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Asymmetric Information in the Hold-Up Problem

A Dissertation

Presented to the Faculty of the Graduate School

of

Yale University

in Candidacy for the Degree of

Doctor of Philosophy

by

Stephanie Lau

Dissertation Director: Professor Dirk Bergemann

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Abstract

Asymmetric Information in the Hold-Up Problem

Stephanie Lau

2003

This dissertation studies the role of asymmetric information in the hold-up problem and consists of three chapters.

Chapter 1 incorporates an information structure with partial information into the canonical hold-up problem. It characterizes explicitly the impact of asymmetric information on the hold-up problem: the tension between *ex ante* efficiency (the "information rent" effect) and *ex post* efficiency (the "bargaining disagreement" effect). The optimal information structure is further identified: with one-shot bargaining, it occurs at an intermediate level of information asymmetry; when there is repeated bargaining, it is attained with perfect asymmetry.

Chapter 2 generalizes the static model in the preceding chapter by allowing for any noisy information structures. It establishes the following properties regarding the optimal information structures: (i) they consist solely of informative signals; (ii) they do not induce "bargaining disagreement" in equilibrium; (iii) for some simple cases, they are either triangular or diagonal, and the monotone likelihood ratio property emerges endogenously.

Chapter 3 complements the previous chapters by characterizing the optimal trading mechanism of the celebrated bilateral-trading problem with *ex ante* invest-

ment. It shows how this mechanism is generally distorted in order to induce investment; however, the expected payoffs at the bottom always remain zero. Furthermore, when the investment constraint is binding, some types necessarily expect more trade while others expect less, relative to the absence of this constraint. Despite the distortion, the investment constraint may actually render the overall trading area bigger.

The results of this dissertation provide a basis for institutional design in settings where the hold-up problem arises in the presence of private information, and shed light on the robustness of existing incomplete-contract models.

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Introduction

In their influential paper, Klein *et al.* (1978) explain vertical integration in terms of reducing transactions costs for "post-contractual opportunistic behavior". Since this seminal contribution, there has been an extensive literature attempting to account for existing economic institutions based on a fundamental phenomenon: the hold-up problem. When one party transacts with another, it often involves some relationship-specific investment. Since contracts are incomplete, these parties have to rely on bargaining to divide the surplus of investment. Typically, however, the agent who makes the *ex ante* investment is not its residual claimant because he does not have all the bargaining power at the *ex post* bargaining stage. Knowing that his sunk cost of investment will not be fully compensated, he will under-invest.

Despite the widespread study of this classic problem and the presence of incomplete information in virtually every economic situation, "[a]symmetric information has played a very limited role in the analysis of the hold-up problem" (Hart, 1995). In the bulk of the existing incomplete-contract literature, all the variables of interest are assumed to be observable among the agents at the bargaining stage. Therefore, *ex post* efficiency is automatically guaranteed and any inefficiency comes from *ex ante* under-investment. Such a simplifying assumption is particularly problematic considering the emphasis of human-capital investment in the literature. Even if investment is purely physical, the degree to which it is non-adjustable is most likely to be private knowledge.

This dissertation studies the role of asymmetric information in the hold-up problem. Chapter 1 incorporates an information structure with partial information into the canonical hold-up model in which the investor has no bargaining power at all. It shows that asymmetric information introduces two counterbalancing forces: the "information rent" effect and "bargaining disagreement"ef fect. On the one hand, asymmetric information raises the inefficiency due to disagreement from *ex post* bargaining. On the other, the information rents created increase investment incentives, lowering the inefficiency due to *ex ante* under-investment. These two effects do not always cancel out and the optimal information structure occurs strictly in the interior.

The analysis in chapter 1 is carried out under a standard class of information structures, whose format is regarded as "hard information" in the sense of Tirole (1986). Chapter 2 enriches the model by allowing for any noisy information structures. In doing so, it shows how the joint surplus can be further improved by properly controlling information flow. Moreover, it establishes richer properties regarding the optimal information structures: they consist solely of informative signals and do not induce "bargaining disagreement" in equilibrium; for some simple cases, furthermore, the monotone likelihood ratio property emerges endogenously.

So far, the trading mechanism has been fixed to one which assigns zero bargaining power to the investor while the information flow between the party varies. Chapter 3 complements the previous chapters by fixing instead the information structure and deriving the optimal bargaining mechanism in a more general environment. More specifically, it characterizes the optimal trading mechanism of the celebrated bilateral-trading problem, when one party has the chance to make an investment to improve his distribution. It shows how the second-best mechanism derived in Myerson and Satterthwaite (1983) is distorted in general in order to induce investment. Contrary to the existing participation constraint which tends to block trade, the investment constraint works to liberate trade. As a consequence, some types necessarily expect more trade while others expect less, relative to original bilateral-trading problem without the possibility of investment. Overall, the investment constraint may actually render the trading area bigger.

The central message of this dissertation is that asymmetric information, the parameter that has been constantly ignored in literature, turns out to play an essential role in the hold-up problem. Its results have implications for institutional design, such as the optimal design of disclosure laws. They also shed light on the robustness of existing incomplete-contract models initiated by Grossman and Hart (1986) and Hart and Moore (1988) regarding the optimal allocation of bargaining power or ownership rights.

Chapter 1
Information and Bargaining in the Hold-Up Problem

1.1 Introduction

The classic hold-up problem has been studied widely, yet almost exclusively, under the assumption of complete information. Recently, asymmetric information does start to appear in the literature, most notably Gibbons (1992) and Gul (2001).[1] They study the static hold-up problem in which the investor has no bargaining power in the one-shot bargaining game that follows. As expected, the complete lack of bargaining power translates into a complete lack of investment incentives. They then ask what happens when the investment decision is not observable. In fact, the joint surplus is exactly the same with or without observability. Like other papers incorporating asymmetric information into the hold-up problem, however, they both assume either *full* information or *no* information.

In contrast, this chapter studies the hold-up problem under *partial* information, which is a more realistic description of real-life examples. By varying the degree of information asymmetry, we characterize explicitly the various effects of incomplete information on the hold-up problem and the sensitivity of its equilibrium outcome. It is tempting to gener-

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[1] Other examples include Rogerson (1992) and Matouschek (2001). Rogerson (1992) analyzes the hold-up problem under a variety of informational environments and illustrates the possibility of achieving the first-best welfare level in each case using a standard mechanism correspondingly. However, his analysis ignores the *interim* individual rationality constraints, contrary to the standard "trade at will" assumption in the literature. Matouschek (2001) focuses exclusively on property rights and does not involve any specific investment.

alize from the results of Gibbons (1992) and Gul (2001) that any degree of information asymmetry necessarily leads to the same welfare level. This conjecture is, surprisingly, false! We show that the parties' joint surplus varies *non*-monotonically with information asymmetry. The key observation is that asymmetric information introduces two counterbalancing forces. On the one hand, it raises the inefficiency due to disagreement from *ex post* bargaining. On the other, the information rents created increase investment incentives, lowering the inefficiency due to *ex ante* under-investment. Within a wide range of information structures, the joint surplus is strictly greater than that under either full information or no information. Put differently, the optimal information structure occurs at an intermediate degree of information asymmetry. Therefore it can be misleading to just look at the two polar cases.

Indeed, we may expect the parties to engage in repeated bargaining in some situations. Yet by making use of a well-established result in infinite-horizon repeated bargaining, it can be shown that repeated bargaining does not add anything to the joint surplus at all, as long as there is full observability. More strikingly, Gul (2001) goes further to show that unobservability, *together with* repeated bargaining, does solve the hold-up problem completely: the parties attain the first-best joint surplus, as the time between successive offers goes to zero. Recognizing that the requirement for perfect asymmetry of information may be unrealistic in this context, he further conjectures that "a small amount of asymmetric information between the buyer and the seller regarding the buyer's investment level may be sufficient" to achieve such a socially efficient outcome (p.344).

We naturally extend our partial-information hold-up model to infinite-horizon repeated bargaining. This extension allows us to look at the interactions between the negative "bargaining disagreement" effect and the positive "information rent" effect in a dynamic context. With repeated bargaining, "bargaining disagreement" is replaced by "bargaining delay". However, as the time between successive offers goes to zero, repeated bargaining kills off any delay, thereby reinforcing the investment incentives due to information rents. Consequently, the joint surplus unambiguously increases with information asymmetry. Furthermore, the first-best outcome can *only* be achieved under the extreme case of perfect asymmetry, whereas the lowest welfare level is obtained for a wide range of information structures. As a by-product, our results demonstrate that for any given information structure, the possibility of repeated bargaining is never Pareto impairing.

We further endogenize the information structure by allowing the less informed party to carry out costly information acquisition. In the more realistic scenario when she cannot commit to a particular information structure, there is always over-acquisition relative to what is socially desirable. Nevertheless, an intermediate level of information asymmetry emerges endogenously in equilibrium, verifying the robustness of our previous result.

This chapter confirms that asymmetric information, the parameter that has been frequently ignored in the literature, turns out to be an important welfare instrument in the hold-up problem. Its implications are two-fold. First, it provides a basis for institutional design regarding the optimal flow of information. Second, it sheds light on the robustness of existing incomplete-contract models in which the assumption of full information is taken for granted. We will demonstrate in section 1.6 that departing from this standard assumption in general gives a distinct prediction regarding the optimal allocation of bargaining power. More startlingly, an immediate consequence of our results is that it may be optimal to assign bargaining power to the party with *less* information.

The rest of this chapter is organized as follows. Section 1.2 describes the partial-information hold-up model, followed by its analysis in section 1.3. Sections 1.4 and 1.5 modify the model to accommodate infinite-horizon repeated bargaining and information acquisition respectively. Finally, section 1.6 concludes this chapter and discusses some applications.

1.2 The Partial-Information Hold-Up Model

We now proceed to describe the partial information hold-up model with one-shot bargaining and exogenous information structure. A risk-neutral seller (she) owns a non-divisible good that is of value to a risk-neutral buyer (he). Before any trade takes place, the buyer can choose the amount of relationship-specific investment I to undertake, which will incur a sunk cost of $C(I)$ and potentially increase his return of the good to $R(I)$. The seller then makes a take-it-or-leave-it offer to exchange the good for a price $p \geq 0$. The buyer can either accept or reject the offer and the overall payoffs of the parties are given by the following, depending on whether they reach agreement to trade or not:

$$U^B = R(I) - p - C(I) \text{ and } U^S = p \text{ if trade takes place;}$$

$$U^B = -C(I) \text{ and } U^S = 0 \text{ otherwise.}$$

The novelty is that we now incorporate an information structure q into this canonical model. More precisely, after the buyer has made his investment decision and before the seller chooses her price offer, the latter observes an *imperfect* signal $\eta \in \{I, X\}$ sent by nature so that with probability $1-q \in [0,1]$, she observes the correct investment level whereas with probability q, she learns nothing. Mathematically, $\forall I$,

$$\Pr(\eta = I|I) = 1-q;$$
$$\Pr(\eta = X|I) = q$$

where I is the true investment level and X is some fixed signal. We will call any $\eta = I$ an *informative signal* and $\eta = X$ the *uninformative signal.* Intuitively, q can be thought of as a measure of the degree of information asymmetry between the parties: as q increases, it becomes less likely that the seller is informed of the buyer's investment choice and there is a bigger gap in terms of the information each possesses. Notice that information of this format is regarded as "hard information" in the sense of Tirole (1986).[2] This exogenous information structure can be interpreted as reflecting the information flow of the existing institution, or the outcome set deliberately by policy makers. In section 1.5, we will endogenize the information structure by allowing the seller to acquire information regarding the true investment level at a cost.

We assume that both $R(I)$ and $C(I)$ behave "nicely": they are strictly increasing, continuously differentiable on a bounded domain $(0, \bar{I})$; $R(I)$ is concave whereas $C(I)$ is

[2] In a related study, we analyze the same problem with a more general information structure so that after any investment I made by the buyer, the seller receives a noisy signal η generated according to the normal distribution:

$$\eta \sim N\left(I, \sigma^2\right)$$

where the variance σ^2 plays the role of q here. Our preliminary study shows, not surprisingly, that the intuition and results in section 1.3 with this more general formulation are analogous.

convex, with one of them being strict. It is also standard to assume that the social surplus function, $R(I) - C(I)$, has an interior optimum (denote by I^*), i.e.,

$$\begin{aligned} R'(0) - C'(0) &= \lim_{I \downarrow 0} R'(I) - \lim_{I \downarrow 0} C'(I) > 0; \\ R'(\bar{I}) - C'(\bar{I}) &= \lim_{I \uparrow \bar{I}} R'(I) - \lim_{I \uparrow \bar{I}} C'(I) < 0. \end{aligned}$$

Moreover, $R(0) > 0$ and $C(0) = 0$ so that the good is valuable to the buyer even without any investment. Finally, we assume $R'(0) < \infty$.[3] The cost and return functions here encompass those in Gibbons (1992) and Gul (2001), therefore their models are our special cases when $q = 0$ and 1.

We denote this static game with exogenous information structure q by $\Gamma(q)$. Throughout, we will use sequential equilibrium as the equilibrium concept, which is standard in the analysis of games with non-trivial information sets.

1.3 Interior Optimal Information Structure

In this section, we analyze the equilibrium outcome of the partial-information hold-up model. We will be interested in how the introduction of information asymmetry might affect the nature of the hold-up problem and hence the welfare of the parties. An essential step is to understand the buyer's investment decision at equilibrium. To this end, we now introduce a few terms which will play an important role in our analysis. First, for any information structure q, we define the *information-truncated return* of the buyer as

$$R^{IT}(I; q) = q \cdot R(I) - C(I). \quad (3.1)$$

[3] This last requirement is needed only for section 1.4.

Intuitively, since the seller is going to extract the whole pie after any informative signal, part of the return, namely $(1-q)\cdot R(I)$, is lost for certain from the buyer's perspective. As a consequence, when he evaluates the cost against benefit of investment, the buyer never takes into account this lost return. Correspondingly, the *optimal information-truncated investment* $\hat{I}(q)$ is defined to be the investment that maximizes the information-truncated return:

$$\hat{I}(q) = \arg\max_{I} R^{IT}(I;q). \tag{3.2}$$

Where there is no confusion, we will frequently suppress the dependence of $\hat{I}$ on q. It is straightforward to check that the information-truncated return is maximized at 0 if q does not exceed $\frac{C'(0)}{R'(0)}$.[4] We call this q the *critical information structure* and denote it by

$$\hat{q} = \frac{C'(0)}{R'(0)}. \tag{3.3}$$

The optimal information-truncated investment exhibits the following properties when there is more information asymmetry than that due to the critical information structure ($q > \hat{q}$):

$$\hat{I}'(q) > 0, 0 < \hat{I}(q) \leq \hat{I}(1) = I^*;$$

$$q = \frac{C'(\hat{I})}{R'(\hat{I})}. \tag{3.4}$$

Clearly, if it is very likely for the seller to observe the informative signal (q not large enough), then a considerable portion of the buyer's investment return will be extracted. As a result, he will not be motivated to invest, for otherwise his utility will be negative. It can be verified that whenever the level of information asymmetry does not exceed that due to

[4] Notice that since $I^* > 0$, $\frac{C'(0)}{R'(0)} < 1$.

the critical information structure ($q \leq \hat{q}$), the buyer never invests in equilibrium. The seller quotes the price $R(0)$ on the equilibrium path and her offer is always accepted. Therefore, the parties receive the trivial expected payoffs given in proposition 1.1.

Proposition 1.1:

When $q \leq \hat{q}$,

$$\begin{aligned}\mathbb{E}\left[U^B(q)\right] &= 0,\\ \mathbb{E}\left[U^S(q)\right] &= R(0)\end{aligned}$$

in the sequential equilibrium of $\Gamma(q)$.

