14.461: Technological Change, Lecture 4
Technology and the Labor Market

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Introduction

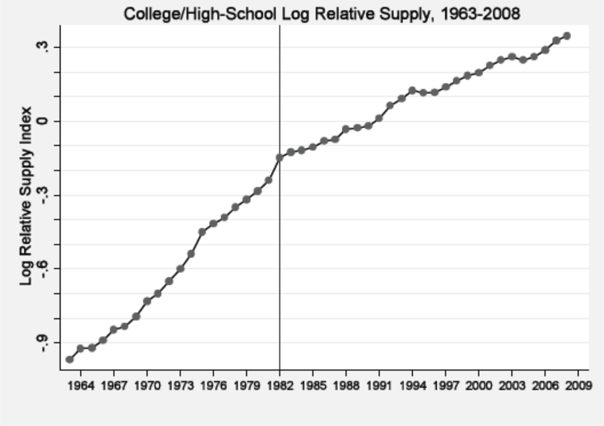
- Rich dynamics of wages and earnings in many developed country labor markets.
- In fact, going beyond what is generally predicted by models based on two types of skills (as we will see).
- A richer framework combining technology (potentially endogenous) with tasks more useful for thinking about these trends.

Skill-biased technological change

- Over the past 60 years, the U.S. relative supply of skills has increased, but:
    1. there has also been an increase in the college premium, and
    2. this increase may have accelerated in the late 1960s, and the skill premium increased very rapidly beginning in the late 1970s.
- Standard explanation: skill bias technical change, and an acceleration that coincided with the changes in the relative supply of skills.

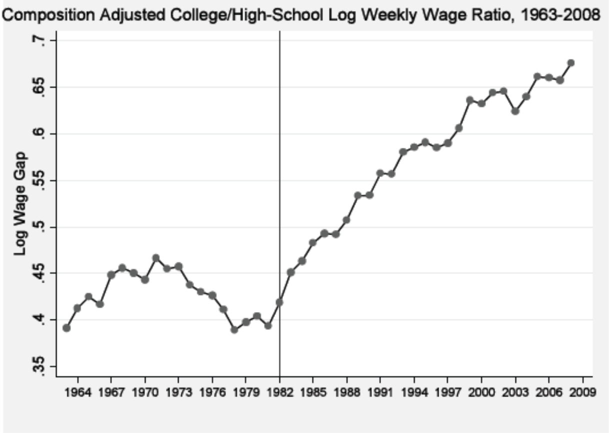
Skill-biased technological change

- Large and possibly accelerating increase in the supply of college workers.



Skill-biased technological change (continued)

- Simultaneously, sharply increasing college wage premium, indicating that the demand for college skills is increasing rapidly.



Constant Elasticity of Substitution Production Function I

- CES production function with two inputs (skilled and unskilled labor):

$$Y(t) = \left[\gamma_L \left(A_L(t) L(t)\right)^{\frac{\sigma-1}{\sigma}} + \gamma_H \left(A_H(t) H(t)\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

  where

  - $A_L(t)$ and $A_H(t)$ are two separate technology terms.
  - γ_is determine the importance of the two factors, $\gamma_L + \gamma_H = 1$.
  - $\sigma \in (0, \infty)$=elasticity of substitution between the two factors.
    - $\sigma = \infty$, perfect substitutes, linear production function is linear.
    - $\sigma = 1$, Cobb-Douglas,
    - $\sigma = 0$, no substitution, Leontieff.
    - $\sigma > 1$, "gross substitutes,"
    - $\sigma < 1$, "gross complements".
- Clearly, $A_L(t)$ is L-augmenting, while $A_H(t)$ is H-augmenting.
- Whether technological change that is L-augmenting (or H-augmenting) is L-biased or H-biased depends on σ.

Constant Elasticity of Substitution Production Function II

- Relative marginal product of the two factors:

$$\frac{MP_H}{MP_L} = \gamma \left(\frac{A_H(t)}{A_L(t)} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H(t)}{L(t)} \right)^{-\frac{1}{\sigma}}, \tag{1}$$

  where $\gamma \equiv \gamma_H / \gamma_L$.
- *substitution effect:* the relative marginal product of H is decreasing in its relative abundance, $H(t) / L(t)$.
- The effect of $A_H(t)$ on the relative marginal product:
  - If $\sigma > 1$, an increase in $A_H(t)$ (relative to $A_L(t)$) increases the relative marginal product of H.
  - If $\sigma < 1$, an increase in $A_H(t)$ reduces the relative marginal product of H.
  - If $\sigma = 1$, Cobb-Douglas case, and neither a change in $A_H(t)$ nor in $A_L(t)$ is biased towards any of the factors.
- Note also that σ is the elasticity of substitution between the two factors.

Constant Elasticity of Substitution Production Function III

- Intuition for why, when $\sigma < 1$, H-augmenting technical change is L-biased:
  - with gross complementarity ($\sigma < 1$), an increase in the productivity of H increases the demand for labor, L, by more than the demand for H, creating "excess demand" for labor.
  - the marginal product of labor increases by more than the marginal product of H.
  - Take case where $\sigma \to 0$ (Leontieff): starting from a situation in which $\gamma_L A_L(t) L(t) = \gamma_H A_H(t) H(t)$, a small increase in $A_H(t)$ will create an excess of the services of the H factor, and its price will fall to 0.

