14.452 Economic Growth: Lecture 4, The Solow Growth Model and the Data

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Solow Growth Model and the Data

- Use Solow model or extensions to interpret both economic growth over time and cross-country output differences.
- Focus on *proximate causes* of economic growth.

Growth Accounting I

- Aggregate production function in its general form:

$$Y(t) = F[K(t), L(t), A(t)].$$

- Combined with competitive factor markets, gives Solow (1957) *growth accounting framework.*
- Continuous-time economy and differentiate the aggregate production function with respect to time.
- Dropping time dependence,

$$\frac{\dot{Y}}{Y} = \frac{F_A A}{Y}\frac{\dot{A}}{A} + \frac{F_K K}{Y}\frac{\dot{K}}{K} + \frac{F_L L}{Y}\frac{\dot{L}}{L}. \tag{1}$$

Growth Accounting II

- Denote growth rates of output, capital stock and labor by $g \equiv \dot{Y}/Y$, $g_K \equiv \dot{K}/K$ and $g_L \equiv \dot{L}/L$.
- Define the contribution of technology to growth as

$$x \equiv \frac{F_A A}{Y} \frac{\dot{A}}{A}$$

- Recall with competitive factor markets, $w = F_L$ and $R = F_K$.
- Define factor shares as $\alpha_K \equiv RK/Y$ and $\alpha_L \equiv wL/Y$.
- Putting all these together, (1) the *fundamental growth accounting* equation

$$x = g - \alpha_K g_K - \alpha_L g_L. \tag{2}$$

- Gives estimate of contribution of technological progress, *Total Factor Productivity* (TFP) or Multi Factor Productivity as

$$\hat{x}(t) = g(t) - \alpha_K(t) g_K(t) - \alpha_L(t) g_L(t). \tag{3}$$

- All terms on right-hand side are "estimates" obtained with a range of assumptions from national accounts and other data sources.

Growth Accounting III

- In continuous time, equation (3) is exact.
- With discrete time, potential problem in using (3): over the time horizon factor shares can change.
- Use beginning-of-period or end-of-period values of α_K and α_L?
  - Either might lead to seriously biased estimates.
  - Best way of avoiding such biases is to use as high-frequency data as possible.
  - Typically use factor shares calculated as the average of the beginning and end of period values.
- In discrete time, the analog of equation (3) becomes

$$\hat{x}_{t,t+1} = g_{t,t+1} - \bar{\alpha}_{K,t,t+1} g_{K,t,t+1} - \bar{\alpha}_{L,t,t+1} g_{L,t,t+1}, \tag{4}$$

- $g_{t,t+1}$ is the growth rate of output between t and $t+1$; other growth rates defined analogously.

Growth Accounting IV

- Moreover,

$$\bar{\alpha}_{K,t,t+1} \equiv \frac{\alpha_K(t) + \alpha_K(t+1)}{2}$$

$$\text{and } \bar{\alpha}_{L,t,t+1} \equiv \frac{\alpha_L(t) + \alpha_L(t+1)}{2}$$

- Equation (4) would be a fairly good approximation to (3) when the difference between t and $t+1$ is small and the capital-labor ratio does not change much during this time interval.
- Solow's (1957) applied this framework to US data: a large part of the growth was due to technological progress.
- From early days, however, a number of pitfalls were recognized.
  - Moses Abramovitz (1956): dubbed the $\hat{x}$ term "the measure of our ignorance".
  - If we mismeasure g_L and g_K we will arrive at inflated estimates of $\hat{x}$.