Proof: See appendix. ■

The results in proposition 1.1 lead to the natural question: will the buyer ever make a positive investment? The answer turns out to be yes, as long as there is enough information asymmetry. We are going to show that in fact, when the level of information asymmetry is higher than that due to the critical information structure ($q > \hat{q}$), there is no pure strategy equilibrium and the equilibrium turns out to be of *mixed* strategies. We know that the seller will always quote a price $R(I)$ upon receiving any informative signal I. In this mixed strategy equilibrium, therefore, the buyer randomizes his investment choice and the seller randomizes her price offer only upon receiving the uninformative signal. Let us introduce the following notations in $\Gamma(q)$:

$$\begin{aligned} I^h(q) &= \text{upper bound of support of buyer's equilibrium investment strategy} \\ I^l(q) &= \text{lower bound of support of buyer's equilibrium investment strategy} \\ G^q(I) &= \text{buyer's equilibrium investment distribution} \\ F^q(p) &= \text{seller's equilibrium price distribution at } \eta = X \end{aligned}$$

Lemma 1.1 characterizes the support of the buyer's investment strategy.

Lemma 1.1:

When $q > \hat{q}$, the buyer randomizes his investment over the support

$$\left[I^l(q), I^h(q)\right] = \left[0, \hat{I}(q)\right]$$

in any sequential equilibrium of $\Gamma(q)$.

Proof: See appendix. ■

Recall that the optimal information-truncated investment $\hat{I}(q)$ is strictly positive whenever $q > \hat{q}$. To see why it is the upper bound of the investment support, notice that the buyer will never invest more that $\hat{I}(q)$, as his maximum possible marginal return cannot compensate for his marginal cost. It is also impossible that $I^h(q) < \hat{I}(q)$, for otherwise price will never exceed $R(I^h)$ at the uninformative signal and the buyer will become the residual claimant for any investment above $I^h(q)$. Therefore, he would prefer $\hat{I}(q)$ to $I^h(q)$. Furthermore, 0 is in the investment support because if $I^l(q) > 0$, then since the seller never charges below $R(I^l)$ and the buyer's payoff at $I^l(q)$ will be negative.

Using the fact that the buyer is indifferent among any investment in the support, we are now ready to derive the price distribution at the uninformative signal and hence the investment distribution using lemma 1.1. Lemma 1.2 below summarizes these distributions.

Lemma 1.2:

When $q > \hat{q}$,

$$G^{q}(I) = \begin{cases} 0 & \text{for} I < 0 \\ 1 - \frac{R(0)}{R(I)} & \text{for } I \in [0, \hat{I}) \\ 1 & \text{for } I \geq \hat{I} \end{cases};$$

$$F^{q}(p) = \begin{cases} 0 & \text{for } p < R(0) \\ \frac{C'(R^{-1}(p))}{q \cdot R'(R^{-1}(p))} & \text{for } p \in [R(0), R(\hat{I})) \\ 1 & \text{for } p \geq R(\hat{I}) \end{cases}$$

in the sequential equilibrium of $\Gamma(q)$.

Proof: See appendix. ■

To give a concrete idea of the buyer's investment decision, we now plot his equilibrium investment distributions for two distinct information structures using an arbitrary pair of return and cost functions. Figure 1.1 compares $G^{q^h}(I)$ against $G^{q^l}(I)$, where $q^h > q^l > \hat{q}$. As shown in the diagram, $G^{q^h}(I)$ and $G^{q^l}(I)$ overlap up to $\hat{I}(q^l)$, the upper bound of the support when $q = q^l$, at which point $G^{q^l}(I)$ jumps to 1. Then $G^{q^h}(I)$ continues to increase smoothly until it reaches $\hat{I}(q^h)$, the upper bound of its support, when it jumps to 1. Therefore, higher level of information asymmetry shifts the investment distribution up by first-order stochastic dominance. Intuitively, with a higher level of information asymmetry, it becomes less likely that the seller will charge her price contingent on investment at the informative signals. Therefore, the buyer's investment has a better chance of being compensated and his investment incentives are enhanced. We call this "information rent" effect.

At the same time, the lower likelihood that price can be contingent on investment, coupled with the wider supports of price and investment at the uninformative signal, also increases the chance of mismatch between price and investment. As a result, it becomes more probable that the parties disagree over the terms of trade and fail to realize the gains from trade. We call this "bargaining disagreement" effect. Let $D(q)$ denote the *ex ante* equilibrium probability of disagreement between the parties in $\Gamma(q)$. Proposition 1.2 below characterizes these two counterbalancing effects formally, using the distributions obtained in lemma 1.2.

Proposition 1.2

(i) (Information Rent Effect)*: For any* q^l, q^h *with* $q^h > q^l > \hat{q}$,

$$G^{q^h}(I) \text{ first-order stochastically dominates } G^{q^l}(I).$$

(ii) (Bargaining Disagreement Effect): *For* $q > \hat{q}$,

$$D'(q) = 1 - \frac{R(0)}{R(\hat{I})} > 0.$$

Proof: (i) By lemma 1.2, any two investment distributions are exactly the same up to the upper-bounds of their supports. Therefore, in order to rank any two $G^q(I)'s$ by $FOSD$, it suffices to compare their $\hat{I}(q)'s$. By definition,

$$q = \frac{C'(\hat{I})}{R'(\hat{I})}.$$

Differentiating with respect to q,

$$1 = \hat{I}' \cdot \frac{C'(\hat{I})}{R'(\hat{I})} \cdot \left[\frac{C''(\hat{I})}{C'(\hat{I})} - \frac{R''(\hat{I})}{R'(\hat{I})}\right].$$

Rearranging,

$$\hat{I}' = \frac{\left(R'(\hat{I})\right)^2}{C''(\hat{I}) \cdot R'(\hat{I}) - R''(\hat{I}) \cdot C'(\hat{I})} > 0.$$

(ii) We use the distributions obtained in lemma 1.2 to compute $D(q)$ explicitly, noticing that disagreement can only occur when $I < \hat{I}\,(q)$ *and* $\eta = X$:[5]

$$\begin{aligned}
D(q) &= q \cdot \int_0^{\hat{I}} [1 - F(R(I))] \cdot g(I)dI \\
&= q \cdot \left[\int_0^{\hat{I}} g(I)dI - \left(\int_0^{\hat{I}} F(R(I)) \cdot g(I)dI\right)\right] \\
&= q \cdot \left[\left(1 - \frac{R(0)}{R(\hat{I})}\right) - \left(\int_0^{\hat{I}} \frac{C'(I)}{q \cdot R'(I)} \cdot \frac{R(0) \cdot R'(I)}{[R(I)]^2} dI\right)\right] \\
&= q \cdot \left(1 - \frac{R(0)}{R(\hat{I})}\right) - R(0) \cdot \left(\int_0^{\hat{I}} \frac{C'(I)}{[R(I)]^2} dI\right).
\end{aligned}$$

Now differentiate $D(q)$ with respect to q,

$$\begin{aligned}
D'(q) &= q \cdot \left(-R(0) \cdot \frac{R'(\hat{I}) \cdot \hat{I}'(q)}{-\left[R(\hat{I})\right]^2}\right) \\
&\quad + \left(1 - \frac{R(0)}{R(\hat{I})}\right) - R(0) \cdot \frac{C'(\hat{I})}{\left[R(\hat{I})\right]^2} \cdot \hat{I}'(q)
\end{aligned}$$

---

5 In the following, $g(I)$ denotes the density of $G(I)$ on $[0, \hat{I})$.

$$
\begin{aligned}
&= \left(1 - \frac{R(0)}{R(\hat{I})}\right) + \frac{R(0) \cdot \hat{I}'(q)}{\left[R(\hat{I})\right]^2} \cdot \left[q \cdot R'(\hat{I}) - C'(\hat{I})\right] \\
&= \left(1 - \frac{R(0)}{R(\hat{I})}\right) > 0. \blacksquare
\end{aligned}
$$

We recall Gibbons (1992) and Gul (2001) have both shown that the parties always jointly receive $R(0)$ with or without observability. This naturally leads us to suspect that the negative "bargaining disagreement" effect and positive "information rent" effect always cancel out each other. In proposition 1.3, we compute explicitly the expected payoff of each party at equilibrium and show that this extrapolation is surprisingly false.

Proposition 1.3:

When $q > \hat{q}$,

$$
\begin{aligned}
\mathbb{E}\left[U^B(q)\right] &= 0, \\
\mathbb{E}\left[U^S(q)\right] &= \left[(1-q) \cdot \ln \frac{R\left(\hat{I}\right)}{R(0)} + 1\right] \cdot R(0)
\end{aligned}
$$

in the sequential equilibrium of $\Gamma(q)$.

Proof: By lemma 1.2, when $q > \hat{q}$,

$$
\begin{aligned}
\mathbb{E}\left[U^S(q)\right] &= (1-q) \cdot \left[\int_0^{\hat{I}} R(I) \cdot g(I) dI\right] \\
&\quad + (1-q) \cdot \frac{R(0)}{R\left(\hat{I}\right)} \cdot R\left(\hat{I}\right) + q \cdot R(0)
\end{aligned}
$$

$$= (1-q)\cdot\left[\int_0^{\hat{I}} \frac{R(I)\cdot R(0)\cdot R'(I)}{(R(I))^2}dI\right] + R(0)$$

$$= (1-q)\cdot R(0)\cdot\left[\int_{R(0)}^{R(\hat{I})} \frac{1}{\tilde{R}}d\tilde{R}\right] + R(0)$$

$$= \left[(1-q)\cdot \ln\frac{R\left(\hat{I}\right)}{R(0)} + 1\right]\cdot R(0)$$

Since 0 is in the support of $G^q(I)$, $\mathbb{E}\left[U^B(q)\right] = 0$. ■

To see why this seemingly surprising result is true, notice that $R(0)$ is in the support of the seller's pricing strategy at the uninformative signal and it will be accepted by the buyer for sure. The seller thus obtains $R(0)$ on average at the uninformative signal. However, she almost always quotes a price *strictly higher* than $R(0)$ at the informative signals. Therefore, the seller's overall expected payoff must be greater than $R(0)$.

When there is perfect asymmetry, however, the seller *always* receives the uninformative signal and her overall expected payoff is *exactly* $R(0)$. Indeed, this is confirmed by our formula: when $q = 1$,

$$(1-q)\cdot\ln\frac{R\left(\hat{I}\right)}{R(0)} = 0 \text{ and } \mathbb{E}\left[U^S(q)\right] = R(0).$$

Let $W(q)$ represent the equilibrium joint surplus in $\Gamma(q)$, and Q^* represent the set of optimal information structures among all the games $\{\Gamma(q)\}_q$:

$$Q^* = \left\{q^* | q^* \in \arg\max_q W(q)\right\}.$$

Corollary 1.1 below is immediate from propositions 1.1 and 1.3.

Corollary 1.1 (Non-Monotonicity and Interior Optimum):

The equilibrium joint surplus as a function of information asymmetry is as follows:

$$W(q)\begin{cases} > R(0) & \text{for } q \in (\hat{q},1); \\ = R(0) & \text{for } q \in [0,1] \setminus (\hat{q},1). \end{cases}$$

Therefore, $Q^* \subset (\hat{q},1)$.

Corollary 1.1 says that within a wide range of information structures ($\hat{q} < q < 1$), the *overall* impact due to the "information rent" effect always outweighs that due to the "bargaining disagreement" effect, using full information ($q = 0$) as the benchmark. As a result, the optimal information structure occurs at an intermediate level of information asymmetry. We might be further interested in knowing the shape of the joint surplus function. Two related questions thus arise naturally: Can we always find a unique optimum? Is the joint surplus function single-peaked? Without further restrictions, however, we are not able to offer explicit answers to these questions. Proposition 1.4 provides the necessary and sufficient condition for the joint surplus function to be strictly concave over $(\hat{q},1)$, in the case of a linear cost function. This condition guarantees that the optimal information structure is unique.

Proposition 1.4 (Unique Optimum):

When $C(I) = I$, $W(q)$ *is strictly concave over* $(\hat{q},1)$ *if and only if the following holds:*

$$R''' < R'' \cdot \left(\frac{3R' - 1}{R' - 1} \cdot \frac{R''}{R'} - \frac{R'}{R} \right), \qquad (P4)$$

where R *and its derivatives are evaluated at* $\hat{I}(q)$. *Therefore,* Q^* *is a singleton if* $(P4)$ *is satisfied.*

It is easily checked that one sufficient condition for $(P4)$ is that the optimal information-truncated investment is concave:

$$\hat{I}''(q) \leq 0. \tag{3.5}$$

In fact, $(P4)$ itself is not restrictive in the sense that it is satisfied by many standard classes of return functions. Two such examples include

$$\begin{aligned}(i)\ R(I) &= k_1 \cdot \ln(I + a_1) \\ (ii)\ R(I) &= k_2 \cdot (I + a_2)^{b_2}\end{aligned}$$

where a_1, a_2 and k_1, k_2 are positive constants and $b_2 \in (0, 1)$.

We conclude this section by plotting the joint surplus function using the following example:

$$R(I) = \sqrt{I + \frac{1}{8}} \quad C(I) = I \tag{E1.1}$$

It can be checked that example $E1.1$ satisfies all the assumptions of our model, with the critical information structure $\hat{q} = \frac{1}{\sqrt{2}} \approx 0.7$. Figure 1.2 depicts $W(q)$ as the solid line. As we have just mentioned, this example satisfies condition $(P4)$. Therefore, its joint surplus is a concave function, giving a unique optimal level of information asymmetry at $q^* \approx 0.85$.

1.4 Repeated Bargaining

In this section, we modify the model to allow for infinite-horizon, repeated bargaining. More precisely, instead of ending the game after a one-shot bargaining, the seller makes a new take-it-or-leave-it offer in every period whenever her old offer is rejected and this continues indefinitely until it is finally accepted (if this ever happens). The parties' payoffs are modified accordingly as follows, depending on the time $t = k \cdot \Delta$ at which trade takes place:

$$U_{\Delta}^{B} = [R(I) - p] \cdot e^{-r \cdot t} - C(I);$$
$$U_{\Delta}^{S} = p \cdot e^{-r \cdot t}$$

where $k = 0, 1, 2, ...,$ indexes the period number; Δ is the length of each period and r is the interest rate. We represent this new game by $\Gamma_{\Delta}(q)$. Any other notation N in the original static game is replaced analogously here by N_{Δ}.

This dynamic game is similar to the original static game in a number of aspects. It is a well-established result in the bargaining literature that under this environment, the seller again extracts all the surplus whenever he observes an informative signal (i.e. the seller quotes a price $p = R(I)$ and is accepted immediately at equilibrium). Therefore, using arguments analogous to those establishing proposition 1, it can be shown that whenever $q \leq \hat{q}$, the buyer again never has incentives to invest at equilibrium and the parties again attain the trivial payoffs given in proposition 1.1*.

Proposition 1.1*:

When $q \leq \hat{q}$,

$$\mathbb{E}\left[U_{\Delta}^{B}(q)\right] = 0,$$
$$\mathbb{E}\left[U_{\Delta}^{S}(q)\right] = R(0)$$

in the sequential equilibrium of $\Gamma_{\Delta}(q)$.

Similarly whenever $q > \hat{q}$ any equilibrium must be of mixed strategy and we obtain the following lemma, which is analogous to lemma 1.1, regarding the support of the buyer's investment strategy.[6]

Lemma 1.1*:

When $q > \hat{q}$, the buyer randomizes his investment over the support

$$\left[I_{\Delta}^{l}(q), I_{\Delta}^{h}(q)\right] = \left[0, \hat{I}(q)\right]$$

in any sequential equilibrium of $\Gamma_{\Delta}(q)$.

Implicit in lemma 1.1* is the conclusion that the buyer again always receives 0 whenever $q > \hat{q}$. What about the seller's equilibrium payoff? In this dynamic setting, bargaining continues even if the seller's offer is rejected. However, the parties forgo part of their joint surplus due to discounting. Therefore, bargaining disagreement is replaced by bargaining delay. What is then the interaction between the "information rent" effect and the "bargaining delay" effect? Lemma 1.3 is central in addressing this question.

Lemma 1.3 (No Delay, Gul (2001): Proposition 5):

Fix $q > \hat{q}$. For every $\epsilon > 0$, there exists $\bar{\Delta} > 0$ such that whenever $\Delta < \bar{\Delta}$, in any sequential equilibrium of $\Gamma_{\Delta}(q)$, the probability that the parties reach agreement by time ϵ is at least $1 - \epsilon$, in the event that the uninformative signal is generated.

