14.461: Technological Change, Lectures 12-14 Network Linkages: Technology, Productivity and Volatility

Daron Acemoglu

MIT

October 10, 14 and 16.

Introduction

- Modern economies rely on complex ("intertwined") interactions between upstream and downstream firms, banks and other financial institutions, innovators, etc.
  - A firm or disaggregated industry supplies inputs to many other industries and receives supplies from yet many others, or the bank will be part of the chain of counterparty relations.
- Does the nature of these "network" linkages matter for productivity, volatility, technology?

Plan

- Start with a framework of input-output linkages, and characterize how these linkages determined in GDP.
- Then study the relationship between input-output linkages and productivity.
- Then use this framework for study of volatility:
  - **Sources of aggregate fluctuations**—from micro shocks.
  - A framework for empirical work for on the interplay between shocks of different industries.
  - A "theory" of **systemic risk**.
- Finally, a glimpse at the innovation network which determines the propagation of ideas and the evolution of technologies.

Production Networks

- Let us consider a simple model of input-output linkages.
- Based on Long and Plosser (*JPE*, 1993) and Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (*Econometrica*, 2012).
- The output of each sector is used by a subset of all sectors as input (intermediate goods) for production.
- A static economy (without capital) consisting of n sectors—generalization to dynamics, including capital accumulation straightforward and omitted for simplicity.

Production Structure

- Cobb-Douglas technologies:

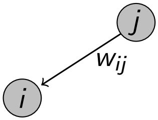
$$x_i = u_i^{\alpha} l_i^{\alpha} \prod_{j=1}^{n} x_{ij}^{(1-\alpha)w_{ij}},$$

with resource constraint: $$\sum_{i=1}^{n} x_{ji} + c_i = x_i,$$

  - l_i: labor employed by sector i;
  - $\alpha \in (0,1)$: share of labor;
  - x_{ij}: the amount of good j used in the production of good i;
  - c_i: final consumption of good i.
  - u_i: idiosyncratic (independent across sectors) shock to sector i—for simplicity introduced as a productivity shock. Let $\epsilon_i \equiv \log(u_i)$ with distribution function F_i and variance $\sigma_i^2 < \infty$.
- w_{ij}: share of good j in input use of sector i;
  - $w_{ij} = 0$ if sector i does not use good j as input for production.
- No aggregate shocks—for simplicity.

Input-Output Structure

- **Input-output structure** represented by a weighted, directed network/graph.



- Suppose that each sector equally relies on the inputs of others:

$$\sum_{j=1}^{n} w_{ij} = 1 \text{ for each } i.$$

Input-Output Structure (continued)

- **Degree of sector** j: (value) share of j's output in the total production of economy

$$d_j = \sum_{i=1}^{n} w_{ij}.$$

  - Formally, this is "out-degree," but since "in-degree" is equal to one for all sectors, we refer to this as "degree".
- Let W be the matrix of w_{ij}'s.
  - the row sums of W are equal to one;
  - the column sums of W are given by the d_j's.
- w_{ij}'s also correspond to the entries of input-output tables. Here Cobb-Douglas is important. Entries of input-output tables are defined as value of spending on input/value of output.
  - With Cobb-Douglas, these values are independent of quantities (price and output effects exactly cancel out), and are given by the exponents w_{ij} of the production function.

Household Maximization

- All sectors are competitive.
  - Identical results with constant elasticity monopolistic competition.
- Representative household with preferences:

$$u(c_1, c_2, \ldots, c_n) = A \prod_{i=1}^{n} (c_i)^{1/n},$$

where A is a normalizing constant.

- Endowed with one unit of labor supplied inelastically, so market clearing implies

$$\sum_{i=1}^{n} l_i = 1.$$

- Consumer maximization:

$$\begin{aligned} \text{maximize} \quad & u(c_1, c_2, \ldots, c_n) \\ \text{subject to} \quad & \sum_{i=1}^{n} p_i c_i = h, \end{aligned}$$

Competitive Equilibrium

- The representative household maximizes utility.
- All firms maximize profits.
- Labor and goods markets clear.

Characterization of Equilibrium

- The structure of equilibrium is straightforward to characterize.
- Log GDP or real value added is given as a *convex combination* of sectoral shocks:

$$y \equiv \log(GDP) = v' \boldsymbol{\epsilon},$$

where $\boldsymbol{\epsilon} \equiv [\epsilon_1 \dots \epsilon_n]'$ is the vector of sectoral shocks, and v the **influence vector** or the vector of **Bonacich centrality** indices defined as

$$v \equiv \frac{\alpha}{n} \left[I - (1-\alpha) W' \right]^{-1} e,$$

where recall that e is the vector of 1's.
- The term $\left[I - (1-\alpha) W' \right]^{-1}$ is also the **Leontief inverse.**
- As noted by Hulten (*Review of Economic Studies*, 1978) and Gabaix (*Econometrica*, 2011), v is also the "sales vector" of the economy, with its elements given by

$$v_i = \frac{p_i x_i}{\sum_{j=1}^{n} p_j x_j}.$$

Why the Leontief Inverse?

- That the Leontief inverse emerges as the relevant measure—and its relationship to Bonacich centrality—is not surprising, though of course the Cobb-Douglas technologies and preferences do matter for the exact functional form.
- Clearly if an industry i is hit by a negative shock, ϵ_i, this will not only reduce x_i, but may also affect downstream and upstream industries.
- First consider upstream industries. It turns out that the impact on upstream industries is zero because price and output effects cancel out due to Cobb-Douglas—as the quantity of good i falls (because of the negative shock) the price of good i increases, leaving $p_i x_i$ unchanged.
- *This implies no upstream impact* in response to productivity shocks.