The Canonical Model

- Suppose that the total supply of low skill labor is L and the total supply of high skill labor is H.
- This could be because different "low skill" and "high skill" workers have different skill endowments, for example aggregated as

$$L = \int_{i \in \mathcal{L}} l_i di \text{ and } H = \int_{i \in \mathcal{H}} h_i di.$$

- The key building block is a constant elasticity of substitution aggregate production function similar to what we have seen already.

$$Y = \left[(A_L L)^{\frac{\sigma-1}{\sigma}} + (A_H H)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where $\sigma \in [0, \infty)$ is the elasticity of substitution between high skill and low skill labor, and A_L and A_H are factor-augmenting technology terms.

The Canonical Model (continued)

- Recall the focal cases are: (i) $\sigma \longrightarrow 0$, when high skill and low skill workers will be Leontieff, and output can be produced only by using high skill and low skill workers in fixed portions; (ii) $\sigma \longrightarrow \infty$ when high skill and low skill workers are perfect substitutes (and thus there is only one skill, which H and L workers possess in different quantities), and (iii) $\sigma \longrightarrow 1$, when the production function tends to the Cobb-Douglas case.
- Also this framework all technologies are *factor-augmenting*, meaning that technological change serves to either increase the productivity of high or low skill workers (or both).
- This implies that there are no explicitly skill-replacing technologies.

The Canonical Model (continued)

- The production function (2) admits three different interpretations.

1. There is only one good, and high skill and low skill workers are imperfect substitutes in the production of this good.
2. The production function (2) is also equivalent to an economy where consumers have utility function $\left[Y_l^{\frac{\sigma-1}{\sigma}} + Y_h^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$ defined over two goods. Good Y_h is produced using only high skill workers, and Y_l is produced using only low skill workers, with production functions $Y_h = A_H H$, and $Y_l = A_L L$.
3. A mixture of the above two whereby different sectors produce goods that are imperfect substitutes, and high and low-education workers are employed in both sectors.

The Canonical Model (continued)

- Since labor markets are competitive, the low skill unit wage is simply given by the value of marginal product of low skill labor, which is obtained by differentiating (2) as

$$w_L = \frac{\partial Y}{\partial L} = A_L^{\frac{\sigma-1}{\sigma}} \left[A_L^{\frac{\sigma-1}{\sigma}} + A_H^{\frac{\sigma-1}{\sigma}} (H/L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}. \tag{3}$$

- Given this unit wage, the earnings of worker $i \in \mathcal{L}$ is simply

$$W_i = w_L l_i.$$

The Canonical Model: Implications

- There are two important implications so far:

1. $\partial w_L / \partial H/L > 0$, that is, as the fraction of high skill workers in the labor force increases, the low skill wage should increase. This is an implication of imperfect substitution between high and low skill workers. An increase in the fraction (or relative supply) of high skill workers increases the demand for the services of low skill workers, pushing up their unit wage.
2. $\partial w_L / \partial A_L > 0$ and $\partial w_L / \partial A_H > 0$, that is, either kind of factor-augmenting technical change *increases* wages of low skill workers (except when $\sigma = 0$ and $\sigma = \infty$, where these inequalities might be weak). This result is intuitive but will also turn out to be important: technological improvements of any sort will lead to higher wages for both skill groups in the canonical model. Thus unless there is "technical regress," the canonical model cannot account for declining (real) wages of a factor whose supply is not shifting outward.

A Two Skill Model: Implications (continued)

- Similarly, the high skill unit wage is

$$w_H = \frac{\partial Y}{\partial H} = A_H^{\frac{\sigma-1}{\sigma}} \left[A_L^{\frac{\sigma-1}{\sigma}} (H/L)^{-\frac{\sigma-1}{\sigma}} + A_H^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}. \qquad (4)$$

- We again have similar comparative statics.
  - First, $\partial w_H / \partial H/L < 0$, so that when high skill workers become more abundant, their wages should fall.
  - Second, $\partial w_H / \partial A_L > 0$ and $\partial w_H / \partial A_H > 0$, so that technological progress of any kind increases high skill (as well as low skill) wages.
  - Also similarly, the earnings of worker $i \in \mathcal{H}$ is simply

$$W_i = w_L h_i.$$

- It can also be verified that an increase in either A_L or A_H (and also an increase in H/L) will raise average wages in this model

A Two Skill Model: Wage Inequality

- Combining (3) and (4), the skill premium—the unit high skill wage divided by the unit low skill wage—is

$$\omega = \frac{w_H}{w_L} = \left(\frac{A_H}{A_L}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H}{L}\right)^{-\frac{1}{\sigma}}. \tag{5}$$

- Equation (5) can be rewritten in a more convenient form by taking logs,

$$\ln \omega = \frac{\sigma-1}{\sigma} \ln\left(\frac{A_H}{A_L}\right) - \frac{1}{\sigma} \ln\left(\frac{H}{L}\right). \tag{6}$$

- Thus

$$\frac{\partial \ln \omega}{\partial \ln H/L} = -\frac{1}{\sigma} < 0. \tag{7}$$

- Related to Tinbergen's race between supply of skills and technology: for a *given skill bias of technology*, captured here by A_H/A_L, an increase in the relative supply of skills H/L reduces the skill premium with an elasticity of $1/\sigma$—via two different types of substitution.