Lemma 1.3 says that when bargaining becomes infinitely frequent, the parties can almost always reach agreement within an arbitrarily short amount of *real* time. In essence,

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6 Although additional arguments involving infinite-horizon repeated bargaining are required to prove proposition 1.1* and lemma 1.1*, they are standard in the literature and are therefore skipped here.

all "bargaining delay" effect is eliminated! This lemma is adapted directly from proposition 5 of Gul (2001), in which the parties are shown to achieve immediate agreement when *ex ante* investment is *always* unobservable. The arguments establishing "no delay" make heavy use of the standard results from the "Coase conjecture", first observed by Coase (1972) and later formulated rigorously by Fudenberg *et al.* (1985) and Gul *et al.* (1986).

In order to give the flavor of the arguments establishing lemma 1.3, we now provide a concise and informal account of the "Coase conjecture". It concerns the pricing of a durable-good monopolist facing a continuum of consumers whose values of the good are private information and are distributed as $H(v)$ over the support $[v^l, v^h]$. This game turns out to be equivalent to the game of bilateral bargaining with one-sided incomplete information. In the latter game, a seller makes infinite-horizon repeated offers to exchange a good for a price with a buyer who has private information about his valuation of the good and the prior beliefs over the valuations are distributed as $H(v)$ over the support $[v^l, v^h]$.[7] The following two assumptions are standard in the literature:

1. The gap condition: $v^l > 0$;

2. The Lipschitz condition: for every sequence $v_n \longrightarrow v^l$ with $v_n > v^l$,

$$\left| \lim_{n\to\infty} \frac{v_n - v^l}{H(v_n) - H(v^l)} \right| < \infty.$$

[7] Strictly speaking, these two games are not exactly equivalent. But since the non-equivalence arises from some subtle technical issue, they are frequently treated as equivalent in the literature. See Gul *et al.* (1986) for the technical issue involved.

With these two assumptions, any such "Coase" game with some fixed period length Δ can be shown to exhibit the following main properties regarding its sequential equilibria (Gul *et al.* (1986): Corollary 1):

1. Equilibrium exists and is generically unique in that all equilibria give rise to the same equilibrium path;
2. The buyer's equilibrium acceptance/rejection rule is stationary in that it depends only on the current price offer;
3. The seller does not randomize her price offer along the equilibrium path after the first period;
4. The game ends in a finite number of periods with probability 1.

In the game with unobservable *ex ante* investment, any investment distribution $G(I)$ induces a corresponding valuation distribution $H(v)$ with v equal to some $R(I)$. However, unlike in the "Coase game" where $H(v)$ is *fixed*, here $G(I)$ and hence $H(v)$ changes with Δ. The question thus boils down to the following: to what extent do the above results carry over to the game with unobservable *ex ante* investment? Observe that after the investment stage, the parties are playing the corresponding "Coase" game with some $H(v)$ and all the above properties are preserved. Gul (2001) thus uses a limiting argument to show that, as the period length tends to 0, the probability that the seller chooses to "price-discriminate arbitrarily finely in an arbitrarily small interval of real time" tends to 1 (p.354). This gives rise to "no delay" in lemma 1.3. To apply it to our current setting, the crucial observation is that after the investment stage, our "bargaining game"at the uninformative signal has exactly the same structure as the "bargaining game" with unobservable investment: in both situations, the parties are involved in the "Coase game". Moreover, the gap condition is fulfilled by $R(0) > 0$ whereas the Lipschitz condition is the consequence of $R'(0) < \infty$. Therefore, in our dynamic game $\Gamma_\Delta(q)$, the time of agreement approaches zero with probability almost 1 as the time between successive offers approaches zero, in the event that the uninformative signal is generated.

With lemma 1.3, we are ready to describe the buyer's investment decision and hence the joint surplus as a function of information asymmetry in the limit. Proposition 1.3* below is a parallel to proposition 1.3.

Proposition 1.3*:

For $q > \hat{q}$,

$$\begin{aligned} \mathbb{E}\left[U_\Delta^B(q)\right] &= 0, \\ \mathbb{E}\left[U_\Delta^S(q)\right] &\to R(\hat{I}) - C(\hat{I}) \end{aligned}$$

in any sequential equilibrium of $\Gamma_\Delta(q)$ *as* $\Delta \to 0$.

Proof: At the beginning of the game, the buyer chooses an investment level $I \in [0, \bar{I}]$ to maximize his expected utility

$$U_\Delta^B(I) = q \cdot \mathbb{E}\left[(R(I) - p(t)) \cdot e^{-r \cdot t}\right] - C(I)$$

where t is the time at which the parties reach agreement, in the event that the uninformative signal is generated. By lemma 1.3, we know that with probability almost 1, $t \to 0$ as

$\Delta \to 0$. In effect, with probability arbitrarily close to 1, the buyer is optimizing an expected payoff arbitrarily close to the information-truncated return,

$$R^{IT}(I;q) = q \cdot \mathbb{E}[R(I)] - C(I),$$

as the period length becomes arbitrarily close to 0. Therefore, the buyer invests arbitrarily close to the optimal information-truncated investment. More formally, we have the following:

Fix $q > \hat{q}$. For every $\epsilon > 0$, there exists $\bar{\Delta} > 0$ such that whenever $\Delta < \bar{\Delta}$, in any sequential equilibrium of $\Gamma_{\Delta}(q)$, the probability that $I \in \left[\hat{I} - \epsilon, \hat{I}\right]$ is at least $1 - \epsilon$.

The above conclusion, coupled with the fact that the parties reach agreement immediately under any informative signal, means that all the gains from trade are realized with probability arbitrarily close to 1. As a consequence, the expected joint surplus is arbitrarily close to $R(\hat{I}) - C(\hat{I})$. By lemma 1.1*, the buyer always receives 0 and our results follow.

■

Let $W_0(q)$ denote $W_{\Delta}(q)$ as $\Delta \to 0$. Corollary 1.1* below, a direct consequence of propositions 1.1* and 1.3*, is the dynamic analog to corollary 1.1.

Corollary 1.1* (Monotonicity and Extreme Optimum):

$W_0(q)$ increases monotonically with q. Therefore, the optimal information structure is attained at perfect asymmetry, with joint surplus arbitrarily close to $R(I^*) - C(I^*)$.

Proof: By proposition 1.1*, $W_0(q)$ is constant for $q \leq \hat{q}$. By proposition 1.3*,

$$W_0(q) \to R(\hat{I}) - C(\hat{I}) \text{ for } q > \hat{q}.$$

$$\frac{d}{dq}\left[R(\hat{I})-C(\hat{I})\right]=\left[R'(\hat{I})-C'(\hat{I})\right]\cdot\hat{I}'(q)>0.\ \blacksquare$$

With perfect asymmetry, therefore, the first-best outcome emerges as a special case. However, this socially efficient outcome is attainable only under this extreme form of information structure. In fact, we have already seen in proposition 1.1* that within a wide range of information structures ($q \leq \hat{q}$), the parties obtain $R(0)$. In essence, the informational requirement to sustain the first-best outcome is extreme.

We now return to example $(E1)$ and depict $W_0(q)$ as the dotted line in figure 1.2. We already know from propositions 1.1 and 1.1* that when there is not enough information asymmetry ($q \leq \hat{q}$), the buyer is never motivated to invest. Therefore, the parties' joint surplus is always $R(0)$, no matter whether bargaining is static or dynamic. It is interesting to compare their joint surpluses when the level of information asymmetry is high enough to induce the buyer to invest ($q > \hat{q}$). As is apparent in figure 1.2, the joint surplus is always higher with repeated bargaining than with one-shot bargaining. This property is in fact not specific to the example, but is a general characteristic. The explanation is simple. When $q > \hat{q}$, the buyer randomizes on $\left[0, \hat{I}(q)\right]$ in the static game and so the parties' joint surplus must be strictly lower than $R(\hat{I}) - C(\hat{I})$, which is what they receive in the dynamic game. In addition, the possible failure to realize gains from trade in the static game makes their joint surplus depart even further from $R(\hat{I}) - C(\hat{I})$. Therefore, repeated bargaining unambiguously makes the parties better off. This final observation is a direct implication of propositions 1.1 and 1.1*, 1.3 and 1.3*.

Corollary 1.2 (Superiority of Repeated Bargaining):

For any given q*,* $W_0(q)$ *is at least as high as* $W(q)$*:*

$$\begin{aligned} W_0(q) &= W(q) \textit{whenever } q \leq \hat{q}; \\ W_0(q) &> W(q) \textit{whenever } q > \hat{q}. \end{aligned}$$

Proof: The first part of the corollary follows immediately from propositions 1.1* and 1.1. To show the second part, for any $q > \hat{q}$, let

$$Y(q) = \left[R\left(\hat{I}\right) - C\left(\hat{I}\right)\right] - \left[\left((1-q)\cdot \ln \frac{R\left(\hat{I}\right)}{R(0)} + 1\right)\cdot R(0)\right].$$

Then

$$\begin{aligned} Y'(q) &= \left[\left(1 - \frac{R(0)}{R\left(\hat{I}\right)}\right)\cdot\left(R'\left(\hat{I}\right) - C'\left(\hat{I}\right)\right)\right]\cdot \hat{I}'(q) \\ &\quad + R(0)\cdot \ln\frac{R\left(\hat{I}\right)}{R(0)} > 0. \end{aligned}$$

But

$$Y(\hat{q}) = \lim_{q \downarrow \hat{q}} Y(q) = 0$$

and therefore

$$Y(q) > 0, \forall q > \hat{q}. \blacksquare$$

1.5 Information Acquisition

Our analysis so far has assumed that information structures are exogenously given. Indeed, the seller is always tempted to find out the true investment level and in some situations, it might not be prohibitively costly to do so. How does the equilibrium outcome change when information structure is determined endogenously? In this section, we extend the original model in section 1.2 to allow for costly information acquisition. More precisely, at the beginning of the game, the seller can choose a particular information structure q at a convex cost $\gamma(q)$ with $\gamma'(q) < 0, \gamma''(q) > 0$. Then the partial-information hold-up game $\Gamma(q)$ follows, with the chosen q governing the probability that each signal is realized. We make the following natural assumptions concerning the information acquisition cost function:

$$\begin{aligned} \gamma(1) &= \gamma'(1) = 0; \\ \lim_{q \downarrow 0} \gamma(q) &= \lim_{q \downarrow 0} \gamma'(q) = \infty. \end{aligned}$$

There are two scenarios to consider. In the benchmark case, the buyer observes the amount of information acquired by the seller before she makes investment. Put differently, the seller will be able to commit to a particular information structure. This game and its corresponding set of equilibrium information structures are represented by Γ^{com} and $\{q^{com}\}$.[8] We recall that the joint surplus in the partial-information hold-up game $\Gamma(q)$ is

$$W(q) = \begin{cases} \left[(1-q)\cdot \ln \frac{R(\hat{I})}{R(0)} + 1\right] \cdot R(0) & \text{for } q > \hat{q} \\ R(0) & \text{for } q \leq \hat{q} \end{cases}.$$

Observe first that any q^{com} must be greater than $\hat{q}$. Otherwise, the seller receives $R(0) - \gamma(q)$. But she can guarantee a payoff of $R(0)$ by not acquiring any information and always quoting the price $R(0)$. Moreover, after any q chosen by the seller, the parties are playing the corresponding $\Gamma(q)$, where q is common knowledge. Consequently, the seller is implicitly choosing $W(q)$ and since she has all the bargaining power, her problem boils

8 To focus our attention on information acquisition incentives, we only consider equilibria in which the seller chooses some q with probability 1.

down to one of comparing marginal cost in acquiring information against marginal gain in joint surplus. As a result, any equilibrium information structure q^{com} must satisfy

$$\gamma'(q^{com}) = W'(q^{com}). \tag{5.1}$$

It should be pointed out that had we incorporated an information cost in the analysis of section 1.3, the set of optimal information structures Q^* there would have coincided with the set of equilibrium information structures $\{q^{com}\}$ here.[9] Effectively, the seller acquires information efficiently when she is able to commit to a particular information structure.

We are interested in the more realistic case when the seller cannot credibly commit to any information structure. In this scenario, the buyer chooses his investment level *without* knowing how much information has been acquired by the seller. This game and its corresponding set of equilibrium information structures will be denoted by Γ^{en} and $\{q^{en}\}$ respectively. Again, any q^{en} must exceed $\hat{q}$. Now, when the seller makes the information acquisition decision, the buyer's investment strategy is taken as *given*. By sequential rationality and consistency, however, if the equilibrium information structure is q, then the joint surplus (disregarding information cost) will still equal $W(q)$. As a result, any equilibrium information structure q^{en} in this game will instead satisfy

$$\gamma'(q^{en}) = \frac{\partial}{\partial q} W\left(q, \hat{I}\right)|_{q=q^{en}} \tag{5.2}$$

where the right hand side denotes the partial derivative of W for fixed $\hat{I}$. Proposition 1.5 summarizes the major properties of the equilibrium information structure in Γ^{en}.

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[9] To capture the idea that information acquisition is not prohibitively costly and hence the problem is non-trivial, we of course assume implicitly that $\exists\tilde{Q} \subset [0,1]$ such that $W(q) - \gamma(q) > R(0), \forall q \in \tilde{Q}$ so that each $q^{com} < 1$.

Proposition 1.5

(i) (Uniqueness): *q^{en} is unique and $q^{en} \in (\hat{q}, 1)$.*

(ii) (Over-Acquisition): $q^{en} < \min\{q^{com}\}$.

Proof: See appendix. ■

When the buyer's investment distribution is taken as fixed, the marginal benefit of acquiring information is simply the marginal likelihood of observing the true investment and hence extracting the whole pie. In Γ^{com}, by contrast, there is also a negative effect: the investment distribution shifts down accordingly with more information acquisition. This explains why the seller is always tempted to acquire more information when the decision of information acquisition is private information. More importantly, proposition 1.5 confirms the robustness of our previous result that the optimal information structure occurs above the critical information structure: it emerges as an equilibrium outcome endogenously.

1.6 Concluding Remarks

To summarize, we now give a big picture to this paper. Let us first return to the partial-information hold-up model in section 2. With full information ($q = 0$), we have the classic hold-up problem: the buyer never invests and the parties jointly receive $R(0)$. What happens when we introduce a significant degree of information asymmetry (by raising q to above $\hat{q}$)? Now the information rents created give the buyer an incentive to invest between 0 and $\hat{I}(q)$, which acts to increase their joint surplus. At the same time, it is more probable that the parties disagree over the terms of trade and this counter effect acts to lower the joint surplus. Overall, the resulting joint surplus is strictly greater than $R(0)$, except when there is perfect asymmetry ($q = 1$). Next, what happens when we further allow for repeated bargaining? As the time between successive offers goes to zero, repeated bargaining guarantees immediate agreement and hence all the gains from trade are realized. The buyer in turn optimally shifts her investment toward $\hat{I}\,(q)$, giving an overall expected joint surplus arbitrarily close to $R(\hat{I}) - C(\hat{I})$.

We conclude this paper by discussing two applications:

Optimal Control of Information Flow

The formulae in propositions 1.3 and 1.3* tell us explicitly how information flow can be used as an instrument to control the welfare in the hold-up problem. Moreover, since the optimal information structure depends crucially on whether bargaining is one-shot or repeated, use of this instrument should depend on the nature of the situation. In particular, if the good being traded is perishable or short-lived and there is no room for repeated bargaining,[10] then the optimal institution will involve an intermediate level of information asymmetry. On the other hand, where repeated bargaining is viable, it will be desirable to ensure its feasibility and limit information flow to the greatest possible extent. This optimal control of information flow can be achieved, for example, by adjusting legal disclosure rules for firms.

Optimal Allocation of Bargaining Power

Consider a slightly richer version of the static model in section 1.2. Instead of calling one party the seller and the other the buyer, we allow each side to play a dual role.

[10] This can mean literally or more broadly that the buyer derives no value from the good after a certain deadline or time limit.

More precisely, each party, with the same cost and return functions, makes a specific investment before bargaining with his trading-partner. If bargaining power can be assigned exclusively to one party only, which side should enjoy this privilege? When either side has the same amount of information regarding his opponent's investment (in the language of our model, the same $q's$), our results in section 1.3 imply that the allocation of bargaining power is irrelevant. In contrast, when both sides have differentiated information (different $q's$), the allocation of bargaining power may matter. For instance, when one party has full information while the other has only a little information, then it is optimal to allocate bargaining power to the party with less information. More generally, the optimal allocation of bargaining power depends on the relative informativeness of each side. Therefore, relaxing the full information assumption which is prevalent in the incomplete-contract literature may give a distinct prediction regarding the optimal allocation of bargaining power.

