Labor Economics, 14.661. Lecture 12: Equilibrium Search and Matching

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Introduction

- To circumvent the Rothschild and the Diamond paradoxes, assume *no wage posting* but instead *wage determination by bargaining*
- Where are the search frictions?
- Reduced form: *matching function*
- Continue to assume *undirected search.*
  $\rightarrow$ Baseline equilibrium model: Diamond-Mortensen-Pissarides (DMP) framework.

Introduction (continued)

- Very tractable framework for analysis of unemployment (level, composition, fluctuations, trends)
- Widely used in macro and labor
- Roughly speaking: flows approach meets equilibrium
- Shortcoming: reduced form matching function.

Setup

- Continuous time, infinite horizon economy with risk neutral agents.
- Matching Function:
    Matches $= x(U, V)$
- Continuous time: $x(U, V)$ as the flow rate of matches.
- Assume that $x(U, V)$ exhibits constant returns to scale.

Matching Function

- Therefore:

$$\text{Matches} = xL = x(uL, vL)$$
$$\implies x = x(u, v)$$

$U =$ unemployment;
$u =$ unemployment rate
$V =$ vacancies;
$v =$ vacancy rate (per worker in labor force)
$L =$ labor force

Evidence and Interpretation

- Existing aggregate evidence suggests that the assumption of x exhibiting CRS is reasonable.
- Intuitively, one might have expected "increasing returns" if the matching function corresponds to physical frictions
    - think of people trying to run into each other on an island.
- But the matching function is to reduced form for this type of interpretation.
- In practice, frictions due to differences in the supply and demand for specific types of skills.

Matching Rates and Job Creation

- Using the constant returns assumption, we can express everything as a function of the *tightness of the labor market*.

$$q(\theta) \equiv \frac{x}{v} = x\left(\frac{u}{v}, 1\right),$$

- Here $\theta \equiv v/u$ is the tightness of the labor market

  $q(\theta)$: Poisson arrival rate of match for a vacancy

  $q(\theta)\theta$: Poisson arrival rate of match for an unemployed worker

- Therefore, job creation is equal to

$$\text{Job creation} = u\theta q(\theta)L$$

Job Destruction

- What about job destruction?
- Let us start with the simplest model of job destruction, which is basically to treat it as "exogenous".
- Think of it as follows, firms are hit by adverse shocks, and then they decide whether to destroy or to continue.

$$\longrightarrow \text{Adverse Shock} \longrightarrow \text{destroy}$$
$$\longrightarrow \text{continue}$$

- Exogenous job destruction: Adverse shock $= -\infty$ with "probability" (i.e., flow rate) s

Steady State of the Flow Approach

- As in the partial equilibrium sequential search model
- Steady State:

$$\text{flow into unemployment} = \text{flow out of unemployment}$$

- Therefore, with exogenous job destruction:

$$s(1 - u) = \theta q(\theta) u$$

- Therefore, steady state unemployment rate:

$$u = \frac{s}{s + \theta q(\theta)}$$

- Intuition

The Beverage Curve

- This relationship is also referred to as the *Beveridge Curve*, or the U-V curve.
- It draws a downward sloping locus of unemployment-vacancy combinations in the U-V space that are consistent with flow into unemployment being equal with flow out of unemployment.
- Some authors interpret shifts of this relationship is reflecting structural changes in the labor market, but we will see that there are many factors that might actually shift at a generalized version of such relationship.

Production Side

- Let the output of each firm be given by neoclassical production function combining labor and capital:

$$Y = AF(K, N)$$

- F exhibits constant returns, K is the capital stock of the economy, and N is employment (different from labor force because of unemployment).
- Let

$$k \equiv K/N$$

be the capital labor ratio, then

$$\frac{Y}{N} = Af(k) \equiv AF(\frac{K}{N}, 1)$$

- Also let
  - r: cost of capital
  - δ: depreciation

Production Side: Two Interpretations

- Each firm is a "job" hires one worker
- Each firm can hire as many worker as it likes
- For our purposes either interpretation is fine

Hiring Costs

- Why don't firms open an infinite number of vacancies?
- Hiring activities are costly.
- Vacancy costs γ_0: fixed cost of hiring

Bellman Equations

J^V : PDV of a vacancy

J^F :PDV of a "job"

J^U :PDV of a searching worker

J^E :PDV of an employed worker

Why is J^F not conditioned on k?