Wage Inequality (continued)

- Also, as we have already observed:

$$\frac{\partial \ln \omega}{\partial \ln(A_H/A_L)} = \frac{\sigma - 1}{\sigma}. \tag{8}$$

- If $\sigma > 1$, then relative improvements in the high skill-augmenting technology (i.e., in A_H/A_L) increase the skill premium—i.e., a shift out of the relative demand curve for skills.
- The converse is obtained when $\sigma < 1$: that is, when $\sigma < 1$, an improvement in the productivity of high skill workers, A_H, relative to the productivity of low skill workers, A_L, shifts the relative demand curve inward and reduces the skill premium.

Bringing Tinbergen's Race to the Data

- To operationalize Tinbergen's hypothesis, suppose the following smooth behavior of skill bias over time

$$\ln\left(\frac{A_{H,t}}{A_{L,t}}\right)=\gamma_0+\gamma_1 t, \tag{9}$$

where t is calendar time and variables written with t subscript refer to these variables at time t.

- Then:

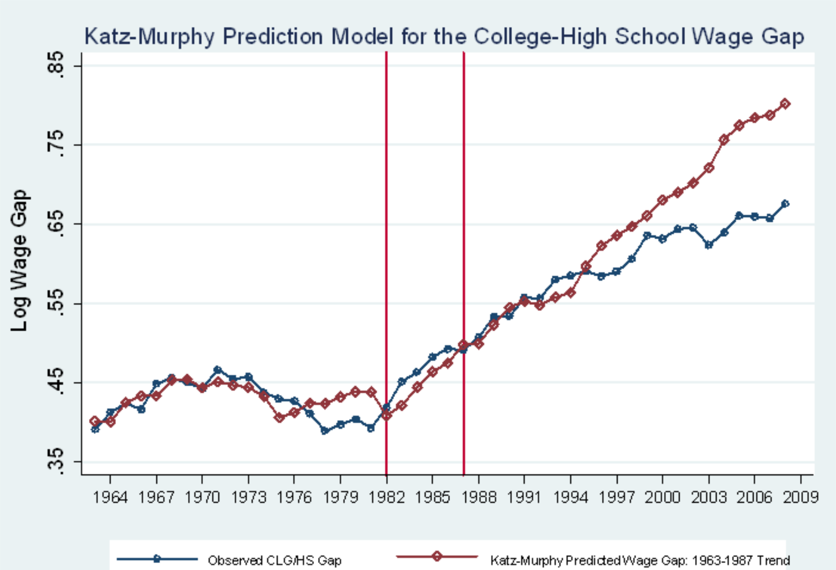
$$\ln\omega_t=\frac{\sigma-1}{\sigma}\gamma_0+\frac{\sigma-1}{\sigma}\gamma_1 t-\frac{1}{\sigma}\ln\left(\frac{H_t}{L_t}\right). \tag{10}$$

- Estimating this for 1963–1987, following Katz and Murphy (1992), we obtain

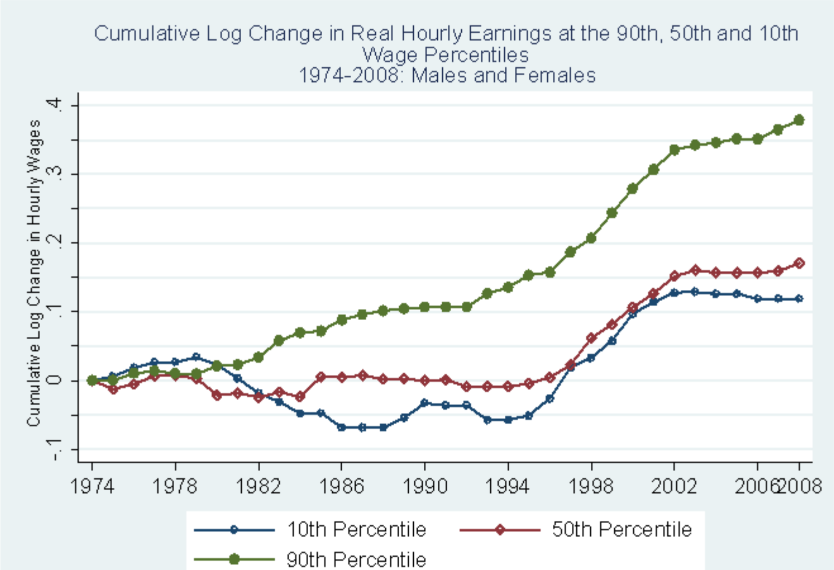
$$\begin{array}{lcccc}\ln\omega_t= & \text{constant} & +\,0.027\times t & -\,0.612\cdot\ln\left(\frac{H_t}{L_t}\right) \\ & & (0.005) & (0.128)\end{array}$$

- However, the fit of the model deteriorates for longer samples.

