14.461: Technological Change, Lecture 1

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Introduction

- The key to understanding *technology* is that R&D and technology adoption are purposeful activities, so improvements in technology often result from endogenous innovation.
- This lecture will review the two most popular macroeconomic models of technological change:
  1. Those with expanding variety of inputs or machines used in production, developed in Romer (1990).
  2. The "Schumpeterian models" with quality improvements and creative destruction as in Aghion and Howitt (1992) and Grossman and Helpman (1991).
- The first set of models should have been covered in 14.452, so this is just a review to fix the notation and bring out the contrasts with the Schumpeterian models. These lecture notes contain material both for the first lecture and the first recitation.

Key Insights

- Innovation as generating new blueprints or *ideas* for production.
- Three important features (Romer):
  1. Ideas and technologies *nonrival*—many firms can benefit from the same idea.
  2. Increasing returns to scale—constant returns to scale to capital, labor, material etc. and then ideas and blueprints are also produced.
  3. Costs of research and development paid as fixed costs upfront.
- We must consider models of *monopolistic competition*, where firms that innovate become monopolists and make profits.
  - Throughout simplify modeling by using the Dixit-Stiglitz constant elasticity structure.
- Major shortcoming (to be addressed in the rest of the course): no microstructure, no firm structure and no easy way of mapping these models to data.

Demographics, Preferences, and Technology

- Infinite-horizon economy, continuous time.
- Representative household with preferences:

$$\int_0^\infty \exp\left(-\rho t\right) \frac{C\left(t\right)^{1-\theta}-1}{1-\theta} dt. \tag{1}$$

- L =total (constant) population of workers. Labor supplied inelastically.
- Representative household owns a balanced portfolio of all the firms in the economy.

Demographics, Preferences, and Technology I

- Unique consumption good, produced with aggregate production function:

$$Y(t) = \frac{1}{1-\beta}\left[\int_0^{N(t)} x(\nu,t)^{1-\beta} d\nu\right] L^{\beta}, \tag{2}$$

  where

  - $N(t)$=number of varieties of inputs (machines) at time t,
  - $x(\nu,t)$=amount of input (machine) type ν used at time t.

- The x's depreciate fully after use.
- They can be interpreted as generic inputs, intermediate goods, machines, or capital.
- Thus machines are *not* additional state variables.
- For given $N(t)$, which final good producers take as given, (2) exhibits constant returns to scale.

Demographics, Preferences, and Technology II

- Final good producers are competitive.
- The resource constraint of the economy at time t is

$$C(t) + X(t) + Z(t) \leq Y(t), \tag{3}$$

  where $X(t)$ is investment on inputs at time t and $Z(t)$ is expenditure on R&D at time t.
- Once the blueprint of a particular input is invented, the research firm can create one unit of that machine at marginal cost equal to $\psi > 0$ units of the final good.

Innovation Possibilities Frontier and Patents I

- *Innovation possibilities frontier*:

$$\dot{N}(t) = \eta Z(t), \tag{4}$$

  where $\eta > 0$, and the economy starts with some $N(0) > 0$.
- There is free entry into research: any individual or firm can spend one unit of the final good at time t in order to generate a flow rate η of the blueprints of new machines.
- The firm that discovers these blueprints receives a *fully-enforced perpetual patent* on this machine.
- There is no aggregate uncertainty in the innovation process.
  - There will be uncertainty at the level of the individual firm, but with many different research labs undertaking such expenditure, at the aggregate level, equation (4) holds deterministically.

Innovation Possibilities Frontier and Patents II

- A firm that invents a new machine variety v is the sole supplier of that type of machine, and sets a profit-maximizing price of $p^x(\nu, t)$ at time t to maximize profits.
- Since machines depreciate after use, $p^x(\nu, t)$ can also be interpreted as a "rental price" or the user cost of this machine.

The Final Good Sector

- Maximization by final the producers:

$$\max_{[x(\nu,t)]_{l\nu\in[0,N(t)]},L} \frac{1}{1-\beta}\left[\int_0^{N(t)} x(\nu,t)^{1-\beta}d\nu\right]L^{\beta} \tag{5}$$
$$-\int_0^{N(t)} p^x(\nu\,,t)\,x(\nu,t)d\nu - w(t)\,L.$$

- Demand for machines:

$$x(\nu,t) = p^x(\nu,t)^{-1/\beta}L, \tag{6}$$

- Isoelastic demand for machines.
- Only depends on the user cost of the machine and on equilibrium labor supply but not on the interest rate, $r(t)$, the wage rate, $w(t)$, or the total measure of available machines, $N(t)$.

