Labor Economics, 14.661. Lectures 11-13: Search, Matching and Unemployment

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December 2, 4 and 9, 2014.

Introduction

- Central question for labor and macro: what determines the level of employment and unemployment in the economy?
- Textbook answer: labor supply, labor demand, and unemployment as "leisure".
- Neither realistic nor a useful framework for analysis.
- Alternative: *labor market frictions*
- Related questions raised by the presence of frictions:
    - is the level of employment efficient/optimal?
    - how is the composition and quality of jobs determined, is it efficient?
    - distribution of earnings across workers.

Introduction (continued)

- Applied questions:
  - why was unemployment around 4-5% in the US economy until the 1970s?
  - why did the increase in the 70s and 80s, and then decline again in the late 90s?
  - why did it then remain high throughout the 90s and 2000s?
  - why did European unemployment increase in the 1970s and remain persistently high?
  - is the unemployment rate the relevant variable to focus on? Or the labor force participation rate? Or the non-employment rate?
  - why is the composition of employment so different across countries?
    - male versus female, young versus old, high versus low wages

Introduction (continued)

- Challenge: how should labor market frictions be modeled?
- Alternatives:
    - incentive problems, efficiency wages
    - wage rigidities, bargaining, non-market clearing prices
    - *search*
- Search and matching: costly process of workers finding the "right" jobs.
- *Theoretical interest:* how do markets function without the Walrasian auctioneer?
- *Empirically important,*
- But how to develop a tractable and rich model?

McCall Partial Equilibrium Search Model

- The simplest model of search frictions.
- Problem of an individual getting draws from a given wage distribution
- Decision: which jobs to accept and when to start work.
- Jobs sampled sequentially.
- Alternative: Stigler, fixed sample search (choose a sample of n jobs and then take the most attractive one).
- Sequential search typically more reasonable.
- Moreover, whenever sequential search is possible, is preferred to fixed sample search (why?).

Environment

- Risk neutral individual in discrete time.
- At time $t = 0$, this individual has preferences given by

$$\sum_{t=0}^{\infty} \beta^t c_t$$

- $c_t =$ consumption.
- Start as unemployed, with consumption equal to b
- All jobs are identical except for their wages, and wages are given by an exogenous stationary distribution of

$$F(w)$$

  with finite (bounded) support $\mathbb{W}$.
- At every date, the individual samples a wage $w_t \in W$, and has to decide whether to take this or continue searching.
- Jobs are for life.
- Draws from $\mathbb{W}$ over time are independent and identically distributed.

Environment (continued)

- *Undirected search*, in the sense that the individual has no ability to seek or direct his search towards different parts of the wage distribution (or towards different types of jobs).
- Alternative: *directed search*.

Environment (continued)

- Suppose search without recall.
- If the worker accepts a job with wage w_t, he will be employed at that job forever, so the net present value of accepting a job of wage w_t is

$$\frac{w_t}{1-\beta}.$$

- Class of decision rules of the agent:

$$a_t : \mathbb{W} \longrightarrow [0,1]$$

as acceptance decision (acceptance probability)

Dynamic Programming Formulation

- Define the value of the agent when he has sampled a job of $w \in \mathbb{W}$:

$$v(w) = \max \left\{ \frac{w}{1-\beta}, \beta v + b \right\}, \tag{1}$$

  where

$$v = \int_{\mathbb{W}} v(\omega) \, dF(\omega) \tag{2}$$

- v is the continuation value of not accepting a job.
- Integral in (2) as a Lebesgue integral, since $F(w)$ could be a mixture of discrete and continuous.
- Intuition.
- We are interested in finding both the value function $v(w)$ and the optimal policy of the individual.

Dynamic Programming Formulation (continued)

- Previous two equations:

$$v\left(w\right) = \max\left\{\frac{w}{1-\beta}, b + \beta \int_{\mathbb{W}} v\left(\omega\right) dF\left(\omega\right)\right\}. \tag{3}$$

- Existence of optimal policies follows from standard theorems in dynamic programming.
- But, even more simply (3) implies that $v\left(w\right)$ must be piecewise linear with first a flat portion and then an increasing portion.
- *Optimal policy*: $v\left(w\right)$ is non-decreasing, therefore optimal policy will take a cutoff form.
  - →*reservation wage R*
    - all wages above R will be accepted and those $w < R$ will be turned down.
- Implication of the reservation wage policy→no recall assumption of no consequence (why?).

Reservation Wage

- Reservation wage given by

$$\frac{R}{1-\beta} = b + \beta \int_{\mathbb{W}} v(\omega) \, dF(\omega). \tag{4}$$

- Intuition?
- Since $w < R$ are turned down, for all $w < R$

$$
\begin{aligned}
v(w) &= b + \beta \int_{\mathbb{W}} v(\omega) \, dF(\omega) \\
&= \frac{R}{1-\beta},
\end{aligned}
$$

and for all $w \geq R$,

$$v(w) = \frac{w}{1-\beta}$$

- Therefore,

$$\int_{\mathbb{W}} v(\omega) \, dF(\omega) = \frac{R F(R)}{1-\beta} + \int_{w \geq R} \frac{w}{1-\beta} dF(w).$$

Reservation Wage (continued)

- Combining this with (4), we have

$$\frac{R}{1-\beta} = b + \beta \left[\frac{RF(R)}{1-\beta} + \int_{w \geq R} \frac{w}{1-\beta} dF(w) \right]$$

- Rewriting

$$\int_{w < R} \frac{R}{1-\beta} dF(w) + \int_{w \geq R} \frac{R}{1-\beta} dF(w) = b + \beta \left[\int_{w < R} \frac{R}{1-\beta} dF(w) \right.$$

- Subtracting $\beta R \int_{w \geq R} dF(w) / (1-\beta) + \beta R \int_{w < R} dF(w) / (1-\beta)$ from both sides,

$$\int_{w < R} \frac{R}{1-\beta} dF(w) + \int_{w \geq R} \frac{R}{1-\beta} dF(w)$$

$$- \beta \int_{w \geq R} \frac{R}{1-\beta} dF(w) - \beta \int_{w < R} \frac{R}{1-\beta} dF(w)$$

$$= b + \beta \left[\int_{w \geq R} \frac{w - R}{1-\beta} dF(w) \right]$$

Reservation Wage (continued)

- Collecting terms, we obtain

$$R - b = \frac{\beta}{1 - \beta} \left[\int_{w \geq R} (w - R) \, dF(w) \right]. \tag{5}$$

- The left-hand side is the cost of foregoing the wage of R.
- The right hand side is the expected benefit of one more search.
- At the reservation wage, these two are equal.

Reservation Wage (continued)

- Let us define the right hand side of equation (5) as

$$g\left(R\right) \equiv \frac{\beta}{1-\beta}\left[\int_{w\geq R}\left(w-R\right)dF\left(w\right)\right],$$

- This is the expected benefit of one more search as a function of the reservation wage.
- Differentiating

$$
\begin{aligned}
g'\left(R\right) &= -\frac{\beta}{1-\beta}\left(R-R\right)f\left(R\right) - \frac{\beta}{1-\beta}\left[\int_{w\geq R}dF\left(w\right)\right] \\
&= -\frac{\beta}{1-\beta}\left[1-F\left(R\right)\right] < 0
\end{aligned}
$$

- Therefore equation (5) has a unique solution.
- Moreover, by the implicit function theorem,

$$\frac{dR}{db} = \frac{1}{1-g'\left(R\right)} > 0.$$

Reservation Wage (continued)

- Suppose that the density of $F(R)$, denoted by $f(R)$, exists (was this necessary until now?).

- Then the second derivative of g also exists and is

$$g''(R) = \frac{\beta}{1 - \beta} f(R) \geq 0.$$

- This implies the right hand side of equation (5) is also convex.

- What does this mean?

Unemployment with Sequential Search

- Suppose that there is now a continuum 1 of identical individuals sampling jobs from the same stationary distribution F.
- Once a job is created, it lasts until the worker dies, which happens with probability s.
- There is a mass of s workers born every period, so that population is constant
- New workers start out as unemployed.
- The death probability means that the effective discount factor of workers is equal to $\beta(1-s)$.
- Consequently, the value of having accepted a wage of w is:

$$v^a(w) = \frac{w}{1 - \beta(1-s)}.$$

Unemployment with Sequential Search (continued)

- With the same reasoning as before, the value of having a job offer at wage w at hand is

$$v(w) = \max\{v^a(w), b + \beta(1-s)v\}$$

with

$$v = \int_{\mathbb{W}} v(w) \, dF.$$

- Therefore, the reservation wages given by

$$R - b = \frac{\beta(1-s)}{1 - \beta(1-s)} \left[\int_{w \geq R} (w - R) \, dF(w)\right].$$

Law of Motion of Unemployment

- Let us start time t with U_t unemployed workers.
- There will be s new workers born into the unemployment pool.
- Out of the U_t unemployed workers, those who survive and do not find a job will remain unemployed.
- Therefore

$$U_{t+1} = s + (1 - s) F(R) U_t.$$

- Here $F(R)$ is the probability of not finding a job, so $(1 - s) F(R)$ is the joint probability of not finding a job and surviving.
- Simple first-order linear difference equation (only depending on the reservation wage R, which is itself independent of the level of unemployment, U_t).
- Since $(1 - s) F(R) < 1$, it is asymptotically stable, and will converge to a unique steady-state level of unemployment.

Flow Approached Unemployment

- This gives us the simplest version of the flow approach to unemployment.

- Subtracting U_t from both sides:

$$U_{t+1} - U_t = s\left(1 - U_t\right) - \left(1 - s\right)\left(1 - F(R)\right)U_t.$$

- If period length is arbitrary, this can be written as

$$U_{t+\Delta t} - U_t = s\left(1 - U_t\right)\Delta t - \left(1 - s\right)\left(1 - F(R)\right)U_t\Delta t + o\left(\Delta t\right).$$

- Dividing by Δt and taking limits as $\Delta t \rightarrow 0$, we obtain the continuous time version

$$\dot{U}_t = s\left(1 - U_t\right) - \left(1 - s\right)\left(1 - F(R)\right)U_t.$$

Flow Approached Unemployment (continued)

- The unique steady-state unemployment rate where $U_{t+1} = U_t$ (or $\dot{U}_t = 0$) given by

$$U = \frac{s}{s + (1-s)(1-F(R))}.$$

- Canonical formula of the flow approach.
- The steady-state unemployment rate is equal to the job destruction rate (here the rate at which workers die, s) divided by the job destruction rate plus the job creation rate (here in fact the rate at which workers leave unemployment, which is different from the job creation rate).
- Clearly, an increases in s will raise steady-state unemployment.
- Moreover, an increase in R, that is, a higher reservation wage, will also depress job creation and increase unemployment.

Aside on Riskiness and Mean Preserving Spreads

- Question: what is the effect of a more unequal (spread out) wage offer distribution on reservation wages, equilibrium wage distribution, and unemployment
  - why a difference between offer distribution and equilibrium distribution?
- Key concept *mean preserving spreads*.
- Loosely speaking, a mean preserving spread is a change in distribution that increases risk.