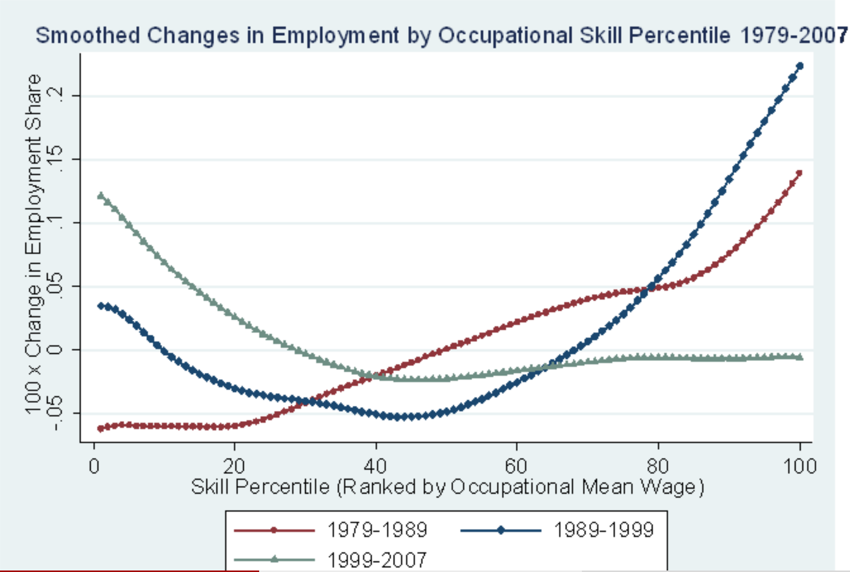
Model Fit


Katz-Murphy Prediction Model for the College-High School Wage Gap
Log Wage Gap
.35
.45
.55
.65
.75
.85
1964 1967 1970 1973 1976 1979 1982 1985 1988 1991 1994 1997 2000 2003 2006 2009
Observed CLG/HS Gap
Katz-Murphy Predicted Wage Gap: 1963-1987 Trend

Beyond Two Skills



Beyond Two Skills (continued)



Environment

- We consider a static environment with a unique final good.
- For now, the economy is closed and there is no trade in tasks (a possibility we allow for later).
- The unique final good is produced by combining a continuum of tasks represented by the unit interval, $[0, 1]$.
- Let us simplify the analysis assuming a Cobb-Douglas technology mapping the services of this range of tasks to the final good:

$$Y = \exp\left[\int_0^1 \ln y(i) di\right]. \tag{11}$$

- Throughout, we choose the price of the final good as the numeraire.

Skills and Tasks

- There are three factors of production, high, medium and low skilled workers. In addition, we will introduce capital or technology (embedded in machines) below.
- Suppose, for now, that there is a fixed, inelastic supply of the three types of workers, L, M and H.
- Skills and tasks are not the same thing. Workers have endowments of skills and use these endowments in order to perform tasks, which are units of activities that are part of the production process.

Tasks

- Each task has the following production function

$$y(i) = A_L \alpha_L(i) \, l(i) + A_M \alpha_M(i) \, m(i) + A_H \alpha_H(i) \, h(i) + A_K \alpha_K(i) \, k(i), \quad (12)$$

  where A terms represent factor-augmenting technology, and $\alpha_L(i)$, $\alpha_M(i)$ and $\alpha_H(i)$ are the task productivity schedules, designating the productivity of low, medium and high skill workers in different tasks.
- For example, $\alpha_L(i)$ is the productivity of low skill workers in task i, and $l(i)$ is the number of low skill workers allocated to task i.
- We will make a *comparative advantage* assumption, ensuring that high skill workers are relatively better at high, more complex tasks:

Assumption 1 $\alpha_L(i) / \alpha_M(i)$ and $\alpha_M(i) / \alpha_H(i)$ are continuously differentiable and strictly decreasing.

Market Clearing

- Factor market clearing requires

$$\int_0^1 l(i)di \leq L, \int_0^1 m(i)di \leq M \text{ and } \int_0^1 h(i)di \leq H. \qquad (13)$$

- When we introduce capital, we will assume that it is available at some constant price r.

Allocation of Skills to Tasks

- In view of the structure of comparative advantage differences in Assumption 1, there will exist some I_L and I_H such that all tasks $i < I_L$ will be performed by low skill workers, and all tasks $i > I_H$ will be performed by high skill workers. Intermediate tasks will be performed by medium skilled workers.
- We can think of these intermediate tasks as the routine tasks performed by workers in many production, clerical, and administrative support occupations.
- More formally, we have:

Lemma

In any equilibrium there exist I_L and I_H such that $0 < I_L < I_H < 1$ and for any $i < I_L$, $m(i) = h(i) = 0$, for any $i \in (I_L, I_H)$, $l(i) = h(i) = 0$, and for any $i > I_H$, $l(i) = m(i) = 0$.

The Law of One Wage (Price)

- Even though workers of the same skill level perform different tasks, in equilibrium they will receive the same wage—a simple "law of one price" that has to hold in any competitive equilibrium,
- Let $p(i)$ denotes the price of services of task i. Since we chose the final good as numeraire (setting its price to 1), we have

$$\exp\left[\int_0^1 \ln p(i)di\right] = 1.$$

- Since all tasks employing low skill workers must pay them the same wage, w_L, we have

$$w_L = p(i)A_L\alpha_L(i) \text{ for any } i < I_L.$$

- And similarly, for other skill groups:

$$w_M = p(i)A_M\alpha_M(i) \text{ for any } I_L < i < I_H.$$

$$w_H = p(i)A_H\alpha_H(i) \text{ for any } i > I_H.$$

Prices

- This observation also determines prices as the price difference between any two tasks produced by the same type of worker must exactly offset the productivity difference of this type of worker in these two tasks. For example:

$$p(i)\alpha_L(i) = p(i')\alpha_L(i') \equiv P_L, \tag{14}$$

for any $i, i' < I_L$, where the last equality defines P_L as the price "index" of tasks performed by low skill workers.