Why the Leontief Inverse? (continued)

- Next consider downstream industries.
- Now the increase in p_i implies that they will cut their demand for x_i, reducing their output.
- The first-order effects (on log outputs) can be captured by $\alpha(1-\alpha)W_i'\epsilon_i$—where W_i is the ith column of the W matrix, and α comes from the fact that the impact of ϵ_i on sector i is $\alpha\epsilon_i$.
- But this is not the end of the adjustment. There will be second-order effects, as downstream industries from i contract and then their downstream industries are also negatively affected. This will be captured by $(1-\alpha)^2\left(W_i'\right)^2\epsilon_i$.

Why the Leontief Inverse? (continued)

- Continuing in this fashion with higher-order effects, we have that the total impact from the shock to sector i is

$$\alpha \sum_{k=1}^{\infty} (1-\alpha)^k \left(W^k\right)'_i \epsilon_i = \alpha \left(\left[I - (1-\alpha) W'\right]^{-1}\right)'_i \epsilon_i = \alpha \left(\sum_{j=1}^{n} l_{ji}\right) \epsilon_i,$$

  where l_{ij}'s are the elements of the Leontief inverse matrix.
- Taking shocks to all sectors into account and the fact that, from the consumer side, sectoral outputs can be logarithmically aggregated with each sector having weight $1/n$, we obtain the total impact on log GDP as

$$\begin{aligned} \frac{\alpha}{n} \sum_{i=1}^{n} \sum_{k=1}^{\infty} (1-\alpha)^k \left(W'\right)_i^k \epsilon_i &= \alpha \sum_{i=1}^{n} \sum_{j=1}^{n} l_{ji} \epsilon_i \\ &= \frac{\alpha}{n} \left(\left[I - (1-\alpha) W'\right]^{-1} e\right)' \epsilon \\ &= v' \epsilon. \end{aligned}$$

Reduced-Form Empirical Approaches

- Do the input-output linkages really matter?
- Consider the effect of all sectoral shocks on sector i, and also look at effects on upstream suppliers in the case of demand/import shocks.
- With the same reasoning (and ignoring constants), the first-order effect can be written as
$$\sum_{j \neq i} w_{ij} \epsilon_i = ((W')_i)' \epsilon_{-i}$$
where W'_i is the ith row of the matrix W and ϵ_{-i} is the column vector of ϵ's with the ith element set to zero.
- Proceeding similarly, the full effects can be obtained as
$$\left([I - (1-\alpha) W]^{-1} e \right)_i' \boldsymbol{\epsilon}_{-i} = \sum_{j \neq i} l_{ij} \epsilon_i$$
where recall that l_{ij}'s are the entries of the Leontief inverse matrix.
- The simplest empirical approach would be to use a measure of the "exogenous" component of $\boldsymbol{\epsilon}$ and study the impact of ϵ_i and $\boldsymbol{\epsilon}_{-i}$ on the output of sector i.

Reduced-Form Empirical Approaches (continued)

- A candidate for such potentially exogenous industry have a level shock is the exogenous component of the increase in (US) imports from China, is exploited by Autor, Dorn, and Hanson (*AER*, 2013).
- This approach is pursued in the context of the study of the impact Chinese trade on aggregate US employment by Acemoglu, Autor, Dorn, Hanson, and Price (mimeo, 2014).
- Exogenous component is obtained, following Autor, Dorn, and Hanson, by using the increase in non-US OECD countries imports from China in that industry.
- Imports from China are measured as imports divided by value of production in the US economy at the four-digit manufacturing industry level.
- The impact of ϵ_{-i} is measured both by first-order effects and the full effects using the Leontief inverse.

Reduced-Form Results

Direct + Indirect Effects of Chinese Import Competition on US Manufacturing Employment

(Acemoglu, Autor, Dorn, Hanson, and Price 2014)

	First-Order I/O Linkages		Full I/O Linkages	
	(1)	(2)	(3)	(4)
Δ in US Exposure to Chinese Imports	-1.16***	-1.26***	-1.18***	-1.27***
	(0.42)	(0.48)	(0.42)	(0.48)
Δ in Downstream Import Exposure	-2.33*	-2.48**	-1.83**	-1.92**
	(1.21)	(1.19)	(0.90)	(0.88)
Δ in Upstream Import Exposure		2.14		1.60
		(2.90)		(2.35)
Time Effect: 1991-1999	0.18	0.12	0.21	0.14
	(0.36)	(0.37)	(0.36)	(0.38)
Time Effect: 1999-2011	-3.06***	-3.23***	-2.95***	-3.16***
	(0.34)	(0.41)	(0.36)	(0.48)
N	784	784	784	784

Reduced-Form Results (continued)

- Consistent with the basic theory exposited here, there are large downstream effects, especially once these are filtered through Leontief inverse.
- The upstream results seem to be much less stable (consistent with the emphasis on downstream effects here).

Motivation

- Most sectors use the output of other sectors in the economy as intermediate goods.
- This introduces interlinkages among sectors.
- Important for understanding several, potentially inter-related phenomena:
  - Inefficiency in one sector will have implications for productivity in others.
  - Shocks to a sector can have aggregate volatility implications.
  - Changes in sectoral composition can affect fundamental volatility in the economy.

