14.999: Topics in Inequality, Lecture 4
Endogenous Technology and Automation

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Introduction

- A common interpretation of the trends we have seen so far is that they are the implications of some inexorable technological trends that:
  1. replace labor by machines, in the process reducing labor share and employment;
  2. reduce the demand for low skill workers, creating unstoppable trends towards greater inequality.
- The main point of this lecture is to argue theoretically that:
  - some of these trends may be endogenous responses to other economic changes — **directed technological change**;
  - there may be self-correcting, equilibriating forces within the economic system — **automation and new tasks**.

Directed Technological Change I

- The first important point is to focus on the response of technological change to various factors, which is the subject matter of the directed technological change literature.
- Two factors of production, say L and H (unskilled and skilled workers).
- Two types of technologies that can complement either one or the other factor.
- Whenever the profitability of H-augmenting technologies is greater than the L-augmenting technologies, more of the former type will be developed by profit-maximizing (research) firms.
- What determines the relative profitability of developing different technologies? It is more profitable to develop technologies...
  1. when the goods produced by these technologies command higher prices (*price effect*);
  2. that have a larger market (*market size effect*).

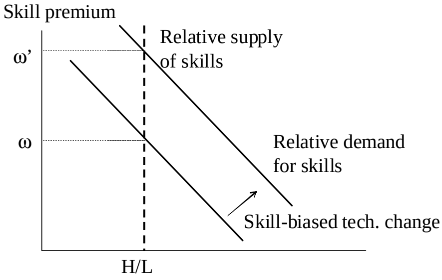
Bias

- Suppose the (inverse) relative demand curve:

$$w_H / w_L = D(H/L, A)$$

  where w_H / w_L is the relative price of the factors and A is a technology term.

- A is H-biased if D is increasing in A, so that a higher A increases the relative demand for the H factor.



What Does Weak Bias Mean? (continued)

- D is *always* decreasing in H/L.
- Equilibrium bias: behavior of A as H/L changes,

$$A\left(H/L\right)$$

- Weak equilibrium bias:
  - $A\left(H/L\right)$ is increasing (nondecreasing) in H/L.
- Strong equilibrium bias:
  - $A\left(H/L\right)$ is sufficiently responsive to an increase in H/L that the total effect of the change in relative supply H/L is to increase w_H/w_L.
  - i.e., let the endogenous-technology relative demand curve be

$$w_H/w_L = D\left(H/L, A\left(H/L\right)\right) \equiv \tilde{D}\left(H/L\right)$$

$\rightarrow$*Strong equilibrium bias:* $\tilde{D}$ **increasing in** H/L.

Equilibrium Relative Bias

- Potentially counteracting effects, but the market size effect will be more powerful often.
- Under fairly general conditions:
  - *Weak Equilibrium (Relative) Bias*: an increase in the relative supply of a factor always induces technological change that is biased in favor of this factor.
  - *Strong Equilibrium (Relative) Bias*: if the elasticity of substitution between factors is sufficiently large, an increase in the relative supply of a factor induces sufficiently strong technological change biased towards itself that the endogenous-technology relative demand curve of the economy becomes *upward-sloping*.

Baseline Model of Directed Technical Change I

- Framework: expanding varieties model with lab equipment specification of the innovation possibilities frontier (so none of the results here depend on technological externalities).
- Constant supply of L and H.
- Representative household with the standard CRRA preferences:

$$\int_0^\infty \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt, \tag{1}$$

- Aggregate production function:

$$Y(t) = \left[\gamma_L Y_L(t)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma_H Y_H(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where intermediate good $Y_L(t)$ is L-intensive, $Y_H(t)$ is H-intensive.

Baseline Model of Directed Technical Change II

- Resource constraint (define $Z(t) = Z_L(t) + Z_H(t)$):

$$C(t) + X(t) + Z(t) \leq Y(t), \tag{3}$$

- Intermediate goods produced competitively with:

$$Y_L(t) = \frac{1}{1-\beta}\left(\int_0^{N_L(t)} x_L(\nu, t)^{1-\beta} d\nu\right) L^{\beta} \tag{4}$$

and

$$Y_H(t) = \frac{1}{1-\beta}\left(\int_0^{N_H(t)} x_H(\nu, t)^{1-\beta} d\nu\right) H^{\beta}, \tag{5}$$

where machines $x_L(\nu, t)$ and $x_H(\nu, t)$ are assumed to depreciate after use.

Baseline Model of Directed Technical Change III

- Differences with baseline expanding product varieties model:
  1. These are production functions for intermediate goods rather than the final good.
  2. (4) and (5) use different types of machines–different ranges $[0, N_L(t)]$ and $[0, N_H(t)]$.
- All machines are supplied by monopolists that have a fully-enforced perpetual patent, at prices $p_L^x(\nu, t)$ for $\nu \in [0, N_L(t)]$ and $p_H^x(\nu, t)$ for $\nu \in [0, N_H(t)]$.
- Once invented, each machine can be produced at the fixed marginal cost ψ in terms of the final good.
- Normalize to $\psi \equiv 1 - \beta$.

Baseline Model of Directed Technical Change IV

- Innovation possibilities frontier:

$$\dot{N}_L(t) = \eta_L Z_L(t) \text{ and } \dot{N}_H(t) = \eta_H Z_H(t), \tag{6}$$

- Value of a monopolist that discovers one of these machines is:

$$V_f(\nu, t) = \int_t^\infty \exp\left[-\int_t^s r(s')\,ds'\right] \pi_f(\nu, s)ds, \tag{7}$$

where $\pi_f(\nu, t) \equiv p_f^x(\nu, t)x_f(\nu, t) - \psi x_f(\nu, t)$ for $f = L$ or H.

