14.461: Technological Change, Lecture 4
Competition and Innovation

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September 19, 2011.

Introduction

- What is the effect of competition on innovation and technological progress?
- The baseline models of endogenous technology suggest that it is negative because greater competition reduces profits from monopoly that innovator will acquire.
- This is both not intuitive and not consistent with several studies, which, if anything, find a negative relationship (e.g., Nickell, 1996, or Blundell, Griffith and Van Reenen, 1999).
- One reason for this may be that benchmark models have too stylized representation of competition.
- This lecture: thinking the richer models of competition and theoretical and empirical analysis of the impact of competition on innovation.

Main Model: Preferences

- Infinite-horizon economy in continuous time.
- Continuum 1 of individuals with preferences over the unique final good:

$$\int_t^\infty \exp\left(-\rho\left(s-t\right)\right)\log C\left(s\right)ds$$

- Supply 1 unit of labor inelastically.
- Closed economy, no investment; Euler equation:

$$g\left(t\right)\equiv\frac{\dot{C}\left(t\right)}{C\left(t\right)}=\frac{\dot{Y}\left(t\right)}{Y\left(t\right)}=r\left(t\right)-\rho$$

Final Good Technology

- Unique final goods produced from continuum 1 of intermediates with Cobb-Douglas production function:

$$\ln Y(t) = \int_0^1 \ln y(j,t)\, dj$$

- We normalize the price of the final good to 1. Then the demand for intermediate good j at time t is:

$$y(j,t) = \frac{Y(t)}{p(j,t)}, \quad \forall j \in [0,1].$$

- In each j, 2 firms compete: perfect substitutes

$$y(j,t) = y_i(j,t) + y_{-i}(j,t).$$

- More generally, imperfect substitutes, so that $y(j,t)$ is an aggregate of two or several imperfectly substitutable products.

Intermediate Good Technology

- Each intermediate good has productivity $q_i(j,t)$ at time t and produced from labor:

$$y(j,t) = q_i(j,t)\, l_i(j,t)$$

- Marginal cost of producing intermediate j for firm i:

$$MC_i(j,t) = \frac{w(t)}{q_i(j,t)}$$

  where $w(t)$ is the wage rate at time t.
- Let firm i be the technological leader:

$$q_i(j,t) \geq q_{-i}(j,t).$$

- Bertrand competition

$$p_i(j,t) = \frac{w(t)}{q_{-i}(j,t)} \text{ and } y(j,t) = \frac{q_{-i}(j,t)}{w(t)} Y(t).$$

Profits

- Firm i's technology: $q_i(j,t) = \lambda^{n_{ij}(t)}$.
- Profit function for leader i:

$$\begin{aligned}\Pi_i(j,t) &= [p_i(j,t) - MC_i(j,t)]\, y_i(j,t) \\ &= \left(\frac{w(t)}{q_{-i}(j,t)} - \frac{w(t)}{q_i(j,t)}\right)\frac{Y(t)}{p_i(j,t)} \\ &= \left(1 - \lambda^{-n_j(t)}\right) Y(t)\end{aligned}$$

  where $n_j(t) \equiv n_{ij}(t) - n_{-ij}(t)$ is the technology gap.
- Profits of followers and neck-and-neck firms equal to 0.

R&D and Innovation Process

- Innovations follow a controlled Poisson Process
- Flow rate of innovation for leader and follower given by

$$x_i\left(j,t\right)=F\left(h_i\left(j,t\right)\right)$$

$h_i\left(j,t\right)$: number of researchers

- $F'\left(\cdot\right)>0,\ F''\left(\cdot\right)<0,\ F'\left(0\right)<\infty$
- The cost for R&D is therefore

$$w\left(t\right)G\left(x_i\left(j,t\right)\right)$$

where $G\left(x_i\left(j,t\right)\right)\equiv F^{-1}\left(x_i\left(j,t\right)\right)$.

R&D by the Leader

- Technology levels: $\underbrace{q_i\left(j,t\right)=\lambda^{n_{ij}(t)}}_{\text{leader}} > \underbrace{q_{-i}\left(j,t\right)=\lambda^{n_{-ij}(t)}}_{\text{follower}}$
- R&D by the leader to improve its technology, $x_i(j,t)$:

$$q_i\left(j,t\right)=\lambda^{n_{ij}(t)} \underset{\text{R\&D}}{\overset{\text{successful}}{\Longrightarrow}} q_i\left(j,t+\Delta t\right)=\lambda^{n_{ij}(t)+1}$$

R&D by the Follower - Quick Catch-up

- R&D by the *follower in quick catch-up regime:*
- Catch-up R&D, $x_{-i}(j,t)$:

$$q_{-i}(j,t) = \lambda^{n_{-ij}(t)} \underset{\text{R\&D}}{\overset{\text{successful}}{\Longrightarrow}} q_{-i}(j,t+\Delta t) = \lambda^{n_{ij}(t)}$$

- Interpretation: follower is developing a related (closely substitutable) but different product.