Big assumption: perfectly reversible capital investments (why is this important?)

Value of Vacancies

- Perfect capital market gives the asset value for a vacancy (in steady state) as

$$rJ^V = -\gamma_0 + q(\theta)(J^F - J^V)$$

- Intuition?

Labor Demand and Job Creation

- Free Entry $\implies$

$$J^V \equiv 0$$

- If it were positive, more firms would enter.
- Important implication: job creation can happen really "fast", except because of the frictions created by matching searching workers to searching vacancies.
- Alternative would be: $\gamma_0 = \Gamma_0(V)$ or $\Gamma_1(\theta)$, so as there are more and more jobs created, the cost of opening an additional job increases.

Characterization of Equilibrium

- Free entry implies that

$$J^F = \frac{\gamma_0}{q(\theta)}$$

- Asset value equation for the value of a field job:

$$r(J^F + k) = Af(k) - \delta k - w - s(J^F - J^V)$$

- Intuitively, the firm has two assets: the fact that it is matched with a worker, and its capital, k.

- So its asset value is $J^F + k$ (more generally, without the perfect reversability, we would have the more general $J^F(k)$).

- Its return is equal to production, $Af(k)$, and its costs are depreciation of capital and wages, δk and w.

- Finally, at the rate s, the relationship comes to an end and the firm loses J^F.

Wage Determination

- Can wages be equal to marginal cost of labor and value of marginal product of labor?
- No because of labor market frictions
- a worker with a firm is more valuable than an unemployed worker.
- How are wages determined?
- *Nash bargaining* over match specific surplus $J^E + J^F - J^U - J^V$
- Where is k?

Implications of Perfect Reversability

- Perfect Reversability implies that w does not depend on the firm's choice of capital

$\implies$ equilibrium capital utilization $f'(k) = r + \delta$

- *Modified Golden Rule*

Digression: Irreversible Capital Investments

- Much more realistic, but typically not adopted in the literature (why not?)
- Suppose k is not perfectly reversible then suppose that the worker captures a fraction β all the output in bargaining.
- Then the wage depends on the capital stock of the firm, as in the holdup models discussed before.

$$
\begin{aligned}
w(k) &= \beta A f(k) \\
A f'(k) &= \frac{r + \delta}{1 - \beta} \; ; \text{ capital accumulation is distorted}
\end{aligned}
$$

Equilibrium Job Creation

- Free entry together with the Bellman equation for filled jobs implies

$$Af(k) - (r - \delta)k - w - \frac{(r + s)}{q(\theta)}\gamma_0 = 0$$

- For unemployed workers

$$rJ^U = z + \theta q(\theta)(J^E - J^U)$$

  where z is unemployment benefits.

- Employed workers:

$$rJ^E = w + s(J^U - J^E)$$

- Reversibility again: w independent of k.

Values For Workers

- Solving these equations we obtain

$$rJ^U = \frac{(r+s)z + \theta q(\theta)w}{r+s+\theta q(\theta)}$$

$$rJ^E = \frac{sz + [r+\theta q(\theta)]\,w}{r+s+\theta q(\theta)}$$

Nash Bargaining

- Consider the surplus of pair i:

$$
\begin{aligned}
rJ_i^F &= Af(k) - (r + \delta)k - w_i - sJ_i^F \\
rJ_i^E &= w_i - s(J_i^E - J_0^U).
\end{aligned}
$$

- Why is it important to do this for pair i (rather than use the equilibrium expressions above)?