- Hamilton-Jacobi-Bellman version:

$$r(t)\,V_f(\nu, t) - \dot{V}_f(\nu, t) = \pi_f(\nu, t). \tag{8}$$

Baseline Model of Directed Technical Change V

- Normalize the price of the final good at every instant to 1, which is equivalent to setting the ideal price index of the two intermediates equal to one, i.e.,

$$\left[\gamma_L^{\varepsilon}\left(p_L\left(t\right)\right)^{1-\varepsilon}+\gamma_H^{\varepsilon}\left(p_H\left(t\right)\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}=1 \text{ for all } t, \tag{9}$$

  where $p_L\left(t\right)$ is the price index of Y_L at time t and $p_H\left(t\right)$ is the price of Y_H.
- Denote factor prices by $w_L\left(t\right)$ and $w_H\left(t\right)$.

Equilibrium I

- Maximization problem of producers in the two sectors:

$$\max_{L,[x_L(\nu,t)]_{\nu\in[0,N_L(t)]}} p_L(t)\, Y_L(t) - w_L(t)\, L - \int_0^{N_L(t)} p_L^x(\nu,t)\, x_L(\nu,t)\, d\nu, \tag{10}$$

and

$$\max_{H,[x_H(\nu,t)]_{\nu\in[0,N_H(t)]}} p_H(t)\, Y_H(t) - w_H(t)\, H - \int_0^{N_H(t)} p_H^x(\nu,t)\, x_H(\nu,t)\, d\nu. \tag{11}$$

- Note the presence of $p_L(t)$ and $p_H(t)$, since these sectors produce intermediate goods.

Equilibrium II

- Thus, demand for machines in the two sectors:

$$x_L(\nu, t) = \left[\frac{p_L(t)}{p_L^x(\nu, t)}\right]^{1/\beta} L \quad \text{for all } \nu \in [0, N_L(t)] \text{ and all } t, \qquad (12)$$

and

$$x_H(\nu, t) = \left[\frac{p_H(t)}{p_H^x(\nu, t)}\right]^{1/\beta} H \quad \text{for all } \nu \in [0, N_H(t)] \text{ and all } t. \qquad (13)$$

- Maximization of the net present discounted value of profits implies a constant markup:

$$p_L^x(\nu, t) = p_H^x(\nu, t) = 1 \text{ for all } \nu \text{ and } t.$$

Equilibrium III

- Substituting into (12) and (13):

$$x_L(\nu, t) = p_L(t)^{1/\beta} L \quad \text{for all } \nu \text{ and all } t,$$
$$x_H(\nu, t) = p_H(t)^{1/\beta} H \quad \text{for all } \nu \text{ and all } t.$$

- Combining these with (4) and (5), *derived* production functions for the two intermediate goods:

$$Y_L(t) = \frac{1}{1-\beta} p_L(t)^{\frac{1-\beta}{\beta}} N_L(t) L \tag{14}$$

$$Y_H(t) = \frac{1}{1-\beta} p_H(t)^{\frac{1-\beta}{\beta}} N_H(t) H. \tag{15}$$

- Profits are also independent of machine type:

$$\pi_L(t) = \beta p_L(t)^{1/\beta} L \text{ and } \pi_H(t) = \beta p_H(t)^{1/\beta} H, \tag{16}$$

and thus the values of monopolists only depend on which sector they are, $V_L(t)$ and $V_H(t)$.

Equilibrium V

- For the prices of the two intermediate goods, (2) imply

$$p(t) \equiv \frac{p_H(t)}{p_L(t)} = \gamma \left(\frac{Y_H(t)}{Y_L(t)} \right)^{-\frac{1}{\varepsilon}} = \gamma^{\frac{\varepsilon\beta}{\sigma}} \left(\frac{N_H(t) H}{N_L(t) L} \right)^{-\frac{\beta}{\sigma}}, \tag{17}$$

  where $\gamma \equiv \gamma_H / \gamma_L$ and $\sigma \equiv \varepsilon - (\varepsilon - 1)(1 - \beta) = 1 + (\varepsilon - 1)\beta$.

- Relative factor prices are given by

$$\omega(t) \equiv \frac{w_H(t)}{w_L(t)} = p(t)^{1/\beta} \frac{N_H(t)}{N_L(t)} = \gamma^{\frac{\varepsilon}{\sigma}} \left(\frac{N_H(t)}{N_L(t)} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H}{L} \right)^{-\frac{1}{\sigma}}. \tag{18}$$

Equilibrium VIII

- Free entry conditions:

$$\eta_L V_L(t) \leq 1 \text{ and } \eta_L V_L(t) = 1 \text{ if } Z_L(t) > 0. \tag{19}$$

and

$$\eta_H V_H(t) \leq 1 \text{ and } \eta_H V_H(t) = 1 \text{ if } Z_H(t) > 0. \tag{20}$$

- Consumer side:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}\left(r(t) - \rho\right), \tag{21}$$

and

$$\lim_{t\to\infty}\left[\exp\left(-\int_0^t r(s)\,ds\right)\left(N_L(t)\,V_L(t) + N_H(t)\,V_H(t)\right)\right] = 0, \tag{22}$$

where $N_L(t)\,V_L(t) + N_H(t)\,V_H(t)$ is the total value of corporate assets in this economy.