Concepts of Riskiness

- Let a family of distributions over some set $X \subset \mathbb{R}$ with generic element x be denoted by $F(x, r)$, where r is a shift variable, which changes the distribution function.
- An example will be $F(x, r)$ to stand for mean zero normal variables, with r parameterizing the variance of the distribution.
- Normal distribution is special in the sense that, the mean and the variance completely describe the distribution, so the notion of risk can be captured by the variance.
- This is generally not true.
- The notion of "riskier" is a more stringent notion than having a greater variance.

Mean Preserving Spreads and Stochastic Dominance

Definition

$F(x, r)$ is less risky than $F(x, r')$, written as $F(x, r) \succeq_R F(x, r')$, if $\int_X x dF(x, r) = \int_X x dF(x, r')$ and for all concave and increasing $u : \mathbb{R} \to \mathbb{R}$, we have

$$\int_X u(x) \, dF(x, r) \geq \int_X u(x) \, dF(x, r').$$

- A related definition is that of second-order stochastic dominance.

Definition

$F(x, r)$ second order stochastically dominates $F(x, r')$, written as $F(x, r) \succeq_{SD} F(x, r')$, if

$$\int_{-\infty}^{c} F(x, r) \, dx \leq \int_{-\infty}^{c} F(x, r') \, dx, \text{ for all } c \in X.$$

Second-Order Stochastic Dominance

- The definition of second-order stochastic dominance requires the distribution function of $F(x, r)$ to start lower and always keep a lower integral than that of $F(x, r')$.
- One easy case where this will be satisfied is when both distribution functions have the same mean and they intersect only once: "single crossing") with $F(x, r)$ cutting $F(x, r')$ from below.
- These definitions could also be stated with strict instead of weak inequalities.
- It can also be established that if $F(x, r)$ second-order stochastic the dominates $F(x, r')$ and $u(\cdot)$ is strictly increasing and concave, then

$$\int_X u(x)\, dF(x, r) \geq \int_X u(x)\, dF(x, r').$$

Riskiness and Mean Preserving Spreads

Theorem

(Blackwell, Rothschild and Stiglitz) *Suppose*
$\int_X x dF(x, r) = \int_X x dF(x, r')$. *Then* $F(x, r) \succeq_R F(x, r')$ *if and only if*
$F(x, r) \succeq_{SD} F(x, r')$.

Wage Dispersion and Search

- Start with equation (5), which is

$$R - b = \frac{\beta}{1-\beta} \left[\int_{w \geq R} (w - R) \, dF(w) \right].$$

- Rewrite this as

$$R - b = \frac{\beta}{1-\beta} \left[\int_{w \geq R} (w - R) \, dF(w) \right] + \frac{\beta}{1-\beta} \left[\int_{w \leq R} (w - R) \, dF \right.$$

$$- \frac{\beta}{1-\beta} \left[\int_{w \leq R} (w - R) \, dF(w) \right],$$

$$= \frac{\beta}{1-\beta} (Ew - R) - \frac{\beta}{1-\beta} \left[\int_{w \leq R} (w - R) \, dF(w) \right],$$

where

$$Ew = \int_{\mathbb{W}} w \, dF(w)$$

is the mean of the distribution.

Wage Dispersion and Search (continued)

- Rearranging the previous equation

$$R - b = \beta \left(Ew - b \right) - \beta \int_{w \leq R} \left(w - R \right) dF \left(w \right).$$

- Applying integration by parts to the integral on the right hand side, i.e., noting that

$$\begin{aligned} \int_{w \leq R} w dF \left(w \right) &= \int_0^R w dF \left(w \right) \\ &= \left. wF \left(w \right) \right|_0^R - \int_0^R F \left(w \right) dw \\ &= RF \left(R \right) - \int_0^R F \left(w \right) dw. \end{aligned}$$

- We obtain

$$R - b = \beta \left(Ew - b \right) + \beta \int_0^R F \left(w \right) dw. \tag{6}$$

Wage Dispersion and Search (continued)

- Now consider a shift from F to $\tilde{F}$ corresponding to a mean preserving spread.
- This implies that Ew is unchanged
- But by definition of a mean preserving spread (second-order stochastic dominance), the last integral increases.
- Therefore, the mean preserving spread induces a shift in the reservation wage from R to $\tilde{R} > R$.
- Intuition?
- Relation to the convexity of $v(w)$?

Paradoxes of Search

- The search framework is attractive especially when we want to think of a world without a Walrasian auctioneer, or alternatively a world with "frictions".
- Search theory holds the promise of potentially answering these questions, and providing us with a framework for analysis.
- But...

The Rothschild Critique

- The key ingredient of the McCall model is non-degenerate wage distribution $F(w)$.
- Where does this come from?
- Presumably somebody is offering every wage in the support of this distribution.
- *Wage posting* by firms.
- The basis of the Rothschild critique is that it is difficult to rationalize the distribution function $F(w)$ as resulting from profit-maximizing choices of firms.

The Rothschild Critique (continued)

- Imagine that the economy consists of a mass 1 of identical workers similar to our searching agent.

- On the other side, there are N firms that can productively employ workers. Imagine that firm j has access to a technology such that it can employ l_j workers to produce

$$y_j = x_j l_j$$

  units of output (with its price normalized to one as the numeraire, so that w is the real wage).

- Suppose that each firm can only attract workers by posting a single vacancy.

- Moreover, to simplify the discussion, suppose that firms post a vacancy at the beginning of the game at $t = 0$, and then do not change the wage from then on. (why is this useful?)

The Rothschild Critique (continued)

- Suppose that the distribution of x in the population of firms is given by $G(x)$ with support $X \subset \mathbb{R}_+$.
- Also assume that there is some cost $\gamma > 0$ of posting a vacancy at the beginning, and finally, that $N >> 1$ (i.e., $N = \int_{-\infty}^{\infty} dG(x) >> 1$) and each worker samples one firm from the distribution of posting firms.
- As before, suppose that once a worker accepts a job, this is permanent, and he will be employed at this job forever.
- Moreover let us set $b = 0$, so that there is no unemployment benefits.
- Finally, to keep the environment entirely stationary, assume that once a worker accepts a job, a new worker is born, and starts search.
- Will these firms offer a non-degenerate wage distribution $F(w)$?

Equilibrium Wage Distribution?

- The answer is no.
- Denote whether the firm is posting a vacancy or not by

$$p : X \rightarrow \{0, 1\},$$

and the wage on for by

$$h : X \rightarrow \mathbb{R}_+.$$

- Intuitively, $h(x)$ should be indecreasing (higher wages are more attractive to high productivity firms).
- Let us suppose that this is so (not necessary).
- Then, the along-the-equilibrium path wage distribution is

$$F(w) = \frac{\int_{-\infty}^{h^{-1}(w)} p(x) \, dG(x)}{\int_{-\infty}^{\infty} p(x) \, dG(x)}.$$

- Intuition?

Equilibrium Wage Distribution? (continued)

- In addition, the strategies of workers can be represented by a function

$$a : \mathbb{R}_+ \longrightarrow [0, 1]$$

  denoting the probability that the worker will accept any wage in the "potential support" of the wage distribution, with 1 standing for acceptance.

- This is general enough to nest non-symmetric or mixed strategies.

- The natural equilibrium concept is subgame perfect Nash equilibrium, whereby the strategies of firms (p, h) and those of workers, a, are best responses to each other in all subgames.

Equilibrium Wage Distribution? (continued)

- Previous analysis: all workers will use a reservation wage, so

$$
\begin{aligned}
a\left(w\right) &= 1 \text{ if } w \geq R \\
&= 0 \text{ otherwise}
\end{aligned}
$$

- Since all workers are identical and the equation above determining the reservation wage, (5), has a unique solution, all workers will all be using the same reservation rule, accepting all wages $w \geq R$ and turning down those $w < R$.
- Workers' strategies are therefore again characterized by a reservation wage R.

Equilibrium Wage Distribution? (continued)

- Now take a firm with productivity x offering a wage $w' > R$.
- Its net present value of profits from this period's matches is

$$\pi\left(p = 1, w' > R, x\right) = -\gamma + \frac{1}{n}\frac{(x - w')}{1 - \beta}$$

where

$$n = \int_{-\infty}^{\infty} p\left(x\right) dG\left(x\right).$$

- This firm can deviate and cut its wage to some value in the interval $[R, w')$.
- All workers will still accept this job since its wage is above the reservation wage, and the firm will increase its profits to

$$\pi\left(p = 1, w \in [R, w'), x\right) = -\gamma + \frac{1}{n}\frac{x - w}{1 - \beta} > \pi\left(p = 1, w', x\right)$$

- Conclusion: there should not be any wages strictly above R.

Equilibrium Wage Distribution? (continued)

- Next consider a firm offering a wage $\tilde{w} < R$.
- This wage will be rejected by all workers, and the firm would lose the cost of posting a vacancy, i.e.,

$$\pi\left(p = 1, w < R, x\right) = -\gamma,$$

  and this firm can deviate to $p = 0$ and make zero profits.
- Therefore, in equilibrium when workers use the reservation wage rule of accepting only wages greater than R, all firms will offer the same wage R, and there is no distribution and no search.

Theorem

(Rothschild Paradox) *When all workers are homogeneous and engage in undirected search, all equilibrium distributions will have a mass point at their reservation wage R.*

The Diamond Paradox

- In fact, the paradox is even deeper.

Theorem

(Diamond Paradox) *For all $\beta < 1$, the unique equilibrium in the above economy is $R = 0$, and all workers accept the first wage offer.*

- **Sketch proof**: suppose $R \geq 0$, and $\beta < 1$.
- The optimal acceptance decision for to worker is

$$
\begin{aligned}
a(w) &= 1 \text{ if } w \geq R \\
&= 0 \text{ otherwise}
\end{aligned}
$$

- Therefore, all firms offering $w = R$ is an equilibrium
- But also...

The Diamond Paradox (continued)

Lemma

There exists $\varepsilon > 0$ such that when "almost all" firms are offering $w = R$, it is optimal for each worker to use the following acceptance strategy:

$$
\begin{aligned}
a(w) &= 1 \text{ if } w \geq \max\{R - \varepsilon, 0\} \\
&= 0 \text{ otherwise}
\end{aligned}
$$

- So for any $R > 0$, a firm can undercut the offers of all other firms and still have its offer accepted.

The Diamond Paradox (continued)

Sketch proof:

- If the worker accepts the wage of $R - \varepsilon$,

$$u^{accept} = \frac{R - \varepsilon}{1 - \beta}$$

- If he rejects and waits until next period, then since "almost all" firms are offering R,

$$u^{reject} = \frac{\beta R}{1 - \beta}$$

- For all $\beta < 1$, there exists $\varepsilon > 0$ such that

$$u^{accept} > u^{reject}.$$

The Diamond Paradox (continued)

- Implication: starting from an allocation where all firms offer R, any firm can deviate and offer a wage of $R - \varepsilon$ and increase its profits.
- This proves that no wage $R > 0$ can be the equilibrium, proving the proposition.
- Is the same true for Nash equilibria?

Solutions to the Diamond Paradox

- How do we resolve this paradox?

1. By assumption: assume that $F(w)$ is not the distribution of wages, but the distribution of "fruits" exogenously offered by "trees". This is clearly unsatisfactory, both from the modeling point of view, and from the point of view of asking policy questions from the model (e.g., how does unemployment insurance affect the equilibrium? The answer will depend also on how the equilibrium wage distribution changes).

