, the writer decides when to stop working on the first chapter without knowing exactly what its quality is, and only learns once

[5]The assumption that $I_2 > I_1$ is made for simplicity but is not necessary. If $I_2 < I_1$, all qualitative results hold with slightly different parameter restrictions.

[6]Of course, the two-stage problem can naturally be extended into several more stages, but all intuitions remain the same.

he has received information from his editor.

Let $\tilde{x}_{it}$ be the observed payoff value in stage i. I assume a discrete, two-point distribution over the noise regarding the payoff processes for tractability. In particular, $\tilde{x}_{it} = (1+\zeta_i)x_{it}$ for $i = 1, 2$, where ζ_i is a discrete random variable with the following known distribution:

$$\zeta_i = \begin{cases} \epsilon & \text{with probability } \frac{1}{2} \\ -\frac{\epsilon}{1+2\epsilon} & \text{with probability } \frac{1}{2}, \end{cases}$$

where $\epsilon \in [0, \infty)$ and $i = 1, 2$. Uncertainty over the true payoff values x_{1t} and x_{2t} increases in ϵ, and perfect observation occurs for $\epsilon = 0$. Given an observed $\tilde{x}_{it}$, the true value of x_{it} is either $(\frac{1}{1+\epsilon})\tilde{x}_{it}$ or $(\frac{1+2\epsilon}{1+\epsilon})\tilde{x}_{it}$ with equal probability. Thus, $E(x_{it}|\tilde{x}_{it}) = \tilde{x}_{it}$, so the agent expects to receive $\tilde{x}_{i\bar{t}_i} - I_i$ if he stops at time $\bar{t}_i$, for $i = 1, 2$.[7] I construct the noise as a mean-preserving spread over the expected outcome in order to isolate the effect of uncertainty alone. Ex ante, nature chooses the realizations of ζ_1 and ζ_2, which are i.i.d. and fixed throughout the problem, but unknown to the agent.[8]

Because the initial value of x_{2t} is some proportion of the payoff value of the first stage process upon its termination, $x_{1\bar{t}}$, the observed processes $\tilde{x}_{1t}$ and $\tilde{x}_{2t}$ are also linked at only one point in time, $\bar{t}_1$. In particular, I assume that $\tilde{x}_{2\bar{t}_1} = k\tilde{x}_{1\bar{t}_1}$ where $k > 0$.

2.3.3 Time Preferences

The agent may have present-biased preferences, creating a self-control problem. I model this present-biasedness by following Harris and Laibson (2004), who formulate

[7]More generally, any distribution of ζ_i such that $E(\frac{1}{1+\zeta_i}) = 1$ yields $E(x_{it}|\tilde{x}_{it}) = \tilde{x}_{it}$. I can also allow the parameter ϵ to differ in each stage, but do not vary it across stages for simplicity.

[8]The assumption that uncertainty is fixed ex ante is a technical necessity, to prevent the observed payoff processes $\tilde{x}_{1t}$ and $\tilde{x}_{2t}$ from jumping discontinuously from one instant to the next.

a continuous time version of quasi-hyperbolic preferences. At any time s, an agent's preferences are divided into a "present," which lasts from time s to time $s+\tau_s$, and a "future," which arrives at time $s+\tau_s$ and persists forever. The length of the present, τ_s, is stochastic and distributed exponentially with parameter $\lambda \in [0, \infty)$. When the future for this self s arrives at time $s+\tau_s$, he is replaced by a new self who takes control of decision-making. Likewise, the preferences of this self $s+\tau_s$ are divided into a "present" of length $\tau_{s+\tau_s}$ and a "future" that arrives at time $(s+\tau_s)+\tau_{s+\tau_s}$ and persists forever. Hence, when each self's "future" arrives, it "dies" and is replaced by a new self.

Each self s has a stochastic discount function $D_s(t)$:

$$D_s(t) = \begin{cases} e^{-\rho(t-s)} & \text{if} \quad t \in [s, s+\tau_s) \\ \beta e^{-\rho(t-s)} & \text{if} \quad t \in [s+\tau_s, \infty). \end{cases} \tag{2.3}$$

where $\beta \in [0, 1]$ and $\rho > 0$. To ensure that the agent never finds it optimal to wait forever in the optimal stopping problem, let $\rho > \mu$. The function $D_s(t)$ decays exponentially at the rate ρ throughout, but drops discontinuously at time $s+\tau_s$ to a fraction β of its prior level. Note that in this continuous time version of quasi-hyperbolic preferences, there are two parameters that determine the degree to which an agent's behavior deviates from that of a time-consistent individual. First, the parameter β retains the same role it plays in the discrete-time version, measuring how much the future is valued relative to the present. Second, the parameter λ determines the arrival rate of the future, and thus how often preferences change. In particular, when $\lambda \to \infty$ and $\beta < 1$, the agent discretely discounts all moments beyond the current instant. Harris and Laibson (2004) describe this limit case as "instantaneous gratification." When $\beta = 1$ or $\lambda = 0$, conversely, the preferences described by Equation (2.3) are equivalent to those of an exponential discounter with discount rate ρ.

I assume that the agent is sophisticated, so he is fully aware of his dynamic inconsistency. Therefore, he would like to employ some regulatory device in order to bring his future selves' behavior in line with his own preferences.

2.3.4 Goals

The agent's preferences are reference-dependent: his utility is composed of both standard consumption utility, which is based on absolute levels, and of comparison utility, which is concerned with gains and losses relative to a reference point, which here corresponds to a goal. In the optimal-stopping context with zero flow payoffs, the agent's expected consumption utility upon completing stage i at time $\bar{t}_i$ is simply his expected net terminal payoff: $E(x_{i\bar{t}_i} - I_i|\tilde{x}_{i\bar{t}_i}) = \tilde{x}_{i\bar{t}_i} - I_i$.

The agent's comparison utility is closely related to his consumption utility. It is derived by comparing his actual net terminal payoff at time $\bar{t}_i$ against his goal at that time, $r_{\bar{t}_i}$, and is governed by a piece-wise linear function $\psi(\cdot)$, given by

$$\psi(y) = \begin{cases} \alpha\eta y & \text{if} \quad y < 0 \\ \eta y & \text{if} \quad y \geq 0, \end{cases}$$

where $\alpha \geq 1$ and $\eta \geq 0$. The parameter η measures the agent's degree of reference dependence, and can be interpreted as the degree to which he *cares about*, or *pays attention to*, the difference between his outcome and his goal. The parameter α captures his degree of loss aversion, where $\alpha = 1$ when loss aversion is absent. If the agent's goal for the completion of stage i at time $\bar{t}_i$ is $r_{\bar{t}_i}$, then the argument y is given by $x_{i\bar{t}_i} - I_i - r_{\bar{t}_i}$ and his expected comparison utility is given by $E_{\bar{t}_i}[\psi(x_{i\bar{t}_i} - I_i - r_{\bar{t}_i})|\tilde{x}_{i\bar{t}_i}]$. Thus, given an observed $\tilde{x}_{i\bar{t}}$, his expected comparison utility upon stopping at time $\bar{t}_i$ is derived by applying the comparison function $\psi(\cdot)$ to the difference between each possible realization of $x_{i\bar{t}_i}$ against this goal and weighting these comparisons linearly by their respective probabilities. However, possible losses relative to the goal are additionally weighted by α. In the absence of loss aversion, mean-zero ex-ante uncertainty over x_{1t} and x_{2t} clearly will have no effect on behavior, since the consumption-utility component of the agent's preferences is linear. Moreover, it is the presence of loss aversion that leads him to prefer aggregated rather than gradual resolution of ex-ante uncertainty over outcomes.[9]

I assume that the agent can only incur comparison utility at the time at which he stops and receives the net terminal payoff. That is, although he is always aware that he will incur comparison utility at the moment of stopping, he does *not* directly experience it while waiting. This assumption accords with the notion from mental accounting that individuals do not necessarily "feel" gains and losses until they have been realized (Thaler, 1999).[10]

For simplicity, overall utility is taken to be additively separable in its two components. Thus, given a goal r_{it_i} to be evaluated upon completion of stage i, the agent's expected total utility upon completing stage i is

$$E_{\bar{t}_i}(x_{i\bar{t}_i}|\tilde{x}_{i\bar{t}_i}) - I_i + E_{\bar{t}_i}[\psi(x_{i\bar{t}_i} - I_i - r_{\bar{t}_i})|\tilde{x}_{i\bar{t}_i}]. \tag{2.4}$$

In the absence of such a goal for stage i, the agent does not make any comparison upon completion of stage i and the second term of Equation (2.4) is omitted.

At any time s, the goal r_s is taken as given by self s and cannot be changed during his entire "lifetime," having been set by his previous self. Similarly, the goal that self $s + \tau_s$ inherits, denoted $r_{s+\tau_s}$, is set by self s, where τ_s, the lifespan of self s, was

[9]Similarly, Kőszegi and Rabin (2009) find that loss aversion leads agents to prefer information to be received in clumps rather than spread apart. Palacios-Huerta (1999) demonstrates that an agent with Gul's (1991) disappointment aversion is also averse to the sequential resolution of uncertainty. Dillenberger (2009) finds that an individual who has recursive, non-expected utility preferences over compound lotteries also exhibits this preferences.

[10]For example, the disposition effect, where stockholders are reluctant to sell losing stocks, and hence realize losses relative to their original buying prices, is consistent with this idea (Odean, 1998; Barberis and Xiong, 2008).

stochastically determined and a priori unknown to self s. The assumption that the agent cannot change an inherited goal implies that such a goal can provide a degree of internal motivation to his (present-biased) future selves. In setting a goal, each self forms an expectation of his immediate "descendant"'s net terminal payoff if he does not stop himself. His descendant inherits this expectation as a given and compares his own net terminal payoff against this inherited goal if he stops.[11] Note, however, that these assumptions do *not* necessarily imply that each self must have the same goal. Each self cannot change the goal that he inherits, but is free to choose a different one for his future self if he so desires, as long as that he perceives, whether accurately or not, that it is realistic.

Because the agent is sophisticated and correctly anticipates his actions, I assume that each self has rational expectations about goal achievement. That is, he cannot consistently fool himself about what he can or cannot achieve - he sets goals that are realistic.[12] Because he has ex-ante uncertainty over the realized terminal payoff when setting a goal, I assume that his goal is the expectation of his net terminal payoff, given the observed payoff upon stopping. For example, if the agent has a goal regarding his stage i payoff and stops at $\bar{t}_i$, then rational expectations require that $r_{\bar{t}_i} = E(x_{i\bar{t}_i}|\tilde{x}_{i\bar{t}_i}) - I_i$. The particular formulation of reference point as a degenerate distribution that is the expectation of his payoff is not essential to the intuitions that drive the main results. Although the formulation of the reference point determines the magnitude of distaste over ex-ante uncertainty, it does not affect the relevant comparative statics. The key requirements are that the agent has distaste over ex-ante

[11]This formulation is consistent with Bandura's (1989) theory that goals serve as both targets to strive for and standards by which outcomes are evaluated, as well as empirical evidence that the degree of self-satisfaction varies depending on goal level. That is, two individuals who attain the same outcome will be unsatisfied or satisfied depending on whether their goals were higher or lower than that outcome, respectively (Mento et al., 1992).

[12]Based on the results of lab and field experiments, Latham and Locke (1991) conclude that goal choice is an integration of what one wants and what one believes is possible, suggesting that goals must be, and are, realistic to the agent.

uncertainty, which arises from loss aversion, and that he holds rational (endogenous) expectations, comparing possible realizations against his reference point.[13]

Because each self inherits his goal from a previous one, it is necessary to specify the source of the agent's goal when he is first able to stop the project. I assume that there exists a "self 0," an ex-ante self, who learns that the sequential investment opportunity will present itself in future and forms an expectation of how he will behave once the option becomes available for exercise.

2.3.5 Goal Bracketing

In addition to setting the level of his goals, the agent can choose when and how he evaluates them. In the two-stage, his options for framing and setting goals are intuitive. First, he can specify incremental goals for each stage of the project, framing the problem narrowly. That is, he can set goals for the net terminal payoffs of stages i, denoted r_i^{inc} for $i = 1, 2$, and evaluate himself against r_i^{inc} upon completion of each stage i. In the context of the writer, he can set goals for each chapter of his book, evaluating himself against individual goals for each. Alternatively, he can specify an aggregate goal for the entire project, framing the problem broadly. That is, he can set a goal regarding the total net payoff from the entire project, denoted r^{agg}, and evaluate the sum of net payoffs from both stages against r^{agg} upon completion of the *entire* project. When the agent sets an aggregate goal, he only derives comparison utility at the end of the second stage. However, when making the stopping decision in the first stage, he is aware that he will make a comparison at the end of the entire

[13]There are a number of other proposed formulations of the reference point. Although there is some evidence supporting the theory of expectations as a reference point (Abeler et al., 2009; Crawford and Meng, 2008), the precise formulation that individuals actually use is an unresolved empirical question. Kőszegi and Rabin (2006) assume that an agent holds a stochastic reference point when there is ex-ante uncertainty, where rational (endogenous) expectations imply that it must be the probability measure over realized outcomes. Gul's (1991) model implies that the reference point is the certainty equivalent of a chosen lottery.

project. For example, the writer can instead set a goal for the overall quality of the book, determined by summing the quality of individual chapters, which he evaluates only upon its completion. But though he does not have an individual goal for each individual chapter, he is aware that each contributes to his overall evaluation.

Just as each self sets the level of his goal(s) for his immediate descendant, who takes this as given, he also determines how and when such a set of goals is evaluated. Because goals are always chosen by predecessors for their descendants, they are chosen to maximize ex-ante welfare at every point in time. Thus, the form of goal bracketing (and its corresponding goals) chosen by a self s for his descendant, self $s + \tau_s$, will be identical to those chosen by self $s + \tau_s$ for his descendant in a stationary equilibrium.

2.4 Equilibrium Construction

To determine the conditions under which setting incremental versus aggregate goals is optimal, I analyze the agent's behavior under each form of bracketing, then consider the intertemporal bracketing choice from an ex-ante perspective. Because the agent is quasi-hyperbolic and sophisticated, the problem takes on the nature of a dynamic game between successive selves. I focus on the most natural equilibrium, namely stationary Markov equilibrium in which each self employs the same threshold strategy in each stage.

In order to construct such an equilibrium, I solve the intrapersonal game backwards in the manner delineated in Grenadier and Wang (2007) and Hsiaw (2009b), which study a single optimal stopping problem.[14] I apply backwards induction to determine the agent's behavior in the second stage upon completion of the first stage, then consider his behavior in the first stage. Each self anticipates that his descendants

[14]Grenadier and Wang (2007) solve for the stationary Markov equilibrium when the agent has quasi-hyperbolic time preferences, which is equivalent to the $\eta = 0$ case, in both this paper and Hsiaw (2009b).

will act according to a threshold that maximizes their own current benefit of waiting, so they will face a problem that is identical to his own. Constructing the stationary solution thus involves, within each stage of the sequential stopping problem, searching for a fixed point such that current and future selves stop at a common threshold.

Since the agent incurs no flow utility while waiting, the Bellman equations describing his decision problem only differ in his total utility upon stopping, given his bracketing and goal choices and the stage he is in. Therefore, I let the function $\Phi_i^b(x_i, r_i^b)$ describe the current self's utility upon stopping, given his bracketing choice b, which is setting incremental goals (inc) or an aggregate goal (agg), his corresponding goal r_i^b, and the stage $i = 1, 2$ he is in. Likewise, I let $\phi_i^b(x_i, r_i^b)$ denote his consideration of future selves' utility from stopping. Using these general stopping values, I will solve for his optimal threshold, then obtain specific expressions for each bracketing choice b and stage i by substituting for $\Phi_i^b(x_i, r_i^b)$ and $\phi_i^b(x_i, r_i^b)$ appropriately in the following sections.

Because each self controls the stopping decision in the present, and cares about - but cannot directly control - those of the future, two value functions are required to describe the intrapersonal problem in a given stage. The continuation value function, denoted $v_i^b(\cdot)$ where $i = \{1, 2\}$ and $b = \{inc, agg\}$, describes each self s's consideration (or internalization) of his future selves, following the random arrival of the future at time τ_s. Denoting the goal inherited by future selves as $\hat{r}_i$, the Bellman equation for the continuation value function in stage i is

$$v_i^b(x_i, \hat{r}_i) = \max\{E[\phi_i^b(x_i, \hat{r}_i^b)|\tilde{x}_i], e^{-\rho dt} E[v_i^b(x_i + dx_i, \hat{r}_i^b)|\tilde{x}_i]\} \tag{2.5}$$

Beyond time τ_s, he discounts any future utility flows exponentially at rate ρ. For this reason, it also describes his preference for future selves to behave as exponential discounters. That is, he prefers that future selves choose the maximum of the current total utility from stopping stage i, described by $\phi_i^b(x_i, \hat{r}_i^b)$, and the expected present discounted value of waiting for a higher realization of $\tilde{x}_i$, where this discounting is exponential. If the agent were time consistent ($\beta = 1$ or $\lambda = 0$), then all selves' preferences would coincide and he would choose the optimal strategy by maximizing v_i^b.

To construct the continuation value function v_i^b, I suppose that all future selves inherit the goal $\hat{r}_i^b$ and employ the threshold $\hat{x}_i^b$ such that they wait if $\tilde{x}_i < \hat{x}_i^b$ and stop if $\tilde{x}_i \geq \hat{x}_i^b$. Because the geometric Brownian motion x_i, and thus $\tilde{x}_i$, changes continuously, I construct v_i^b by considering its behavior in the "wait" and "stop" regions separately, then joining them using the appropriate boundary conditions.

The threshold strategy implies that the value of Equation (2.5) in its stop region ($\tilde{x}_i \geq \hat{x}_i^b$) is given by $E[\phi_i^b(x_i, \hat{r}_i^b)|\tilde{x}_i]$. In its wait region ($\tilde{x}_i < \hat{x}_i^b$), standard results imply that v_i^b obeys the following linear differential equation:

$$\rho v_i^b(x_i, \hat{r}_i^b) = \mu x_i(\frac{\partial v_i^b}{\partial x_i}) + \frac{1}{2}\sigma^2 x_i^2(\frac{\partial^2 v_i^b}{\partial x_i^2}). \tag{2.6}$$

Because x_i is a geometric Brownian motion, $x_i = 0$ is an absorbing barrier. Clearly, the agent should never stop the process if $x_i = 0$. Moreover, the continuation value must be continuous everywhere, including at the threshold between waiting and stopping. Because there is no optimal decision embodied in the continuation value function, the smooth pasting condition does not apply to $v_i^b(x_i, \hat{r}_i^b)$ if the agent is present-biased. The stopping decision is never made by future selves, only by current selves. Thus, there are two relevant boundary conditions for v_i^b:

$$\text{Boundary: } E[v_i^b(x_i, \hat{r}_i^b)|\tilde{x}_i = 0] = 0, \tag{2.7}$$

$$\text{Value Matching: } E[v_i^b(x_i, \hat{r}_i^b)|\tilde{x}_i = \hat{x}_i^b] = E[\phi_i^b(x_i, \hat{r}_i^b)|\tilde{x}_i = \hat{x}_i^b]. \tag{2.8}$$

However, if the agent is present-biased ($\beta < 1$ and $\lambda > 0$), he maximizes the current value function, denoted $w_i^b(\cdot)$ where $i = 1, 2$, which overweights the present relative to the future. Denoting the goal inherited by the current self as r_i^b, the Bellman equation for the current value function is

$$w_i^b(x_i, r_i^b) = \max\{E[\Phi_i(x_i, r_i^b)|\tilde{x}_i], (1 - e^{-\lambda dt})e^{-\rho dt}\beta E[v_i^b(x_i + dx_i, \hat{r}_i^b)|\tilde{x}_i] + (e^{-\lambda dt})e^{-\rho dt} E[w_i^b(x_i + dx_i, r_i^b)|\tilde{x}_i]\}. \quad (2.9)$$

Given the observed $\tilde{x}_i$ and an inherited goal r_i^b, and anticipating that his future selves will inherit $\hat{r}_i^b$ (with the knowledge that he sets $\hat{r}_i^b$ for his immediate descendant), the current self chooses the maximum of the current total utility from stopping, described by $\Phi_i^b(x_i, r_i^b)$, and the expected present discounted value of waiting for a higher realization of $\tilde{x}_i$, where this discounting discontinuously drops by the factor β upon the random arrival of the future. A future self arrives in the next instant dt with probability $1 - e^{-\lambda dt}$, while the current self remains in control with probability $e^{-\lambda dt}$.

To construct the current value function w_i^b, I suppose that all current selves inherit the goal r_i^b and employ the threshold $\overline{x}_i^b$ such that they wait if $\tilde{x}_i < \overline{x}_i^b$ and stop if $\tilde{x}_i \geq \overline{x}_i^b$. The threshold strategy implies that the value of w_i^b in its "stop" region ($\tilde{x}_i \geq \overline{x}_i^b$) is given by $E[\Phi_i^b(x_i, r_i^b)|\tilde{x}_i]$. In its wait region ($\tilde{x}_i < \overline{x}_i^b$), standard results imply that w_i^b obeys the following linear differential equation:

$$\rho w_i^b(x_i, r_i^b) = \lambda(\beta v_i^b(x_i, \hat{r}_i^b) - w_i^b(x_i, r_i^b)) + \mu x_i(\frac{\partial w_i^b}{\partial x_i}) + \frac{1}{2}\sigma^2 x_i^2(\frac{\partial^2 w_i^b}{\partial x_i^2}). \quad (2.10)$$

Comparing Equation (2.10) to Equation (2.6), the additional term $\lambda(\beta v_i^b(x_i, \hat{r}_i^b) - w_i^b(x_i, r_i^b))$ is the expected value of the change in the current value w_i^b that occurs through the stochastic arrival of a transition from the present to the future.

As with v_i^b, $x_i = 0$ is an absorbing barrier and w_i^b must be continuous everywhere. Since the optimal threshold is chosen to maximize the current value function by the current self, the smooth pasting condition, that the marginal value of waiting equals that of stopping, must apply to w_i^b with respect to x_i. This yields the boundary conditions for w_i^b:

$$\text{Boundary: } E[w_i^b(x_i, r_i^b)|\tilde{x}_i = 0] = 0, \tag{2.11}$$

$$\text{Value Matching: } E[w_i^b(x_i, r_i^b)|\tilde{x}_i = \overline{x}_i^b] = E[\Phi_i^b(x_i, r_i^b)|\tilde{x}_i = \overline{x}_i^b], \tag{2.12}$$

$$\text{Smooth Pasting: } E[\frac{\partial w_i^b}{\partial x_i}(x_i, r_i^b)|\tilde{x}_i = \overline{x}_i^b] = E[\frac{\partial \Phi_i^b}{\partial x_i}(x_i, r_i^b)|\tilde{x}_i = \overline{x}_i^b]. \tag{2.13}$$

Applying conditions (2.7) and (2.8) to Equation (2.6) yields the solution to the continuation value function v_i^b. Under the assumption that $x_i \leq \hat{x}_i^b$, which the fixed point condition will satisfy, it is the value of v_i^b in its wait region that applies to Equation (2.10). Combining v_i^b in its wait region with Equation (2.10), along with conditions (2.11), (2.12), (2.13), we obtain the solution to the optimal threshold $\overline{x}_i^b$ as a function of goal r_i^b and the conjectured future goals $\hat{r}_i^b$ and threshold $\hat{x}_i^b$. Moreover, stationarity implies that $\overline{x}_i^b = \hat{x}_i^b$ and $r_i^b = \hat{r}_i^b$, allowing us to obtain $\overline{x}_i^b$ as a function of the goal r_i^b.

In the following sections, I derive the optimal threshold $\overline{x}_i^b$ for each bracketing choice b and stage i by substituting the appropriate expressions for the total utility upon stopping, $\phi_i^b(x_i, r_i^b)$ and $\Phi_i^b(x_i, r_i^b)$.

2.5 Incremental Goals

First, consider the case in which the agent sets incremental goals for the net terminal payoffs of stages i, denoted r_i^{inc} for $i = 1, 2$. He evaluates himself against a goal r_i^{inc} only upon completion of stage i. Given that he sets incremental goals for himself, let $\overline{x}_i^{inc}$ be the stopping threshold that the agent employs to complete stage $i = 1, 2$. I apply backwards induction to obtain the optimal thresholds employed in each stage.

2.5.1 Stage 2

In the second stage, the agent's problem is identical to a standard, single-stage optimal stopping problem. Because his goal in this stage only pertains to the outcome of stage 2, his behavior in the first stage is irrelevant at this point.[15] Since he evaluates himself against the goal r_2^{inc} upon completion of stage 2, the current self's total utility upon stopping stage 2 is given by $\Phi_2^{inc}(x_2, r_2^{inc})$:

$$\Phi_2^{inc}(x_2, r_2^{inc}) = x_2 - I_2 + \psi(x_2 - I_2 - r_2^{inc}), \tag{2.14}$$

which is simply the sum of his net terminal payoff and his expected comparison utility, and enters into Equations (2.9) and its corresponding boundary conditions (2.11) - (2.13). Likewise, he anticipates that future selves obtain the same utility from stopping:

$$\phi_2^{inc}(x_2, r_2^{inc}) = x_2 - I_2 + \psi(x_2 - I_2 - r_2^{inc}), \tag{2.15}$$

which enters into Equations (2.5) and its corresponding boundary conditions (2.7) and (2.8).

Because the comparison utility function is kinked at the origin, I derive $\overline{x}_2^{inc}$ under the assumption that r_2^{inc} is such that

$$(\frac{1}{1+\epsilon})\overline{x}_2^{inc} - I_2 \leq r_2^{inc} \leq (\frac{1+2\epsilon}{1+\epsilon})\overline{x}_2^{inc} - I_2, \tag{2.16}$$

which will be satisfied in equilibrium when expectations are rational. Under this assumption and given the presence of unresolved uncertainty when the stopping decision

[15] In the standard problem without reference dependent preferences, it is also the case that stage 1 behavior is irrelevant to the stage 2 decision. However, stage 1 behavior will *not* be irrelevant when the agent sets an aggregate goal.

is made, his expected comparison utility upon stopping at $\overline{x}_2^{inc}$ is given by

$$E[\psi(x_2 - I_2 - r_2)|\tilde{x}_2 = \overline{x}_2^{inc}] = \frac{1}{2}\alpha\eta[(\frac{1}{1+\epsilon})\overline{x}_2^{inc} - I_2 - r_2] + \frac{1}{2}\eta[(\frac{1+2\epsilon}{1+\epsilon})\overline{x}_2^{inc} - I_2 - r_2]. \tag{2.17}$$

Given this assumption, the agent employs the stopping threshold $\overline{x}_2^{inc}$ in the second stage:[16]

$$\overline{x}_2^{inc} = \frac{\overline{\gamma}[I_2 + \frac{1}{2}\eta(\alpha+1)(r_2^{inc} + I_2)]}{(\overline{\gamma}-1)[1 + \frac{1}{2}\eta(\frac{1+\alpha+2\epsilon}{1+\epsilon})]}, \tag{2.18}$$

where $\overline{\gamma} \equiv \beta\gamma_1 + (1-\beta)\gamma_2$, and $\gamma_1 > 1$ is the positive root[17] of the quadratic equation

$$\frac{1}{2}\sigma^2\gamma_1^2 + (\mu - \frac{1}{2}\sigma^2)\gamma_1 - \rho = 0, \tag{2.19}$$

and $\gamma_2 \geq \gamma_1$ is the positive root[18] of the quadratic equation

$$\frac{1}{2}\sigma^2\gamma_2^2 + (\mu - \frac{1}{2}\sigma^2)\gamma_2 - (\rho + \lambda) = 0. \tag{2.20}$$

As in Hsiaw (2009b), the parameter γ_1 reflects the fact that the agent discounts the future exponentially at the rate ρ, while the parameter γ_2 reflects the fact that each self's expected "lifetime" ends with hazard rate λ. The degree to which this feature affects behavior is determined by his degree of present-biasedness, measured by $1-\beta$. Thus, the parameter $\overline{\gamma} = \beta\gamma_1 + (1-\beta)\gamma_2$ serves as a sufficient statistic for measuring the agent's self-control problem, which is determined by both β and λ.

[16]Unsurprisingly, in the absence of loss aversion ($\alpha = 1$), the threshold $\overline{x}_2^{inc}$ reduces to the threshold $\overline{x}^{SE}$ found in Hsiaw (2009b), which describes the sophisticate agent's optimal stopping threshold in a single-stage stopping problem in the absence of loss aversion.

[17]The negative root is ruled out by the boundary condition for $x = 0$. Writing out γ_1 explicitly, we have $\gamma_1 = -\frac{\mu}{\sigma^2} + \frac{1}{2} + \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}}$. To see that $\gamma_1 > 1$, note that $\sigma^2 > 0$ and the left-hand side of the quadratic is negative when evaluated at $\gamma_1 = 0$ and $\gamma_1 = 1$, implying that the negative root is strictly negative and the positive root is strictly greater than 1 if $\mu < \rho$.

[18]Again, the negative root is ruled out by the boundary condition for $x = 0$. It follows that $\gamma_2 \geq \gamma_1$ because $\lambda \geq 0$, with equality only if $\lambda = 0$.

Because the goal represents a penalty that the agent wants to avoid, he waits for a higher expected value of the second-stage payoff process when the goal r_2^{inc} is higher. In the absence of reference dependence, the agent's behavior is unaffected by any goal: if $\eta = 0$, then $x_2^{inc} = (\frac{\overline{\gamma}}{\overline{\gamma}-1})I_2$. His present-biasedness leads him to undervalue the option to wait, so he stops earlier than he would in its absence: $(\frac{\overline{\gamma}}{\overline{\gamma}-1})I_2 < (\frac{\gamma_1}{\gamma_1-1})I_2 \equiv x_2^*$, where x_2^* is the stopping threshold he employs if he is neither reference dependent nor present-biased ($\beta = 1, \eta = 0$). Consequently, his stopping threshold decreases as the degree of impulsiveness increases.

If the agent is reference-dependent ($\eta > 0$), the goal does mitigate his impatience because it presents a potential penalty, assessed upon stopping, that provides an additional incentive to wait for a higher realization of the project value. His incentive to wait increases with his degree of reference dependence, since he puts more weight on the comparative disutility from falling short.

It is only in the presence of loss aversion ($\alpha > 1$) that the agent dislikes ex-ante, mean-zero uncertainty over outcomes, since it leads him to overweight the possibility of a loss. In this case, this expected comparative disutility, given by Equation (2.17), increases with the degree of uncertainty, measured by ϵ, leading the agent to wait for a higher realization of the project value in order to compensate for the anticipated loss. Similarly, the expected comparative disutility arising from any given amount of ex-ante uncertainty increases with the degree of loss aversion, leading the agent to wait longer on average.

Proposition 12. *In a stationary equilibrium with any given incremental goal for the second stage, the agent's stopping threshold in the second stage exhibits the following properties:*

1. *The threshold increases with the goal level:* $\frac{\partial \overline{x}_2^{inc}}{\partial r_2^{inc}} > 0$.

2. *The threshold decreases with impulsiveness:* $\frac{\partial \overline{x}_2^{inc}}{\partial \overline{\gamma}} < 0$.

3. *The threshold increases with reference dependence:* $\frac{\partial \overline{x}_2^{inc}}{\partial \eta} > 0$.

4. *The threshold increases with ex-ante uncertainty if the agent is loss averse:* $\frac{\partial \overline{x}_2^{inc}}{\partial \epsilon} \geq 0$, *with equality only if* $\alpha = 0$.

5. *The threshold increases with the degree of loss aversion:* $\frac{\partial \overline{x}_2^{inc}}{\partial \alpha} > 0$.

The agent's expectation of his terminal payoff, and hence his goal, is dependent on whether he completes the second stage simultaneously with or strictly after the first. In this paper, I assume that he completes stages sequentially, then impose the conditions required for such a strategy to be optimal. Because the results in the case of simultaneous completion rely on the same intuitions and offer no additional insights, I omit that analysis and focus on the more natural scenario.

If the agent completes the second stage strictly after the first, he expects to receive a terminal payoff that is determined by his stopping threshold in this case. Thus, his incremental goal r_2^{inc} is given by $r_2^{inc} = \overline{x}_2^{inc} - I_2$ when he holds rational expectations. Substituting this condition into Equation (2.18) yields the second-stage threshold $\overline{x}_2^{inc}$ when he sets incremental goals and stops sequentially:[19]

$$\overline{x}_2^{inc} = \frac{\overline{\gamma} I_2}{(\overline{\gamma} - 1)[1 - \frac{1}{2}\eta(\alpha - 1)(\frac{\epsilon}{1+\epsilon})] - \frac{1}{2}\eta(\alpha + 1)}, \tag{2.21}$$

with $\overline{\gamma} \equiv \beta\gamma_1 + (1-\beta)\gamma_2$ and $\frac{1}{2}\eta(\alpha+1) < (\overline{\gamma} - 1)[1 - \frac{1}{2}\eta(\alpha - 1)(\frac{\epsilon}{1+\epsilon})]$. With rational expectations, the equilibrium value functions for the second stage, w_2^{inc} and v_2^{inc}, are

[19]Clearly, condition (2.16) is satisfied here.

given by

$$E[w_2^{inc}(x_2, r_2^{inc})|\tilde{x}_2] = \begin{cases} \beta[\overline{x}_2^{inc}(1 - \frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})) - I_2](\frac{\tilde{x}_2}{\overline{x}_2^{inc}})^{\gamma_1} + \\ (1-\beta)[\overline{x}_2^{inc}(1 - \frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})) - I_2](\frac{\tilde{x}_2}{\overline{x}_2^{inc}})^{\gamma_2} & \text{if} \quad \tilde{x}_2 < \overline{x}_2^{inc} \\ \tilde{x}_2 - I_2 + E[\psi(x_2 - r_2^{inc})|\tilde{x}_2] & \text{if} \quad \tilde{x}_2 \geq \overline{x}_2^{inc} \end{cases} \tag{2.22}$$

$$E[v_2^{inc}(x_2, r_2^{inc})|\tilde{x}_2] = \begin{cases} [\overline{x}_2^{inc}(1 - \frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})) - I_2](\frac{\tilde{x}_2}{\overline{x}_2^{inc}})^{\gamma_1} & \text{if} \quad \tilde{x}_2 < \overline{x}_2^{inc} \\ \tilde{x}_2 - I_2 + E[\psi(x_2 - I_2 - r_2^{inc})|\tilde{x}_2] & \text{if} \quad \tilde{x}_2 \geq \overline{x}_2^{inc}. \end{cases} \tag{2.23}$$

The value of Equation (2.22) in its wait region is the expected present value of the option to stop, given the current value of the projects payoff, $x < \overline{x}_2^{inc}$. This is essentially the weighted average of two time-consistent option values, where the first, weighted by β, uses the discount rate ρ, and the second, weighted by $1-\beta$, uses the discount rate $\rho + \lambda$. Moreover, the expected present value of the stopping option reflects the comparative disutility that the agent expects to incur upon stopping and evaluating himself against his goal. Because it reflects his preferences from an ex-ante perspective, the value of Equation (2.23) in its wait region is the expected present value of the option to stop, using only the discount rate ρ.