R&D by the Follower - Quick Catch-up

- IPR Policy:
  - **Patent enforcement**: $\boldsymbol{\eta} \equiv \{\eta_1, \eta_2, ...\}$, such that $\eta_n \in \mathbb{R}_+$ for each $n \in \mathbb{N}$ is the flow rate at which a follower that is $n - step$ behind can copy the leader's technology.

- Will also introduce *Regulated infringement fee* and *Regulated licensing fee*.

R&D and the Law of Motion

- $n_j(t) \equiv n_{ij}(t) - n_{-ij}(t)$: the technology gap between firms

$$n_j(t+\Delta t) = \begin{cases} \overbrace{n_j(t)+1}^{\text{next state}} & \overbrace{x_i(j,t)\,\Delta t}^{\text{with approx. probability}} \\ 0 & (x_{-i}(j,t)+\eta_{n_j(t)})\Delta t \\ n_j(t) & 1-(x_i(j,t)+x_{-i}(j,t)+\eta_{n_j(t)})\Delta t \end{cases}$$

Allocations and Equilibrium

- Let $[\eta_n]_{n \in \mathbb{Z}_+,\ t \geq 0}$ be the IPR policy sequence. Then an **allocation** is a sequence of price, output and R&D decisions for a leader that is $n = 0, 1, ..., \infty$ step ahead,

$$\left[p_n(t), y_n(t), x_n(t)\right]_{n \in \mathbb{Z}_+,\ t \geq 0}$$

a sequence of R&D decisions for a follower that is $n = 0, 1, ..., \infty$ step behind,

$$\left[x_{-n}(t)\right]_{n \in \mathbb{Z}_+,\ t \geq 0}$$

a sequence of **wage rates** $[w(t)]_{t \geq 0}$ and a sequence of **industry distributions** over technology gaps $[\mu_n(t)]_{n \in \mathbb{Z}_+,\ t \geq 0}$.

Steady State Value Functions

- Defined normalized value: $v_n \equiv V_n / Y$, where V_n is net present discounted value of profits.
- For **leaders**:

$$\rho v_n = \max_{x_n \geq 0} \left\{ \begin{array}{c} \left(1 - \lambda^{-n}\right) - \omega^* G\left(x_n\right) + x_n \left[v_{n+1} - v_n\right] \\ +x^*_{-n} \left[v_0 - v_n\right] + \eta_n \left[v_0 - v_n\right] \end{array} \right\}$$

- For **followers**

$$\rho v_{-n} = \max_{x^c_{-n} \geq 0} \left\{ \begin{array}{c} -\omega^* G\left(x_{-n}\right) + x_{-n} \left[v_0 - v_{-n}\right] \\ +x^*_n \left[v_{-n-1} - v_{-n}\right] + \eta_n \left[v_0 - v_{-n}\right] \end{array} \right\}$$

- For **neck-and-neck firms**

$$\rho v_0 = \max_{x_0 \geq 0} \left\{ -\omega^* G\left(x_0\right) + x_0 \left[v_1 - v_0\right] + x^*_0 \left[v_{-1} - v_0\right] \right\}$$

Labor Market Clearing

- Labor Supply $= 1$; thus

$$1 \geq \sum_{n=0}^{\infty} \mu_n^* \left[\frac{1}{\omega^* \lambda^n} + G\left(x_n^*\right) + G\left(x_{-n}^*\right) \right]$$

Major Simplification to Start With

- Suppose that $n_{ij}(t)$ is "technologically" restricted to be 0 or 1.
  - This might be because if the leader gets two steps ahead of the follower, the follower automatically copies the previous step.
  - In actuality, however, this is just a trick to get simple expressions.
- IPR policy for now implies the possibility that the follower can copy the one step ahead leader.
- Suppose also that R&D uses final good rather than labor, so that the general equilibrium interaction is also shut off.

Structure of Equilibrium in the Simplified Economy

- From the analysis so far, we have that the normalized profits of the leader (who is necessarily one step ahead) is

$$\pi_1 \equiv \frac{\Pi_1}{Y} = 1 - \lambda.$$

- Naturally, also,

$$\pi_{-1} = 0.$$

- Suppose also that when two firms are neck and neck, they can "collude" and obtain some fraction of π_1, i.e.,

$$\pi_0 = \varepsilon \pi_1,$$

  where we can then think of ε as an inverse measure of competition. (What could be wrong with this reasoning?)