Growth Accounting Results

- Example from Barro and Sala-i-Martin's textbook

Table 10.1
Growth Accounting for a Sample of Countries

Country	(1) Growth Rate of GDP	(2) Contribution from Capital	(3) Contribution from Labor	(4) TFP Growth Rate
		Panel A: OECD Countries, 1947–73		
Canada ($\alpha = 0.44$)	0.0517	0.0254 (49%)	0.0088 (17%)	0.0175 (34%)
France[a] ($\alpha = 0.40$)	0.0542	0.0225 (42%)	0.0021 (4%)	0.0296 (54%)
Germany[b] ($\alpha = 0.39$)	0.0661	0.0269 (41%)	0.0018 (3%)	0.0374 (56%)
Italy[b] ($\alpha = 0.39$)	0.0527	0.0180 (34%)	0.0011 (2%)	0.0337 (64%)
Japan[b] ($\alpha = 0.39$)	0.0951	0.0328 (35%)	0.0221 (23%)	0.0402 (42%)
Netherlands[c] ($\alpha = 0.45$)	0.0536	0.0247 (46%)	0.0042 (8%)	0.0248 (46%)
U.K.[d] ($\alpha = 0.38$)	0.0373	0.0176 (47%)	0.0003 (1%)	0.0193 (52%)
U.S. ($\alpha = 0.40$)	0.0402	0.0171 (43%)	0.0095 (24%)	0.0135 (34%)
		Panel B: OECD Countries, 1960–95		
Canada ($\alpha = 0.42$)	0.0369	0.0186 (51%)	0.0123 (33%)	0.0057 (16%)
France ($\alpha = 0.41$)	0.0358	0.0180 (53%)	0.0033 (10%)	0.0130 (38%)
Germany ($\alpha = 0.39$)	0.0312	0.0177 (56%)	0.0014 (4%)	0.0132 (42%)
Italy ($\alpha = 0.34$)	0.0357	0.0182 (51%)	0.0035 (9%)	0.0153 (42%)
Japan ($\alpha = 0.43$)	0.0566	0.0178 (31%)	0.0125 (22%)	0.0265 (47%)
U.K. ($\alpha = 0.37$)	0.0221	0.0124 (56%)	0.0017 (8%)	0.0080 (36%)
U.S. ($\alpha = 0.39$)	0.0318	0.0117 (37%)	0.0127 (40%)	0.0076 (24%)

Table continued

Growth Accounting Results (continued)

Table 10.1
(Continued)

Country	(1) Growth Rate of GDP	(2) Contribution from Capital	(3) Contribution from Labor	(4) TFP Growth Rate
Panel C: Latin American Countries, 1940–90				
Argentina ($\alpha = 0.54$)	0.0279	0.0128 (46%)	0.0097 (35%)	0.0054 (19%)
Brazil ($\alpha = 0.45$)	0.0558	0.0294 (53%)	0.0150 (27%)	0.0114 (20%)
Chile ($\alpha = 0.52$)	0.0362	0.0120 (33%)	0.0103 (28%)	0.0138 (38%)
Colombia ($\alpha = 0.63$)	0.0454	0.0219 (48%)	0.0152 (33%)	0.0084 (19%)
Mexico ($\alpha = 0.69$)	0.0522	0.0259 (50%)	0.0150 (29%)	0.0113 (22%)
Peru ($\alpha = 0.66$)	0.0323	0.0252 (78%)	0.0134 (41%)	−0.0062 (−19%)
Venezuela ($\alpha = 0.55$)	0.0443	0.0254 (57%)	0.0179 (40%)	0.0011 (2%)
Panel D: East Asian Countries, 1966–90				
Hong Kong[c] ($\alpha = 0.37$)	0.073	0.030 (41%)	0.020 (28%)	0.023 (32%)
Singapore ($\alpha = 0.49$)	0.087	0.056 (65%)	0.029 (33%)	0.002 (2%)
South Korea ($\alpha = 0.30$)	0.103	0.041 (40%)	0.045 (44%)	0.017 (16%)
Taiwan ($\alpha = 0.26$)	0.094	0.032 (34%)	0.036 (39%)	0.026 (28%)

Interpreting the Results

- Reasons for mismeasurement:
  - what matters is not labor hours, but effective labor hours
    - important—though difficult—to make adjustments for changes in the *human capital* of workers.
  - measurement of capital inputs:
    - in the theoretical model, capital corresponds to the final good used as input to produce more goods.
    - in practice, capital is machinery, need assumptions about how relative prices of machinery change over time.
    - typical assumption was to use capital expenditures but if machines become cheaper would severely underestimate g_K

A World of Augmented Solow Economies I

- Mankiw, Romer and Weil (1992) used regression analysis to take the augmented Solow model, with human capital, to data.
- Use the Cobb-Douglas model and envisage a world consisting of $j = 1, ..., N$ countries.
- "Each country is an island": countries do not interact (perhaps except for sharing some common technology growth).
- Country $j = 1, ..., N$ has the aggregate production function:

$$Y_j(t) = K_j(t)^{\alpha} H_j(t)^{\beta} (A_j(t) L_j(t))^{1-\alpha-\beta}.$$

- Nests the basic Solow model without human capital when $\alpha = 0$.
- Countries differ in terms of their saving rates, $s_{k,j}$ and $s_{h,j}$, population growth rates, n_j, and technology growth rates $\dot{A}_j(t) / A_j(t) = g_j$.
- Define $k_j \equiv K_j / A_j L_j$ and $h_j \equiv H_j / A_j L_j$.