Balanced Growth Path I

- Consumption grows at the constant rate, g^*, and the relative price $p(t)$ is constant. From (9) this implies that $p_L(t)$ and $p_H(t)$ are also constant.
- Let V_L and V_H be the BGP net present discounted values of new innovations in the two sectors. Then (8) implies that

$$V_L = \frac{\beta p_L^{1/\beta} L}{r^*} \text{ and } V_H = \frac{\beta p_H^{1/\beta} H}{r^*}, \tag{23}$$

- Taking the ratio of these two expressions, we obtain

$$\frac{V_H}{V_L} = \left(\frac{p_H}{p_L}\right)^{\frac{1}{\beta}} \frac{H}{L}.$$

Balanced Growth Path II

- Note the two effects on the direction of technological change:
  1. The price effect: V_H/V_L is increasing in p_H/p_L. Tends to favor technologies complementing scarce factors.
  2. The market size effect: V_H/V_L is increasing in H/L. It encourages innovation for the more abundant factor.
- The above discussion is incomplete since prices are endogenous. Combining (23) together with (17):

$$\frac{V_H}{V_L} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{\sigma}} \left(\frac{N_H}{N_L}\right)^{-\frac{1}{\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-1}{\sigma}} . \tag{24}$$

- Note that an increase in H/L will increase V_H/V_L as long as $\sigma > 1$ and it will reduce it if $\sigma < 1$. Moreover,

$$\sigma \gtreqless 1 \iff \varepsilon \gtreqless 1.$$

- The two factors will be gross substitutes when the two intermediate goods are gross substitutes in the production of the final good.

Balanced Growth Path III

- Next, using the two free entry conditions (19) and (20) as equalities, we obtain the following BGP "technology market clearing" condition:

$$\eta_L V_L = \eta_H V_H. \tag{25}$$

- Combining this with (24), BGP ratio of relative technologies is

$$\left(\frac{N_H}{N_L}\right)^* = \eta^\sigma \gamma^\varepsilon \left(\frac{H}{L}\right)^{\sigma-1}, \tag{26}$$

  where $\eta \equiv \eta_H / \eta_L$.

- Note that relative productivities are determined by the innovation possibilities frontier and the relative supply of the two factors. In this sense, this model totally endogenizes technology.

Equilibrium Characterization

Proposition There exists a unique BGP. Moreover, starting with any $N_H(0) > 0$ and $N_L(0) > 0$, there exists a unique equilibrium path. If $N_H(0)/N_L(0) < (N_H/N_L)^*$ as given by (26), then we have $Z_H(t) > 0$ and $Z_L(t) = 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$. If $N_H(0)/N_L(0) > (N_H/N_L)^*$, then $Z_H(t) = 0$ and $Z_L(t) > 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$.

- Summary: the dynamic equilibrium path always tends to the BGP and during transitional dynamics, there is only one type of innovation.

Directed Technological Change and Factor Prices

- In BGP, there is a positive relationship between H/L and N_H^*/N_L^* only when $\sigma > 1$.
- But this does not mean that depending on σ (or ε), changes in factor supplies may induce technological changes that are biased in favor or against the factor that is becoming more abundant.
- Why?
  - N_H^*/N_L^* refers to the ratio of factor-augmenting technologies, or to the ratio of *physical* productivities.
  - What matters for the bias of technology is *the value of marginal product* of factors, affected by relative prices.
  - The relationship between factor-augmenting and factor-biased technologies is reversed when σ is less than 1.
  - When $\sigma > 1$, an increase in N_H^*/N_L^* is relatively biased towards H, while when $\sigma < 1$, a *decrease* in N_H^*/N_L^* is relatively biased towards H.

Weak Equilibrium (Relative) Bias Result

Proposition Consider the directed technological change model described above. There is always **weak equilibrium (relative) bias** in the sense that an increase in H/L always induces relatively H-biased technological change.

- The results reflect the strength of the market size effect: it always dominates the price effect.
- But it does not specify whether this induced effect will be strong enough to make the endogenous-technology relative demand curve for factors upward-sloping.

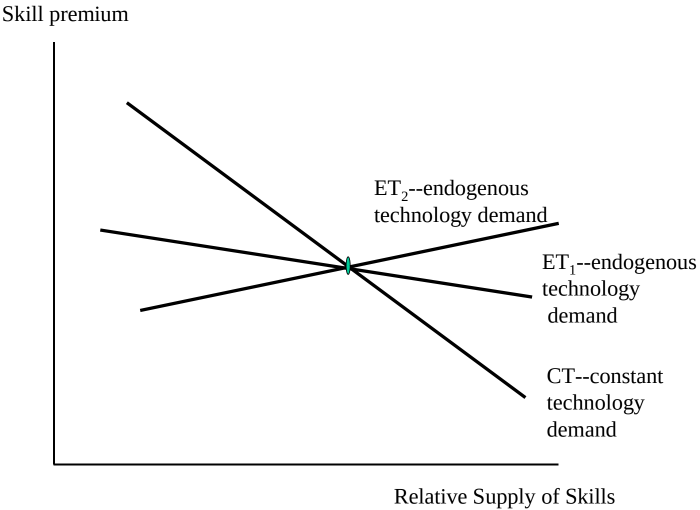
Strong Equilibrium (Relative) Bias Result

- Substitute for $(N_H / N_L)^*$ from (26) into the expression for the relative wage given technologies, (18), and obtain:

$$\omega^* \equiv \left(\frac{w_H}{w_L}\right)^* = \eta^{\sigma-1}\gamma^{\varepsilon}\left(\frac{H}{L}\right)^{\sigma-2}. \tag{27}$$

Proposition Consider the directed technological change model described above. Then if $\sigma > 2$, there is **strong equilibrium (relative) bias** in the sense that an increase in H/L raises the relative marginal product and the relative wage of the factor H compared to factor L.