Input-Output Linkages and Sectoral Misallocation

- Consider next the model of Jones (2010), which is a slight generalization of the setup presented about:

$$x_i = A_i \left(k_i^{\alpha_i} l_i^{1-\alpha_i} \right)^{1-w_i} x_{i1}^{w_{i1}} x_{i2}^{w_{i2}} \cdot \ldots \cdot x_{iN}^{w_{iN}}$$

  where:

  - $A_i \equiv A\eta_i$,
  - K_i and L_i are the quantities of physical and human capital used in sector i,
  - x_{ij}'s are intermediates (output of other sectors).

- Moreover, $w_i \equiv \sum_{j=1}^{N} w_{ij}$ and and $0 < \alpha_i < 1$, so the production function features constant returns to scale.

Sectoral Misallocation

- As before, output of each industry used for final consumption, c_j, or as inputs to other industries:

$$c_j + \sum_{i=1}^{N} x_{ij} = x_j, \quad j = 1, \ldots, N.$$

- Suppose that there is a single final good, combining the output of different sectors is Cobb-Douglas:

$$Y = c_1^{\beta_1} \cdot \ldots \cdot c_N^{\beta_N},$$

where $\sum_{i=1}^{N} \beta_i = 1$.

- This aggregate final good can itself be used in one of two ways, as consumption or exported to the rest of the world:

$$C + X = Y.$$

Sectoral Misallocation (continued)

- Finally, factors are supplied inelastically:

$$\sum_{i=1}^{N} k_i = K,$$

$$\sum_{i=1}^{N} l_i = L. \tag{1}$$

Equilibrium with Misallocation

- Why will there be "misallocation"?
- Jones assumes "sector specific wedges" causing sector-specific reductions in revenue in proportion to τ_i.
- Then equilibrium is defined as a competitive equilibrium given these distortions.

Equilibrium

- In the competitive equilibrium with misallocation, the solution for total production of the aggregate final good is

$$Y = A^{\tilde{\mu}} K^{\tilde{\alpha}} L^{1-\tilde{\alpha}} \epsilon,$$

where

- $\mu' \equiv \beta' (I - W)^{-1}$ where β is the vector of β_i's and W is the matrix of w_{ij}
- $\tilde{\mu} \equiv \mu' \mathbf{1}$
- $\tilde{\alpha} \equiv \mu' (1 - w_i)$
- $\log \epsilon \equiv \omega + \mu' \bar{\eta}$ where $\bar{\eta}$ is the vector of $\log (\eta_i (1 - \tau_i))$'s and ω is a constant depending on the other parameters.

Discussion

- Aggregate TFP, ϵ, depends on both sectoral TFPs and the underlying distortions, which is intuitive in light of the input-output linkages.
- The Leontief inverse again plays a major role in determining the multipliers associated with the distortions

$$\mu' \equiv \beta' (I - W)^{-1}.$$

  - The typical element ℓ_{ij} of the Leontief inverse matrix gives us the following information: a 1% increase in productivity in sector j raises output in sector i by ℓ_{ij}% — because of the indirect effects working to input-output linkages.

Discussion (continued)

- Multiplying this Leontief inverse matrix by the vector of value-added weights in β essentially amounts to adding up the effects of sector j on all the other sectors in the economy, weighting by their shares in aggregate value-added.
  - So the elements of this multiplier matrix show how a change in productivity in sector i affects overall value-added in the economy.
- Moreover, the elasticity of final output with respect to aggregate TFP is $\tilde{\mu} \equiv \mu' \mathbf{1}$.
  - Intuitively, this is obtained by adding up all the multipliers in μ because an increase in aggregate TFP affects all sectors through input-output linkages.

Further Intuition

- Consider the following simplification: $w_i \equiv \sum_{j=1}^{N} w_{ij} = \hat{w}$ for all i.
- Then
$$\frac{\partial \log Y}{\partial \log A} = \mu' \mathbf{1} = \beta' \left(I - W\right)^{-1} = \frac{1}{1 - \hat{w}}.$$
- This special case shows that the "sparseness" of the input-output matrix W is not important.
  - All that matters are the "out-degrees".
- Secondly, the common out-degree across sectors is all that matters for the multiplier with respect to aggregate TFP shock A.
- These results are also present in the general model — though naturally in a more complicated form.
- This result suggests a large amount of amplification of distortions.
  - But what happens when we look at "appropriately measured" TFP?

Distortions in the Symmetric Case

- Now consider the following special case:
  - $w_{ij} = \hat{w}/N$, $\beta_i = 1/N$, and $\alpha_i = \alpha$
  - $\log(1-\tau_i) \sim N(\theta, v^2)$ and let $1 - \bar{\tau} \equiv e^{\theta + \frac{1}{2}v^2}$ (which is the average distortion in this case).
- Then as $N \to \infty$, $\log C$ almost surely converges to

$$\text{Constant} + \frac{\hat{w}}{1-\hat{w}}(1-\bar{\tau}) + \log\left(1 - \hat{w}(1-\bar{\tau})\right) - \frac{1}{2}\frac{1}{1-\hat{w}}v^2.$$

- Therefore, what matters in this case is simply the dispersion of distortions.
- This is parallel to the dispersion of firm-level misallocations determining sectoral productivities in Hsieh and Klenow's accounting exercise.