1.A Appendix of Omitted Proofs

In this appendix, where there is no confusion, we will frequently suppress the dependence of the various symbols on q.

A Few Observations: Before proceeding to the actual proofs, we make the following simple observations regarding the behavior and payoff of each party in any sequential equilibrium of $\Gamma(q)$:

- after investing I, the buyer accepts any price p if and only if $p \leq R(I)$;

- upon receiving any informative signal $\eta = \tilde{I}$, the seller's posterior beliefs about the buyer's investment choice are given by

$$\Pr\left(I|\eta = \tilde{I}\right) = \begin{cases} 1 & \text{for } I = \tilde{I} \\ 0 & \text{for } I \neq \tilde{I} \end{cases}.$$

  He therefore optimally quotes the price $R(I)$ and the buyer accepts.

- upon receiving the uninformative signal $\eta = X$, the seller's posterior beliefs about the buyer's investment choice coincide with the actual investment distribution.

- The seller's price offer upon receiving the uninformative signal is between $R(I^l)$ and $R(I^h)$, where I^l and I^h are the upper and lower bounds of the buyer's investment support.

- the buyer's *ex ante* expected payoff from investing I is therefore given by

$$U^B(I) = q \cdot \Pr\left[p \leq R(I)\right] \cdot \mathbb{E}_{p \leq R(I)}\left[R(I) - p\right] - C(I)$$

  where $\mathbb{E}_Z[\cdot]$ denotes the expectation *conditional* on event Z.

We will utilize these observations without explicitly referring to them in the proofs of proposition 1.1, lemmas 1.1 and 1.2.

Proof of Proposition 1.1: For $q \leq \hat{q}$, the buyer's payoff from investing any $I > 0$ is given by

$$U^B(I) = q \cdot \Pr[p \leq R(I)] \cdot \mathbb{E}_{p \leq R(I)}[R(I) - p] - C(I).$$

Since $I \geq 0$, it follows that $p \geq R(0)$ and

$$\begin{aligned} U^B(I) &\leq q \cdot \Pr[p \leq R(I)] \cdot [R(I) - R(0)] - C(I) \\ &\leq q \cdot [R(I) - R(0)] - C(I) \\ &= q \cdot R(I) - C(I) - q \cdot R(0). \end{aligned}$$

Notice that $\arg\max_I R^{IT}(I;q) = 0$ for $q \leq \hat{q}$, so

$$\begin{aligned} U_B(I) &< q \cdot R(0) - C(0) - q \cdot R(0) \\ &= 0. \end{aligned}$$

But the buyer can avoid this loss by not investing, so she will not be motivated to invest. To sum up, the unique sequential equilibrium of $\Gamma(q)$ for $q \leq \hat{q}$ is as follows:

- the buyer always invests 0; after having invested I, he accepts any price p if and only if $p \leq R(I)$;
- the seller's posterior beliefs after any informative signal are given by

$$\Pr\left(I|\eta = \tilde{I}\right) = \begin{cases} 1 & \text{for } I = \tilde{I} \\ 0 & \text{for } I \neq \tilde{I} \end{cases}$$

while those after the uninformative signal are given by

$$\Pr(I|\eta = X) = \begin{cases} 1 & \text{for } I = 0 \\ 0 & \text{for } I \neq 0 \end{cases};$$

- the seller's pricing rule is

$$p(\eta) = \begin{cases} R(0) & \text{for } \eta = X \\ R(I) & \text{for } \eta = I \end{cases}.$$

Therefore, the expected payoffs are those given in proposition 1.1. ■

Proof of Lemma 1.1: We first show that there is no pure strategy equilibrium whenever $q > \hat{q}$. Assume on the contrary that there is a pure strategy equilibrium in which the buyer always invests $\tilde{I}$. If $\tilde{I} = 0$, then $p = R(0)$ at the uninformative signal and the buyer receives 0. But since

$$q \cdot R(\hat{I}) - C(\hat{I}) > q \cdot R(0) - C(0)$$

or

$$q \cdot \left[R(\hat{I}) - R(0)\right] - C(\hat{I}) > 0,$$

the buyer has a profitable deviation to $\hat{I}(q)$. If, on the other hand, $\tilde{I} > 0$, then $p = R(\tilde{I})$ at the uninformative signal and the buyer's payoff is negative. So he would rather not to invest. In any case, there cannot be a pure strategy equilibrium.

Having shown that there is no pure strategy equilibrium, any equilibrium must be of mixed strategy. We now proceed to establish the support of the buyer's investment choice in any such equilibrium using successive claims.

Claim 1: $I^l(q) = 0$.

Proof: Suppose on the contrary that $I^l > 0$, then

$$\begin{aligned} & U^B\left(I^l\right) \\ = \;& q \cdot \Pr\left[p \leq R(I^l)\right] \cdot \mathbb{E}_{p \leq R(I^l)}\left[R(I^l) - p\right] - C(I^l). \end{aligned}$$

Since $\Pr\left[p < R(I^l)\right] = 0$, it follows that

$$U^B\left(I^l\right) = -C(I^l) < 0.$$

Therefore, the buyer would rather not to invest. ■

Claim 2: $I^h\left(q\right) = \hat{I}\left(q\right)$.

Proof: Assume on the contrary that $I^h \neq \hat{I}\left(q\right)$, then

$$\begin{aligned} U^B(I^h) &= q \cdot \mathbb{E}\left[R(I^h) - p\right] - C(I^h) \\ &= q \cdot R(I^h) - C(I^h) - q \cdot \mathbb{E}\left[p\right]. \end{aligned}$$

Notice that $\arg\max_I R^{IT}\left(I; q\right) = \hat{I}$, so

$$\begin{aligned} U^B(I^h) &< q \cdot R(\hat{I}) - C(\hat{I}) - q \cdot \mathbb{E}\left[p\right] \\ &= q \cdot \mathbb{E}\left[R(\hat{I}) - p\right] - C(\hat{I}) \end{aligned}$$

$$\begin{aligned} = \;& q \cdot \Pr\left[p \leq R(\hat{I})\right] \cdot \mathbb{E}_{p \leq R(\hat{I})}\left[R(\hat{I}) - p\right] \\ & + q \cdot \Pr\left[p > R(\hat{I})\right] \cdot \mathbb{E}_{p > R(\hat{I})}\left[R(\hat{I}) - p\right] - C(\hat{I}). \end{aligned}$$

Since $\Pr\left[p > R(\hat{I})\right] \cdot \mathbb{E}_{p > R(\hat{I})}\left[R(\hat{I}) - p\right] \leq 0$, we must have

$$\begin{aligned} U^B(I^h) &< q \cdot \Pr\left[p \leq R(\hat{I})\right] \cdot \mathbb{E}_{p \leq R(\hat{I})}\left[R(\hat{I}) - p\right] - C(\hat{I}) \\ &= U^B(\hat{I}). \end{aligned}$$

Therefore, the buyer would prefer $\hat{I}(q)$ to $I^h(q)$. ■

With the preceding claims, we are now ready to show that the buyer is randomizing her investment over a connected support $\left[0, \hat{I}(q)\right]$. Suppose the support is not connected and has a gap. WLOG, suppose $supp\,[G] = [0, I^a] \cup \left[I^b, \hat{I}(q)\right]$, with $I^a < I^b$. Then clearly the seller will never quote any price $p \in \left[R(I^a), R(I^b)\right)$. Now let $F^- = \lim_{p \uparrow R(I^a)} F(p) = \lim_{p \uparrow R(I^b)} F(p)$ and $\tilde{p} = \mathbb{E}_{p<R(I^a)}[p] = \mathbb{E}_{p<R(I^b)}[p]$. Then

$$\begin{aligned} U^B(I^a) &= q \cdot F^- \cdot [R(I^a) - \tilde{p}] - C(I^a) \\ U^B(I^b) &= q \cdot F^- \cdot \left[R(I^b) - \tilde{p}\right] - C(I^b). \end{aligned}$$

$\forall \tilde{I} \in \left(I^a, I^b\right)$,

$$U_B\left(\tilde{I}\right) = q \cdot F^- \cdot \left[R(\tilde{I}) - \tilde{p}\right] - C(\tilde{I}).$$

Since $I^a, I^b \in supp\,[G]$ and $\tilde{I} \notin supp\,[G]$, we must have

$$\begin{aligned} & q \cdot F^- \cdot [R(I^a) - \tilde{p}] - C(I^a) \\ = & \; q \cdot F^- \cdot \left[R(I^b) - \tilde{p}\right] - C(I^b) \\ \geq & \; q \cdot F^- \cdot \left[R(\tilde{I}) - \tilde{p}\right] - C(\tilde{I}) \end{aligned}$$

which is a contradiction to the fact that $R(I) - C(I)$ and hence $q \cdot F^- \cdot [R(I) - \tilde{p}] - C(I)$ is strictly concave. ■

Proof of Lemma 1.2: By lemma 1.1, the buyer is randomizing his is randomizing over $[0, \hat{I}(q)]$. Therefore, $\forall\, I \in [0, \hat{I}(q)]$,

$$U^B(I) = U^B(0)$$

$$q \cdot \left[\begin{array}{c} F(R(0)) \cdot (R(I) - R(0)) + \\ \int_{R(0)}^{R(I)} [(R(I) - p)] \cdot dF(p) \end{array} \right] - C(I) = 0.$$

Using integration by parts on $\int_{R(0)}^{R(I)} p \cdot dF(p)$ and after some simple manipulations, we obtain

$$q \cdot \int_{R(0)}^{R(I)} F(p) \cdot dp = C(I).$$

Differentiating with respect to I,

$$\frac{C'(I)}{q} = F(R(I)) \cdot R'(I).$$

Therefore,

$$\frac{C'(R^{-1}(p))}{q} = F(p) \cdot R'(R^{-1}(p)).$$

Clearly, $F(p)$ is strictly increasing on $[R(0), R(\hat{I}))$ and so the support of $F(p)$ is $[R(0), R(\hat{I})]$. Since the seller in indifferent among any price $p \in [R(0), R(\hat{I}))$,

$$\begin{aligned} U_S(p) &= U_S(R(0)) \\ \Pr(I &\geq R^{-1}(p)) \cdot p = R(0) \\ \left[1 - G(R^{-1}(p))\right] \cdot p &= R(0) \end{aligned}$$

Therefore,

$$[1 - G(I)] \cdot R(I) = R(0).$$

For completeness, we now give the unique sequential equilibrium of $\Gamma(q)$ for $q > \hat{q}$ as follows:

- the buyer randomizes his investment according to $G(I)$; after having invested I, she accepts any price p if and only if $p \leq R(I)$;

- the seller's posterior beliefs after any informative signal are given by

$$\Pr\left(I|\eta=\tilde{I}\right)=\begin{cases}1 & \text{for } I=\tilde{I}\\ 0 & \text{for } I\neq\tilde{I}\end{cases}$$

whereas those after the uninformative signal coincide with $G(I)$;

- the seller's quotes the price $R(I)$ at any informative signal I, and she randomizes his price according to $F(p)$ at the uninformative signal. ■

Proof of Proposition 1.4: First, we take the second derivative of the joint surplus function with respect to q,

$$W''(q)=\left[(1-q)\frac{R'}{R}\hat{I}''+(1-q)\left(\frac{R''}{R}-\frac{(R')^2}{R^2}\right)\left(\hat{I}'\right)^2-2\frac{R'}{R}\hat{I}'\right]\cdot R(0)$$

where the dependence of R, R' and R'' on $\hat{I}\,(q)$ have been suppressed. Next, we express the above in terms of R and its derivatives,

$$W''(q)=\left[\left(1-\frac{1}{R'}\right)\left(\begin{array}{c}\frac{(R')^4}{RR''}-\frac{(R')^6}{R^2(R'')^2}\\ +2\frac{(R')^4}{RR''}-\frac{(R')^5R'''}{R(R'')^3}\end{array}\right)+2\frac{(R')^3}{RR''}\right]\cdot R(0).$$

Therefore, the joint surplus is strictly concave if and only if

$$\left(1-\frac{1}{R'}\right)\left(\begin{array}{c}\frac{(R')^4}{RR''}-\frac{(R')^6}{R^2(R'')^2}\\ +2\frac{(R')^4}{RR''}-\frac{(R')^5R'''}{R(R'')^3}\end{array}\right)+2\frac{(R')^3}{RR''}<0.$$

After some rearrangements, we obtain the equivalent expression in $(P4)$. ■

Proof of Proposition 1.5: (i) $\forall q>\hat{q}$, let

$$\begin{aligned}Z(q) &= \frac{\partial}{\partial q}W(q)-\gamma'(q)\\ &= -R(0)\cdot\ln\frac{R\left(\hat{I}\right)}{R(0)}-\gamma'(q).\end{aligned}$$

Then

$$\begin{aligned} Z(1) &= R(0) \cdot \ln \frac{R(I^*)}{R(0)} - \gamma'(1) \\ &= -R(0) \cdot \ln \frac{R(I^*)}{R(0)} < 0. \end{aligned}$$

$$\begin{aligned} \lim_{q \downarrow \hat{q}} Z(q) &= -R(0) \cdot \ln \frac{R(0)}{R(0)} - \gamma'(\hat{q}) \\ &= -\gamma'(\hat{q}) > 0. \end{aligned}$$

$$Z'(q) = -R(0) \cdot \frac{R'(\hat{I}) \cdot \hat{I}'(q)}{R(\hat{I})} - \gamma''(q) < 0,$$

Therefore, $\exists$ unique $\tilde{q} \in (\hat{q}, 1)$ such that $Z(\tilde{q}) = 0$. But this $\tilde{q}$ is the equilibrium information structure q^{en}.

(ii)

$$\frac{\partial}{\partial q} W(q) = -R(0) \cdot \ln \frac{R\left(\hat{I}\right)}{R(0)}.$$

$$\begin{aligned} W'(q) &= -R(0) \cdot \ln \frac{R\left(\hat{I}\right)}{R(0)} + (1-q) \cdot R(0) \cdot \frac{R'(\hat{I}) \cdot \hat{I}'(q)}{R(\hat{I})} \\ &> \frac{\partial}{\partial q} W(q), \forall q \in (\hat{q}, 1). \end{aligned}$$

Now let

$$\tilde{Z}(q) = W'(q) - \gamma'(q).$$

Then

$$\begin{aligned} 0 &= \tilde{Z}(q^{com}) \\ &= W'(q^{com}) - \gamma'(q^{com}) \\ &> \frac{\partial}{\partial q} W(q^{com}) - \gamma'(q^{com}) \\ &= Z(q^{com}). \end{aligned}$$

Therefore,

$$Z\left(q^{com}\right) < 0 = Z\left(q^{en}\right).$$

Since $Z\left(q\right)$ is strictly decreasing, it follows that

$$q^{com} > q^{en}. \blacksquare$$

1.B Appendix of Figures

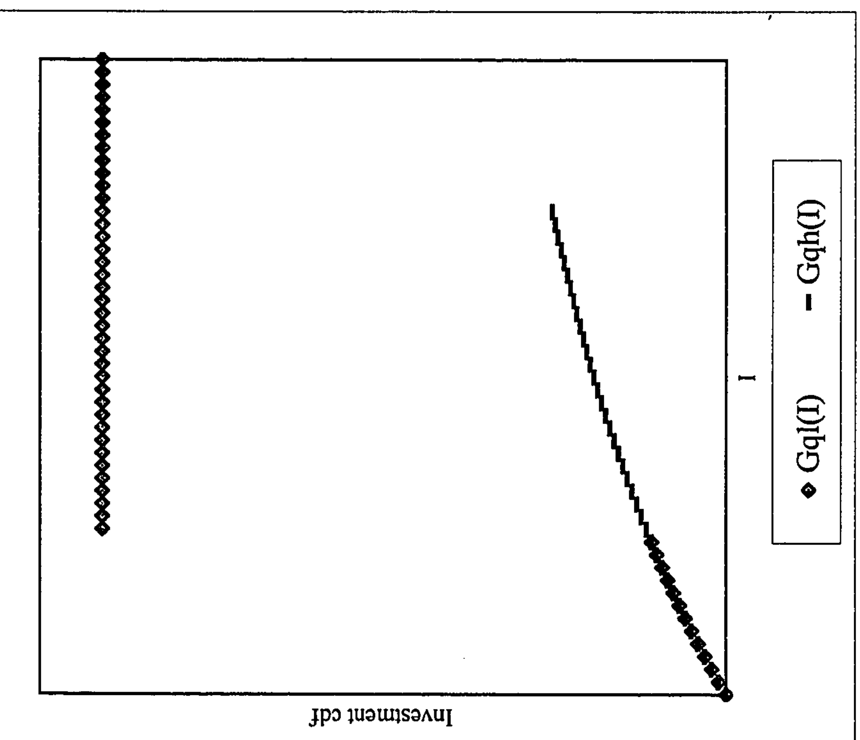

Figure 1.1: Comparing Investment Distributions.