Relative Supply of Skills and Skill Premium



Discussion

- Analogous to Samuelson's *LeChatelier principle*: think of the endogenous-technology demand curve as adjusting the "factors of production" corresponding to technology.
- But, the effects here are caused by general equilibrium changes, not on partial equilibrium effects.
- Moreover ET_2, which applies when $\sigma > 2$ holds, is upward-sloping.
- A complementary intuition: importance of non-rivalry of ideas:
  - leads to an aggregate production function that exhibits increasing returns to scale (in all factors including technologies).
  - the market size effect can create sufficiently strong induced technological change to increase the relative marginal product and the relative price of the factor that has become more abundant.

Implications I

- Recall we have the following stylized facts:
  - Secular skill-biased technological change increasing the demand for skills throughout the 20th century.
  - Possible acceleration in skill-biased technological change over the past 25 years.
  - A range of important technologies biased against skill workers during the 19th century.
- The current model gives us a way to think about these issues.
  - The increase in the number of skilled workers should cause steady skill-biased technical change.
  - Acceleration in the increase in the number of skilled workers should induce an acceleration in skill-biased technological change.
  - Available evidence suggests that there were large increases in the number of unskilled workers during the late 18th and 19th centuries.

Implications II

- The framework also gives a potential interpretation for the dynamics of the college premium during the 1970s and 1980s.
  - It is reasonable that the equilibrium skill bias of technologies, N_H / N_L, is a sluggish variable.
  - Hence a rapid increase in the supply of skills would first reduce the skill premium as the economy would be moving along a constant technology (constant N_H / N_L).
  - After a while technology would start adjusting, and the economy would move back to the upward sloping relative demand curve, with a relatively sharp increase in the college premium.

Implications III

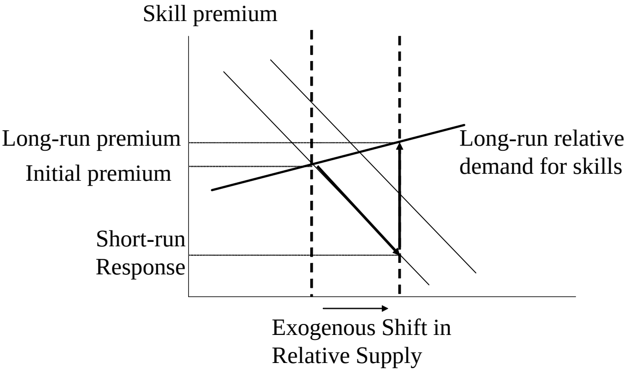


Figure: Dynamics of the skill premium in response to an exogenous increase in the relative supply of skills, with an upward-sloping endogenous-technology relative demand curve.

Implications IV

- If instead $\sigma < 2$, the long-run relative demand curve will be downward sloping, though again it will be shallower than the short-run relative demand curve.
- An increase in the relative supply of skills leads again to a decline in the college premium, and as technology starts adjusting the skill premium will increase.
- But it will end up below its initial level. To explain the larger increase in the college premium in the 1980s, in this case we would need some exogenous skill-biased technical change.

Implications V

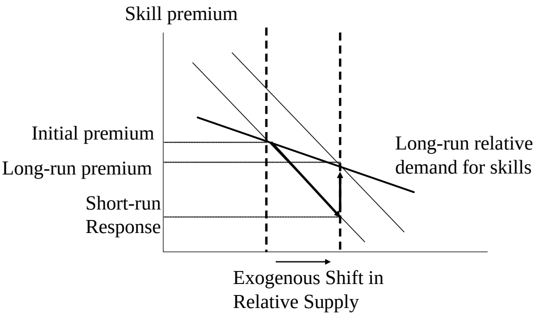


Figure: Dynamics of the skill premium in response to an increase in the relative supply of skills, with a downward-sloping endogenous-technology relative demand curve.

New Tasks

- Going beyond factor-augmenting technological changes — incentives and implications.
- **Main new ingredient**: in addition to capital simplifying and replacing tasks previously performed by labor, new more complex tasks relying on labor are created.
- E.g.:
    - the replacement of the stagecoach by the railroad is a clear example of capital-labor substitution, but it was preceded and also followed by the creation of several entirely new labor-intensive tasks, including a new class of engineers, new types of managers and financiers, machinists, repairmen, conductors, and so on;
    - today, as computers and machines replace labor, we are also creating new design tasks ranging from engineering tasks based on new machines to programming and apps design.

Plan

- Following Acemoglu and Restrepo (2015), we will embed this new ingredient in a dynamic model in which two types of innovations create different and countervailing forces:
    1. Labor-replacing innovations enable capital to replace previously labor-intensive tasks.
    2. Labor-intensive innovations create new, more complex versions of existing tasks.
- Growth will take place due to productivity upgrading of a fixed set of tasks as well as from the substitution of capital for labor.
- We will see how different types of technological changes create different distributional forces and how the economy generates incentives for “self-correction”.
- Also, implications for employment and unemployment (only sketched).

Static Model: Production

- There is a unique final good Y produced by combining a continuum of tasks $y(i)$, with $i \in [N-1, N]$.

$$Y = \left(\int_{N-1}^{N} y(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \text{ with } \epsilon = \text{elasticity of substitution.}$$

- Set the resulting ideal price index as numeraire.
- The range $N-1$ to N implies that the set of tasks is constant, but older tasks might be replaced by new versions thereof (and thus an increase in N adds a new task at the top while simultaneously replacing one at the bottom).