R&D Technology and Value Functions

- Suppose also that the R&D cost function is

$$G(x) = \frac{x^2}{2}.$$

- Then value functions can be written as (with x' denoting the other firm's R&D)

$$\begin{aligned} rv_1 &= \pi_1 + (x_{-1} + \eta)(v_0 - v_1) \\ rv_0 &= \pi_0 + x_0(v_1 - v_0) + x_0'(v_{-1} - v_0) - x_0^2/2. \\ rv_{-1} &= (x_{-1} + \eta)(v_0 - v_{-1}) - x_{-1}^2/2. \end{aligned}$$

Equilibrium

- Characterizing this equilibrium is now trivial.
- Most importantly, as intermediate step, we have the following intricate equations:

$$
\begin{aligned}
x_0 &= v_1 - v_0 \\
x_{-1} &= v_0 - v_{-1}.
\end{aligned}
$$

- Thus anything that increases the gap between neck and neck values and monopoly values increases R&D of neck and neck firms, and anything that increases the gap between this value and the value of a follower increases the R&D of a follower.

Equilibrium (continued)

- Now completing the algebra, we obtain:

$$
\begin{aligned}
x_0 &= -\eta + \sqrt{\eta^2 + 2(1-\varepsilon)(1-\lambda)} \\
x_{-1} &= -(\eta + x_0) + \sqrt{\eta^2 + x_0^2 + 2(1-\lambda)}
\end{aligned}
$$

- This implies that a higher ε (lower competition) reduces x_0 and increases x_{-1}, which is what we might have expected intuitively.
- The first is due to the **escape competition effect**, which was absent in the benchmark endogenous growth models. The second effect is the standard **appropriability effect**, which was already present in both input variety and Schumpeterian models.

Equilibrium (continued)

- Which effect dominates?
- One natural measure of aggregate rate of innovation would be

$$\mu x_{-1} + 2\left(1 - \mu\right) x_0,$$

where μ is the fraction of sectors that are in a neck and neck state.

- Aghion et al. actually define it closer to

$$\mu\left(x_{-1} + \eta\right) + 2\left(1 - \mu\right) x_0.$$

For now, this doesn't matter.

- It can be shown that, provided that η and λ take intermediate values, the effect of competition on aggregate innovation, has an inverted U-shape.

Empirical Evidence

- Aghion et al. provide evidence from UK firms consistent with such an inverted U-shaped relationship.
- They use measured price-cost ratios (the Lerner index) as a proxy for competition (in particular, operating profit minus financial costs divided by sales).

$$\text{Lerner index} = \frac{\text{operating profit - financial cost}}{\text{sales}}$$

  - What might be wrong with this measure?
- Using this measure, they find a robustly inverted U between citation weighted patents and the Lerner index.
- They also play in the same results when they instrument for the Lerner index using "policy instruments" meaning interventions either due to single markets for Monopoly and Merger Commission investigations, presumably reducing monopoly power in the industry.

Empirical Evidence (continued)

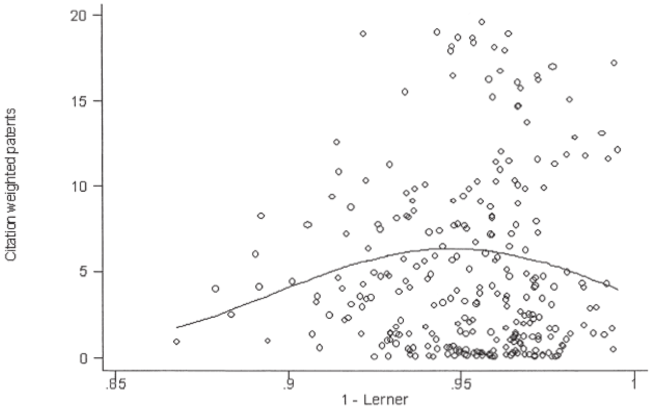


FIGURE I
Scatter Plot of Innovation on Competition

Empirical Evidence (continued)

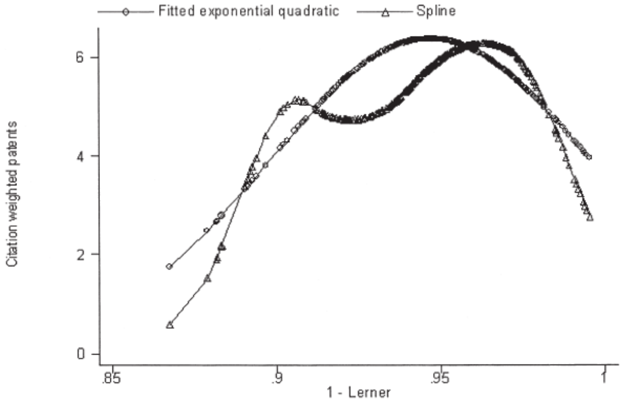


FIGURE II
Innovation and Competition: Exponential Quadratic and the Semiparametric Specifications with Year and Industry Effects