2.5.2 Stage 1

In the first stage, the agent faces a problem that is very similar to that of the second, since his goal pertains only to the outcome of stage 1 and he evaluates himself upon its completion. The only difference is that in addition to receiving the project payoff $x_{1\bar{t}_1}$ upon completing stage 1 at time $\bar{t}_1$, he obtains the option to complete the second stage of the project. Thus, the current self's total utility upon stopping stage 1 is given by $\Phi_1^{inc}(x_1, r_1^{inc})$, where

$$\Phi_1^{inc}(x_1, r_1^{inc}) = x_1 - I_1 + \psi(x_1 - I_1 - r_1^{inc}) + w_2^{inc}(kx_1, r_2^{inc}), \tag{2.24}$$

which enters into Equation (2.9) and its corresponding boundary conditions (2.11), (2.12), and (2.13). Equation (2.24) only differs from the second-stage stopping utility, given by (2.14), in its last term, the option to complete stage 2. When evaluating the possibility that a future self will complete the first stage, the agent considers the option to complete stage 2 by discounting it exponentially. Thus, his consideration of future selves' stopping utility for stage 1 is given by $\phi_1^{inc}(x_1, r_1^{inc})$, where

$$\phi_1^{inc}(x_1, r_1^{inc}) = x_1 - I_1 + \psi(x_1 - I_1 - r_1^{inc}) + v_2^{inc}(kx_1, r_2^{inc}), \tag{2.25}$$

which enters into Equation (2.5) and its respective boundary conditions (2.7) and (2.8). The stopping values $\Phi_1^{inc}(x_1, r_1^{inc})$ and $\phi_1^{inc}(x_1, r_1^{inc})$ differ only in their last terms, since the agent considers the option to complete stage 2 differently depending on whether it is obtained in the present or the future.

The value of the option to complete stage 2 depends on whether it is optimal for him to stop it immediately upon completion of stage 1. If he stops immediately, it is the stop region of w_2^{inc} that applies to Equation (2.24). Likewise, it is the stop region v_2^{inc} that applies to (2.25) when considering future behavior. If he does not, it is the wait region of the stage 2 option value that is applicable. Here, I assume that it is optimal for him to wait and construct the stage 1 strategy accordingly, then analyze the conditions necessary for this to be optimal in equilibrium.

Finally, imposing the requirement that $r_1^{inc} = \overline{x}_1^{inc} - I_1$, gives the optimal threshold when the goal is self-set and expectations are rational:

$$\overline{x}_1^{inc} = \frac{\overline{\gamma} I_1}{(\overline{\gamma} - 1)[1 - \frac{1}{2}\eta(\alpha - 1)(\frac{\epsilon}{1+\epsilon})] - \frac{1}{2}\eta(\alpha + 1)}, \tag{2.26}$$

with $\overline{\gamma} \equiv \beta\gamma_1 + (1-\beta)\gamma_2$ and $\frac{1}{2}\eta(\alpha+1) < (\overline{\gamma}-1)[1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})]$, where γ_1 and γ_2 are defined by Equations (2.19) and (2.20), respectively.

When the agent sets incremental goals for each stage, he employs thresholds in each as if they were separate stopping decisions, just as in a standard setting without goals. The incremental goals induce more patience in their respective stages, but do not affect behavior in other stages. Thus, the stopping decision in stage 2 is completely independent of the first stage, once its option has been acquired. Since the second stage value functions are determined independent of the first stage stopping threshold $\overline{x}_1^{inc}$, and his stage 1 goal only pertains to the outcome of stage 1, they have no effect on the decision in the first stage. For this reason, $\overline{x}_2^{inc}$ and $\overline{x}_1^{inc}$ differ only in the fixed stopping cost I_i where $i = 1, 2$, and all properties described in Proposition (12) apply to the stopping threshold in the first stage, $\overline{x}_1^{inc}$.

The expected equilibrium value functions, w_1^{inc} and v_1^{inc} in the first stage are provided in Appendix B.1. Because the agent completes the second stage strictly after the first, the stop regions of the current and continuation value functions, w_1^{inc} and v_1^{inc}, are composed of two regions. When the observed first-stage payoff $\tilde{x}_1$ is such that $k\tilde{x}_1 < \overline{x}_2^{inc}$, the option value of stage 2 is determined by the value of waiting to stop the process x_{2t}. When $\tilde{x}_1$ is sufficiently high that $k\tilde{x}_1 \geq \overline{x}_2^{inc}$, the option value of stage 2 is simply the value of stopping x_{2t} immediately. Since the agent evaluates himself against a goal at the end of each stage when he sets incremental goals, he expects to incur comparative disutility at the end of each.

We can easily verify that the agent stops the second-stage process x_{2t} strictly after completing the first stage when $\overline{x}_2^{inc} > k\overline{x}_1^{inc}$, which reduces to the following upper bound on k:

$$k < \frac{I_2}{I_1}. \tag{2.27}$$

Unsurprisingly, the second-stage process must start at a sufficiently low value for the agent not to stop it immediately. The factor k must compensate for the cost of stopping the second stage, I_2. The upper bound on the initial value of the second-stage process is increasing in I_2 because he waits longer to compensate for the cost of stopping the second stage. Moreover, the agent's stopping rule for completing the first stage is proportional to its cost I_1 and to the start value of x_{2t}, so the upper bound for k is inversely proportional to I_1.

2.5.3 Welfare

From an ex-ante perspective, no matter how severe his degree of present-biasedness or reference dependence and regardless of how he brackets his goals, the agent prefers that his future selves behave according to a time-consistent, optimal strategy. Therefore, I use the preferences of self 0, which determine the ex-ante optimum, to evaluate the agents welfare.

Given that the agent decides to set incremental goals for each stage, the knowledge of his loss aversion leads him to expect, ex ante, to incur some comparative disutility upon stopping. For this reason, a welfare analysis of his behavior given incremental goals must include both comparison and consumption utility. The ex-ante self wants to maximize the expected present discounted value of his overall utility, as if he were time consistent. That is, he wants to choose the stopping thresholds for each stage that maximize v_1^{inc} in its wait region.[20]

Proposition 13. *Let the agent set incremental goals. Given his preferences, described by $(\eta, \alpha, \beta, \lambda)$, he stops earlier than is first-best when ex-ante uncertainty is sufficiently low, and later than is first-best when ex-ante uncertainty is sufficiently high. In particular, the agent stops earlier the ex-ante first-best in each stage $i = 1, 2$ if $F(\cdot) > 0$ and waits longer than the first-best if $F(\cdot) < 0$, where the function $F(\cdot)$ is*

[20] Because each self shares the same ex-ante preferences over future selves' behavior, this is equivalent to working backward and having the ex-ante self choose a second-stage threshold to maximize v_2^{inc} in the second stage, then a first-stage threshold to maximize v_1^{inc}.

given by

$$F = (\frac{\overline{\gamma} - \gamma_1}{\gamma_1})[1 - \frac{1}{2}\eta(\alpha - 1)(\frac{\epsilon}{1+\epsilon})] - \frac{1}{2}\eta\gamma_1(\alpha + 1). \tag{2.28}$$

At the appropriate combinations of reference dependence and loss aversion, the agent can stop at the first-best threshold in each stage when he sets incremental goals. Clearly, F is decreasing in η and α. That is, his reference dependence and loss aversion can overcompensate for the conflict in time preferences between present and future selves.[21] If the agent is not present-biased ($\overline{\gamma} = \gamma_1$), he behaves in a time-consistent manner, so reference dependence and loss aversion offer no beneficial value, and $F < 0$ whenever $\eta > 0$. Despite the realization that he should not wait too long, his reference dependence and loss aversion distort the marginal value of waiting and stopping at the first-best threshold. Because he has an incentive to avoid incurring comparative disutility, the marginal value of waiting at the first best threshold exceeds the marginal value of stopping, so the agent waits longer ex post. Moreover, F is also decreasing in ϵ whenever $\overline{\gamma} > \gamma_1$. This implies that whenever he is present-biased whether the agent stops too early or too late relative to the first-best, which is defined given characteristics (η, α) and uncertainty ϵ, depends on the degree of uncertainty present in his environment. As uncertainty increases, loss aversion becomes less beneficial for the agent by generating so much comparative disutility that it leads him to wait longer than is optimal.[22] That is, an impulsive agent may stop too early when uncertainty is low, but stop too late when uncertainty is sufficiently high. If he is not present-biased ($\overline{\gamma} = \gamma_1$), he always stops too late relative to the first-best if he is reference-dependent, regardless of the degree of uncertainty.

Proposition 14. *Let the agent set incremental goals. Given a degree of reference dependence $\eta < \eta^*$ and environmental uncertainty ϵ, some degree of loss aversion*

[21] This result accords with that of Hsiaw (2009b), which is essentially the case of $\alpha = 1$ and $\epsilon = 0$.

[22] Although the first-best thresholds are also increasing as a function of uncertainty, loss aversion overcompensates when uncertainty is sufficiently high.

is optimal ($\alpha > 1$) when uncertainty is sufficiently small; but when uncertainty is sufficiently large, having no loss aversion ($\alpha = 1$) is optimal. In particular, there exists an interior constrained optimum $\alpha > 1$ if $\eta < \eta^*$ and

$$\epsilon < \frac{\overline{\gamma} - \gamma_1(1+\eta)}{\overline{\gamma}(\gamma_1 - 1)(1+\eta)},$$

where $\eta^* = \frac{\overline{\gamma}-\gamma_1}{\gamma_1}$. Otherwise, the constrained optimal level of loss aversion is $\alpha = 1$.

Although loss aversion is costly because it leads the agent to incur comparative disutility upon stopping, it also contributes to patient behavior for precisely this reason, as he waits longer in order to compensate for such expected disutility. Thus, if his degree of reference dependence is at or beyond the level necessary to achieve the (global) ex-ante optimum, given by η^*, any amount of loss aversion is unambiguously detrimental relative to its absence. That is, when $\eta \geq \eta^*$, there is no need for additional patience, so additional loss aversion is detrimental in terms of both self-regulation, contributing to excessive patience, and expectations, contributing to comparative disutility. However, if η is fixed at some level below the global optimum η^*, some amount of loss aversion may be beneficial, even in the presence of uncertainty. When $\eta < \eta^*$, the additional regulatory power derived from loss aversion can bring the agent closer to a constrained first-best, which is defined relative to fixed η and ϵ. But when uncertainty is sufficiently high, *any* degree of loss aversion makes the agent strictly worse off relative to its absence, even when $\eta < \eta^*$. In this case, the benefits of increased self-regulation are sufficiently offset by the additional expected comparative disutility from uncertainty that any loss aversion is detrimental. Unsurprisingly, the threshold of uncertainty for which an interior constrained optimal level of loss aversion can exist increases with $\overline{\gamma}$, which reflects the need for self-regulation, and decreases with the level of reference dependence η, which amplifies the effects of loss aversion.

2.6 Aggregate Goals

Now, consider the case in which the agent specifies an aggregate goal for the entire project, framing the problem broadly. He sets a goal regarding the total net payoff from the entire project, denoted r^{agg}, and evaluates the sum of net payoffs from both stages against r^{agg} upon completion of the entire project. When the agent sets an aggregate goal, he has no goal against which to evaluate himself in the first stage and derives no comparison utility upon its completion. Given that he sets an aggregate goal for himself, let $\overline{x}_i^{agg}$ be the stopping threshold that the agent employs to complete stage $i = 1, 2$.

In the standard case without reference dependent preferences (i.e., $\eta = 0$), the outcome of stage 1 is irrelevant to the decision in stage 2. Likewise, when the agent sets incremental goals for each stage, he employs thresholds in each as if they were separate stopping decisions. The incremental goals induce more patience in their respective stages, but do not affect behavior in other stages. But when the agent sets an aggregate goal, information about the stage 1 outcome becomes relevant to his behavior in stage 2, because it enters into his evaluation relative to r^{agg}. Thus, there are two key differences between incremental and aggregate goals: the first is the timing of goal evaluation, and the second is the relevance of information regarding the outcome of the completed first stage. In order to isolate the effect of the former, I assume that the agent does not learn the true realization of x_1 until he completes the project in its entirety, upon completion of stage 2.

2.6.1 Stage 2

In the second stage, the agent's problem differs in that he compares the sum of his net payoffs in each stage against his goal for it, rather than making a comparison regarding the outcome of stage 2 alone. At this point, he is aware that he has employed the threshold $\overline{x}_1^{agg}$ to complete the first stage. Therefore, the current self's total utility upon stopping stage 2 is given by $\Phi_2^{agg}(x_2, r^{agg})$:

$$\Phi_2^{agg}(x_2, r^{agg}) = x_2 - I_2 + \psi(x_{1\bar{t}} - I_1 + x_2 - I_2 - r^{agg}), \tag{2.29}$$

which enters into Equations (2.9) and its corresponding boundary conditions (2.11)-(2.13). Likewise, he anticipates that future selves obtain the same utility from stopping:

$$\phi_2^{agg}(x_2, r^{agg}) = x_2 - I_2 + \psi(x_{1\bar{t}} - I_1 + x_2 - I_2 - r^{agg}), \tag{2.30}$$

which enters into Equations (2.5) and its corresponding boundary conditions (2.7) and (2.8). In comparison to Equations (2.14) and (2.15), which describe the utility upon stopping stage 2 when he sets incremental goals, Equations (2.29) and (2.30) differ only in the goal evaluation that occurs upon completion of the project. Upon completing the project, it is the *sum* of the payoffs that the agent expects from both stages that he compares against his aggregate goal.

Because the comparison utility function is kinked at the origin, I derive $\overline{x}_2^{agg}$ under the assumption that r^{agg} is such that

$$(\frac{1+2\epsilon}{1+\epsilon})\overline{x}_1^{agg} - I_1 + (\frac{1}{1+\epsilon})\overline{x}_2^{agg} - I_2 \leq r^{agg} \leq (\frac{1}{1+\epsilon})\overline{x}_1^{agg} - I_1 + (\frac{1+2\epsilon}{1+\epsilon})\overline{x}_2^{agg} - I_2, \tag{2.31}$$

which will be satisfied in equilibrium under rational expectations. Under this assumption, the threshold employed in stage 2 is

$$\overline{x}_2^{agg} = \frac{\overline{\gamma}[I_2 - \frac{1}{2}\eta(\alpha+1)\overline{x}_1^{agg} + \frac{1}{2}\eta(\alpha+1)(r^{agg} + I_1 + I_2)]}{(\overline{\gamma}-1)[1 + \frac{1}{2}\eta(\frac{1+\alpha+2\epsilon}{1+\epsilon})]}. \tag{2.32}$$

Because the agent compares the sum of net project payoffs from both stages against a given goal, the threshold $\overline{x}_2^{agg}$ is decreasing in $\overline{x}_1^{agg}$. Expecting to receive a larger payoff from the first stage brings the agent closer to his aggregate goal and decreases the potential penalty from falling short of it for any $x_{2\bar{t}_2}$, weakening the agent's motivation to wait longer in the second stage. Since both $\overline{x}_2^{agg}$ and $\overline{x}_1^{agg}$ contribute to his comparison against a given goal r^{agg}, they act as motivational substitutes in the agent's stopping behavior across stages.

The equilibrium value functions for the second stage, w_2^{agg} and v_2^{agg}, are provided in Appendix B.1. They differ from those of incremental goals only in the goal comparison that is being made.

2.6.2 Stage 1

In the first stage, the agent has no goal against which to compare himself upon its completion. However, he *is* aware that he will be comparing the sum of net payoffs from *both* stages to the aggregate goal r^{agg} upon completion of the second stage. This knowledge is reflected in the option value of stage 2 that he obtains upon completion of the first stage. Thus, the current self's total utility upon stopping stage 2 is given by $\Phi_1^{agg}(x_1, r^{agg})$, where

$$\Phi_1^{agg}(x_1, r^{agg}) = x_1 - I_1 + w_1^{agg}(kx_1, r^{agg}), \tag{2.33}$$

which enters into Equation (2.9) and its corresponding boundary conditions (2.11), (2.12), and (2.13). In contrast to the case of incremental goals described by Equation (2.24), the agent makes no direct evaluation against a goal upon completing the first stage. His consideration of future selves' stopping utility for stage 1 is given by $\phi_1^{agg}(x_1, r^{agg})$, where

$$\phi_1^{agg}(x_1, r^{agg}) = x_1 - I_1 + w_1^{agg}(kx_1, r^{agg}), \tag{2.34}$$

which enters into Equation (2.5) and its respective boundary conditions (2.7) and (2.8). Again, the stopping values $\Phi_1^{agg}(x_1, r^{agg})$ and $\phi_1^{agg}(x_1, r^{agg})$ differ only in their last terms, since the agent considers the option to complete stage 2 differently depending on whether it is obtained in the present or the future.

The key difference between aggregate and incremental goals is that the second stage threshold $\overline{x}_2^{agg}$ is now a function of $\overline{x}_1^{agg}$, a fact that the agent takes into account when making the stopping decision in the first stage. Just as with incremental goals, the agent's stopping threshold in the first stage is dependent on whether or not he stops the second stage immediately upon the completion of stage 1, since he must consider the option value of the second stage.

Assuming that the agent completes the second stage strictly after the first, he expects to receive a terminal payoff that is determined by his stopping threshold, denoted $\overline{x}_2^{agg}$, so his aggregate goal r^{agg} is given by $r^{agg} = \overline{x}_1^{agg} - I_1 + \overline{x}_2^{agg} - I_2$ when expectations are rational. Applying this condition yields the second-stage threshold $\overline{x}_2^{agg}$ when he sets an aggregate goal and stops sequentially:[23]

$$\overline{x}_2^{agg} = \frac{\overline{\gamma} I_2}{(\overline{\gamma} - 1)[1 - \frac{1}{2}\eta(\alpha - 1)(\frac{\epsilon}{1+\epsilon})] - \frac{1}{2}\eta(\alpha + 1)}, \tag{2.35}$$

with $\overline{\gamma} \equiv \beta\gamma_1 + (1 - \beta)\gamma_2$ and $\frac{1}{2}\eta(\alpha + 1) < (\overline{\gamma} - 1)[1 - \frac{1}{2}\eta(\alpha - 1)(\frac{\epsilon}{1+\epsilon})]$.

When goal-setting is bound by rational expectations, the stopping threshold employed in stage 2 is identical regardless of the bracketing he employs: $\overline{x}_2^{agg} = \overline{x}_2^{inc}$. Given that he will not know the true outcome of stage 1 when he is making the stopping decision in stage 2 and he (correctly) expects to have neither fallen short of nor exceeded his expectation of the outcome of stage 1, this component of his goal exerts no influence on his decision in stage 2. As a result, he behaves as though the outcome of stage 1 is irrelevant to his decision, just as he does when he sets incremental goals for each stage.[24]

[23]Clearly, condition (2.31) is satisfied here.

[24]Note that this result is reliant on the imposition of rational expectations, but does not require that ϵ be the same across stages. Denoting ϵ_i as the degree of the ex-ante uncertainty in stage i, this result holds whenever $\epsilon_1 \leq \epsilon_2$ or as long as $\epsilon_1 - \epsilon_2 > 0$ is

Imposing the requirement that $r^{agg} = \overline{x}_1^{agg} - I_1 + \overline{x}_2^{agg} - I_2$ also yields the following nonlinear equation to describe the optimal stationary threshold that the agent employs in stage 1, $\overline{x}_1^{agg}$, given an aggregate goal and rational expectations:

$$\overline{x}_1^{agg} = (\frac{\overline{\gamma}}{\overline{\gamma}-1})I_1 + \frac{1}{2}\eta(\alpha+1)(\frac{1}{(\overline{\gamma}-1)^2})[\beta k^{\gamma_1}(\gamma_1 - 1)(\frac{1}{\overline{x}_2^{agg}})^{\gamma_1}(\overline{x}_1^{agg})^{\gamma_1+1} + (1-\beta)k^{\gamma_2}(\gamma_2 - 1)(\frac{1}{\overline{x}_2^{agg}})^{\gamma_2}(\overline{x}_1^{agg})^{\gamma_2+1}]. \quad (2.36)$$

Proposition 15. *Setting an aggregate goal is less effective at curbing impatience than setting incremental goals:* $\overline{x}_1^{agg} < \overline{x}_1^{inc}$ *and* $\overline{x}_2^{agg} = \overline{x}_2^{inc}$*. However, the aggregate goal does induce patience even in the first stage, relative to having no goals.*

Although the agent does not directly evaluate himself upon completion of the first stage, he anticipates that he will evaluate the sum of both stages' payoffs at the end of stage 2 when he sets an aggregate goal. Because being too impatient in the first stage and settling for a lower stage 1 payoff detrimentally affects his comparison utility in the future, the aggregate goal provides motivation for him to be more patient in the first stage as well. Recall that when he sets incremental goals, he expects to incur some comparative disutility, arising from evaluation of the goal r_1^{inc}, upon completing stage 1. Consequently, he waits for a higher payoff to compensate for this disutility. But when he sets an aggregate goal, this disutility is absent from the first stage, so he does have this immediate motivation to wait longer. Moreover, because it is evaluated at the end of stage 2, the potential disutility from evaluating the aggregate goal is incurred in the relatively distant future, so it is discounted more heavily in the first stage. In contrast, an incremental goal for the first stage is a source of expected comparative disutility and is evaluated sooner on average, so it provides a stronger incentive to practice patience. Thus, the aggregate goal is less effective at curbing impatience than incremental goals.

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not too large.

The agent stops the second-stage process strictly after completing the first stage when $\overline{x}_2^{agg} > k\overline{x}_1^{agg}$. Since $\overline{x}_1^{agg} < \overline{x}_1^{inc}$ by Proposition (15) and $\overline{x}_2^{agg} = \overline{x}_2^{inc}$, he stops sequentially given an aggregate goal whenever k satisfies Equation (2.27).

Proposition 16. *In a stationary equilibrium with an endogenous, aggregate goal, the agent's stopping threshold in the first stage decreases with ex-ante uncertainty if the agent is loss averse:* $\frac{\partial x_{1b}^{agg}}{\partial \epsilon} \leq 0$*, with equality only if* $\alpha = 1$ *or* $\eta = 0$*.*

When the agent sets an aggregate goal and stops sequentially, his reaction to ex-ante uncertainty differs markedly from that of incremental goals depending upon which stage he is in. While his behavior in the second stage is identical under both forms of bracketing, his stopping threshold in the first stage *decreases*, rather than increases, as uncertainty increases. Knowing that he will evaluate the aggregate goal upon completion of the second stage, he waits longer to complete stage 2 in order to realize a higher project value as compensation for anticipated comparative disutility. In the first stage, when there is no direct goal evaluation, then, he anticipates that the potential disutility from evaluating the aggregate goal is incurred even farther in the future, and thus discounts it more heavily. For this reason, the agent chooses a *lower* stopping threshold in the first stage in reaction to more uncertainty.

In equilibrium, his current and continuation value functions, denoted w_1^{agg} and v_1^{agg} respectively, are provided in Appendix B.1. The first two terms of the expected current value function, given by Equation (B.6), in its wait region reflect the option value of stopping in the first stage, while the second two reflect that of stopping in the second. In contrast to the case of incremental goals, the disutility from ex-ante uncertainty is absent from the first two terms, as the agent does not directly evaluate himself against a goal in the first stage. Likewise, the first term of the expected continuation value function, given by Equation (B.7), in its wait region reflects the option value of the first stage from an ex ante perspective, while the second term reflects that of the second.

2.6.3 Welfare

Because the second-stage threshold that the agent employs is identical regardless of whether he sets incremental or aggregate goals, all of the analysis described in Section (2.5.3) applies to the agent's behavior in the second stage when he sets an aggregate goal. However, comparing his behavior across both stages to an ex-ante first-best becomes more complex because the agent's behavior in the first stage is quite different when he sets an aggregate goal. In contrast to the case of incremental goals, he stops *earlier* as uncertainty increases because the effect of the aggregate goal weakens. Moreover, it is quite intuitive that he stops too early in both the first and second stages, relative to their respective first-best thresholds, whenever he stops too early the second stage. The aggregate goal has the strongest effect on improving patience in the second stage, when it is evaluated. Thus, if it is insufficient to overcome impulsiveness in the second stage, then it will certainly be insufficient to do so in the first stage, when its effect is weaker.

Proposition 17. *Let the agent set aggregate goals. Given his preferences, described by* $(\eta, \alpha, \beta, \lambda)$ *and environmental uncertainty* ϵ*, he stops earlier than the ex-ante first-best in the first stage whenever* $F(\cdot) \geq 0$*, where* F *is given by Equation (2.28). His second-stage behavior is described in Proposition 13. Thus, he stops earlier than the ex-ante first-best in both stages whenever* $F(\cdot) \geq 0$.

Unsurprisingly, given some degree of reference dependence, the welfare effects of loss aversion when the agent sets an aggregate goal are analogous to those described in Proposition (14) with respect to incremental goals. Since the intuition is identical, the details are provided in the Appendix.

2.7 Optimal Bracketing

Having separately considered the agent's behavior given that he sets incremental and aggregate goals, I now determine the conditions under which each type of bracketing is optimal from an ex-ante perspective. Because he prefers that future selves behave in a time consistent manner, he chooses to bracket such that the *continuation* value of the option is maximized.

First, consider the case in which, in addition to his bracketing and goal choices, the agent's reference-dependent preferences (η, α) may be chosen ex-ante to optimize his welfare. Is loss aversion ever optimal in this case, and what type of bracketing should he employ?

Regardless of how he brackets his goals, the agent expects to incur some comparative disutility once he completes the project, if he is loss averse and there exists ex-ante uncertainty over outcomes. Since both η and α contribute to patient behavior, reference dependence and loss aversion essentially act as substitutes in self-regulation. Thus, appropriately increasing his level of reference dependence while decreasing his degree of loss aversion enables him to maintain the same stopping thresholds in each stage in which he directly evaluates a goal. But in the absence of loss aversion, he expects to incur no comparative disutility from mean-zero uncertainty. Since he thus receives the same material payoff, but expects to incur no comparative disutility when he is not loss averse, he is better off in this case. Recall that the aggregate goal is always weaker source of motivation than a set of incremental goals, because he does *not* evaluate a goal directly upon completion of the first stage. Because the agent incurs no comparative disutility if he is not loss averse, then setting incremental goals is strictly better than setting an aggregate goals in this case. Thus, his ex-ante welfare is maximized when he sets incremental goals, is not loss averse, and his level of reference dependence is given by η^*.

Proposition 18. *In the presence of uncertainty ($\epsilon > 0$), the agent's ex-ante welfare is maximized when*

1. *he sets incremental goals,*

2. *he is not loss averse ($\alpha = 1$), and*

3. *his level of reference dependence is given by η^*, where $\eta^* = \frac{\overline{\gamma}-\gamma_1}{\gamma_1}$.*

However, the more relevant situation occurs when the agent's reference-dependent preferences (η, α) are not necessarily globally optimal and cannot be changed. In this case, he can only choose how to bracket his goals and his goal level(s), and he may face a trade-off between the benefits of additional motivation and the costs of additional comparative disutility from frequent goal evaluation. That is, the use of an aggregate goal, rather than incremental goals, arises as a way to compensate for the presence of loss aversion or excessive reference dependence.

In the absence of outcome uncertainty ($\epsilon = 0$), the agent's only relevant consideration is how much motivation he needs to counteract his impulsiveness. Thus, a necessary, but not sufficient[25], condition for choosing an aggregate goal in this case is that his degree of reference dependence is so high relative to his self-control problem that he waits longer than is ex-ante optimal when he sets incremental goals.[26] Here, incremental goals become increasingly undesirable relative to an aggregate goal as outcome uncertainty increases - aggregate goals are already preferable in terms of motivation alone, and more frequent evaluation leads to more expected disutility from uncertainty. This case is rather extreme, insofar as the agent is so reference-dependent that the degree of outcome uncertainty is irrelevant to his choice of goal bracketing.

[25]Clearly, this is not sufficient because he must also be better off under aggregate goals, which provide a weaker source of motivation.

[26]This intuition is explained in more detail in Hsiaw (2009b).

Therefore, I consider the more interesting, and arguably more realistic, situation in which the agent's choice of goal bracketing depends on the interaction between motivation and outcome uncertainty. That is, the agent's self-control is sufficiently poor (i.e., β is sufficiently low and $\lambda > 0$) relative to his reference dependence and loss aversion that incremental goals are preferred to the aggregate goal in the absence of outcome uncertainty: $E[v_1^{agg}(x_1, r^{agg})|\tilde{x}_1] \leq E[v_1^{inc}(x_1, r_1^{inc})|\tilde{x}_1]$ when $\epsilon = 0$. Since the second-stage stopping threshold is the same regardless of how he brackets, this condition is certainly satisfied whenever $\lim_{\epsilon \to 0} \overline{x}_1^{inc} \leq x_1^*$, which is equivalent to the following condition:

$$\frac{1}{2}\eta(\alpha + 1) \leq \frac{\overline{\gamma} - \gamma_1}{\gamma_1}. \tag{2.37}$$

In accordance with intuition, condition (2.37) requires that (η, α) must be sufficiently low relative to some self-control problem described by (β, λ), so that the agent does not wait longer than the ex-ante optimum $x_1^* = (\frac{\gamma_1}{\gamma_1 - 1})I_1$ in the absence of uncertainty. When the self-control problem is exacerbated (i.e., $\overline{\gamma} - \gamma_1$ increases), a larger degree of reference dependence and loss aversion can be welfare-improving.

When Equation (2.37) is satisfied, incremental goals provide desired motivation that is stronger than that of the aggregate goal.[27] However, as outcome uncertainty increases, the expected disutility incurred upon evaluation of a goal increases, making more frequent goal evaluation less desirable. On the other hand, increasing outcome uncertainty also indirectly weakens the motivational power of the aggregate goal. Because the agent expects more disutility from evaluation of the aggregate goal upon stopping, he requires a higher project value in the second stage in order to compensate

[27] Note that Equation (2.37) is a more stringent condition than is strictly necessary. Since $\overline{x}_1^{inc} > \overline{x}_1^{agg}$, there is clearly a range of (η, α) such that the agent waits strictly longer than the ex-ante optimum when he sets incremental goals (i.e., $\overline{x}_1^{inc} > x_1^*$), but may still be better off than under the aggregate goal. The exact condition can be obtained by equating the value functions under each form of bracketing when $\epsilon \to 0$, but requires an implicit function, since $\overline{x}_1^{agg}$ is only implicitly defined. Equation (2.37) illustrates the same intuition, and all cases in which it is satisfied must satisfy the exact condition.

for this expected loss. This implies that on average, he waits longer to invest in the second stage, and consequently evaluates his aggregate goal later. Thus, the power of the aggregate goal to induce more patient behavior in the first stage is weakened, because he anticipates that it will be evaluated even farther in the future (that is, only upon completion of the second stage). Because the former, direct effect of incremental goals is stronger than the latter, indirect effect of the aggregate goal, the difference in option values under each form of bracketing changes monotonically in the degree of outcome uncertainty.

Proposition 19. *Given that the agent is sufficiently reference dependent and loss averse, he will set incremental goals whenever uncertainty is sufficiently low, and an aggregate goal when it is sufficiently high. That is, for (η, α) sufficiently high, there exists a unique threshold $\tilde{\epsilon}$ such that $E[v_1^{inc}(x_1)|\tilde{x}_1] > E[v_1^{agg}(x_1)|\tilde{x}_1]$ when $\epsilon < \tilde{\epsilon}$, and $E[v_1^{agg}(x_1)|\tilde{x}_1] > E[v_1^{inc}(x_1)|\tilde{x}_1]$ when $\epsilon > \tilde{\epsilon}$.*

Since outcome uncertainty is proportional to the observed payoff, the expected disutility from goal evaluation is finite even as $\epsilon \to \infty$. For this reason, the disutility arising from outcome uncertainty may not be large enough to make setting an aggregate goal more desirable, if the agent's degree of loss aversion, in combination with his reference dependence, is sufficiently small. Thus, setting an aggregate goal is more likely to occur among more loss averse agents in the face of outcome uncertainty.

2.8 Conclusion

This paper addresses the role of goal bracketing as a source of internal motivation to attenuate the self-control problem of a hyperbolic discounter with reference-dependent preferences. When setting non-binding goals in a sequential stopping problem, an individual must decide how and when to evaluate himself against such goals. In particular, he can bracket broadly by setting an aggregate goal for the entire project, or he can bracket narrowly by setting incremental goals for individual stages. In the presence of loss aversion and uncertainty over outcomes, the intertemporal bracketing decision involves a trade-off between motivation and comparative disutility due to ex ante uncertainty. I find that if the agent is sufficiently loss averse and ex-ante uncertainty is high, he will choose to bracket broadly; otherwise, he brackets narrowly despite the disutility from frequent goal evaluation.