- The Nash solution will solve

$$
\max(J_i^E - J^U)^\beta (J_i^F - J^V)^{1-\beta}
$$

$$
\beta = \quad \text{bargaining power of the worker}
$$

- Since we have linear utility, thus "transferable utility", this implies

$$
J_i^E - J^U = \beta(J_i^F + J_i^E - J^V - J^U)
$$

Nash Bargaining

- Using the expressions for the value functions

$$w = (1 - \beta)z + \beta \left[Af(k) - (r + \delta)k + \theta\gamma_0 \right]$$

- Here

$$Af(k) - (r + \delta)k + \theta\gamma_0$$

  is the quasi-rent created by a match that the firm and workers share.
- Why is the term $\theta\gamma_0$ there?

Steady State Equilibrium

- Steady State Equilibrium is given by four equations

  1. The Beveridge curve:
  $$u = \frac{s}{s + \theta q(\theta)}$$

  2. Job creation leads zero profits:
  $$Af(k) - (r + \delta)k - w - \frac{(r + s)}{q(\theta)}\gamma_0 = 0$$

  3. Wage determination:
  $$w = (1 - \beta)z + \beta\left[Af(k) - (r + \delta)k + \theta\gamma_0\right]$$

  4. Modified golden rule:
  $$Af'(k) = r + \delta$$

Steady State Equilibrium (continued)

- These four equations define a block recursive system

$$(4) + r \longrightarrow k$$

$$k + r + (2) + (3) \longrightarrow \theta, w$$

$$\theta + (1) \longrightarrow u$$

Steady State Equilibrium (continued)

- Alternatively, combining three of these equations we obtain the zero-profit locus, the VS curve.
- Combine this with the Beveridge curve to obtain the equilibrium.

$$(2),(3),(4) \implies \qquad \text{the VS curve}$$

$$(1 - \beta)\left[Af(k) - (r+\delta)k - z\right] - \frac{r + \delta + \beta\theta q(\theta)}{q(\theta)}\gamma_0 = 0$$

- Therefore, the equilibrium looks very similar to the intersection of "quasi-labor demand" and "quasi-labor supply".

Steady State Equilibrium in a Diagram
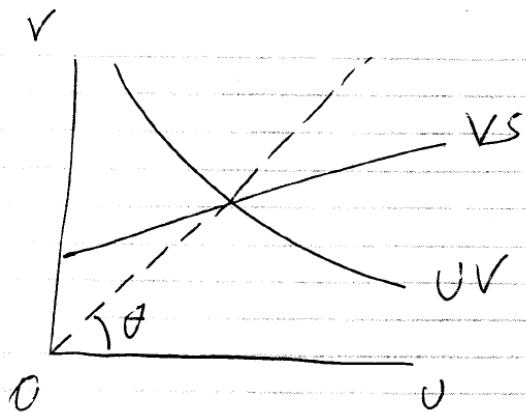
Comparative Statics of the Steady State

- From the figure:

$s \uparrow$	$U \uparrow$	$V \uparrow$	$\theta \downarrow$	$w \downarrow$
$r \uparrow$	$U \uparrow$	$V \downarrow$	$\theta \downarrow$	$w \downarrow$
$\gamma_0 \uparrow$	$U \uparrow$	$V \downarrow$	$\theta \downarrow$	$w \downarrow$
$\beta \uparrow$	$U \uparrow$	$V \downarrow$	$\theta \downarrow$	$w \uparrow$
$z \uparrow$	$U \uparrow$	$V \downarrow$	$\theta \downarrow$	$w \uparrow$
$A \uparrow$	$U \downarrow$	$V \uparrow$	$\theta \uparrow$	$w \uparrow$

- Can we think of any of these factors is explaining the rise in unemployment in Europe during the 1980s, or the lesser rise in unemployment in 1980s in in the United States?

Efficiency?