Empirical Evidence: Regression Evidence

TABLE I
EXPONENTIAL QUADRATIC: BASIC SPECIFICATION

Dependent variable: citation-weighted patents	(1)	(2)	(3)	(4)
Data frequency	Annual	Annual	5-year averages	Annual
Competition_{jt}	152.80	387.46	819.44	385.13
	(55.74)	(67.74)	(265.63)	(67.56)
$\text{Competition squared}_{jt}$	−80.99	−204.55	−434.43	−204.83
	(29.61)	(36.17)	(141.43)	(36.06)
Significance of: Competition_{jt}, $\text{Competition squared}_{jt}$	7.60	38.34	9.97	32.59
	(0.02)	(0.00)	(0.01)	(0.00)
Significance of policy instruments in reduced form				10.11
				(0.002)
Significance of other instruments in reduced form				5.00
				(0.000)
Control functions in regression				4.38
				(4.04)
R^2 of reduced form				0.801
Year effects	Yes	Yes	Yes	Yes
Industry effects		Yes	Yes	Yes
Observations	354	354	67	354

Empirical Evidence: Further Evidence

TABLE III
TECHNOLOGY GAP AND EXPONENTIAL QUADRATIC WITH NECK-AND-NECK SPLIT

	(1)	(2)	(3)	(4)
Dependent variable:	Technology gap	Technology gap	Citation-weighted patents	Citation-weighted patents
Estimation procedure	Linear regression	Linear regression	Poisson	Poisson
Competition_{jt}	2.858	0.942	183.81	424.46
	(0.400)	(0.419)	(58.99)	(69.5)
$\text{Competition squared}_{jt}$			−96.35	−222.9
			(31.01)	(36.9)
$\text{Competition}_{jt} * \text{Technology gap}_{jt}$			1.43	3.82
			(2.48)	(2.66)
$\text{Competition squared}_{jt} * \text{Technology gap}_{jt}$			−1.30	−3.84
			(2.59)	(2.78)
Significance of: Competition_{jt}, $\text{Competition squared}_{jt}$			16.59	39.21
			(0.00)	(0.00)
Significance of: $\text{Competition}_{jt} * \text{Technology gap}_{jt}$,			9.74	7.93
$\text{Competition squared}_{jt} * \text{Technology gap}_{jt}$			(0.01)	(0.02)
Year effects	Yes	Yes	Yes	Yes
Industry effects		Yes		Yes

IPR and Competition

- Even at this level of generality, it is clear that IPR and competition are related. Changes in η will affect the degree of competition in the economy.
- Aghion et al. suggest that higher η (less strict IPR) should increase aggregate innovation because it pushes more industries into neck and neck state, where R&D tends to be higher (necessarily higher when $\varepsilon = 0$).
- But there measure of aggregate R&D directly depends on due to IPR relaxation, which is not really innovation.
- We need a more detailed analysis for understanding the effect of IPR on innovation.
- For this, let us return to the general framework.

R&D by the Follower - Quick Catch-up

- Let us start with IPR policy, that is more general, potentially state dependent.
- IPR Policy:
  - **Patent enforcement**: $\boldsymbol{\eta} \equiv \{\eta_1, \eta_2, ...\}$, such that $\eta_n \in \mathbb{R}_+$ for each $n \in \mathbb{N}$ is the flow rate at which a follower that is $n -$ *step* behind can copy the leader's technology.
- Let us also introduce *Regulated infringement fee* and *Regulated licensing fee*.

R&D and the Law of Motion

- Recall: $n_j(t) \equiv n_{ij}(t) - n_{-ij}(t)$: the technology gap between firms

$$
n_j(t+\Delta t) = \left\{ \begin{array}{cc} \overbrace{}^{\text{next state}} & \overbrace{\phantom{1-(x_i(j,t)+x_{-i}(j,t)+\eta_{n_j(t)})\Delta t}}^{\text{with approx. probability}} \\ n_j(t)+1 & x_i(j,t)\,\Delta t \\ 0 & (x_{-i}(j,t)+\eta_{n_j(t)})\Delta t \\ n_j(t) & 1-(x_i(j,t)+x_{-i}(j,t)+\eta_{n_j(t)})\Delta t \end{array} \right.
$$

Steady State Value Functions

- Recall also the normalized value: $v_n \equiv V_n / Y$:
- For **leaders**:

$$\rho v_n = \max_{x_n \geq 0} \left\{ \begin{array}{c} \left(1 - \lambda^{-n}\right) - \omega^* G\left(x_n\right) + x_n\left[v_{n+1} - v_n\right] \\ +x^*_{-n}\left[v_0 - v_n\right] + \eta_n\left[v_0 - v_n\right] \end{array} \right\}$$

- For **followers**

$$\rho v_{-n} = \max_{x^c_{-n} \geq 0} \left\{ \begin{array}{c} -\omega^* G\left(x_{-n}\right) + x_{-n}\left[v_0 - v_{-n}\right] \\ +x^*_n\left[v_{-n-1} - v_{-n}\right] + \eta_n\left[v_0 - v_{-n}\right] \end{array} \right\}$$

- For **neck-and-neck firms**

$$\rho v_0 = \max_{x_0 \geq 0} \left\{ -\omega^* G\left(x_0\right) + x_0\left[v_1 - v_0\right] + x^*_0\left[v_{-1} - v_0\right] \right\}$$