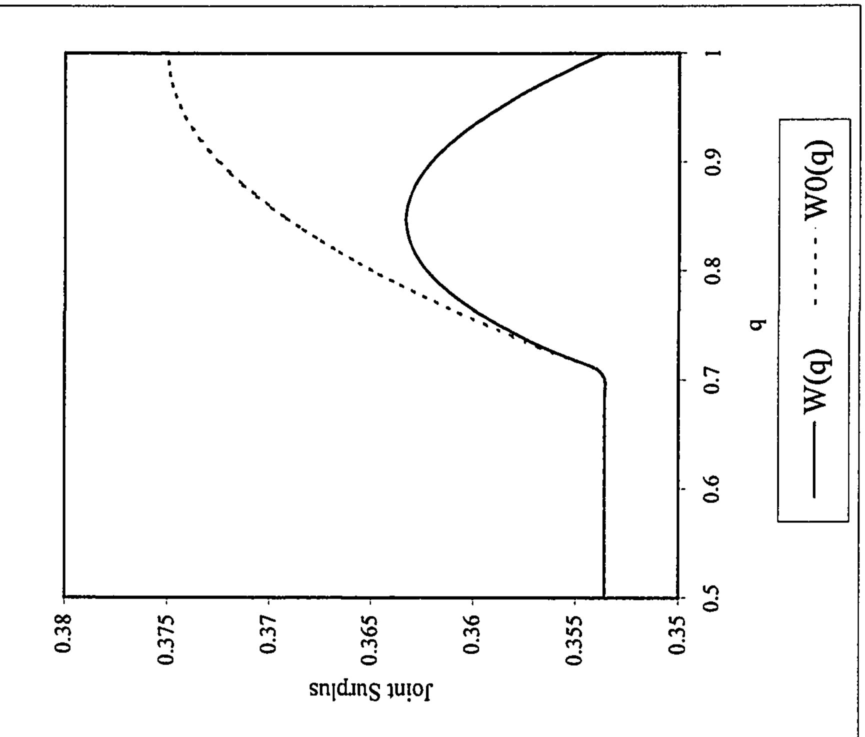

Figure 1.2: Joint Surplus against Information Asymmetry.

Chapter 2
Optimal Information Structures in the Static Hold-Up Problem

2.1 Introduction

In the previous chapter, we study the canonical hold-up problem with varying degrees of information asymmetry. We show that asymmetric information introduces a tension between *ex ante* efficiency (the "information rent" effect) and *ex post* efficiency (the "bargaining disagreement" effect), and illustrate how to attain the desired welfare level by properly controlling information flow.

The above analysis is carried out under a standard class of information structures so that all the signals are either completely informative or completely uninformative. In this chapter, we take one step further and ask the following question: how do the optimal information structures look like if we allow for any noisy information structures? The importance of this question is two-fold. From the theoretical point of view, it gives us an idea of the maximum extent to which information can be used to solve the hold-up problem. From the practical point of view, it provides a more general basis on institutional design regarding the optimal control of information flow.

Our model consists of a finite number of investment levels. As a first step, it is established that we can limit attention to only a finite number of signals without loss of generality, which significantly simplifies our analysis. We further show that any optimal information structure must involve only informative signals, in the sense that the seller does not randomize her price offers upon receiving the signals. Moreover, trade always occurs in equilibrium under any optimal information structure. In other words, the "bargaining disagreement" effect can be completely killed off, leaving only the "information rent" effect.

Any optimal information structure induces an equilibrium in which the buyer mixes among different investment levels. Having killed off the "bargaining disagreement" effect, the problem thus boils down to that of inducing the buyer to choose the most favorable equilibrium investment distribution. Indeed, changing the information structure in general involves a trade-off between the benefits of increasing the probabilities of some investment levels against the costs of decreasing others. In some simple cases, we are able to completely resolve this trade-off and characterize the optimal information structures. In particular, the monotone likelihood ratio property emerges endogenously in these cases.

The rest of this chapter is organized as follows. Section 2.2 outlines our model, followed by the general analysis in section 2.3. Section 2.4 gives the complete characterization of a simple case. Finally, section 2.5 concludes this chapter.

2.2 The Model

This section describes our model.

2.2.1 The Hold-Up Problem

A risk-neutral seller (she) owns a non-divisible good that is of value to a risk-neutral buyer (he). Before any trade takes place, the buyer can choose to undertake some relationship-specific investment from a finite set of investment levels:

$$I_i \in \mathcal{I} = \{I_0, I_1, ..., I_n\}.$$

This will incur a sunk cost of I_i and increase his return of the good to

$$R(I) \in \mathcal{R} = \{R_0, R_1, ..., R_n\},$$

where

$$\{R_0, R_1, ..., R_n\} = \{R(I_0), R(I_1), ..., R(I_n)\}.$$

The seller then makes a take-it-or-leave-it offer to exchange the good for a price $p \geq 0$. This is to capture the idea that the investor has no bargaining power at all, which is at the heart of the hold-up problem. The buyer can either accept or reject the offer.

We make the following standard assumptions regarding R_i and I_i:

(i) $R_0 > 0$ and $I_0 = 0$ so that the good is valuable to the buyer without any investment;

(ii) The return function is strictly increasing and concave: $R_{i+1} > R_i$ and $\frac{R_{i+1}-R_i}{I_{i+1}-I_i} > \frac{R_{i+2}-R_{i+1}}{I_{i+2}-I_{i+1}}, \forall i$.

Without loss of generality, we also assume the following to simplify notations:

(iii) The net surplus is strictly increasing: $R_{i+1} - I_{i+1} > R_i - I_i, \forall i$, so that the first-best investment level is I_n;

(iv) $I_i < R_i - R_{i-1}, \forall i > 0$.

2.2.2 Information Structures

The buyer's investment decision is private knowledge; however, the seller may elicit this piece of private information through a signal. More precisely, after the buyer has invested I_i, the seller observes an imperfect signal η according to a conditional distribution denoted by

$$Q(\eta|I_i)$$

over the space of signal realizations $\mathcal{S}$.[11] In general, the signal space can be infinite or uncountable. However, we will show later that it is without loss of generality to limit our attention to a finite set of signals.

Intuitively, the set of conditional distributions over signals $\{Q(\eta|I_i)\}_i$ determine the information flow between the parties, they are naturally interpreted as information structures.

2.2.3 Payoffs

The overall payoffs of the parties are given by the following, depending on whether they reach agreement to trade or not:

$$\text{The buyer's payoff: } U^B = \begin{cases} R(I_i) - p - I_i & \text{if trade takes place} \\ -I_i & \text{otherwise} \end{cases};$$

$$\text{The seller's payoff: } U^S = \begin{cases} p & \text{if trade takes place} \\ 0 & \text{otherwise} \end{cases},$$

where I_i is the investment made and p is the trading price respectively.

[11] This notation implicitly assumes that the signal space is independent of I_i. This is without loss of generality, as will be clear later. Therefore, our formulation is compatible with the standard formulation adopted by Laffont (1989).

Throughout, we use sequential equilibrium as the solution concept. Our problem is to adjust the information structure so that the parties jointly obtain the highest joint surplus in equilibrium.

2.3 No Bargaining Disagreement

In order to attain a particular equilibrium, two sets of constraints must be satisfied. First, the buyer must be given the right incentives to choose the equilibrium investment level(s). Second, the seller must be willing to play the equilibrium pricing strategy. To facilitate exposition, let us first introduce a few notations:

$$\begin{aligned} G(I_i) &= \text{buyer's equilibrium investment distribution;} \\ g(I_i) &= \text{probability the buyer invests } I_i \text{ in equilibrium;} \\ I_h, I_l &= \text{upper, lower bounds of support of equilibrium investment strategy;} \\ F(p|\eta) &= \text{seller's equilibrium price distribution at signal } \eta; \\ \mu(R|\eta) &= \text{seller's equilibrium posterior distribution over returns at signal } \eta. \end{aligned}$$

With these notations, the above two sets of constraints are translated mathematically to:

$\forall I_i \in supp\,[G(\cdot)]$,

$$I_i \in \arg\max_{I_k \in \mathcal{I}} \Pr[p \leq R_k] \cdot \mathbb{E}[R_k - p | p \leq R_k] - I_k;$$

$\forall p \in supp\,[F(\cdot|\eta)]$,

$$p \in \arg\max_{p \in [0,\infty]} [1 - \mu(p|\eta)] \cdot p.$$

Before proceeding further, it is illuminating to understand how the agents behave in equilibrium. Observe first that since the seller has all the bargaining power, the buyer will accept any price as long as it is no less than the return from his investment. As a result, the seller will never quote a price below the return due to the lower bound of the support of the buyer's investment strategy, I_l. Now if I_l is strictly positive, then the buyer will incur a loss at I_l. However, he could avoid this loss by not investing at all. Under *any* information structure, therefore, I_l must be equal to 0. This general result regarding the buyer's equilibrium behavior is recorded in lemma 2.1.

Lemma 2.1:

Irrespective of the information structure, the buyer always invests 0 *with positive probability in equilibrium. Therefore, his equilibrium payoff is* 0.

Lemma 2.1 greatly simplifies the buyer's investment constraints: he is willing to invest at any level as long as it gives him an expected payoff of 0. For this reason, we will call the buyer's investment incentive constraints *individual rationality* constraints ($IR's$), to differentiate them from the *incentive compatibility* constraints ($IC's$) governing the seller's pricing incentives.

Next, observe that any price that is strictly between two consecutive elements of $\mathcal{R}$ will be dominated by one that is equal to the larger of these two elements, for the latter price will give a strictly higher payoff without lowering the chance of being accepted. Consequently, the seller will never quote a price that is *not* in $\mathcal{R}$. Now fix an information structure and a corresponding equilibrium. If we combine together those signals upon which the seller's pricing distributions have the same support, then it is easily shown that none of the constraints will be affected. Therefore, the original equilibrium investment distribution can still be supported in equilibrium after this "merging" of signals. Since there are only a finite number of possible prices, the possible number of supports is finite as well. We can therefore without loss of generality limit attention to only a finite number of signals, each being completely determined by the support of the pricing distribution. This observation is summarized in lemma 2.2.

Lemma 2.2:

Without loss of generality, it suffices to consider only a finite number of signals in the optimal information structure.

We now introduce more notations to simplify our analysis. Let $q_{i\eta}$ denote the probability that signal η is realized conditional on investment I_i. If a signal induces the seller to quote price R_j with probability 1 in equilibrium, then we will denote this signal as η_j and the corresponding conditional probability as q_{ij}. Signals such as this will be called *informative* signals. Those signals that induce the seller to mix her prices are called *uninformative* signals. We will show below that the optimal information structures consist solely of informative signals.

Proposition 2.1, which is stated more intuitively in corollary 2.1, argues that the buyer never has to reject any offer in an optimal information structure. Observe first that it is never optimal to assign positive probability to a signal such that the seller will quote a higher price than the return of investment. To understand why this is true, let us fix some information structure and a corresponding equilibrium. If there is some i such that $q_{ij} > 0$ for some $j > i$, we can redistribute the signals by moving all such q_{ij} to q_{ii}. This obviously will not affect the buyer's investment incentives, and may actually reinforce the seller's pricing incentives at each signal. Consequently, the original equilibrium investment distribution can still be sustained. However, this redistribution lowers the chance of disagreement between the parties, thereby improving the joint surplus. This explains the first half of the proposition. The second part is a little more involved because it is trickier to move positive probabilities from uninformative signals to informative ones in such a way that none of the constraints is affected. However, the principle is analogous.

Proposition 2.1:

Any optimal information structure has the following properties:

(i) $q_{ij} = 0$ *whenever* $j > i$;

(ii) the seller is never induced to mix his price offers in equilibrium.

Proof: See appendix. ■

Corollary 2.1 (No Bargaining Disagreement):

Any optimal information structure does not induce "bargaining disagreement" between the parties in equilibrium.

Proposition 2.1 further reduces the information structure to the following set of probabilities:

$$\{q_{ij}\}_{i \in \mathcal{I}, j \leq i}.$$

Ideally, we would like to shift the equilibrium investment distribution toward higher levels as much as possible. Yet lemma 2.1 shows that the lower bound of the support of the buyer's investment strategy must be 0. How about the upper bound of the support? Is it possible for it to be I_n? Using the results in proposition 2.1, we argue that the buyer actually must invest I_n with positive probability.

Proposition 2.2:

Any optimal information structure induces the buyer to mix his investment choices with lower and upper bounds equal to 0 *and* I_n *in equilibrium.*

Proof. See appendix. ■

To summarize, propositions 2.1 and 2.2 further reduce the set of individual rationality constraints (of the buyer) to:

$\forall i \in \{0, 1, ..., n\}$,

$$\sum_{j<i} q_{ij} (R_i - R_j) - I_i = 0. \qquad (IR_i)$$

Moreover, they also substantially simplify the set of incentive compatibility constraints (of the seller) to:

$\forall j \in \{0, 1, ..., n\}$,

$$R_j \geq \left(\frac{\sum_{i \geq k} g_i \cdot q_{ij}}{\sum_{i \geq j} g_i \cdot q_{ij}} \right) \cdot R_k, \forall k > j. \qquad (IC_{j,k})$$

For the case of binary-investment, propositions 2.1 and 2.2 together in fact have completely characterized the optimal information structure.

Corollary 2.2 (Binary-Investment):

When $n = 1$, the optimal information structure is as follows:

$q_{00} = 1$	$q_{10} = \frac{I_1}{R_1 - R_0}$
$q_{01} = 0$	$q_{11} = 1 - \frac{I_1}{R_1 - R_0}$

.

In passing, notice that the above information structure trivially satisfies the monotone likelihood ratio property. We will show in section 2.3 that this property continues to hold for $n = 2$.

2.4 Complete Characterization: A Simple Case

In this section, we assume that $n = 2$. This special case will turn out to be a parameterized linear programming problem in one parameter, which is readily tractable. From the previous section, we can limit attention to information structures with 3 signals η_1, η_2 and η_3 upon which the seller quotes prices R_0, R_1 and R_2 respectively.