The model makes predictions regarding the instrumental use of bracketing that could be tested in a controlled environment. In particular, the comparative statics regarding the agent's response to environmental uncertainty in the first stage differ with his bracketing choice. This implies that an observer can distinguish when and whether the bracketing choice changes by varying the degree of ex-ante outcome uncertainty in the environment. Moreover, the model can be distinguished from an alternative specification, in which the agent's consumption utility is concave rather than linear and his comparison utility is either linear or non-existent. In this case, risk aversion in consumption utility implies that the agent dislikes mean-zero uncertainty, so he waits longer as uncertainty increases, just as if he has linear consumption utility but is loss averse and sets incremental goals. However, this alternative specification predicts that the bracketing choice does *not* vary in response to ex-ante uncertainty over outcomes, since there is no benefit from aggregating anticipated comparative disutility. Thus, the agent's response to environmental uncertainty does not change in this case.

The study of instrumental bracketing in intertemporal choice has been relatively unexplored by economists thus far. This paper offers several testable implications regarding the interaction between bracketing and uncertainty when the agent has a self-control problem due to present-biased preferences. Empirical tests of the theory would greatly contribute to our understanding of how, why, and when individuals bracket decisions and set goals for themselves.

Chapter 3

Lifestyle Brands

3.1 Introduction

In its narrowest definition, a brand is a good or class of goods that can be attributed to a specific firm. In recent years, the popular press has championed the redefinition of a brand as a device through which to "foster a sense of shared experience and of belonging" (Brady et al., 2004). Likewise, marketing researchers claim that "brands facilitate the community-forming process by making the identification of like-minded others visible and vivid . . . The why of consumption here lies in recognizing that brands say much about the groups that use them. Brands demonstrate shared beliefs, beliefs consumers like to recognize" (O'Guinn and Muñiz, 2005). In this era of mass consumption, "[consumers are] beginning to act like and feel like owners or members of a community . . . Newly empowered consumers can appropriate and manipulate the brand in whatever way they want" (Brady et al., 2004). While brands have traditionally been considered tools for identification of *firms*, these statements emphasize that branded goods can become *consumer*-oriented objects.

There are many horizontally differentiated goods for which there exist numerous varieties of comparable quality, yet some are branded and some are not. Many of these brands are known as "lifestyle brands," whereby the identification of a firm and its products centers on the characteristics of its customers rather than the product itself - they are "brand communities" whose members share a clear common identity and communicate with one another through their consumption choices. For example, Harley Davidson is known as a cult brand for motorcyclists who consider themselves rugged individualists; Crumpler sells assorted bags that appeal to non-conformist hipsters.[1] Furthermore, an increasing number of goods and services are bundled with social networking services that enable a firm's customers to share information amongst themselves in some capacity. For example, wine retailer WineStyles creates

[1] Crumpler's offbeat product names and descriptions clearly align with this segment of the population, with the "Salary Sacrifice" laptop bag and "The Manchild" wallet, as well as a logo that is a dread-locked stick figure.

clubs and organizes events for its customers to meet in person, while Harley Davidson organizes events and operates a members-only Internet forum for Harley Davidson owners. Netflix, an online DVD rental service, has a "Friends and Community" feature that allows its customers to share movie ratings and interests with friends and strangers with similar tastes. In addition, pure social networking services, such as Facebook, have grown increasingly popular because they serve as a platform for sharing interests with friends. There exist numerous applications, called widgets, that users can download and use through the Facebook platform, ranging from travel maps to online bookshelves to virtual fashion shows. Some of these widgets also enable users to connect to others whom they do not already know, but who may ostensibly share some common interests given their common use of the widget.[2] More recently, Facebook instituted an application called Beacon, which allows users to see their "friends'" online purchase activities with various third party vendors. Despite controversy regarding privacy issues, Beacon illustrates that social networking sites have begun to realize that they are offering a valuable information service and can attempt to capitalize on it.

While the rise of lifestyle brands and social networking services may appear disparate, this paper provides a common motivation for both phenomena and also considers firms' strategic brand investment and pricing choices. I propose a model in which agents wish to "meet" with similar types in order to obtain information. They each have imperfect information about certain dimensions of their own tastes, but are aware that their preferences are correlated with those of others in the population. I assume that agents can only (or, more easily) communicate with those who chose common actions. This constraint could be technological or physical.[3] For example,

[2]For example, the Scrabulous widget allows "friends" who both have the application to play online Scrabble together, and also allows access to the entire community of Scrabulous users, who might otherwise only be accessible to their existing "friends." Hence, the Scrabulous widget is one channel through which a user can find more potential "friends."

[3]More generally, I could assume that agents can communicate more easily with

only customers of Netflix can view information about the movie ratings of other users. When agents meet at a physical location, such as at Winestyles tasting event, they can only communicate with others who are present. Alternatively, a firm's logo or style, like Crumpler's dread-locked stick figure, may be more likely to be remembered and recognized by "insiders," i.e. consumers of that firm's goods, than by outsiders. More generally, this assumption captures the aforementioned observation that the buying and displaying of brands *facilitates* the exchange of information among consumers, insofar as it enables easier identification of like-minded individuals. Given that actions, not preferences, are publicly observable to others, agents can correctly infer that others are more likely to be similar to themselves if they share common actions. Thus, the desire to "meet" others with similar preferences results in an endogeous value of matching and the formation of reference groups. As a result, in equilibrium peers exhibit conformity of behavior in one dimension in order to identify one another and learn about their tastes along other dimensions.

Given the demand for coordination to expedite information exchange, firms have an obvious opportunity to supply some coordination service for their customers by increasing the recognizability of their consumption decisions. I assume that firms can invest in branding their goods in a way that facilitates their customers' "meeting" with one another and sharing information. Thus, I model the brand as a "meeting" or "recognition" technology that is accessible only to the firm's customers. In essence, this coordination service is bundled with the good or service itself, as in the preceding examples. I assume that this brand investment is an initial, one-period fixed cost that has no effect on the future marginal costs of production: designing a logo or style for a brand, or setting up the physical or virtual infrastructure of the meeting technology. I analyze equilibrium branding in a duopoly and, surprisingly, find that all consumers are worse off when a brand exists than when it does not. In contrast to Economides

those who chose common actions than those who did not. In this case, the qualitative results would be unchanged.

(1993), I also find that firms differentiate maximally in both vertical (i.e., brand strength) and horizontal attributes (i.e., product variety). Only one of the two firms chooses to brand its product, and yet both firms are able to charge sufficiently high prices that consumers are actually worse off than if brands could not exist.

Consider the following concrete example. A person who considers himself a rugged individualist wants to buy a motorcycle, and among other things, he is interested in taking road trips and does not know much about them. Although he can discover this for himself by making his best guess about what itinerary he would most enjoy, he might also be able to communicate with someone who has similar tastes to reduce the risk of going on a terrible road trip. He can buy a motorcycle that fits his ideal, but is generally non-descript and offers no external benefits, or he can buy a motorcycle somewhat farther from his ideal from Harley-Davidson, which organizes owners' events and operates a members-only Internet forum.[4] Thus, if he buys the Harley, he will be more likely to meet other Harley owners who recognize him as a fellow Harley owner and wish to discuss common interests, including road trips. Like him, the other Harley owners wish to talk to others who have similar tastes in order to learn more about their own tastes. Perhaps some of them have been on many good road trips but are not sure what local bars they should go to, and would like recommendations from someone knowledgeable like him. When he and others buy Harley-Davidsons for this reason, they endogenously become riders with similar tastes in other dimensions, like road trips and bar hangouts, and they all benefit from sharing this information amongst themselves. However, I show that Harley-Davidson is able to extract this surplus from its customers through pricing, so consumers are actually worse off than if they had been unable to recognize and meet one another in this way.

[4]The Harley-Davidson website proclaims, "The Harley Owners Group® is much more than a motorcycle organization. It's one million people around the world united by a common passion: making the Harley Davidson® dream a way of life." (Harley-Davidson, 2010)

The paper is organized as follows. Section 3.2 links this paper to related lines of research. Section 3.3 describes the basic model. Section 3.4 solves for a pooling equilibrium, where groups of agents choose identical actions that may diverge from their known tastes in order to learn from one another. Section 3.5 extends the model to include supply of the information-sharing mechanism through branding. In a duopoly setting, I examine the effects of agents' taste uncertainty on firms' profit-maximizing behavior and consumer welfare. Section 3.6 discusses the model's results and limitations and suggests avenues for future research. Proofs are gathered in the Appendix.

3.2 Literature Review

This paper connects several lines of research. First, it contributes to the economics literature on branding. Second, it relates to the body of work on consumption and identity. Thirdly, much of this literature is closely tied to the influence of peer effects as a source of consumption externalities. Lastly, this paper relates to the study of word-of-mouth and social networks.

The economics literature has traditionally placed the practice of branding (and similarly, of brand extension) in the context of a signaling game for "experience goods," goods whose value cannot be observed until they are actually consumed. Because consumers cannot observe the objective quality of a firm's product before purchase, a brand name attached to the product serves as a signal of quality. One interpretation is that the brand name is equivalent to the posting of a bond - if the branded product is of poor quality, then the firm is reneging on its bond and suffers a loss of reputation (Telser, 1980; Wernerfelt, 1988). Another, imported from the advertising literature, is that expenditures on a brand name credibly signal high quality because only the high quality firm can afford to "burn money" (Milgrom and Roberts, 1986). However, these theories are better suited to describe markets with vertically differentiated goods than horizontally differentiated goods. There are many goods for which there exist numerous varieties of comparable quality, yet some are branded and some are not, even if consumers do not have uncertainty about the goods' characteristics. Furthermore, they are hard-pressed to address the growth of brand communities and "lifestyle brands."

Marketing research discusses consumption as a medium of expression (Kleine, III et al., 1993) or an instrument for construction of identity (Elliott and Wattanasuwan, 1998). In this vein, Muller and Shachar (2008) consider firms' optimal choice of functional and self-expressive attributes when consumers use products as a means of self-expression. More recently, much attention has been devoted to consumption in a social context. Muñiz and O'Guinn (2001; 2005) discuss the role of brands as a channel for community formation because they serve as visual identification of others with similar tastes or beliefs. A recent series of papers in marketing discusses conformity and divergence of consumption choices to signal similarity to peers and dissimilarity from other groups, respectively (Berger et al., 2005; Heath et al., 2006; Berger and Heath, 2007). Kuksov (2007) considers the value of brands as a signaling device when agents engage in costly search for partnerships. In economics, consumption has been interpreted as a form of status signaling when identity is known and social preferences are a primitive of the model (Pesendorfer, 1995; Bagwell and Bernheim, 1996). Though factors such as status may also play an important role, I propose an alternative motivation for the existence of branded, horizontally differentiated goods that endogenizes the costs and benefits of matching with others, leading to the formation of reference groups and linking them to peer groups and subcultures.

This paper is also closely tied to the literature on peer effects as a source of consumption externalities. Bernheim (1994) finds that conformity can arise when individuals are concerned with others' views of them, as inferred by their actions.

Austen-Smith and Fryer (2005) study the influence of cultural norms and expectations on racial differences in school performance. Rather than entering preferences directly, peer effects can be endogenized when individuals have imperfect information about themselves, but can learn more about themselves from observing others (Banerjee and Besley, 1990; Battaglini et al., 2005). They can also arise from a technological innovation, such as a recommender system that improves information about product quality by aggregating individual signals (Bergemann and Ozmen, 2004). Here, I consider how a brand, or meeting technology, enables individuals with imperfect self-knowledge to observe and learn from others, and how firms make branding and pricing decisions accordingly.

Finally, this paper relates to the study of word-of-mouth and social networks by studying endogenous reference group formation in order to expedite communication about tastes. The study of word of mouth has primarily examined the role of sequential social learning in the creation of herding and information cascades (Bikhchandani et al., 1998). One line of research studies the effect of communication structure on information aggregation and efficiency when agents are boundedly rational (Ellison and Fudenberg, 1995; Bala and Goyal, 1998, 2001). Another studies firms' decisions and consumer welfare when fully rational consumers can communicate amongst themselves about the quality of goods through word of mouth (Vettas, 1997; Alcalá et al., 2006; Navarro, 2006). While the aforementioned work has studied the effects of given social structures, social network theory studies the stability and efficiency of high structured social networks when link formation is endogenous. Jackson and Wolinsky (1996) determine the structure of stable networks when bilateral agreement is required for link formation, while Galeotti et al. (2006) account for heterogeneity among agents.

3.3 The Model

Let the agents in a society choose a vector of actions $\mathbf{x}$ from a set $\mathbf{X}$. For simplicity, I assume that this vector has only three components, $\mathbf{x} = (x_1, x_2, x_3)$, where the full action set is the Cartesian product of three action spaces, $\mathbf{X} = X_1 \times X_2 \times X_3$. The set of actions in each action space is $X_j = \{S^1 \cup \emptyset\}$ for $j = 1, 2, 3$, where numerical actions lie on a circle with unit circumference ($S^1 = \{x \in \mathbb{R}^2 : \|x\| = \frac{1}{2\pi}\}$) and the empty set denotes the action of not choosing a number. Each of these action spaces can be interpreted quite broadly, where there exists no vertical differentiation within them. In the context of the previous example, we can imagine that agents choose a motorcycle, a road trip itinerary, and a bar hangout. Within each of these action spaces, the variety of choices of x_j emcompasses scooters to off-road bikes, touring California wine country to touring Louisiana backcountry, and patronizing nightclubs to sportsbars, respectively, or refraining from any of the respective activities. The circular model implies that there are no extremes and no "middle" in the realm of agents' preferences.

An agent receives utility v_j from choosing any action $x_j \in [0, 1]$, and utility 0 if $x_j = \emptyset$. I assume that this utility is the same across individuals, $v_j^i = v_j \ \forall i$. This utility can be interpreted as the baseline utility from taking the action - having a motorcycle, enjoying a road trip, or going to a bar. In addition, each agent i has an ideal variety over each of these sets X_j, $j = 1, 2, 3$, so he can be described by a vector of tastes $\boldsymbol{\theta}^i = (\theta_1^i, \theta_2^i, \theta_3^i)$, where $\theta_j^i \in S^1 \ \forall i, j$, which will denote his type. Then the full type space Θ is the Cartesian product of the taste spaces, $\Theta = \Theta_1 \times \Theta_2 \times \Theta_3$, where $\Theta_j = S^1$ for all $j = 1, 2, 3$. Hence, I assume that no agent's ideal is to refrain from an activity. An agent incurs disutility from choosing a numerical action that differs from his ideal, where the disutility is, without loss of generality, a quadratic function of the distance z from his ideal taste. To guarantee that individuals never abstain from an activity entirely, let $v_j \geq \frac{1}{4} \ \forall j$.

Assumption 1. *An individual's period utility is additively separable and decreases quadratically in the distance between his action and his taste:*
$U_i(x^i_1, x^i_2, x^i_3) = \sum_{j=1}^{3} [v_j - (\theta^i_j - x^i_j)^2]$.

There are two periods in which an agent i takes actions in X, but each agent only has unit demand in each action space over the two period span. For a given j, if he chooses $x_j \in [0,1]$ in the first period, then he must choose $x^i_j = \emptyset$ in the second period. If he chooses $x^i_j = \emptyset$ in the first period, then he can choose $x^i_j \in \{[0,1] \cup \emptyset\}$ in the second period. There is the usual discount factor $\delta \in [0,1]$.

Furthermore, I assume that agents have imperfect information about their types. Specifically, each agent only knows two components of his type.

Assumption 2. *Imperfect information: Each agent knows only two components of his type, and his information set is drawn from* $\{(\theta_1, \theta_2), (\theta_1, \theta_3)\}$. *Each of these information types is equally likely.*

Clearly, there is no reason that all agents should know their tastes in Θ_1 rather than Θ_2 or Θ_3. The general point is that there must exist at least one taste space, and corresponding action space, in which agents who have mutually beneficial information can identify one another through their choices.[5] In the X_1 action space, both information types might be able to infer one another's tastes through their choices of x_1. When both types have information that is useful to the other, there is an incentive for both parties to take actions that facilitate this possibility. Beyond having a common taste space over actions that can coincide, each agent must have information about another taste space that the other does not, so that communication is mutally desirable. Thus, a minimum of three taste spaces, with corresponding action spaces, is required for differing information types to pool in an action space.

[5]Alternatively, we could suppose that there are three information types, drawn from the set $\{(\theta_1, \theta_2), (\theta_1, \theta_3), (\theta_2, \theta_3)\}$. This specification leads to the same qualitative results when we have the perfect correlation structure stated below, but is technically incompatible with imperfect correlation.

Assumption 3. *Agents' preferences are perfectly correlated, but agents do not know the direction of the correlation. Suppose that an agent knows* θ_j*. A priori,* $P(\theta_k = \theta_j) = P(\theta_k = \theta_j + \frac{1}{2}) = \frac{1}{2}$ *for any unknown* θ_k*, where* $j \neq k$*.*

I assume that agents' preferences are correlated, but that they do not know the exact realization of this correlation in the population, though they are aware that it exists and that it is either $+1$ or -1. The assumption of perfect correlation is made for simplicity and without loss of generality. We can obtain the same qualitative results with imperfect correlation, as shown in Appendix C.1.[6] Suppose an agent knows that his ideal variety in action space j is θ_j but he does not know his ideal variety θ_k in action space $k \neq j$. Ex ante, it is equally likely that $\theta_k = \theta_j$ or that $\theta_k = \theta_j + \frac{1}{2}$, for *all* agents.[7] More formally, there are equally likely states of the world, and no agent knows the true state. A state $\omega \in \Omega$ is composed of three correlations, $\boldsymbol{\omega} = (\rho_{12}, \rho_{13}, \rho_{23})$, where we define ρ_{jk}, where $j \neq k$, as the true $P(\theta_j = \theta_k)$. Perfect correlation implies that $\rho_{jk} \in \{0, 1\}$ for $j \neq k$.[8] Clearly, knowing two correlations pins down the third (e.g., $\rho^{12} = \rho^{13} = 1$ implies $\rho^{23} = 1$), so not every permutation is possible. In fact, there are four possible states when there is perfect correlation: $\Omega = \{(1,1,1), (1,0,0), (0,1,0), (0,0,1)\}$.

Assumption 4. *The distribution of agents' tastes is uniform in each taste space* $\Theta_j,\ j = 1, 2, 3$ *over the unit circle, normalized to the interval* $[0, 1]$.

Because the agents' incentives to pool on an action is clearly influenced by the underlying distribution of the population, I assume uniformity in order to ensure that such an equilibrium is not driven trivially by the nature of the distribution. The same

[6]In particular, changing the absolute value of ρ is equivalent to decreasing the ease of communication (i.e., rescaling the parameter α downward), as expected.

[7]This assumption does not need to be interpreted literally. It is clearly equivalent to supposing that either $\theta_k = \theta_j + a_j$ or $\theta_k = \theta_j + \frac{1}{2} + a_j$, where the a_j's are known constants with appropriate restrictions on $a_j,\ j = 1, 2, 3$.

[8]Since $\rho_{jk} \equiv P(\theta_j = \theta_k)$, then $\rho_{jk} = P(\theta_j = \theta_k) = 1 - P(\theta_j = \theta_k + \frac{1}{2}),\ \forall j \neq k$.

motivation underlies the assumption that each information type is equally likely to occur in the population.

If agent i chooses an $x_j^i \neq \emptyset$ in period 1, then he might be able to receive useful information about the behavior of others in the population, which can inform him about his own preferences along other dimensions. Figure 3.1 depicts the information mechanism for any action $x_j \neq \emptyset$.

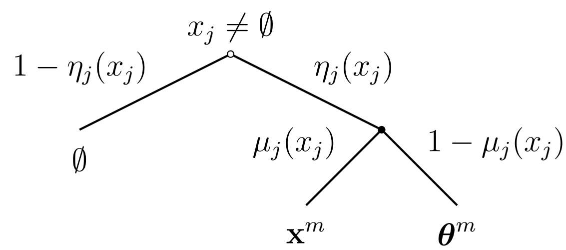

Figure 3.1: The Information Mechanism

Let $\eta_j(x_j) = \min\{\alpha\mu_j(x_j), 1\}$, where $\alpha \geq 0$ is a constant, and let $\mu_j(x_j)$ be the proportion of agents who chose x_j. The parameter α reflects the ease of communication within the community. The agent is more likely to encounter others who chose the same action x_j if their numbers in the population are relatively large. We can interpret this probability as the result of search success. Specifically, he meets with another agent m who has chosen the same action x_j with probability η_j.[9] However, he is unsuccessful in meeting another agent with probability $1 - \eta_j$. In this case, choosing action x_j does not produce new information about his unknown tastes. Denoting the signal he receives from choosing action x_j by $s_j(x_j)$, acquiring no information is equivalent to $s_j(x_j) = \emptyset$. Likewise, if agent i chooses no action ($x_j^i = \emptyset$) in period 1, then he observes nothing, i.e. $s_j(x_j) = \emptyset$ with certainty.

[9]Strictly speaking, we could suppose that there are n agents in the population and $m \leq n$ agents who chose action x_j. If each individual agent's probability of being found or available is $\frac{1}{n}$, then an agent's probability of meeting another agent who chose x_j is $1 - (1 - \frac{1}{n})^m$. As n and m become infinitely large, then this probability converges to $1 - e^{-n/m} = 1 - e^{-\mu_j(x_j)}$. We can take $\mu_j(x_j)$ as a linear approximation of this probability, for tractability.

If he chooses action x_j and does meet another agent m, then agent i learns either m's ideal *tastes* ($s_j(x_j) = \boldsymbol{\theta}^m$) with probability $1 - \mu_j(x_j)$, or m's first-period *actions* ($s_j(x_j) = \mathbf{x}^m$) with probability $\mu_j(x_j)$.[10] While information about tastes is fully revealing, actions are less informative since they can deviate from ideal tastes. For example, if i is able to talk to m, they can exchange information about their preferences; if i can only observe m, he can only see his actions, such as hairstyle and clothing choices, but not how much he likes or dislikes them. Thus, the likelihood of successful information acquisition increases in community size, but its expected quality coarsens. I interpret this structure as a feature of congestion. This may be due to a technological limitation, like a server space constraint such that there is a trade-off between allowing higher volume of less detailed information, such as a list of other products an agent has purchased, or a lower volume of richer information, such as a detailed review of each purchase. A more psychological explanation is that if there is a potentially vast quantity of information to parse through, an individual might reduce search costs by looking for coarser information, which requires less effort to acquire. I assume this particular information structure for tractability.[11] Hence, the probability of obtaining information increases with the proportion of others who chose the same action, but the value of information decreases if this proportion becomes too large. In the most extreme case, an agent obtains no information if all agents of the same information type choose the same action.[12]

Although I have assumed the constraint, whether technological or physical, that

[10]I assume that when two consumers meet and exchange information about tastes, they always report these tastes truthfully. Given that neither agent has any incentive to lie about either his actions or his tastes, truthful disclosure is the informative equilibrium in a game where agents can choose whether and what to disclose private information costlessly upon meeting.

[11]This particular information structure enables us to obtain an explicit analytical solution in the duopoly setting that follows, but is not necessary for qualitative results to hold. More generally, a necessary condition is that agent i cannot always learn m's tastes, which are fully revealing.

[12]If all agents of the same information types choose the same action x_j, i is just as likely to draw the wrong inference about his true θ_k^i as he is to draw the correct one if he does not account for this fact when interpreting observation $\mathbf{x}^m$.

agents can only communicate with others who share a common action, I could more generally assume that agents can communicate *more easily* with those who chose common actions than those who did not. Since it is the relative benefit of coordination that is relevant to an agent's action decision, increasing the ease of communication among agents who did not choose common actions is qualitatively equivalent to decreasing α, the ease of communication among agents who chose a common action, in the current model. Thus the qualitative results are unchanged as long as such a relative benefit of coordination exists, though they weaken as this relative benefit decreases. If agents communicate freely (and honestly) with anybody, regardless of their actions, then there is no additional informational benefit to coordination over actions, which is equivalent to the standard case of $\alpha = 0$.

Figure 3.2 describes the series of events.

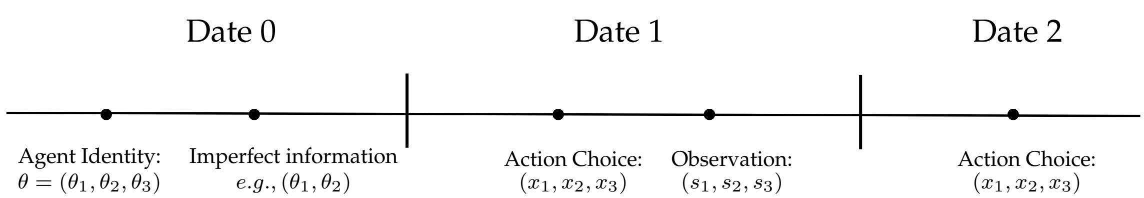

Figure 3.2: Timeline of Events

3.4 Equilibria

Because of the expected information benefit of learning the true correlation structure (i.e., the true state), an agent might decide to postpone his action x_k and choose an $x_j \neq \theta_j$ in order to learn his unknown θ_k. Then he will be able to select the x_k in the second period that is exactly his ideal. In other words, he (and others) might decide to pool on an action that is not necessarily their known ideal, so that they can gain information about their tastes in another taste space.

Clearly, no agent has an incentive to pool in the second period, since there is no information gain from pooling at that time. Hence, I look for pooling in a "symmetric-by-types" equilibrium in the first period in the following sense: The two information types act identically and pool in a single action space. For example, suppose that (θ_1, θ_2) and (θ_1, θ_3) types pool on the X_1 action space by acting symmetrically. These two types clearly will not pool in either the X_2 or X_3 spaces. The (θ_1, θ_2) types will only pool in X_2 if there is a benefit from doing so, by obtaining information about θ_3 from (θ_1, θ_3) types. But since the (θ_1, θ_3) types want to learn about θ_2, they have no incentive to pool in X_2. Hence, the (θ_1, θ_2) types will not pool in X_2 either. A similar argument holds for pooling in X_3. Since we are assuming that (θ_1, θ_2) and (θ_1, θ_3) types act symmetrically, we will solve the (θ_1, θ_2) type's maximization problem without loss of generality. In fact, I show that any equilibrium must be "symmetric-by-types" (see Appendix).

Moreover, pools can also be classified through symmetry or asymmetry in variety choice within a given dimenson. A pool at the point x_1 is symmetric if $|\underline{\theta}_1 - x_1| = |\overline{\theta}_1 - x_1|$, where $\underline{\theta}_1$ and $\overline{\theta}_1$ are the marginal agents at the endpoints of the pool, and asymmetric otherwise. I characterize the former case in detail by constructing an individual pool, then considering the complete set of pools in an action space.

3.4.1 Symmetric Equilibria

If an agent i does not pool at a given point x_1, then he can either choose not pool at all or pool at some other point x_1'. To characterize the full set of symmetric equilibria, I first consider the case in which an agent's best alternative to pooling at a given point x_1 is the former option, then analyze the latter case.

If agent i's best alternative is not to pool, then he should instead follow his ideal tastes and choose $x_1 = \theta_1, x_2 = \theta_2$ in period 1. Since he will acquire no additional information about θ_3, then he chooses x_3 in period 1 as well. In order to choose x_3, he solves the following decision problem,

$$\max_{x_3} \; -\frac{1}{2}(\theta_1 - x_3)^2 - \frac{1}{2}(\theta_1 + \frac{1}{2} - x_3)^2,$$

for which the solution is $x_3 = \theta_1 + \frac{1}{4}$ and his expected payoff is $v_1 + v_2 + v_3 - (\frac{1}{4})^2$.[13]

If the agent pools, with probability η_1 he observes the actions or learn the tastes of another agent. The correlation structure implies that i can perfectly infer θ_k^i given any $\mathbf{x}^m$ when up to half of the population of information types pools monotonically at x_j. When a majority of information types pools monotonically at x_j, signal dilution arises from the presence of those whose tastes diverge too much from the rest of the pool and the fact that the agent cannot always learn others' *tastes*. If he observes another's actions instead, then it is possible that he may draw the wrong inference about his own tastes because the other's (unobserved) tastes differ too much.

Suppose that $\mu_1 \leq \frac{1}{2}$, so that any $\mathbf{x}^m$ from a different information type is fully informative. Then he is equally likely to meet another (θ_1, θ_2) agent, from whom he learns nothing about θ_3, or a (θ_1, θ_3) agent, from whom he will learn his own θ_3 perfectly. But there is still a chance $(1 - \frac{\eta_1}{2})$ that he learns nothing from pooling, so his payoff is $v_1 - (\theta_1 - x_1)^2 + v_2 + \delta[v_3 - (1 - \frac{\eta_1}{2})(\frac{1}{4})^2]$. Therefore, in order to be willing to pool, his expected utility from pooling must exceed his utility from deviating:

$$v_1 - (\theta_1 - x_1)^2 + v_2 + \delta[v_3 - (1 - \frac{\eta_1}{2})(\frac{1}{4})^2] \geq v_1 + v_2 + v_3 - (\frac{1}{4})^2$$

$$\delta \geq \frac{v_3 - (\frac{1}{4})^2 + (\theta_1 - x_1)^2}{v_3 - (1 - \frac{\eta_1}{2})(\frac{1}{4})^2}.$$

For an agent to pool at a particular action x_1, we must have that $\frac{v_3 - (\frac{1}{4})^2 + (\theta_1 - x_1)^2}{v_3 - (1 - \frac{\eta_1}{2})(\frac{1}{4})^2} \leq 1$. This implies that

$$(\frac{\eta_1}{2})(\frac{1}{4})^2 \geq (\theta_1 - x_1)^2. \tag{3.1}$$

[13]Alternatively, he could select $x_3 = \theta_2 + \frac{1}{4}$ and his expected payoff would be identical.

Equation (3.1) states that in order to be willing to pool, the net expected information gain from pooling must exceed the loss from choosing an action that differs from one's own ideal.

Let $\delta = 1$ and α be sufficiently low that $\alpha\mu_1 \leq 1$.[14] Then we can use the above equation to solve for pooling equilibria by finding the indifference conditions at the edges of the pool at any point $x_1 \in [0, 1]$, assuming that the marginal types on both ends of the pool are indifferent between pooling at x_1 and not pooling anywhere. Let $\overline{\theta}_1(x_1)$ be the highest type in the pool and $\underline{\theta}_1(x_1)$ be the lowest type in the pool. Then $\mu_1(x_1) = \overline{\theta}_1(x_1) - \underline{\theta}_1(x_1)$. The following conditions must hold for an interior solution with $\mu_1(x_1) \leq \frac{1}{2}$:

$$\frac{\alpha(\overline{\theta}_1 - \underline{\theta}_1)}{2}(\frac{1}{16}) = (\overline{\theta}_1 - x_1)^2 \tag{3.2}$$

$$\frac{\alpha(\overline{\theta}_1 - \underline{\theta}_1)}{2}(\frac{1}{16}) = (\underline{\theta}_1 - x_1)^2, \tag{3.3}$$

where $\underline{\theta}_1 \leq x_1 \leq \overline{\theta}$. There is obviously a multiplicity of equilibria, since we have two equations and three unknowns. Since agents need to coordinate on an action, this is not surprising. However, for any given x_1, we can solve for the endpoints $\overline{\theta}_1$ and $\underline{\theta}_1$ to obtain pooling solutions. There are two pooling solutions for this system. The first

[14] For now, I am fixing this exogenous parameter so that I can work directly with $\alpha\mu_j(x_j)$ without worrying about a non-linear probability function, since I am supposing that $\mu_1(x_1) \leq \frac{1}{2}$ in this equilibrium. When α is an endogenous variable in a later application, this assumption will no longer be necessary, since it is clearly suboptimal to choose an α such that $\alpha\mu_1 > 1$ if α is costly.

is the pooling solution,

$$\underline{\theta}_1 = x_1 - \frac{\alpha}{16} \tag{3.4}$$
$$\overline{\theta}_1 = x_1 + \frac{\alpha}{16} \tag{3.5}$$
$$\mu_1(x_1) = \frac{\alpha}{8}. \tag{3.6}$$

The second is the separating equilibrium, $x_1 = \overline{\theta}_1 = \underline{\theta}_1$. However, it can be shown that only the pooling solution is stable (proof provided in the Appendix). Therefore, when the best alternative to pooling at x_1 is not to pool at all, the unique equilibrium is a pool of size $\mu_1(x_1) = \frac{\alpha}{8}$, where the pool is also symmetric in the x_1 action space about the point x_1. Since $\alpha\mu_1 \leq 1$, we can verify that $\mu_1(x_1) \leq \frac{1}{2}$ in equilibrium, as surmised. This implies that any pool is composed only of sufficiently similar types, who observe one another's actions to learn about their own tastes. Based on the common action x_1, they correctly infer that all agents who chose this action have similar tastes to their own, and they benefit from sharing information with one another. Hence, peers exhibit conformity of behavior in order to identify one another and form reference groups to learn their tastes. Figure 3.3 illustrates the symmetric pooling equilibria characterized by Equations (3.2) and (3.3), since the marginal agents' problems are symmetric about the pooling point.

Note that it is entirely possible that more than one pool of this form exists in a given action space, as long as the pools do not overlap. Moreover, the fact that only the pooling equilibrium is stable implies that *all* agents in the Θ_1 taste space will pool on the action space X_1. If every pool is exactly of size $\frac{\alpha}{8}$, then this presents an n-integer problem, if the parameter α is a value such that the number of pools required to "fill" the space completely is not an integer. In this event, the remaining space in X_1 can neither stably exist as a set of points at which the agents are not pooling, nor as a pool of size smaller than $\frac{\alpha}{8}$ (proof provided in Appendix), so this

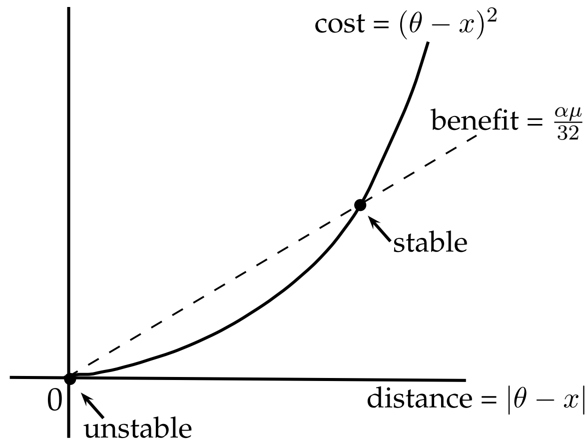

Figure 3.3: Symmetric Equilibria

pooling equilibrium is not sustainable if $\frac{\alpha}{8}$ is not an integer.