2. Introduce other dimensions of heterogeneity.

3. Modify the wage determination assumptions→bargaining rather than *wage posting*: the most common and tractable alternative (though is it the most realistic?)

Introduction

- To circumvent the Rothschild and the Diamond paradoxes, assume *no wage posting* but instead *wage determination by bargaining*
- Where are the search frictions?
- Reduced form: *matching function*
- Continue to assume *undirected search.*
  $\rightarrow$ Baseline equilibrium model: Diamond-Mortensen-Pissarides (DMP) framework.

Introduction (continued)

- Very tractable framework for analysis of unemployment (level, composition, fluctuations, trends)
- Widely used in macro and labor
- Roughly speaking: flows approach meets equilibrium
- Shortcoming: reduced form matching function.

Setup

- Continuous time, infinite horizon economy with risk neutral agents.
- Matching Function:
  Matches $= x(U, V)$
- Continuous time: $x(U, V)$ as the flow rate of matches.
- Assume that $x(U, V)$ exhibits constant returns to scale.

Matching Function

- Therefore:

$$\text{Matches} \;\; = \;\; xL = x(uL, vL)$$
$$\implies \;\; x = x\left(u, v\right)$$

$U =$ unemployment;
$u =$ unemployment rate
$V =$ vacancies;
$v =$ vacancy rate (per worker in labor force)
$L =$ labor force

Evidence and Interpretation

- Existing aggregate evidence suggests that the assumption of x exhibiting CRS is reasonable.
- Intuitively, one might have expected "increasing returns" if the matching function corresponds to physical frictions
  - think of people trying to run into each other on an island.
- But the matching function is to reduced form for this type of interpretation.
- In practice, frictions due to differences in the supply and demand for specific types of skills.

Matching Rates and Job Creation

- Using the constant returns assumption, we can express everything as a function of the *tightness of the labor market.*

$$q(\theta) \equiv \frac{x}{v} = x\left(\frac{u}{v}, 1\right),$$

- Here $\theta \equiv v/u$ is the tightness of the labor market

  $q(\theta)$: Poisson arrival rate of match for a vacancy

  $q(\theta)\theta$: Poisson arrival rate of match for an unemployed worker

- Therefore, job creation is equal to

$$\text{Job creation} = u\theta q(\theta)L$$

Job Destruction

- What about job destruction?
- Let us start with the simplest model of job destruction, which is basically to treat it as "exogenous".
- Think of it as follows, firms are hit by adverse shocks, and then they decide whether to destroy or to continue.

$$\longrightarrow \text{Adverse Shock} \longrightarrow \text{destroy}$$
$$\longrightarrow \text{continue}$$

- Exogenous job destruction: Adverse shock $= -\infty$ with "probability" (i.e., flow rate) s

Steady State of the Flow Approach

- As in the partial equilibrium sequential search model
- Steady State:

  flow into unemployment = flow out of unemployment

- Therefore, with exogenous job destruction:

$$s(1 - u) = \theta q(\theta) u$$

- Therefore, steady state unemployment rate:

$$u = \frac{s}{s + \theta q(\theta)}$$

- Intuition

The Beverage Curve

- This relationship is also referred to as the *Beveridge Curve*, or the U-V curve.
- It draws a downward sloping locus of unemployment-vacancy combinations in the U-V space that are consistent with flow into unemployment being equal with flow out of unemployment.
- Some authors interpret shifts of this relationship is reflecting structural changes in the labor market, but we will see that there are many factors that might actually shift at a generalized version of such relationship.

Production Side

- Let the output of each firm be given by neoclassical production function combining labor and capital:

$$Y = AF(K, N)$$

- F exhibits constant returns, K is the capital stock of the economy, and N is employment (different from labor force because of unemployment).
- Let

$$k \equiv K/N$$

be the capital labor ratio, then

$$\frac{Y}{N} = Af(k) \equiv AF(\frac{K}{N}, 1)$$

- Also let
  - r: cost of capital
  - δ: depreciation

Production Side: Two Interpretations

- Each firm is a "job" hires one worker
- Each firm can hire as many worker as it likes
- For our purposes either interpretation is fine

Hiring Costs

- Why don't firms open an infinite number of vacancies?
- Hiring activities are costly.
- Vacancy costs γ_0: fixed cost of hiring

Bellman Equations

J^V : PDV of a vacancy

J^F :PDV of a "job"

J^U :PDV of a searching worker

J^E :PDV of an employed worker

Why is J^F not conditioned on k?

Big assumption: perfectly reversible capital investments (why is this important?)

Value of Vacancies

- Perfect capital market gives the asset value for a vacancy (in steady state) as

$$rJ^V = -\gamma_0 + q(\theta)(J^F - J^V)$$

- Intuition?

Labor Demand and Job Creation

- Free Entry $\Longrightarrow$

$$J^V \equiv 0$$

- If it were positive, more firms would enter.
- Important implication: job creation can happen really "fast", except because of the frictions created by matching searching workers to searching vacancies.
- Alternative would be: $\gamma_0 = \Gamma_0(V)$ or $\Gamma_1(\theta)$, so as there are more and more jobs created, the cost of opening an additional job increases.

Characterization of Equilibrium

- Free entry implies that

$$J^F = \frac{\gamma_0}{q(\theta)}$$

- Asset value equation for the value of a field job:

$$r(J^F + k) = Af(k) - \delta k - w - s(J^F - J^V)$$

- Intuitively, the firm has two assets: the fact that it is matched with a worker, and its capital, k.

- So its asset value is $J^F + k$ (more generally, without the perfect reversability, we would have the more general $J^F(k)$).

- Its return is equal to production, $Af(k)$, and its costs are depreciation of capital and wages, δk and w.

- Finally, at the rate s, the relationship comes to an end and the firm loses J^F.

Wage Determination

- Can wages be equal to marginal cost of labor and value of marginal product of labor?
- No because of labor market frictions
- a worker with a firm is more valuable than an unemployed worker.
- How are wages determined?
- *Nash bargaining* over match specific surplus $J^E + J^F - J^U - J^V$
- Where is k?

Implications of Perfect Reversability

- Perfect Reversability implies that w does not depend on the firm's choice of capital

$\implies$ equilibrium capital utilization $f'(k) = r + \delta$

- *Modified Golden Rule*

Digression: Irreversible Capital Investments

- Much more realistic, but typically not adopted in the literature (why not?)
- Suppose k is not perfectly reversible then suppose that the worker captures a fraction β all the output in bargaining.
- Then the wage depends on the capital stock of the firm, as in the holdup models discussed before.

$$
\begin{aligned}
w(k) &= \beta A f(k) \\
A f'(k) &= \frac{r + \delta}{1 - \beta} \; ; \text{capital accumulation is distorted}
\end{aligned}
$$

Equilibrium Job Creation

- Free entry together with the Bellman equation for filled jobs implies

$$Af(k) - (r - \delta)k - w - \frac{(r + s)}{q(\theta)}\gamma_0 = 0$$

- For unemployed workers

$$rJ^U = z + \theta q(\theta)(J^E - J^U)$$

where z is unemployment benefits.

- Employed workers:

$$rJ^E = w + s(J^U - J^E)$$

- Reversibility again: w independent of k.

Values For Workers

- Solving these equations we obtain

$$rJ^U = \frac{(r+s)z + \theta q(\theta)w}{r + s + \theta q(\theta)}$$

$$rJ^E = \frac{sz + [r + \theta q(\theta)]\,w}{r + s + \theta q(\theta)}$$

Nash Bargaining

- Consider the surplus of pair i:

$$
\begin{aligned}
rJ_i^F &= Af(k) - (r+\delta)k - w_i - sJ_i^F \\
rJ_i^E &= w_i - s(J_i^E - J_0^U).
\end{aligned}
$$

- Why is it important to do this for pair i (rather than use the equilibrium expressions above)?

- The Nash solution will solve

$$
\max(J_i^E - J^U)^\beta (J_i^F - J^V)^{1-\beta}
$$

$$
\beta = \quad \text{bargaining power of the worker}
$$

- Since we have linear utility, thus "transferable utility", this implies

$$
J_i^E - J^U = \beta(J_i^F + J_i^E - J^V - J^U)
$$

Nash Bargaining

- Using the expressions for the value functions

$$w = (1 - \beta)z + \beta \left[Af(k) - (r + \delta)k + \theta\gamma_0 \right]$$

- Here

$$Af(k) - (r + \delta)k + \theta\gamma_0$$

  is the quasi-rent created by a match that the firm and workers share.
- Why is the term $\theta\gamma_0$ there?

Steady State Equilibrium

- Steady State Equilibrium is given by four equations

  1. The Beveridge curve:
  $$u = \frac{s}{s + \theta q(\theta)}$$

  2. Job creation leads zero profits:
  $$Af(k) - (r + \delta)k - w - \frac{(r + s)}{q(\theta)}\gamma_0 = 0$$

  3. Wage determination:
  $$w = (1 - \beta)z + \beta\left[Af(k) - (r + \delta)k + \theta\gamma_0\right]$$

  4. Modified golden rule:
  $$Af'(k) = r + \delta$$

Steady State Equilibrium (continued)

- These four equations define a block recursive system

$(4) + r \longrightarrow k$

$k + r + (2) + (3) \longrightarrow \theta, w$

$\theta + (1) \longrightarrow u$

Steady State Equilibrium (continued)

- Alternatively, combining three of these equations we obtain the zero-profit locus, the VS curve.
- Combine this with the Beveridge curve to obtain the equilibrium.

$(2), (3), (4) \implies$ the VS curve

$$(1 - \beta) \left[Af(k) - (r + \delta)k - z \right] - \frac{r + \delta + \beta \theta q(\theta)}{q(\theta)} \gamma_0 = 0$$

- Therefore, the equilibrium looks very similar to the intersection of "quasi-labor demand" and "quasi-labor supply".

Steady State Equilibrium in a Diagram
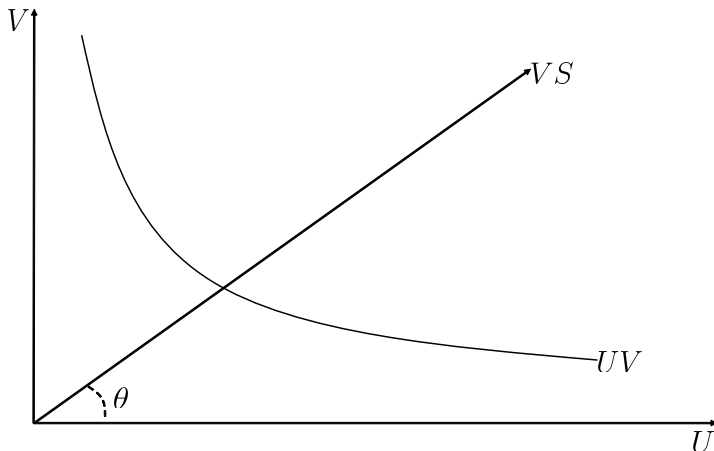
Steady state comparative statics

Comparative Statics of the Steady State

- From the figure:

$s \uparrow$	$U \uparrow$	$V \uparrow$	$\theta \downarrow$	$w \downarrow$
$r \uparrow$	$U \uparrow$	$V \downarrow$	$\theta \downarrow$	$w \downarrow$
$\gamma_0 \uparrow$	$U \uparrow$	$V \downarrow$	$\theta \downarrow$	$w \downarrow$
$\beta \uparrow$	$U \uparrow$	$V \downarrow$	$\theta \downarrow$	$w \uparrow$
$z \uparrow$	$U \uparrow$	$V \downarrow$	$\theta \downarrow$	$w \uparrow$
$A \uparrow$	$U \downarrow$	$V \uparrow$	$\theta \uparrow$	$w \uparrow$

- Can we think of any of these factors is explaining the rise in unemployment in Europe during the 1980s, or the lesser rise in unemployment in 1980s in in the United States?