Profit Maximization by Technology Monopolists I

- Consider the problem of a monopolist owning the blueprint of a machine of type ν invented at time t.
- Maximize value discounted profits:

$$V(\nu, t) = \int_t^\infty \exp\left[-\int_t^s r\left(s'\right) ds'\right] \pi(\nu, s)\, ds \tag{7}$$

where

$$\pi(\nu, t) \equiv p^x(\nu, t) x(\nu, t) - \psi x(\nu, t)$$

and $r\left(t\right)$ is the market interest rate at time t.

- Value function in the alternative Hamilton-Jacobi-Bellman form:

$$r\left(t\right) V(\nu, t) - \dot{V}(\nu, t) = \pi(\nu, t). \tag{8}$$

Characterization of Equilibrium I

- Since (6) defines isoelastic demands, the solution to the maximization problem of any monopolist $\nu \in [0, N(t)]$ involves setting the same price in every period:

$$p^x(\nu, t) = \frac{\psi}{1-\beta} \text{ for all } \nu \text{ and } t. \tag{9}$$

- Normalize $\psi \equiv (1-\beta)$, so that

$$p^x(\nu, t) = p^x = 1 \text{ for all } \nu \text{ and } t.$$

- Profit-maximization also implies that each monopolist rents out the same quantity of machines in every period, equal to

$$x(\nu, t) = L \text{ for all } \nu \text{ and } t. \tag{10}$$

Characterization of Equilibrium II

- Monopoly profits:

$$\pi(\nu, t) = \beta L \text{ for all } \nu \text{ and } t. \tag{11}$$

- Substituting (6) and the machine prices into (2) yields:

$$Y(t) = \frac{1}{1-\beta} N(t) L. \tag{12}$$

- Even though the aggregate production function exhibits constant returns to scale from the viewpoint of final good firms (which take $N(t)$ as given), there are *increasing returns to scale* for the entire economy;
- An increase in $N(t)$ raises the productivity of labor and when $N(t)$ increases at a constant rate so will output per capita.

Characterization of Equilibrium III

- Equilibrium wages:

$$w(t) = \frac{\beta}{1-\beta} N(t). \tag{13}$$

- Free entry

$$\begin{aligned} \eta V(\nu, t) &\leq 1,\ Z(\nu, t) \geq 0 \text{ and} \\ (\eta V(\nu, t) - 1) Z(\nu, t) &= 0,\ \text{for all } \nu \text{ and } t, \end{aligned} \tag{14}$$

  where $V(\nu, t)$ is given by (7).

- For relevant parameter values with positive entry and economic growth:

$$\eta V(\nu, t) = 1.$$

Characterization of Equilibrium IV

- Since each monopolist $\nu \in [0, N(t)]$ produces machines given by (10), and there are a total of $N(t)$ monopolists, the total expenditure on machines is

$$X(t) = N(t) L. \tag{15}$$

- Finally, the representative household's problem is standard and implies the usual Euler equation:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho) \tag{16}$$

and the transversality condition

$$\lim_{t \to \infty} \left[\exp\left(-\int_0^t r(s)\, ds \right) N(t)\, V(t) \right] = 0. \tag{17}$$

Equilibrium and Balanced Growth Path I

- An equilibrium is given by time paths
  - $[C(t), X(t), Z(t), N(t)]_{t=0}^{\infty}$, such that (3), (15), (16), (17) and (14) are satisfied;
  - $[p^x(\nu, t), x(\nu, t)]_{\nu \in N(t), t=0}^{\infty}$ that satisfy (9) and (10),
  - $[r(t), w(t)]_{t=0}^{\infty}$ such that (13) and (16) hold.
- A *balanced growth path (BGP)* as an equilibrium path where $C(t), X(t), Z(t)$ and $N(t)$ grow at a constant rate. Such an equilibrium can alternatively be referred to as a "steady state", since it is a steady state in transformed variables.