- Is the search equilibrium efficient?
- Clearly, it is inefficient relative to a first-best alternative, e.g., a social planner that can avoid the matching frictions.
- Instead look at "surplus-maximization" subject to search constraints (why not constrained Pareto optimality?)

Search Externalities

- There are two major externalities

$$\theta \uparrow \quad \Longrightarrow \quad \text{workers find jobs more easily}$$
$$\hookrightarrow \quad \text{thick-market externality}$$
$$\Longrightarrow \quad \text{firms find workers more slowly}$$
$$\hookrightarrow \quad \text{congestion externality}$$

- Why are these externalities?
- Pecuniary or nonpecuniary?
- Why should we care about the junior externalities?

Efficiency of Search Equilibrium

- The question of efficiency boils down to whether these two externalities cancel each other or whether one of them dominates.
- To analyze this question more systematically, consider a social planner subject to the same constraints, intending to maximize "total surplus", in other words, pursuing a utilitarian objective.
- First ignore discounting, i.e., $r \rightarrow 0$, then the planner's problem can be written as

$$\max_{u,\theta} SS = (1-u)y + uz - u\theta\gamma_0.$$

$$\text{s.t.}$$
$$u = \frac{s}{s + \theta q(\theta)}.$$

  where we assumed that z corresponds to the utility of leisure rather than unemployment benefits (how would this be different if z were unemployment benefits?)
- Intuition?

Efficiency of Search Equilibrium

- Why is $r = 0$ useful?
- It turns this from a dynamic into a static optimization problem.
- Form the Lagrangian:

$$\mathcal{L} = (1 - u)y + uz - u\theta\gamma_0 + \lambda \left[u - \frac{s}{s + \theta q(\theta)} \right]$$

- The first-order conditions with respect to u and θ are straightforward:

$$(y - z) + \theta\gamma_0 = \lambda$$

$$u\gamma_0 = \lambda s \frac{\theta q'(\theta) + q(\theta)}{(s + \theta q(\theta))^2}$$

Efficiency of Search Equilibrium (continued)

- The constraint will clearly binding (why?)
- Then substitute for u from the Beveridge curve, and obtain:

$$\lambda = \frac{\gamma_0 \left(s + \theta q\left(\theta\right)\right)}{\theta q'\left(\theta\right) + q\left(\theta\right)}$$

- Now substitute this into the first condition to obtain

$$\left[\theta q'\left(\theta\right) + q\left(\theta\right)\right]\left(y - z\right) + \left[\theta q'\left(\theta\right) + q\left(\theta\right)\right]\theta\gamma_0 - \gamma_0\left(s + \theta q\left(\theta\right)\right) = 0$$

- Simplifying and dividing through by $q\left(\theta\right)$, we obtain

$$\left[1 - \eta(\theta)\right]\left[y - z\right] - \frac{s + \eta(\theta)\theta q(\theta)}{q(\theta)}\gamma_0 = 0.$$

where

$$\eta\left(\theta\right) = -\frac{\theta q'\left(\theta\right)}{q\left(\theta\right)} = \frac{\frac{\partial M(U,V)}{\partial U}U}{M\left(U,V\right)}$$

is the elasticity of the matching function respect to unemployment.

Comparison to Equilibrium

- Recall that in equilibrium (with $r = 0$) we have

$$(1 - \beta)(y - z) - \frac{s + \beta\theta q(\theta)}{q(\theta)}\gamma_0 = 0.$$

- Comparing these two conditions we find that efficiency obtains if and only if *the Hosios condition*

$$\beta = \eta(\theta)$$

is satisfied

- In other words, efficiency requires the bargaining power of the worker to be equal to the elasticity of the matching function with respect to unemployment.
- This is only possible if the matching function is constant returns to scale.
- What happens if not?
- Intuition?