Static Model: Intermediates

- Each task produced with an intermediate good $x(i)$ embodying new (labor- or capital-intensive) technology.
- Tasks with $i > I$ are not yet automated (or are new tasks) can only be produced with labor according to the following CES-type production function.

$$y(i) = B\left[\eta q(i)^{\frac{\xi-1}{\xi}} + (1-\eta)\left(\gamma(i)l(i)\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}},$$

  where ξ is the elasticity of substitution between labor and the intermediate good embodying the technology.
- Tasks with $i \leq I$ are **automated**, and can be produced with labor or capital:

$$y(i) = B\left[\eta q(i)^{\frac{\xi-1}{\xi}} + (1-\eta)\left(k(i)+\gamma(i)l(i)\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}.$$

- $\gamma(i) =$ "comparative advantage schedule".
- I will be determined by endogenous technological change.

Static Model: Simplification

- For this lecture, let us take $\zeta = 1$, so that we end up with the following Cobb-Douglas production functions:

$$y(i) = Bx(i)^{\eta}\left(k(i) + \gamma(i)l(i)\right)^{1-\eta} \text{ for } i \leq I \text{ (automated)}$$

$$y(i) = Bx(i)^{\eta}\left(\gamma(i)l(i)\right)^{1-\eta} \text{ for } i > I \text{ (non-automated).}$$

- This is with little loss of generality (except for one thing mentioned below).

Static Model: Comparative Advantage

- We assume
$$\gamma(i) = e^{Ai}, \text{ with } A > 0.$$
- This implies that labor is more productive in new more complex tasks and will build growth through quality improvements.
- It also implies **comparative advantage** between capital and labor, so that more complex tasks will be produced with labor.

Static Model: Pricing

- Intermediates, $x(i)$, are produced by monopolist firms at a constant marginal cost $\psi\mu$, with $\mu < 1$.
- Two kinds of competition:
  1. from the fringe of non-innovative copiers, which can produce any intermediate at cost ψ;
  2. for labor- (capital-) intensive tasks there is also potential competition from the just-replaced capital- (labor-) intensive tasks. We assume that these innovative firms will need to pay a small cost $\varepsilon > 0$ (formally $\varepsilon \to 0$) not to shut down entirely.
- This structure implies that the next-best efficient innovative firm will always shut down (suppose not, then it never wins the market and incurs the cost $\varepsilon > 0$).
- The fringe is always around, so equilibrium prices for intermediates are always at ψ. This applies even when demand elasticity is < 1.

Static Model: Factor Supplies

- In the static model, we take capital to be fixed at K and rented at a price r (determined endogenously).
- Total labor used is given by

$$L^s\left(\frac{w}{rK}\right),$$

where L^s is a weakly increasing function, and w is the wage rate.

  - This is a reduced form for many different models of labor supply and quasi-labor supply behavior, including labor supply under balanced growth preferences or efficiency wages.
  - Later, this will be derived endogenously as the equilibrium representation in a search-matching model.

Equilibrium: Factor Demands

- Let $\widetilde{I}$ be the endogenous threshold at which, given factor prices, firms are indifferent between using capital and labor. It is given by

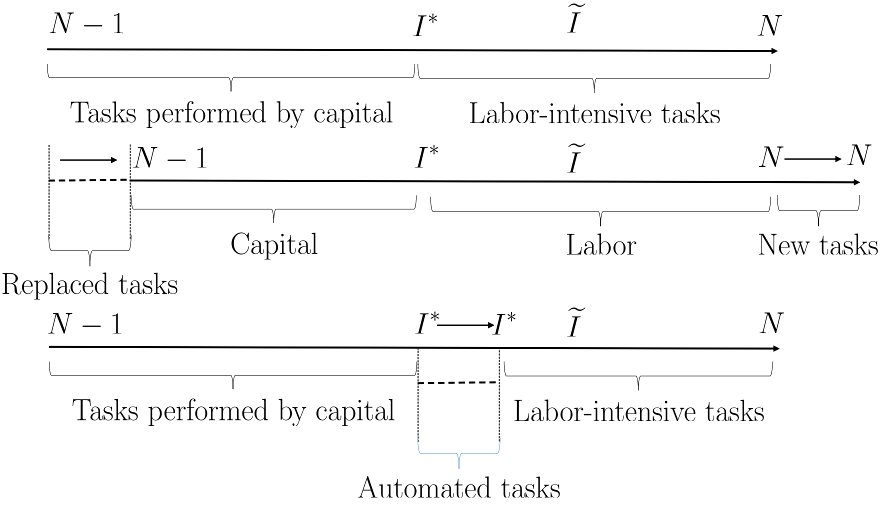
$$\frac{w}{r} = \gamma(\widetilde{I}).$$

- Then, tasks with $i \leq I^* \equiv \min\{I, \widetilde{I}\}$ will be produced with capital and their output priced at $r^{1-\eta}$ (since if $I < \widetilde{I}$, the binding threshold is the technologically-determined one, I). Tasks with $i > I^*$ will be produced with labor and priced at $\left(\frac{w}{\gamma(i)}\right)^{1-\eta}$.
- With the appropriate normalization of B, factor demands are

$$k(i) = r^{-\sigma} Y, \text{ and } l(i) = \gamma(i)^{\sigma-1} w^{-\sigma} Y,$$

where $\sigma \equiv (\epsilon - 1)(1 - \eta) + 1$ is the short-run elasticity of substitution between capital and labor.
- In general, σ is a weighted average of ϵ and ξ, but in this case $\xi = 1$, and thus $\sigma > 1$ iff $\epsilon > 1$. Relevant for interpretation below.