In the preceding construction, I have characterized the set of possible equilibria based on the assumption that marginal agents' best alternative to pooling at a point x_1 was not to pool anywhere. Suppose, instead, that the best alternative to pooling at x_1 is to pool at some other point x_1'. Since pooling at x_1' must also be better than not pooling anywhere in order to be the best alternative to x_1, this implies that x_1' must be sufficiently close to x_1 in order to be the best alternative. Restricting our attention to symmetric equilibria (i.e., equilibria of the form $|\underline{\theta}_1 - x_1| = |x_1 - \overline{\theta}_1|$), it follows that the maximum pool size is $\mu_1 = \frac{\alpha}{8}$, but pools of size $\mu_1 \in (0, \frac{\alpha}{8}]$ can stably exist if the n-integer condition is satisfied.

Clearly, Equation (3.1) implies that any configuration in which the X_1 space is completely filled with pools of identical size, where $\mu_1 \in (0, \frac{\alpha}{8}]$ and the value of α allows for the number of pools of size μ_1 to be an integer, is a stable equilibrium. Any marginal agent between two adjacent pooling points is indifferent between joining the two pools, and is weakly better off joining a pool than not pooling anywhere (and is strictly better off if $\mu_1 < \frac{\alpha}{8}$). This implies that for any $\alpha > 0$, there exists at least one equilibrium where all pools are symmetric and identical in size. Furthermore, there also exist equilibria in which adjacent symmetric pools can be of different size, but must satisfy certain conditions. In particular, the complete set of symmetric pooling equilibria is characterized in the following proposition, where the proof is provided in the Appendix.

Proposition 20. *1. For all $\alpha > 0$, there exists a unique type of pooling equilibrium, which is characterized by the following behavior:*

(a) No agent pools in more than one action space.

(b) In period 1, the two information types pool on an action $x_j \in X_j$ where θ_j is known and choose $x_k = \emptyset$ for unknown θ_k.

(c) In any action space where an agent knows his taste and does not pool, he chooses his known ideal.

2. For any value of α such that $\alpha\mu(x_j) \leq 1$, there exists at least one equilibrium in which the action space X_j is completely filled with symmetric pools. Furthermore, only two configurations are possible when all pools are symmetric:

(a) All pools in X_j are identical in size μ, where $\mu \in (0, \frac{\alpha}{8}]$.

(b) Exactly two differing pool sizes, μ and μ', coexist in an action space X_j, where $\mu \in (0, \frac{\alpha}{8})$ and $\mu' = \frac{\alpha}{8} - \mu$.

The above statement implies that at most, two differing pool sizes can simultaneously exist in an action space when all pools are symmetric across a pooling point. Clearly, this allows for a configuration in which adjacent pools are always different, which implies that adjacent pools alternate in size between $\mu_j(x_j)$ and $\mu_j(x'_j)$ subject to $|x_j - x'_j| = \frac{\alpha}{16}$. There can also exist equilibria in which some adjacent pools are differing in size, while other adjacent pools are identical in size in X_j. However, as long as there are adjacent pools of differing size somewhere along X_1, it must be that they are bounded in size on the interval $(0, \frac{\alpha}{8})$ and that the distance between their pooling points is exactly $\frac{\alpha}{16}$.

The comparative statics of an individual symmetric pool can be summarized in the following proposition:

Proposition 21. *In any action space X_j, a symmetric pool $\mu_j(x_j)$ about the action x_j exhibits the following properties:*

1. *Pool size $\mu_j(x_j)$ is invariant to location x_j.*

2. *Pool size increases with α, attaining a maximum of $\frac{\alpha}{8}$.*

Thus, agents with similar tastes may endogenously exhibit conformity of behavior in order to identify one another and learn about their tastes in other dimensions.

3.4.2 Asymmetric Equilibria

Thus far, I have only characterized equilibria in which pools are symmetric about the point x_1. However, there is also the possibility that $|\underline{\theta}_1 - x_1| \neq |\overline{\theta}_1 - x_1|$, where $\underline{\theta}_1$ and $\overline{\theta}_1$ are the marginal agents at the endpoints of the pool. Clearly, any asymmetric pool is also a continuous interval. Moreover, it follows from Equation (3.1) that the size of any asymmetric pool is bounded above by $\frac{\alpha}{8}$. However, the range of asymmetric configurations that can occur is far less restricted than in the symmetric case. In the ensuing application to brand investment, asymmetric equilibria will not occur in equilibrium, so I do not characterize the entire set of asymmetric equilibria.

3.5 Duopoly with Brand Investment

A natural extension of this framework is to consider consumption of a good as a specific type of action, so that an action space X_j becomes the variety space of an "experience good" j, over which there is a uniform distribution of consumers with ideal varieties θ_j. Books, music, clothing, and automobiles are among typical experience goods for which consumers have ideal tastes, even holding objective quality constant. In this context, firms can serve as the suppliers of the information mechanism that allows consumers to meet one another more easily.

Suppose that one action space (say, X_1) is actually a goods space in which agents must choose from the set of varieties that firms have chosen to offer. This restriction reasonably reflects the limited availability of varieties for most goods in the real world. More specifically, I assume that there are only two varieties of the good available, each of which is offered by a different firm at some price. All consumers have the same reservation price v_1 for the good, $v_1 > 0$. Because it is arguably more difficult for agents to coordinate on common actions when there is an infinite spectrum of possible actions, the limited variety of a consumption good can serve as a coordinating constraint for consumers to try to meet one another, an opportunity that firms offering these varieties can try to exploit.

There are two firms, each offering a variety of the good, denoted by x_1 and x_2, at prices p_1 and p_2. Firms simultaneously choose the locations of the varieties, then set prices. However, before choosing price, they have the option of investing in a coordination service, whose efficacy is measured by the parameter α, that enables their own customers to learn about each other's preferences over other goods. This coordination service is a "meeting" or "recognition" technology, such as a physical club, an Internet community, a logo, or a style. For example, the wine retailer WineStyles creates clubs and organizes events for customers to meet in person, while Harley Davidson operates an Internet forum for Harley Davidson owners and also sells its own clothing line. Each firm n simultaneously chooses an investment level $\alpha_n \geq 0$, $n = 1, 2$, in the first stage, according to a quadratic cost function. The information mechanism for the brand is as I described in the basic model. The parameter α measures the "effectiveness" of the brand. If the firm does not invest in a brand, then its customers cannot "meet" or "recognize" each other and exchange information, so choosing $\alpha = 0$ is equivalent to choosing not to invest in the brand. The marginal cost of producing one unit of any variety is identical for both firms, denoted c where $c \geq 0$, and is independent of the level of brand investment. This assumption is plausible in the preceding examples, and allows us to isolate the effect of brand investment. Firms cannot price discriminate.

Although we are only studying the market in one goods space, the information benefits from learning about others' preferences will be realized in other action spaces, so the presence of the brand will certainly affect behavior in this market. To simplify the exposition, I assume that the firms' varieties x_n where $n = 1, 2$ are exogenously given and equidistant ($x_1 - x_2 = \frac{1}{2}$). Endogenizing location leads to equidistant location choices and the same results (proof provided in Appendix).

Timeline

1. Firms n simultaneously choose a variety x_n (i.e., location), where $x_n \in [0, 1]$ for $n = 1, 2$.

2. Firms n simultaneously choose α_n, where $n = 1, 2$ and incur investment costs $c_\alpha \alpha_n^2 \geq 0$ where $c_\alpha \geq 0$.

3. Firms simultaneously choose prices $p_n,\ n = 1, 2$.

4. Consumers choose whether or not to buy variety x_1 or x_2. Before making a decision, consumers know whether a firm has the brand or not, and how effective this brand is.

5. (Consumers choose to buy goods in other markets/dimensions.)

I proceed by solving the model backwards in pure strategies, considering each pair of strategies in turn. I assume that the market is always covered, so marginal cost c is sufficiently low and the reservation price v is sufficiently high. Because I assume that firms' locations are equidistant, consumers' behavior will be symmetric. I denote

$d_1 = \overline{\theta}_1 - x_1 = x_1 - \underline{\theta}_1, d_2 = \underline{\theta}_1 - x_2 = x_2 - \underline{\theta}_2$, where $\underline{\theta}_2 \leq x_2 \leq \underline{\theta}_1 \leq x_1 \leq \overline{\theta_1}$ and $2d_1 + 2d_2 = 1$. Since firms' locations are equidistant, then $x_1 - x_2 = \frac{1}{2}$. This can be described graphically by Figure 3.4.

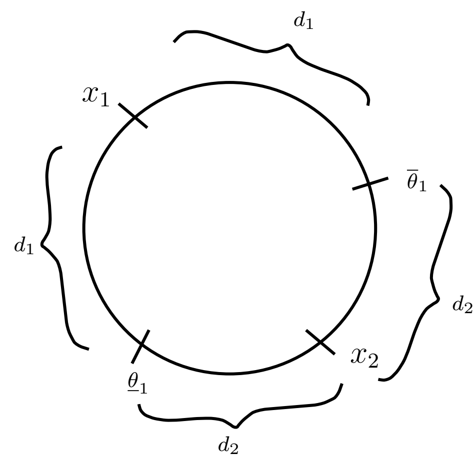

Figure 3.4: Duopoly

Let μ_n denote firm n's market share. Without loss of generality, let $\alpha_1 \geq \alpha_2$. I assume that $\mu_1 \geq \frac{1}{2}$ and later verify that this holds in equilibrium. I also assume that firms' locations are equidistant ($x_1 - x_2 = \frac{1}{2}$) to simplify the exposition, but it can be shown that equidistant locations is the unique equilibrium in this game (see Appendix).

3.5.1 Demand

Suppose that the firm with a stronger brand captures the majority of the market: $\mu_1 \geq \frac{1}{2}$. With probability $1 - \frac{\eta_1}{2}$, a customer of firm 1 acquires no new information about his own tastes, because he either meets no one or he meets another (θ_1, θ_2) agent, from whom he learns nothing about θ_3. In this case, he chooses action $\theta_1 + \frac{1}{4}$ and his expected payoff is $-(\frac{1}{4})^2$. With probability $\frac{\eta_1}{2}$, he meets an agent m of type (θ_1, θ_3) who chose x_1. If he observes m's tastes (θ^m), which occurs with conditional probability $1 - \mu_1$, then he can infer his own tastes perfectly, so his expected payoff will be zero. But if he observes m's actions ($\mathbf{x}^m$), which occurs with conditional probability μ_1, then he may draw the wrong inference about his own tastes because m's (unobserved) tastes differ too much. Since $\mu_1 \geq \frac{1}{2}$, there is a proportion, $1-\frac{1}{2\mu_1}$, of firm 1's customers whose tastes differ sufficiently from the others that the observation of actions $\mathbf{x}^m$ can lead to the wrong inference. To account for this possibility, he chooses action x_1 by solving the maximization problem

$$\max_{x_1}\{-\frac{1}{2\mu_1}(\theta_1 - x_1)^2 - (1-\frac{1}{2\mu_1})(\theta_1 + \frac{1}{2} - x_1)^2\},$$

where the first term is the expected utility from observing an action that leads to the correct inference about θ_3 and the second is the expected utility from observing an action that leads to the wrong inference about θ_3.[15] Due to this signal dilution, he hedges by choosing action $x_1 = \theta_1 + \frac{1}{2}(1-\frac{1}{2\mu_1})$ when he observes m's actions $\mathbf{x}^m$. Therefore, his expected information value from patronizing firm 1 is

$$\begin{aligned} -(1-\frac{\alpha_1\mu_1}{2})[\frac{1}{16}] - (\frac{\alpha_1\mu_1}{2})\left\{\mu_1[\frac{1}{2\mu_1}((\frac{1}{2}-\frac{1}{4\mu_1}))^2 + (1-\frac{1}{2\mu_1})(\frac{1}{4\mu_1})^2] + (1-\mu_1)(0)\right\} \\ = -\frac{1}{16} + \frac{\alpha_1}{32}(1-\mu_1), \end{aligned}$$

where the first term is the expected benefit if he does not meet another agent m, the next two terms are the expected benefit if he observes m's actions ($\mathbf{x}^m$), and the last term is the expected benefit if he observes m's tastes ($\boldsymbol{\theta}^m$).

Since $\mu_2 = 1 - \mu_1 \leq \frac{1}{2}$, a customer of firm 2 knows that any observation of a differing information type is fully informative, so his expected information value from

[15]This formulation assumes that the true state is that $\theta_1 = \theta_3$ without loss of generality. If we assume instead that $\theta_1 = \theta_3 + \frac{1}{2}$, the optimal action changes, but its expected payoff and the expected information benefit from patronizing firm 1 are identical.

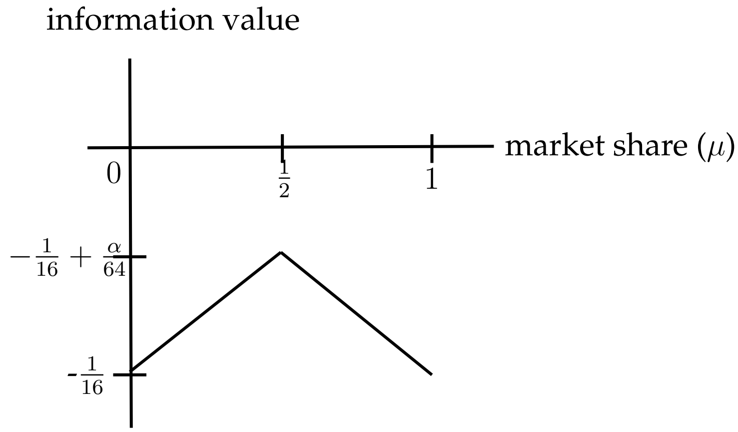

Figure 3.5: Expected Information Value

patronizing firm 2 is

$$-(1-\frac{\alpha_2\mu_2}{2})[\frac{1}{16}]-(\frac{\alpha_2\mu_2}{2})[\mu_2(0)+(1-\mu_2)(0)]$$
$$=-\frac{1}{16}+\frac{\alpha_2}{32}(\mu_2).$$

Thus, there is a non-monotonic relationship between the equilibrium information value from patronizing a firm and its market share μ, depicted in Figure 3.5. This hump-shaped relationship arises endogenously due to the trade-off between search cost, since having a larger community increases the ease of communication, and signal dilution, since agents cannot always observe others' preferences perfectly. The assumption that an agent observes another's actions with probability μ or another's preferences with probability $1-\mu$ upon meeting is not necessary to obtain a decreasing relationship between information value and market share when $\mu > \frac{1}{2}$; this will occur whenever the information structure allows signal dilution to occur sufficiently quickly. Rather, this particular assumption generates the *linear* decreasing relationship when $\mu > \frac{1}{2}$, allowing for an explicit analytic solution for firm 1's demand.

The marginal consumers' indifference conditions can be described by the following equation:

$$p_1 + d_1^2 - \frac{\alpha_1}{32}(1 - \mu_1) = p_2 + d_2^2 - \frac{\alpha_2}{32}(\mu_2), \tag{3.7}$$

where $d_2 = \frac{1}{2} - d_1$ and $\mu_1 + \mu_2 = 1$. Then we can obtain firm 1's demand when the investment level is endogeneous, μ_1^{EI}, as

$$\mu_1^{EI} = \frac{\alpha_1 - \alpha_2 + 8}{\alpha_1 - \alpha_2 + 16} + \frac{32(p_2 - p_1)}{\alpha_1 - \alpha_2 + 16}. \tag{3.8}$$

The first term of Equation (3.8) indicates that firm 1 would have a larger market share if prices were equal. The second indicates that demand is less sensitive to the price differential than it is in the standard case when brands do not exist, which is equivalent to no investment by either firm ($\alpha_1 = \alpha_2 = 0$). Neither of these features is surprising, since firm 1 has the technological advantage. But the fact that the information benefit of a brand is determined by consumers' equilibrium behavior, rather than exogenous, is the reason that price competition is directly weakened by the difference in brand effectiveness. Firm 1 can raise its prices to some extent without decreasing the total value of its good, because the information benefit from patronizing firm 1 actually increases when market share decreases at the margin.[16] That is, firm 1's product is so popular that its customer base contains a subset whose taste is too divergent from the others'. Increasing the price of its product decreases the size of this fringe so that the remaining customer base is more homogeneous in taste. For this reason, the information value of the brand increases for these customers and they are willing to pay a higher price for it. Hence, firm 1's price elasticity of

[16]The information value of firm 1's good decreases in market share at the margin due to the combination of two factors. First, the specified correlation structure implies that signal dilution occurs whenever a firm's market share is greater than $\frac{1}{2}$. Second, the duopoly setting implies that the investing firm should will have a market share that exceeds $\frac{1}{2}$. When either of these conditions does not hold, then the information benefit will not decrease with market share at the margin. More generally, however, we can obtain this phenomenon whenever alternative correlation and market structures are specified such that signal dilution increases with market share and occurs at the margin.

demand, ϵ_{11}^{EI} decreases as the technology gap $(\alpha_1 - \alpha_2)$ between the two firms widens, so firm 1's markup will be higher:

$$\epsilon_{11}^{EI} = \frac{4p_1}{4(p_2 - p_1) + 1 + \frac{1}{8}(\alpha_1 - \alpha_2)}.$$

3.5.2 Prices

Given demand, firm 1 chooses price to maximize profit, taking p_2 as given:

$$\max_{p_1}(p_1 - c)(\frac{\alpha_1 - \alpha_2 + 8}{\alpha_1 - \alpha_2 + 16} + \frac{32(p_2 - p_1)}{\alpha_1 - \alpha_2 + 16}) - k\alpha_1^2.$$

Verifying the second order condition, we obtain

$$p_1 = \frac{1}{2}[p_2 + c + \frac{1}{4} + \frac{1}{32}(\alpha_1 - \alpha_2)].$$

Likewise, firm 2 maximizes profit, where $\mu_2^{EI} = 1 - \mu_1^{EI}$, to obtain

$$p_2 = \frac{1}{2}[p_1 + c + \frac{1}{4}].$$

Hence, the Nash equilibrium in prices is $p_1^{EI} = c + \frac{1}{4} + \frac{1}{48}(\alpha_1 - \alpha_2)$ and $p_2^{EI} = c + \frac{1}{4} + \frac{1}{96}(\alpha_1 - \alpha_2)$. Firms' market shares are $\mu_1^{EI} = \frac{2}{3}(\frac{\alpha_1 - \alpha_2 + 12}{\alpha_1 - \alpha_2 + 16})$ and $\mu_2^{EI} = \frac{1}{3}(\frac{\alpha_1 - \alpha_2 + 24}{\alpha_1 - \alpha_2 + 16})$, where we can easily verify that $\mu_1^{EI} > \frac{1}{2} > \mu_2^{EI}$ when $\alpha_1 > \alpha_2$, as we supposed.

3.5.3 Brand Investment

Given equilibrium prices and demand, the firms' total profits Π_n^{EI} as functions of brand investment are as follows:

$$\Pi_1^{EI} = \frac{1}{72}(\frac{(\alpha_1 - \alpha_2 + 12)^2}{\alpha_1 - \alpha_2 + 16}) - c_\alpha \alpha_1^2$$
$$\Pi_2^{EI} = \frac{1}{288}(\frac{(\alpha_1 - \alpha_2 + 24)^2}{\alpha_1 - \alpha_2 + 16}) - c_\alpha \alpha_2^2.$$

When $\alpha_1 > \alpha_2$, it is clear that the first terms of both Π_1^{EI} and Π_2^{EI} are strictly increasing in α_1 and strictly decreasing in α_2. Given that increasing α_2 is also costly (and even if it is costless!), firm 2 optimizes by choosing $\alpha_2^* = 0$. We can see why firm 2 makes this somewhat counterintuitive choice by decomposing its profit:

$$\frac{d\Pi_2^{EI}}{d\alpha_2} = (p_2^{EI} - c)(\frac{\partial \mu_2^{EI}}{\partial \alpha_2} + \frac{\partial \mu_2^{EI}}{\partial p_1}\frac{dp_1^{EI}}{d\alpha_2}) - 2c_\alpha \alpha_2.$$

Using our equilibrium demand and prices, I obtain

$$\frac{\partial \mu_2^{EI}}{\partial \alpha_2} = \frac{\alpha_1 - \alpha_2 + 24}{3(\alpha_1 - \alpha_2 + 16)^2} > 0 \tag{3.9}$$

and

$$\frac{\partial \mu_2^{EI}}{\partial p_1}\frac{dp_1^{EI}}{d\alpha_2} = -\frac{2}{3(\alpha_1 - \alpha_2 + 16)} < 0. \tag{3.10}$$

Equation (3.9) is the demand effect, where increasing α_2 directly increases firm 2's demand by increasing its expected informational benefits. Equation (3.10) is the strategic effect, where increasing α_2 indirectly decreases firm 2's demand by causing its competitor to lower its price. When α_2 increases, then firm 1's brand advantage diminishes, so demand is relatively more responsive to prices and price competition intensifies. Summing equations (3.9) and (3.10), I find that the strategic effect dominates, so $\frac{d\Pi_2^{EI}}{d\alpha_2} < 0$. The incentive to weaken price competition is stronger than the incentive to increase demand. The relaxation of price competition through increased brand differentiation is also the reason that $\frac{d\Pi_2^{EI}}{d\alpha_1} > 0$.

Since the second term of Π_1^{EI} is decreasing in α_1, then firm 1's optimal α_1 depends on k, the degree to which brand improvements are costly.

Proposition 22. *When brand investment is a continuous choice, only one firm invests in a brand, and its optimal investment is weakly decreasing in the cost of brand improvements. In particular, there exist a cost* $\underline{c}_\alpha$ *and an upper bound* $\overline{\alpha}_1$ *such that the investing firm chooses* $\alpha_1^* = \overline{\alpha}_1$ *if* $c_\alpha \leq \underline{c}_\alpha$ *and a unique* $\alpha_1^*(c_\alpha) \in (0, \overline{\alpha}_1)$ *if* $c_\alpha > \underline{c}_\alpha$*, where* $\alpha_1^*(c_\alpha)$ *is strictly decreasing with* c_α*.*

Unsurprisingly, the firm's investment in a brand is inversely related to its cost. Since investment is increasingly costly and $\eta_n = \min\{\alpha_n \mu_n(x_n), 1\}$, firm 1 never invests such that $\alpha_1 \mu_1 > 1$. Thus, there exists an upper bound on equilibrium brand strength, α_1, even if the cost of investment is low. But since it is never an equilibrium for neither firm to invest, it is clear that firm 1 chooses some $\alpha_1^* > 0$ for *any* finite k, no matter how large (see Appendix).

Hence, there exists no equilibrium in which both firms choose to invest in a brand, even if investment is costless. The investing firm possesses a "lifestyle" brand whose consumers possess a common set of preferences and communicate with one another, while the other firm sells a good without the added benefit of a brand community.

3.5.4 Welfare

When only one firm invests, *both* firms charge higher prices than in the no-investment case. Firm 1 is better off, since it can charge a higher price and claim a larger share of the market. Since investing in a brand is optimal given that firm 2 does not invest, firm 1's profit must be strictly greater than in the no-investment case. Customers of firm 2 are definitely worse off, since they pay a higher price for the same good. However, it is not obvious how firm 2 and customers of firm 1 are affected.

Proposition 23. *In the duopoly setting, all consumers are strictly worse off and both* *firms are strictly better off when only one firm invests in a brand.*

Firm 2's profit is

$$
\begin{aligned}
\Pi_2^{EI} &= \frac{1}{4}(1+\frac{\alpha_1}{24})(\frac{\alpha_1+24}{3(\alpha_1+16)}) \\
&= \frac{1}{288}(\frac{(\alpha_1+24)^2}{\alpha_1+16}),
\end{aligned}
$$

which exceeds its profit of $\frac{1}{8}$ in the no-investment case whenever $\alpha_1 > 0$. Hence, the benefits from softened price competition outweigh the loss of market share.

I define consumer surplus as the aggregate benefits derived from good 1 minus the aggregate costs. Here, the aggregate benefit is the sum of the utility derived from good 1 and the information value derived from patronizing a firm that offers the coordination service. The aggregate cost is the sum of the price paid by consumers and the transportation cost incurred by consumers. In the no-investment case, there is no information value, so total consumer surplus is simply $CS^{NI} = v-(c+\frac{1}{4})-\frac{1}{48} = v-c-\frac{13}{48}$.[17]

Consumer surplus for customers of firm 1, denoted CS_1^{EI}, is

$$
\begin{aligned}
CS_1^{EI} &= \mu_I^{EI}[v-p_I^{EI}+\frac{\alpha_1}{32}(1-\mu_1^{EI})] - 2\int_0^{\frac{\mu_I^{EI}}{2}} y^2 dy \\
&= \mu_I^{EI}[v-(c+\frac{1}{4}(1+\frac{\alpha_1}{12}))+\frac{\alpha_1}{32}(1-\mu_I^{EI})] - \frac{(\mu_I^{EI})^3}{12}.
\end{aligned}
$$

In contrast, the surplus of these consumers in the no-investment case, denoted CS_1^{NI},

[17] The two agents who lie at the midpoint between the two firms' locations must travel the maximal distance of $\frac{1}{4}$ to buy the good. Hence, the aggregate transportation cost is given by $4\int_0^{\frac{1}{4}} x^2 dx = \frac{1}{48}$.

is

$$\begin{aligned} CS_1^{NI} &= \mu_1^{EI}(v - p^{NI}) - 2\int_0^{\frac{1}{4}} y^2 dy - 2\int_{\frac{\mu_2^{EI}}{2}}^{\frac{1}{4}} y^2 dy \\ &= \mu_1^{EI}(v - (\frac{1}{4} + c)) - \frac{1}{48} + \frac{(\mu_2^{EI})^3}{12}, \end{aligned}$$

where $\mu_2^{EI} < \frac{1}{2} < \mu_1^{EI}$. Comparing the two, we find that $CS_1^{EI} < CS_1^{NI}$ for all $\alpha_1 > 0$. Firm 1 is actually able to extract *more* than the information value from its customers, because it is the monopolist over a pure bundle (the good and the information) whose information value is decreasing in market share at the margin. Although customers of firm 1 have lost surplus, they are still better off patronizing firm 1 than switching to firm 2. The information benefit from buying variety x_1 rather than x_2 is

$$\frac{\alpha_1}{32}(1 - \mu_I^{EI}),$$

while firm 1's additional markup is $\frac{\alpha_1}{48}$. Comparing the two terms, we can see that the markup exceeds the information benefit. Due to the weakened price competition, firm 2 can charge a markup of $\frac{\alpha_1}{96}$. Comparing the net loss from x_1 to that of x_2, we obtain that

$$0 > \frac{\alpha_1}{32}(1 - \mu_I^{EI}) - \frac{\alpha_1}{48} > -\frac{\alpha_1}{96}.$$

Absent transportation costs, variety x_1 is more appealing than x_2, and consumers of firm 1 are better off than consumers of firm 2, but they are both worse off than in the no- and dual-investment cases. We can also compare the average consumer surplus of firm 1 customers, $\overline{CS}_1^{EI}$, who pay a higher price but reap information benefits, against the average consumer surplus of firm 2 customers, $\overline{CS}_2^{EI}$, who pay a lower price than firm 1 customers but reap no information benefits:

$$\overline{CS}_1^{EI} = CS_1^{EI}/\mu_1^{EI}$$
$$\overline{CS}_2^{EI} = [\mu_2^{EI}(v - p_2^{EI}) - \frac{(\mu_2^{EI})^3}{12}]/\mu_2^{EI}.$$

Using the equilibrium prices and demands, I find that $\overline{CS}_1^{EI} > \overline{CS}_2^{EI}$ for $\alpha_1 > 0$. Customers of firm 1 are, on average, better off than customers of firm 2.

Hence, summing together consumer surplus and firms' profits, total surplus is higher when brands can invest in brands than when they cannot, but total *consumer* surplus is lower. Maximal differentiation in brand investment weakens price competition to such an extent that the investing firm is able to extract all of its consumers' information surplus through pricing. In addition, the weakened price competition allows the non-investing firm can charge a sufficiently high markup to compensate for its lower market share.

3.5.5 Endogenizing Location

When both firms simultaneously choose location before choosing brand investment levels and setting prices, they differentiate maximally in location. The full proof with both endogenous location and brand investment is provided in the Appendix.

Proposition 24. *In a duopoly where firms choose variety before level of brand investment α, the unique pure strategy equilibrium is such that they differentiate maximally in both location and brand investment.*

- *Firms choose equidistant locations in the product space ($x_1 - x_2 = \frac{1}{2}$).*
- *For any $c_\alpha \geq 0$, one firm chooses some $\alpha > 0$ and the other firm chooses $\alpha = 0$.*
- *The investing firm has a larger market share and charges a higher price than the other firm.*

- *Both firms are strictly better off than when brand investment is not possible, while all consumers are strictly worse off.*

There exists no equilibrium in which both firms choose to invest in brands, even if investment is costless. Similar to the standard two-stage location-then-price game, firms' incentives to weaken price competition are stronger than the incentive to increase demand, so maximal differentiation in brand strength occurs. However, consumers are worse off and firms are better off than if brands could not exist. Thus, when location, brand investment, and price are endogenous in the duopoly setting, I find that firms differentiate maximally in *both* location and brand strength.

Here, investment in brand strength is a form of vertical product differentiation, but its properties differ in two key respects from the classic conception. While vertical differentiation has usually been considered a feature fixed by firms alone (Shaked and Sutton, 1982), here a brand's quality is also a function of consumer behavior. Moreover, there is a non-monotonic relationship between a branded good's information quality and its market share. Because the information benefit is determined by equilibrium consumer behavior, a brand difference directly weakens price competition. The information benefit from patronizing the investing firm actually increases when market share decreases in the relevant region, so the investing firm can raise its prices to some extent without decreasing the information quality of its good. The incentive to weaken price competition is sufficiently strong that firms engage in maximal differentiation in brand choice, just as in the case of "pure" vertical differentiation (Shaked and Sutton, 1982). In contrast, if there were no consumption externality, then it would have no direct effect on price competition and firms would differentiate minimally in brand choice, as in Economides (1993), who studies firms' decisions when both vertical and horizontal product differentiation are possible.

In a duopoly setting, due to the combination of population correlation in tastes and market structures, the model makes a strong prediction of maximal differentiation in brand strength that is detrimental to consumer welfare, compared to an environment where brands cannot exist. It predicts that consumers should gravitate, for example, to firms that offer bundled social networking services, and that those firms should consequently have larger market shares and higher prices than counterparts without such services.[18]

The prediction of maximal differentiation in brand strength stems from the fact that the information value of a brand is declining in market share at the margin, allowing the branding firm to simultaneously raise its price *and* increase its brand's information value at the expense of consumers. Here, this occurs due to two assumptions, namely the structure of correlated tastes in the population and the presence of a duopoly in the product space. Given the specific population correlation structure posited, where the brand's information value is maximized when it captures half of the market, the brand's information value is strictly decreasing at the margin in the duopoly setting, since the branding firm covers the majority of the market in equilibrium.

Thus, a relevant question is whether and how this prediction generalizes to other settings. In reality, a market can certainly contain $n > 2$ firms, and the correlation in tastes among individuals in the population is almost certainly more localized. While the model's predictions may not hold when there are $n > 2$ firms given the specific population correlation structure posited, the key requirement that a brand's information value be declining in market share at the margin can certainly apply if correlation among agents' tastes is sufficiently localized. The market share threshold at which there is signal dilution from observing a common action lowers as the degree

[18] Anecdotally, it appears that Netflix and Blockbuster Online, competing online DVD rental services, roughly fit this description a few years ago. Part of the appeal of Netflix was the quality of its user review service, and it tended to attract cinephiles. It had a larger market share and charged higher prices than Blockbuster Online, whose review service was significantly less developed. The recent addition of streaming services to Netflix, but not Blockbuster, makes the comparison of market shares and prices less straightforward now.

of similarity required for agents to learn from one another increases. If the necessary degree of similarity is sufficiently high, then a brand's information value can be declining in market share at the margin when there are $n > 2$ firms. Of course, as the number of firms increases, the necessary degree of localization also increases. Though it is unlikely that the model would predict a single brand in such a market, the predictions that firms differentiate in brand strength to the detriment of consumers and that branded firms should have larger market shares as well as higher prices than their un-branded counterparts are likely to hold.

3.6 Conclusion

This paper has offered an information-based explanation for lifestyle brands and social networking services and considers firms' optimal brand investment and pricing choices in a duopoly setting. If agents have uncertainty over their tastes but are aware that their tastes are correlated with others in the population, there exists an incentive to communicate with others in order to learn what is best for oneself. When communication is tied to action, similar agents may choose common actions in order to learn from each other. Hence, peer groups endogenously form reference groups by exhibiting conformity of behavior. Because agents have this desire for information, firms have an opportunity to provide mechanisms that facilitate this coordination. I argue that one natural channel for providing this service is bundling it with the goods themselves, since they present an obvious sorting mechanism for heterogeneous tastes. In this way, goods (and firms) can be associated with specific subsets of the population who wish to and can communicate with one another, leading to the formation of brand communities and lifestyle brands. I show that in a duopoly setting, only one firm chooses to provide this service, because the incentive to weaken price competition is stronger than the incentive to increase demand. Surprisingly, although total surplus increases with the provision of this mechanism, consumer surplus decreases. Although consumers benefit from learning from one another, all of this surplus is extracted by the firms through pricing.