Efficiency with Discounting

- Exactly the same result holds when we have discounting, i.e., $r > 0$
- In this case, the objective function is

$$SS^* = \int_0^\infty e^{-rt} \left[Ny - zN - \gamma_0 \theta(L - N) \right] dt$$

and will be maximized subject to

$$\dot{N} = q(\theta)\theta(L - N) - sN$$

- Simple optimal control problem.

Efficiency with Discounting (continued)

- Solution:

$$y - z - \frac{r + s + \eta(\theta)q(\theta)\theta}{q(\theta)\left[1 - \eta(\theta)\right]}\gamma_0 = 0$$

- Compared to the equilibrium where

$$(1 - \beta)[y - z] + \frac{r + s + \beta q(\theta)\theta}{q(\theta)}\gamma_0 = 0$$

Efficiency with Discounting

- Again, $\eta(\theta) = \beta$ would decentralize the constrained efficient allocation.
- Does the surplus maximizing allocation to zero unemployment?
- Why not?
- What is the social value unemployment?

Endogenous Job Destruction

- So far we treated the rate at which jobs get destroyed as a constant, s, giving us the simple flow equation

$$\dot{u} = s(1 - u) - \theta q(\theta) u$$

- But in practice firms decide when to expand and contract, so it's a natural next step to endogenize s.
- Suppose that each firm consists of a single job (so we are now taking a position on for size).

Endogenous Job Destruction (continued)

- Also assume that the productivity of each firm consists of two components, a common productivity and a firm-specific productivity.

$$\text{productivity for firm } i = \underbrace{p}_{\text{common productivity}} + \underbrace{\sigma \times \varepsilon_i}_{\text{firm-specific}}$$

where

$$\varepsilon_i \sim F(\cdot)$$

over support $\underline{\varepsilon}$ and $\bar{\varepsilon}$, and σ is a parameter capturing the importance of firm-specific shocks.

- Moreover, suppose that each new job starts at $\varepsilon = \bar{\varepsilon}$, but does not necessarily stay there.
- In particular, there is a new draw from $F(\cdot)$ arriving at the flow the rate λ.

Endogenous Job Destruction (continued)

- To further simplify the discussion, let us ignore wage determination and set

$$w = b$$

- This then gives the following value function (written in steady state) for a an active job with productivity shock ε (though this job may decide not to be active):

$$rJ^F(\varepsilon) = p + \sigma\varepsilon - b + \lambda \left[\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max\{J^F(x), J^V\} dF(x) - J^F(\varepsilon) \right]$$

where J^V is the value of a vacant job, which is what the firm becomes if it decides to destroy.

- The max operator takes care of the fact that the firm has a choice after the realization of the new shock, x, whether to destroy or to continue.

Endogenous Job Destruction (continued)

- Since with free entry $J^V = 0$, we have

$$rJ^F(\varepsilon) = p + \sigma\varepsilon - b + \lambda\left[E(J^F) - J^F(\varepsilon)\right] \tag{1}$$

where $J^F(\varepsilon)$ is the value of employing a worker for a firm as a function of firm-specific productivity.

- Also

$$E(J^F) = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max\left\{J^F(x), 0\right\} dF(x) \tag{2}$$

is the expected value of a job after a draw from the distribution $F(\varepsilon)$.

- Given the Markov structure, the value conditional on a draw does not depend on history.

- Intuition?

Endogenous Job Destruction (continued)

- Differentiation of (1) immediately gives

$$\frac{dJ^F(\varepsilon)}{d\varepsilon} = \frac{\sigma}{r+\lambda} > 0 \qquad (3)$$

- Greater productivity gives greater values the firm.
- When will job destruction take place?
- Since (3) establishes that J^F is monotonic in ε, job destruction will be characterized by a cut-off rule, i.e.,

$$\exists \varepsilon_d : \varepsilon < \varepsilon_d \longrightarrow \text{ destroy}$$

- Clearly, this cutoff threshold will be defined by

$$rJ^F(\varepsilon_d) = 0$$

Endogenous Job Destruction (continued)

- But we also have

$$rJ^F(\varepsilon_d) = p + \sigma\varepsilon_d - b + \lambda\left[E(J^F) - J^F(\varepsilon_d)\right],$$

which yields an equation for the value of a job after a new draw:

$$E(J^F) = -\frac{p + \sigma\varepsilon_d - b}{\lambda} > 0$$

- $E\left(J^F\right) > 0$ implies that the expected value of continuation is positive (remember equation (2)).
- Therefore, the flow profits of the marginal job, $p + \sigma\varepsilon_d - b$, must be negative.
- Interpretation?