Balanced Growth Path I

- A balanced growth path (BGP) requires that consumption grows at a constant rate, say g_C. This is only possible from (16) if

$$r(t) = r^* \text{ for all } t$$

- Since profits at each date are given by (11) and since the interest rate is constant, $\dot{V}(t) = 0$ and

$$V^* = \frac{\beta L}{r^*}. \tag{18}$$

Balanced Growth Path II

- Let us next suppose that the (free entry) condition (14) holds as an equality, in which case we also have

$$\frac{\eta \beta L}{r^*} = 1$$

This equation pins down the steady-state interest rate, r^*, as:

$$r^* = \eta \beta L$$

- The consumer Euler equation, (16), then implies that the rate of growth of consumption must be given by

$$g_C^* = \frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r^* - \rho). \tag{19}$$

Balanced Growth Path III

- Note the current-value Hamiltonian for the consumer's maximization problem is concave, thus this condition, together with the transversality condition, characterizes the optimal consumption plans of the consumer.
- In BGP, consumption grows at the same rate as total output

$$g^* = g_C^*.$$

Therefore, given r^*, the long-run growth rate of the economy is:

$$g^* = \frac{1}{\theta}\left(\eta\beta L - \rho\right) \tag{20}$$

- Suppose that

$$\eta\beta L > \rho \text{ and } \left(1-\theta\right)\eta\beta L < \rho, \tag{21}$$

which ensures $g^* > 0$ and the transversality condition is satisfied.

Transitional Dynamics

- There are no transitional dynamics in this model.
- Substituting for profits in the value function for each monopolist, this gives

$$r(t)\,V(\nu,t) - \dot{V}(\nu,t) = \beta L.$$

- The key observation is that positive growth at any point implies that $\eta V(\nu,t) = 1$ for all t. In other words, if $\eta V(\nu,t') = 1$ for some t', then $\eta V(\nu,t) = 1$ for all t.
- Now differentiating $\eta V(\nu,t) = 1$ with respect to time yields $\dot{V}(\nu,t) = 0$, which is only consistent with $r(t) = r^*$ for all t, thus

$$r(t) = \eta\beta L \text{ for all } t.$$

Summary

Proposition Suppose that condition (21) holds. Then, in this model there exists a unique balanced growth path in which technology, output and consumption all grow at the same rate, g^*, given by (20). Moreover, there are no transitional dynamics. That is, starting with initial technology stock $N(0) > 0$, there is a unique equilibrium path in which technology, output and consumption always grow at the rate g^* as in (20).

Social Planner Problem I

- Monopolistic competition implies that the competitive equilibrium is not necessarily Pareto optimal. The model exhibits a version of the *aggregate demand externalities*:
  1. There is a markup over the marginal cost of production of inputs.
  2. The number of inputs produced at any point in time may not be optimal.
- The first inefficiency is familiar from models of static monopoly, while the second emerges from the fact that in this economy the set of traded (Arrow-Debreu) commodities is endogenously determined.
- This relates to the issue of endogenously incomplete markets (there is no way to purchase an input that is not supplied in equilibrium).

Social Planner Problem II

- Given $N(t)$, the social planner will choose

$$\max_{[x(\nu,t)]_{\nu\in[0,N(t)]},L} \frac{1}{1-\beta}\left[\int_0^{N(t)} x(\nu,t)^{1-\beta}d\nu\right]L^{\beta}-\int_0^{N(t)}\psi x(\nu,t)d\nu,$$

- Differs from the equilibrium profit maximization problem, (5), because the marginal cost of machine creation, ψ, is used as the cost of machines rather than the monopoly price, and the cost of labor is not subtracted.
- Recalling that $\psi\equiv 1-\beta$, the solution to this program involves

$$x^S(\nu,t)=(1-\beta)^{-1/\beta}L,$$

Social Planner Problem III

- The *net* output level (after investment costs are subtracted) is

$$\begin{aligned} Y^S(t) &= \frac{(1-\beta)^{-(1-\beta)/\beta}}{1-\beta} N^S(t) L \\ &= (1-\beta)^{-1/\beta} N^S(t) L, \end{aligned}$$

- Therefore, the maximization problem of the social planner can be written as

$$\max \int_0^\infty \frac{C(t)^{1-\theta} - 1}{1-\theta} \exp(-\rho t)\, dt$$

subject to

$$\dot{N}(t) = \eta (1-\beta)^{-1/\beta} \beta N(t) L - \eta C(t).$$

where $(1-\beta)^{-1/\beta} \beta N^S(t) L$ is net output.