- Similarly, for medium skill workers, i.e., for any $I_H > i, i' > I_L$, we have

$$p(i)\alpha_M(i) = p(i')\alpha_M(i') \equiv P_M, \tag{15}$$

and for high skill workers and any $i, i' > I_H$,

$$p(i)\alpha_H(i) = p(i')\alpha_H(i') \equiv P_H. \tag{16}$$

Prices (continued)

- The Cobb-Douglas technology (the unitary elasticity of substitution between tasks) in (11) implies that "expenditure" across all tasks should be equalized, and given our choice of numeraire, this expenditure should be equal to the value of total output.
- More specifically, the first-order conditions for cost minimization in the production of the final good imply that $p(i)y(i) = p(i')y(i')$ for any i, i', or

$$p(i)y(i) = Y, \text{ for any } i \in [0,1]. \tag{17}$$

Labor Allocation

- Now consider two tasks $i, i' < I_L$ (performed by low skill workers), then using the definition of the productivity of low skill workers in these tasks, we have

$$p(i)\alpha_L(i)\,l(i) = p(i')\alpha_L(i')\,l(i').$$

- Therefore, for any $i, i' < I_L$, we have $l(i) = l(i')$, and from labor market clearing,

$$l(i) = \frac{L}{I_L} \text{ for any } i < I_L. \tag{18}$$

- Similarly for other skill groups

$$m(i) = \frac{M}{I_H - I_L} \text{ for any } I_H > i > I_L. \tag{19}$$

$$h(i) = \frac{H}{1 - I_H} \text{ for any } i > I_H. \tag{20}$$

Relative Prices

- Now comparing tasks performed by high and medium skill workers ($I_L < i < I_H < i'$), we also obtain:

$$\frac{P_M A_M M}{I_H - I_L} = \frac{P_H A_H H}{1 - I_H},$$

or

$$\frac{P_H}{P_M} = \left(\frac{A_H H}{1 - I_H}\right)^{-1} \left(\frac{A_M M}{I_H - I_L}\right). \quad (21)$$

- Similarly, comparing two tasks performed by medium and high skill workers, we obtain

$$\frac{P_M}{PL} = \left(\frac{A_M M}{I_H - I_L}\right)^{-1} \left(\frac{A_L L}{I_L}\right). \quad (22)$$

Equilibrium Wages

- Given the threshold tasks, I_L and I_H, wage levels and earnings differences across skill groups can be found in a straightforward manner.
- In particular,

$$\begin{aligned} w_L &= P_L A_L \\ w_M &= P_M A_M \\ w_H &= P_H A_H. \end{aligned} \tag{23}$$

Wage Inequality

- Wage inequality and skill premia are also straightforward.
- Comparing high and medium skill wages, we have

$$\frac{w_H}{w_M} = \frac{P_H A_H}{P_M A_M}.$$

- Substituting out relative prices, this gives

$$\frac{w_H}{w_M} = \left(\frac{1 - I_H}{I_H - I_L}\right) \left(\frac{H}{M}\right)^{-1}. \tag{24}$$

- Similarly, the wage of medium relative to low skill workers is given by

$$\frac{w_M}{w_L} = \left(\frac{I_H - I_L}{I_L}\right) \left(\frac{M}{L}\right)^{-1}. \tag{25}$$

- These expressions highlight the central role that allocation of tasks to skills plays in the model.
- Crucially, relative wages are simply functions of relative supplies and equilibrium task assignments (in particular, the threshold tasks, I_L and I_H).

No Arbitrage

- Now the threshold tasks must be determined such that there is no possible "arbitrage" from having a task produced by different type of worker.
- To ensure this, in view of Assumption 1, it is sufficient to check threshold tasks. (Why?)
- No arbitrage between high and medium skills implies that threshold tasks should be produced at the same cost with both types of workers, i.e.,

$$\frac{w_H}{A_H \alpha_H(I_H)} = \frac{w_M}{A_M \alpha_M(I_H)}.$$

  That is, wage divided by productivity of the two types of workers at task I_H must be equalized.
- Substituting from (23), we have

$$\frac{P_H}{\alpha_H(I_H)} = \frac{P_M}{\alpha_M(I_H)}.$$

No Arbitrage (continued)

- Then using (21), we obtain the no arbitrage condition between high and medium skills as

$$\frac{A_M \alpha_M (I_H) M}{I_H - I_L} = \frac{A_H \alpha_H (I_H) H}{1 - I_H}. \quad (26)$$

- Intuitively, this condition requires that a marginal medium skill and a marginal high skill worker both produce the threshold task with equal productivity.
- Similarly, no arbitrage between medium and low skills gives

$$\frac{A_L \alpha_L (I_L) L}{I_L} = \frac{A_M \alpha_M (I_L) M}{I_H - I_L}. \quad (27)$$

Prices

- Finally, using the choice of the numeraire, $\int_0^1 \ln p(i) di = 0$, together with (14)-(16), we also have

$$\int_0^{I_L} (\ln P_L - \ln \alpha_L(i))\, di + \int_{I_L}^{I_H} (\ln P_M - \ln \alpha_M(i))\, di \quad (28)$$
$$+ \int_{I_H}^{1} (\ln P_H - \ln \alpha_H(i))\, di = 0.$$

- Equations (24) and (25) give the relative wages of high to medium and medium to low skill workers.
- To obtain the wage *level* for any one of these three groups, we need to use the price normalization in (28) together with (21) and (22) to solve out for one of the price indices, for example, P_L, and then (**??**) will give w_L and the levels of w_M and w_H can be readily obtained from (24) and (25).