Endogenous Job Destruction (continued)

- Furthermore, we have a tractable equation for $J^F(\varepsilon)$:

$$J^F(\varepsilon) = \frac{\sigma}{r + \lambda}(\varepsilon - \varepsilon_d)$$

- To characterize $E(J^F)$, note that

$$E(J^F) = \int_{\varepsilon_d}^{\bar{\varepsilon}} J^F(x) dF(x)$$

- Integration by parts

$$
\begin{aligned}
E(J^F) &= \int_{\varepsilon_d}^{\bar{\varepsilon}} J^F(x) dF(x) = J^F(x)F(x)\Big|_{\varepsilon_d}^{\bar{\varepsilon}} - \int_{\varepsilon_d}^{\bar{\varepsilon}} F(x) \frac{dJ^F(x)}{dx} dx \\
&= J^F(\bar{\varepsilon}) - \frac{\sigma}{\lambda + r} \int_{\varepsilon_d}^{\bar{\varepsilon}} F(x) dx \\
&= \frac{\sigma}{\lambda + r} \int_{\varepsilon_d}^{\bar{\varepsilon}} \left[1 - F(x)\right] dx
\end{aligned}
$$

where the last line use the fact that $J^F(\varepsilon) = \frac{\sigma}{\lambda + r}(\varepsilon - \varepsilon_d)$.

Endogenous Job Destruction (continued)

- Next, we have that

$$\underbrace{p + \sigma\varepsilon_d - b}_{\text{profit flow from marginal job}} = -\frac{\lambda\sigma}{r + \lambda} \int_{\varepsilon_d}^{\bar{\varepsilon}} [1 - F(x)] \, dx$$

$$< \quad 0 \text{ due to option value}$$

- Again "hoarding".
- More importantly, we have

$$\frac{d\varepsilon_d}{d\sigma} = \frac{p - b}{\sigma} \left[\sigma\left(\frac{r + \lambda F(\varepsilon_d)}{r + \lambda}\right) \right]^{-1} > 0.$$

- Therefore, when there is more dispersion of firm-specific shocks, there will be more job destruction

Endogenous Job Destruction (continued)

- The job creation part of this economy is similar to before.
- In particular, since firms enter at the productivity $\bar{\varepsilon}$, we have

$$
\begin{aligned}
q\left(\theta\right) J^{F}(\bar{\varepsilon}) &= \gamma_0 \\
\implies \frac{\gamma_0(r+\lambda)}{\sigma(\bar{\varepsilon}-\varepsilon_d)} &= q(\theta)
\end{aligned}
$$

- Recall that as in the basic search model, job creation is "sluggish", in the sense that it is dictated by the matching function; it cannot jump it can only increase by investing more resources in matching.
- On the other hand, job destruction is a jump variable so it has the potential to adjust much more rapidly (but of course the relative importance of job creation and job destruction in practice is an empirical matter)

Endogenous Job Destruction (continued)

- The Beveridge curve is also different now.
- Flow into unemployment is also endogenous, so in steady-state we need to have

$$\lambda F(\varepsilon_d)(1 - u) \quad = \quad q(\theta)\theta u$$

- In other words:

$$u = \frac{\lambda F(\varepsilon_d)}{\lambda F(\varepsilon_d) + q(\theta)\theta},$$

which is very similar to our Beveridge curve above, except that $\lambda F(\varepsilon_d)$ replaces s.