Social Planner Problem IV

- In this problem, $N(t)$ is the state variable, and $C(t)$ is the control variable. The current-value Hamiltonian is:

$$\begin{aligned}\hat{H}(N, C, \mu) &= \frac{C(t)^{1-\theta} - 1}{1-\theta} \\ &\quad + \mu(t)\left[\eta(1-\beta)^{-1/\beta}\beta N(t) L - \eta C(t)\right].\end{aligned}$$

- The conditions for a candidate Pareto optimal allocation are:

$$\begin{aligned}\hat{H}_C(N, C, \mu) &= C(t)^{-\theta} - \eta\mu(t) = 0 \\ \hat{H}_N(N, C, \mu) &= \mu(t)\eta(1-\beta)^{-1/\beta}\beta L \\ &= \rho\mu(t) - \dot{\mu}(t) \\ \lim_{t\to\infty}\left[\exp(-\rho t)\mu(t) N(t)\right] &= 0.\end{aligned}$$

Comparison of Equilibrium and Pareto Optimum

- The current-value Hamiltonian is (strictly) concave, thus these conditions are also sufficient for an optimal solution.
- Combining these conditions:

$$\frac{\dot{C}^S(t)}{C^S(t)} = \frac{1}{\theta}\left(\eta (1-\beta)^{-1/\beta} \beta L - \rho\right). \tag{22}$$

- The comparison to the growth rate in the decentralized equilibrium, (20), boils down to that of

$$(1-\beta)^{-1/\beta} \beta \text{ to } \beta.$$

- The socially-planned economy *always has a higher growth rate* than the decentralized economythe former is always greater since $(1-\beta)^{-1/\beta} > 1$ by virtue of the fact that $\beta \in (0,1)$.
- Why? Because of a *pecuniary externality*: the social planner values innovation more because she is able to use the machines more intensively after innovation, and this encourages more innovation.

The Effects of Competition I

- Recall that the monopoly price is:

$$p^x = \frac{\psi}{1-\beta}.$$

- Imagine, instead, that a fringe of competitive firms can copy the innovation of any monopolist.
  - But instead of a marginal cost ψ, the fringe has marginal cost of $\gamma\psi$ with $\gamma > 1$.
- If $\gamma > 1/(1-\beta)$, no threat from the fringe.
- If $\gamma < 1/(1-\beta)$, the fringe would forced the monopolist to set a "*limit price*",

$$p^x = \gamma\psi. \tag{23}$$

The Effects of Competition II

- Why? If $p^x > \gamma\psi$, the fringe could undercut the price of the monopolist, take over to market and make positive profits.
  If $p^x < \gamma\psi$, the monopolist could increase price and make more profits.
  Thus, there is a unique equilibrium price given by (23).
- Profits under the limit price:

$$\text{profits per unit } = (\gamma - 1)\,\psi = (\gamma - 1)\,(1 - \beta) < \beta,$$

- Therefore, growth with competition:

$$\hat{g} = \frac{1}{\theta}\left(\eta\gamma^{-1/\beta}\,(\gamma - 1)\,(1 - \beta)^{-(1-\beta)/\beta}\,L - \rho\right) < g^*.$$

Growth with Knowledge Spillovers

- In the lab equipment model, growth resulted from the use of final output for R&D. This is similar to the endogenous growth model of Rebelo (1991), since the accumulation equation is linear in accumulable factors. In equilibrium, output took a linear form in the stock of knowledge (new machines), thus a AN form instead of Rebelo's AK form.
- An alternative is to have "scarce factors" used in R&D: we have scientists as the key creators of R&D.
- With this alternative, there cannot be endogenous growth unless there are knowledge spillovers from past R&D, making the scarce factors used in R&D more and more productive over time.

Innovation Possibilities Frontier I

- Innovation possibilities frontier in this case:

$$\dot{N}(t) = \eta N(t) L_R(t) \tag{24}$$

  where $L_R(t)$ is labor allocated to R&D at time t.
- The term $N(t)$ on the right-hand side captures spillovers from the stock of existing ideas.
- Notice that (24) imposes that these spillovers are proportional or linear. This linearity will be the source of endogenous growth in the current model.
- In (24), $L_R(t)$ comes out of the regular labor force.The cost of workers to the research sector is given by the wage rate in final good sector.