Question

- Similar issues could be important in thinking about the origins of aggregate fluctuations.
- Aggregate shocks to productivity or demand (except for monetary policy shocks) seem less than fully compelling.
- Could they be the result of more microeconomic shocks, hitting disaggregated sectors?
- Conventional wisdom: No
    - "Diversification argument": firm-level or disaggregated sectoral shocks washed up at the rate $\sqrt{n}$ and for large n, they would be trivial.
- But intersectoral linkages introduce "network effects"
    - Shocks to some sectors may propagate to the rest of the economy and may even create "cascade effects".

Aggregate Volatility

- Let us go back to the general framework presented above and consider **aggregate volatility**—meaning the volatility of log GDP—measured as

$$\sigma_{agg} \equiv \sqrt{\mathrm{var}\ y}.$$

- Recall that

$$y \equiv \log(GDP) = v'\epsilon,$$

- Hence:

$$\sigma_{agg} = \sqrt{\sum_{i=1}^{n} \sigma_i^2 v_i^2}.$$

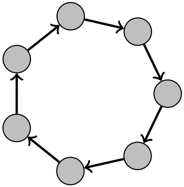
- From this expression, the "conventional wisdom"—e.g., as articulated by Lucas (*Theories of Business Cycles,* 1984)—can be understood:
  - suppose $v_i \approx \frac{1}{n}$ and n is large (so that the economy is "well diversified"), then σ_{agg} this trivial—*no aggregate fluctuations without aggregate shocks.*

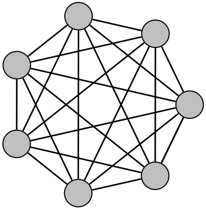
Some Theoretical Results

- We first start with some simple theoretical observations questioning the above "diversification argument" and then link the structure of the input-output network to aggregate volatility.
- We next turn to a structural empirical strategy to shed more light on the relationship between aggregate volatility and sectoral shocks.
- Finally, we provide sharper results by studying "large" (highly diversified) economies—i.e., those with n large.

Macroeconomic Irrelevance of Micro Shocks

- We say that the network is **regular** if $d_i = d$ for each i.
  - That is, each sector has a similar degree of importance as a supplier to other sectors.
- Examples of regular networks:
  - **rings:** the most "*sparse*" input-output matrix, where each sector draws all of its inputs from a single other sector.
  - **complete graphs**: where each sector equally draws inputs from all other sectors.



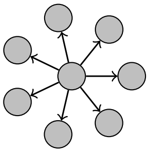


Irrelevance of Micro Shocks (continued)

- Suppose also that
$$\sigma_i = \sigma \text{ for each } i.$$
- Then we have that for all regular networks:
$$\sigma_{agg} = \frac{\sigma}{\sqrt{n}}$$
(see also Dupor, *Journal of Monetary Economics*,1999).
- Intuition: with the (log) linearity implied by the Cobb-Douglas technologies, shocks average out exactly *provided that all sectors have the same degree.*
- This result is particularly interesting because rings are often conjectured to be unstable or prone to "domino effects" (or other types of contagion).

Asymmetric Networks Are Fragile

- However, this irrelevance is not generally correct.
- In particular, Lucas's argument is incorrect when v_i's are far from $1/n$, which happens when the network is highly asymmetric—in terms of degrees.
- The extreme example is the **star network**:



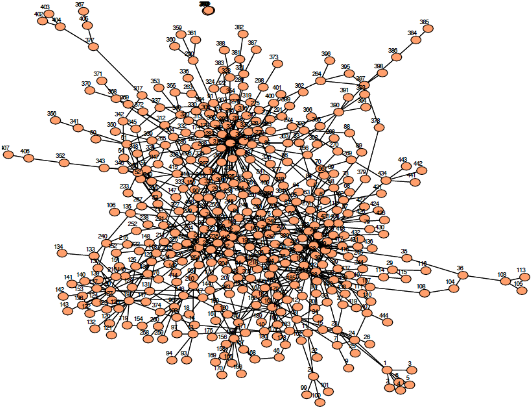
Asymmetric Networks Are Fragile (continued)

- In fact, it can be shown that the highest level of aggregate volatility is generated by the **star network** and is equal to

$$\sigma_{agg} = \frac{\sigma}{\sqrt{1 - \left(\frac{n-1}{n}\right) \alpha \left(1 - \alpha\right)}},$$

  which is much greater than $\sigma / \sqrt{n}$ when n is large.
- In fact, this is not just high volatility, but **systemic volatility** ($\approx$ "system-wide" volatility: shocks to the central sector spread to the rest, creating system-wide co-movement—we return to systemic volatility below.
- Intuition: the shock to the central sector of the star does not "wash out".
- More general result: **unequal degrees**—or asymmetric networks—create additional volatility.

What Does the US Input-Output Network Look Like?



- Intersectoral network corresponding to the US input-output matrix in 1997. For every input transaction above 5% of the total input purchases of the destination sector, a link between two vertices is drawn.

Towards a Structural Approach

- The observation about the systemic nature of volatility here also provides a useful direction about empirical work based on more fine-grained predictions of the framework here.
- If aggregate productivity is driven by inter-sectoral linkages, then there should be a specific pattern of co-movement across sectors (as a function of the input-output network).
- For example, if the input-output network is given by the star network, all sectors should co-move with the star sector, *but not* with each other conditional on the star sector.
- If the input-output network is given by the ring network, then sector i should co-move with sector $i-1$ etc.