A World of Augmented Solow Economies II

- Focus on a world in which each country is in their steady state
- Assuming that human capital also has depreciation, at the rate δ_h, and it is accumulated with the saving rate s_h, steady state values for country j would be (to be derived in recitation):

$$
\begin{aligned}
k_j^* &= \left(\left(\frac{s_{k,j}}{n_j+g_j+\delta_k}\right)^{1-\beta}\left(\frac{s_{h,j}}{n_j+g_j+\delta_h}\right)^{\beta}\right)^{\frac{1}{1-\alpha-\beta}} \\
h_j^* &= \left(\left(\frac{s_{k,j}}{n_j+g_j+\delta_k}\right)^{\alpha}\left(\frac{s_{h,j}}{n_j+g_j+\delta_h}\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha-\beta}} .
\end{aligned}
$$

- Consequently:

$$
\begin{aligned}
y_j^*(t) &\equiv \frac{Y(t)}{L(t)} && (5) \\
&= A_j(t)\left(\frac{s_{k,j}}{n_j+g_j+\delta_k}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{s_{h,j}}{n_j+g_j+\delta_h}\right)^{\frac{\beta}{1-\alpha-\beta}} .
\end{aligned}
$$

A World of Augmented Solow Economies II

- Here $y_j^*(t)$ stands for output per capita of country j along the balanced growth path.
- Note if g_j's are not equal across countries, income per capita will diverge.
- Mankiw, Romer and Weil (1992) make the following assumption:

$$A_j(t) = \bar{A}_j \exp(gt).$$

- Countries differ according to technology *level*, (initial level $\bar{A}_j$) but they share the same common technology growth rate, g.

A World of Augmented Solow Economies III

- Using this together with (5) and taking logs, equation for the balanced growth path of income for country $j = 1, ..., N$:

$$\begin{aligned} \ln y_j^*(t) &= \ln \bar{A}_j + gt + \frac{\alpha}{1-\alpha-\beta} \ln \left(\frac{s_{k,j}}{n_j + g + \delta_k} \right) \\ &\quad + \frac{\beta}{1-\alpha-\beta} \ln \left(\frac{s_{h,j}}{n_j + g + \delta_h} \right). \end{aligned} \tag{6}$$

- Mankiw, Romer and Weil (1992) take:
  - $\delta_k = \delta_h = \delta$ and $\delta + g = 0.05$.
  - $s_{k,j}$=average investment rates (investments/GDP).
  - $s_{h,j}$=fraction of the school-age population that is enrolled in secondary school.

A World of Augmented Solow Economies IV

- Even with all of these assumptions, (6) can still not be estimated consistently.
- $\ln \bar{A}_j$ is unobserved (at least to the econometrician) and thus will be captured by the error term.
- Most reasonable models would suggest $\ln \bar{A}_j$'s should be correlated with investment rates.
- Thus an estimation of (6) would lead to omitted variable bias and inconsistent estimates.
- Implicitly, MRW make another *crucial* assumption, the **orthogonal technology assumption**:

$$\bar{A}_j = \varepsilon_j A, \text{ with } \varepsilon_j \text{ orthogonal to all other variables.}$$

Cross-Country Income Differences: Regressions I

- MRW first estimate equation (6) without the human capital term for the cross-sectional sample of non-oil producing countries

$$\ln y_j^* = \text{constant} + \frac{\alpha}{1-\alpha} \ln \left(s_{k,j} \right) - \frac{\alpha}{1-\alpha} \ln \left(n_j + g + \delta_k \right) + \varepsilon_j .$$

Cross-Country Income Differences: Regressions II

Estimates of the Basic Solow Model			
	MRW	Updated data	
	1985	1985	2000
$\ln(s_k)$	1.42	1.01	1.22
	(.14)	(.11)	(.13)
$\ln(n+g+\delta)$	-1.97	-1.12	-1.31
	(.56)	(.55)	(.36)
Adj R^2	.59	.49	.49
Implied α	.59	.50	.55
No. of observations	98	98	107