Dynamics

- It can be verified that in this basic model there are no dynamics in θ. (Why is that? How could this be generalized?)
- But there will still be dynamics nonemployment because job creation is slow.
- We will later see how important these dynamics could be.

Efficiency?

- Is the search equilibrium efficient?
- Clearly, it is inefficient relative to a first-best alternative, e.g., a social planner that can avoid the matching frictions.
- Instead look at "surplus-maximization" subject to search constraints (why not constrained Pareto optimality?)

Search Externalities

- There are two major externalities

$$\theta \uparrow \implies \text{workers find jobs more easily}$$
$$\hookrightarrow \quad \text{thick-market externality}$$
$$\implies \quad \text{firms find workers more slowly}$$
$$\hookrightarrow \quad \text{congestion externality}$$

- Why are these externalities?
- Pecuniary or nonpecuniary?
- Why should we care about the junior externalities?

Efficiency of Search Equilibrium

- The question of efficiency boils down to whether these two externalities cancel each other or whether one of them dominates.
- To analyze this question more systematically, consider a social planner subject to the same constraints, intending to maximize "total surplus", in other words, pursuing a utilitarian objective.
- First ignore discounting, i.e., $r \rightarrow 0$, then the planner's problem can be written as

$$\max_{u,\theta} SS = (1 - u)y + uz - u\theta\gamma_0.$$

$$\text{s.t.}$$

$$u = \frac{s}{s + \theta q(\theta)}.$$

where we assumed that z corresponds to the utility of leisure rather than unemployment benefits (how would this be different if z were unemployment benefits?)

- Intuition?

Efficiency of Search Equilibrium

- Why is $r = 0$ useful?
- It turns this from a dynamic into a static optimization problem.
- Form the Lagrangian:

$$\mathcal{L} = (1-u)y + uz - u\theta\gamma_0 + \lambda\left[u - \frac{s}{s + \theta q(\theta)}\right]$$

- The first-order conditions with respect to u and θ are straightforward:

$$(y - z) + \theta\gamma_0 = \lambda$$

$$u\gamma_0 = \lambda s \frac{\theta q'(\theta) + q(\theta)}{(s + \theta q(\theta))^2}$$

Efficiency of Search Equilibrium (continued)

- The constraint will clearly binding (why?)
- Then substitute for u from the Beveridge curve, and obtain:

$$\lambda = \frac{\gamma_0 \left(s + \theta q \left(\theta\right)\right)}{\theta q' \left(\theta\right) + q \left(\theta\right)}$$

- Now substitute this into the first condition to obtain

$$\left[\theta q' \left(\theta\right) + q \left(\theta\right)\right] \left(y - z\right) + \left[\theta q' \left(\theta\right) + q \left(\theta\right)\right] \theta \gamma_0 - \gamma_0 \left(s + \theta q \left(\theta\right)\right) = 0$$

- Simplifying and dividing through by $q \left(\theta\right)$, we obtain

$$\left[1 - \eta(\theta)\right] \left[y - z\right] - \frac{s + \eta(\theta)\theta q(\theta)}{q(\theta)} \gamma_0 = 0.$$

  where

$$\eta \left(\theta\right) = -\frac{\theta q' \left(\theta\right)}{q \left(\theta\right)} = \frac{\frac{\partial M(U,V)}{\partial U} U}{M \left(U, V\right)}$$

  is the elasticity of the matching function respect to unemployment.

Comparison to Equilibrium

- Recall that in equilibrium (with $r = 0$) we have

$$(1 - \beta)(y - z) - \frac{s + \beta\theta q(\theta)}{q(\theta)}\gamma_0 = 0.$$

- Comparing these two conditions we find that efficiency obtains if and only if *the Hosios condition*

$$\beta = \eta(\theta)$$

is satisfied

- In other words, efficiency requires the bargaining power of the worker to be equal to the elasticity of the matching function with respect to unemployment.
- This is only possible if the matching function is constant returns to scale.
- What happens if not?
- Intuition?

Efficiency with Discounting

- Exactly the same result holds when we have discounting, i.e., $r > 0$
- In this case, the objective function is

$$SS^* = \int_0^\infty e^{-rt} \left[Ny - zN - \gamma_0 \theta (L - N) \right] dt$$

and will be maximized subject to

$$\dot{N} = q(\theta)\theta(L - N) - sN$$

- Simple optimal control problem.

Efficiency with Discounting (continued)

- Solution:

$$y - z - \frac{r + s + \eta(\theta)q(\theta)\theta}{q(\theta)\left[1 - \eta(\theta)\right]}\gamma_0 = 0$$

- Compared to the equilibrium where

$$(1 - \beta)[y - z] + \frac{r + s + \beta q(\theta)\theta}{q(\theta)}\gamma_0 = 0$$

Efficiency with Discounting

- Again, $\eta(\theta) = \beta$ would decentralize the constrained efficient allocation.
- Does the surplus maximizing allocation to zero unemployment?
- Why not?
- What is the social value unemployment?

Endogenous Job Destruction

- So far we treated the rate at which jobs get destroyed as a constant, s, giving us the simple flow equation

$$\dot{u} = s(1 - u) - \theta q(\theta) u$$

- But in practice firms decide when to expand and contract, so it's a natural next step to endogenize s.
- Suppose that each firm consists of a single job (so we are now taking a position on for size).

Endogenous Job Destruction (continued)

- Also assume that the productivity of each firm consists of two components, a common productivity and a firm-specific productivity.

$$\text{productivity for firm } i = \underbrace{p}_{\text{common productivity}} + \underbrace{\sigma \times \varepsilon_i}_{\text{firm-specific}}$$

where

$$\varepsilon_i \sim F(\cdot)$$

over support $\underline{\varepsilon}$ and $\bar{\varepsilon}$, and σ is a parameter capturing the importance of firm-specific shocks.

- Moreover, suppose that each new job starts at $\varepsilon = \bar{\varepsilon}$, but does not necessarily stay there.
- In particular, there is a new draw from $F(\cdot)$ arriving at the flow the rate λ.

Endogenous Job Destruction (continued)

- To further simplify the discussion, let us ignore wage determination and set

$$w = b$$

- This then gives the following value function (written in steady state) for a an active job with productivity shock ε (though this job may decide not to be active):

$$rJ^F(\varepsilon) = p + \sigma\varepsilon - b + \lambda \left[\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max\{J^F(x), J^V\} dF(x) - J^F(\varepsilon) \right]$$

where J^V is the value of a vacant job, which is what the firm becomes if it decides to destroy.

- The max operator takes care of the fact that the firm has a choice after the realization of the new shock, x, whether to destroy or to continue.

Endogenous Job Destruction (continued)

- Since with free entry $J^V = 0$, we have

$$rJ^F(\varepsilon) = p + \sigma\varepsilon - b + \lambda \left[E(J^F) - J^F(\varepsilon) \right] \tag{7}$$

  where $J^F(\varepsilon)$ is the value of employing a worker for a firm as a function of firm-specific productivity.

- Also

$$E(J^F) = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max\left\{ J^F(x), 0 \right\} dF(x) \tag{8}$$

  is the expected value of a job after a draw from the distribution $F(\varepsilon)$.

- Given the Markov structure, the value conditional on a draw does not depend on history.

- Intuition?

Endogenous Job Destruction (continued)

- Differentiation of (7) immediately gives

$$\frac{dJ^F(\varepsilon)}{d\varepsilon} = \frac{\sigma}{r+\lambda} > 0 \qquad (9)$$

- Greater productivity gives greater values the firm.
- When will job destruction take place?
- Since (9) establishes that J^F is monotonic in ε, job destruction will be characterized by a cut-off rule, i.e.,

$$\exists\, \varepsilon_d : \varepsilon < \varepsilon_d \longrightarrow \text{ destroy}$$

- Clearly, this cutoff threshold will be defined by

$$rJ^F(\varepsilon_d) = 0$$

Endogenous Job Destruction (continued)

- But we also have

$$rJ^F(\varepsilon_d) = p + \sigma\varepsilon_d - b + \lambda \left[E(J^F) - J^F(\varepsilon_d) \right],$$

which yields an equation for the value of a job after a new draw:

$$E(J^F) = -\frac{p + \sigma\varepsilon_d - b}{\lambda} > 0$$

- $E(J^F) > 0$ implies that the expected value of continuation is positive (remember equation (8)).
- Therefore, the flow profits of the marginal job, $p + \sigma\varepsilon_d - b$, must be negative.
- Interpretation?

Endogenous Job Destruction (continued)

- Furthermore, we have a tractable equation for $J^F(\varepsilon)$:

$$J^F(\varepsilon) = \frac{\sigma}{r + \lambda}(\varepsilon - \varepsilon_d)$$

- To characterize $E(J^F)$, note that

$$E(J^F) = \int_{\varepsilon_d}^{\bar{\varepsilon}} J^F(x) dF(x)$$

- Integration by parts

$$
\begin{aligned}
E(J^F) &= \int_{\varepsilon_d}^{\bar{\varepsilon}} J^F(x) dF(x) = J^F(x)F(x)\Big|_{\varepsilon_d}^{\bar{\varepsilon}} - \int_{\varepsilon_d}^{\bar{\varepsilon}} F(x)\frac{dJ^F(x)}{dx} dx \\
&= J^F(\bar{\varepsilon}) - \frac{\sigma}{\lambda + r}\int_{\varepsilon_d}^{\bar{\varepsilon}} F(x) dx \\
&= \frac{\sigma}{\lambda + r}\int_{\varepsilon_d}^{\bar{\varepsilon}} [1 - F(x)] \, dx
\end{aligned}
$$

where the last line use the fact that $J^F(\varepsilon) = \frac{\sigma}{\lambda + r}(\varepsilon - \varepsilon_d)$.