Towards a Structural Approach (continued)

- This is related to the approach taken by Foerster, Sarte and Watson (*JPE*, 2011) (see also Shea, *Journal of Money, Credit and Banking*, 2002), but they use additional structure on the model coming from a specific real business cycle model instead of the full covariance structure just coming from the input-output interactions.
- Recall that the impact of input-output linkages on sector i is

$$\sum_{j=1}^{n} l_{ij}\epsilon_i$$

(now including the effect of sector i on itself through input-output linkages).
- Now suppose that

$$\epsilon_i = \eta + \varepsilon_i,$$

where η is an aggregate shock and ε_i is a sector-specific shock orthogonal to all other shocks.

Towards a Structural Approach (continued)

- This implies that the variance of log output of sector i can be written as

$$\sigma_\eta^2 + \alpha^2 \sum_{k=1}^{n} l_{ij}^2 \sigma_k^2,$$

where σ_η^2 is the variance of the aggregate shock and σ_i^2 is the variance of the ith sectoral shock.

- Since the vector v can be computed from the input-output table, this structure implies a close link between sectoral variances.
- More importantly, the correlation between sector i and k is

$$\sigma_\eta^2 + \alpha^2 \sum_{k=1}^{n} l_{ik} l_{jk} \sigma_k^2,$$

so the entire variance-covariance structure of sectoral outputs can be used to recover the underlying shocks.

Asymptotic Results

- To obtain sharper theoretical results, consider a sequence of economies with $n \to \infty$.
- So we will be looking at "*law of large numbers*"-type results.
- Suppose that $\sigma_i \in (\underline{\sigma}, \overline{\sigma})$.
- Then the greatest degree of "*stability*" or "*robustness*" (least systemic risk) corresponds to

$$\sigma_{agg} \sim 1/\sqrt{n}$$

  (as in standard law of large numbers for independent variables).
- Define the **coefficient of variation of degrees** (of an economy with n sectors) as

$$\mathrm{CV}_n \equiv \frac{1}{d_{\mathrm{avg}}} \left[\frac{1}{n-1} \sum_{i=1}^{n} (d_i - d_{\mathrm{avg}}) \right]^{1/2},$$

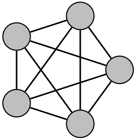
  where $d_{\mathrm{avg}} = \frac{1}{n} \sum_i d_i$ is the average degree.

First-Order Results

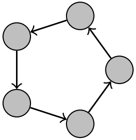
- Just considering the first-order downstream impacts,

$$\sigma_{agg} = \Omega\left(\frac{1}{\sqrt{n}} + \frac{\mathrm{CV}_n}{\sqrt{n}}\right)$$

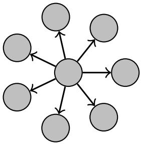
  where the Ω means $\sigma_{agg} \to 0$ as $n \to 0$ no faster than $\frac{1+\mathrm{CV}_n}{\sqrt{n}}$.
- For regular networks, $\mathrm{CV}_n = 0$, so $\sigma_{agg} \to 0$ at the rate $\frac{1}{\sqrt{n}}$.
- Ford the star network, $\mathrm{CV}_n \not\to 0$ as $n \to 0$, so $\sigma_{agg} \not\to 0$ and the law of large numbers fails.



$\mathrm{CV}_n = 0$



$\mathrm{CV}_n = 0$



$\mathrm{CV}_n \sim \sqrt{n}$

First-Order Results (continued)

- We can also make these results easier to apply.
- We say that the degree distribution for a sequence of economies has **power law tail** if, there exists $\beta > 1$ such that for each n and for large k,

$$P_n(k) \propto k^{-\beta},$$

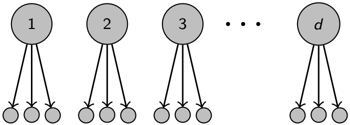
where $P_n(k)$ is the counter-cumulative distribution of degrees and β is the shape parameter.
- It can be shown that if a sequence of economies has power law tail with shape parameter $\beta \in (1, 2)$, then

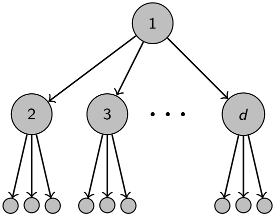
$$\sigma_{agg} = \Omega\left(n^{-\frac{\beta-1}{\beta}-\varepsilon}\right)$$

where $\varepsilon > 0$ is arbitrary.
- A smaller β corresponds to a "thicker" tail and thus higher coefficient of variation, and greater fragility.

Higher-Order Results

- In the same way that first-order downstream effects do not capture the full implications of negative shocks to a sector, the degree distribution does not capture the full extent of asymmetry/inequality of "connections".
- Two economies with the same degree distribution can have very different structures of connections and very different nature of volatility:



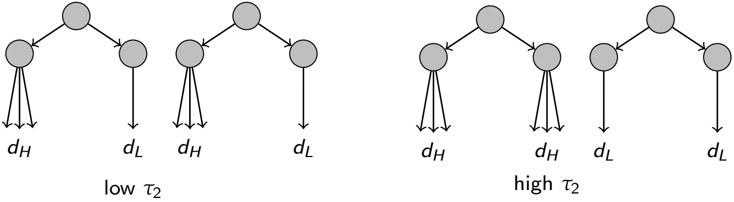


Higher-Order Results (continued)