Summary of Equilibrium

Proposition

There exists a unique equilibrium summarized by $(I_L, I_H, P_L, P_M, P_H, w_L, w_M, w_H)$ *given by equations (21), (22), (26), (27), (23), (24), (25) and (28).*

- Uniqueness follows from Assumption 1.

Diagrammatic Representation

- To do this, we can rewrite (26) as follows:

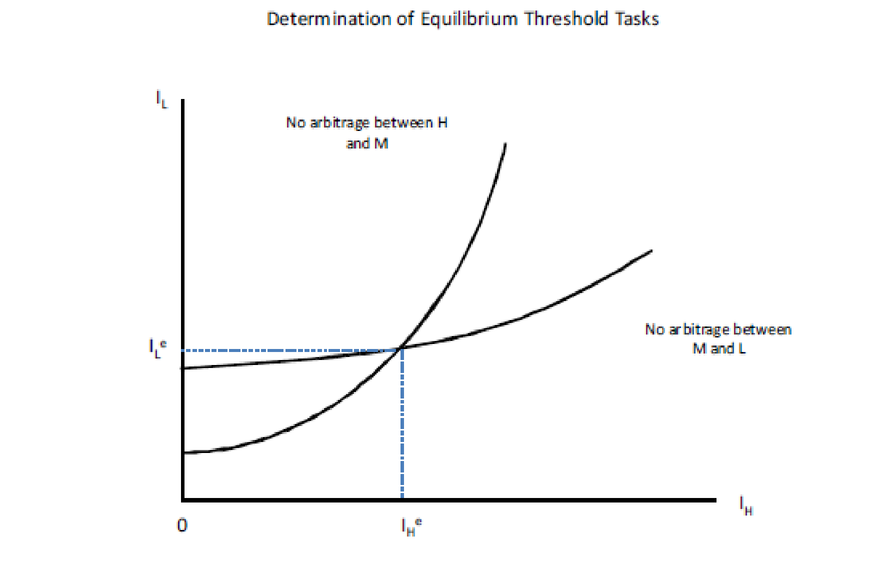
$$\frac{1 - I_H}{I_H - I_L} \frac{\alpha_M (I_H)}{\alpha_H (I_H)} = \frac{A_H H}{A_M M}. \tag{29}$$

- The right-hand side of this equation corresponds to the relative effective supply of high to medium skills.
- The left-hand side, on the other hand, can be interpreted as the effective demand for high relative to medium skills.
- Similarly, we rewrite (27) as:

$$\frac{I_H - I_L}{I_L} \frac{\alpha_L (I_H)}{\alpha_M (I_H)} = \frac{A_M M}{A_L L}$$

  for given I_H, and this expression has the same relative effective demand and supply interpretation.

Diagrammatic Determination of Equilibrium Special Tasks



Special Cases

- If there are no medium skill workers, then Assumption 1 simply implies that $\alpha_L(i)/\alpha_H(i)$ is strictly decreasing in i and we are back to a version of the two-factor standard framework.
- One example is Acemoglu and Zilibotti (2001), who also assume the following form of comparative advantage:

$$\alpha_L(i) = 1 - i \text{ and } \alpha_H(i) = i. \tag{30}$$

- This then gives the threshold task I as

$$\frac{1-I}{I} = \left(\frac{A_H H}{A_L L}\right)^{1/2}.$$

Special Cases (continued)

- We can then obtain the following:
- Relative prices:

$$\frac{P_H}{P_L} = \left(\frac{A_H H}{A_L L}\right)^{-1/2}.$$

- And relative wages:

$$\frac{w_H}{w_L} = \left(\frac{A_H}{A_L}\right)^{1/2} \left(\frac{H}{L}\right)^{-1/2}.$$

- Thus the model is isomorphic to the standard framework with less is the of substitution equal to 2.

Comparative Statics

- To derive these comparative statics, we return to the general model, and take logs in equations (26) and (27) to obtain slightly simpler expressions, given by the following two equations:

$$\ln A_M - \ln A_H + \beta_H(I_H) + \ln M - \ln H - \ln(I_H - I_L) + \ln(1 - I_H) = 0, \tag{31}$$

and

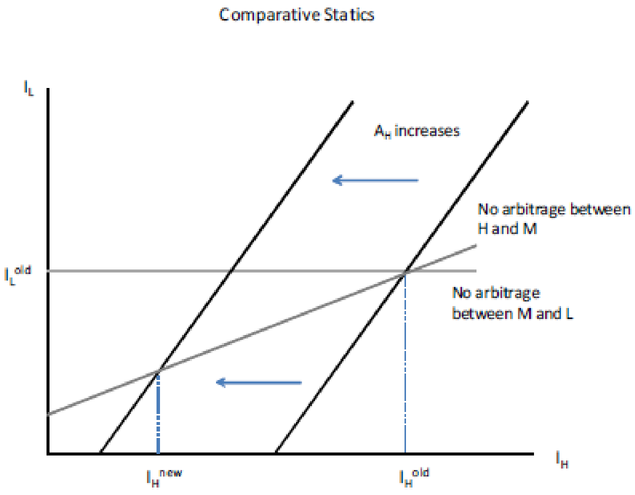
$$\ln A_L - \ln A_M + \beta_L(I_L) + \ln L - \ln M + \ln(I_H - I_L) - \ln(I_L) = 0, \tag{32}$$