Characterization of Equilibrium I

- Labor market clearing:

$$L_R(t) + L_E(t) \leq L.$$

- Aggregate output of the economy:

$$Y(t) = \frac{1}{1-\beta} N(t) L_E(t), \tag{25}$$

and profits of monopolists from selling their machines is

$$\pi(t) = \beta L_E(t). \tag{26}$$

- The net present discounted value of a monopolist (for a blueprint ν) is still given by $V(\nu, t)$ as in (7) or (8), with the flow profits given by (26).

Characterization of Equilibrium II

- Free entry now implies:

$$\eta N(t) V(\nu, t) = w(t), \tag{27}$$

where $N(t)$ is on the left-hand side because it parameterizes the productivity of an R&D worker, while the flow cost of undertaking research is hiring workers for R&D, thus is equal to the wage rate $w(t)$.

- The equilibrium wage rate must be the same as before:

$$w(t) = \beta N(t) / (1 - \beta)$$

Characterization of Equilibrium III

- Balanced growth again requires that the interest rate must be constant at some level r^*, and in particular

$$\eta N(t) \frac{\beta L_E(t)}{r^*} = \frac{\beta}{1-\beta} N(t). \tag{28}$$

and thus

$$r^* = (1-\beta)\eta L_E^*,$$

where $L_E^* = L - L_R^*$. The fact that the number of workers in production must be constant in BGP follows from (28).

- From the Euler equation, (16), for all t:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}\left((1-\beta)\eta L_E^* - \rho\right) \equiv g^*. \tag{29}$$

Characterization of Equilibrium IV

- But also, in BGP, (24):

$$\frac{\dot{N}(t)}{N(t)} = \eta L_R^* = \eta \left(L - L_E^* \right)$$

This implies that the BGP level of employment is

$$L_E^* = \frac{\theta \eta L + \rho}{(1-\beta)\eta + \theta \eta}. \tag{30}$$

Summary of Equilibrium in the Model with Knowledge Spillovers

Proposition Consider the above-described expanding input-variety model with knowledge spillovers and suppose that

$$(1-\theta)(1-\beta)\,\eta L_E^* < \rho < (1-\beta)\,\eta L_E^*, \tag{31}$$

where L_E^* is the number of workers employed in production in BGP, given by (30).Then there exists a unique balanced growth path in which technology, output and consumption grow at the same rate, $g^* > 0$, given by (29) starting from any initial level of technology stock $N(0) > 0$.

- As in the lab equipment model, the equilibrium allocation is Pareto suboptimal, but now more severely because of uninternalized knowledge spillovers. (Why?)

Introduction

- Most process innovations either increase the quality of an existing product or reduce the costs of production.
- *Competitive* aspect of innovations: a newly-invented superior computer often *replaces* existing vintages.
- Realm of Schumpeterian *creative destruction*.
- Schumpeterian growth raises important issues:
  1. Direct price competition between producers with different vintages of quality or different costs of producing
  2. Competition between incumbents and entrants: *business stealing effect.*

Preferences and Technology I

- Again:
  - Continuous time;
  - Representative household with standard CRRA preferences;
  - Constant population L, and labor supplied inelastically.
- Resource constraint:

$$C(t) + X(t) + Z(t) \leq Y(t), \tag{32}$$

- Normalize the measure of inputs to 1, and denote each machine line by $\nu \in [0, 1]$.

Preferences and Technology II

- Engine of economic growth: *quality improvement*.
- $q(\nu, t)$ =quality of machine line ν at time t.
- "Quality ladder" for each machine type:

$$q(\nu, t) = \lambda^{n(\nu,t)} q(\nu, 0) \text{ for all } \nu \text{ and } t, \tag{33}$$

where:

  - $\lambda > 1$
  - $n(\nu, t)$ =innovations on this machine line between 0 and t.

- Production function of the final good:

$$Y(t) = \frac{1}{1-\beta} \left[\int_0^1 q(\nu, t) x(\nu, t \mid q)^{1-\beta} d\nu \right] L^{\beta}, \tag{34}$$

where $x(\nu, t \mid q)$=quantity of machine of type ν quality q.

Preferences and Technology III

- Implicit assumption in (34): at any point in time only one quality of any machine is used.
- *Creative destruction*: when a higher-quality machine is invented it will replace ("destroy") the previous vintage of machines.
- Why?