- We define the **second-order interconnectivity coefficient** as

$$\tau_2(W_n) \equiv \sum_{i=1}^{n} \sum_{j \neq i} \sum_{k \neq i,j} w_{ji} w_{ki} d_j d_k .$$

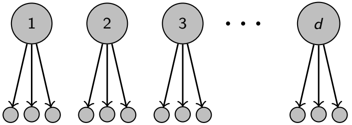
- This will be higher when high degree sectors share "upstream parents":



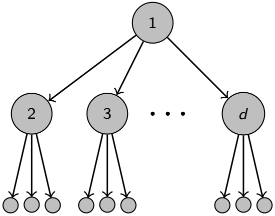
Higher-Order Results (continued)

- It can be shown that

$$\sigma_{agg} = \Omega\left(\frac{1}{\sqrt{n}} + \frac{\mathrm{CV}_n}{\sqrt{n}} + \frac{\sqrt{\tau_2(W_n)}}{n}\right).$$



$\tau_2 = 0$



$\tau_2 \sim n^2$

Higher-Order Results (continued)

- Define **second-order degree** as

$$q_i \equiv \sum_{j=1}^{n} d_j w_{ji}.$$

- For a sequence of economies with a power law tail for the second-order degree with shape parameter $\zeta \in (1, 2)$, we have

$$\sigma_{agg} = \Omega\left(n^{-\frac{\zeta-1}{\zeta}-\varepsilon}\right),$$

  for any $\varepsilon > 0$.
- If both first and second-order degrees have power laws, then

$$\sigma_{agg} = \Omega\left(n^{-\frac{\zeta-1}{\zeta}-\varepsilon} + n^{-\frac{\beta-1}{\beta}}\right),$$

  i.e., dominant term: $\min\{\beta, \zeta\}$.

When Network Structure Does Not Matter

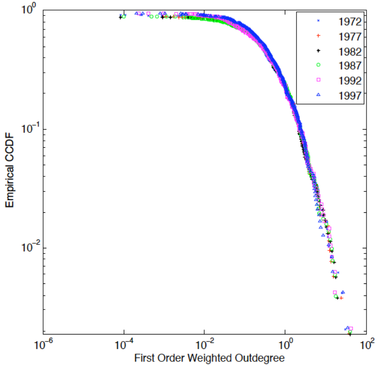
- We say that a sequence of economies is **balanced** if $\max_i d_i < c$ for some c.
- This is clearly much weaker than regularity.
- It can be shown that, for any sequence of balanced economies,

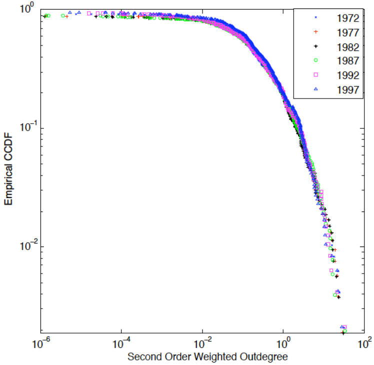
$$\sigma_{agg} \sim \frac{1}{\sqrt{n}}.$$

- Once again rings and complete networks are equally stable (emphasizing that sparseness of the input-output matrix has little to do with aggregate volatility).

Another Look at the US Input-Output Network

- Empirical counter-cumulative distribution of first-order and second-order degrees
- Linear tail in the log-log scale $\longrightarrow$ power law tail





Higher-Order Results (continued)

- Average (across years) estimates: $\hat{\beta} = 1.38$, $\hat{\zeta} = 1.18$.
- $\hat{\zeta} < \hat{\beta}$: second-order effects dominate first-order effects.
- Average (annual) standard deviation of total factor productivity across 459 four-digit (SIC) manufacturing industries between 1958 and 2005 is 0.058.
- Since manufacturing is about 20% of the economy, for the entire economy this corresponds to $5 \times 459 = 2295$ sectors at a comparable level of disaggregation.
- Had the structure been balanced: $\sigma_{\text{agg}} = 0.058/\sqrt{2295} \simeq 0.001$.
- But from the lower bound from the second-order degree distribution:

$$\sigma_{\text{agg}} \sim \sigma/\sqrt{n} \approx 0.018.$$

Back to Basics

- Carvalho and Gabaix (2013) observe that changes in sectoral and firm-size distribution can impact "fundamental" volatility in the economy.
- With the same reasoning as before (see also Gabaix (2011) and Hulten (1978)),

$$\log(GDP) = v'\epsilon,$$

where v and ϵ are n-dimensional vectors (where n is the number of firms are sectors in the economy).

- Now if the n elements of ϵ are independent, aggregate volatility can be written as

$$\sigma_{agg} = \sqrt{\sum_{i=1}^{n} v_i^2 \sigma_i^2},$$

where σ_i^2 is the variance of the ith firm or sector, and v_i is its sale to GDP ratio:

$$v_i = \frac{S_i}{GDP_i}.$$

Fundamental Volatility

- Carvalho and Gabaix define this object computed at time t (which can be defined even when sectoral shocks are not independent) as the economy's *fundamental volatility* at time t:

$$\sigma_{Ft} = \sqrt{\sum_{i=1}^{n} v_{it}^2 \sigma_i^2},$$

  where σ_i is taken to be time-invariant.

- This object can be easily computed from available data (Carvalho and Gabaix do it using a sectoral breakdown at the level of 88 sectors).