where we have defined

$$\beta_H(I) \equiv \ln \alpha_M(I) - \ln \alpha_H(I) \text{ and } \beta_L(I) \equiv \ln \alpha_L(I) - \ln \alpha_M(I),$$

both of which are strictly decreasing in view of Assumption 1.

Comparative Statics: An Increase in H Productivity



Comparative Statics: Summary

Proposition

(The response of task allocation to technology and skill supplies):

$$
\begin{aligned}
\frac{dI_H}{d\ln A_H} &= \frac{dI_H}{d\ln H} < 0,\ \frac{dI_L}{d\ln A_H} = \frac{dI_L}{d\ln H} < 0 \text{ and} \\
\frac{d\left(I_H - I_L\right)}{d\ln A_H} &= \frac{d\left(I_H - I_L\right)}{d\ln H} < 0; \\
\frac{dI_H}{d\ln A_L} &= \frac{dI_H}{d\ln L} > 0,\ \frac{dI_L}{d\ln A_L} = \frac{dI_L}{d\ln L} > 0 \text{ and} \\
\frac{d\left(I_H - I_L\right)}{d\ln A_L} &= \frac{d\left(I_H - I_L\right)}{d\ln L} < 0; \\
\frac{dI_H}{d\ln A_M} &= \frac{dI_H}{d\ln M} > 0,\ \frac{dI_L}{d\ln A_M} = \frac{dI_L}{d\ln M} < 0 \text{ and} \\
\frac{d\left(I_H - I_L\right)}{d\ln A_M} &= \frac{d\left(I_H - I_L\right)}{d\ln M} > 0.
\end{aligned}
$$

Comparative Statics: Summary (continued)

Proposition

(The response of relative wages to skill supplies):

$$
\begin{aligned}
\frac{d \ln \left(w_H / w_L\right)}{d \ln H} &< 0, \ \frac{d \ln \left(w_H / w_M\right)}{d \ln H} < 0, \ \frac{d \ln \left(w_H / w_L\right)}{d \ln L} > 0, \\
\frac{d \ln \left(w_M / w_L\right)}{d \ln L} &> 0, \ \frac{d \ln \left(w_H / w_M\right)}{d \ln M} > 0, \text{ and} \\
\frac{d \ln \left(w_H / w_L\right)}{d \ln M} &\lesseqqgtr 0 \text{ if and only if } \left|\beta_L'\left(I_L\right) I_L\right| \gtreqqless \left|\beta_H'\left(I_H\right)\left(1-I_H\right)\right| .
\end{aligned}
$$

- Intuition for last result: the impact of middle skill supply on relative wages of high and low skill workers depends on whether middle skill workers are more substitutable for high or low skill workers, captured by the slopes of the comparative advantage schedules near the two thresholds.

Comparative Statics: Summary (continued)

Proposition

(The response of wages to factor-augmenting technologies):

$$\frac{d\ln\left(w_H/w_L\right)}{d\ln A_H} > 0, \quad \frac{d\ln\left(w_M/w_L\right)}{d\ln A_H} < 0, \quad \frac{d\ln\left(w_H/w_M\right)}{d\ln A_H} > 0;$$

$$\frac{d\ln\left(w_H/w_L\right)}{d\ln A_L} < 0, \quad \frac{d\ln\left(w_M/w_L\right)}{d\ln A_L} < 0, \quad \frac{d\ln\left(w_H/w_M\right)}{d\ln A_L} > 0;$$

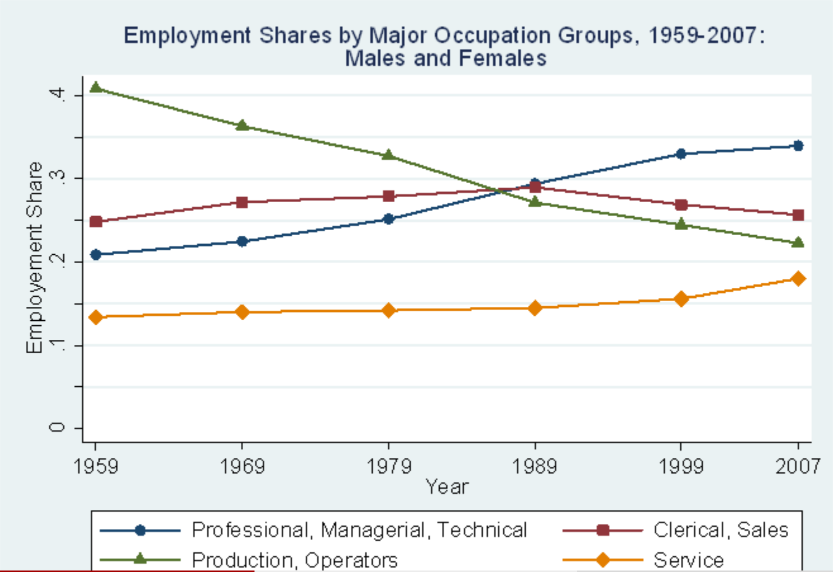
$$\frac{d\ln\left(w_H/w_M\right)}{d\ln A_M} < 0, \quad \frac{d\ln\left(w_M/w_L\right)}{d\ln A_M} > 0, \text{ and}$$