Endogenous Job Destruction (continued)

- Next, we have that

$$\underbrace{p + \sigma\varepsilon_d - b}_{\text{profit flow from marginal job}} = -\frac{\lambda\sigma}{r+\lambda} \int_{\varepsilon_d}^{\bar{\varepsilon}} [1 - F(x)] \, dx$$

$$< \quad 0 \text{ due to option value}$$

- Again "hoarding".
- More importantly, we have

$$\frac{d\varepsilon_d}{d\sigma} = \frac{p-b}{\sigma} \left[\sigma \left(\frac{r + \lambda F(\varepsilon_d)}{r+\lambda} \right) \right]^{-1} > 0.$$

- Therefore, when there is more dispersion of firm-specific shocks, there will be more job destruction

Endogenous Job Destruction (continued)

- The job creation part of this economy is similar to before.
- In particular, since firms enter at the productivity $\bar{\varepsilon}$, we have

$$
\begin{aligned}
q\left(\theta\right) J^{F}(\bar{\varepsilon}) &= \gamma_0 \\
&\implies \frac{\gamma_0 (r + \lambda)}{\sigma(\bar{\varepsilon} - \varepsilon_d)} = q(\theta)
\end{aligned}
$$

- Recall that as in the basic search model, job creation is "sluggish", in the sense that it is dictated by the matching function; it cannot jump it can only increase by investing more resources in matching.
- On the other hand, job destruction is a jump variable so it has the potential to adjust much more rapidly (but of course the relative importance of job creation and job destruction in practice is an empirical matter)

Endogenous Job Destruction (continued)

- The Beveridge curve is also different now.
- Flow into unemployment is also endogenous, so in steady-state we need to have

$$\lambda F(\varepsilon_d)(1 - u) \ = \ q(\theta)\theta u$$

- In other words:

$$u = \frac{\lambda F(\varepsilon_d)}{\lambda F(\varepsilon_d) + q(\theta)\theta},$$

which is very similar to our Beveridge curve above, except that $\lambda F(\varepsilon_d)$ replaces s.
- The most important implication of this is that shocks (for example to productivity) now also shift the Beveridge curve shifts.
- E.g., an increase in p will cause an inward shift of the Beveridge curve; so at a given level of creation, unemployment will be lower.
- How does endogenous job destruction affects efficiency?

A Two-Sector Search Model

- Now consider a two-sector version of the search model, where there are skilled and unskilled workers.
- Suppose that the labor force consists of L_1 and L_2 workers, i.e.

$L_1 :$ unskilled worker
$L_2 :$ skilled worker

- Firms decide whether to open a skilled vacancy or an unskilled vacancy.

$$\left. \begin{array}{l} M_1 = x(U_1, V_1) \\ M_2 = x(U_2, V_2) \end{array} \right\}$$ the same matching function in both sectors.

- Opening vacancies is costly in both markets with

$\gamma_1 :$ cost of vacancy for unskilled worker
$\gamma_2 :$ cost of vacancy for skilled worker.

A Two-Sector Search Model (continued)

- As before, shocks arrive at some rate, here assumed to be exogenous and potentially different between the two types of jobs

$$s_1, s_2 : \text{ separation rates}$$

- Finally, we allow for population growth of both skilled unskilled workers to be able to discuss changes in the composition of the labor force.

- In particular, let the rate of population growth of L_1 and L_2 be n_1 and n_2 respectively.

$$n_1, n_2 : \text{ population growth rates}$$

- This structure immediately implies that there will be two separate Beveridge curves for unskilled and skilled workers, given by

$$u_1 = \frac{s_1 + n_1}{s_1 + n_1 + \theta_1 q(\theta_1)} \qquad u_2 = \frac{s_2 + n_2}{s_2 + n_2 + \theta_2 q(\theta_2)}.$$

- Intuition?

A Two-Sector Search Model (continued)

- Implication: different unemployment rates are due to three observable features,
  1. separation rates,
  2. population growth
  3. job creation rates.

A Two-Sector Search Model (continued)

- The production side is largely the same as before

$$\text{output} \quad Af(K, N)$$

where N is the effective units of labor, consisting of skilled and unskilled workers.

- We assume that each unskilled worker has one unit of effective labor, while each skilled worker has $\eta > 1$ units of effective labor.
- Finally, the interest rate is still r and the capital depreciation rate is δ.

Bellman Equations

- Parallel to before.
- For filled jobs

$$
\begin{aligned}
rJ_1^F &= Af(k) - (r+\delta)k - w_1 - s_1 J_1^F \\
rJ_2^F &= Af(k)\eta - (r+\delta)k\eta - w_2 - s_2 J_2^F
\end{aligned}
$$

- For vacancies

$$
\begin{aligned}
rJ_1^V &= -\gamma_1 + q(\theta_1)(J_1^F - J_1^V) \\
rJ_2^V &= -\gamma_2 + q(\theta_2)(J_2^F - J_2^V)
\end{aligned}
$$

- Free entry:

$$
J_1^V = J_2^V = 0
$$

Equilibrium

- Using this, we have the value of filled jobs in the two sectors

$$J_1^F = \frac{\gamma_1}{q(\theta_1)} \quad \text{and} \quad J_2^F = \frac{\gamma_2}{q(\theta_2)}$$

- The worker side is also identical, especially since workers don't have a choice affecting their status. In particular,

$$
\begin{aligned}
rJ_1^U &= z + \theta_1 q(\theta_1)(J_1^E - J_1^U) \\
rJ_2^U &= z + \theta_2 q(\theta_2)(J_2^E - J_2^U)
\end{aligned}
$$

where we have assumed the unemployment benefit is equal for both groups (is this reasonable? Important?).

- Finally, the value of being employed for the two types of workers are

$$rJ_i^E = w_i - s(J_i^E - J_i^U)$$

Equilibrium (continued)

- The structure of the equilibrium is similar to before, in particular the modified golden rule and the two wage equations are:

$$
\begin{aligned}
Af'(k) &= r + \delta \qquad \text{M.G.R.} \\
w_1 &= (1 - \beta)z + \beta\left[Af(k) - (r + \delta)k + \theta_1\gamma_1\right] \\
w_2 &= (1 - \beta)z + \delta\left[Af(k)\eta - (r + \delta)k\eta + \theta_2\gamma_2\right]
\end{aligned}
$$

- The most important result here is that wage differences between skilled unskilled workers are compressed.
- To illustrate this, let us take a simple case and suppose first that

$$
\gamma_1 = \gamma_2, \ n_1 = n_2, \ s_1 = s_2, \ z = 0.
$$

- Thus there are no differences in costs of creating vacancies, separation rates, unemployment benefits, and population growth rates between skilled and unskilled workers.

Unemployment Differences

- In this special case, we have

$$u_2 > u_1$$

- Why?

$$J_1^F = \frac{\gamma}{q(\theta_1)} \quad \text{and} \quad J_2^F = \frac{\gamma}{q(\theta_2)}$$
$$J_2^F > J_1^F \implies \theta_1 < \theta_2 \implies u_1 > u_2.$$

- High skill jobs yield higher rents, so everything else equal firms will be keener to create these types of jobs, and the only thing that will equate their marginal profits is a slower rate of finding skilled workers, i.e., a lower rate of unemployment for skilled than unskilled workers

Unemployment Differences More Generally

- There are also other reasons for higher unemployment for unskilled workers.
- Also, $s_1 > s_2$ but lately $n_1 < n_2$ so the recent fall in n_1 and increase in n_2 should have helped unskilled unemployment.
- But $z \uparrow$ has more impact on unskilled wages.
- $\eta \uparrow \implies$ "skill-biased" technological change.

$$\implies \quad u_1 = cst, \; w_1 = cst$$
$$u_2 \downarrow, \; w_2 \uparrow$$

Does the Cost of Capital Matter?

- A set of interesting effects happen when r are endogenous.
- Suppose we have $\eta \uparrow$, this implies that demand for capital goes up, and this will increase the interest rate, i.e., $r \uparrow$
- The increase in the interest rate will cause

$$u_1 \uparrow, w_1 \downarrow .$$

Labor Force Participation

- Can this model explain non-participation?
- Suppose that workers have outside opportunities distributed in the population, and they decide to take these outside opportunities if the market is not attractive enough.
- Suppose that there are N_1 and N_2 unskilled and skilled workers in the population.
- Each unskilled worker has an outside option drawn from a distribution $G_1(v)$, while the same distribution is $G_2(v)$ for skilled workers.
- In summary:

$$G_1(v) \quad N_1 : \text{ unskilled}$$
$$G_2(v) \quad N_2 : \text{ skilled}$$

Labor Force Participation (continued)

- Given v; the worker has a choice between J_i^U and v.
- Clearly, only those unskilled workers with

$$J_1^U \geq v$$

will participate and only skilled workers with

$$J_2^U \geq v$$

(why are we using the values of unemployed workers and not employed workers?)

Labor Force Participation (continued)

- Since L_1 and L_2 are irrelevant to steady-state labor market equilibrium above (because of constant returns to scale), the equilibrium equations are unchanged. Then,

$$L_1 = N_1 \int_0^{J_1^U} dG_1(v)$$

$$L_2 = N_2 \int_0^{J_2^U} dG_2(v).$$

$\eta \uparrow, \ r \uparrow \Longrightarrow u_1 \uparrow, \ w_1 \downarrow \ J_1^U \downarrow$

$\Longrightarrow$ unskilled participation falls (consistent with the broad facts).

- But this mechanism requires an interest rate response (is this reasonable?)

Equilibrium in the Search Model

- Recall that the steady state equilibrium involves

$$(1 - \beta) \left[A f(k) - (r + \delta) k - z \right] - \frac{r + \delta + \beta \theta q(\theta)}{q(\theta)} \gamma_0 = 0$$

- In fact, this condition holds at all points in time not just in steady state
    - because θ is a "jump" variable and can thus immediately take the value consistent with this equation, and all other variables are either constant or parameters;
    - slow dynamics are only in the unemployment rate

Can the Model Generate Significant Employment Fluctuations?

- The answer is no.
- Shimer (AER 2005) for example has shown that the model generates significant movements in unemployment only when shocks are implausibly large.
- In other words, to generate movements in u and θ similar to those in the data, we need much bigger changes in $Af(k) - (r + \delta)k$, A or p (labor productivity or TFP) than is in the data.

Understanding Fluctuations in the Search Model

- When A increases, so do the net present discounted value of wages, and this leaves less profits, and therefore there is less of a response from vacancies.

- To see this point in greater detail, let us combine data and the basicc search model developed above.

- In particular, note that in the data the standard deviation of $\ln p$ (log productivity) is about 0.02, while the standard deviation of $\ln \theta$ is about 0.38.

- Therefore, to matched abroad facts with productivity-driven shocks, one needs an elasticity $d \ln \theta / d \ln p$ of approximately 20.

- Can the model generate this?

Understanding Fluctuations in the Search Model (continued)

- To investigate this, consider the equilibrium condition

$$(1 - \beta)\left[Af(k) - (r + \delta)k - z\right] - \frac{r + s + \beta\theta q(\theta)}{q(\theta)}\gamma_0 = 0.$$

- Rewrite this as

$$\frac{r + s + \beta\theta q(\theta)}{q(\theta)}\gamma_0 = (1 - \beta)\,p \tag{10}$$

where p is "net productivity", $p = Af(k) - (r + \delta)k - z$, the net profits that the firm makes over and above the outside option of the worker. This is not exactly the same as labor productivity, but as long as z is small, it is going to be very similar to labor productivity.

- The quantitative predictions of the model will therefore depend on whether the elasticity implied by (10) comes close to a number like 20 or not.