Innovation Possibilities Frontier I

- Cumulative R&D process.
- $Z(\nu, t)$ units of the final good for research on machine line ν, quality $q(\nu, t)$ generate a flow rate

$$\eta Z(\nu, t) / q(\nu, t)$$

  of innovation.
- Note one unit of R&D spending is proportionately less effective when applied to a more advanced machine.
- Free entry into research.
- The firm that makes an innovation has a perpetual patent.
- But other firms can undertake research based on the product invented by this firm.

Innovation Possibilities Frontier II

- Once a machine of quality $q(\nu, t)$ has been invented, any quantity can be produced at the marginal cost $\psi q(\nu, t)$.
- New entrants undertake the R&D and innovation:
    - The incumbent has weaker incentives to innovate, since it would be replacing its own machine, and thus destroying the profits that it is already making (*Arrow's replacement effect*).

Equilibrium: Innovations Regimes

- Demand for machines similar to before:

$$x(\nu, t \mid q) = \left(\frac{q(\nu, t)}{p^x(\nu, t \mid q)}\right)^{1/\beta} L \quad \text{for all } \nu \in [0, 1] \text{ and all } t, \quad (35)$$

  where $p^x(\nu, t \mid q)$ refers to the price of machine type ν of quality $q(\nu, t)$ at time t.
- Two regimes:
  1. innovation is "drastic" and each firm can charge the unconstrained monopoly price,
  2. limit prices have to be used.
- Assume drastic innovations regime: λ is sufficiently large

$$\lambda \geq \left(\frac{1}{1-\beta}\right)^{\frac{1-\beta}{\beta}}. \quad (36)$$

- Again normalize $\psi \equiv 1 - \beta$

Monopoly Profits

- Profit-maximizing monopoly:

$$p^x (\nu, t \mid q) = q (\nu, t) . \tag{37}$$

- Combining with (35)

$$x (\nu, t \mid q) = L. \tag{38}$$

- Thus, flow profits of monopolist:

$$\pi (\nu, t \mid q) = \beta q (\nu, t) L.$$

Characterization of Equilibrium I

- Substituting (38) into (34):

$$Y(t) = \frac{1}{1-\beta} Q(t) L, \tag{39}$$

where

$$Q(t) \equiv \int_0^1 q(\nu, t) d\nu. \tag{40}$$

- Equilibrium wage rate:

$$w(t) = \frac{\beta}{1-\beta} Q(t). \tag{41}$$

Characterization of Equilibrium II

- Value function for monopolist of variety ν of quality $q(\nu, t)$ at time t:

$$r(t)\,V(\nu, t \mid q) - \dot{V}(\nu, t \mid q) = \pi(\nu, t \mid q) - z(\nu, t \mid q)\,V(\nu, t \mid q), \tag{42}$$

  where:

  - $z(\nu, t \mid q)$=rate at which new innovations occur in sector ν at time t,
  - $\pi(\nu, t \mid q)$=flow of profits.

- Last term captures the essence of Schumpeterian growth:
  - when innovation occurs, the monopolist loses its monopoly position and is replaced by the producer of the higher-quality machine.
  - From then on, it receives zero profits, and thus has zero value.
  - Because of Arrow's replacement effect, an entrant undertakes the innovation, thus $z(\nu, t \mid q)$ is the flow rate at which the incumbent will be replaced.

Characterization of Equilibrium III

- Free entry:

$$\eta V(\nu, t \mid q) \leq \lambda^{-1} q(\nu, t) \tag{43}$$
$$\text{and } \eta V(\nu, t \mid q) = \lambda^{-1} q(\nu, t) \text{ if } Z(\nu, t \mid q) > 0.$$

- Note: Even though the $q(\nu, t)$'s are stochastic as long as the $Z(\nu, t \mid q)$'s, are nonstochastic, average quality $Q(t)$, and thus total output, $Y(t)$, and total spending on machines, $X(t)$, will be nonstochastic.
- Consumer maximization implies the Euler equation,

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho), \tag{44}$$

- Transversality condition:

$$\lim_{t \to \infty} \left[\exp\left(-\int_0^t r(s)\, ds \right) \int_0^1 V(\nu, t \mid q)\, d\nu \right] = 0 \tag{45}$$

for all q.