Cross-Country Income Differences: Regressions III

- Their estimates for $\alpha / (1 - \alpha)$, implies that α must be around $2/3$, but should be around $1/3$.
- The most natural reason for the high implied values of α is that ε_j is correlated with $\ln(s_{k,j})$, either because:
  1. the orthogonal technology assumption is not a good approximation to reality or
  2. there are also human capital differences correlated with $\ln(s_{k,j})$.
- Mankiw, Romer and Weil favor the second interpretation and estimate the augmented model,

$$\begin{aligned} \ln y_j^* &= \text{cst} + \frac{\alpha}{1-\alpha-\beta}\ln(s_{k,j}) - \frac{\alpha}{1-\alpha-\beta}\ln(n_j + g + \delta_k) \\ &\quad + \frac{\beta}{1-\alpha-\beta}\ln(s_{h,j}) - \frac{\beta}{1-\alpha-\beta}\ln(n_j + g + \delta_h) + \varepsilon_j. \end{aligned} \quad (7)$$

Estimates of the Augmented Solow Model

	MRW	Updated data	
	1985	1985	2000
$\ln(s_k)$	.69	.65	.96
	(.13)	(.11)	(.13)
$\ln(n+g+\delta)$	-1.73	-1.02	-1.06
	(.41)	(.45)	(.33)
$\ln(s_h)$	.66	.47	.70
	(.07)	(.07)	(.13)
Adj R^2	.78	.65	.60
Implied α	.30	.31	.36
Implied β	.28	.22	.26
No. of observations	98	98	107

Cross-Country Income Differences: Regressions IV

- If these regression results are reliable, they give a big boost to the augmented Solow model.
    - Adjusted R^2 suggests that three quarters of income per capita differences across countries can be explained by differences in their physical and human capital investment.
- Immediate implication is technology (TFP) differences have a somewhat limited role.
- But this conclusion should not be accepted without further investigation.

Challenges to Regression Analyses I

- **Technology differences across countries are not orthogonal to all other variables.**
- $\bar{A}_j$ is correlated with measures of s_j^h and s_j^k for two reasons.
  1. *omitted variable bias*: societies with high $\bar{A}_j$ will be those that have invested more in technology for various reasons; same reasons likely to induce greater investment in physical and human capital as well.
  2. *reverse causality*: complementarity between technology and physical or human capital imply that countries with high $\bar{A}_j$ will find it more beneficial to increase their stock of human and physical capital.
- In terms of (7), implies that key right-hand side variables are correlated with the error term, ε_j.
- OLS estimates of α and β and R^2 are biased upwards.

Challenges to Regression Analyses II

- **β is too large relative to what we should expect on the basis of microeconometric evidence.**
- The working age population enrolled in school ranges from 0.4% to over 12% in the sample of countries.
- Predicted log difference in incomes between these two countries is

$$\frac{\beta}{1-\alpha-\beta}\left(\ln 12-\ln\left(0.4\right)\right)=0.66\times\left(\ln 12-\ln\left(0.4\right)\right)\approx 2.24.$$

- Thus a country with schooling investment of over 12 should be about $\exp\left(2.24\right)-1\approx 8.5$ times richer than one with investment of around 0.4.

Challenges to Regression Analyses III

- Take Mincer regressions of the form:

$$\ln w_i = \mathbf{X}_i' \gamma + \phi S_i, \tag{8}$$

- Microeconometrics literature suggests that ϕ is between 0.06 and 0.10.
- Can deduce how much richer a country with 12 if we assume:
  1. That the micro-level relationship as captured by (8) applies identically to all countries.
  2. That there are no *human capital externalities*.
- Then: a country with 12 more years of average schooling should have between $\exp(0.10 \times 12) \simeq 3.3$ and $\exp(0.06 \times 12) \simeq 2.05$ times the stock of human capital of a county with fewer years of schooling.