Understanding Fluctuations in the Search Model (continued)

- To see this, let us differentiate (10) totally with respect to θ and p.
- We obtain:

$$-\frac{(r+s)}{q(\theta)}\gamma_0 q'(\theta)\,d\theta + \beta\gamma_0 d\theta = (1-\beta)\,dp$$

- Now dividing the left-hand side by θ and the right hand side by p, and using the definition of the elasticity of the matching function $\eta(\theta) \equiv -\theta q'(\theta)/q(\theta)$ and the value of p from (10), we have

$$(r+s)\,\gamma_0\eta(\theta)\,\frac{d\theta}{\theta} + \beta\gamma_0\theta\frac{d\theta}{\theta} = (1-\beta)\,\frac{dp}{p}\frac{r+s+\beta\theta q(\theta)}{(1-\beta)\,q(\theta)}\gamma_0,$$

and since $dx/x = d\ln x$, we have the elasticity of the vacancy to unemployment ratio with respect to p as

$$\frac{d\ln\theta}{d\ln p} = \frac{r+s+\beta\theta q(\theta)}{(r+s)\,\eta(\theta)+\beta\theta q(\theta)}.$$

Understanding Fluctuations in the Search Model (continued)

- Therefore, this crucial elasticity depends on the interest rate, the separation rate, the bargaining power of workers, the elasticity of the matching function $(\eta(\theta))$, and the job finding rate of workers $(\theta q(\theta))$.

- Let us take one period to correspond to a month. Then the numbers Shimer estimates imply that $\theta q(\theta) \simeq 0.45$, $s \simeq 0.034$ and $r \simeq 0.004$.

- These numbers imply that the monthly interest rate is about 0.4%, an unemployed worker finds a job with a probability of about 0.45 within a month and an employed worker transits into unemployment with probability 0.034. Moreover, like other papers in the literature, Shimer estimates a constant returns to scale Cobb-Douglas matching function,

$$m(u, v) \propto u^{0.72} v^{0.28}.$$

Understanding Fluctuations in the Search Model (continued)

- As a first benchmark, suppose that we have efficiency, so that the Hosios condition holds. In this case, we would have $\beta = \eta(\theta) = 0.72$. In that case, we have

$$
\begin{aligned}
\frac{d \ln \theta}{d \ln p} &\simeq \frac{0.034 + 0.004 + 0.72 \times 0.45}{(0.034 + 0.004) \times 0.72 + 0.72 \times 0.45} \\
&\simeq 1.03,
\end{aligned}
$$

which is substantially smaller than the 20-fold number that seems to be necessary.

- One way to increase this elasticities to reduce the bargaining power of workers below the Hosios level. But this does not help that much.

- The upper bound on the elasticity is reached when $\beta = 0$ is

$$
\begin{aligned}
\frac{d \ln \theta}{d \ln p} &\simeq \frac{0.034 + 0.004}{(0.034 + 0.004) \times 0.72} \\
&\simeq 1.39,
\end{aligned}
$$

Understanding Fluctuations in the Search Model (continued)

- Yet another way would be to make p more variable than labor productivity, which is possible because p includes z, but it's unlikely that this will go very far.

- In fact, what's happening is related to the cyclicality of wages. Recall that in the steady-state equilibrium,

$$w = (1 - \beta)z + \beta \left[Af(k) - (r + \delta)k + \theta\gamma_0 \right].$$

- Therefore, the elasticity of the wage with respect to p is in fact greater than 1 (since it comes both from the direct affect and from the changes in θ); much of the productivity fluctuations are absorbed by the wage.

Understanding Fluctuations in the Search Model (continued)

- Naturally, in matching the data we may not want to limit our focus to just productivity shocks, especially since the correlation between productivity shocks and vacancy-unemployment rate is not that high (roughly about 0.4). So there must be other shocks, but what could those be?

- One possibility is shocks to separation rates, for example because of plant closings. Hall (REStat 2005) also further argues that there is no way to resolve this puzzle by looking at the side of worker inflows into unemployment. He suggests that most of the action is in job creation. To a first approximation, job destruction or worker inflows into unemployment can be ignored. There is debate on this point, those like Steve Davis working on job destruction rates disagreeing, but it's an interesting perspective.

Wage Rigidity

- Another possibility is to assume that wages are "rigid".
- This is the avenue pursued by Hall (AER 2005).
- He argues that any distribution of the quasi-rent created by search frictions can be an equilibrium, and the exact level of wages is determined by "social norms," and take this to be constant.
- If wages are constant, changes in p will translate into bigger changes in J^F, and thus consequently to bigger changes in θ.

Real Wage Rigidity

- The type of wager rigidity that Hall has in mind is "real wage rigidity," meaning that real wages are rigid.

- This may result from fairness type considerations found in the data by Bewley.

- It may also result from other bargaining protocols as emphasized by Hall.

- For example, can firms pay different wages to workers doing the same tasks? Can they create a very unequal wage distribution within a firm?

Quantitative Effects of Real Wage Rigidity

- How large can the impact of wage rigidity be?
- If all other assumptions of search and matching model are maintained, then this is bounded by the labor demand elasticity.
- Consistent with the results that the search model does not generate much fluctuations when $\beta = 0$, the extent of labor market fluctuations will be quite limited in this case also.
- Most estimates of the labor demand elasticity (with respect to wage) are less than 1 (see, e.g., Hamermesh, 1993).
  - In general equilibrium, this follows because labor and capital are often estimated to be complements, which implies demand elasticities less than 1 for both.
  - But it's also true when industry-level demand curves are estimated
- How can we measure the extent of real wage rigidity?
  - Difficult, since how wages would move without "rigidity" is unclear.

Nominal Wage Rigidity

- Another possibility is downward nominal wage rigidity.
- This was popular in the early Keynesian models, but was then dismissed based on the argument that it would imply countercyclical real wages
  - it would only do so if there are no other shocks than productivity/demand shocks changing labor demand and downward nominal wage rigidity is complete.
- One advantage of this is that its empirical extent can be assessed in a more careful manner.

Nominal Wage Rigidity (continued)

- There is continuing debate on the extent of nominal wage rigidity.
  - Many economists are biased against it because it is difficult to understand why workers should be so averse to nominal wage cuts when they are not as averse to real wage cuts.

- What does the evidence say?

- Little doubt that there is nominal wage rigidity but there is no consensus on its quantitative importance.

Nominal Wage Rigidity (continued)

- Two kinds of studies:
  1. Studies using panel data on individual wages, e.g., McLaughlin, 1994, Kahn, 1997, Card and Hyslop, 1997, Nickell and Quintini, 2003:
     - all find evidence for nominal wage rigidity except McLaughlin;
     - but quantitative magnitude probably difficult to ascertain because of endemic measurement error—with measurement error, downward rigid wages will lead to the impression of wages that are being increased or decreased nominally.
  2. Few studies using administrative data, e.g., Baker, Gibbs and Holmstrom, 1994, Lebow, Saks and Wilson, 2003, Fehr and Goette, 2005:
     - find much clearer evidence of nominal wage rigidities;
     - but still conceptual issues in estimating the at date of importance of nominal rigidities.

Nominal Wage Rigidity (continued)
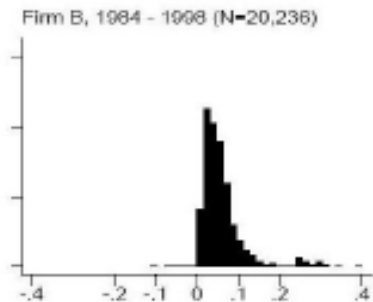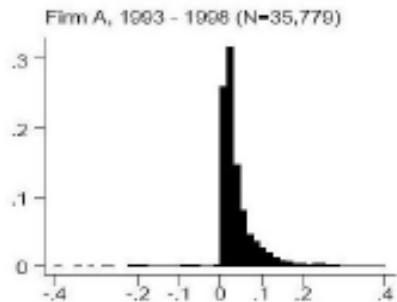
- The picture from the PSID (Kahn, 1997)
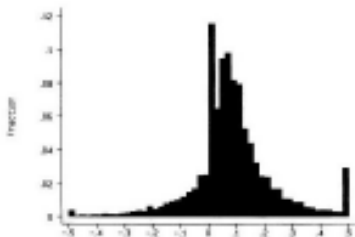
FIGURE 1. DISTRIBUTION OF ANNUAL PROPORTIONATE
CHANGE IN PAY, 1970–1988

Nominal Wage Rigidity (continued)

- The picture from administrative data (Fehr and Goette, 2005)

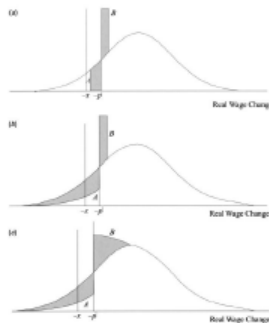# Nominal Wage Rigidity (continued)

- Conceptual issues:

Fig. 4. *Real Wage Change Frequency Distortions Due to Nominal Rigidities*

- But in fact, the impact of nominal wage rigidity can be much greater if firms also try to maintain relative wage changes in line with some notional distribution.

Composition of Jobs

- Search models can also be useful for studying the endogenous (quality) composition of jobs and whether this is sufficient or not.
- I will now sketch two related models of endogenous job composition.
  1. A model in which identical workers in equilibrium accept jobs of different qualities, and quality differences arise endogenously from two types of jobs that differ in terms of their capital intensity.
  2. A model in which heterogeneous workers look for heterogeneous jobs and the distribution of worker types determines the distribution of job types.

Good Jobs Versus Bad Jobs: Model

- Suppose that there is is a continuum of identical workers with measure normalized to 1.
- All workers are infinitely lived and risk-neutral with discount rate r.
- The technology of production for the final good is:

$$Y = \left(\alpha Y_b^\rho + (1-\alpha) Y_g^\rho\right)^{1/\rho} \tag{11}$$

where Y_g is the aggregate production of the first input, and Y_b is the aggregate production of the second input, and $\rho < 1$. The elasticity of substitution between Y_g and Y_b is $1/(1-\rho)$.

- The two intermediate goods are sold in competitive markets, so

$$
\begin{aligned}
p_b &= \alpha Y_b^{\rho-1} Y^{1-\rho} \\
p_g &= (1-\alpha) Y_g^{\rho-1} Y^{1-\rho}
\end{aligned}
$$

Good Jobs Versus Bad Jobs (continued)

- The technology of production for the inputs is Leontieff.
- When matched with a firm with the necessary equipment (capital k_b or k_g), a worker produces 1 unit of the respective good.
- The equipment required to produce the first input costs k_g while the cost of equipment for the second input is $k_b < k_g$. In equilibrium, capital-intensive jobs will pay higher wages (thus correspond to "good" jobs).
- Firms can choose either one of two types of vacancies: *(i)* a vacancy for a intermediate good 1 - a *good job*; *(ii)* a vacancy for an intermediate good 2 - *a bad job*.
- Search is undirected between the two types of jobs (in the single labor market).

Bellman Equations

- Denoting the proportion of bad job vacancies among all vacancies by ϕ and focusing on steady state, we have

$$rJ^U = z + \theta q(\theta) \left[\phi J_b^E + (1 - \phi) J_g^E - J^U \right] \qquad (12)$$

- The steady state discounted present value of employment for two types of jobs can be written as:

$$rJ_i^E = w_i + s(J^U - J_i^E) \qquad (13)$$

for $i = b, g$.