Optimal R&D Decisions

- R&D decisions:

$$
\begin{aligned}
x_n^* &= \max\left\{G'^{-1}\left(\frac{[v_{n+1}-v_n]}{\omega^*}\right),0\right\} \\
x_{-n}^* &= \max\left\{G'^{-1}\left(\frac{[v_0-v_{-n}]}{\omega^*}\right),0\right\} \\
x_0^* &= \max\left\{G'^{-1}\left(\frac{[v_1-v_0]}{\omega^*}\right),0\right\},
\end{aligned}
$$

Optimal R&D Decisions (cont'd)

- Relaxation of patent enforcement will reduce $v_{n+1} - v_n$, and thus x_n^*: **disincentive effects.**
- Because typically $x_0^* > x_n^*$, relaxation of IPR also tends to shift more industries into neck-and-neck state and increase overall R&D - **composition effect.**

Steady-State Equilibrium

Proposition

A steady-state equilibrium exists.

-

Proposition

If IPR policy is uniform, then:

$$x_n^* > x_{n+1}^*, \ \forall n \in \mathbb{Z}_{++}.$$

Proposition

The equilibrium growth rate is:

$$g^* = \ln \lambda \left[2\mu_0^* x_0^* + \sum_{n=1}^{\infty} \mu_n^* x_n^* \right].$$

Impact of IPR Policy

- IPR policy in this more general model creates two effects:
  1. **Composition effect:** relaxation of patent protection increases fraction of industries where firms are *neck-and-neck*.
     Neck-and-neck firms perform more R&D, thus innovation and growth increases.
  2. **Disincentive effect:** relaxation of patent protection reduces the value of innovation, thus discourages R&D both for leaders and followers.
- Optimal level and structure of IPR determined by the interaction of these two forces.

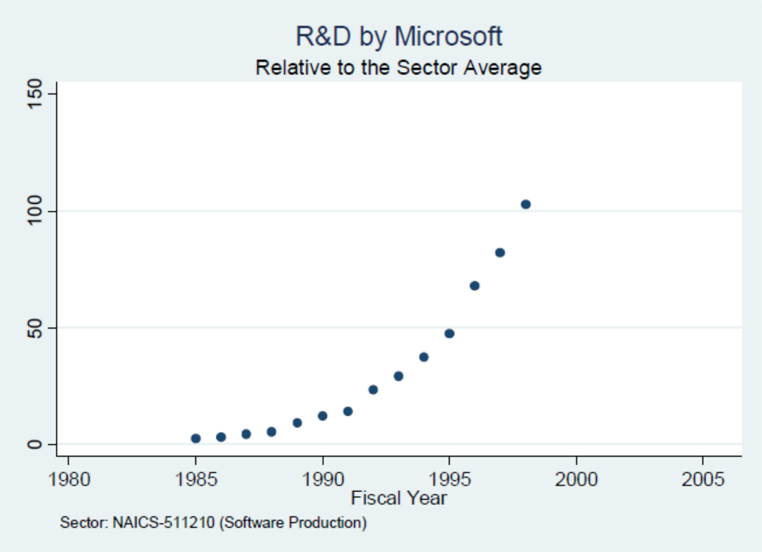
A Thought Experiment

- Which company should be protected more:
  - A company with a bigger lead, or
  - A company with a more limited lead over its rival?
- **Naive intuition:** less protection for technologically advanced firms to exploit the composition effect.
- But this intuition is motivated by uniform IPR.
- With state-dependent relaxation of IPR it is possible to create incentives.
  - **positive incentive effects** instead of disincentive effects

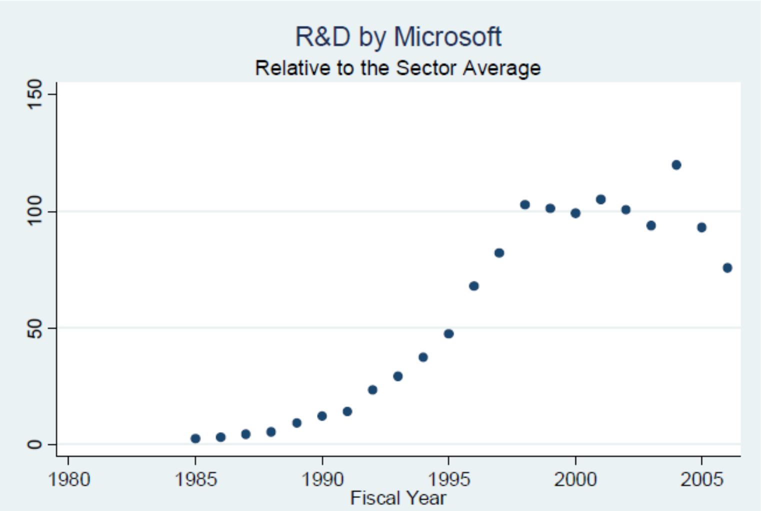
Optimal IPR Policy

- Both growth-maximizing and (steady-state) welfare-maximizing.
- State-dependent IPR can create significant gains:
  - Low patent protection for leaders that are a few steps ahead.
  - Full patent protection for leaders that are suffciently advanced.
  - Opposite of the above *naive* intuition.
  - **Trickle-down of incentives**: full protection for a leader that is n^* steps ahead gives better incentives to all leaders $n < n^*$ steps ahead.