Challenges to Regression Analyses IV

- Thus holding other factors constant, this country should be about 2-3 times as rich as the country with zero years of average schooling.
- Much less than the 8.5 fold difference implied by the Mankiw-Romer-Weil analysis.
- Thus β in MRW is too high relative to the estimates implied by the microeconometric evidence and thus likely upwardly biased.
- Overestimation of β is, in turn, most likely related to correlation between the error term ε_j and the key right-hand side regressors in (7).
- We have so far discussed cross-country "levels" regressions, similar issues apply to "growth regressions" but we have also seen in the first lecture how one might make partial progress here.

Calibrating Productivity Differences I

- The problems with regression analysis with cross-country data have motivated some macroeconomists to turn to "calibration"-type exercises.
- Suppose each country has access to the Cobb-Douglas aggregate production function:

$$Y_j = K_j^{\alpha} \left(A_j H_j \right)^{1-\alpha}, \tag{9}$$

- Each worker in country j has S_j years of schooling.
- Then using the Mincer equation (8) ignoring the other covariates and taking exponents, H_j can be estimated as

$$H_j = \exp\left(\phi S_j\right) L_j,$$

- Does not take into account differences in other "human capital" factors, such as experience.

Calibrating Productivity Differences II

- Let the rate of return to acquiring the Sth year of schooling be $\phi(S)$.
- A better estimate of the stock of human capital can be constructed as

$$H_j = \sum_S \exp\{\phi(S) S\} L_j(S)$$

- $L_j(S)$ now refers to the total employment of workers with S years of schooling in country j.
- Series for K_j can be constructed from Summers-Heston dataset using investment data and the perpetual inventory method.

$$K_j(t+1) = (1-\delta) K_j(t) + I_j(t),$$

- Assume, following Hall and Jones that $\delta = 0.06$.
- With same arguments as before, choose a value of $1/3$ for α.

Calibrating Productivity Differences III

- Given series for H_j and K_j and a value for α, construct "predicted" incomes at a point in time using

$$\hat{Y}_j = K_j^{1/3} \left(A_{US} H_j\right)^{2/3}$$

- A_{US} is computed so that $Y_{US} = K_{US}^{1/3} \left(A_{US} H_{US}\right)^{2/3}$.
- Once a series for $\hat{Y}_j$ has been constructed, it can be compared to the actual output series.
- Gap between the two series represents the contribution of technology.
- Alternatively, could back out country-specific technology terms (relative to the United States) as

$$\frac{A_j}{A_{US}} = \left(\frac{Y_j}{Y_{US}}\right)^{3/2} \left(\frac{K_{US}}{K_j}\right)^{1/2} \left(\frac{H_{US}}{H_j}\right).$$

Calibrating Productivity Differences IV

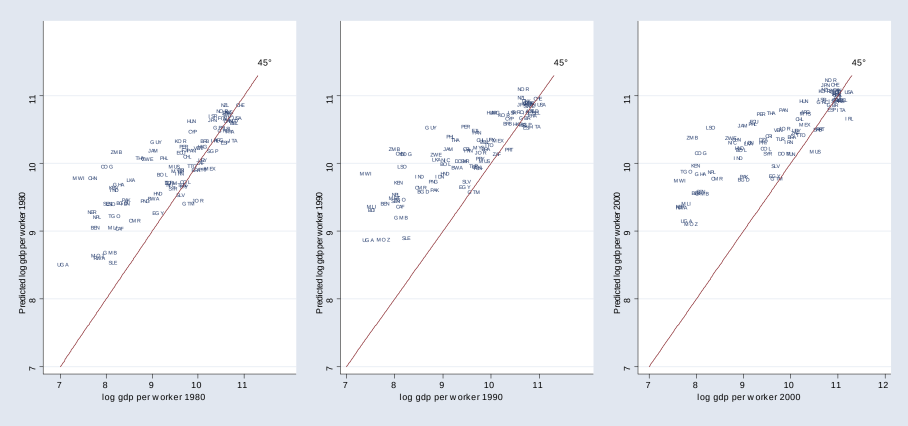

Figure: Calibrated technology levels relative to the US technology (from the Solow growth model with human capital) versus log GDP per worker, 1980, 1990 and 2000.