Two alternative explanations for desired coordination among agents are conformity, a preference to behave like others, perhaps out of status concerns (Bernheim, 1994), and homophily, a inherent preference for like-minded others. While homophily may be another component of people's preferences, the prevalence of services like Facebook Beacon suggests that learning about others' tastes in *other* dimensions is also important. The need or desire to do so due to homophily is arguably less obvious. While conformity in the presence of subcultures (Bernheim, 1994) can also generate similar results, this paper offers an information-based, rather than preference-based, explanation for the same phenomenon. Conformity and learning motives could be distinguished by examining whether behavior differs when actions in other dimensions are public or private information. The former predicts a disinterest in information about others in other dimensions when one's own actions in such dimensions are private, while the latter predicts that such information is valued in both cases.

Here, the coordination service provided by brands is similar to certain features of social networking services, such as Facebook. However, the reality is that Facebook users rarely pay for the use of these widgets; Facebook widgets aim instead to earn revenue through advertisers, who may pay to have exposure to specific groups of users through the widgets. Extending the model to a two-sided market structure, such that the widget creator acts as an intermediary to link advertisers to specific groups of users and prices accordingly, is an interesting direction for further research.

Appendix A

Appendix for Chapter 1

A.1 Proof of Proposition 3

First, I show that the first best threshold x^* is identical to the optimal threshold chosen by a standard agent ($\beta = 1$, $\eta = 0$): $x^* = (\frac{\gamma_1}{\gamma_1 - 1})I$.

Proof. Each self wants his future selves to behave as a time-consistent agent. Equivalently, each self prefers that his future selves choose their thresholds in order to maximize the continuation value v rather than the current value w. Therefore, we can look for the optimal threshold such that the option value of waiting according to v is maximized. This is equivalent to looking for the threshold $\overline{x}$ that maximizes the wait region of Equation (1.24), the equilibrium continuation value function. The first order condition $\frac{\partial v^{SI}}{\partial \overline{x}} = 0$ gives us $\overline{x} = (\frac{\gamma_1}{\gamma_1 - 1})I$, which is identical to the solution for a time-consistent agent without reference dependence ($\beta = 1, \eta = 0$). Hence, the first best threshold x^* is identical to the optimal threshold chosen by a standard agent ($\beta = 1$, $\eta = 0$): $x^* = (\frac{\gamma_1}{\gamma_1 - 1})I$. We can also see this feature by inspecting the wait region of the Equation (1.24), which is the same function of the threshold as the usual value function for a standard agent. This is because utility incurred upon stopping is not directly distorted, so the option value of waiting, given a stopping threshold, is not distorted by η directly. □

However, the presence of $\eta > 0$ affects the equilibrium threshold by changing the marginal value of stopping. Given that every current self actually chooses the threshold by maximizing w rather than v, the current self's optimal threshold is $\overline{x}^{SI} = (\frac{\overline{\gamma}}{\overline{\gamma} - 1 - \eta})I$. Hence, the first best can only be achieved for η^* such that $\frac{\overline{\gamma}}{\overline{\gamma} - 1 - \eta^*} = \frac{\gamma_1}{\gamma_1 - 1}$. Given that $x^* = (\frac{\gamma_1}{\gamma_1 - 1})I$, the lower bound for waiting too long follows directly from the comparison between the actual threshold $\overline{x}$ and the first best, such that $\overline{x}^{SI} > x^*$:

$$\frac{\overline{\gamma}}{\overline{\gamma} - 1 - \eta} > \frac{\gamma_1}{\gamma_1 - 1},$$

where $\overline{\gamma} = \beta\gamma_1 + (1 - \beta)\gamma_2$. This condition holds if $\eta > \frac{\overline{\gamma} - \gamma_1}{\gamma_1}$. Finally, we can verify that

$$\frac{\overline{\gamma}-\gamma_1}{\gamma_1} < \overline{\gamma}-1$$

by noting that $\frac{\overline{\gamma}-\gamma_1}{\gamma_1} = \frac{\overline{\gamma}}{\gamma_1} - 1$. Since $\gamma_1 > 1$, then this inequality is satisfied for any $\beta \in [0,1]$.

A.2 Proof of Corollary 1

The first part follows from the above proof of Proposition 3. The second statement follows by comparing the value functions when $\eta > 0$ versus $\eta = 0$. When $\eta = 0$, the agent stops at threshold $(\frac{\overline{\gamma}}{\overline{\gamma}-1})I$. When $\eta > 0$, the agent stops at threshold $(\frac{\overline{\gamma}}{\overline{\gamma}-1-\eta})I$. Thus, goal-setting is detrimental when

$$\begin{aligned}
[(\frac{\overline{\gamma}}{\overline{\gamma}-1-\eta})I - I](\frac{1}{(\frac{\overline{\gamma}}{\overline{\gamma}-1-\eta})I})^{\gamma_1} &< [(\frac{\overline{\gamma}}{\overline{\gamma}-1})I - I](\frac{1}{(\frac{\overline{\gamma}}{\overline{\gamma}-1})I})^{\gamma_1} \\
[(\frac{1+\eta}{\overline{\gamma}-1-\eta})I](\frac{1}{(\frac{\overline{\gamma}}{\overline{\gamma}-1-\eta})I})^{\gamma_1} &< [(\frac{1}{\overline{\gamma}-1})I](\frac{1}{(\frac{\overline{\gamma}}{\overline{\gamma}-1})I})^{\gamma_1} \\
0 &< (\frac{\overline{\gamma}-1}{\overline{\gamma}-1-\overline{\eta}})^{\gamma_1-1}(\frac{1}{1+\overline{\eta}}) - 1,
\end{aligned}$$

and the lower bound $\overline{\eta}$ satisfies the above condition with equality. Define the function $H(\eta)$ such that

$$H(\eta) = (\frac{\overline{\gamma}-1}{\overline{\gamma}-1-\overline{\eta}})^{\gamma_1-1}(\frac{1}{1+\overline{\eta}}) - 1.$$

We can verify that $\overline{\eta}$ exists and is unique by noting that $H(0) = 0$, $H(\overline{\gamma}-1) \to \infty$, and $H(\eta)$ strictly decreases for $\eta < \eta^*$ and strictly increases for $\eta > \eta^*$. Thus, we have shown existence and uniqueness of $\overline{\eta}$ for any $\gamma \in (1,\infty)$, as well as the fact that $\eta^* < \overline{\eta} < \overline{\gamma}-1$. Finally, the implicit function theorem yields

$$\frac{\partial\overline{\eta}}{\partial\overline{\gamma}} = -\frac{-(\frac{\overline{\eta}}{1+\overline{\eta}})(\frac{\gamma_1-1}{(\overline{\gamma}-1-\overline{\eta})^2})(\frac{\overline{\gamma}-1}{(\overline{\gamma}-1-\overline{\eta})^{\gamma_1-2}})}{(\frac{1}{1+\overline{\eta}})(\frac{\overline{\gamma}-1}{(\overline{\gamma}-1-\overline{\eta})^{\gamma_1-1}})[-\frac{1}{1+\overline{\eta}} + (\frac{\gamma_1-1}{\overline{\gamma}-1-\overline{\eta}})]} > 0,$$

which is positive since the numerator is always negative and the denominator is positive since $\overline{\eta} > \eta^*$.

A.3 Naivete

A.3.1 Existence and Uniqueness of $\overline{x}^{NE}$

First, I prove the existence and uniqueness of $\overline{x}^{NE}$ for any given $r \geq 0$. Define the following function $G(x)$:

$$G(x) = \frac{1}{(\gamma_2 - 1)(1+\eta)}[\beta(\gamma_2 - \gamma_1)(\hat{x} - I + \eta(\hat{x} - I - r))(\frac{x}{\hat{x}})^{\gamma_1} + \gamma_2(1+\eta)I + \gamma_2 \eta r] - x, \tag{A.1}$$

where $\hat{x} = (\frac{\gamma_1}{\gamma_1 - 1})I + r(\frac{\eta}{1+\eta})(\frac{\gamma_1}{\gamma_1 - 1})$. Note that when $r = \hat{x} - I$, then $\hat{x} = \hat{x}^{NI}$.

Proof. Consider the function $G(x)$, given by Equation (A.1). Clearly, $\overline{x}^{NE}$ must satisfy $G(\overline{x}^{NE}) = 0$, where I have assumed that $\overline{x}^{NE} \leq \hat{x}$ by construction. Since $G(0) = \gamma_2[(1+\eta)I + \eta r] > 0$ and

$$G(\hat{x}) = -\frac{(1-\beta)(\gamma_2 - \gamma_1)(r\eta + (1+\eta)I)}{(1+\eta)(\gamma_1 - 1)(\gamma_2 - 1)} < 0,$$

then $\overline{x}^{NE}$ exists for any $r \geq 0$. Next, note that

$$G'(x) = \frac{1}{(\gamma_2 - 1)(1+\eta)}[\beta(\gamma_2 - \gamma_1)(\hat{x} - I + \eta(\hat{x} - I - r))(\frac{1}{\hat{x}})^{\gamma_1}(\gamma_1)(x)^{\gamma_1 - 1}] - 1$$

$$G''(x) = \frac{1}{(\gamma_2 - 1)(1+\eta)}[\beta(\gamma_2 - \gamma_1)(\hat{x} - I + \eta(\hat{x} - I - r))(\frac{1}{\hat{x}})^{\gamma_1}(\gamma_1)(\gamma_1 - 1)(x)^{\gamma_1 - 2}],$$

so $G''(x) > 0$ for all $x > 0$. Then $G'(x) < 0$ for any x such that $G(x) = 0$. Thus, there exists a unique $\overline{x}^{NE} \leq \hat{x}$ such that $G(\overline{x}^{NE}) = 0$.

The expression for $\overline{x}^{NE}$, which is given by Equation 1.26, was constructed by assuming that $\overline{x}^{NE} \leq \hat{x}$. To ensure uniqueness of $\overline{x}^{NE}$, we must rule out the case where $\overline{x}^{NE} > \hat{x}$. Suppose that there exists another threshold $\tilde{x}$ such that it is optimal for the naif to stop when $x \geq \tilde{x}$ and wait otherwise, where $\tilde{x} > \hat{x}$. To construct the value function, note that the naif still believes that all future selves will employ the threshold $\hat{x} = (\frac{\gamma_1}{\gamma_1 - 1})I + r(\frac{\eta}{1+\eta})(\frac{\gamma_1}{\gamma_1 - 1})$, and the continuation value v is still given by Equation (1.11). When $x \in [0, \hat{x})$, then $v(x, \hat{r}) = [\hat{x} - I + \eta(\hat{x} - I - \hat{r})](\frac{x}{\hat{x}})^{\gamma_1} \equiv v_1(x, \hat{r})$. When $x \in [\hat{x}, \infty)$, then $v(x, \hat{r}) = x - I + \eta(x - I - \hat{r})$. When $x \in [0, \tilde{x})$, then w satisfies the differential equation given by Equation (1.7). When $x \in [\tilde{x}, \infty)$, then $w(x, r) = x - I + \eta(x - I - r)$. Since $0 \leq \hat{x} < \tilde{x}$, the continuation value function w now has three regions. Let w_1 describe w when $x \in [0, \hat{x})$, w_2 describe w when $x \in [\hat{x}, \tilde{x})$, and w_3 describe w when $x \in [\tilde{x}, \infty)$. Let $r = \hat{r}$.

When $x \in [0, \hat{x})$, then $v(x, r) = v_1(x, r) = A_1 x^{\gamma_1}$, where $A_1 \equiv [\hat{x} - I + \eta(\hat{x} - I - \hat{r})](\frac{x}{\hat{x}})^{\gamma_1}$, and w satisfies the differential equation given by Equation (1.7). Substituting $v(x, r) = A_1 x^{\gamma_1}$ into (1.7) yields the solution

$$w_1(x, r) = \beta A_1 x^{\gamma_1} + A_2 x^{\gamma_3}, \tag{A.2}$$

where γ_1 and γ_3 are the positive and negative roots of the quadratic equation $\frac{1}{2}\sigma^2\gamma +$

$(\mu - \frac{1}{2}\sigma^2)\gamma - \rho = 0$, as before, where $\gamma_1 > 1$ and $\gamma_3 < 0$.

When $x \in [\hat{x}, \tilde{x})$, then $v(x, r) = x - I + \eta(x - I - r)$ and w again satisfies Equation (1.7). Substituting $v(x, r) = x - I + \eta(x - I - r)$ into (1.7) yields the solution

$$w_2(x, r) = \beta A_3 x^{\gamma_2} + A_2 x^{\gamma_4} + \beta(1 + \eta)(\frac{\lambda}{\rho + \lambda - \mu})x - \beta[(1 + \eta)I + \eta r], \tag{A.3}$$

where γ_2 and γ_4 are the positive and negative roots of the quadratic equation $\frac{1}{2}\sigma^2\gamma^2 + (\mu - \frac{1}{2}\sigma^2)\gamma - (\rho + \lambda) = 0$, as before, where $\gamma_2 \geq \gamma_1 > 1$ and $\gamma_4 < 0$.

The agent stops for any $x \in [\tilde{x}, \infty)$, so

$$w_3(x, r) = x - I + \eta(x - I - r). \tag{A.4}$$

By definition of the geometric Brownian motion x, $x = 0$ is an absorbing barrier of the project value process. Furthermore, the continuation value function v must be continuous, while the current value function w must be continuous and smooth everywhere. Since the naif *believes* that future selves will behave in the optimal manner, v is smooth as well. This gives the following boundary conditions:

$$\text{Boundary: } v_1(0, r) = 0, \tag{A.5}$$
$$w_1(0, r) = 0, \tag{A.6}$$
$$\text{Value Matching: } v_1(\hat{x}, r) = x - I + \eta(\hat{x} - I - r), \tag{A.7}$$
$$w_1(\hat{x}, r) = w_2(\hat{x}, r), \tag{A.8}$$
$$w_2(\tilde{x}, r) = w_3(\tilde{x}, r), \tag{A.9}$$
$$\text{Smooth Pasting: } \frac{\partial v_1}{\partial x}(\hat{x}, r) = 1 + \eta, \tag{A.10}$$
$$\frac{\partial w_1}{\partial x}(\hat{x}, r) = \frac{\partial w_2}{\partial x}(\hat{x}, r), \tag{A.11}$$
$$\frac{\partial w_2}{\partial x}(\tilde{x}, r) = 1 + \eta. \tag{A.12}$$

Using the above boundary conditions in conjunction with $v_1(x, r) = A_1 x^{\gamma_1}$ and Equations (A.2), (A.3), and (A.4), we obtain the following non-linear equation for $\tilde{x}$:

$$\tilde{x} = \frac{\gamma_2(1 + \beta)[(1 + \eta)I + \eta r]}{(\gamma_2 - 1)(1 + \eta)(1 - \frac{\lambda\beta}{\rho + \lambda - \mu})} + (\frac{\tilde{x}}{\hat{x}})^{\gamma_4}\left(\frac{\beta(\rho - \mu)\hat{x}}{\rho + (1 - \beta)\lambda - \mu}\right), \tag{A.13}$$

where $\hat{x} = (\frac{\gamma_1}{\gamma_1})I + r(\frac{\gamma_1}{\gamma_1 - 1})(\frac{\eta}{1 + \eta})$. Now, define the function $F(x)$ as follows:

$$F(x) = \frac{\gamma_2(1 + \beta)[(1 + \eta)I + \eta r]}{(\gamma_2 - 1)(1 + \eta)(1 - \frac{\lambda\beta}{\rho + \lambda - \mu})} + (\frac{x}{\hat{x}})^{\gamma_4}(\hat{x})\left(\frac{\beta - \frac{\lambda\beta}{\rho + \lambda - \mu}}{1 - \frac{\lambda\beta}{\rho + \lambda - \mu}}\right) - x, \tag{A.14}$$

where the first term and second terms are positive since $\gamma_2 > 1$, $\beta < 1$, $\mu < \rho$, and $r \geq 0$. Clearly, $\tilde{x}$ satisfies $F(\tilde{x}) = 0$. Since $\gamma_4 < 0$, then $\lim_{x \to 0} F(x) \to \infty$ and $\lim_{x \to \infty} F(x) \to -\infty$. Thus, $\tilde{x}$ exists. Moreover, $\gamma_4 < 0$ implies that $F'(x) < 0$ and

$F''(x) < 0$, so $\tilde{x}$ is unique. Evaluating $F(\cdot)$ at $\hat{x}$, we obtain

$$F(\hat{x}) = \frac{\gamma_2(1+\beta)[(1+\eta)I + \eta r]}{(\gamma_2 - 1)(1+\eta)(1 - \frac{\lambda\beta}{\rho+\lambda-\mu})} + \left(\frac{\beta - \frac{\lambda\beta}{\rho+\lambda-\mu}}{1 - \frac{\lambda\beta}{\rho+\lambda-\mu}}\right)(\hat{x}) - \hat{x}$$
$$= \frac{-(1-\beta)(\gamma_2-\gamma_1)[(1+\eta)I + \eta r]}{(\gamma_1 - 1)(\gamma_2 - 1)(1+\eta)(1 - \frac{\lambda\beta}{\rho+\lambda-\mu})},$$

which is negative since $\gamma_2 > \gamma_1$. Since $G(\hat{x}) < 0$, then $\tilde{x} < \hat{x}$, which violates the assumption used to construct $\tilde{x}$, that $\tilde{x} > \hat{x}$. Thus, there does not exist an optimal threshold $\tilde{x}$ such that $\tilde{x} > \hat{x}$ and the naif stops when $x \geq \tilde{x}$ and waits otherwise. □

A.3.2 Comparative Statics for $\overline{x}^{NE}$

Having shown existence and uniqueness of $\overline{x}^{NE}$, we can use the implicit function theorem to derive the relevant comparative statics.

Proof. By the implicit function theorem,

$$\frac{\partial x}{\partial \beta} = -\frac{\frac{\partial G}{\partial \beta}}{\frac{\partial G}{\partial x}}.$$

From the preceding proof of uniqueness, it is clear that $G'(x) < 0$ for all $x \in [0, \hat{x}]$. Turning to the numerator,

$$\frac{\partial G}{\partial \beta} = \frac{1}{(\gamma_2 - 1)(1+\eta)}(\gamma_2 - \gamma_1)(\hat{x} - I + \eta(\hat{x} - I - r))(\frac{x}{\hat{x}})^{\gamma_1} > 0.$$

Therefore, $\frac{\partial \overline{x}^{NE}}{\partial \beta} > 0$. The implicit function theorem also gives us

$$\frac{\partial x}{\partial r} = -\frac{\frac{\partial G}{\partial r}}{\frac{\partial G}{\partial x}},$$

Turning to the numerator,

$$\frac{\partial G}{\partial r} = \frac{1}{(\gamma_2 - 1)(1+\eta)}[\beta(\gamma_2 - \gamma_1)(\frac{x}{\hat{x}})^{\gamma_1}(\frac{1}{\hat{x}})[\hat{x}(1+\eta)(\frac{\partial \hat{x}}{\partial r}) - \eta\hat{x}$$
$$- \gamma_1(\hat{x} - I + \eta(x - I - r))(\frac{\partial \hat{x}}{\partial r})] + \gamma_2\eta]$$
$$= \frac{1}{(\gamma_2 - 1)(1+\eta)}\left(\beta(\gamma_2 - \gamma_1)(\frac{x}{\hat{x}})^{\gamma_1}(-\eta) + \gamma_2\eta\right).$$

Since $x \leq \hat{x}$, then $\frac{\partial G}{\partial r} > 0$, since $\beta(\gamma_2 - \gamma_1)(\frac{x}{\hat{x}})^{\gamma_1} < \gamma_2$. Thus, $\frac{\partial \overline{x}^{NE}}{\partial r} > 0$.

Consider the effect of increasing β on $\frac{\partial \overline{x}^{NE}}{\partial r}$. It is clear that $\frac{\partial G}{\partial r}$ is decreasing in β, while $\frac{\partial G}{\partial x}$ is increasing in β. Therefore, $\frac{\partial^2 \overline{x}^{NE}}{\partial r \partial \beta} > 0$.

Now, consider the effect of increasing η in equilibrium. Let $\hat{r} = \hat{x} - I$. Since it is shown in the following proof that $\frac{\partial \overline{x}^{NE}}{\partial \hat{x}} > 0$ and clearly $\frac{\partial \hat{x}}{\partial \eta} > 0$, it is sufficient to show that $\frac{\partial G}{\partial \eta} > 0$ holding $\hat{x}$ fixed. By the implicit function theorem,

$$\frac{\partial x}{\partial \eta} = -\frac{\frac{\partial G}{\partial \eta}}{\frac{\partial G}{\partial x}},$$

where $\frac{\partial G}{\partial x} < 0$. Turning to the numerator,

$$\begin{aligned}\frac{\partial G}{\partial \eta} &= \frac{1}{(\gamma_2 - 1)}\left(-(\frac{1}{1+\eta})^2[\beta(\gamma_2-\gamma_1)(\hat{x}-I)(\frac{x}{\hat{x}})^{\gamma_1} + \gamma_2 I + \gamma_2\eta\hat{x}] + (\frac{1}{1+\eta})(\gamma_2\hat{x})\right)\\ &= (\frac{1}{(\gamma_2-1)})(\frac{1}{1+\eta})^2(\hat{x}-I)[-\beta(\gamma_2-\gamma_1)(\frac{x}{\hat{x}})^{\gamma_1} + \gamma_2] > 0,\end{aligned}$$

since $\overline{x}^{NE} < \hat{x}$. Thus, $\frac{\partial \overline{x}^{NE}}{\partial \eta} > 0$. □

A.3.3 Proof of Proposition 4

Since the result that $\overline{x}^{SE} < \overline{x}^{NE} < \hat{x}^{NE}$ is slightly more general and implies that $\overline{x}^{SI} < \overline{x}^{NI} < \hat{x}^{NI}$, I will show the former. Note that in "rational expectations" equilibrium ($r = \hat{x} - I$), the naif's goal is the same as that of the time-consistent agent due to his mistaken beliefs, and is higher than that of the sophisticate.

Proof. In the preceding proof of uniqueness of $\overline{x}^{NE}$, I already showed that $\overline{x}^{NE} < \hat{x}^{NE}$ when $\beta < 1$ for any $r \geq 0$. Since $r = \hat{x} - I > 0$, then it follows that $\overline{x}^{NI} < \hat{x}^{NI}$ as well. Thus, the naif falls short of his reference point.

Now, consider the function $G(x)$ as defined by Equation (A.1) and reproduced here:

$$G(x) = \frac{1}{(\gamma_2-1)(1+\eta)}[\beta(\gamma_2-\gamma_1)(\hat{x}-I+\eta(x-I-r))(\frac{x}{\hat{x}})^{\gamma_1} + \gamma_2(1+\eta)I + \gamma_2\eta r] - x,$$

The sophisticate's threshold solves $G(x) = 0$ when $\hat{x} = (\frac{\overline{\gamma}}{\overline{\gamma}-1})I + r(\frac{\eta}{1+\eta})(\frac{\overline{\gamma}}{\overline{\gamma}-1}) \equiv \hat{x}^{SE}$, while the naif's threshold solves $G(x) = 0$ when $\hat{x} = (\frac{\gamma_1}{\gamma_1-1})I + r(\frac{\eta}{1+\eta})(\frac{\gamma_1}{\gamma_1-1}) \equiv \hat{x}^{NE}$. Thus, $\hat{x}^{SE} < \hat{x}^{NE}$ and it is sufficient to show that $\frac{\partial x}{\partial \hat{x}}|_{\hat{x}=\hat{x}^{SE}} > 0$. By the implicit function theorem,

$$\frac{\partial x}{\partial \hat{x}} = -\frac{\frac{\partial G}{\partial \hat{x}}}{\frac{\partial G}{\partial x}}.$$

From the preceding proof, we have that $\frac{\partial G}{\partial x} < 0$. Turning to the numerator,

$$\frac{\partial G}{\partial \hat{x}} = \frac{1}{(\gamma_2 - 1)(1+\eta)} \left(\beta(\gamma_2 - \gamma_1)(\frac{x}{\hat{x}})^{\gamma_1 + 1}(\frac{1}{\hat{x}})[(1+\eta)\hat{x} - \gamma_1(\hat{x} - I + \eta(\hat{x} - I - r))] \right)$$

which is strictly positive for any $\hat{x} < \hat{x}^{NE}$. Thus, $\frac{\partial x}{\partial \hat{x}} > 0$, so $\overline{x}^{NE} > \overline{x}^{SE}$ for any fixed r. Moreover, when the goal is endogenous, the naif's goal is greater than the sophisticate's ($r^{NI} > r^{SI}$). Since we have already shown that the threshold is increasing in r, then $\overline{x}^{NI} > \overline{x}^{SI}$ when $r = \hat{x} - I$ for each type of agent. □

A.3.4 Proof of Proposition 5

Proof. In Proposition 3, I showed that the sophisticate's first-best is achieved by stopping at the threshold $x^* = (\frac{\gamma_1}{\gamma_1 - 1})I$. When he stops at x^*, the first-best ex ante value of the option (v^{S^*}) is given by

$$v^{S^*}(x) = (x^* - I)(\frac{x}{x^*})^{\gamma_1}.$$

Given that the naif sets the goal $\hat{x}^{NI} - I$ where $\hat{x}^{NI} = (\frac{\gamma_1}{\gamma_1 - 1 - \eta})I$, his true ex-ante option value of waiting is given by Equation (1.29), reproduced below:

$$\tilde{v}^N(x, r = \hat{x}^{NI} - I) = [\overline{x}^N - I + \eta(\overline{x}^N - \hat{x}^{NI})](\frac{x}{\overline{x}^N})^{\gamma_1}.$$

To find the threshold $\overline{x}^N$ that would maximize $\tilde{v}^N$, we have the first order condition:

$$\begin{aligned}\frac{\partial \tilde{v}^N}{\partial \overline{x}^N} = 0 &= (x)^{\gamma_1} \left((1+\eta)(\frac{1}{\overline{x}^N})^{\gamma_1} - \gamma_1(\frac{1}{\overline{x}^N})^{\gamma_1+1}[\overline{x}^N - I + \eta(\overline{x}^N - \hat{x}^{NI})] \right) \\ &= (x)^{\gamma_1}(\frac{1}{\overline{x}^N})^{\gamma_1+1}[\overline{x}^N(1+\eta) - \gamma_1(\overline{x}^N - I) - \gamma_1\eta(\overline{x}^N - \hat{x}^{NI})] \\ &= (x)^{\gamma_1}(\frac{1}{\overline{x}^N})^{\gamma_1+1}(1+\eta)(\gamma_1 - 1)(\hat{x}^{NI} - \overline{x}^N),\end{aligned}$$

which is satisfied when $\hat{x}^N = \overline{x}^{NI}$ (and clearly, second order conditions are satisfied as well). Thus, the upper bound on the naif's option value of waiting is given by

$$\tilde{v}^N(x) = (\hat{x}^{NI} - I)(\frac{x}{\hat{x}^{NI}})^{\gamma_1}.$$

But we have already shown previously that the option value of waiting when there is zero comparative utility is maximized at x^*. Since $\hat{x}^{NI} > x^*$, then $\tilde{v}^{N^*}(x) < v^{S^*}(x)$. □

A.4 Proof of Proposition 6

Since the linear comparison utility function is a special case of the general comparison utility function $\psi(\cdot)$, I address this case in the following proof of Proposition 7.

A.5 Proof of Proposition 7

Because the agent always sets a goal for future selves, and he anticipates future behavior in response to any given goal, choosing a goal to maximize total utility from an ex-ante perspective is equivalent to self 0's problem of setting the goal $\hat{r}$ that maximizes the value of the wait region of the continuation value function $v(x, \hat{r})$ described by Equation (1.11), where the anticipated threshold employed by future selves is the optimal threshold given an exogenous goal, $\hat{x}$. When the comparison utility function is linear, then $\hat{x} = \overline{x}^{SE}$ and it is described by Equation (1.20).

When $\psi(\cdot)$ is non-linear, we must derive the optimal stopping threshold given a goal r. As in the linear case, the optimal threshold is determined by joining the waiting and stopping regions of the value function. It is only the stopping region that now changes. The the optimal threshold $\overline{x}^{NL}$ is given by the implicit function described by Equation (1.30) and replicated here:

$$0 = (\overline{\gamma} - 1)\overline{x}^{NL} - \overline{\gamma} I + \overline{\gamma}\psi(\overline{x}^{NL} - I - r) - \overline{x}^{NL}\psi'(\overline{x}^{NL} - I - r). \tag{A.15}$$

An alternative way to express this is:

$$\overline{x}^{NL} = \frac{\overline{\gamma}[I - \psi(\overline{x}^{NL} - I - r)]}{\overline{\gamma} - 1 - \psi'(\overline{x}^{NL} - I - r)}.$$

For $\overline{x}$ to be defined and positive, I will assume that $\psi'(\overline{x} - I - r) < \overline{\gamma} - 1$ for all $\overline{x} - I - r \in [-I, \infty)$. To see that $I - \psi(\overline{x}^{NL} - I - r) > 0$, suppose that it is negative, implying that $\overline{x}^{NL} < 0$. But if $\overline{x}^{NL} < 0$, then $\psi(\overline{x}^{NL} - I - r) < 0$ becaues $\psi(0) = 0$ and $\psi'(y) \geq 0$ for all y. This implies that $I - \psi(\overline{x}^{NL} - I - r) > 0$, since $I > 0$, which is a contradiction. Note that when the comparison utility function is linear with slope $\eta \geq 0$ and $\eta < \overline{\gamma} - 1$, then $\overline{x}^{NL}$ is equivalent to Equation (1.20).

Using the implicit function theorem and suppressing the argument for ψ for brevity, we have that $\frac{\partial \overline{x}}{\partial r}$ is:

$$\frac{\partial \overline{x}}{\partial r} = \frac{\overline{\gamma}\psi'(\overline{x} - I - r) - \overline{x}\psi''(\overline{x} - I - r)}{(\overline{\gamma} - 1)[1 + \psi'(\overline{x} - I - r)] - \overline{x}\psi''(\overline{x} - I - r)}, \tag{A.16}$$

which is positive whenever $\psi''(y) \leq 0$, so $\frac{\partial \overline{x}}{\partial r} > 0$ for all $r \in [-I, \infty)$. We can also verify that $\frac{\partial \overline{x}}{\partial r} < 1$. The implicit function theorem also yields $\frac{\partial \overline{x}}{\partial \overline{\gamma}}$:

$$\frac{\partial \overline{x}}{\partial \overline{\gamma}} = -\frac{\overline{x} - I + \psi(\overline{x} - I - r)}{(\overline{\gamma} - 1)[1 + \psi'(\overline{x} - I - r)] - \overline{x}\psi''(\overline{x} - I - r)}, \tag{A.17}$$

which is clearly negative since $\overline{x} - I + \psi(\overline{x} - I - r) \geq 0$ (otherwise, the agent would be better off never stopping).

Differentiating the continuation value function v, given by Equation (1.11), with respect to r, and suppressing the argument for $\psi(\cdot)$ for brevity we obtain:

$$\begin{aligned}\frac{\partial v}{\partial r} =& [\frac{1}{(\overline{\gamma}-1)(1+\psi') - \overline{x}\psi''}][\frac{I-\psi}{\overline{\gamma}-1-\psi'}](\frac{x}{\overline{x}})^{\gamma_1}\,(\overline{x}\psi''[-\overline{\gamma}+\gamma_1(1+\psi')]\\ &-\overline{\gamma}(\gamma_1-1)\psi'(1+\psi'))\,.\end{aligned}$$

The first third terms are positive given that our regularity conditions are satisfied. Therefore, the sign of $\frac{\partial v}{\partial r}$ is determined by the sign of the last term. Note that if $\psi(y)$ is linear, i.e. $\psi''(y) = 0$ for all $y \in \mathbb{R}$, then $\frac{\partial v}{\partial r} < 0$ so the optimal r^* is the lowest value of r possible.

Rewriting the last term, we have the condition given in Equation (1.32):

$$\overline{x}\psi''[-\overline{\gamma}+\gamma_1(1+\psi')] - \overline{\gamma}(\gamma_1-1)\psi'(1+\psi'),$$

which must equal zero at r^* in order to obtain some $r^* > -I$. The second term of (1.32) is negative, so the first term must be positive to satisfy the first-order condition. Since $\psi''(\overline{x} - I - r) < 0$, then the first term is only positive if $-\overline{\gamma} + \gamma_1(1 + \psi') > 0$. That is, a necessary condition for a non-degenerate $r^* > -I$ is that

$$\overline{\gamma} > \gamma_1[1 + \psi'(\overline{x}^{NL} - I - r^*)], \tag{A.18}$$

where the right-hand side is greater than γ_1. That is, the agent must have a self-control problem and it must be sufficiently severe to counteract the marginal benefit of setting very low goals. Note that the left-hand side of Equation (A.18) is increasing in r for $\overline{x} - I - r > 0$, since we can show that $\overline{x} - I - r$ is decreasing in r. So $\overline{\gamma}$ must be even higher to choose some positive optimal goal $r^* > 0$.

Suppose that Equation (A.18) holds at r^*. In order for Equation (1.32) to be non-negative, we also require that the first term of (1.32) be sufficiently positive to counteract the second. This is equivalent to the requirement that ψ'' be sufficiently negative at r^* such that Equation (1.32) holds:

$$\psi'' = \left(\frac{(\gamma_1-1)\psi'(1+\psi')}{-\overline{\gamma}+\gamma_1(1+\psi')}\right)\left(\frac{\overline{\gamma}-1-\psi'}{I-\psi}\right).$$

To verify that r^* is a maximum, we need to verify that the second-order condition is satisfied. The first derivative can be written as

$$\frac{\partial v}{\partial r} = (\frac{x}{\overline{x}^{NL}})^{\gamma_1}\left(\overline{x}^{NL}[\frac{\partial \overline{x}^{NL}}{\partial r}(1+\psi') - \psi'] - \gamma_1(\frac{\partial \overline{x}^{NL}}{\partial r})(\overline{x}^{NL} - I + \psi)\right). \tag{A.19}$$

Evaluated at r^*, the sign of $\frac{\partial v^2}{\partial^2 r}$ is given by the sign of the second term of (A.19).