- Similarly, when matched, both vacancies produce 1 unit of their goods, so:

$$rJ_i^F = p_i - w_i + s \left(J_i^V - J_i^F \right) \qquad (14)$$

$$rJ_i^V = q(\theta) \left(J_i^F - J_i^V \right) \qquad (15)$$

for $i = b, g$.

Wage Bargaining

- Since workers and firms are risk-neutral and have the same discount rate, Nash Bargaining implies that w_b and w_g will be chosen so that:

$$
\begin{aligned}
(1-\beta)(J_b^E - J^U) &= \beta(J_b^F - J_b^V) \\
(1-\beta)(J_g^E - J^U) &= \beta(J_g^F - J_g^V)
\end{aligned}
\tag{16}
$$

- Important feature: workers cannot pay to be employed in high wage jobs.
- Free entry:

$$
J_i^V = k_i. \tag{17}
$$

- Finally, the steady state unemployment rate is as usual

$$
u = \frac{s}{s + \theta q(\theta)}. \tag{18}
$$

Steady-State Equilibrium

- Let ϕ be the fraction of good job vacancies.
- Then $Y_b = (1-u)\phi$ and $Y_g = (1-u)(1-\phi)$, and thus

$$
\begin{align}
p_g &= (1-\alpha)(1-\phi)^{\rho-1}\left[\alpha\phi^\rho + (1-\alpha)(1-\phi)^\rho\right]^{\frac{1-\rho}{\rho}} \quad (19) \\
p_b &= \alpha\phi^{\rho-1}\left[\alpha\phi^\rho + (1-\alpha)(1-\phi)^\rho\right]^{\frac{1-\rho}{\rho}}.
\end{align}
$$

- Wages can then be determined as

$$
w_i = \beta\left(p_i - rk_i\right) + (1-\beta)rJ^U. \quad (20)
$$

- Zero profit conditions can be written as

$$
\frac{q(\theta)(1-\beta)\left(p_b - rJ^U\right)}{r + s + (1-\beta)q(\theta)} = rk_b \quad (21)
$$

$$
\frac{q(\theta)(1-\beta)\left(p_g - rJ^U\right)}{r + s + (1-\beta)q(\theta)} = rk_g. \quad (22)
$$

- Intuition?

Steady-State Equilibrium (continued)

- Now combining (20), (21) and (22), we get :

$$w_g - w_b = \frac{(r+s)(rk_g - rk_b)}{q(\theta)} > 0 \qquad (23)$$

- Therefore, wage differences are related to the differences in capital costs and also to the average duration of a vacancy. In particular, when $q(\theta) \to \infty$, the equilibrium converges to the Walrasian limit point, and both w_g and w_b converge to rJ^U, so wage differences disappear. (Why?)

Steady-State Equilibrium (continued)

- Let us first start with the case in which $\rho \leq 0$, so that good and bad jobs are gross complements.

- In this case:

Proposition

*Suppose that $\rho \leq 0$. Then, a steady state equilibrium with $\phi \in (0,1)$ always exists and is characterized by (19), (20), (21), (22) and (**??**). In equilibrium, for all $k_g > k_b$, we have $p_g > p_b$ and $w_g > w_b$.*

- When $\rho > 0$, an equilibrium continues to exist, but does not need to be interior, so one of (21) and (22) may not hold. A particular example of this is discussed in the next subsection.

- There can also be multiple equilibria.

Welfare

- Consider utilitarian welfare:

$$TS = (1-u)\left[\phi(p_b - rk_b) + (1-\phi)(p_g - rk_g)\right] - \theta u\left(\phi rk_b + (1-\phi)rk_g\right)$$
(24)

- Intuitively, this is equal to total flow of net output, which consists of the number of workers in good jobs $((1-\phi)(1-u))$ times their net output (p_g minus the flow cost of capital rk_g), plus the number of workers in bad jobs $(\phi(1-u))$ times their net product $(p_b - rk_b)$, minus the flow costs of job creation for good and bad vacancies (respectively, $\theta u(1-\phi)rk_g$ and $\theta u\phi rk_b$).

- Constrained optimal allocations would be given by solutions to the maximization of (24) subject to (18), and we have one source of some optimality due to the failure of Hosios conditions.

Welfare (continued)

- But more interesting is distortions in the composition of jobs.
  Consider:

$$\frac{dTS}{d\phi} = (1-u) \cdot \left[\frac{d(\phi p_b + (1-\phi)p_g)}{d\phi} \right] - (1-u+u\theta) \cdot \{ rk_b - rk_g \} .$$

- Using (18), (19), (21) and (22) to substitute out u, and k_i, we obtain

$$\left. \frac{dTS}{d\phi} \right|_{dec.\ eq.} = \frac{\theta q(\theta)}{s + \theta q(\theta)} \cdot \left(1 + \frac{(s+q(\theta))(1-\beta)}{r + s + (1-\beta)q(\theta)} \right) \cdot (p_b - p_g) < 0.$$

- This expression is always negative, irrespective of the value of θ, so
  starting from laissez-faire equilibrium, a reduction in ϕ will increase
  social surplus. Therefore:

Welfare (continued)

Proposition

Let $\phi^s(\theta)$ be the value of ϕ that the social planner would choose at labor market tightness θ, and $\phi^*(\theta)$ be the laissez-faire equilibrium with $z = 0$, then $\phi^*(\theta) > \phi^s(\theta)$ for all θ. That is, in the laissez-faire equilibrium, the proportion of bad jobs is too high.

- The intuition is in terms of pecuniary externalities.
- In equilibrium, good jobs always create greater positive pecuniary externalities workers (why?).

Minimum Wages and Unemployment Benefit

Proposition

Both the introduction of a minimum wage $\underline{w}$ and an increase in unemployment benefit z decrease θ and ϕ. Therefore, they improve the composition of jobs and average labor productivity, but increase unemployment. The impact on overall surplus is ambiguous.

- Intuition.

Heterogeneity and Composition of Jobs

- A different and richer set of issues arise when there is heterogeneity among workers.
- Such models are more complicated, so I will present a simplified static version with two types of workers (and endogenous distribution of job qualities).

Static Model

- There is a mass 1 of risk-neutral workers and a mass 1 of risk-neutral and profit-maximizing firms. The economy lasts for one period, and contains workers in two education groups, high and low (e.g. high school and college graduates).

- Let the fraction of skilled workers by ϕ, and normalize the human capital of unskilled workers to $h = 1$ and the human capital of skilled workers to $h = \eta > 1$.

- Production takes place in one firm-one worker pairs. A worker with human capital h and a firm with capacity k produce:

$$y(h, k) = k^{1-\alpha} h^{\alpha}. \tag{25}$$

Timing of Events

- The timing of events is as follows.
- First, each firm has to choose its capacity ("physical capital"), k, irreversibly. This choice captures the type of job the firm has designed and the line of business it has chosen, and is assumed to be costless. At this point, the firm does not know the type of the worker it will recruit.
- Next, the firm matches with a worker, finds out his type, and decides whether to shut down or continue. If it continues, it installs the required equipment and incurs a cost ck.
- Finally, wages are determined.

Frictional Equilibrium

- Suppose that there is random matching with one firm meeting one worker, and once they meet, the worker-firm pair have to decide whether to produce together, with zero earnings if they separate.
- As usual, bargaining implies the following wages

$$w^H(k) = \beta k^{1-\alpha}\eta^{\alpha} \text{ and } w^L(k) = \beta k^{1-\alpha}.$$

- To simplify the algebra, let us normalize $c \equiv (1 - \beta)$.
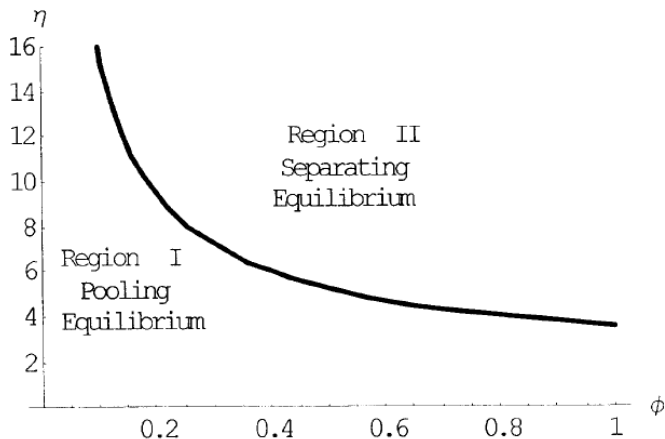
Frictional Equilibrium (continued)

- The expected value of a firm choosing capacity k can be written as:

$$V(k, x^H, x^L) = \phi x^H (1-\beta) \left[k^{1-\alpha} \eta^\alpha - k \right] + (1-\phi) x^L (1-\beta) \left[k^{1-\alpha} - k \right]$$

$$\tag{26}$$

  where x^j is the equilibrium probability that the firm produces with a worker of type $j = L$ or H, conditional on matching with this worker.

- Since a fraction ϕ of workers are skilled and there is random matching, the firm produces with a skilled worker with probability ϕx^H, and obtains $(1-\beta)y(\eta, k) - ck = (1-\beta) \left[k^{1-\alpha} \eta^\alpha - k \right]$. The second part of (26) is explained similarly.

- Note that when the firm decides not to produce with the worker, i.e. $x = 0$, it does not incur the cost of capital, ck.

Frictional Equilibrium (continued)

# Frictional Equilibrium (continued)

Proposition

If

$$\eta < \left(\frac{1-\phi}{\phi^\alpha - \phi} \right)^{1/\alpha} \tag{27}$$

then, there is a unique equilibrium which is pooling. All firms choose $k^P = a \left[\phi \eta^\alpha + (1-\phi) \right]^{1/\alpha}$, and $x^H(k^P) = x^L(k^P) = 1$.
If $\eta > \left[(1-\phi) / (\phi^\alpha - \phi) \right]^{1/\alpha}$, there is a unique equilibrium which is separating. All firms choose capacity $k^H = a\eta$, $x^H(k) = 1$ and $x^L(k) = 0$.

- In the dynamic version, in the separating equilibrium there will also be some fraction of firms that create low physical capital jobs paying low wages directed at low skill workers.

Dynamic Equilibrium in a Picture
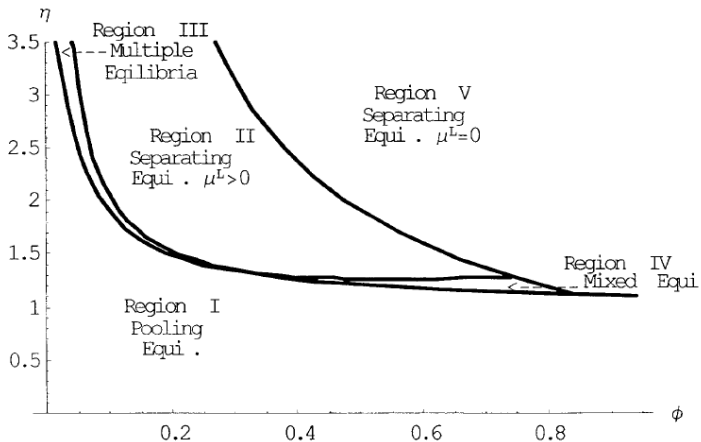
Implications

- An increase in η (corresponding to skill-biased technological change or changing human capital of skilled workers or globalization) will increase inequality, but this effect will be muted inside region 1 because search frictions create pooling.