For each information structure $\{q_{ij}\}$, our first objective is to find the best equilibrium. Therefore, we solve the linear programming problem:

$$\max_{0 \leq g_0, g_1, g_2 \leq 1} g_0 \cdot (R_0 - 0) + g_1 \cdot (R_1 - I_1) + g_2 \cdot (R_2 - I_2)$$

subject to

$$R_0 \geq \left(\frac{g_1 \cdot q_{10} + g_2 \cdot q_{20}}{g_0 \cdot q_{00} + g_1 \cdot q_{10} + g_2 \cdot q_{20}} \right) \cdot R_1 \qquad (IC_{0,1})$$

$$R_0 \geq \left(\frac{g_2 \cdot q_{20}}{g_0 \cdot q_{00} + g_1 \cdot q_{10} + g_2 \cdot q_{20}} \right) \cdot R_2 \qquad (IC_{0,2})$$

$$R_1 \geq \left(\frac{g_2 \cdot q_{21}}{g_1 \cdot q_{11} + g_2 \cdot q_{21}} \right) \cdot R_2 \qquad (IC_{1,2})$$

and

$$q_{10} \cdot (R_1 - R_0) = I_1 \qquad (IR_1)$$

$$q_{20} \cdot (R_2 - R_0) + q_{21} \cdot (R_2 - R_1) = I_2 \qquad (IR_2)$$

and

$$g_0 + g_1 + g_2 = 1.$$

Our ultimate objective is to find the information structure $\{q_{ij}\}$ that gives the highest value to the linear programming problem. Notice that all

$$qij, \forall i < 2, \forall j$$

are completely defined by lemma 2.1 and proposition 2.1. Moreover, (IR_2) implies that each information structure is essentially identified by only one parameter. We choose q_{20} to be this parameter and frequently refer to q_{20} as the information structure. For each information structure q_{20}, the corresponding linear programming problem $L(q_{20})$ can be simplified as :

$$\max_{g_1, g_2} W(q_{20}) = R_0 + (R_1 - I_1 - R_0) \cdot g_1 + (R_2 - I_2 - R_0) \cdot g_2$$

subject to

$$\begin{aligned} \Lambda_1(q_{20}) &= R_0 - [I_1 + R_0] \cdot g_1 - [(R_1 - R_0) \cdot q_{20} + R_0] \cdot g_2 \geq 0 \\ \Lambda_2(q_{20}) &= R_0 - R_0 \cdot \left(\frac{R_1 - R_0 - I_1}{R_1 - R_0}\right) \cdot g_1 - [(R_2 - R_0) \cdot q_{20} + R_0] \cdot g_2 \geq 0 \\ \Lambda_3(q_{20}) &= R_1 \cdot \left(\frac{R_1 - R_0 - I_1}{R_1 - R_0}\right) \cdot g_1 - [I_2 - (R_2 - R_0) \cdot q_{20}] \cdot g_2 \geq 0 \end{aligned}$$

In passing, notice that the constraint

$$g_1 + g_2 \leq 1$$

is automatically satisfied by $(IC_{0,1})$. Our ultimate goal is to find the q_{20} that gives the highest value to the linear programming problem. An essential step is to understand which constraints are binding in the solution to each $L(q_{20})$. To this end, we denote q^b to be the information structure such that $\Lambda_1(q^b)$, $\Lambda_2(q^b)$ and $\Lambda_3(q^b)$ intersect, that is,

$$q^b = \frac{I_1 \cdot I_2}{(R_1 - R_0) \cdot (R_2 - R_1 + I_1)}.$$

It is worth pointing out that only at $q_{20} = q^b$ can the $IC's$ be all binding in the solution. Proposition 2.3 below gives the complete characterization of the optimal information structure, which makes use of the following condition:

$$\frac{R_1 - R_0 - I_1}{R_2 - R_0 - I_2} - \frac{(I_1 + R_0) \cdot (R_2 - R_0) - R_1 \cdot (R_1 - R_0 - I_1)}{I_2 \cdot (R_1 - R_0) + R_0 \cdot (R_2 - R_0)}. \tag{P3}$$

Proposition 2.3:

When $n = 2$,

$$q_{20} \begin{cases} = 0 & \text{if } (P3) > 0 \\ \in [0, q^b] & \text{if } (P3) = 0 \\ = q^b & \text{if } (P3) < 0 \end{cases}$$

in the optimal information structure.

Proof. See appendix. ■

The appendix outlines the mathematical proof to proposition 2.3. We now give an intuition regarding the trade-off involved. Start with the information structure $q_{20} = 0$. In the best equilibrium, only $(IC_{0,1})$ and $(IC_{1,2})$ are binding. When we raise q_{20} (and lower q_{21} at the same time), the benefit is the increase in g_2 while the cost is the decrease in g_1. The second term in $P3$ in fact gives the ratio of magnitudes to these benefit and cost. If this ratio is larger than $\frac{R_1 - R_0 - I_1}{R_2 - R_0 - I_2}$ (i.e. $P3 < 0$), then obviously it is worthwhile to raise

q_{20}. This trade-off continues to hold as long as $q_{20} < q^b$. However $(IC_{0,2})$ starts to be binding when q_{20} reaches q^b, after which point this trade-off is no longer valid. It becomes unambiguously worst off to further raise q_{20} beyond q^b.

Proposition 2.3, together with the previous results in section 2.2, says that the optimal information structure is one of the following (except in the knife-edged case when $P3$ is exactly equal to 0) :

$q_{00} = 1$	$q_{10} = \frac{I_1}{R_1 - R_0}$	$q_{20} = 0$
$q_{01} = 0$	$q_{11} = \frac{R_1 - R_0 - I_1}{R_1 - R_0}$	$\frac{I_2}{R_2 - R_1}$
$q_{02} = 0$	$q_{12} = 0$	$\frac{R_2 - R_1 - I_2}{R_2 - R_1}$

,

$q_{00} = 1$	$q_{10} = \frac{I_1}{R_1 - R_0}$	$q_{20} = \frac{I_1 I_2}{(R_1 - R_0)(R_2 - R_1 + I_1)}$
$q_{01} = 0$	$q_{11} = \frac{R_1 - R_0 - I_1}{R_1 - R_0}$	$q_{21} = \frac{I_1(R_1 - R_0 - I_1)}{(R_1 - R_0)(R_2 - R_1 + I_1)}$
$q_{02} = 0$	$q_{12} = 0$	$q_{22} = \frac{(R_2 - R_1) - (I_2 - I_1)}{R_2 - R_1 + I_1}$

.

Moreover, it is easily verified that either of these information structures satisfies the monotone likelihood ratio property. Therefore, we have corollary 2.3:

Corollary 2.3 (MLRP):

When $n = 2$, the optimal information structure has the following properties:

(i) it is either triangular or diagonal;

(ii) it exhibits monotone likelihood ratio property.

We conclude this section by comparing the joint surplus due to the optimal information structure in the current setting to that in chapter 1, which adopts the "hard information" format.

$$I_0 = 0, I_1 = 1, I_2 = 2; \qquad (E2.1)$$

$$R_0 = 50, R_1 = 100, R_2 = 149.$$

$$I_0 = 0, I_1 = 1, I_2 = 2; \quad (E2.2)$$

$$R_0 = 0.8, R_1 = 3, R_2 = 5.1.$$

The table below shows the best joint surplus in each case, using examples 2.1 and 2.1.[12] As is apparent, allowing for more general noisy information structure can raise the joint surplus substantially.

	Hard Information	**General Noisy Information**	**% Increase**
$E2.1$	1.29	1.807	+40%
$E2.2$	90.6	146	+61.14%

Table 2.1: Gains in Welfare with General Noisy Information.

2.5 Concluding Remarks

The endogenous choice of information structure has become an emerging topic in economic theory recently. For example, Bergemann and Pesendorfer (2002) study the properties of the optimal information structures in an auction environment. This paper adds to this literature by making a step in characterizing the optimal information structures of the canonical hold-up problem. We establish some general properties and obtain the complete characterization for some simple cases.

12 The computations are done using *Mathematica*.

Indeed, several individual arguments in section 2.4 can be extended to the general case. To what extent do our results carry over to a more general setting is the topic for further research. Let us conclude by making two conjectures regarding the general case:

(i) the monotone likelihood ratio property will continue to hold in any optimal information structure;

(ii) the diagonal and triangular information structures will continue to be among the set of optimal information structures. An obvious extension is to find the sufficient conditions for the optimal information structure to be of each of these formats.

2.A Appendix of Omitted Proofs

Proof of Proposition 2.1: The argument establishing part (i) is given in the text preceding the proposition. We now give the argument for part (ii). Fix an information structure and a corresponding equilibrium investment distribution. If this information structure involves a signal x upon which the seller mixes among prices $\{R_k | k \in K\}$. Let f_k denote the probability the seller quotes R_k upon this signal. Now, consider redistributing signals in the following manner:

$\forall i \in \mathcal{I}$,

$$\text{move } f_k \cdot q_{ix} \text{ to} \begin{cases} q_{ik} & \text{if } k \leq i \\ q_{ii} & \text{if } k > i \end{cases}, \forall k \in K.$$

By construction, all the $IR's$ are satisfied and this redistribution can only strengthen the $IC's$. Therefore the original investment distribution can still be sustained as an equilibrium. But we have eliminated signal x, thus reducing the chance of disagreement and increasing the joint surplus. ■

Proof of Proposition 2.2: Again, fix an information structure $\{q_{ij}\}$, which induces a corresponding equilibrium in which the buyer invests I_i with probability g_i. If $g_n = 0$ so that $I_h < I_n$, then consider the following alternative information structure $\{\hat{q}_{ij}\}$ and alternative probability $\hat{g}_i$ for I_i:

$$\begin{aligned} \forall i &\neq n, \hat{q}_{ij} = q_{ij}, \forall j; \\ q_{nj} &= 0, \forall j \neq h, n; q_{nh} = \frac{I_n}{R_n - R_h}, q_{n,n} = 1 - \frac{I_n}{R_n - R_h}; \end{aligned}$$

$$\begin{aligned} \hat{g}_i &= g_i, \forall i < h; \\ \hat{g}_h &= g_h - \epsilon; \hat{g}_n = \epsilon. \end{aligned}$$

By construction, all the $IR's$ are satisfied. The only (IC) that may potentially be affected is $(IC_{h,n})$, however this will not be a problem if ϵ is small enough. Therefore, we have constructed an alternative information structure that can sustain a better equilibrium. We conclude that $I_h = I_n$ in any optimal information structure. Also, by lemma 2.1, $I_l = 0$. ■

Proof of Proposition 2.3: We give an outline to the proof. The detailed calculations involved are tedious, though conceptually straight-forward and are therefore skipped here.

Step 1: When $q_{20} \leq q^b$, $(IC_{0,1})$ and $(IC_{1,2})$ are binding in the solution to $L(q_{20})$. We solve $\Lambda_1(q_{20})$ and $\Lambda_3(q_{20})$ to obtain its corresponding value

$$W(q_{20}), \text{which is} \begin{cases} \text{increasing in } q_{20} & \text{if } (P3) < 0 \\ \text{constant in } q_{20} & \text{if } (P3) = 0 \\ \text{decreasing in } q_{20} & \text{if } (P3) > 0 \end{cases}.$$

(Note: Only when $q_{20} = q^b$ are all the $IC's$ binding at the same time in the solution.)

Step 2: When $q_{20} > q^b$, $(IC_{0,2})$ is always binding in the solution to $L(q_{20})$; however, irrespective of which of the other constraints is/are binding, the corresponding value $W(q_{20})$ is unambiguously decreasing in q_{20}. ■

Chapter 3
Investment Incentives in Bilateral Trading

3.1 Introduction

In a seminal paper, Myerson and Satterthwaite (1983) show that any mechanism satisfying budget balance, incentive compatibility and *interim* individual rationality cannot be *ex post* efficient. They then derive the surplus-maximizing mechanism (thereafter, the MS mechanism) passing these three qualifications . In order for the worse types of each side to avoid a negative payoff, some profitable trade must be blocked. As a consequence, both agents necessarily expect less trade in this second-best mechanism, no matter what their type realizations are.

This chapter is motivated by the observation that in some situations, the parties may have the chance to make an investment to improve his distribution before participating in the trading game. A classic example that fits into this paradigm is the hold-up model, when there is asymmetric information at the bargaining stage. Indeed, the hold-up model is distinguished by the relationship-specific investments to potentially improve the cost and return distributions, as well as the contractual incompleteness so that the agents have to rely on bargaining to realize and split the surplus. With the possibility of a distributional-upgrade investment, gains from trade are no longer fixed. To ensure better potential cost and return conditions, the parties must be given appropriate incentives to invest. As a result, the second-best trading mechanism may have to be further compromised and more surplus may have to be forgone.

In this chapter, we investigate the tension between investment incentive and bargaining efficiency in a bilateral-trading problem with one-sided investment. We characterize the optimal mechanism in the presence of the extra investment constraint and see how the MS mechanism is further distorted in general. Contrary to the participation constraint which tends to block trade, the investment constraint works to liberate trade. These opposing effects imply that the former constraint is always binding, so that the worst type of each agent is again never rewarded with a positive payoff. This conclusion allows us to compute the optimal mechanism completely by solving the two constraints with equality simultaneously. Relative to the MS mechanism, trade tends to shift from areas where the investor is rewarded less for investment to those where he is rewarded more. As a result, some types of each agent necessarily expect more trade while others expect less. We will illustrate with an example that it is possible for the overall area of trade to grow.

Rogerson (1992) and Schmitz (2002) show that if *interim* individual rationality in the bilateral-trading problem is weakened to *ex ante* individual rationality, then *ex ante* and *ex post* efficiencies can be jointly achieved.[13] Our results demonstrate that departing from this weaker requirement generally brings about some trade-offs. The tension between investment incentive and bargaining efficiency in the hold-up problem has been recognized in the previous chapters. There, the trading mechanism is fixed to one which assigns zero bargaining power to the investor. We then derive the welfare-maximizing information

[13] In Schmitz's model, cost is fixed while return only takes on two possible values. But his problem is made non-trivial by assuming investment to be *cooperative*.

structure by varying the information flow between the agents. The current chapter complements the previous ones by fixing instead the information structure so there is never any information flow between the parties, and deriving the optimal trading mechanism.

The rest of this chapter is organized as follows. Section 3.2 describes the model, followed by its analysis in section 3.3. Finally, section 3.4 concludes the paper by discussing some extensions and applications.

3.2 The Model

This section outlines the model. A seller owns a good which is to be traded with a buyer according to some *given* trading mechanism. At the beginning of the game, this mechanism is announced to the players. The seller's cost of production s is distributed according to some *fixed* distribution G with continuous and strictly positive density g over the support $\mathcal{S} = \left[s^l, s^h\right]$. The buyer's valuation of the good, denoted by b, has distribution F^o over the support $\mathcal{B} = \left[b^l, b^h\right]$. After the announcement of the trading mechanism, the buyer has the chance to make an observable investment $I > 0$, which will shift the valuation distribution to F^* over the same support by first order stochastic dominance; mathematically,

$$F^*(b) - F^o(b) \leq 0,$$

with the inequality being strict for some range in $\mathcal{B}$. At the same time, this investment will also incur a sunk cost $K > 0$. Again, both F^o and F^* have continuous and strictly positive densities, denoted by f^o and f^* respectively. The cost and valuation distributions are independently distributed. To make our problem non-trivial, we also assume that $\mathcal{B}$

and S have a non-empty intersection. Before the agents participate in the mechanism, s and b have been realized to the seller and buyer respectively, but remain unknown to their opponents.

Before proceeding further, an important point is worth noting. We have assumed investment to be observable. A natural question arises: to what extent is our analysis valid in settings with unobservable investment? Observe that the incentives of a party to report honestly and participate willingly at the *interim* stage depend only on his opponent's distribution, but not on his own. In a game with unobservable investment, the seller has correct expectation about the buyer's investment level at equilibrium. As a result, provided a pure-strategy equilibrium is induced, whether investment is observable or not does not make any material difference. Put differently, our analysis carries over to settings with unobservable investment as long as it is optimal to induce a pure-strategy equilibrium in settings where investment is unobservable.

We will restrict our attention to direct mechanisms. This is without loss of generality due to the well-known revelation principle (see, for example, Myerson (1979)). To facilitate exposition, it is useful to introduce some notations. $(x(s,b), t(s,b))$ will denote the direct mechanism with probability of trade $x(s,b)$ and expected payment $t(s,b)$,[14] which are functions of reported valuation and cost types respectively. Moreover, we have the following notations for the expected probabilities of trade, expected payoffs and expected

[14] This notation implicitly assumes budget balance, since we do not differentiate between the expected payment for the buyer and that for the seller.

payments from the mechanism:

$$p(b) = \int_{s^l}^{s^h} x(s,b) \cdot g(s) \cdot ds;$$
$$q(s) = \int_{b^l}^{b^h} x(s,b) \cdot f(b) \cdot db,$$

$$U(b) = \int_{s^l}^{s^h} [b \cdot x(s,b) - t(s,b)] \cdot g(s) \cdot ds;$$
$$V(s) = \int_{b^l}^{b^h} [t(s,b) - s \cdot x(s,b)] \cdot f(b) \cdot db,$$

$$c(b) = \int_{s^l}^{s^h} t(s,b) \cdot g(s) \cdot ds;$$
$$d(s) = \int_{b^l}^{b^h} t(s,b) \cdot f(b) \cdot db,$$

where $f \in \{f^o, f^*\}$. When there is no confusion, we will frequently suppress the dependence of various functions on s and b.