Given that Equation (1.32) must hold at r^*, the following must hold at r^* if it exists:

$$\frac{\partial \overline{x}^{NL}}{\partial r}|_{r=r^*} = (\frac{\partial \psi'(\overline{x}^{NL} - I - r^*)}{1+\psi'(\overline{x}^{NL} - I - r^*)})(\frac{\overline{\gamma}}{\overline{\gamma} - \gamma_1}) \tag{A.20}$$

$$\overline{x}^{NL}(1+\psi'(\overline{x}^{NL} - I - r^*)) = \overline{\gamma}[\overline{x}^{NL} - I + \psi(\overline{x}^{NL} - I - r^*)]. \tag{A.21}$$

Note that $\overline{\gamma} > \gamma_1$ at r^*. Using these two facts and Equation (1.32), we need $\frac{\partial v^2}{\partial^2 r}(r^*) < 0$, where I suppress the argument for $\psi(\cdot)$ for brevity:

$$\frac{\partial v^2}{\partial^2 r}|_{r=r^*} = -\frac{\overline{\gamma}(\gamma_1 - 1)\psi'}{(\overline{\gamma} - \gamma_1)(1+\psi')} + \frac{\partial^2 \overline{x}^{NL}}{\partial r^2}|_{r=r^*}[(\overline{\gamma} - \gamma_1)(\overline{x}^{NL} - I + \psi)] < 0,$$

where the first term is negative and the second is positive. Evaluating $\frac{\partial^2 \overline{x}^{NL}}{\partial r^2}|_{r=r^*}$, the second-order condition is satisfied when the following upper bound on $\psi'''(\overline{x}^{NL} - I - r^*)$ holds:

$$\psi'''(\overline{x}^{NL} - I - r^*)K(r^*) < L(r^*)M(r^*), \tag{A.22}$$

where

$$K(r^*) = \overline{x}^{NL}(\frac{\overline{x}^{NL} - I - \psi}{\overline{\gamma} - 1 - \psi'})\left(\frac{[\overline{\gamma} - \gamma_1(1+\psi')]^3}{(\overline{\gamma} - \gamma_1)^2(1+\psi')^3}\right),$$

$$L(r^*) = \frac{\overline{\gamma}(\gamma_1 - 1)\psi'}{(\overline{\gamma} - \gamma_1)^2(1+\psi')(\overline{\gamma} - 1 - \psi')},$$

$$M(r^*) = (\overline{\gamma} - \gamma_1)(\overline{\gamma} - 1 - \psi') - [\overline{\gamma} - \gamma_1(1+\psi')[2\psi' + \overline{\gamma} - \gamma_1(1+\psi')].$$

We can verify that $K(r^*), L(r^*)$, and $M(r^*)$ are positive, so the upper bound on $\psi'''(\overline{x}^{NL} - I - r^*)$ is positive. Given properties A3, A4, and A5, a $\psi(\cdot)$ satisfying the second-order condition exists. Thus, an optimal $r^* > -I$ exists when the Equations (1.32) and (A.22) are satisfied.

A.6 Proof of Corollary 2

Evaluating $\overline{x}^{NL}$ at $\tilde{r}$ such that $\tilde{r} = \overline{x}_I^{NL}$ yields

$$\overline{x}^{NL}(\tilde{r}) = (\frac{\overline{\gamma}}{\overline{\gamma} - 1 - \psi'(0)})I,$$

which is clearly unique for any $\overline{\gamma}$. Therefore, $\tilde{r}$ is given by

$$\tilde{r} = (\frac{1+\phi'(0)}{\overline{\gamma} - 1 - \psi'(0)})I,$$

which is unique for any $\overline{\gamma}$. The second part of Corollary 2 follows from verifying whether Equations (A.18) and (1.32) are satisfied at $\tilde{r}$.

A.7 Proof of Proposition 8

To show the first part of Proposition 8, we can apply the implicit function theorem to Equation (1.32). Since the second-order condition must be satisfied at the optimal r^*, it is sufficient to sign the partial derivative of (1.32) with respect to $\overline{\gamma}$ at r^*, which is positive only if $\frac{\partial^2 \overline{x}^{NL}}{\partial r \partial \overline{\gamma}}$ is sufficiently high.[1] This implies that $\psi'''(\overline{x}^{NL} - I - r^*)$ must be sufficiently high:

$$\psi'''(\overline{x}^{NL} - I - r^*)P(r^*) > Q(r^*) + S(r^*) + U(r^*), \tag{A.23}$$

where

$$P(r^*) = \frac{[\overline{x}^{NL}(\overline{\gamma} - \gamma_1(1+\psi'))]^3}{\overline{\gamma}^2(\overline{\gamma} - \gamma_1)^2(1+\psi')(\overline{\gamma} - 1 - \psi')^2} > 0,$$
$$Q(r^*) = \frac{\overline{x}^{NL}(\gamma_1 - 1)[-\overline{\gamma}\psi' - \gamma_1(1+f')]\psi'}{\overline{\gamma}(\overline{\gamma} - \gamma_1)(\overline{\gamma} - 1 - \psi')} < 0,$$
$$S(r^*) = -\frac{\overline{x}^{NL}(\overline{\gamma} - 1)(\gamma_1 - 1)[\overline{\gamma} - \gamma_1(1+\psi')]^2\psi'}{\overline{\gamma}(\overline{\gamma} - \gamma_1)^2(\overline{\gamma} - 1 - \psi')^2} < 0,$$
$$U(r^*) = -\frac{\overline{\gamma}^2(\gamma_1 - 1)(\psi')^2}{(\overline{\gamma} - \gamma_1)[\overline{\gamma} - \gamma_1(1+\psi')]} < 0.$$

Thus, given that r^* exists, $\frac{\partial \overline{x}^{NL}}{\partial \overline{\gamma}} > 0$ if $\psi'''(\overline{x}^{NL} - I - r^*)$ exceeds a negative lower bound. Given that $\psi(\cdot)$ must satisfy A5, such a $\psi(\cdot)$ exists that satisfies both Equations (A.22) and (A.23).

To show the second part of Proposition 8, recall from the preceding proof that $\overline{x}^{NL}(r^*)$ must satisfy Equation (A.21). Applying the implicit function, we have

$$\frac{\partial \overline{x}^{NL}}{\partial \overline{\gamma}} = -\frac{-[\overline{x}^{NL} - I + \psi(\overline{x}^{NL} - I - r^*)]}{-(\overline{\gamma} - 1)(1 + \psi'(\overline{x}^{NL} - I - r^*)) + \overline{x}^{NL}\psi''(\overline{x}^{NL} - I - r^*)}.$$

The numerator is strictly negative by the optimality of $\overline{x}^{NL}$. The denominator is strictly negative since $\overline{\gamma} > 1$ and $\psi'(y) \geq 0$ and $\psi''(y) \leq 0$ for all $y \in \mathbb{R}$. Thus, $\frac{\partial \overline{x}^{NL}}{\partial \overline{\gamma}} < 0$.

A.8 Verification

To verify that the constructed current value function w is optimal for any given $r \geq 0$, note that Equation (1.7) implies that any solution must satisfy the following

[1] Algebraic details are omitted for brevity.

two conditions for all $x \in (0, \infty)$, whether the agent is sophisticated or naive:

$$w(x,r) \geq x - I + \eta(x - I - r) \tag{A.24}$$

$$0 \geq -\rho w(x,r) + \lambda(\beta v(x,\hat{r}) - w(x,r)) + \mu x(\frac{\partial w}{\partial x}) + \frac{1}{2}\sigma^2 x^2(\frac{\partial^2 w}{\partial x^2}). \tag{A.25}$$

Let $\overline{x}$ denote current self's stopping threshold and $\hat{x}$ denote the (perceived) future self's stopping threshold. By construction, $w(x,r) = x - I + \eta(x - I - r)$ when $x \geq \overline{x}$ so equation (A.24) holds with equality. When $x < \overline{x}$, $w(x,r)$ is of the form $w(x,r) = A_1 x^{\gamma_1} + A_2 x^{\gamma_2}$, where $\gamma_2 \geq \gamma_1 > 1$, $A_1 > 0$, and $A_2 > 0$. Since $w(x,r)$ is convex and increasing, it must lie above the line $x - I + \eta(x - I - r)$ for all $x < \overline{x}$.

Whether the agent is sophisticated or naive, Equation (A.25) holds with equality when $x < \overline{x}$ by construction. Define the function $J(x)$ as follows:

$$J(x) = -\rho w(x,r) + \lambda(\beta v(x,\hat{r}) - w(x,r)) + \mu x(\frac{\partial w}{\partial x}) + \frac{1}{2}\sigma^2 x^2(\frac{\partial^2 w}{\partial x^2})$$

When $x \geq \overline{x}$, we have $w(x,r) = x - I + \eta(x - I - r)$. Since we have shown that $\overline{x} \leq \hat{x}$ (with equality only if the individual is sophisticated), then $v(x,r) = x - I + \eta(x - I - r)$ if $x \geq \overline{x}$. Then we have

$$\begin{aligned} J(x) = &- \rho[x - I + \eta(x - I - r)] + \lambda\left(\beta[x - I + \eta(x - I - r)]\right. \\ &\left. -[x - I + \eta(x - I - r)]\right) + \mu x(1 + \eta) \\ =&(1 + \eta)[\mu - \rho - \lambda(1 - \beta)]x + [\rho + \lambda(1 - \beta)][(1 + \eta)I + \eta r], \end{aligned}$$

which is strictly decreasing in x since $\mu < \rho$. We have previously shown that $\frac{\partial \overline{x}}{\partial \hat{x}} > 0$, and recall that $\hat{x}^{SE} < \hat{x}^{NE}$ and $\overline{x} = \hat{x}^{SE}$ when $\hat{x} = \hat{x}^{SE}$. So it is sufficient to show that $J(\hat{x}^{SE}) < 0$ to satisfy Equation (A.25).

$$\begin{aligned} J(x) &\leq J(\hat{x}^S) \\ &= [(1 + \eta)I + r\eta](\frac{1}{\overline{\gamma} - 1})[\overline{\gamma}\mu - \rho - \lambda(1 - \beta)] \\ &= [(1 + \eta)I + r\eta](\frac{1}{\overline{\gamma} - 1})[\beta(\mu\gamma_1 - \rho) + (1 - \beta)(\mu\gamma_2 - \rho - \lambda)]. \end{aligned}$$

Recall that $\gamma_1 > 1$ satisfies $0 = -\rho + \mu\gamma_1 + \frac{1}{2}\sigma^2\gamma_1(\gamma_1 - 1)$. Then $\mu\gamma_1 - \rho = -\frac{1}{2}\sigma^2\gamma_1(\gamma_1 - 1) < 0$. Likewise, $\gamma_2 > 1$ satisfies $0 = -(\rho + \lambda) + \mu\gamma_2 + \frac{1}{2}\sigma^2\gamma_2(\gamma_2 - 1)$, so $\mu\gamma_2 - \rho - \lambda = -\frac{1}{2}\sigma^2\gamma_2(\gamma_2 - 1) < 0$. Thus, $J(x) \leq J(\hat{x}^S) < 0$. Since $\overline{x}^{SE} < \overline{x}^{NE}$, then Equation (A.25) is also satisfied when the agent is naive. Therefore, the constructed value function w is at least as good as the value function generated by any alternative Markov strategy.

A.9 Proof of Proposition 10

The equilibrium defined by agents i and j's optimal threshold functions, given by Equations (1.36) and (1.37), is

$$\overline{x}_i = \frac{\overline{\gamma}_i I[\eta\overline{\gamma}_j + (1+\eta)(\overline{\gamma}_j - 1)]}{(1+\eta)^2(\overline{\gamma}_i - 1)(\overline{\gamma}_j - 1) - \eta^2\overline{\gamma}_i\overline{\gamma}_j} \tag{A.26}$$

$$\overline{x}_j = \frac{\overline{\gamma}_j I[\eta\overline{\gamma}_i + (1+\eta)(\overline{\gamma}_i - 1)]}{(1+\eta)^2(\overline{\gamma}_i - 1)(\overline{\gamma}_j - 1) - \eta^2\overline{\gamma}_i\overline{\gamma}_j}, \tag{A.27}$$

where $\overline{\gamma}_k = \beta_k\gamma_1 + (1-\beta_k)\gamma_2$ for $k = i, j$. Note that when $\overline{\gamma_i} = \overline{\gamma_j}$, the agents exhibit identical behavior in the absence of a peer, so the equilibrium thresholds are identical to those shown in Proposition 9. The first two parts of Proposition 10 are obtained by differentiating the equilibrium thresholds (A.26) and (A.27) directly. In particular, note that

$$\lim_{\overline{\gamma}_j \to \infty} \overline{x}_i = \left(\frac{(1+2\eta)\overline{\gamma}_i}{(1+2\eta)\overline{\gamma}_i - (1+\eta)^2}\right)I \geq \left(\frac{\overline{\gamma}_i}{\overline{\gamma}_i - 1}\right)I$$

$$\lim_{\overline{\gamma}_j \to \infty} \overline{x}_j = \left(\frac{(1+2\eta)\overline{\gamma}_i - (1+\eta)^2}{(1+2\eta)\overline{\gamma}_i - (1+\eta)}\right)I \geq I,$$

with inequality only if $\overline{\gamma}_i \to \infty$ as well. This implies that as long as $\overline{\gamma}_i$ is finite, both agents behave more patiently than they would in the absence of a goal.

The third part of the proposition follows by noting that

$$\overline{x}_i - \overline{x}_j = \frac{(\overline{\gamma}_j - \overline{\gamma}_i)(1+\eta)I}{(1+\eta)^2(\overline{\gamma}_i - 1)(\overline{\gamma}_j - 1) - \eta^2\overline{\gamma}_i\overline{\gamma}_j},$$

so $\overline{x}_i - \overline{x}_j > 0$ whenever $\overline{\gamma}_j - \overline{\gamma}_i > 0$.

A.10 Proof of Proposition 11

In an interpersonal equilibrium, agent i's continuation value function is given by

$$v_i(x, r_i = \overline{x}_j - I) = \begin{cases} [\overline{x}_i - I + \eta(\overline{x}_i - \overline{x}_j)](\frac{x}{\overline{x}_i})^{\gamma_1} & \text{if} \quad x < \overline{x}_i \\ x - I + \eta(x - \overline{x}_j) & \text{if} \quad x \geq \overline{x}_i, \end{cases}$$

where $\overline{x}_i$ and $\overline{x}_j$ are given by (A.26) and (A.27), respectively. To find the peer j^* who maximizes ex ante welfare, we can find the γ_j^* such that the value of v_i in its wait region is maximized given equilibrium behavior. The first order condition is

$$\frac{\partial v_i}{\partial \overline{\gamma}_j} = \left(\frac{x}{\overline{x}_i}\right)^{\gamma_1}\left(\frac{\partial \overline{x}_i}{\partial \overline{\gamma}_j} + \eta\left(\frac{\partial \overline{x}_i}{\partial \overline{\gamma}_j} - \frac{\partial \overline{x}_j}{\partial \overline{\gamma}_j}\right) - \left(\frac{\gamma_1}{\overline{x}_i}\right)\left(\frac{\partial \overline{x}_i}{\partial \overline{\gamma}_j}\right)[\overline{x}_i - I + \eta(\overline{x}_i - \overline{x}_j)]\right).$$

After making the appropriate substitutions and simplifying, the first order condition is of the following form:

$$\frac{\partial v_i}{\partial \overline{\gamma}_j} = (\frac{x}{\overline{x}_i})^{\gamma_1} \frac{A}{B(\overline{\gamma}_j)}[C + \overline{\gamma}_j D],$$

where

$$\begin{aligned} A &= \eta(1+\eta)^2(\gamma_1 - 1)[(\overline{\gamma}_i - 1)(1+\eta) + \eta\overline{\gamma}_i]I \\ B(\overline{\gamma}_j) &= \Big(\frac{1}{(\overline{\gamma}_j - 1)(1+\eta) + \eta\overline{\gamma}_j}\Big)\Big(\frac{1}{(1+\eta)^2(\overline{\gamma}_i - 1)(\overline{\gamma}_j - 1) - \eta^2\overline{\gamma}_i\overline{\gamma}_j}\Big)^2 \\ C &= -(1+\eta) \\ D &= (1+2\eta). \end{aligned}$$

Since $A > 0$, $B(\delta) > 0$, $C < 0$, and $D > 0$, it is clear that v_i is an asymmetric function of $\overline{\gamma}_j$, with a unique minimum at $\hat{\gamma}_j < 0$ such that $\frac{\partial v_i}{\partial \overline{\gamma}_j}(\hat{\gamma}_j) = 0$. Since $\hat{\gamma}_j = \frac{1+\eta}{1+2\eta} < 1$, then v_i is monotonically increasing for all $\overline{\gamma}_j \in (1, \infty)$. Hence, the value function is maximized as $\overline{\gamma}_j \to \infty$. Since $\overline{\gamma}_j = \beta_j\gamma_1 + (1 - \beta_j)\gamma_2$, this is equivalent to desiring a peer such that $\beta_j^* < 1$ and $\lambda_j^* \to \infty$.

Appendix B

Appendix for Chapter 2

B.1 Equilibrium Value Functions

This section collects the equilibrium current and continuation value functions that arise for each bracketing choice, with its corresponding goals, and each stage. Because the case of incremental goals in stage 2 is provided in the text, it is not repeated here. For ease of reference, it reiterates the key features of each.

B.1.1 Incremental Goals: Stage 1

In the first stage, the equilibrium current and future value functions when the agent sets incremental goals are given by w_1^{inc} and v_1^{inc}, described by Equations (B.1) and (B.2) respectively.

Because the agent completes the second stage strictly after the first, the stop regions of the current and continuation value functions, w_1^{inc} and v_1^{inc}, are composed of two regions. When the observed first-stage payoff $\tilde{x}_1$ is such that $k\tilde{x}_1 < \overline{x}_2^{inc}$, the option value of stage 2 is determined by the value of waiting to stop the process x_{2t}. When $\tilde{x}_1$ is sufficiently high that $k\tilde{x}_1 \geq \overline{x}_2^{inc}$, the option value of stage 2 is simply the value of stopping x_{2t} immediately. Since the agent evaluates himself against a goal at the end of each stage when he sets incremental goals, he expects to incur comparative disutility at the end of each.

$$E[w_1^{inc}(\tilde{x}_1, r_1^{inc})|\tilde{x}_1] = \begin{cases} \beta[\overline{x}_1^{inc}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{\overline{x}_1^{inc}})^{\gamma_1} + (1-\beta)[\overline{x}_1^{inc}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{\overline{x}_1^{inc}})^{\gamma_2} & \\ +k^{\gamma_1}\beta[\overline{x}_2^{inc}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{\overline{x}_2^{inc}})^{\gamma_1} & \\ +k^{\gamma_1}(1-\beta)[\overline{x}_2^{inc}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{\overline{x}_2^{inc}})^{\gamma_2} & \text{if} \quad \tilde{x}_1 < \overline{x}_1^{inc} \\ \tilde{x}_1 - I_1 + E[\psi(x_1-\overline{x}_1^{inc})|\tilde{x}_1] + k^{\gamma_1}\beta[\overline{x}_2^{inc}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{\overline{x}_2^{inc}})^{\gamma_1} & \\ +k^{\gamma_1}(1-\beta)[\overline{x}_2^{inc}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{\overline{x}_2^{inc}})^{\gamma_2} & \text{if} \quad \overline{x}_1^{inc} \le \tilde{x}_1 < \frac{\overline{x}_2^{inc}}{k} \\ (1+k)\tilde{x}_1 - I_1 - I_2 + E[\psi(x_1-\overline{x}_1^{inc})|\tilde{x}_1] + E[\psi(x_2-\overline{x}_2^{inc})|\tilde{x}_1] & \text{if} \quad \tilde{x}_1 \ge \frac{\overline{x}_2^{inc}}{k} \end{cases} \tag{B.1}$$

$$E[v_1^{inc}(\tilde{x}_1, r_1^{inc})|\tilde{x}_1] = \begin{cases} [\overline{x}_1^{inc}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{\overline{x}_1^{inc}})^{\gamma_1} + k^{\gamma_1}[\overline{x}_2^{inc}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{\overline{x}_2^{inc}})^{\gamma_1} & \text{if} \quad \tilde{x}_1 < \overline{x}_1^{inc}, \\ \tilde{x}_1 - I_1 + E[\psi(x_1-\overline{x}_1^{inc})|\tilde{x}_1] + k^{\gamma_1}[\overline{x}_2^{inc}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{\overline{x}_2^{inc}})^{\gamma_1} & \text{if} \quad \overline{x}_1^{inc} \le \tilde{x}_1 < \frac{\overline{x}_2^{inc}}{k}, \\ (1+k)\tilde{x}_1 - I_1 - I_2 + E[\psi(x_1-\overline{x}_1^{inc})|\tilde{x}_1] + E[\psi(x_2-\overline{x}_2^{inc})|\tilde{x}_1] & \text{if} \quad \tilde{x}_1 \ge \frac{\overline{x}_2^{inc}}{k}. \end{cases} \tag{B.2}$$

(B.3)

B.1.2 Aggregate Goals: Stage 2

In the second stage, the equilibrium current and future value functions when the agent sets aggregate goals are given by w_2^{agg} and v_2^{agg}, described by Equations (B.4) and (B.5) respectively. They differ from those of incremental goals only in the goal comparison that is being made.

B.1.3 Aggregate Goals: Stage 1

The first two terms of the expected current value function, given by Equation (B.6), in its wait region reflect the option value of stopping in the first stage, while the second two reflect that of stopping in the second. In contrast to the case of incremental goals, the disutility from ex-ante uncertainty is absent from the first two terms, as the agent does not directly evaluate himself against a goal in the first stage. Likewise, the first term of the expected continuation value function, given by Equation (B.7), in its wait region reflects the option value of the first stage from an ex ante perspective, while the second term reflects that of the second.

$$E[w_2^{agg}(\tilde{x}_2, r_2^{agg})|\tilde{x}_2] = \begin{cases} \beta[\overline{x}_2^{agg} - I_2 + \frac{1}{2}\eta(\frac{1+2\epsilon+\alpha}{1+\epsilon})(\overline{x}_2^{agg} + \overline{x}_1^{agg}) - \frac{1}{2}\eta(\alpha+1)(r_2^{agg} + I_1 + I_2)](\frac{\tilde{x}_2}{\overline{x}_2^{agg}})^{\gamma_1} & \\ +(1-\beta)[\overline{x}_2^{agg} - I_2 + \frac{1}{2}\eta(\frac{1+2\epsilon+\alpha}{1+\epsilon})(\overline{x}_2^{agg} + \overline{x}_1^{agg}) & \\ -\frac{1}{2}\eta(\alpha+1)(r_2^{agg} + I_1 + I_2)](\frac{\tilde{x}_2}{\overline{x}_2^{agg}})^{\gamma_2} & \text{if} \quad \tilde{x}_2 < \overline{x}_2^{agg} \\ \tilde{x}_2 - I_2 + E[\psi(x_{1\bar{t}_1} - I_1 + x_2 - I_2 - r_2^{agg})|\tilde{x}_2] & \text{if} \quad \tilde{x}_2 \geq \overline{x}_2^{agg} \end{cases} \tag{B.4}$$

$$E[v_2^{agg}(\tilde{x}_2, r_2^{agg})|\tilde{x}_2] = \begin{cases} [\overline{x}_2^{agg} - I_2 + \frac{1}{2}\eta(\frac{1+2\epsilon+\alpha}{1+\epsilon})(\overline{x}_2^{agg} + \overline{x}_1^{agg}) - \frac{1}{2}\eta(\alpha+1)(r_2^{agg} + +I_1 + I_2)](\frac{\tilde{x}_2}{\overline{x}_2^{agg}})^{\gamma_1} & \text{if} \quad \tilde{x}_2 < \overline{x}_2^{agg} \\ \tilde{x}_2 - I_2 + E[\psi(x_{1\bar{t}_1} - I_1 + x_2 - I_2 - r_2^{agg})|\tilde{x}_2] & \text{if} \quad \tilde{x}_2 \geq \overline{x}_2^{agg}. \end{cases} \tag{B.5}$$

$$E[w_1^{agg}(\tilde{x}_1, r_1^{agg})|\tilde{x}_1] = \begin{cases} \beta[\overline{x}_1^{agg} - I_1](\frac{\tilde{x}_1}{\overline{x}_1^{agg}})^{\gamma_1} + (1-\beta)[\overline{x}_1^{agg} - I_2](\frac{\tilde{x}_1}{\overline{x}_1^{agg}})^{\gamma_2} & \\ +k^{\gamma_1}\beta[\overline{x}_2^{agg}(1 - \frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})) - I_2](\frac{\tilde{x}_1}{\overline{x}_2^{agg}})^{\gamma_1} & \\ +k^{\gamma_1}(1-\beta)[\overline{x}_2^{agg}(1 - \frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})) - I_2](\frac{\tilde{x}_1}{\overline{x}_2^{agg}})^{\gamma_2} & \text{if} \quad \tilde{x}_1 < \overline{x}_1^{agg} \\ \tilde{x}_1 - I_1 + k^{\gamma_1}\beta[\overline{x}_2^{agg}(1 - \frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})) - I_2](\frac{\tilde{x}_1}{\overline{x}_2^{agg}})^{\gamma_1} & \\ +k^{\gamma_1}(1-\beta)[\overline{x}_2^{agg}(1 - \frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})) - I_2](\frac{\tilde{x}_1}{\overline{x}_2^{agg}})^{\gamma_2} & \text{if} \quad \overline{x}_1^{agg} \leq \tilde{x}_1 < \frac{\overline{x}_2^{agg}}{k} \\ (1+k)\tilde{x}_1 - I_1 - I_2 + E[\psi(x_1 + x_2 - \overline{x}_1^{agg} - \overline{x}_2^{agg})|\tilde{x}_1] & \text{if} \quad \tilde{x}_1 \geq \frac{\overline{x}_2^{agg}}{k}, \end{cases} \tag{B.6}$$

$$E[v_1^{agg}(\tilde{x}_1, r^{agg})|\tilde{x}_1] = \begin{cases} [\overline{x}_1^{agg} - I_1](\frac{\tilde{x}_1}{\overline{x}_1^{agg}})^{\gamma_1} + k^{\gamma_1}[\overline{x}_2^{agg}(1 - \frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})) - I_2](\frac{\tilde{x}_1}{\overline{x}_2^{agg}})^{\gamma_1} & \text{if} \quad \tilde{x}_1 < \overline{x}_1^{agg} \\ \tilde{x}_1 - I_1 + k^{\gamma_1}[\overline{x}_2^{agg}(1 - \frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})) - I_2](\frac{\tilde{x}_1}{\overline{x}_2^{agg}})^{\gamma_1} & \text{if} \quad \overline{x}_1^{agg} \leq \tilde{x}_1 < \frac{\overline{x}_2^{agg}}{k} \\ (1+k)\tilde{x}_1 - I_1 - I_2 + E[\psi(x_1 + x_2 - \overline{x}_1^{agg} - \overline{x}_2^{agg})|\tilde{x}_1] & \text{if} \quad \tilde{x}_1 \geq \frac{\overline{x}_2^{agg}}{k}. \end{cases} \tag{B.7}$$

B.2 Proof of Proposition 13

Given parameters $(\eta, \alpha, \beta, \lambda, \epsilon)$, the stopping thresholds that maximize v_1^{inc} in its wait region solve the following problem:

$$\max_{\overline{x}_1,\overline{x}_2}[\overline{x}_1(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{\overline{x}_1})^{\gamma_1}+k^{\gamma_1}[\overline{x}_2(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_2](\frac{\tilde{x}_1}{\overline{x}_2})^{\gamma_1}.$$

For each stage, the first order conditions $\frac{\partial v_1^{inc}}{\partial \overline{x}_1}=0$ and $\frac{\partial v_1^{inc}}{\partial \overline{x}_2}=0$ yield the following two first-best thresholds, $\overline{x}_1^*$ and $\overline{x}_2^*$:

$$\overline{x}_1^* = \frac{\gamma_1 I_1}{(\gamma_1-1)[1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})]},$$
$$\overline{x}_2^* = \frac{\gamma_1 I_2}{(\gamma_1-1)[1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})]}.$$

When $\overline{x}_i^{inc}=\overline{x}_i^*$, the agent achieves the first-best; when $\overline{x}_i^{inc}>\overline{x}_i^*$ he waits longer than the first-best. Solving for the condition such that $\overline{x}_i^{inc}\geq\overline{x}_i^*$ for $i=1,2$ yields:

$$(\frac{\overline{\gamma}-\gamma_1}{\gamma_1})[1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})]-\frac{1}{2}\eta\gamma_1(\alpha+1)\leq 0.$$

B.3 Proof of Proposition 14

To show Proposition 14, I will find the ϵ such that $\frac{\partial}{\partial\alpha}(E[v_1^{inc}(x_1)|\tilde{x}_1])|_{\alpha=1}\leq 0$. Since the two components of $E[v_1^{inc}(x_1)|\tilde{x}_1]$ regarding the first and second stages differ only by I_i and the constant k^{γ_1}, I differentiate only the first stage $E[v_1^{inc}(x_1)|\tilde{x}_1]$ with respect to α for brevity. This yields,

$$\begin{aligned}
\frac{\partial}{\partial\alpha}(E[v_1^{inc}(x_1)|\tilde{x}_1]) =& \frac{\partial}{\partial\alpha}\left([\overline{x}_1^{inc}(1-\frac{1}{2}\eta(\hat{\alpha}-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{\overline{x}_1^{inc}})^{\gamma_1}\right)\\
=&(\frac{1}{\overline{x}_1^{inc}})^{\gamma_1}\left((\frac{\partial\overline{x}_1^{inc}}{\partial\alpha})[-(\gamma_1-1)(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})+\frac{\gamma_1 I_1}{\overline{x}_1^{inc}})]\right.\\
&\left.-\frac{1}{2}\eta(\frac{\epsilon}{1+\epsilon})\overline{x}_1^{inc}\right)\\
=&(\frac{1}{\overline{x}_1^{inc}})^{\gamma_1}(\frac{\eta I_1}{2})(\frac{1}{(\overline{\gamma}-1)[1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})]-\frac{1}{2}\eta(\alpha+1)})^2\\
&\{[1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})][-\gamma_1(\overline{\gamma}-1)(\frac{\epsilon}{1+\epsilon})+\overline{\gamma}-\gamma_1]\\
&-\frac{1}{2}\eta(\alpha+1)[(\gamma_1(\overline{\gamma}-1)-\overline{\gamma})(\frac{\epsilon}{1+\epsilon})+\gamma_1]\}.
\end{aligned}$$

Since the first three terms of the above expression are positive for all feasible (η, α), I consider only the fourth term, which is clearly monotonic in α, when evaluating the partial derivative at $\alpha = 1$ for brevity. This yields,

$$\frac{\partial}{\partial \alpha}(E[v_1^{inc}(x_1)|\tilde{x}_1])|_{\alpha=1} = -\gamma_1(\overline{\gamma}-1)(1+\eta)(\frac{\epsilon}{1+\epsilon}) + \overline{\gamma} - \gamma_1 + \eta\overline{\gamma}(\frac{\epsilon}{1+\epsilon}) - \eta\gamma_1$$
$$= \epsilon[\overline{\gamma}(\gamma_1 - 1)(1+\eta)] - \overline{\gamma} + (1+\eta)\gamma_1.$$

Thus, $\frac{\partial}{\partial \alpha}(E[v_1^{inc}(x_1)|\tilde{x}_1])|_{\alpha=1} > 0$ (i.e., there exists an interior constrained optimum for α) when $\eta < \eta^* = \frac{\overline{\gamma}-\gamma_1}{\gamma_1}$ and

$$\epsilon < \frac{\overline{\gamma} - \gamma_1(1+\eta)}{\overline{\gamma}(\gamma_1 - 1)(1+\eta)}.$$

Otherwise, $\frac{\partial}{\partial \alpha}(E[v_1^{inc}(x_1)|\tilde{x}_1])|_{\alpha=1} \leq 0$, so the constrained optimum is $\alpha = 1$.

When the agent sets an aggregate goal, the analogous result holds. Recall that $E[v_1^{agg}(x_1)|\tilde{x}_1]$ is given by

$$E[v_1^{agg}(x_1)|\tilde{x}_1] = [\overline{x}_1^{agg} - I_1](\frac{\tilde{x}_1}{\overline{x}_1^{agg}})^{\gamma_1} + k^{\gamma_1}[\overline{x}_2^{agg}(1 - \frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})) - I_2](\frac{\tilde{x}_1}{\overline{x}_2^{agg}})^{\gamma_1}.$$

Since $\overline{x}_2^{agg} = \overline{x}_2^{inc}$, then the preceding analysis with respect to incremental goals applies to the expected option value of the second stage, i.e. the second term of the expected option value of the entire project, $E[v_1^{agg}(x_1)|\tilde{x}_1]$. Consider the η' such that $E[v_1^{agg}(x_1)|\tilde{x}_1]$ is maximized when $\eta = \eta'$ and $\alpha = 1$. In the proof of Proposition 18, I show that $\eta' > \eta^*$ and that $\overline{x}_1^{agg} \leq x_1^*$ in this case. Let $\eta < \eta^*$ and ϵ be sufficiently small that

$$\epsilon < \frac{\overline{\gamma} - \gamma_1(1+\eta)}{\overline{\gamma}(\gamma_1 - 1)(1+\eta)}.$$

Then our previous result on incremental goals implies that the second term of $E[v_1^{agg}(x_1)|\tilde{x}_1]$ is increasing in α when $\alpha = 1$. To show that the first term of $E[v_1^{agg}(x_1)|\tilde{x}_1]$ is increasing, note that $\overline{x}_1^{agg} < x_1^*$ when $\eta < \eta^*$, so the first term is increasing as long as $\frac{\partial \overline{x}_1^{agg}}{\partial \eta} > 0$. Differentiating $\overline{x}_1^{agg}$, this is guaranteed whenever

$$\eta \leq \frac{\overline{\gamma} - 1}{\gamma_2(\overline{\gamma}-1)(\frac{\epsilon}{1+\epsilon}) + \gamma_2 + 1}.$$

Thus, $\frac{\partial}{\partial \alpha}(E[v_1^{agg}(x_1)|\tilde{x}_1])|_{\alpha=1} > 0$ whenever η and ϵ are sufficiently small, and an interior optimum $\alpha > 1$ exists. Otherwise, the optimal level of loss aversion is $\alpha = 1$.