$$\frac{d\ln\left(w_H/w_L\right)}{d\ln A_M} \lesseqqgtr 0 \text{ if and only if } \left|\beta_L'\left(I_L\right)I_L\right| \gtreqqless \left|\beta_H'\left(I_H\right)\left(1-I_H\right)\right|.$$

Machines Replacing Labor

- We can now investigate indications of new technologies/machines directly displacing workers from tasks that they previously performed.
- Autor, Levy and Murnane (2003): this should be thought of as machines replacing workers performing "routine tasks", closer related to tasks performed by medium skill (semi-skilled) workers.
- For this reason, let us suppose that there now exists a range of tasks $[I', I''] \subset [I_L, I_H]$ for which $\alpha_K(i)$ increases sufficiently (with fixed cost of capital r) so that are now more economically preformed by machines than middle skill workers.
- For all the remaining tasks, i.e., for all $i \notin [I', I'']$, we continue to assume that $\alpha_K(i) = 0$.

Suggestive Evidence



Machines Replacing Labor (continued)

Proposition

Suppose we start with an equilibrium characterized by thresholds $[I_L, I_H]$ *and technical change implies that the tasks in the range* $[I', I''] \subset [I_L, I_H]$ *are now performed by machines. Then after the introduction of machines, there exists new unique equilibrium characterized by new thresholds* $\hat{I}_L$ *and* $\hat{I}_H$ *such that* $0 < \hat{I}_L < I' < I'' < \hat{I}_H < 1$ *and for any* $i < \hat{I}_L$, $m(i) = h(i) = 0$ *and* $l(i) = L/\hat{I}_L$*; for any* $i \in (\hat{I}_L, I') \cup (I'', \hat{I}_H)$, $l(i) = h(i) = 0$ *and* $m(i) = M/\left(\hat{I}_H - I'' + I' - \hat{I}_L\right)$*; for any* $i \in (I', I'')$, $l(i) = m(i) = h(i) = 0$*; and for any* $i > \hat{I}_H$, $l(i) = m(i) = 0$ *and* $h(i) = H/\left(1 - \hat{I}_H\right)$.

- Implication: reallocation of tasks in the economy as medium skill workers will now start performing some of the tasks previously allocated to low skill workers, thus increasing the supply of these tasks

Machines Replacing Labor: Wage Inequality

Proposition

Suppose we start with an equilibrium characterized by thresholds $[I_L, I_H]$ *and technical change implies that the tasks in the range* $[I', I''] \subset [I_L, I_H]$ *are now performed by machines. Then:*

1. w_H / w_M *increases;*
2. w_M / w_L *decreases;*
3. w_H / w_L *increases if* $\left|\beta_L'(I_L) I_L\right| < \left|\beta_H'(I_H)(1 - I_H)\right|$ *and* w_H / w_L *decreases if* $\left|\beta_L'(I_L) I_L\right| > \left|\beta_H'(I_H)(1 - I_H)\right|$.

Machines Replacing Labor: Wage Inequality (continued)

- Intuition for the first two parts is straightforward.
- The impact of this type of technical change on the wage of high skill relative to low skill workers is also naturally ambiguous.
- The condition $\left|\beta_L'(I_L)\, I_L\right| < \left|\beta_H'(I_H)\,(1 - I_H)\right|$ is also intuitive as it implies that medium skill workers are closer substitutes for low than high skill workers in the sense that, around I_H, there is a stronger comparative advantage of high skill relative to medium skill workers than there is comparative advantage of low relative to medium skill workers around I_L.

Machines Replacing Labor: Special Case

- Suppose

$$\alpha_H(i) = \begin{cases} \theta^{\tilde{I}_H - i}\tilde{\alpha}_H(i) & \text{if } i \leq \tilde{I}_H \\ \tilde{\alpha}_H(i) & \text{if } i > \tilde{I}_H \end{cases} \tag{33}$$

  where $\tilde{\alpha}_H(i)$ is a function that satisfies Assumption 1 and $\theta \geq 1$, and suppose that $\tilde{I}_H$ is in the neighborhood of the equilibrium threshold task for high skill workers, I_H.

- The presence of the term $\theta^{\tilde{I}_H - i}$ in (33) implies that an increase in θ creates an upward twist in the task productivity schedule for high skill workers around $\tilde{I}_H$.

Machines Replacing Labor: Special Case (continued)

- Consider the implications of an increase in θ. High skill workers can now successfully perform tasks previously performed by medium skill workers, and hence will replace them in tasks close to $\tilde{I}_H$ (or close to the equilibrium threshold I_H).
- Therefore, even absent machine-substitution for medium skill tasks, the model generates comparative static results similar to those discussed above, highlighting the parallel roles that technology (embodied in machinery) and task productivity schedules (represented by $\alpha(\cdot)$) play.