- The most important implication of this is that shocks (for example to productivity) now also shift the Beveridge curve shifts.
- E.g., an increase in p will cause an inward shift of the Beveridge curve; so at a given level of creation, unemployment will be lower.
- How does endogenous job destruction affects efficiency?

A Two-Sector Search Model

- Now consider a two-sector version of the search model, where there are skilled and unskilled workers.
- Suppose that the labor force consists of L_1 and L_2 workers, i.e.

L_1 :   unskilled worker

L_2 :   skilled worker

- Firms decide whether to open a skilled vacancy or an unskilled vacancy.

$$\left. \begin{array}{l} M_1 = x(U_1, V_1) \\ M_2 = x(U_2, V_2) \end{array} \right\} \text{ the same matching function in both sectors.}$$

- Opening vacancies is costly in both markets with

$$\gamma_1 : \quad \text{cost of vacancy for unskilled worker}$$
$$\gamma_2 : \quad \text{cost of vacancy for skilled worker.}$$

A Two-Sector Search Model (continued)

- As before, shocks arrive at some rate, here assumed to be exogenous and potentially different between the two types of jobs

$$s_1, s_2 : \text{ separation rates}$$

- Finally, we allow for population growth of both skilled unskilled workers to be able to discuss changes in the composition of the labor force.
- In particular, let the rate of population growth of L_1 and L_2 be n_1 and n_2 respectively.

$$n_1, n_2 : \text{ population growth rates}$$

- This structure immediately implies that there will be two separate Beveridge curves for unskilled and skilled workers, given by

$$u_1 = \frac{s_1 + n_1}{s_1 + n_1 + \theta_1 q(\theta_1)} \qquad u_2 = \frac{s_2 + n_2}{s_2 + n_2 + \theta_2 q(\theta_2)}.$$

- Intuition?

A Two-Sector Search Model (continued)

- Implication: different unemployment rates are due to three observable features,

    1. separation rates,
    2. population growth
    3. job creation rates.

A Two-Sector Search Model (continued)

- The production side is largely the same as before

$$\text{output} \quad Af(K, N)$$

where N is the effective units of labor, consisting of skilled and unskilled workers.

- We assume that each unskilled worker has one unit of effective labor, while each skilled worker has $\eta > 1$ units of effective labor.
- Finally, the interest rate is still r and the capital depreciation rate is δ.

Bellman Equations

- Parallel to before.
- For filled jobs

$$
\begin{aligned}
rJ_1^F &= Af(k) - (r+\delta)k - w_1 - s_1 J_1^F \\
rJ_2^F &= Af(k)\eta - (r+\delta)k\eta - w_2 - s_2 J_2^F
\end{aligned}
$$

- For vacancies

$$
\begin{aligned}
rJ_1^V &= -\gamma_1 + q(\theta_1)(J_1^F - J_1^V) \\
rJ_2^V &= -\gamma_2 + q(\theta_2)(J_2^F - J_2^V)
\end{aligned}
$$

- Free entry:

$$
J_1^V = J_2^V = 0
$$

Equilibrium

- Using this, we have the value of filled jobs in the two sectors

$$J_1^F = \frac{\gamma_1}{q(\theta_1)} \quad \text{and} \quad J_2^F = \frac{\gamma_2}{q(\theta_2)}$$

- The worker side is also identical, especially since workers don't have a choice affecting their status. In particular,

$$
\begin{aligned}
rJ_1^U &= z + \theta_1 q(\theta_1)(J_1^E - J_1^U) \\
rJ_2^U &= z + \theta_2 q(\theta_2)(J_2^E - J_2^U)
\end{aligned}
$$

where we have assumed the unemployment benefit is equal for both groups (is this reasonable? Important?).