B.4 Proof of Proposition 15

Proof. First, I show existence and uniqueness of $\overline{x}_1^{agg}$.

Proof. Define the following function, $G(x)$:

$$G(x) = \overline{\gamma} I_1 + \frac{1}{2}\eta(\alpha+1)(\frac{1}{\overline{\gamma}-1})[\beta k^{\gamma_1}(\gamma_1 - 1)(\frac{1}{\overline{x}_2^{agg}})^{\gamma_1}(x)^{\gamma_1+1}$$
$$+ (1-\beta)k^{\gamma_2}(\gamma_2 - 1)(\frac{1}{\overline{x}_2^{agg}})^{\gamma_2}(x)^{\gamma_2+1}] - (\overline{\gamma}-1)x. \quad \text{(B.8)}$$

Clearly, $G(\overline{x}_1^{agg}) = 0$, so it is sufficient to verify that $G(x)$ has a unique root in the range $0 < \overline{x}_1^{agg} < \overline{x}_1^{inc}$. First, $G(0) = \overline{\gamma} I_1 > 0$. Second, G is clearly increasing in k, where $k < \frac{I_2}{I_1}$. Therefore,

$$
\begin{aligned}
G(\overline{x}_1^{inc}) &= \overline{\gamma} I_1 + \frac{1}{2}\eta(\alpha+1)\frac{1}{(\overline{\gamma}-1)}[\beta k^{\gamma_1}(\gamma_1 - 1)(\frac{\overline{x}_1^{inc}}{\overline{x}_2^{agg}})^{\gamma_1}(\overline{x}_1^{inc}) \\
&+ (1-\beta)k^{\gamma_2}(\gamma_2 - 1)(\frac{\overline{x}_1^{inc}}{\overline{x}_2^{agg}})^{\gamma_2}(\overline{x}_1^{inc})] - (\overline{\gamma}-1)(\overline{x}_1^{inc}) \\
&= \overline{\gamma} I_1 + \frac{1}{2}\eta(\alpha+1)\frac{1}{(\overline{\gamma}-1)}[\beta k^{\gamma_1}(\gamma_1 - 1)(\frac{I_1}{I_2})^{\gamma_1}(\overline{x}_1^{inc}) \\
&+ (1-\beta)k^{\gamma_2}(\gamma_2 - 1)(\frac{I_1}{I_2})^{\gamma_2}(\overline{x}_1^{inc})] - (\overline{\gamma}-1)(\overline{x}_1^{inc}) \\
&\leq \overline{\gamma} I_1 + \frac{1}{2}\eta(\alpha+1)\frac{1}{(\overline{\gamma}-1)}[\beta(\frac{I_2}{I_1})^{\gamma_1}(\gamma_1 - 1)(\frac{I_1}{I_2})^{\gamma_1}(\overline{x}_1^{inc}) \\
&+ (1-\beta)(\frac{I_2}{I_1})^{\gamma_2}(\gamma_2 - 1)(\frac{I_1}{I_2})^{\gamma_2}(\overline{x}_1^{inc})] - (\overline{\gamma}-1)(\overline{x}_1^{inc}) \\
&\leq \overline{\gamma} I_1 + \frac{1}{2}\eta(\alpha+1)\overline{x}_2^{agg} - (\overline{\gamma}-1)\overline{x}_2^{inc} \\
&< \overline{\gamma} I_1 + \frac{1}{2}\eta(\alpha+1)\overline{x}_2^{agg} - (\overline{\gamma}-1)\overline{x}_2^{inc} + \frac{1}{2}\eta(\alpha-1)(\overline{\gamma}-1)(\frac{\epsilon}{1+\epsilon})\overline{x}_2^{inc} \\
&= \overline{\gamma}(I_1 - I_I) = 0,
\end{aligned}
$$

where the second line follows from the fact that $\overline{x}_2^{agg} = \overline{x}_2^{inc}$ and $\overline{x}_1^{inc} = (\frac{I_1}{I_2})\overline{x}_2^{inc}$, and the third from the fact that G is increasing in k, where $k < \frac{I_2}{I_1}$.

Since $G(0) > 0$ and $G(\overline{x}_1^{inc}) < 0$, then there exists at least one root in this range. Suppose that there exists more than one root in $[0, \overline{x}_{1b}^{inc}]$. Since $G(0) > 0$ and $G(\overline{x}_1^{inc}) < 0$, then there must exist a local maximum x' such that $G'(x') = 0$ and $G''(x') < 0$. However, we can verify that $\frac{d^2G(x)}{dx^2} > 0$ for all $x > 0$. Therefore, no local maximum exists in the interval $[0, \overline{x}_1^{inc}]$, so $G(x)$ has a unique root in the range $0 < x < \overline{x}_1^{inc}$. To rule out any roots x'' where $x'' > \overline{x}_1^{inc}$, note that it must be that $G'(x'') > 0$ since $\frac{d^2G(x)}{dx^2} > 0$ for all $x > 0$. However, this implies that $\frac{\partial \overline{x}_1^{agg}}{\partial \overline{x}_2^{agg}} > 0$, which is nonsensical. If the agent never completes the second stage, then he will never evaluate himself against the aggregate goal, so it is essentially ineffectual. Given this, his stopping threshold in the first stage cannot increase as $\overline{x}_2^{agg}$ increases. □

Thus, $\overline{x}_1^{agg}$ exists and is unique, and $\overline{x}_1^{agg} < \overline{x}_1^{inc}$. The fact that $\overline{x}_1^{agg} > (\frac{\overline{\gamma}}{\overline{\gamma}-1})I_1$, where the right-hand side of the inequality is the agent's stopping threshold in the absence of reference dependence ($\eta = 0$), is obvious by inspection of Equation (2.36). Since $\overline{x}_2^{agg} = \overline{x}_2^{inc}$, we have proven the proposition. □

B.5 Proof of Proposition 16

To find $\frac{\partial \overline{x}_1^{agg}}{\partial \epsilon}$, we apply the implicit function theorem to Equation (B.8). First,

$$\frac{\partial G}{\partial \epsilon} = -\frac{1}{2}\eta(\alpha+1)(\frac{1}{\overline{\gamma}-1})[\beta k^{\gamma_1}(\gamma_1-1)(x)^{\gamma_1+1}(\gamma_1)(\frac{1}{\overline{x}_2^{agg}})^{\gamma_1+1}(\frac{\partial \overline{x}_2^{agg}}{\partial \epsilon})$$
$$-(1-\beta)k^{\gamma_2}(\gamma_2-1)(x)^{\gamma_2+1}(\gamma_2)(\frac{1}{\overline{x}_2^{agg}})^{\gamma_2+1}(\frac{\partial \overline{x}_2^{agg}}{\partial \epsilon})].$$

Since $\frac{\partial \overline{x}_2^{agg}}{\partial \epsilon} > 0$, then $\frac{\partial G}{\partial \epsilon} \leq 0$, with equality only when $\eta = 0$ or $\alpha = 1$. The latter condition arises from the fact that $\frac{\partial \overline{x}_2^{agg}}{\partial \epsilon} = 0$ if $\alpha = 1$. In the preceding proof of the existence of $\overline{x}_1^{agg}$, I have shown that $\frac{\partial G}{\partial x} < 0$ when $0 < x < \overline{x}_1^{inc}$. By the implicit function theorem,

$$\frac{\partial \overline{x}_1^{agg}}{\partial \epsilon} = -\frac{\frac{\partial G}{\partial \epsilon}}{\frac{\partial G}{\partial x}}.$$

Thus, $\frac{\partial \overline{x}_1^{agg}}{\partial \epsilon} \leq 0$, with equality only when $\eta = 0$ or $\alpha = 1$.

B.6 Proof of Proposition 17

Since $\overline{x}_2^{agg} = \overline{x}_2^{inc}$, the expected option value of the second stage is the same regardless of whether the agent sets incremental or aggregate goals. Therefore, the results from Proposition 13 apply to the second stage. Since he incurs no comparison utility in the first stage, the first-stage first-best is given by $x_1^* = (\frac{\gamma_1}{\gamma_1-1})I_1$, the stopping threshold he would employ in the absence of reference dependence ($\eta = 0$) and present-biasedness ($\beta = 1$ or $\lambda = 0$). When $F \geq 0$, then it must be true that

$$(\frac{\overline{\gamma}-\gamma_1}{\gamma_1}) - \frac{1}{2}\eta\gamma_1(\alpha+1) \geq 0, \tag{B.9}$$

with equality only if $\epsilon = 0$, since F is decreasing in ϵ. Note that when $\epsilon = 0$, $\overline{x}_1^{inc} \leq x_1^*$ whenever (B.9) is true. But since $\overline{x}_1^{agg} < \overline{x}_1^{inc}$ and $\frac{\partial \overline{x}_1^{agg}}{\partial \epsilon} < 0$, then $\overline{x}_1^{agg} < x_1^*$ whenever $F > 0$.

B.7 Proof of Proposition 18

To show the proposition, I find the optimal combinations of (η, α) given that the agent sets incremental and aggregate goals separately, then compare the two value functions under each form of bracketing.

Incremental Goals

I show that $\arg\max_{\eta,\alpha} E[v_1^{inc}(x_1)|\tilde{x}_1] = (\eta^*, 1)$. First, $E[v_1^{inc}(x_1)|\tilde{x}_1]$ is given by

$$E[v_1^{inc}(x_1)|\tilde{x}_1] =[\overline{x}_1^{inc}(1-\frac{1}{2}\eta(\hat{\alpha}-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{\overline{x}_1^{inc}})^{\gamma_1} + k^{\gamma_1}[\overline{x}_2^{inc}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_2](\frac{\tilde{x}_1}{\overline{x}_2^{inc}})^{\gamma_1}.$$

Suppose that $\arg\max_{\eta,\alpha} E[v_1^{inc}(x_1)|\tilde{x}_1] = (\hat{\eta}, \hat{\alpha})$, where $\hat{\eta} \neq \eta^*$ and $\hat{\alpha} > 1$. Let $\hat{x}_i^{inc}$ be the threshold that the agent uses when $\eta = \hat{\eta}$ and $\alpha = \hat{\alpha}$, where $i = 1, 2$. Clearly, $\hat{x}_i^{inc}$ is a function of $\hat{\eta}$ and $\hat{\alpha}$. We can construct a threshold x_i^{*inc} such that $\alpha = 1$ and $\eta = \eta^*$, and $\hat{x}_i^{inc} = x_i^{*inc}$:

$$\begin{aligned}
\hat{x}_i^{inc} &= x_i^{*inc} \\
\frac{\overline{\gamma} I_i}{(\overline{\gamma}-1)[1-\frac{1}{2}\hat{\eta}(\hat{\alpha}-1)(\frac{\epsilon}{1+\epsilon})]-\frac{1}{2}\hat{\eta}(\hat{\alpha}+1)} &= \frac{\overline{\gamma} I_i}{(\overline{\gamma}-1)[1-\frac{1}{2}\eta^*(1-1)(\frac{\epsilon}{1+\epsilon})]-\frac{1}{2}\eta^*(2)} \\
\frac{1}{(\overline{\gamma}-1)[1-\frac{1}{2}\hat{\eta}(\hat{\alpha}-1)(\frac{\epsilon}{1+\epsilon})]-\frac{1}{2}\hat{\eta}(\hat{\alpha}+1)} &= \frac{1}{\overline{\gamma}-1-\eta^*} \\
\eta^* &= \frac{1}{2}\hat{\eta}[(\overline{\gamma}-1)(\hat{\alpha}-1)(\frac{\epsilon}{1+\epsilon})+\hat{\alpha}+1].
\end{aligned}$$

Thus, there exists $(\eta^*, 1)$, where η^* is clearly non-negative, such that $\hat{x}_i^{inc} = x_i^{*inc}$. Unsurprisingly, inspection of the equation describing η^* yields that η^* is increasing in $\hat{\eta}$ and $\hat{\alpha}$, as well as ϵ. Let $\hat{v}_1^{inc}$ be the option value of the project with $(\hat{\eta}, \hat{\alpha})$ and v_1^{*inc} be the option value of the project with $(\eta^*, 1)$. Since $\hat{x}_i^{inc} = x_i^{*inc}$, then

$$\begin{aligned}
E[\hat{v}_1^{inc}(x_1)|\tilde{x}_1] =&[\hat{x}_1^{inc}(1-\frac{1}{2}\hat{\eta}(\hat{\alpha}-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{\hat{x}_1^{inc}})^{\gamma_1} \\
&+ k^{\gamma_1}[\hat{x}_2^{inc}(1-\frac{1}{2}\hat{\eta}(\hat{\alpha}-1)(\frac{\epsilon}{1+\epsilon}))-I_2](\frac{\tilde{x}_1}{\hat{x}_2^{inc}})^{\gamma_1} \\
=&[x_1^{*inc}(1-\frac{1}{2}\hat{\eta}(\hat{\alpha}-1)(\frac{\epsilon}{1+\epsilon}))-I_1](\frac{\tilde{x}_1}{x_1^{*inc}})^{\gamma_1} \\
&+ k^{\gamma_1}[x_2^{*inc}(1-\frac{1}{2}\hat{\eta}(\hat{\alpha}-1)(\frac{\epsilon}{1+\epsilon}))-I_2](\frac{\tilde{x}_1}{x_2^{*inc}})^{\gamma_1} \\
&< [x_1^{*inc}-I_1](\frac{\tilde{x}_1}{x_1^{*inc}})^{\gamma_1} + k^{\gamma_1}[x_2^{*inc}-I_2](\frac{\tilde{x}_1}{x_2^{*inc}})^{\gamma_1} \\
=&E[v_1^{*inc}(x_1)|\tilde{x}_1].
\end{aligned}$$

Thus, we have shown that $\arg\max_{\eta,\alpha} E[v_1^{inc}(x_1)|\tilde{x}_1] = (\eta^*, 1)$ when the agent sets incremental goals.

Aggregate Goals

I show that $\arg\max_{\eta,\alpha} E[v_1^{agg}(x_1)|\tilde{x}_1] = (\eta', 1)$ where $\eta' > \eta^*$. First, $E[v_1^{agg}(x_1)|\tilde{x}_1]$ is given by

$$E[v_1^{agg}(x_1)|\tilde{x}_1] = [\overline{x}_1^{agg}-I_1](\frac{\tilde{x}_1}{\overline{x}_1^{agg}})^{\gamma_1} + k^{\gamma_1}[\overline{x}_2^{agg}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_2](\frac{\tilde{x}_1}{\overline{x}_2^{agg}})^{\gamma_1}.$$

Suppose that $\arg\max_{\eta,\alpha} E[v_1^{agg}(x_1)|\tilde{x}_1] = (\hat{\eta}, \hat{\alpha})$, where $\hat{\eta} \neq \eta'$ and $\hat{\alpha} > 1$. Let $\hat{x}_i^{agg}$ be the threshold that the agent uses when $\eta = \hat{\eta}$ and $\alpha = \hat{\alpha}$, where $i = 1, 2$. As in the case of incremental goals, we can construct a threshold $x_2'^{agg}$ such that $\alpha = 1$ and $\eta = \eta^*$, and $\hat{x}_2^{agg} = x_2'^{agg}$. Since $\overline{x}_2^{agg} = \overline{x}_2^{inc}$, the same procedure yields

$$\eta' = \frac{1}{2}\hat{\eta}[(\overline{\gamma} - 1)(\hat{\alpha} - 1)(\frac{\epsilon}{1+\epsilon}) + \hat{\alpha} + 1].$$

Thus, there exists $(\eta', 1)$, where η' is clearly non-negative, such that $\hat{x}_2^{agg} = x_2'^{agg}$. Unsurprisingly, inspection of the equation describing η' yields that η' is increasing in $\hat{\eta}$ and $\hat{\alpha}$, as well as ϵ. Let $\hat{v}_1^{agg}$ be the option value of the project with $(\hat{\eta}, \hat{\alpha})$ and $v_1'^{agg}$ be the option value of the project with $(\eta', 1)$. Since $\hat{x}_2^{agg} = x_2'^{agg}$, then

$$\begin{aligned}
E[\hat{v}_1^{agg}(x_1)|\tilde{x}_1] &= [\hat{x}_1^{agg} - I_1](\frac{\tilde{x}_1}{\hat{x}_1^{agg}})^{\gamma_1} + k^{\gamma_1}[\hat{x}_2^{agg}(1 - \frac{1}{2}\hat{\eta}(\hat{\alpha}-1)(\frac{\epsilon}{1+\epsilon})) - I_2](\frac{\tilde{x}_1}{\hat{x}_2^{agg}})^{\gamma_1} \\
&= [\hat{x}_1^{agg} - I_1](\frac{\tilde{x}_1}{\hat{x}_1^{agg}})^{\gamma_1} + k^{\gamma_1}[x_2'^{agg}(1 - \frac{1}{2}\hat{\eta}(\hat{\alpha}-1)(\frac{\epsilon}{1+\epsilon})) - I_2](\frac{\tilde{x}_1}{x_2'^{agg}})^{\gamma_1} \\
&< [\hat{x}_1^{agg} - I_1](\frac{\tilde{x}_1}{\hat{x}_1^{agg}})^{\gamma_1} + k^{\gamma_1}[x_2'^{agg} - I_2](\frac{\tilde{x}_1}{x_2'^{agg}})^{\gamma_1} \\
&< [x_1'^{agg} - I_1](\frac{\tilde{x}_1}{x_1'^{agg}})^{\gamma_1} + k^{\gamma_1}[x_2'^{agg} - I_2](\frac{\tilde{x}_1}{x_2'^{agg}})^{\gamma_1} \\
&= E[v_1'^{agg}(x_1)|\tilde{x}_1].
\end{aligned}$$

The second inequality follows from the fact that $\hat{x}_1^{agg} < x_1'^{agg}$, since $\hat{x}_2^{agg} = x_2'^{agg}$ and clearly $\eta' > \frac{1}{2}\hat{\eta}(\hat{\alpha}+1)$. Note that the first term of $E[\hat{v}_1^{agg}(x_1)|\tilde{x}_1]$ is maximized when $\overline{x}_1^{agg} = (\frac{\gamma_1}{\gamma_1 - 1})I_1 \equiv x_1^*$. Since we know that $x_1'^{agg} < (\frac{\overline{\gamma}}{\overline{\gamma}-1-\eta'})I_1$, then the second inequality follows as long as η' is sufficiently low that $x_1'^{agg} \leq x_1^*$. If η' is so high that $x_1'^{agg} > x_1^*$, then we must have $\eta' > \eta^*$. But since the second term of $E[v_1^{agg}(x_1)|\tilde{x}_1]$ is maximized when $\eta = \eta^*$ and $\alpha = 1$, then both the first and second terms of his option value can be increased by decreasing his degree of reference dependence if η' is so high that $x_1'^{agg} > x_1^*$. Thus, we must have that the optimal η' is sufficiently low that $x_1'^{agg} \leq x_1^*$, so the second inequality follows. Thus, $\arg\max_{\eta,\alpha} E[v_1^{agg}(x_1)|\tilde{x}_1] = (\eta', 1)$. It follows that $\eta' > \eta^*$ by noting that given that $\alpha = 1$, $\frac{\partial}{\partial\eta}(E[v_1^{agg}(x_1)|\tilde{x}_1]) > 0$ for all $\eta \leq \eta^*$.

I have shown that given either form of goal bracketing, the agent's option value is maximized when $\alpha = 1$. When he sets incremental goals, then the optimal degree of reference dependence is given by η^*. When he sets an aggregate goal, the optimal degree of reference dependence is given by $\eta' > \eta^*$. Then the optimized value functions are given by

$$\begin{aligned}
E[v_1'^{agg}(x_1)|\tilde{x}_1] &= [x_1'^{agg} - I_1](\frac{\tilde{x}_1}{x_1'^{agg}})^{\gamma_1} + k^{\gamma_1}[x_2'^{agg} - I_2](\frac{\tilde{x}_1}{x_2'^{agg}})^{\gamma_1} \\
E[v_1^{*inc}(x_1)|\tilde{x}_1] &= [x_1^{*inc} - I_1](\frac{\tilde{x}_1}{x_1^{*inc}})^{\gamma_1} + k^{\gamma_1}[x_2^{*inc} - I_2](\frac{\tilde{x}_1}{x_2^{*inc}})^{\gamma_1}.
\end{aligned}$$

I have already shown that each term of $E[v_1^{*inc}(x_1)|\tilde{x}_1]$ is maximized when $\eta = \eta^*$. Since $\eta' > \eta^*$, then this implies that $E[v_1'^{agg}(x_1)|\tilde{x}_1] < E[v_1^{*inc}(x_1)|\tilde{x}_1]$, so the option value is globally maximized when the agent sets incremental goals, $\alpha = 1$, and $\eta = \eta^*$.

B.8 Proof of Proposition 19

If $k < \frac{I_2}{I_1}$, then the agent completes each stage sequentially regardless of how he brackets. So we need to compare the expectations of v_1^{agg} and v_1^{inc}.

Proof. **Existence**

I assume that $E[v_1^{agg}(x_1, r^{agg})|\tilde{x}_1] < E[v_1^{inc}(x_1, r_1^{inc})|\tilde{x}_1]$ when $\epsilon = 0$, and in particular that condition (2.37) is satisfied: $\frac{1}{2}\eta(\alpha+1) \leq \frac{\overline{\gamma}-\gamma_1}{\gamma_1}$. To demonstrate existence, I consider conditions such that $E[v_1^{agg}(x_1, r^{agg})|\tilde{x}_1] > E[v_1^{inc}(x_1, r_1^{inc})|\tilde{x}_1]$ when $\epsilon \to \infty$:

$$\lim_{\epsilon \to \infty} E[v_1^{inc}(x_1, r_1^{inc}) - v_1^{agg}(x_1, r^{agg})|\tilde{x}_1] = [\overline{x}_1^{inc}(1 - \frac{1}{2}\eta(\alpha - 1)) - I_1](\frac{\tilde{x}_1}{\overline{x}_1^{inc}})_1^{\gamma} - [\overline{x}_1^{agg} - I_1](\frac{\tilde{x}_1}{\overline{x}_1^{agg}})^{\gamma_1}. \tag{B.10}$$

Since $\overline{x}_1^{agg}(\epsilon = 0) < \overline{x}_1^{inc}(\epsilon = 0) < x_1^*(\epsilon = 0)$, and $\frac{\partial \overline{x}_1^{agg}}{\partial \epsilon} < 0$, then the second term is positive. Although we do not have an explicit expression for $\overline{x}_1^{agg}$, we know that it is bounded below: $\overline{x}_1^{agg} > (\frac{\overline{\gamma}}{\overline{\gamma}-1})I_1 \equiv \overline{x}_h$. Then the second term in Equation (B.10) is bounded below by

$$[\overline{x}_h - I_1](\frac{\tilde{x}_1}{\overline{x}_h})^{\gamma_1}.$$

Thus, any (η, α) such that

$$0 \geq [\overline{x}_1^{inc}(1 - \frac{1}{2}\eta(\alpha - 1)) - I_1](\frac{\tilde{x}_1}{\overline{x}_1^{inc}})_1^{\gamma} - [\overline{x}_h - I_1](\frac{\tilde{x}_1}{\overline{x}_h})^{\gamma_1}$$

will also satisfy the condition that Equation (B.10) is negative. Since the left-hand term is strictly decreasing in α and η, then Equation (B.10) will be negative for any (η, α) sufficiently high that this inequality is satisfied. This inequality can be reduced to the following:

$$M(\eta, \alpha) \equiv (\overline{\gamma} - 1)^{\gamma_1 - 1} - (1 + \eta)\left((\overline{\gamma} - 1)[1 - \frac{1}{2}\eta(\alpha - 1)] - \frac{1}{2}\eta(\alpha + 1)\right)^{\gamma_1 - 1} \geq 0,$$

where $M(0, 1) = 0$. Also, consider the maximum permissible combination(s) $(\overline{\eta}, \overline{\alpha})$, which satisfies

$$(\overline{\gamma} - 1)[1 - \frac{1}{2}\overline{\eta}(\overline{\alpha} - 1)(\frac{\epsilon}{1 + \epsilon})] - \frac{1}{2}\overline{\eta}(\overline{\alpha} + 1) = 0,$$

and where $\overline{\eta} \leq \overline{\gamma} - 1$. Evaluating M at such a point, we must have that $M(\overline{\eta}, \overline{\alpha}) > 0$ since $\epsilon \geq 0$. Furthermore, $\frac{\partial M}{\partial \alpha} > 0$ for $\eta > 0$. Differentiating M with respect to η, we have that M is increasing if

$$\overline{\gamma}(\alpha+1) - 2\gamma_1(1+\eta) - \gamma_1\overline{\gamma}(\alpha-1)(1+\eta) \leq 0, \tag{B.11}$$

and decreasing otherwise. Since the left-hand side of Equation (B.11) is decreasing in both η and α, we require (η, α) sufficiently large so that Equation (B.11) is satisfied in order for Equation (B.10) to be negative. Since $M(0,1) = 0$, $M(\overline{\eta}, \overline{\alpha}) > 0$, $\frac{\partial M}{\partial \alpha} > 0$ when $\eta > 0$, and M is initially decreasing (and therefore negative) before increasing thereafter in η, then there exist some combinations (η', α') such that $M(\eta', \alpha') = 0$ and $M(\eta, \alpha) > 0$ for all $\eta' \leq \eta \leq \overline{\eta}$ and $\alpha' \leq \alpha \leq \overline{\alpha}$, where $\eta' > 0$ and $\alpha' > 1$.

To verify that there exist values of (η, α) that satisfy both (2.37) and $M(\eta, \alpha) > 0$, let us consider whether (B.11) can be satisfied when (2.37) holds with equality. If not, then such values (η, α) do not exist and such a threshold ϵ does not exist. Suppose that α is sufficiently high (denoted $\hat{\alpha}$), given some η, that $\frac{1}{2}\eta(\hat{\alpha}+1) = \frac{\overline{\gamma}-\gamma_1}{\gamma_1}$. Does there exist some range of η such that $M(\eta, \alpha) > 0$ is still satisfied? When $\frac{1}{2}\eta(\hat{\alpha}+1) = \frac{\overline{\gamma}-\gamma_1}{\gamma_1}$, then $\frac{1}{2}\eta(\hat{\alpha}-1) = \frac{\overline{\gamma}-\gamma_1}{\gamma_1} - \eta$ and M becomes

$$M(\eta, \hat{\alpha}) = (\overline{\gamma}-1)^{\gamma_1 - 1} - (1+\eta)[(\overline{\gamma}-1)(1+\eta) - \overline{\gamma}(\frac{\overline{\gamma}-\gamma_1}{\gamma_1})]^{\gamma_1}. \tag{B.12}$$

Since the second term is strictly positive and less than $\overline{\gamma} - 1$ when $\eta = 0$, then $M(0, \hat{\alpha})$ is strictly positive whenever $\eta = 0$. We can also verify that $\frac{\partial M}{\partial \eta}|_{(0,\hat{\alpha})} < 0$ and $\frac{\partial M}{\partial \alpha} > 0$ for $\eta > 0$. Therefore, there exists some range of η such that $M(\eta, \hat{\alpha})$ is satisfied when (2.37) holds with equality. Thus, for (η, α) sufficiently large, there exists some threshold $\tilde{\epsilon}$ such that $E[v_1^{agg}(x_1, r^{agg})|\tilde{x}_1] < E[v_1^{inc}(x_1, r_1^{inc})|\tilde{x}_1]$ when $\epsilon < \tilde{\epsilon}$, and $E[v_1^{agg}(x_1, r^{agg})|\tilde{x}_1] > E[v_1^{inc}(x_1, r_1^{inc})|\tilde{x}_1]$ when $\epsilon > \tilde{\epsilon}$. In particular, (η, α) must satisfy both (2.37) and $M(\eta, \alpha) > 0$.

Uniqueness

I have shown that there exists an $\tilde{\epsilon}$ such that $E[v_1^{agg}(x_1, r^{agg}) - v_1^{inc}(x_1, r_1^{inc})|\tilde{x}_1] = 0$ if (η, α) is sufficiently large. Since we know that $E[v_1^{agg}(x_1, r^{agg}) - v_1^{inc}(x_1, r_1^{inc})|\tilde{x}_1] < 0$ when $\epsilon = 0$ and $E[v_1^{agg}(x_1, r^{agg}) - v_1^{inc}(x_1, r_1^{inc})|\tilde{x}_1] > 0$ when $\epsilon \to \infty$, it is sufficient to show that $\frac{\partial}{\partial \epsilon}(E[v_1^{agg}(x_1, r^{agg}) - v_1^{inc}(x_1, r_1^{inc})|\tilde{x}_1])$ is monotonic in ϵ. It must be true that at $\tilde{\epsilon}$, the following condition must hold:

$$[\overline{x}_1^{inc}(1 - \frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon})) - I_1](\frac{\tilde{x}_1}{\overline{x}_1^{inc}})^{\gamma_1} = [\overline{x}_1^{agg} - I_1](\frac{\tilde{x}_1}{\overline{x}_1^{agg}})^{\gamma_1}. \tag{B.13}$$

Since $\tilde{x}_1$ simply carries through, I suppress the $\tilde{x}_1$ term in the following derivations.

Direct differentiation of $E[v_1^{inc}|\tilde{x}_1]$ yields

$$\begin{aligned}
\frac{\partial}{\partial\epsilon}(E[v_1^{inc}|\tilde{x}_1]) =&(\frac{1}{\overline{x}_1^{inc}})^{\gamma_1+1}[-(\overline{x}_1^{inc})^2(\frac{1}{2}\eta(\alpha-1))(\frac{1}{(1+\epsilon)^2})\\
&+\frac{\partial\overline{x}_1^{inc}}{\partial\epsilon}[-(\gamma_1-1)\overline{x}_1^{inc}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))+\gamma_1 I_1]]\\
=&-(\frac{1}{\overline{x}_1^{inc}})^{\gamma_1}(\frac{1}{2}\eta(\alpha-1))[\frac{1}{(\overline{\gamma}-1)(1+\frac{1}{2}\eta(\frac{2\epsilon+\alpha+1}{1+\epsilon}))-\frac{\overline{\gamma}}{2}\eta(\alpha+1)}]^2\\
&(\frac{I_1}{(1+\epsilon)^2})[\overline{\gamma}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))+(1+\frac{1}{2}\eta(\frac{\alpha+2\epsilon+1}{1+\epsilon}))]\\
=&-(\frac{1}{\overline{x}_1^{inc}})^{\gamma_1-1}(\frac{1}{2}\eta(\alpha-1))[\frac{1}{(\overline{\gamma}-1)(1+\frac{1}{2}\eta(\frac{2\epsilon+\alpha+1}{1+\epsilon}))-\frac{\overline{\gamma}}{2}\eta(\alpha+1)}]\\
&(\frac{1}{\overline{\gamma}(1+\epsilon)^2})[\overline{\gamma}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))+(1+\frac{1}{2}\eta(\frac{\alpha+2\epsilon+1}{1+\epsilon}))]
\end{aligned}$$

Differentiation of $E[v_1^{agg}|\tilde{x}_1]$ yields

$$\begin{aligned}
\frac{\partial}{\partial\epsilon}(E[v_1^{agg}|\tilde{x}_1]) &= (\frac{1}{\overline{x}_1^{agg}})^{\gamma_1+1}(\frac{\partial\overline{x}_1^{agg}}{\partial\epsilon})[\overline{x}_1^{agg}-\gamma_1(\overline{x}_1^{agg}-I_1)]\\
&= (\frac{\partial\overline{x}_1^{agg}}{\partial\epsilon})[(\frac{1}{\overline{x}_1^{agg}})^{\gamma_1}-\gamma_1(\frac{1}{\overline{x}_1^{agg}})^{\gamma_1+1}(\overline{x}_1^{agg}-I_1)],
\end{aligned}$$

where $\frac{\partial\overline{x}_1^{agg}}{\partial\epsilon}<0$ and the last term is positive, since $\overline{x}_1^{agg}<(\frac{\gamma_1}{\gamma_1-1})I_1=x_1^*(\epsilon=0)$. Using Equation (B.13), we can rewrite this as

$$\begin{aligned}
\frac{\partial}{\partial\epsilon}(E[v_1^{agg}|\tilde{x}_1]) =&(\frac{\partial\overline{x}_1^{agg}}{\partial\epsilon})(\frac{1}{\overline{x}_1^{inc}})^{\gamma_1}[\overline{x}_1^{inc}(1-\frac{1}{2}\eta(\alpha-1)(\frac{\epsilon}{1+\epsilon}))-I_1]\\
&[\frac{\overline{x}_1^{agg}-\gamma_1(\overline{x}_1^{agg}-I_1)}{\overline{x}_1^{agg}(\overline{x}_1^{agg}-I_1)}],
\end{aligned}$$

For brevity, let us define

$$H(\epsilon)=\frac{\frac{1}{2}\eta(\alpha+1)[\beta k^{\gamma_1}(\frac{\gamma_1-1}{\overline{\gamma}-1})\gamma_1(\frac{\overline{x}_1^{agg}}{\overline{x}_2^{agg}})^{\gamma_1+1}+(1-\beta)k^{\gamma_2}(\frac{\gamma_2-1}{\overline{\gamma}-1})\gamma_2(\frac{\overline{x}_1^{agg}}{\overline{x}_2^{agg}})^{\gamma_2+1}]}{\overline{\gamma}-1-\frac{1}{2}\eta(\alpha+1)[\beta k^{\gamma_1}(\frac{\gamma_1-1}{\overline{\gamma}-1})(\gamma_1+1)(\frac{\overline{x}_1^{agg}}{\overline{x}_2^{agg}})^{\gamma_1}+(1-\beta)k^{\gamma_2}(\frac{\gamma_2-1}{\overline{\gamma}-1})(\gamma_2+1)(\frac{\overline{x}_1^{agg}}{\overline{x}_2^{agg}})^{\gamma_2}]}.$$

We can write out $\frac{\partial\overline{x}_1^{agg}}{\partial\epsilon}$ as

$$\frac{\partial\overline{x}_1^{agg}}{\partial\epsilon}=-(\frac{\partial\overline{x}_2^{agg}}{\partial\epsilon})H(\epsilon),$$

where the last term must be positive because we have already shown that $\frac{\partial\overline{x}_1^{agg}}{\partial\epsilon}<0$.