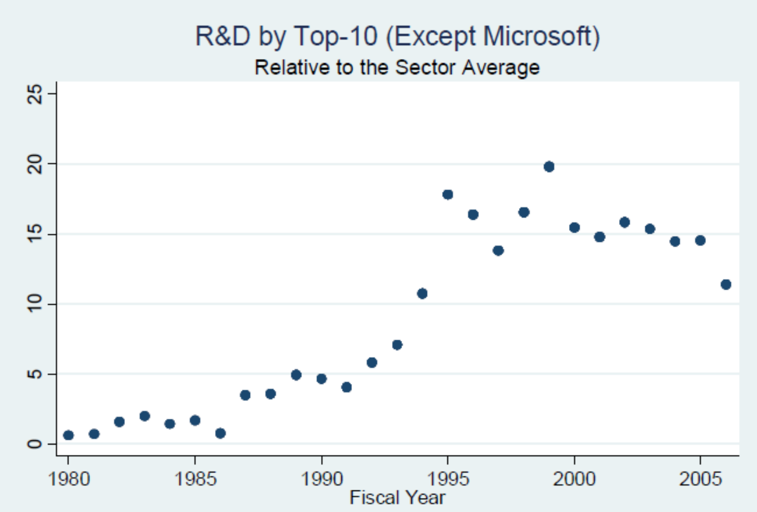
Microsoft R&D Relative to Sector Average



Microsoft R&D Relative to Sector Average - After DOJ Case



R&D by Other Technology Leaders Relative to Sector Average



Parameterization of IPR Policy

Uniform IPR

$$\underset{\text{Technology gap: } n \rightarrow}{\text{IPR policy} \rightarrow} \quad \overset{\textit{none}}{\underset{0}{\text{—}}} \quad \overbrace{\underset{1}{\text{—}}}^{\eta} \quad \overbrace{\underset{2}{\text{—}}}^{\eta} \quad \overbrace{\underset{3}{\text{—}}}^{\eta} \quad \overbrace{\underset{4}{\text{—}}}^{\eta} \quad \overbrace{\underset{5}{\text{—}}\ \underset{6}{\text{—}}\ \underset{7}{\text{—}}\ \underset{8}{\text{—}}\ \underset{9}{\text{—}}\ \underset{10}{\text{—}}\ \underset{11}{\text{—}}\ \underset{.}{\text{—}}\ \underset{.}{\text{—}}\ \underset{\infty}{\text{—}}}^{\eta}$$

State-Dependent IPR

$$\underset{\text{Technology gap: } n \rightarrow}{\text{IPR policy} \rightarrow} \quad \overset{\textit{none}}{\underset{0}{\text{—}}} \quad \overbrace{\underset{1}{\text{—}}}^{\eta_1} \quad \overbrace{\underset{2}{\text{—}}}^{\eta_2} \quad \overbrace{\underset{3}{\text{—}}}^{\eta_3} \quad \overbrace{\underset{4}{\text{—}}}^{\eta_4} \quad \overbrace{\underset{5}{\text{—}}\ \underset{6}{\text{—}}\ \underset{7}{\text{—}}\ \underset{8}{\text{—}}\ \underset{9}{\text{—}}\ \underset{10}{\text{—}}\ \underset{11}{\text{—}}\ \underset{.}{\text{—}}\ \underset{.}{\text{—}}\ \underset{\infty}{\text{—}}}^{\eta_5}$$

Parameters:

- List of parameters: r, γ, λ, B
- $r_{year} = 5\%$
- The empirical literature usually assumes

$$Innovation(t) = B_0 \exp(\kappa t)(R\&D\_inputs)^{\gamma}$$

where B_0 : constant, $\exp(\kappa t)$: trend term. Thus we assume

$$x = Bh^{\gamma}$$

- Kortum (1993) ; Pakes and Griliches (1980) ; Hall, Hausmann and Griliches (1988) report:

$$\gamma \in [0.1, \quad 0.6]$$

Set benchmark elasticity as $\gamma = 0.35$. Do robustness check with $\gamma = 0.1$ and $\gamma = 0.6$.