Calibrating Productivity Differences V

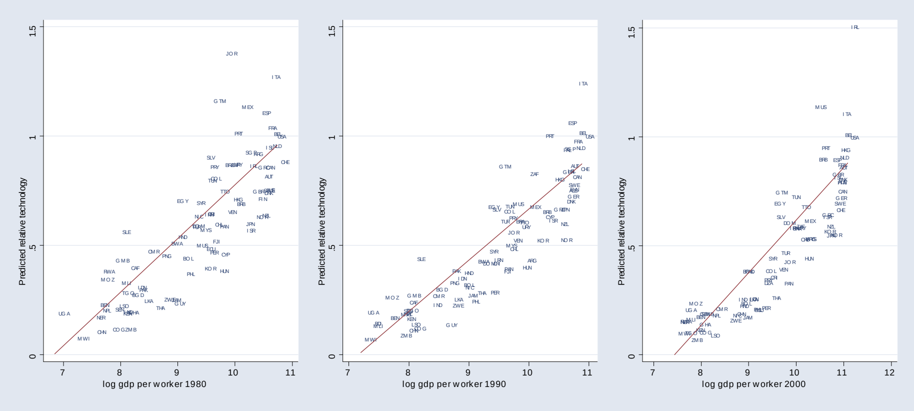

Figure: Calibrated technology levels relative to the US technology (from the Solow growth model with human capital) versus log GDP per worker, 1980, 1990 and 2000.

Calibrating Productivity Differences VI

The following features are noteworthy:

1. Differences in physical and human capital still matter a lot.
2. However, differently from the regression analysis, this exercise also shows significant *technology (productivity) differences*.
3. Same pattern visible in the next three figures for the estimates of the technology differences, A_j / A_{US}, against log GDP per capita in the corresponding year.
4. Also interesting is the pattern that the empirical fit of the neoclassical growth model seems to deteriorate over time.

Challenges to Callibration I

- In addition to the standard assumptions of competitive factor markets, we had to assume :
  - no human capital externalities, a Cobb-Douglas production function, and a range of approximations to measure cross-country differences in the stocks of physical and human capital.
- The calibration approach is in fact a close cousin of the growth-accounting exercise (sometimes referred to as "levels accounting").
- Imagine that the production function that applies to all countries in the world is

$$F(K_j, H_j, A_j),$$

- Assume countries differ according to their physical and human capital as well as technology—but not according to F.

Challenges to Callibration II

- Rank countries in descending order according to their physical capital to human capital ratios, K_j/H_j Then

$$\hat{x}_{j,j+1} = g_{j,j+1} - \bar{\alpha}_{K,j,j+1} g_{K,j,j+1} - \bar{\alpha}_{Lj,j+1} g_{H,j,j+1}, \tag{10}$$

- where:
  - $g_{j,j+1}$: proportional difference in output between countries j and $j+1$,
  - $g_{K,j,j+1}$: proportional difference in capital stock between these countries and
  - $g_{H,j,j+1}$: proportional difference in human capital stocks.
  - $\bar{\alpha}_{K,j,j+1}$ and $\bar{\alpha}_{Lj,j+1}$: average capital and labor shares between the two countries.
- The estimate $\hat{x}_{j,j+1}$ is then the proportional TFP difference between the two countries.

Challenges to Callibration III

- Levels-accounting faces two challenges.
  1. Data on capital and labor shares across countries are not widely available. Almost all exercises use the Cobb-Douglas approach (i.e., a constant value of α_K equal to $1/3$).
  2. The differences in factor proportions, e.g., differences in K_j / H_j, across countries are large. An equation like (10) is a good approximation when we consider small (infinitesimal) changes.

From Correlates to Fundamental Causes

- In this lecture, the focus has been on proximate causes— importance of human capital, physical capital and technology.
- Let us now return to the list of potential fundamental causes discussed in the first lecture:
  1. luck (or multiple equilibria)
  2. geographic differences
  3. institutional differences
  4. cultural differences
- Do we need to worry about the relationship between these fundamental causes and the correlates of growth? In what way? Where is theory useful?

Conclusions

- Message is somewhat mixed.
  - On the positive side, despite its simplicity, the Solow model has enough substance that we can take it to data in various different forms, including TFP accounting, regression analysis and calibration.
  - On the negative side, however, no single approach is entirely convincing.
- Complete agreement is not possible, but safe to say that consensus favors the interpretation that cross-country differences in income per capita cannot be understood solely on the basis of differences in physical and human capital
- Differences in TFP are not necessarily due to technology in the narrow sense.
- It is also useful and important to think about *fundamental causes*, what lies behind the factors taken as given either Solow model.