Since $\overline{x}_2^{agg} = \overline{x}_2^{inc}$, then $\frac{\partial \overline{x}_1^{agg}}{\partial \epsilon} = -(\frac{\partial \overline{x}_1^{inc}}{\partial \epsilon})(\frac{I_2}{I_1})H(\epsilon)$. Then we have

$$\begin{aligned}
\frac{\partial}{\partial \epsilon}(E[v_1^{agg}|\tilde{x}_1]) =& -(\frac{\partial \overline{x}_1^{inc}}{\partial \epsilon})(\frac{I_2}{I_1})H(\epsilon)(\frac{1}{\overline{x}_1^{inc}})(\frac{1}{\overline{x}_1^{inc}})^{\gamma_1 - 1}[\overline{x}_1^{inc}(1 - \frac{1}{2}\eta(\alpha - 1)(\frac{\epsilon}{1+\epsilon})) - I_1] \\
& [\frac{\overline{x}_1^{agg} - \gamma_1(\overline{x}_1^{agg} - I_1)}{\overline{x}_1^{agg}(\overline{x}_1^{agg} - I_1)}] \\
=& -(\overline{\gamma} - 1)\frac{1}{2}\eta(\alpha - 1)(\frac{1}{1+\epsilon})^2[\frac{1}{(\overline{\gamma} - 1)(1 + \frac{1}{2}\eta(\frac{2\epsilon + \alpha + 1}{1+\epsilon})) - \frac{\overline{\gamma}}{2}\eta(\alpha + 1)}] \\
& (\frac{I_2}{I_1})H(\epsilon)(\frac{1}{\overline{x}_1^{inc}})^{\gamma_1 - 1}[\overline{x}_1^{inc}(1 - \frac{1}{2}\eta(\alpha - 1)(\frac{\epsilon}{1+\epsilon})) - I_1] \\
& [\frac{\overline{x}_1^{agg} - \gamma_1(\overline{x}_1^{agg} - I_1)}{\overline{x}_1^{agg}(\overline{x}_1^{agg} - I_1)}].
\end{aligned}$$

Thus,

$$\frac{\partial v_1^{inc}}{\partial \epsilon} - \frac{\partial v_1^{agg}}{\partial \epsilon} = A(\epsilon)[B(\epsilon) + (\frac{I_2}{I_1})(\overline{\gamma} - 1)H(\epsilon)C(\epsilon)D(\epsilon)],$$

where

$$\begin{aligned}
A(\epsilon) &= -(\frac{1}{\overline{x}_1^{inc}})^{\gamma_1 - 1}(\frac{1}{2}\eta(\alpha - 1))(\frac{1}{1+\epsilon})^2[\frac{1}{(\overline{\gamma} - 1)(1 + \frac{1}{2}\eta(\frac{2\epsilon + \alpha + 1}{1+\epsilon})) - \frac{\overline{\gamma}}{2}\eta(\alpha + 1)}] \\
B(\epsilon) &= \overline{\gamma}(1 - \frac{1}{2}\eta(\alpha - 1)(\frac{\epsilon}{1+\epsilon})) + (1 + \frac{1}{2}\eta(\frac{\alpha + 2\epsilon + 1}{1+\epsilon})) \\
C(\epsilon) &= \overline{x}_1^{inc}(1 - \frac{1}{2}\eta(\alpha - 1)(\frac{\epsilon}{1+\epsilon})) - I_1 \\
D(\epsilon) &= -\frac{\overline{x}_1^{agg} - \gamma_1(\overline{x}_1^{agg} - I_1)}{\overline{x}_1^{agg}(\overline{x}_1^{agg} - I_1)}
\end{aligned}$$

Since $A(\epsilon)$ is always negative, we need only consider how the second term, $[B(\epsilon) + (\frac{I_2}{I_1})(\overline{\gamma} - 1)H(\epsilon)C(\epsilon)D(\epsilon)]$, varies with ϵ. By direct differentiation, $B(\epsilon)$, $H(\epsilon)$ and $D(\epsilon)$ are monotonically decreasing in ϵ. Although $C(\epsilon)$ is monotonically increasing in ϵ, its rate of increase is not sufficient to counteract $H(\epsilon)$ and $D(\epsilon)$, and $H(\epsilon)C(\epsilon)D(\epsilon)$ is also monotonically decreasing in ϵ. Since $\frac{\partial}{\partial \epsilon}(v_1^{inc} - v_1^{agg})$ is monotonic in ϵ, then any threshold $\tilde{\epsilon}$ such that $E[v_1^{agg}(x_1, r^{agg})|\tilde{x}_1] < E[v_1^{inc}(x_1, r_1^{inc})|\tilde{x}_1]$ when $\epsilon < \tilde{\epsilon}$, and $E[v_1^{agg}(x_1, r^{agg})|\tilde{x}_1] > E[v_1^{inc}(x_1, r_1^{inc})|\tilde{x}_1]$ when $\epsilon > \tilde{\epsilon}$ is unique. □

Appendix C

Appendix for Chapter 3

C.1 Imperfect Correlation

Suppose that the correlation among agents' tastes is not necessarily perfect. I will show that noisier information (i.e., increasing $\rho_{jk}(1-\rho_{jk})$) has the same qualitative effect as decreasing parameter α.

For example, suppose that $\rho_{13} \in \{\rho, 1-\rho\}$, where $0 \leq \rho \leq 1$. A priori, $P(\rho_{13} = \rho) = P(\rho_{13} = 1-\rho) = \frac{1}{2}$. If a (θ_1, θ_2) agent i observes the tastes of a (θ_1, θ_3) agent j such that $\theta_3^j = \theta_1^j = \theta_1^i$, then his posterior is that $P(\rho_{13} = \rho) = \rho$. Agent i makes the same inference if $\mu_1 \leq \frac{1}{2}$ and he only observes the actions of agent j, since he knows that everyone in the pool is like-minded. Suppose that $\mu \leq \frac{1}{2}$. Then agent i chooses x_3 to solve the following problem,

$$\max_{x_3} -\rho[\rho(\theta_1 - x_3)^2 + (1-\rho)(\theta_1 - \frac{1}{2} - x_3)^2] - (1-\rho)[(1-\rho)(\theta_1 - x_3)^2 + \rho(\theta_1 - \frac{1}{2} - x_3)^2],$$

for which the solution is $x_3 = \theta_1 - \rho(1-\rho)$ and the payoff is $b_1 = -\rho(1-\rho)(\frac{1}{2} - \rho(1-\rho))$. Therefore, his expected benefit from pooling is

$$\begin{aligned} &(1 - \frac{\alpha\mu_1}{2})(-\frac{1}{16}) + \frac{\alpha\mu}{2}(-\rho(1-\rho)(\frac{1}{2} - \rho(1-\rho))) \\ &= -\frac{1}{16} + \frac{\alpha\mu}{2}[\frac{1}{16} - \rho(1-\rho)(\frac{1}{2} - \rho(1-\rho))]. \end{aligned}$$

Clearly, as $\rho \to \frac{1}{2}$ from the right or the left (or equivalently, as $\rho(1-\rho)$ increases), the second term decreases. This is the same qualitative effect as decreasing α.

Suppose that $\mu > \frac{1}{2}$. Then there is the possibility that agent i will only observe the actions of an "unlike" type. Because he cannot distinguish a "like" from an "unlike" type based on actions, then his posterior $P(\rho_{13} = \rho) = \frac{1}{2\mu_1}\rho + (1 - \frac{1}{2\mu_1})(1-\rho)$. Solving the problem

$$\max_{x_3} -[\frac{1}{2\mu_1}\rho + (1-\frac{1}{2\mu_1})(1-\rho)][\rho(\theta_1 - x_3)^2 + (1-\rho)(\theta_1 - \frac{1}{2} - x_3)^2]$$
$$-[(1-\frac{1}{2\mu_1})\rho + \frac{1}{2\mu_1}(1-\rho)][(1-\rho)(\theta_1 - x_3)^2 + \rho(\theta_1 - \frac{1}{2} - x_3)^2],$$

he chooses $x_3 = \theta_1 - \frac{1}{4\mu_1}[4(1-\mu)\rho(1-\rho) + 2\mu_1 - 1]$, which yields a payoff of $b_2 = \frac{1}{16\mu_1^2}[1 - 4(1-\mu_1)\rho(1-\rho)][4(1-\mu_1)\rho(1-\rho) + 2\mu_1 - 1]$. Hence, his expected benefit from pooling is

$$(1 - \frac{\alpha\mu_1}{2})(-\frac{1}{16}) + \frac{\alpha\mu_1}{2}[(\mu_1)(b_1) + (1-\mu_1)(b_2)]$$
$$= -\frac{1}{16} + \frac{\alpha}{32}(1-\mu_1)(2\rho-1)^2(4\mu_1\rho^2 - 4\mu_1\rho + 1).$$

As $\rho \to \frac{1}{2}$ from the right or the left (or equivalently, as $\rho(1-\rho)$ increases), the second term decreases. This is the same qualitative effect as decreasing α. Hence, increasing the noisiness of the information is equivalent to decreasing α.

C.2 Stability of Pooling Equilibria

A stable solution for pooling at any given x_1 must be robust to slight perturbations. In particular, we can convert our static equilibrium into a dynamic system to check the stability of our equilibria, where location x_1 is taken as given:

$$\dot{\overline{\theta}}_1(x_1) = x_1 + \sqrt{\alpha(\frac{1}{32})(\overline{\theta}_1 - \underline{\theta}_1)} - \overline{\theta}_1 \quad (C.1)$$

$$\dot{\underline{\theta}}_1(x_1) = x_1 - \sqrt{\alpha(\frac{1}{32})(\overline{\theta}_1 - \underline{\theta}_1)} - \underline{\theta}_1. \quad (C.2)$$

The Jacobian for this system is

$$J = \begin{pmatrix} \frac{\alpha}{2}(\frac{1}{32})[\alpha(\frac{1}{32})(\overline{\theta}_1 - \underline{\theta}_1)]^{-\frac{1}{2}} - 1 & -\frac{\alpha}{2}(\frac{1}{32})[\alpha(\frac{1}{32})(\overline{\theta}_1 - \underline{\theta}_1)]^{-\frac{1}{2}} \\ -\frac{\alpha}{2}(\frac{1}{32})[\alpha(\frac{1}{32})(\overline{\theta}_1 - \underline{\theta}_1)]^{-\frac{1}{2}} & \frac{\alpha}{2}(\frac{1}{32})[\alpha(\frac{1}{32})(\overline{\theta}_1 - \underline{\theta}_1)]^{-\frac{1}{2}} - 1 \end{pmatrix}$$

Evaluating the Jacobian at the pooling solution from (3.4) and (3.5), I obtain eigenvalues that are strictly negative, so the solution is stable. Evaluating the Jacobian at the separating solution $\underline{\theta}_1 = x_1 = \overline{\theta}_2$, I obtain one eigenvalue that is strictly positive and another that is strictly negative, so the solution is unstable.

C.3 Proof of Proposition 20

I eliminate all other pooling equilibria by proving a series of claims to rule out various classes of equilibria, and then consider the remaining type of pooling configuration.

Claim 1. *In a given action space, the point at which an interval of agents pool must be lie inside that interval. No pool can be composed of disjoint intervals, and no pools overlap.*

Proof. Without loss of generality, consider an interval of agents $[\underline{\theta}_1, \overline{\theta}_1]$, where $0 < \underline{\theta}_1 < \overline{\theta}_1 \leq 1$, that pool at some x_1. Suppose x_1 lies outside this interval, e.g. $0 < x_1 < \underline{\theta}_1$. Then there exists some $\theta_1' \in [x_1, \underline{\theta}_1)$ such that the θ_1' agent prefers not to join the pool, even though his travel costs to pool are strictly lower than that of the $\underline{\theta}_1$ agent and the information benefits are the same. This cannot hold simultaneously in equilibrium. Thus, an interval of agents $[\underline{\theta}_1, \overline{\theta}_1]$ must be pooling at an $x_1 \in [\underline{\theta}_1, \overline{\theta}_1]$.

Suppose that the set of agents that pool at x_1 is the union of two disjoint intervals, $[\underline{\theta}_1, \overline{\theta}_1] \cup [\underline{\theta}_1', \overline{\theta}_1']$, where $0 \leq \underline{\theta}_1 < \overline{\theta}_1 < \underline{\theta}_1' < \overline{\theta}_1' < 1$. Without loss of generality, suppose $x_1 \in [\underline{\theta}_1, \overline{\theta}_1]$. Since x_1 lies outside $[\underline{\theta}_1', \overline{\theta}_1']$, then we can apply the same argument as above to rule out this possibility.

Suppose that two pools overlap. For example, let agents in the interval $[\underline{\theta}_1, \overline{\theta}_1]$ pool at $x_1 \in [\underline{\theta}_1, \overline{\theta}_1]$ and agents in the interval $[\underline{\theta}_1', \overline{\theta}_1']$ pool at $x_1' \in [\underline{\theta}_1', \overline{\theta}_1']$, where $x_1 < \underline{\theta}_1' < \overline{\theta}_1 < x_1'$. Any agent whose type lies the interval $(\underline{\theta}_1', \overline{\theta}_1')$ would incur strictly lower travel costs to pool at x_1' and enjoy the same benefits as the $\underline{\theta}_1'$ type, but prefers to pool at x_1 rather than x_1', which is not possible. Thus, no pools can overlap. □

Second, no pool exists such that $\mu_j(x_j) > \frac{1}{2}$.

Claim 2. *When $g(\theta_j - x_j) = -(\theta_j - x_j)^2$, $\mu_j \leq \frac{1}{2}$ when $\alpha\mu_j \leq 1$.*

Proof. Obviously, there exists no pool such that $\mu_1 = 1$, since this would be completely uninformative yet costly. If $1 > \mu_1 > \frac{1}{2}$, then we have to worry about signal dilution, since an agent cannot be certain that the signal he receives will give him correct information about his preferences. If $\mu_1 > \frac{1}{2}$, then the probability that the signal $\boldsymbol{\theta}^m$ is informative is 1, so the agent learns his own θ_3 with certainty. But the signal $\mathbf{x}^m$ is not guaranteed to be correct, so the agent's posterior probability that his true taste is θ_3 rather than $\theta_3 + \frac{1}{2}$ is $\frac{1}{4\mu_1}$. In the event that the agent receives the signal $\mathbf{x}^m$, he chooses x_3 to solve the following problem

$$\max_{x_3} \; -\frac{1}{2\mu_1}(\theta_1 - x_3)^2 - (1 - \frac{1}{2\mu_1})(\theta_1 + \frac{1}{2} - x_3)^2,$$

to which the solution is $x_3 = \theta_1 + \frac{1}{2}(1 - \frac{1}{2\mu_1})$. Then pooling surplus relative to not pooling is

$$-(\theta_1 - x_1)^2 - \{(1 - \frac{\eta_1}{2})(\frac{1}{4})^2 + \frac{\eta_1}{2}[\mu_1[(\frac{\mu_1 - \frac{1}{2}}{\mu_1})(\frac{1}{4\mu_1})^2 + \frac{1}{2\mu_1}(\frac{1}{2}(\frac{\mu_1 - \frac{1}{2}}{\mu_1}))^2] + (1 - \mu_1)(0)]\} + (\frac{1}{4})^2.$$

As before, I require that $\eta_1 = \alpha\mu_1 \leq 1$. Pooling surplus must be non-negative for all agents in the pool in equilibrium, which reduces to the following condition:

$$(\theta_1 - x_1)^2 \leq \frac{\alpha}{32}(1 - \mu_1). \tag{C.3}$$

When $\mu_1 > \frac{1}{2}$, then there exists some θ_1 in the pool such that $|x_1 - \theta_1| = \frac{1}{4}$ and surplus is non-negative. From (C.3), this implies that for such an agent,

$$\frac{\alpha}{32}(1 - \mu_1) > (\theta_1 - x_1)^2 = \frac{1}{16}.$$

Since $\mu_1 > \frac{1}{2}$ and $\alpha\mu_1 \leq 1$, then this requires that $\mu_1 < 1 - \mu_1$, which cannot be true when $\mu_1 > \frac{1}{2}$. Hence, $\mu_1 \geq \frac{1}{2}$ when $g(\cdot)$ is a symmetric quadratic loss function with $\alpha\mu_1 \geq 1$. □

Third, the composition of a pool is always "symmetric-by-types" in the following sense.

Claim 3. *For any pool at a given point in an action space X_j, agents of differing information types pool symmetrically. For example, suppose that $\overline{\theta}_1^a(x_1)$ and $\underline{\theta}_1^a(x_1)$ are the marginal agents with information (θ_1, θ_2) who pool at x_1, and $\overline{\theta}_1^b(x_1)$ and $\underline{\theta}_1^b(x_1)$ are the marginal agents with information (θ_1, θ_3) who pool at x_1. Then $\overline{\theta}_1^a(x_1) = \overline{\theta}_1^b(x_1)$ and $\underline{\theta}_1^a(x_1) = \underline{\theta}_1^b(x_1)$.*

Proof. In order to prove this claim, I will show that $\mu_1^a(x_1) = \mu_1^b(x_1)$. Due to the symmetry of the circular model, there is clearly a unique solution $(\underline{\theta}_j(x_j), \overline{\theta}_j(x_j))$ for any $\mu_j(x_j)$, so showing that $\mu_1^a(x_1) = \mu_1^b(x_1)$ is sufficient to establish the claim.

Suppose that $\mu_1^a(x_1) \neq \mu_1^b(x_1)$. Without loss of generality, let $\mu_1^a(x_1) < \mu_1^b(x_1)$. This implies that for the same location, there are (θ_1, θ_3) agents who are willing to incur a larger cost to pool for a relatively lower expected benefit from information, while there are (θ_1, θ_2) agents who are unwilling to incur a smaller cost to pool for a relatively higher expected benefit from information. Given that (θ_1, θ_2) and (θ_1, θ_3) agents face an identical optimization problem, this cannot hold in equilibrium. Therefore, $\mu_1^a(x_1) = \mu_1^b(x_1)$ at x_1. □

The following claim demonstrates the n-integer problem.

Claim 4. *Let n be the number of pools that exist in the action space X_1. Suppose that the parameter α is a value such that n is not an integer when it satisfies the condition $n(\frac{\alpha}{8}) = 1$. Then there does not exist a stable equilibrium in which all pools have size $\frac{\alpha}{8}$ except the remaining space, where either the remaining agents do not pool anywhere or they form a pool of size less than $\frac{\alpha}{8}$.*

Proof. Suppose that there exists some open interval of length $l < \frac{\alpha}{8}$ on X_1 such that any agent with $\theta_1 \in l$ does not pool anywhere. Let agents at the endpoints of interval l be denoted $\underline{\theta}_l$ and $\overline{\theta}_l$. Because the separating equilibrium is not stable, this interval of non-pooling agents is not robust to small perturbations. Without loss of generality, suppose that the agents in this interval pool at some $x_1^l \in l$. Since $l < \frac{\alpha}{8}$, the requirement that the information benefits of pooling outweigh the costs is clearly satisfied, that is the inequality in Equation (3.1) is satisfied for all agents in the interval l. Furthermore, Equation (3.1) must hold with inequality for both for the marginal agents $\underline{\theta}_l$ and $\overline{\theta}_l$, regardless of whether the pool at x_1^l is symmetric on either side of the point x_1^l. Let x_1 be the center of a symmetric pool of size $\frac{\alpha}{8}$ adjacent to x_1^l, such that $x_1 < \underline{\theta}_l < x_1^l$. Then the $\underline{\theta}_l$ agent is also the marginal agent for the pool at the point x_1. By construction, Equation (3.1) must hold with equality for the $\underline{\theta}_l$ agent with respect to pooling at x_1 rather than not pooling, so there is zero surplus from pooling at x_1. Likewise, this situation applies to the $\overline{\theta}_l$ agent. But since we have just argued that Equation (3.1) holds with inequality for this agent with respect to pooling at x_1^l, then he cannot be the marginal agent for both pools, since pooling at the point x_1^l yields strictly positive surplus. Hence, the proposed configuration unravels and cannot exist. □

Furthermore, there can exist equilibria in which adjacent symmetric pools are of alternating size, where the distance between any two pooling points is exactly $\frac{\alpha}{16}$.

Claim 5. *There can exist an equilibrium in which any two adjacent pooling points x_j and x_j', with pools of size $\mu_j(x_j)$ and $\mu_j(x_j')$ respectively, are symmetric and either $|x_j - x_j'| = \frac{\alpha}{16}$ or the two adjacent pools are identical in size. Furthermore, this is the only configuration for which more than one size of symmetric pool can coexist in equilibrium.*

Proof. Let x_1 and x_1' be two adjacent pooling points on the X_1 space with pool sizes $\mu_1(x_1)$ and $\mu_1(x_1')$ respectively, and let the marginal agent between the two points be denoted $\overline{\theta}'$, so that $x_1 < \overline{\theta}' < x_1'$. Let $d_1 = |\overline{\theta}' - x_1|$ and $d_1' = |\overline{\theta}' - x_1'|$. Without loss of generality, let $d_1 \geq d_1'$. Since each pool is symmetric across its pooling point, then $\mu_1(x_1) = 2d_1$ and $\mu_1(x_1') = 2d_1'$. Combining this with Equation (3.1), the marginal $\overline{\theta}'$ agent must be indifferent between pooling at x_1 and x_1':

$$\begin{aligned}\frac{\alpha\mu_1}{32} - (\overline{\theta}' - x_1)^2 &= \frac{\alpha\mu_1'}{32} - (\overline{\theta}' - x_1')^2 \\ \frac{\alpha}{16}(2d_1) - (d_1)^2 &= \frac{\alpha}{16}(2d_1') - (d_1')^2 \\ (d_1 - d_1')[d_1 + d_1' - \frac{\alpha}{16}] &= 0.\end{aligned}$$

Therefore, the marginal agent is only indifferent if $d_1 = d_1'$ or $d_1 + d_1' = \frac{\alpha}{16}$. This implies that two adjacent symmetric pools of unequal size can only exist if their pooling points are spaced exactly $\frac{\alpha}{16}$ apart; otherwise, the adjacent pools must be identical in size. Because this condition must hold for *every* marginal agent in the X_1 space, this means that exactly two differing pool sizes can coexist in an action space X_j, and they are bounded above by $\frac{\alpha}{16}$. □

C.4 Proof of Proposition 22

Because $\eta_n = \min\{\alpha_n \mu_n(x_n), 1\}$, firm 1 will never invest in $\alpha_1 > \overline{\alpha}_1$ where $\overline{\alpha}_1(\mu_1^{EI}) = 1$. Since $\alpha_2^* = 0$ in equilibrium, I obtain an upper bound on equilibrium brand strength, $\overline{\alpha}_1$:

$$\overline{\alpha}_1(\mu_1^{EI}) = 1$$

$$\overline{\alpha}_1 \left(\frac{2}{3}(\frac{\overline{\alpha}_1 + 12}{\overline{\alpha}_1 + 16}) \right) = 1$$

$$\overline{\alpha}_1 = -\frac{21}{4} + \frac{5}{4}\sqrt{33} \ (< 2).$$

Because Π_1^{EI} is concave for $\alpha_1 \in (0, \overline{\alpha}_1]$, there may exist $0 < \underline{c}_\alpha < \overline{c}_\alpha$ such that the optimal choice is $\alpha_1 = \overline{\alpha}_1$ if $k \leq \underline{c}_\alpha$ and $\alpha_1 = 0$ if $c_\alpha > \overline{c}_\alpha$. Then for $\underline{c}_\alpha < c_\alpha < \overline{c}_\alpha$, there is a unique interior solution $\alpha_1^*(c_\alpha) \in (0, \overline{\alpha}_1)$, where $\alpha_1^*(c_\alpha)$ is the value of α_1 such that firm 1's first order condition equals zero, given that $\alpha_2^* = 0$. The first order condition for firm 1 is

$$\frac{\partial \Pi_1^{EI}}{\partial \alpha_1} = \frac{1}{72}[\frac{(\alpha_1 - \alpha_2 + 12)(\alpha_1 - \alpha_2 + 20)}{(\alpha_1 - \alpha_2 + 16)^2}] - 2k\alpha_1.$$

The lower bound $\underline{c}_\alpha$ is the maximum c_α such that the first order condition is positive for all $\alpha_1 \in [0, \overline{\alpha}_1]$. This holds for all c_α such that

$$c_\alpha < \frac{1}{144}[\frac{(\alpha_1 - \alpha_2 + 12)(\alpha_1 - \alpha_2 + 20)}{\alpha_1(\alpha_1 - \alpha_2 + 16)^2}].$$

Since $\alpha_2^* = 0$ in equilibrium and the right-hand side is strictly decreasing in α_1, I evaluate the right-hand side of this expression at $\alpha_1 = \overline{\alpha}_1, \alpha_2 = 0$ to obtain $\underline{c}_\alpha$:

$$\underline{c}_\alpha = \frac{8401}{4718592} + \frac{1345}{4718592}(\sqrt{33}).$$

The upper bound $\overline{c}_\alpha$ is the minimum c_α such that the first order condition is negative for all $\alpha_1 \in [0, \overline{\alpha}_1]$. This holds for all c_α such that

$$c_\alpha > \frac{1}{144}[\frac{(\alpha_1 - \alpha_2 + 12)(\alpha_1 - \alpha_2 + 20)}{\alpha_1(\alpha_1 - \alpha_2 + 16)^2}].$$

But since the right-hand side is unbounded as $\alpha_1 \to 0$, then this $\overline{c}_\alpha$ does not exist. That is, no matter how finitely large c_α is, there always exists an interior solution $\alpha_1 > 0$ (though as c_α increases, the optimal α_1 asymptotically approaches zero). Thus, firm 1 chooses $\alpha_1^* = \overline{\alpha}_1$ if $c_\alpha \leq \underline{c}_\alpha$ and a unique $\alpha_1^*(c_\alpha) \in (0, \overline{\alpha}_1)$ if $c_\alpha > \underline{c}_\alpha$, where $\alpha_1^*(c_\alpha)$ strictly decreases with c_α in this region.

C.5 Endogenizing Location and Brand Investment

In the previous section, I assumed that firms' locations were equidistant, so that $x_1 - x_2 = \frac{1}{2}$. Here, I will demonstrate that the configuration of locations and technologies that exhibits maximal differentiation in both of those dimensions is the unique equilibrium when location is also endogenous. The game becomes a three-stage game in which firms simultaneously choose locations, then brand strengths, and lastly prices.

C.5.1 Timeline

1. Firms simultaneously choose a variety x_n (i.e., location), where $x_n \in [0,1]$ for $n = 1,2$.

2. Firms simultaneously choose α_n, where $n = 1,2$ and they incur investment costs $c_\alpha \alpha_n^2 \geq 0$ where $c_\alpha \geq 0$.

3. Firms simultaneously choose prices $p_n,\ n = 1,2$.

4. Consumers choose whether or not to buy x_1 or x_2. Note that before making a decision, consumers know whether a firm has a branded product or not.

5. (Consumers choose to buy goods in other markets/dimensions.)

In reference to the previous graph, we no longer assume that $\overline{d}_1 = \underline{d}_1$ and $\overline{d}_1 = \underline{d}_1$.

C.5.2 Demand

Without loss of generality, suppose that $\alpha_1 \geq \alpha_2$ and let $d = x_1 - x_2$, where $d \in [0,1]$. The marginal consumers' indifference conditions can be described by the following equations:

$$p_1 + (\overline{\theta}_1 - x_1)^2 - \frac{\alpha_1}{16}(1-\mu_1) = p_2 + (1 + x_2 - \overline{\theta}_1)^2 - \frac{\alpha_2}{16}(\mu_2) \qquad \text{(C.4)}$$

$$p_1 + (x_1 - \underline{\theta}_1)^2 - \frac{\alpha_1}{16}(1-\mu_1) = p_2 + (\underline{\theta}_1 - x_2)^2 - \frac{\alpha_2}{16}(\mu_2), \qquad \text{(C.5)}$$

where $\mu_1 = \overline{\theta}_1 - \underline{\theta}_1$ and $\mu_1 + \mu_2 = 1$. Then we can obtain firm 1's demand as

$$\mu_1 = \frac{\alpha_1 - \alpha_2 + 32(p_2 - p_1) + 32d(1-d)}{\alpha_1 - \alpha_2 + 64d(1-d)}. \qquad \text{(C.6)}$$

C.5.3 Prices

Given demand, firm 1 chooses price to maximize profit, taking p_2 as given:

$$\max_{p_1}(p_1 - c)\left(\frac{\alpha_1 - \alpha_2 + 32(p_2 - p_1) + 32d(1-d)}{\alpha_1 - \alpha_2 + 64d(1-d)}\right) - k\alpha_1^2.$$

Verifying the second order condition, we obtain

$$p_1 = \frac{1}{2}[p_2 + c + d(1-d) + \frac{1}{32}(\alpha_1 - \alpha_2)].$$

Likewise, firm 2 maximizes profit, where $\mu_2 = 1 - \mu_1$, to obtain

$$p_2 = \frac{1}{2}[p_1 + c + d(1-d)].$$

Hence, the Nash equilibrium in prices is $p_1 = c + d(1-d) + \frac{1}{48}(\alpha_1 - \alpha_2)$ and $p_2 = c + d(1-d) + \frac{1}{96}(\alpha_1 - \alpha_2)$. Firms' market shares are

$$\mu_1 = \frac{2}{3}(\frac{\alpha_1 - \alpha_2 + 48d(1-d)}{\alpha_1 - \alpha_2 + 64d(1-d)})$$
$$\mu_2 = \frac{1}{3}(\frac{\alpha_1 - \alpha_2 + 96d(1-d)}{\alpha_1 - \alpha_2 + 64d(1-d)}).$$

C.5.4 Brand Choice

Given equilibrium prices and demand, firms' total profits are as follows:

$$\Pi_1 = \frac{1}{72}[\frac{(\alpha_1 - \alpha_2 + 48d(1-d))^2}{\alpha_1 - \alpha_2 + 64d(1-d)}] - c_\alpha \alpha_1^2$$
$$\Pi_2 = \frac{1}{288}[\frac{(\alpha_1 - \alpha_2 + 96d(1-d))^2}{\alpha_1 - \alpha_2 + 64d(1-d)}] - c_\alpha \alpha_2^2.$$

Firm 2

When $\alpha_1 > \alpha_2$, it is clear that the first terms of both Π_1^{EI} and Π_2^{EI} are strictly increasing in α_1 and strictly decreasing in α_2. Given that increasing α_2 is also costly (and even if it is costless!), firm 2 optimizes by choosing $\alpha_2^* = 0$.

We can see why firm 2 makes this choice by decomposing its profit. Note that

$$\frac{d\Pi_2}{d\alpha_2} = (p_2 - c)(\frac{\partial \mu_2}{\partial \alpha_2} + \frac{\partial \mu_2}{\partial p_1}\frac{dp_1}{d\alpha_2}) - 2c_\alpha \alpha_2.$$

Using our equilibrium demand and prices, we obtain

$$\frac{\partial \mu_2}{\partial \alpha_2} = \frac{1}{3}[\frac{\alpha_1 - \alpha_2 + 96d(1-d)}{(\alpha_1 - \alpha_2 + 64d(1-d))^2}] > 0 \tag{C.7}$$

and

$$\frac{\partial \mu_2}{\partial p_1}\frac{dp_1}{d\alpha_2} = -\frac{2}{3}[\frac{1}{\alpha_1 - \alpha_2 + 64d(1-d)}] < 0. \tag{C.8}$$

The first term is the demand effect, where increasing α_2 directly increases firm 2's demand by increasing its expected informational benefits. The second term is the strategic effect, where increasing α_2 indirectly decreases firm 2's demand by causing its competitor to lower its price. When α_2 increases, then firm 1's brand advantage diminishes, so demand is relatively more responsive to prices and price competition intensifies. Summing equations (C.7) and (C.8), we find that the strategic effect dominates, so $\frac{d\Pi_2^{EI}}{d\alpha_2} < 0$. The incentive to weaken price competition is stronger than the incentive to increase demand. Likewise, the relaxation of price competition through increased brand differentiation is the reason that $\frac{d\Pi_2^{EI}}{d\alpha_1} > 0$, and firm 2 was better off in the single investment case than in the dual investment case, previously.

Firm 1

Since the second term of Π_1^{EI} is decreasing in α_1, then firm 1's optimal α_1 depends on c_α, the degree to which brand improvements are costly. By the same argument from the previous section, in which location was exogenous, firm 1 will choose an $\alpha_1 > 0$ for any $c_\alpha \geq 0$. Since the expression for the equilibrium α_1 is algebraically messy when equilibrium location has not been pinned down, I will simply denote it as α_1^* for now, where we know that $\alpha_1^* > 0$.

C.5.5 Location

Given equilibrium brand investments $\alpha_1 = \alpha_1^*$ and $\alpha_2 = 0$, firms simultaneously maximize profit with respect to location:

$$\max_{x_1}\{\frac{1}{72}[\frac{(\alpha_1^* + 48d(1-d))^2}{\alpha_1^* + 64d(1-d)}] - c_\alpha \alpha_1^{*2}\}$$
$$\max_{x_2}\{\frac{1}{288}[\frac{(\alpha_1^* + 96d(1-d))^2}{\alpha_1^* + 64d(1-d)}]\}.$$

, where $d = x_1 - x_2$. The unique solution for which $d \in [0, 1]$ is that $d^* = \frac{1}{2}$. Therefore, we obtain that firms differentiate maximally in location. Since $d^* = \frac{1}{2}$, then we can refer the results from the previous section, where we had assumed that $d = \frac{1}{2}$, to obtain the equilibrium demands, prices, and brand investments.

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