3.3 Analysis

This section analyzes the model and is composed of 3 sub-sections. Section 3.3.1 characterizes the optimal mechanism to induce investment. Section 3.3.2 provides some comparative statics between the mechanism with and without the investment constraint. Finally, section 3.3.3 illustrates our results with an example.

3.3.1 Optimal Mechanism to Induce Investment

Our objective is to derive the surplus-maximizing mechanism that induces the buyer to invest while still preserving incentive compatibility and *interim* individual rationality. Put differently, both parties will report their types honestly and be willingly to participate irrespective of their type realizations. In a world without incentive problems, we would ideally like the parties to trade as long as valuation exceeds cost. Myerson and Satterthwaite (1983) show that this is impossible even with exogenously fixed cost and valuation distributions. Moreover, the second-best mechanism always involves less trade for each party, no matter what their type realizations are. The extra requirement to provide investment incentives presents yet another difficulty. Two related questions thus arise: how is the second-best mechanism further distorted? Is the amount of trade for each type further depressed?

Before answering the various questions, let us first formulate our problem formally (call this problem $\mathcal{P}$):

$$\max_{(x,t)} \int_{b^l}^{b^h} \int_{s^l}^{s^h} (b-s) \cdot x \cdot g \cdot f^* \cdot ds \cdot db$$

such that (incentive compatibility and individual rationality):

$\forall b,$

$$\begin{aligned} U(b) &\geq 0; \\ U(b) &\geq b \cdot p\left(\hat{b}\right) - c\left(\hat{b}\right), \forall \hat{b} \in \mathcal{B}; \end{aligned}$$

$\forall s,$

$$\begin{aligned} V(s) &\geq 0; \\ V(s) &\geq d(\hat{s}) - s \cdot p(\hat{s}), \forall \hat{s} \in \mathcal{S}, \end{aligned}$$

and (investment incentives):

$$\int_{b^l}^{b^h} U(b) \cdot f^* \cdot db - \int_{b^l}^{b^h} U(b) \cdot f^o \cdot db \geq K.$$

Our model differs from the original bilateral-trading problem essentially in that there is an *ex ante* investment stage. Since investment is observable, the valuation and cost distributions are common knowledge at the bargaining stage. As a result, the parties are engaged in the standard bilateral-trading game in the *interim* stage and their incentives to report honestly and participate willingly remain unchanged.[15] Myerson and Satterthwaite (1983) show that a direct mechanism jointly satisfying incentive compatibility and *interim* individual rationality is equivalent to the following:

$$p(b) \text{ is non-decreasing and } q(s) \text{ is non-increasing;} \qquad (C1)$$

$$\int_{b^l}^{b^h} \int_{s^l}^{s^h} \left[\left(b - \frac{1-F^*}{f^*} \right) - \left(s + \frac{G}{g} \right) \right] \cdot x \cdot g \cdot f^* \cdot db \cdot ds \geq 0. \qquad (C2)$$

The idea is to first show that incentive compatibility and individual rationality together imply $C1$ and $C2$. On the other hand, given $x(\cdot,\cdot)$ satisfying $C1$ and $C2$ with $p(b)$ non-decreasing and $q(s)$ non-increasing, they construct a payment function $t^m(\cdot,\cdot)$ so that (x, t^m) is incentive compatible and individually rational.[16]

15 Please refer back to section 2 for what happens when investment is unobservable.

16 The payment function they construct is

$$\begin{aligned} t^m &= \int_{b^l}^{b} y \cdot d[p(y)] - \int_{s^l}^{s} z \cdot d[-q(z)] \\ &\quad + b^l \cdot p(b^l) + \int_{s^l}^{s^h} z \cdot [1 - G(z)]\, d[-q(z)]. \end{aligned}$$

Let us now consider the investment constraint. The expected payoff of the buyer with valuation distribution F and density f can be re-written as the following, using a standard argument regarding incentive compatibility:

$$\int_{b^l}^{b^h} U(b) \cdot f \cdot db = \int_{b^l}^{b^h} \left[U\left(b^l\right) + \int_{b^l}^{b} p\left(y\right) \cdot dy \right] \cdot f \cdot db$$

where $p(\cdot)$ is non-decreasing. Applying integration by parts, it can be further expressed as

$$\begin{aligned} \int_{b^l}^{b^h} U\left(b^l\right) \cdot f \cdot db &= \int_{b^l}^{b^h} \left[U\left(b^l\right) + \int_{b^l}^{b} p\left(y\right) \cdot dy \right] \cdot dF \\ &= U(b^l) + \left[\int_{b^l}^{b} p\left(y\right) \cdot dy \cdot F \right]_{b^l}^{b^h} - \int_{b^l}^{b^h} F \cdot p(b) \cdot db \\ &= U(b^l) + \int_{b^l}^{b^h} (1 - F) \cdot p \cdot db \\ &= U(b^l) + \int_{b^l}^{b^h} \int_{s^l}^{s^h} (1 - F) \cdot x \cdot g \cdot ds \cdot db. \end{aligned}$$

This implies the investment constraint is simply

$$\int_{b^l}^{b^h} \int_{s^l}^{s^h} \left(F^o - F^*\right) \cdot x \cdot g \cdot ds \cdot db \geq K. \qquad (C3)$$

On the other hand, consider the mechanism (x, t^m) where $x\left(\cdot, \cdot\right)$ satisfies $C3$ with $p\left(\cdot\right)$ non-decreasing and $q\left(\cdot\right)$ non-increasing, and $t^m\left(\cdot, \cdot\right)$ is the accompanying payment function constructed by Myerson and Satterthwaite (1983). It is straight-forward to show that the buyer will be motivated to invest under this mechanism. In other words, the investment constraint can be replaced by $C3$.

To sum up, problem $\mathcal{P}$ is equivalent to:

$$\max_{x} \int_{b^l}^{b^h} \int_{s^l}^{s^h} (b - s) \cdot x \cdot g \cdot f^* \cdot ds \cdot db$$

such that

$$p(b) \text{ is non-decreasing and } q(s) \text{ is non-increasing;}$$

$$\int_{b^l}^{b^h}\int_{s^l}^{s^h}\left[\left(b-\frac{1-F^*}{f^*}\right)-\left(s+\frac{G}{g}\right)\right]\cdot x\cdot g\cdot f^*\cdot db\cdot ds\geq 0;$$

and

$$\int_{b^l}^{b^h}\int_{s^l}^{s^h}(F^o-F^*)\cdot x\cdot g\cdot ds\cdot db\geq K.$$

In passing, it is obvious from the expressions above that x plays the fundamental role in the mechanism (x,t). For this reason, we will frequently ignore t and simply refer to x as the mechanism.

In deriving the optimal mechanism, we will adopt the usual trick and ignore the monotonicity constraints in $C1$ first. In order to guarantee that the resulting solution indeed satisfies these monotonicity properties, extra assumptions regarding the behavior of distributions are required. Therefore, we will assume $A1$ and $A2$ for the rest of this chapter:

$$b-\frac{1-F^*}{f^*} \text{ is non-decreasing in } b \text{ and } s+\frac{G}{g} \text{ is non-decreasing in } s; \qquad (A1)$$

$$\frac{F^o-F^*}{f^*} \text{ is non-decreasing in } b. \qquad (A2)$$

$A1$ holds for many standard classes of distributions and is by now standard in the optimal mechanism design literature (see, for example, Myerson (1981)). $A2$ is relatively stronger, but is far more than necessary. We will come back to this point later. Lemma 3.1 provides a crucial step toward characterizing the optimal mechanism.

Lemma 3.1 (Partial Characterization):

The optimal mechanism that induces investment has the following format:

$$x(s,b) = \begin{cases} 1 & \textit{if} \left[\begin{array}{l} b - s - \alpha \cdot \left(\frac{1-F^*}{f^*} + \frac{G}{g} \right) \\ +\beta \cdot \frac{F^o - F^*}{f^*} \end{array} \right] > 0; \\ 0 & \textit{otherwise.} \end{cases}$$

where α and β are some constants with $\alpha \in [0,1]$ and $\beta \geq 0$.

Proof: First, let us ignore the monotonicity constraints in $C1$ and form the Lagrangian of $\mathcal{P}$:

$$\begin{aligned} \mathcal{L} &= \int_{b^l}^{b^h} \int_{s^l}^{s^h} (b-s) \cdot x \cdot g \cdot f^* \cdot ds \cdot db \\ &+\lambda \cdot \int_{b^l}^{b^h} \int_{s^l}^{s^h} \left[\left(b - \frac{1-F^*}{f^*} \right) - \left(s + \frac{G}{g} \right) \right] \cdot x \cdot g \cdot f^* \cdot ds \cdot db \\ &+\mu \cdot \left[\int_{b^l}^{b^h} \int_{s^l}^{s^h} (F^o - F^*) \cdot x \cdot g \cdot ds \cdot db - K \right] \end{aligned}$$

where λ and μ are the multipliers for $C2$ and $C3$ respectively. Re-arranging,

$$\mathcal{L} = (1+\lambda) \cdot \int_{b^l}^{b^h} \int_{s^l}^{s^h} \left[\begin{array}{c} \left(b - \frac{\lambda}{1+\lambda} \frac{1-F^*}{f^*} \right) - \left(s + \frac{\lambda}{1+\lambda} \frac{G}{g} \right) \\ + \frac{\mu}{1+\lambda} \cdot \frac{F^o - F^*}{f^*} \end{array} \right] \cdot x \cdot g \cdot f^* \cdot ds \cdot db - \mu \cdot K.$$

The above is not maximized unless

$$x(s,b) = \begin{cases} 1 & \text{if} \left[\begin{array}{l} b - s - \frac{\lambda}{1+\lambda} \cdot \left(\frac{1-F^*}{f^*} + \frac{G}{g} \right) \\ + \frac{\mu}{1+\lambda} \cdot \frac{F^o - F^*}{f^*} \end{array} \right] > 0; \\ 0 & \text{otherwise.} \end{cases}$$

By $A1$ and $A2$, $x(\cdot, b)$ and hence $p(b)$ is non-decreasing whereas $x(s, \cdot)$ and hence $q(s)$ is non-increasing. Notice also that $\frac{\lambda}{1+\lambda} \in [0,1]$ and μ and hence $\frac{\mu}{1+\lambda} \geq 0$ because $C3$ is an inequality constraint. This completes the proof. ■

The second-best mechanism in lemma 3.1 departs from the *ex post* efficient mechanism through two forms of distortion: the first term $\alpha \cdot \left(\frac{1-F^*}{f^*} + \frac{G}{g} \right)$ comes from the participation constraint while the second term $\beta \cdot \frac{F^o - F^*}{f^*}$ is due to the need to provide investment incentives. Interestingly, since α, $\left(\frac{1-F^*}{f^*} + \frac{G}{g}\right)$, β, $\frac{F^o - F^*}{f^*}$ are all non-negative, the two forms of distortion have opposing effects on the amount of trade: the former tends to block trade whereas the latter tends to liberate trade.

In the proof, $A1$ and $A2$ are required to guarantee monotonicity and hence incentive compatibility. In many examples, however, both α and β are small and therefore $A1$ alone is enough to guarantee these monotonicity properties.

When the investment cost is so small that the investment constraint is slack ($\beta = 0$), the MS mechanism emerges as the optimal mechanism. In this scenario, the participation constraint must be binding ($\lambda > 0$) so that the worst type of each party obtains zero, by virtue of the well-known impossibility theorem. This leads us to the following question: can the participation constraint ever be relaxed so that the worst types are rewarded with a strictly positive payoff? The answer is negative. This is not surprising, for otherwise the optimal mechanism will generate more trade than the *ex post* efficient mechanism and violate the participation constraint even more. This result is shown formally in lemma 3.2 below.

Lemma 3.2 (Payoffs at the Bottom):

In the optimal mechanism that induces investment, the payoffs of the worst types of both the buyer and the seller are zero:

$$U\left(b^l\right) = V\left(s^h\right) = 0.$$

Proof: Proving this lemma is equivalent to proving that the $C2$ constraint is binding in the optimal mechanism. Assume on the contrary that this constraint is not binding, then $\lambda = 0$ and the optimal mechanism becomes

$$x = \begin{cases} 1 & \textit{if } b - s + \beta \cdot \frac{F^o - F^*}{f^*} > 0; \\ 0 & \textit{otherwise.} \end{cases}$$

We will show a contradiction by comparing x to the *ex post* efficient mechanism x^0:

$$x^0 = \begin{cases} 1 & \textit{if } b - s > 0; \\ 0 & \textit{otherwise.} \end{cases}$$

First, observe that

$$\int_{b^l}^{b^h} \int_{s^l}^{s^h} (b - s) \cdot x \cdot g \cdot f^* \cdot db \cdot ds \leq \int_{b^l}^{b^h} \int_{s^l}^{s^h} (b - s) \cdot x^0 \cdot g \cdot f^* \cdot db \cdot ds$$

and

$$\int_{b^l}^{b^h} \int_{s^l}^{s^h} \left(\frac{1 - F^*}{f^*} + \frac{G}{g} \right) \cdot x \cdot g \cdot f^* \cdot db \cdot ds \geq \int_{b^l}^{b^h} \int_{s^l}^{s^h} \left(\frac{1 - F^*}{f^*} + \frac{G}{g} \right) \cdot x^0 \cdot g \cdot f^* \cdot db \cdot ds.$$

Next, consider the $C2$ constraint:

$$\begin{aligned}
& \int_{b^l}^{b^h} \int_{s^l}^{s^h} \left[\left(b - \frac{1 - F^*}{f^*} \right) - \left(s + \frac{G}{g} \right) \right] \cdot x \cdot g \cdot f^* \cdot db \cdot ds \\
= & \int_{b^l}^{b^h} \int_{s^l}^{s^h} (b - s) \cdot x \cdot g \cdot f^* \cdot db \cdot ds - \int_{b^l}^{b^h} \int_{s^l}^{s^h} \left(\frac{1 - F^*}{f^*} + \frac{G}{g} \right) \cdot x \cdot g \cdot f^* \cdot db \cdot ds \\
\leq & \int_{b^l}^{b^h} \int_{s^l}^{s^h} (b - s) \cdot x^0 \cdot g \cdot f^* \cdot db \cdot ds - \int_{b^l}^{b^h} \int_{s^l}^{s^h} \left(\frac{1 - F^*}{f^*} + \frac{G}{g} \right) \cdot x^0 \cdot g \cdot f^* \cdot db \cdot ds \\
= & \int_{b^l}^{b^h} \int_{s^l}^{s^h} \left[\left(b - \frac{1 - F^*}{f^*} \right) - \left(s + \frac{G}{g} \right) \right] \cdot x^0 \cdot g \cdot f^* \cdot db \cdot ds \\
< & \ 0.
\end{aligned}$$

The last strict inequality follows from the impossibility of *ex post* efficiency. But this means that x does not satisfy constraint $C2$. ■

With lemmas 3.1 and 3.2, we are now ready to completely characterize the optimal mechanism that induces investment.

Theorem 3.1 (Optimal Mechanism):

When the investment constraint is binding, the optimal mechanism is as follows:

$$x^*(s,b) = \begin{cases} 1 & \text{if } \left[\begin{array}{l} b - s - \alpha \cdot \left(\frac{1-F^*}{f^*} + \frac{G}{g}\right) \\ +\beta \cdot \frac{F^o - F^*}{f^*} \end{array}\right] > 0; \\ 0 & \text{otherwise,} \end{cases} \tag{T1}$$

where α *and* β *solve*

$$U\left(b^l\right) = V\left(s^h\right) = 0$$

and

$$\int_{b^l}^{b^h} \int_{s^l}^{s^h} \left(F^o - F^*\right) \cdot x^* \cdot g \cdot ds \cdot db = K.$$

Proof: This follows immediately from lemmas 3.1 and 3.2. ■

3.3.2 Some Comparative Statics

The results in the previous sub-section allow us to compute the optimal mechanism explicitly. What can we say about this mechanism in general with and without the investment constraint? In this section, we will answer this question by deriving some comparative statics. In doing so, it should be pointed out that while there is only one optimal mechanism without the investment constraint, the optimal mechanism when the investment constraint is binding depends on the value of investment cost. Our aim is to obtain some general conclusions between the former and any of the latter.