- Finally, the value of being employed for the two types of workers are

$$rJ_i^E = w_i - s(J_i^E - J_i^U)$$

Equilibrium (continued)

- The structure of the equilibrium is similar to before, in particular the modified golden rule and the two wage equations are:

$$
\begin{aligned}
Af'(k) &= r + \delta \qquad \text{M.G.R.} \\
w_1 &= (1-\beta)z + \beta\left[Af(k) - (r+\delta)k + \theta_1\gamma_1\right] \\
w_2 &= (1-\beta)z + \delta\left[Af(k)\eta - (r+\delta)k\eta + \theta_2\gamma_2\right]
\end{aligned}
$$

- The most important result here is that wage differences between skilled unskilled workers are compressed.
- To illustrate this, let us take a simple case and suppose first that

$$
\gamma_1 = \gamma_2, \ n_1 = n_2, \ s_1 = s_2, \ z = 0.
$$

- Thus there are no differences in costs of creating vacancies, separation rates, unemployment benefits, and population growth rates between skilled and unskilled workers.

Unemployment Differences

- In this special case, we have

$$u_2 > u_1$$

- Why?

$$
\begin{aligned}
J_1^F &= \frac{\gamma}{q(\theta_1)} \quad \text{and} \quad J_2^F = \frac{\gamma}{q(\theta_2)} \\
J_2^F &> J_1^F \implies \theta_1 < \theta_2 \implies u_1 > u_2.
\end{aligned}
$$

- High skill jobs yield higher rents, so everything else equal firms will be keener to create these types of jobs, and the only thing that will equate their marginal profits is a slower rate of finding skilled workers, i.e., a lower rate of unemployment for skilled than unskilled workers

Unemployment Differences More Generally

- There are also other reasons for higher unemployment for unskilled workers.
- Also, $s_1 > s_2$ but lately $n_1 < n_2$ so the recent fall in n_1 and increase in n_2 should have helped unskilled unemployment.
- But $z \uparrow$ has more impact on unskilled wages.
- $\eta \uparrow \implies$ "skill-biased" technological change.

$\implies \quad u_1 = cst, \ w_1 = cst$
$\quad\quad u_2 \downarrow, \ w_2 \uparrow$

Does the Cost of Capital Matter?

- A set of interesting effects happen when r are endogenous.
- Suppose we have $\eta \uparrow$, this implies that demand for capital goes up, and this will increase the interest rate, i.e., $r \uparrow$
- The increase in the interest rate will cause

$$u_1 \uparrow, w_1 \downarrow .$$

Labor Force Participation

- Can this model explain non-participation?
- Suppose that workers have outside opportunities distributed in the population, and they decide to take these outside opportunities if the market is not attractive enough.
- Suppose that there are N_1 and N_2 unskilled and skilled workers in the population.
- Each unskilled worker has an outside option drawn from a distribution $G_1(v)$, while the same distribution is $G_2(v)$ for skilled workers.
- In summary:

$$
\begin{array}{lll}
G_1(v) & N_1 : & \text{unskilled} \\
G_2(v) & N_2 : & \text{skilled}
\end{array}
$$

Labor Force Participation (continued)

- Given v; the worker has a choice between J_i^U and v.
- Clearly, only those unskilled workers with

$$J_1^U \geq v$$

will participate and only skilled workers with

$$J_2^U \geq v$$

(why are we using the values of unemployed workers and not employed workers?)

Labor Force Participation (continued)

- Since L_1 and L_2 are irrelevant to steady-state labor market equilibrium above (because of constant returns to scale), the equilibrium equations are unchanged. Then,

$$L_1 = N_1 \int_0^{J_1^U} dG_1(v)$$

$$L_2 = N_2 \int_0^{J_2^U} dG_2(v).$$

$\eta \uparrow, \ r \uparrow \Longrightarrow u_1 \uparrow, \ w_1 \downarrow \ J_1^U \downarrow$

$\Longrightarrow$ unskilled participation falls (consistent with the broad facts).

- But this mechanism requires an interest rate response (is this reasonable?)