Equilibrium: Task space



Equilibrium: Market Clearing

- Thus, capital market market clearing can be written as

$$r^{-\sigma}(I^* - N + 1)Y = K.$$

- Note: $I^* - N + 1$ is the range of tasks produced by capital.
- Labor market clearing can be similarly written as

$$\left(\int_{I^*}^{N} \gamma(i)^{\sigma-1} di\right) w^{-\sigma} Y = L^s\left(\frac{w}{rK}\right).$$

- Both σ less than and greater than 1 are allowed for now.

Equilibrium in the Static Model

- Let $\omega = \frac{w}{rK}$. Then the capital-labor indifference condition can be written as

$$\omega K = \gamma(\widetilde{I}).$$

- The equilibrium value of ω can be found by combining capital and labor market clearing, which yields

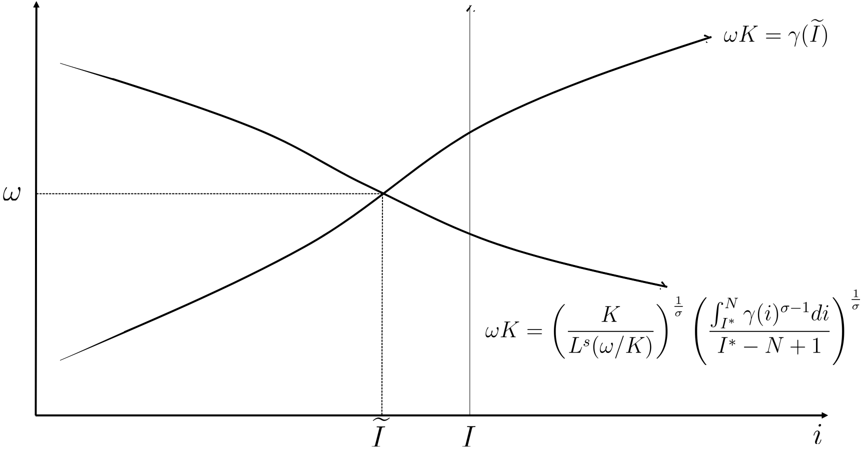
$$\omega K = \left(\frac{K}{L^s(\omega)} \right)^{\frac{1}{\sigma}} \left(\frac{\int_{I^*}^{N} \gamma(i)^{\sigma-1} di}{I^* - N + 1} \right)^{\frac{1}{\sigma}} .$$

- Aggregate output can now be obtained, using market clearing, as

$$Y = \left[(I^* - N + 1)^{\frac{1}{\sigma}} K^{\frac{\sigma-1}{\sigma}} + \left(\int_{I^*}^{N} \gamma(i)^{\sigma-1} di \right)^{\frac{1}{\sigma}} L^s(\omega)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} ,$$

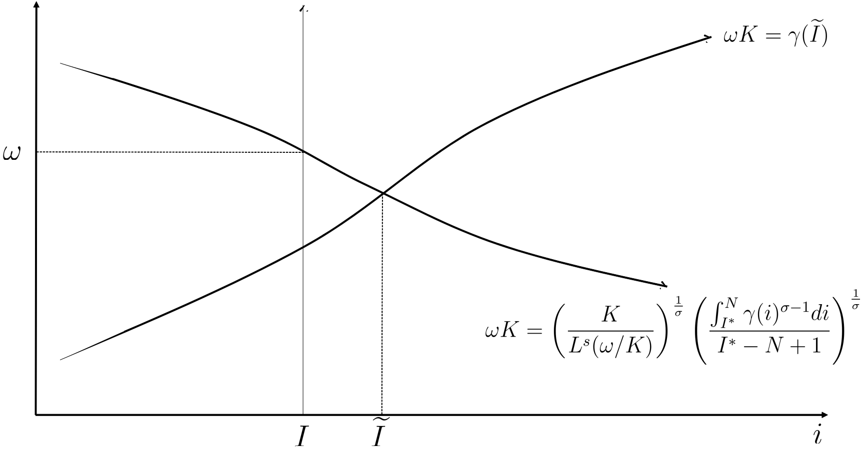
with factor prices given by the ideal price index equation.

Diagrammatic Representation



- Equilibrium determined by capital-labor indifference $(\widetilde{I})$.

Diagrammatic Representation



- Equilibrium determined by technological constraint on automation (I).

Summary

Proposition

Let $\omega = \frac{w}{rK}$. *For any range of tasks* $[N-1, N]$, *level of automation* $I > N-1$ *and capital* K, *there is a unique equilibrium. The equilibrium is characterized by* ω *and threshold* $I^* = \min\{I, \widetilde{I}\} \in (N-1, N)$ *that are given by the (unique) solution to the system of equations:*

$$\omega K = \gamma(\widetilde{I}),$$

and

$$\omega K = \left(\frac{K}{L^s(\omega)}\right)^{\frac{1}{\sigma}} \left(\frac{\int_{I^*}^{N} \gamma(i)^{\sigma-1} di}{I^* - N + 1}\right)^{\frac{1}{\sigma}}.$$

Comparative Statics

Proposition

Let $\zeta \equiv \frac{d\ln L^s}{d\ln \omega} > 0$ *be the quasi-labor supply elasticity.*
If $I < \widetilde{I}$, *we have:*

$$\begin{aligned}(\sigma+\zeta)\, d\ln\omega &= (1-\sigma) d\ln K + \left[\frac{N\gamma(N)^{\sigma-1}}{\int_I^N \gamma(i)^{\sigma-1} di} + \frac{N}{I-N+1}\right] d\ln N \\ &\quad - \left[\frac{I\gamma(I)^{\sigma-1}}{\int_I^N \gamma(i)^{\sigma-1} di} + \frac{I}{I-N+1}\right] d\ln I.\end{aligned}$$