Fundamental and Actual Volatility

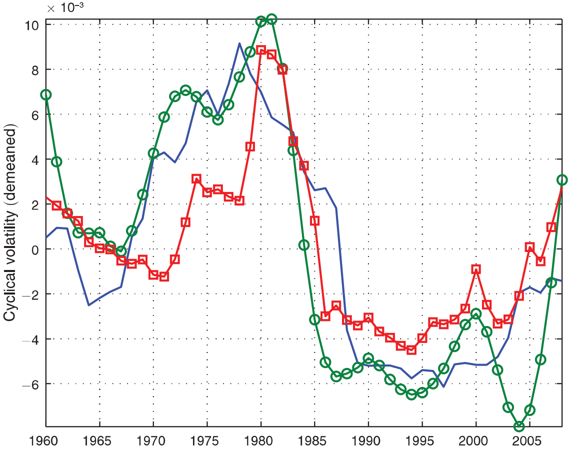


FIGURE 1. FUNDAMENTAL VOLATILITY AND GDP VOLATILITY

Notes: The squared line gives the fundamental volatility ($4.5\sigma_{Ft}$, demeaned). The solid and circle lines are annualized (and demeaned) estimates of GDP volatility, using respectively a rolling-window estimate and an HP trend of instantaneous volatility.

Fundamental and Actual Volatility (continued)

- Regression of actual volatility (computed from residuals at annual frequency or from a regression) on fundamental volatility show that much of the variation in actual annual volatility is explained by annual fundamental volatility (between 43 and 60%).
- Moreover, there does not seem to be a trend break in actual volatility once we control for fundamental volatility.
- This implies that the great moderation and the recent increase in aggregate volatility are due to changes in sectoral composition of output.
  - Great moderation driven by the declining share of highly volatile heavy manufacturing industries.
  - Greater aggregate volatility more recently due to the increasing share of finance.

Financial Contagion

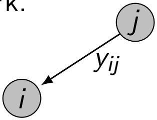
- An at-first surprising implication of the analysis so far is the result that aggregate volatility is the *same* in complete and ring networks.
- Is this a general result?
- *The answer is no*, and underscores that the implications of different network structures crucially depend on what types of interactions are taking place over the network.
- In particular, the linearity (log-linearity) is responsible for this result—positive and negative shocks cancel out when all units have similar "influence".
- But linearity may be a good approximation for input-output that works, but not for finance—where, in the presence of debt-like contracts, **default** (and bankruptcy) creates a major nonlinearity.

A Simple Model of Counterparty Relations

- Based on Acemoglu, Ozdaglar and Tahbaz-Salehi (mimeo, 2014). See also Allen and Gale (*JPE*, 2000) and Elliott, Golub and Jackson (mimeo, 2013) on a non-linear financial model due to cross-firm shareholdings and bankruptcy.
- Consider a network of banks (financial institutions) potentially borrowing and lending to each other (as well as from outside creditors and senior creditors).
- All borrowing and lending is through short-term, uncollateralized debt contracts.
- Suppose that all contracts are signed at date $t = 0$.
- Banks have long-term assets that will pay out at date $t = 2$, but are illiquid, and cannot be liquidated at date $t = 1$.
- Banks are hit by liquidity shocks at date $t = 1$ and also receive and make payments on their interbank contracts.

A Simple Model of Counterparty Relations (continued)

- More specifically, banks lend to one another at $t = 0$ through **standard debt contracts** to be repaid at $t = 1$.
- Face values of debt of bank j to bank i: y_{ij}.
- $\{y_{ij}\}$ defines a financial network.



- Related problem: chains of trade credit—Kiyotaki and Moore (mimeo, 1997) for theory and Jacobson and von Schedvin (mimeo, 2013) for evidence.

A Simple Model of Counterparty Relations (continued)

- Bank i invests in a project with returns at $t = 1, 2$.
- Random return of z_i at $t = 1$.
- Deterministic return of A at $t = 2$ if the entire project is held to maturity.
- In addition, bank i has **senior** obligations in the amount $v > 0$.
- If the bank cannot meet its obligations, it will be in bankruptcy and has to liquidate its project with ζA.
- If it still has insufficient funds, the bank will have to **default** on its creditors, which will be paid on pro rata basis.
- Simplify the discussion here by assuming that $\zeta \approx 0$, so that liquidation of long-term assets is never sufficient to stave off default.

Payment Equilibrium

- From the above description, we have that bank j's actual payments are:

$$x_{ij} = \begin{cases} y_{ij} & \text{if } z_j + \sum_s x_{js} \geq v + \sum_s y_{sj} \\ \frac{y_{ij}}{\sum_s y_{sj}}(z_j - v + \sum_s x_{js}) & \text{if } v \leq z_j + \sum_s x_{js} < v + \sum_s y_{sj} \\ 0 & \text{if } z_j + \sum_s x_{js} < v. \end{cases}$$

- The first branch is when the bank is not in default.
- The second is when the bank is in default but senior creditors are not hurt.
- The third is when senior creditors are not paid in full (and the rest are not paid at all).

Payment Equilibrium (continued)

- A **payment equilibrium** is a fixed point $\{x_{ij}\}$ of the above set of equations (one for each bank j).
- *A payment equilibrium exists and is generically unique.*
- This generalizes Eisenberg and Noe (*Management Science*, 2001).