Parameters (cont'd)

- For step size λ, similar reasoning to Stokey (1995) :

$$\text{Expected duration between consecutive innovations} = 2.5 \text{ years}$$

$$+$$

$$\text{Annual growth rate} = 2\%$$

$$\Downarrow$$

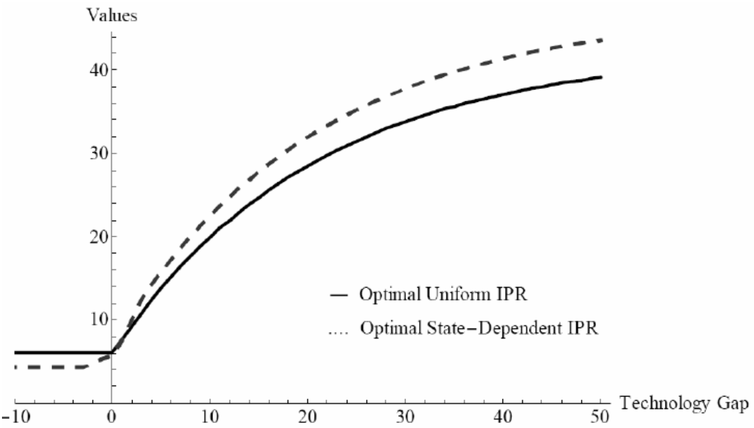
$$\lambda = 1.05$$

- Robustness checks with $\lambda = 1.01$ and $\lambda = 1.2$ ($= 10$ yr)

Table 1: Patent Length in Quick Catch-up...

$\lambda = 1.05, \ \gamma = 0.35$	Full IPR	Optimal Uniform IPR	Optimal State-dependent IPR
η_1	0	0	0.71
η_2	0	0	0.08
η_3	0	0	0
η_4	0	0	0
η_5	0	0	0
$v_1 - v_0$	2.71	2.71	1.52
x^*_{-1}	0.22	0.22	0.12
x^*_0	0.35	0.35	0.25
μ^*_0	0.24	0.24	0.46
μ^*_1	0.33	0.33	0.19
μ^*_2	0.20	0.20	0.13
ω^*	0.95	0.95	0.96
Researcher ratio	0.032	0.032	0.028
$\ln C(0)$	33.78	33.78	34.20
g^*	0.0186	0.0186	0.0204
Welfare	683.0	683.0	692.1

Figure 1: Value Function



Slow Catch-up Value Functions

- Slow catch-up$\longrightarrow$

$$q_{-i}\left(j,t\right)=\lambda^{n_{-ij}(t)}\ \underset{\text{R\&D}}{\overset{\text{successful}}{\Longrightarrow}}\ q_{-i}\left(j,t+\Delta t\right)=\lambda^{n_{-ij}(t)+1}$$

- For **leaders**:

$$\rho v_n=\max_{x_n\geq 0}\left\{\begin{array}{c}\left(1-\lambda^{-n}\right)-\omega^* G\left(x_n\right)+x_n\left[v_{n+1}-v_n\right]\\ \\ +x^*_{-n}\left[v_{n-1}-v_n\right]+\eta_n\left[v_0-v_n\right]\end{array}\right\}$$

- For **followers**

$$\rho v_{-n}=\max_{\substack{x_{-n}\geq 0,\\ a_{-n}\in[0,1]}}\left\{\begin{array}{c}-\omega^* G\left(x_{-n}\right)+x_{-n}\left[v_{-n+1}-v_{-n}\right]\\ \\ +x^*_n\left[v_{-n-1}-v_{-n}\right]+\eta_n\left[v_0-v_{-n}\right]\end{array}\right\}$$

Table 2: Patent Length in Slow Catch-up...

$\lambda = 1.05,\ \gamma = 0.35$ $\zeta_n = \infty,\ \vartheta_n = \infty$	Full IPR	Optimal Uniform IPR	Optimal State-dependent IPR
η_1	0.02	0.11	0.69
η_2	0.02	0.11	0.20
η_3	0.02	0.11	0.14
η_4	0.02	0.11	0.12
η_5	0.02	0.11	0.08
$v_1 - v_0$	11.4	3.74	2.27
x^*_{-1}	0.75	0.27	0.17
x^*_0	0.99	0.14	0.32
μ^*_0	0.02	0.16	0.30
μ^*_1	0.03	0.19	0.15
μ^*_2	0.03	0.14	0.10
ω^*	0.56	0.90	0.90
Researcher ratio	0.150	0.055	0.059
$\ln C(0)$	31.31	34.31	34.57
g^*	0.0252	0.0229	0.0247
Welfare	636.3	695.3	701.2

Licensing with Slow Catch-up

- Suppose that followers can close the gap between themselves and the leader by "reverse engineering" the current leading-edge technology, and in return, they will have to pay a license fee to leader.
- IPR Policy:
  - **Regulated licensing fee**: (normalized values) $\zeta = (\zeta_1(t), ..., \zeta_\infty(t))$, such that $\zeta_n(t) \in \mathbb{R}_+$ for each $n \in \mathbb{N}$ at time t is the patent fee that the follower should pay in order to use leader's technology.
  - Interpretation: licensing fees are bargained between leader and follower, and government policy determinants outside options in this bargaining.