Otherwise, if $\widetilde{I} < I$, *we have:*[a]

$$(\sigma+\zeta+\Lambda/A)\, d\ln\omega = (1-\sigma-\Lambda/A) d\ln K + \left[\frac{N\gamma(N)^{\sigma-1}}{\int_I^N \gamma(i)^{\sigma-1} di} + \frac{N}{I-N+1}\right] d\ln N.$$

[a] $\Lambda \equiv \frac{(\sigma-1)\widetilde{I}}{e^{(\sigma-1)A(N-\widetilde{I})}-1} + \frac{\widetilde{I}}{\widetilde{I}-(N-1)} > 0.$

Comparative Statics: Interpretation

- Note that when $\widetilde{I} < I$, the elasticity of substitution between capital and labor is $\sigma + \Lambda / A$ rather than σ because of the endogenous changes in the set of tasks produced by capital.
- Labor-intensive technology, N, increases ω, the labor share, and total employment.
- Labor-replacing technology, I, reduces ω, the labor share and total employment (if $I^* = I$).
  - Note this is different from models with factor-augmenting technological change — automation always increases the share of capital (and new labor-intensive tasks always increase the share of labor).
  - This is related to the feature emphasized in Acemoglu and Autor (2011).
- In the (very) short run, capital accumulation increases or reduces the labor share depending on $\sigma < 1$ or > 1. But capital-labor substitution (increase in $\widetilde{I}$ for given technology) may (further) reduce it in the medium run.

The Structure of Balanced Growth Path

- Define the BGP as an equilibrium in which w, K and Y grow at a common rate and the interest rate, r, is constant.
- Suppose $I = \widetilde{I}$.
- We now characterize what types of technological changes will be consistent with BGP.
- Suppose $\dot{N} = \dot{I} = \dot{\widetilde{I}} = \Delta$.
- Then from

$$\frac{w}{r} = \gamma(\widetilde{I}) = e^{A\widetilde{I}},$$

so wages grow at a rate $g^* = A\Delta$ if the interest rate, r, is constant.

The Structure of Balanced Growth Path (continued)

- The labor market clearing condition, with the integral solved out,

$$\frac{e^{(\sigma-1)AN}-e^{(\sigma-1)AI}}{(\sigma-1)A}w^{-\sigma}Y=L^{s}\left(\omega\right),$$

  holds over time when w, K and Y grow at the rate $g^{*}=A\Delta$.
- Finally, the capital market clearing condition,

$$(I^{*}-N+1)r^{-\sigma}Y=K,$$

  implies that r is constant (at some r^{*}) over time in this case.

The Structure of Balanced Growth Path: Summary

Proposition

Suppose N and I grow as $\dot{N} = \dot{I} = \Delta$. For any $N(0)$ and $I(0)$, the economy has a unique BGP where wages, w, output Y and capital, K, grow at a common rate

$$g^* = A\Delta,$$

and the interest rate r is constant at r^*, and $N(t) - I(t) = n^*$.

Dynamic Model: Preferences and Resource Constraint

- We now move to a dynamic model with capital accumulation and endogenous technological change.
- A representative household economy with preferences over consumption

$$\int_0^\infty e^{-\rho t} \frac{C(t)^{1-\chi} - 1}{1-\chi} dt,$$

who saves by acquiring capital or claims over monopolists' profits.
- Capital earns an interest rate $r(t)$ and depreciation is normalized to zero.
- The aggregate resource constraint is then given by

$$\dot{K}(t) = Y(t) - C(t) - X(t),$$

where $C(t)$ denotes consumption, $X(t)$ resources used in the production of intermediates $x(i)$, and $K(t)$ capital.

Dynamic Model: Innovation Possibilities Frontier

- Technology is endogenous and developed by S scientists.
- If $S_N(t) \geq 0$ scientists are allocated to creating new more complex tasks, they increase N by

$$\dot{N}(t) = a_N \varphi(n(t)) S_N(t) > 0,$$

where $n = N - I$, and φ is a weakly diminishing function, capturing the fact that replacing capital-intensive tasks with new more complex tasks may become harder when there are only a few capital-intensive tasks.

- As specified above, new complex tasks replace capital-intensive tasks, so that the range of tasks remains fixed (just shifts to the right).
- Similarly, if $S_I(t) \geq 0$ scientists are allocated to automating older tasks, they increase I by

$$\dot{I}(t) = a_I \varphi(1 - n(t)) S_I(t) > 0,$$

and replace previously labor-intensive tasks.

Dynamic Model: Profits and Value Functions

- Profits of capital-intensive tasks can be written as

$$\pi_K(i,t) = \eta(1-\mu)Y(t)r(t)^{1-\sigma},$$

where recall that $1-\mu$ is the markup determined by the competitive fringe.

- Note that these profits are decreasing in r only when $\sigma > 1$, which is intuitive.
- The value function of such firms can be written as follows:
    - There will exist some equilibrium time such that intermediate i has been created, $T_0^I(i)$, and there is some time $T_1^I(i)$ at which it will be replaced by a more complex labor-intensive task.
    - In the interim its value is given by the following differential equation

$$r(t)V_I(i,t) - \dot{V}_I(i,t) = \pi_K(i,t).$$

- The boundary condition is $V_I(i, T_1^I(i))) = 0$.