For the rest of this paper, we will frequently represent the optimal mechanism as x^m or x^*, depending on whether the investment constraint is slack or binding (therefore, x^m is the MS mechanism) . Any other notation N is differentiated analogously by denoting it as N^m or N^*. Moreover, let us introduce the following notations:

$$B^m = \{(s,b) \in \mathcal{S} \times \mathcal{B} | x^m = 1\};$$
$$B^* = \{(s,b) \in \mathcal{S} \times \mathcal{B} | x^* = 1\}.$$

$$\hat{K} = \max\left\{K | \int_{b^l}^{b^h} \int_{s^l}^{s^h} (F^o - F^*) \cdot x^m \cdot g \cdot ds \cdot db \geq K\right\}.$$

Therefore, B^m and B^* represent the sets of cost-valuation pairs in which trade occurs under x^m and x^* respectively. $\hat{K}$ is the cutoff value of investment cost beyond which the investment constraint is binding. Before the investment cost exceeds this critical value, the optimal mechanism coincides with the MS mechanism. Some of the potential gains from trade are blocked by the participation constraint. As the cost of investment increases, the investment constraint tends to liberate trade so as to realize more values of $(F^o - F^*) \cdot g$. At the same time, the participation constraint works to depress trade even more. Overall, trade would shift from areas where $(F^o - F^*) \cdot g$ is small to areas where $(F^o - F^*) \cdot g$ is large. In theorem 3.2 below, we will show formally that the investment constraint blocks trade in some areas while liberates it in others. Consequently, some types necessarily expect more trade while others expect less (corollary 3.2).

Theorem 3.2 (Areas of Trade):

The sets of cost-valuation pairs where trade takes place under x^m and x^* cross each other:

$$(i) \int\int_{B^* \backslash B^m} ds \cdot db > 0;$$

$$\text{(ii)} \int\int_{B^m \backslash B^*} ds \cdot db > 0.$$

Proof: (i) Assume on the contrary that

$$\int\int_{B^* \backslash B^m} ds \cdot db = 0,$$

Since $F^o - F^* \geq 0$, we must have

$$\int_{b^l}^{b^h} \int_{s^l}^{s^h} (F^o - F^*) \cdot x^m \cdot g \cdot ds \cdot db \geq \int_{b^l}^{b^h} \int_{s^l}^{s^h} (F^o - F^*) \cdot x^* \cdot g \cdot ds \cdot db$$

as at least as many values of $F^o - F^*$ are realized under x^m than x^*. But $\forall K > \hat{K}$,

$$\int_{b^l}^{b^h} \int_{s^l}^{s^h} (F^o - F^*) \cdot x^m \cdot g \cdot ds \cdot db < K = \int_{b^l}^{b^h} \int_{s^l}^{s^h} (F^o - F^*) \cdot x^* \cdot g \cdot ds \cdot db. \blacksquare$$

(ii) Again, we establish this part by contradiction. Assume on the contrary that

$$\int\int_{B^m \backslash B^*} ds \cdot db = 0.$$

Suppose first that trade never occurs in the *ex post* inefficient area A under x^* so that $B^* \cap A = \phi$, where

$$A = \{(s, b) \in \mathcal{S} \times \mathcal{B} |\, b < s\}.$$

Then by hypothesis and by part (i),

$$\begin{aligned} & \int_{b^l}^{b^h} \int_{s^l}^{s^h} (b - s) \cdot x^* \cdot g \cdot f^* \cdot db \cdot ds \\ = & \int\int_{\bar{A}} (b - s) \cdot x^* \cdot g \cdot f^* \cdot db \cdot ds \end{aligned}$$

$$\begin{aligned} > & \int\int_{\bar{A}} (b - s) \cdot x^m \cdot g \cdot f^* \cdot db \cdot ds \qquad (T2) \\ = & \int_{b^l}^{b^h} \int_{s^l}^{s^h} (b - s) \cdot x^m \cdot g \cdot f^* \cdot db \cdot ds \end{aligned}$$

where $\bar{A}$ is the complement of A. Moreover, $\forall K \leq \hat{K}$,

$$\begin{aligned} & \int_{b^l}^{b^h} \int_{s^l}^{s^h} (F^o - F^*) \cdot x^* \cdot g \cdot ds \cdot db \\ > & \int_{b^l}^{b^h} \int_{s^l}^{s^h} (F^o - F^*) \cdot x^m \cdot g \cdot ds \cdot db \\ \geq & \ K. \end{aligned}$$

Therefore, x^* cannot satisfy constraint $C2$, for otherwise x^* would have been a better mechanism than x^m and would have replaced x^m as the MS mechanism.

If trade does occur in the *ex post* inefficient area A under x^*, then $T2$ may hold with equality, but in this situation

$$\begin{aligned} & \int_{b^l}^{b^h} \int_{s^l}^{s^h} \left[\left(b - \frac{1-F^*}{f^*} \right) - \left(s + \frac{G}{g} \right) \right] \cdot x^* \cdot g \cdot f^* \cdot db \cdot ds \\ = & \int \int_A \left[\left(b - \frac{1-F^*}{f^*} \right) - \left(s + \frac{G}{g} \right) \right] \cdot x^* \cdot g \cdot f^* \cdot db \cdot ds \\ & + \int \int_{\bar{A}} \left[\left(b - \frac{1-F^*}{f^*} \right) - \left(s + \frac{G}{g} \right) \right] \cdot x^* \cdot g \cdot f^* \cdot db \cdot ds \\ = & \int \int_A \left[\left(b - \frac{1-F^*}{f^*} \right) - \left(s + \frac{G}{g} \right) \right] \cdot x^* \cdot g \cdot f^* \cdot db \cdot ds \\ & + \int \int_{\bar{A}} \left[\left(b - \frac{1-F^*}{f^*} \right) - \left(s + \frac{G}{g} \right) \right] \cdot x^m \cdot g \cdot f^* \cdot db \cdot ds \\ = & \int \int_A \left[\left(b - \frac{1-F^*}{f^*} \right) - \left(s + \frac{G}{g} \right) \right] \cdot x^* \cdot g \cdot f^* \cdot db \cdot ds \\ & + \int_l^h \int_n^m \left[\left(b - \frac{1-F^*}{f^*} \right) - \left(s + \frac{G}{g} \right) \right] \cdot x^m \cdot g \cdot f^* \cdot db \cdot ds \\ = & \int \int_A \left[\left(b - \frac{1-F^*}{f^*} \right) - \left(s + \frac{G}{g} \right) \right] \cdot x^* \cdot g \cdot f^* \cdot db \cdot ds + 0 \\ < & \ 0. \end{aligned}$$

Therefore, x^* still does not satisfy constraint $C2$. ■

Corollary 3.2 (Probabilities of Trade):

(i) There exist intervals $B_1, B_2 \subset \mathcal{B}$ such that

$$\forall b \in B_1,\ p^*(b) > p^m(b);$$
$$\forall b \in B_2,\ p^*(b) < p^m(b).$$

(ii) There exist intervals $S_1, S_2 \subset \mathcal{S}$ such that

$$\forall s \in S_1,\ q^*(s) > q^m(s);$$
$$\forall s \in S_2,\ q^*(b) < q^m(s).$$

Proof: This is a direct consequence of theorem 3.2 and the fact that $x^*(\cdot, b)$, $x^m(\cdot, b)$ are non-decreasing whereas $x^*(s, \cdot)$, $x^m(s, \cdot)$ are non-increasing. ■

With theorem 3.2, one is led to wonder what happens to the overall area of trade. We will show with an example in the next section that the trading area may grow under the investment constraint.

3.3.3 An Illustrative Example

We now illustrate the results in this paper with the following example:[17]

$$F^o = b, F^* = b^2, G = s; \qquad (E3.1)$$
$$\mathcal{B} = \mathcal{S} = [0,1].$$

17 All the computations are done using *Mathematica*.

Substitute the various expressions in $T1$ and we obtain the optimal mechanisms with and without the investment constraint respectively:

$$x^* = \begin{cases} 1 & \textit{if } s < \frac{1}{1+\alpha}\left(b - \alpha \cdot \frac{1-b^2}{2b} + \beta\frac{b-b^2}{2b}\right); \\ 0 & \textit{otherwise,} \end{cases}$$

$$x^m = \begin{cases} 1 & \textit{if } s < \frac{1}{1+\alpha^m}\left(b - \alpha^m \cdot \frac{1-b^2}{2b}\right); \\ 0 & \textit{otherwise.} \end{cases}$$

α^m is computed by solving

$$\int_{\sqrt{\frac{\alpha^m}{1+\alpha^m}}}^{1} \int_0^{\frac{(2+\alpha^m)\cdot b - \alpha^m \cdot b^{-1}}{2\cdot(1+\alpha^m)}} \left(b - \frac{1-b^2}{2b} - 2s\right) \cdot 2b \cdot ds \cdot db = 0,$$

which gives

$$\alpha^m = 0.2990.$$

The critical investment cost above which the investment constraint becomes binding is equal to

$$\begin{aligned} \hat{K} &= \int_{\sqrt{\frac{\alpha^m}{1+\alpha^m}}}^{1} \int_0^{\frac{(2+\alpha^m)\cdot b - \alpha^m \cdot b^{-1}}{2\cdot(1+\alpha^m)}} \left(b - b^2\right) \cdot ds \cdot db \\ &= 0.0401. \end{aligned}$$

Above this critical value, the investment constraint becomes binding and we can solve for α and β in x^* using the following system of equations:

$$\begin{aligned} \int_{\frac{-\beta+\sqrt{\beta^2+4\alpha\cdot(2+\alpha-\beta)}}{2\cdot(2+\alpha-\beta)}}^{1} \int_0^{\frac{(2+\alpha-\beta)\cdot b+\beta-\alpha\cdot b^{-1}}{2\cdot(1+\alpha)}} \left(b - \frac{1-b^2}{2b} - 2s\right) \cdot 2b \cdot ds \cdot db &= 0; \\ \int_{\frac{-\beta+\sqrt{\beta^2+4\alpha\cdot(2+\alpha-\beta)}}{2\cdot(2+\alpha-\beta)}}^{1} \int_0^{\frac{(2+\alpha-\beta)\cdot b+\beta-\alpha\cdot b^{-1}}{2\cdot(1+\alpha)}} \left(b - b^2\right) \cdot ds \cdot db &= 0. \end{aligned}$$

Table 3.1 below shows how some important values vary with investment cost (K increases down the column). β increases with K as the investment constraint becomes "more binding" with higher cost of investment. At the same time, α also increases so as to restore the participation constraint. As expected, both the gross surplus and the net surplus (gross surplus minus cost of investment) decrease down the column because the optimal mechanism becomes more distorted.

Figure 3.1 compares the areas where trade takes place before and after the investment constraint becomes binding. Let us look at the top diagram first. The thin line divides the diagram into two parts below which trade occurs under x^m. Similarly, trade occurs below the thick line under x^*. The middle diagram depicts the same graphs when the investment constraint is more binding (when K is higher). As is apparent in these diagrams, the investment constraint opens trade in some areas and closes trade in others. To see how the trading areas are moved, the bottom diagram plots $(F^o - F^*) \cdot g = b - b^2$ as a function of b. Not surprisingly, trade tends to move to areas to take advantage of higher values of $b - b^2$ as the investment constraint becomes more binding. The last column in table 3.1 shows that the overall trading area may either grow or shrink.

Although $A2$ is violated in our example, it can be verified that $p(b)$ is non-decreasing and $q(s)$ is non-increasing in any of these optimal mechanisms. It can be checked that the joint surplus obtained from the MS mechanism without investment is only 0.140625. Therefore, inducing investment, together with the corresponding mechanism in table 3.1, indeed constitutes the overall optimal solution.

α	β	**Gross Surplus**	**Net Surplus**	**Area of Trade**
$\alpha^m(K \leq 0.0401)$	0	0.2175	$0.17736 - K$	0.2675
0.3	0.0061	0.2175	0.17734	0.2676
0.4	0.5787	0.2167	0.1747	0.2699
0.5	1.1196	0.2149	0.1717	0.2704
0.6	1.6398	0.2125	0.1685	0.2696
0.7	2.1452	0.2097	0.1653	0.2680
0.8	2.6395	0.2069	0.1624	0.2660
0.9	3.1251	0.2039	0.1590	0.2636
$1(K = 0.0450)$	3.6035	0.2010	0.1561	0.2611

Table 3.1: Numerical Comparative Statics for Example E3.1.

3.4 Concluding Remarks

We conclude this chapter by discussing some extensions and applications.

Although we have concentrated on one-sided investment, the analysis for two-sided investment is completely analogous. In addition to the various existing constraints, there is also an investment constraint for the seller

$$\int_{b^l}^{b^h} \int_{s^l}^{s^h} (G^o - G^*) \cdot x \cdot f \cdot db \cdot ds \geq J$$

where G^*, G^o are the cost distributions with and without investment and J is the investment cost for the seller.

Similarly, our assumption of binary-investment choice can be relaxed so that the investment cost I can take on a continuum of values. In this scenario, $C2$ may be replaced by the first-order condition:[18]

$$-\int_{b^l}^{b^h}\int_{s^l}^{s^h} F_I \cdot x \cdot g \cdot db \cdot ds - K'(I) = 0$$

where $K(I)$ is the investment cost function.

Finally, as mentioned in the introduction, our research is motivated by the hold-up model with asymmetric information. A direct application of our results involves designing the best institution for trade to take place in such settings. In reality, however, the optimal direct mechanism derived in this paper may not be executed for practical reasons. Another potential application is therefore to see, within a class of indirect/simple trading mechanisms frequently observed in the real world, which is closest to the optimal direct mechanism derived in this paper.

[18] In general, additional conditions are required to assure that the first order approach is valid. This is analogous to the critique of the first order approach used in the moral hazard problem (see Rogerson (1985)).

3.A Appendix of Figures

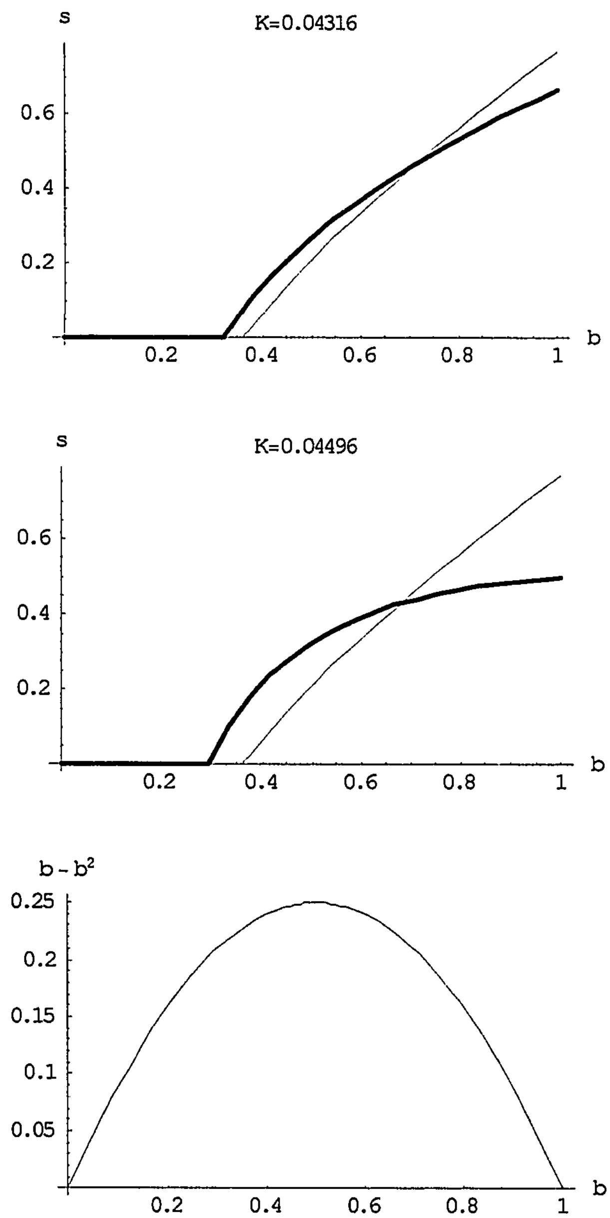

Figure 3.1: Graphical Comparative Statics for Example E3.1.

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