Licensing with Slow Catch-up: Value Functions

- For **leaders**:

$$\rho v_n = \max_{x_n \geq 0} \left\{ \begin{array}{c} \left(1 - \lambda^{-n}\right) - \omega^* G\left(x_n\right) + \\ x_n \left[v_{n+1} - v_n\right] + a^*_{-n} x^*_{-n} \left[v_0 - v_n + \zeta_n\right] + \\ \left(1 - a^*_{-n}\right) x^*_{-n} \left[v_{n-1} - v_n\right] + \eta_n \left[v_0 - v_n\right] \end{array} \right\}$$

- For **followers** - choose R&D and also licensing:

$$\rho v_{-n} = \max_{\substack{x_{-n} \geq 0, \\ a_{-n} \in [0,1]}} \left\{ \begin{array}{c} -\omega^* G\left(x_{-n}\right) + \\ a_{-n} x_{-n} \left[v_0 - v_{-n} - \zeta_n\right] + \left(1 - a_{-n}\right) x_{-n} \left[v_{-n+1} - v_{-n}\right] \\ +x^*_n \left[v_{-n-1} - v_{-n}\right] + \eta_n \left[v_0 - v_{-n}\right] \end{array} \right\}$$

Table 3: Licensing in Slow Catch-up...

$\lambda = 1.05,\ \gamma = 0.35$ $\eta_n = 0.02,\ \vartheta_n = \infty$	Full IPR	Optimal Uniform IPR	Optimal State-dependent IPR
ζ_2	∞	1.61	1.54
ζ_3	∞	1.61	2.45
ζ_4	∞	1.61	2.92
ζ_5	∞	1.61	3.32
$v_1 - v_0$	11.4	3.48	4.18
x^*_{-1}	0.75	0.27	0.31
x^*_0	0.99	0.39	0.45
μ^*_0	0.02	0.21	0.18
μ^*_1	0.03	0.25	0.20
μ^*_2	0.03	0.20	0.12
ω^*	0.56	0.94	0.91
Researcher ratio	0.150	0.043	0.071
$\ln C(0)$	31.31	34.13	34.47
g^*	0.0252	0.0209	0.0248
Welfare	636.3	690.9	699.3

Regulated Infringement Fee in Slow Catch-up

- Suppose that followers can engage in frontier R&D and "leapfrog" the technology leader, but might have to pay an infringement fee.
- IPR Policy:
  - **Regulated infringement fee**: (normalized values) $\vartheta = (\vartheta_1(t), ..., \vartheta_\infty(t))$, such that $\vartheta_n(t) \in \mathbb{R}_+$ for each $n \in \mathbb{N}$ at time t is the penalty that the follower should pay if the follower's frontier innovation turns out to be infringing the existing patent.
- τ : Lanjouw and Schankerman (1998) reports that 10% of the patents are filed for infringement

$$\Longrightarrow \tau = 0.1$$

Value Functions under Infringement Fees

- For **leaders**:

$$\rho v_n = \max_{x_n \geq 0} \left\{ \begin{array}{c} \left(1-\lambda^{-n}\right) - \omega^* G\left(x_n\right) + x_n\left[v_{n+1} - v_n\right] \\ + x_{-n}^{c*}\left[v_{n-1} - v_n\right] + x_{-n}^{f*}\left[v_{-1} - v_n + \tau\vartheta_n\right] \\ + \eta_n\left[v_0 - v_n\right] \end{array} \right\}$$

- For **followers** - choose catch-up and R&D levels:

$$\rho v_{-n} = \max_{x_{-n}^c \geq 0, x_{-n}^f \geq 0} \left\{ \begin{array}{c} -\omega^* G\left(x_{-n}\right) + \\ x_{-n}^c\left[v_{-n+1} - v_{-n}\right] + x_{-n}^f\left[v_1 - v_{-n} - \tau\vartheta_n\right] \\ + x_n^*\left[v_{-n-1} - v_{-n}\right] + \eta_n\left[v_0 - v_{-n}\right] \end{array} \right\}$$

Table 4: Infringement Fee in Slow Catch-up...

$\lambda = 1.05,\ \gamma = 0.35$ $\eta_n = 0.02,\ \zeta_n = \infty$	Full IPR	Optimal Uniform IPR	Optimal State-dependent IPR
ϑ_1	∞	14	0
ϑ_2	∞	14	18.1
ϑ_3	∞	14	31.3
ϑ_4	∞	14	36.6
ϑ_5	∞	14	43.7
$v_1 - v_0$	11.4	2.09	1.91
x_{-1}^{f*}	0.75	0.23	0.33
x_0^*	0.99	0.30	0.29
μ_0^*	0.02	0.14	0.12
μ_1^*	0.03	0.42	0.35
μ_2^*	0.03	0.22	0.17
ω^*	0.56	0.95	0.94
Researcher ratio	0.150	0.028	0.058
$\ln C(0)$	31.31	35.48	36.17
g^*	0.0252	0.0260	0.0311
Welfare	636.3	720.0	735.9