- But an increase in η or in ϕ (increasing the fraction of skilled workers) can switch the economy from region wanted region 2, i.e., to the separating equilibrium. This will create a discrete jump in inequality as well as in the equilibrium composition of jobs.

- It is particularly interesting that an increase in the supply of skilled workers can change the equilibrium in their favor (we see here a flavor of induced skill biased technological change and directed technological change models).

Evidence

- Acemoglu (1999) presented several different types of evidence consistent with this:

  1. A hollowing of the wage and occupation distributions (shown in greater detail later in Autor, Levy and Murnane, 2003, Acemoglu and Autor, 2011).
  2. An increase in the heterogeneity of jobs in terms of physical capital to labor ratios.
  3. A different pattern of mismatch between firms and workers (from PSID).

The Burdett-Mortensen Model

- A simple variant of search models can also generate endogenous wage dispersion. Consider the following model due to Burdett and Mortensen.
- There is a continuum 1 of workers, who can be employed or unemployed.
- Think of the workers as in the McCall sequential search world, observing wages from a given distribution (except that, imagine we are in continuous time, so workers see a wage at some flow rate).
- Moreover assume that both employed and unemployed workers receive wage offers at the flow rate p.
- An employed worker who receives an offer can leave his job and immediately start at the new job if he so wishes. The important assumption is that the future distribution of job offers and rate of job offers is unaffected by whether the worker is employed or not (this is not an assumption in the original Burdett-Mortensen model, but it simplifies life).

The Burdett-Mortensen Model (continued)

- There is a continuum $m < 1$ of firms.
- They post wages and their wage offers are seen by a worker at the flow rate q. Thus p and q are exactly as in our standard search model, except that they are not what "matching" probabilities but flow rates of a worker seeing a wage, and a wage being seen by a worker.
- For simplicity, let us take p and q as exogenous.
- Unemployed workers receive a benefit of $b < 1$. Employed workers produce output equal to 1, and there is no disutility of work.
- There is exogenous separation at the rate s, and also potentially endogenous separation if workers receive a better wage offer.
- Both workers and firms are risk-neutral and discount the future at the rate r.

The Burdett-Mortensen Model (continued)

- Wage posting corresponds to a promise by the firm to employ a worker at some prespecified wage untill the job is destroyed exogenously.
- Workers observe promised wages before making their decisions.
- Let us denote the offered wage distribution by $F(w)$, and let us restrict attention to steady states, assuming that this distribution is stationary.

Equilibrium

- First let's look at the search behavior of an unemployed worker.
- As usual, the worker is solving a straightforward dynamic programming problem, and his search behavior will be characterized by a reservation wage.
- Moreover, in this case the reservation wage is easy to pin down.
- Since there is no disutility and accepting a job does not reduce the future opportunities, an unemployed worker will accept all wages

$$w \geq b$$

Equilibrium (continued)

- Let's now look at the behavior of an employed worker, currently working at the wage w_0.

- By the same reasoning, this worker will take any job that offers

$$w \geq w_0$$

- Therefore, firms get workers from other firms that have lower wages and lose workers to exogenous separation and to firms that offer higher wages.

Equilibrium (continued)

- Now with this structure, we can show that there will exist a non-degenerate continuous connected wage distribution over some range $[b, \bar{w}]$.

- Why? (Proceeding informally).

- First, it is easy to check that $\bar{w} \leq 1$. If $\bar{w} > 1$, the firm would make negative profits. This implies that employing a (one more) worker is always profitable.

- Suppose that the wage distribution is not continuous, meaning that there is an atom at some point w'. Then it is a more profitable to offer a wage of $w' + \varepsilon$ than w' for ε sufficiently small, since with positive probability a worker will end up with two wages of w', thus accepting each with probability $1/2$. A wage of $w' + \varepsilon$ wins the worker for sure in this case.

Equilibrium (continued)

- Suppose that the wage distribution is not connected, so that there is zero mass in some range (w', w''). Then all firms offering w'' can cut their wages to $w' + \varepsilon$, and receive the same number of workers.

- The lower support has to be $\underline{w} = b$. Suppose not, i.e., suppose $\underline{w} > b$. Then firms offering $\underline{w}$ can cut their wages without losing any workers.

- Let's now look at the differential equations determining the number of workers employed in each firm and workers in unemployment.

Equilibrium (continued)

- Unemployment dynamics are given by

$$\dot{u} = s\left(1 - u\right) - pu$$

  since workers receive wage offers at the rate p, and all of them take their offers.

- Therefore, steady state unemployment is fixed by technology as

$$u = \frac{s}{s + p}$$

- However, employment rate of firms is endogenous. Imagine that the equilibrium wage distribution is given by $G\left(\tilde{w}\right)$ and the offered wage distribution is $F\left(\tilde{w}\right)$.

- Let us continue to restrict attention to steady states, where both of those are stationary. It is important that these two are not the same (why?).

Equilibrium (continued)

- Then the level of employment of a firm offering wage w (now and forever) follows the law of motion

$$\dot{N}(w) = q(u + (1-u) G(w)) - (s + p(1 - F(w))) N(w)$$

where the explanation is intuitive; the offer of this firm is seen by a worker at the flow rate q, and if he is unemployed, which has probability u, he takes it, and otherwise he is employed at some wage distribution G. His wage is lower than the offered wage with probability $G(w)$, in which case he takes the job.

- The outflow is explained similarly, bearing in mind that now what is relevant is not the actual wage distribution but the offered wage distribution $F(w)$.

Equilibrium (continued)

- To find the steady state, we need to set $\dot{N}(w) = 0$, which implies

$$N(w) = \frac{q(u + (1-u)G(w))}{(s + p(1 - F(w)))} \tag{28}$$

- Moreover, we have a similar law of motion for the distribution function $G(w)$. In particular, the total fraction of workers employed and getting paid the wage of less than or equal to w is

$$(1 - u)G(w)$$

- The outflow of workers from this group is equal to

$$[s + p(1 - F(w))](1 - u)G(w)$$

by the same reasoning as above.

Equilibrium (continued)

- The inflow of workers into the status of employed and being paid a wage less than w only come from unemployment (when a worker upgrades from the wage w' to $w'' \in (w', w]$, this does not change $G(w)$.

- Hence the inflow is

$$pF(w) u,$$

which is the measure of unemployed workers receiving an offer, pu, times the probability that this offer is less than w.

- Equating the outflow and the inflow, we obtain the cumulative density function of actual wages as

$$G(w) = \frac{pF(w) u}{[s + p(1 - F(w))](1 - u)}$$

Equilibrium (continued)

- Now using the steady-state unemployment rate:

$$G(w) = \frac{psF(w)}{p[s + p(1 - F(w))]} \qquad (29)$$

- The important thing to note is that

$$G(w) < F(w),$$

meaning that the fraction of jobs in the equilibrium wage distribution below wage w is always lower than the fraction of offers below w, so that F first-order stochastically dominates G.

- Stated differently, this means that low wages have a lower probability of being accepted and, once accepted, a lower probability of surviving. Thus equilibrium wages are **positively selected** from offered wages.

Equilibrium (continued)

- Now combining (29) this with (28), we obtain

$$
\begin{aligned}
N(w) &= \frac{q\left(\frac{s}{s+p} + \frac{p}{s+p}\frac{psF(w)}{p[s+p(1-F(w))]}\right)}{(s+p(1-F(w)))} \\
&= \frac{psq}{(s+p(1-F(w)))^2}
\end{aligned}
$$

- Thus we now have to solve for $F(w)$ only, or for $G(w)$ only.
- In equilibrium, all firms have to make equal profits, which means equal discounted profits. This is a complicated problem, since a new firm accumulates workers slowly. Rather than solve this problem, let us look at the limit where $r \to 0$. This basically means that we can simply focus on state state, and equal discounted profits is equivalent to equal profits in the steady state.

Equilibrium (continued)

- The profits of a firm offering wage w (when the offer wage distribution is given by F), is

$$\pi(w) = (1 - w) N(w)$$

- In other words, an equilibrium satisfies

$$\pi(w) = \bar{\pi} \text{ for all } w \in \text{supp} F,$$

where $\bar{\pi}$ is also determined as part of the equilibrium.

- Now solving these equations:

$$\pi(w) = (1 - w) \frac{psq}{(s + p(1 - F(w)))^2} = \bar{\pi}$$

Equilibrium (continued)

- Inverting this:

$$F(w) = 1 - \sqrt{\frac{(1-w)\,sq}{p\bar{\pi}}} + \frac{s}{p}$$

  over the support of F.

- Moreover, we know that $w = b$ is in the support of F, and $F(b) = 0$, and this implies

$$0 = 1 - \sqrt{\frac{(1-b)\,sq}{p\bar{\pi}}} + \frac{s}{p}$$

  or

$$\bar{\pi} = \frac{(1-b)\,psq}{(p+s)^2}$$

Equilibrium (continued)

- Now substituting, we have

$$F\left(w\right) = 1 - \sqrt{\frac{\left(1 - w\right)\left(p + s\right)^2}{\left(1 - b\right)p^2}} + \frac{s}{p},$$

which is a well-behaved distribution function that is increasing everywhere.

- Moreover, since $F\left(\bar{w}\right) = 1$, we also obtain that

$$\bar{w} < 1,$$

so even the highest wages less than the full marginal product of the worker.

- From here, observed wage distribution is quite easy to calculate.

Discussion

- Why is there wage dispersion? The answer is similar to the models we have already seen.
- The Burdett-Mortensen model is sometimes interpreted as a model of "monopsony".
- The reasoning is that firms do not face a flat labor supply curve, but can increase their "labor supply" by increasing their wages (they attract more workers and avoid losing workers).
- The reason why wages are lower than full marginal product is argued to be this monopsony power. This is in fact quite **misleading**. In the baseline search model, there is both monopoly power and monopsony power (there is bilateral monopoly, that's why there is bargaining!). In fact, we saw that in the Diamond's Paradox, even though in equilibrium firms do not face an upward-sloping labor supply, they have full monopsony power, and can hold workers down to their reservation utility (unemployment benefit or zero).

Discussion (continued)

- The nice thing about the Burdett-Mortensen model is that it generates a wage dispersion together with employer-size dispersion, and it matches a very well-known stylized fact that larger employers pay higher wages.

- The typical interpretation for this is that larger employers attract higher-quality workers or somehow workers have greater bargaining power against such employers.

- Burdett-Mortensen turn this on its head; they argued that it is not that larger employers pay higher wages, it is that employers that pay higher wages become larger in equilibrium!

Discussion (continued)

- Nevertheless, there are theoretical objections to the Burdett-Mortensen model. The most important is that it is not optimal for firms to post wages. Instead they should post "contracts" that make workers pay upfront and receive their full marginal products. If this is not possible (because negative wages or bonding contracts are not allowed), they can make workers receive a low-wage early on, and increase their wages later. Why is this?

- Finally, note that even though there is wage posting here, there isn't *directed search*. In the next lecture, we will see that directed search is the essence. The previous model of wage dispersion in fact may have mimicked directed search, for reasons we are going to see more clearly soon.