Definition of Equilibrium

- $V(\nu, t \mid q)$, is nonstochastic: either q is not the highest quality in this machine line and $V(\nu, t \mid q)$ is equal to 0, or it is given by (42).
- An equilibrium can then be represented as time paths of
  - $[C(t), X(t), Z(t)]_{t=0}^{\infty}$ that satisfy (32), (**??**), (45),
  - $[Q(t)]_{t=0}^{\infty}$ and $[V(\nu, t \mid q)]_{\nu\in[0,1],t=0}^{\infty}$ consistent with (40), (42) and (43),
  - $[p^{x}(\nu, t \mid q), x(\nu, t)]_{\nu\in[0,1],t=0}^{\infty}$ given by (37) and (38), and
  - $[r(t), w(t)]_{t=0}^{\infty}$ that are consistent with (41) and (44)
- Balanced Growth Path defined similarly to before (constant growth of output, constant interest rate).

Balanced Growth Path I

- In BGP, consumption grows at the constant rate g_C^*, that must be the same rate as output growth, g^*.
- From (44), $r(t) = r^*$ for all t.
- If there is positive growth in BGP, there must be research at least in some sectors.
- Since profits and R&D costs are proportional to quality, whenever the free entry condition (43) holds as equality for one machine type, it will hold as equality for all of them.
- Thus,

$$V(\nu, t \mid q) = \frac{q(\nu, t)}{\lambda \eta}. \tag{46}$$

- Moreover, if it holds between t and $t + \Delta t$, $\dot{V}(\nu, t \mid q) = 0$, because the right-hand side of equation (46) is constant over time—$q(\nu, t)$ refers to the quality of the machine supplied by the incumbent, which does not change.

Balanced Growth Path II

- Since R&D for each machine type has the same productivity, constant in BGP:

$$z(\nu, t) = z(t) = z^*$$

- Then (42) implies

$$V(\nu, t \mid q) = \frac{\beta q(\nu, t) L}{r^* + z^*}. \tag{47}$$

- Note the *effective discount rate* is $r^* + z^*$.
- Combining this with (46):

$$r^* + z^* = \lambda \eta \beta L. \tag{48}$$

- From the fact that $g_C^* = g^*$ and (44), $g^* = (r^* - \rho)/\theta$, or

$$r^* = \theta g^* + \rho. \tag{49}$$

Balanced Growth Path III

- To solve for the BGP equilibrium, we need a final equation relating g^* to z^*. From (39)

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{Q}(t)}{Q(t)}.$$

- Note that in an interval of time Δt, $z(t)\Delta t$ sectors experience one innovation, and this will increase their productivity by λ.
- The measure of sectors experiencing more than one innovation within this time interval is $o(\Delta t)$—i.e., it is second-order in Δt, so that

$$\text{as } \Delta t \to 0,\ o(\Delta t)/\Delta t \to 0.$$

- Therefore, we have

$$Q(t+\Delta t) = \lambda Q(t) z(t) \Delta t + (1 - z(t)\Delta t) Q(t) + o(\Delta t).$$

Balanced Growth Path IV

- Now subtracting $Q(t)$ from both sides, dividing by Δt and taking the limit as $\Delta t \to 0$, we obtain

$$\dot{Q}(t) = (\lambda - 1) z(t) Q(t).$$

- Therefore,

$$g^* = (\lambda - 1) z^*. \tag{50}$$

- Now combining (48)-(50), we obtain:

$$g^* = \frac{\lambda \eta \beta L - \rho}{\theta + (\lambda - 1)^{-1}}. \tag{51}$$

Summary

Proposition In the model of Schumpeterian growth, suppose that

$$\lambda\eta\beta L > \rho > (1-\theta)\frac{\lambda\eta\beta L-\rho}{\theta+(\lambda-1)^{-1}}. \tag{52}$$

Then, there exists a unique BGP in which average quality of machines, output and consumption grow at rate g^* given by (51). The rate of innovation is $g^*/(\lambda-1)$. Moreover, starting with any average quality of machines $Q(0)>0$, there are no transitional dynamics and the equilibrium path always involves constant growth at the rate g^* given by (51).

- Note only the average quality of machines, $Q(t)$, matters for the allocation of resources.
  - In fact, little discipline on firm or micro innovation structure.
- Moreover, the incentives to undertake research are identical for two machine types ν and ν', with different quality levels $q(\nu,t)$ and $q(\nu',t)$.

Pareto Optimality

- This equilibrium is typically Pareto suboptimal.
- But now distortions more complex than the expanding varieties model.
  - monopolists are not able to capture the entire social gain created by an innovation.
  - Business stealing effect.
- The equilibrium rate of innovation and growth can be too high or too low.