Dynamic Model: Profits and Value Functions (continued)

- In a BGP, there will exist some n^* that is constant, and as described above, both N and I grow by the same increments, Δ.
- Then we have that

$$T_1^I(i) - T_0^I(i) = \frac{1-n^*}{\Delta}.$$

- Therefore, the value of a new capital-intensive intermediate at inception time $t = T_0^I(i)$ can be solved out as

$$V_I((T_0^I)^{-1}(t), t) = \eta(1-\mu)Y(t)(r^*)^{1-\sigma} \int_0^{\frac{1-n^*}{\Delta}} e^{-(r^*-g^*)\tau} d\tau$$

$$= \frac{\eta(1-\mu)Y(t)(r^*)^{1-\sigma}[1 - e^{-(r^*-g^*)\frac{1-n^*}{\Delta}}]}{r^* - g^*}.$$

- Key observation: in BGP $V_I(i, T_0^I(i))) = V_I((T_0^I)^{-1}(t), t)$ grows at the rate g^* in $T_0^I(i) = t$.

Dynamic Model: Profits and Value Functions (continued)

- The argument for labor-intensive intermediates is similar, with the value function between the boundary points, $T_0^N(i)$ and $T_1^N(i)$, being given by

$$r(t)V_N(i,t) - \dot{V}_N(i,t) = \pi_L(i,t),$$

with boundary condition $V_N(i, T_1^N(i)) = 0$.

- Then again using the BGP properties, we have

$$V_N((T_0^N)^{-1}(t),t)) = \eta(1-\mu)Y(t)\left(\frac{w(t)}{\gamma((T_0^N)^{-1}(t))}\right)^{1-\sigma}\int_0^{n^*/\Delta} e^{-(r^*-g^*-g^*(1-\sigma))\tau}d\tau$$

$$= \frac{\eta(1-\mu)Y(t)\left(\frac{w(t)}{\gamma((T_0^N)^{-1}(t))}\right)^{1-\sigma}[1-e^{-(r^*-g^*-g^*(1-\sigma))\frac{n^*}{\Delta}}]}{r^*-g^*-g^*(1-\sigma)}.$$

- This expression also grows at the rate g^* in $T_0^N(i) = t$ (the growth of $w(t)$ is fully neutralized by the growth of $\gamma(i)$ in $i = (T_0^N)^{-1}(t)$).

Dynamic Model: BGP

- This derivation implies that at the appropriate value of n, the incentives for the two types of technological change will be balanced, generating a BGP.
- Then we can look for an allocation in which $\dot{I} = \dot{N} = \Delta$, and $I^* = I$ (otherwise, there are no incentives for automation).
- Balanced growth will be achieved at n^* when

$$S_N = \frac{a_I \varphi(1 - n^*)}{a_N \varphi(n^*) + a_I \varphi(1 - n^*)} S \text{ and } S_I = \frac{a_N \varphi(n^*)}{a_N \varphi(n^*) + a_I \varphi(1 - n^*)} S.$$

Dynamic Equilibrium: Summary

Proposition

There exists a unique BGP with endogenous, directed technological change. In this BGP, output $Y(t)$, wages $w(t)$ and capital $K(t)$ grow at the common rate

$$g^* = \frac{a_I a_N \varphi(1-n^*)\varphi(n^*)}{a_I \varphi(1-n^*) + a_N \varphi(n^*)} S.$$

There is simultaneous creation of labor-replacing technologies and labor-intensive tasks. In particular, we have

$$\dot{N}(t) = \dot{I}(t) = \frac{g^*}{A},$$

and $N(t) - I(t) = n^*$ is fixed over time. Moreover, $I^*(t) = I\ (t)$ so that there are incentives for automation.

The Race between the Two Types of Technologies

Proposition

Suppose the economy is in the BGP described in the previous proposition.

- *An unanticipated increase in N increases the labor share and total employment.*
- *An unanticipated increase in I reduces the labor share and total employment.*
  If the increase is large enough, the economy will switch for a while to the case $\widetilde{I} < I$, until $I^* > I$ and the economy starts automating more tasks again.

- What happens following such increase? The answer depends on dynamics, which we turn to the next.

Dynamics

- Let us develop the heuristic argument here. Suppose $\sigma > 1$ (or more generally $\epsilon > \xi$), which ensures that incentive to produce with the cheaper factor are greater.
- In the neighborhood of the BGP,

$$\frac{V_N}{V_I} \propto \left(\frac{w(t)/r(t)}{\gamma((T_0^N)^{-1}(t))} \right)^{1-\sigma}.$$

- From this, we have that if $N(t) - I(t) > n^*$, then $\frac{w}{r}$ will be higher than its BGP value and thus $\frac{V_N}{V_I}$ will be lower than its BGP value, encouraging further increases in $I(t)$, and thus pushing the economy towards n^*.
- The argument for the case in which $N(t) - I(t) < n^*$ is analogous.

Dynamics (continued)

- If $K(t)$ is above its BGP value, then we have $\frac{w}{r}$ higher than its BGP value, and this will discourage further capital accumulation and encourage faster increases in $I(t)$, restoring the economy towards the BGP.
- Thus (with details ongoing):

Proposition

The above-characterized BGP is (saddle-path) stable.

- This implies that shocks to different types of technological changes will ultimately self-correct.
- So we may have an extended period in which the share of capital is higher, but this will ultimately trigger faster creation of new more complex tasks, restoring the capital share back to its BGP value.
- As noted above, this result is stated here for $\sigma > 1$ but can be extended in the general case to the configuration where $\epsilon > \xi$.

Search and Matching

- To allow a discussion of the implications of different types of technological changes for unemployment, let us now embed the setup in a search-matching framework.
- This essentially entails combining the model of capital-labor substitution described above combined with a standard Diamond-Mortensen-Pissarides setup.
- Main trick: use the simple relationship implied by this framework between labor market tightness (vacancy-unemployment ratio) and profitability of new jobs.
- Main result: all of the results above generalize, but plus when labor share declines, unemployment increases, so that the two different types of technologies now, respectively, increase and reduce unemployment.