Social Planner's Problem I

- Quantities of machines used in the final good sector: no markup.

$$\begin{aligned} x^S(\nu, t \mid q) &= \psi^{-1/\beta} L \\ &= (1-\beta)^{-1/\beta} L. \end{aligned}$$

- Substituting into (34):

$$Y^S(t) = (1-\beta)^{-1/\beta} Q^S(t) L,$$

Social Planner's Problem II

- Maximization problem of the social planner:

$$\max \int_0^\infty \frac{C^S(t)^{1-\theta} - 1}{1-\theta} \exp(-\rho t)\, dt$$

subject to

$$\dot{Q}^S(t) = \eta(\lambda - 1)(1-\beta)^{-1/\beta} \beta Q^S(t) L - \eta(\lambda - 1) C^S(t),$$

where $(1-\beta)^{-1/\beta} \beta Q^S(t) L$ is net output.

Social Planner's Problem III

- Current-value Hamiltonian:

$$\hat{H}\left(Q^S, C^S, \mu^S\right) = \frac{C^S(t)^{1-\theta} - 1}{1-\theta} + \mu^S(t)\left[\begin{array}{c} \eta(\lambda-1)(1-\beta)^{-1/\beta}\beta Q^S(t)L \\ -\eta(\lambda-1)C^S(t) \end{array}\right].$$

Social Planner's Problem IV

- Necessary conditions:

$$\begin{aligned}\hat{H}_C(\cdot) &= C^S(t)^{-\theta} - \mu^S(t)\,\eta(\lambda-1) \\ &= 0 \\ \hat{H}_Q(\cdot) &= \mu^S(t)\,\eta(\lambda-1)(1-\beta)^{-1/\beta}\beta L \\ &= \rho\mu^S(t) - \dot{\mu}^S(t)\end{aligned}$$

$$\lim_{t\to\infty}\left[\exp(-\rho t)\,\mu^S(t)\,Q^S(t)\right] = 0$$

- Combining:

$$\frac{\dot{C}^S(t)}{C^S(t)} = g^S \equiv \frac{1}{\theta}\left(\eta(\lambda-1)(1-\beta)^{-1/\beta}\beta L - \rho\right). \tag{53}$$

Summary of Social Planner's Problem

- Total output and average quality will also grow at the rate g^S.
- Comparing g^S to g^*, either could be greater.
  - When λ is very large, $g^S > g^*$. As $\lambda \to \infty$, $g^S/g^* \to (1-\beta)^{-1/\beta} > 1$.

Proposition In the model of Schumpeterian growth, the decentralized equilibrium is generally Pareto suboptimal, and may have a higher or lower rate of innovation and growth than the Pareto optimal allocation.

Policies I

- Creative destruction implies a natural *conflict of interest*, and certain types of policies may have a constituency.
- Suppose there is a tax τ imposed on R&D spending.
- This has no effect on the profits of existing monopolists, and only influences their net present discounted value via replacement.
- Since taxes on R&D will discourage R&D, there will be replacement at a slower rate, i.e., z^* will fall.
- This increases the steady-state value of all monopolists given by (47):

$$V(q) = \frac{\beta q L}{r^*(\tau) + z^*(\tau)},$$

- The free entry condition becomes

$$V(q) = \frac{(1+\tau)}{\lambda \eta} q.$$

Policies II

- $V(q)$ is clearly increasing in the tax rate on R&D, τ.
- Combining the previous two equations, we see that in response to a positive rate of taxation, $r^*(\tau) + z^*(\tau)$ must adjust downward.
- Intuitively, when the costs of R&D are raised because of tax policy, the value of a successful innovation, $V(q)$, must increase to satisfy the free entry condition. This can only happen through a decline the effective discount rate $r^*(\tau) + z^*(\tau)$.
- A lower effective discount rate, in turn, is achieved by a decline in the equilibrium growth rate of the economy:

$$g^*(\tau) = \frac{(1+\tau)^{-1}\lambda\eta\beta L - \rho}{\theta + (\lambda - 1)^{-1}}.$$

- This growth rate is strictly decreasing in τ, but incumbent monopolists would be in favor of increasing τ.

Conclusion

- Two different conceptions of aggregate technological change.
- But in either case, no plausible microstructure or ability to use the model with microdata.
- Also limited or counterfactual comparative statics.
- We will address these issues and the rest of the course.