Volatility in the Financial Network

- To discuss volatility in this financial network, let us focus on the case in which:
  - The financial network is **regular**, i.e., $\sum_s y_{sj} = y$ for all j. (We know from our analysis of input-output networks that asymmetries in this quantity create one source of stemic volatility, so we are abstracting from this).
  - $z_j = a$ or $z_j = a - \varepsilon$, so that banks are potentially hit by a negative liquidity shock at time $t = 1$.
  - Suppose also that only one bank in the network is hit by the negative liquidity shock, $-\varepsilon$.
  - Throughout, focus on the network of size n (i.e., no asymptotic results).

Volatility in the Financial Network (continued)

- How to quantify volatility?
- The following observation gives us a simple way:

$$\text{Social surplus } = na - \varepsilon - (\text{number of defaults})A.$$

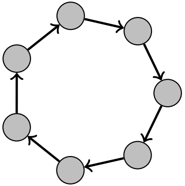
- Thus social surplus clearly related to how systemic the shock that hits one bank becomes, suggesting a natural measure of volatility and **stability** in this financial network.
- We say that a network is **less stable** than another if it has greater number of expected defaults.

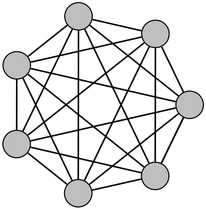
Small Shock vs. Large Shock Regimes

- It will turn out that the size of the negative shock (or more generally the size and the number of shocks) will matter greatly for what types of networks are stable.
- For this, let us call a regime in which $\varepsilon < \varepsilon^*$ the **small shock regime**, and the regime in which $\varepsilon > \varepsilon^*$ the **large shock regime.**

Stability in the Small Shock Regime

- Suppose that $\varepsilon < \varepsilon^*$ and $y > y^*$ (so that the liabilities of banks are not too small). Then:
- *The complete financial network is the most stable network.*
- *The ring financial network is the least stable network.*



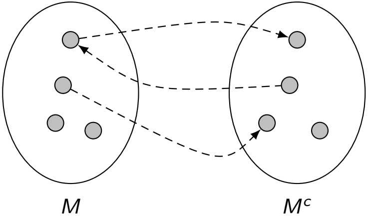


Stability in the Small Shock Regime (continued)

- In addition, it can be shown that if we take a γ convex combination of the complete and the ring networks (so that $y_{ij} = (1-\gamma)y_{ij}^{\text{ring}} + \gamma y_{ij}^{\text{complete}}$), then *as γ increases, the network becomes more stable.*
- *Intuition*: more links out from a bank implies that liabilities of that bank are held in a more *diversified* manner, and losses of that bank can be better absorbed by the financial system.
- The ring is the least diversified network structure, leading to the greatest amount of systemic volatility/instability.
- In the linear/log-linear case, positive shocks and negative shocks in different parts of the regular network canceled out. This no longer happens because of **default.**
- Rather, default creates **domino effects**.
  - If a bank is negatively hit, then it is unable to make payments on its debt, and this puts its creditors (that are highly exposed to it) in potential default, and so on.

Stability in the Large Shock Regime

- The picture is sharply different in the large shock regime.
- We say that a financial network δ-**connected** if there exists a subset M of banks such that the linkages between this subset and its complement is never greater than δ—i.e., $y_{ij} \leq \delta$ for any to banks from this upset and its complement.



Stability in the Large Shock Regime (continued)

- Suppose that $\varepsilon > \varepsilon^*$ and $y > y^*$. Then:
- *The complete and the ring financial networks are the least stable networks.*
- *For δ sufficiently small, a δ-connected network is more stable than the complete and the ring networks.*

Stability in the Large Shock Regime (continued)

- This is a type of **phase transition**—meaning that the network properties and comparative statics change sharply at a threshold value.
- *Network Intuition*: When shocks are large, they cannot be contained even with full diversification and spread through the network like an "epidemic". In that case, insulating parts of the network from others increases stability.
- *Economic Intuition*: weakly connected networks make better use of the liquidity of senior creditors.
    - The complete network uses the excess liquidity of non-distressed banks, $a - v > 0$, very effectively, but does not use the resources of senior creditors at all. Weakly connected networks do not utilize the liquidity of non-distressed banks much, but do make good use of the resources of senior creditors when needed.

Innovation Networks

- In addition to input-output and financial pathways, shocks the one part of the economy propagate to the rest because of the **innovation network**.
- Ideas in one part of the economy (in one sector, process or technology class) become the basis of innovation or technological improvement in some other part of the economy—"building on the shoulders of giants".
- Suppose, for example, that we represent innovation relations as a network between n "technology classes" G (again with G_i denoting the ith row of this matrix).
- In the data, G corresponds to the matrix given by citation patterns.

Innovation Networks (continued)

- Then let us posit the following relationship:

$$x_{i,t} = \alpha_i x_{i,t-1} + \phi G_i' x_{t-1} + \varepsilon_i,$$

  where $x_{i,t}$ is the innovation rate in technology class i at time t and x_t denotes the vector of $x_{i,t}$'s.
- This implies that successful innovations in sectors that i cites translate into higher innovations in the future by sector i.
- In practice, important to estimate G from past data (to avoid mechanical biases).

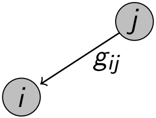
The US Innovation Network

- Acemoglu, Akcigit and Kerr (mimeo, 2014) perform this task using US citation data for the baseline period, 1975-1984.
- First construct the matrix G as

$$g_{jj'} = \sum_{k \neq j} \frac{\text{Citations}_{j \to j'}^{1975-1984}}{\text{Citations}_{j \to k}^{1975-1984}}$$

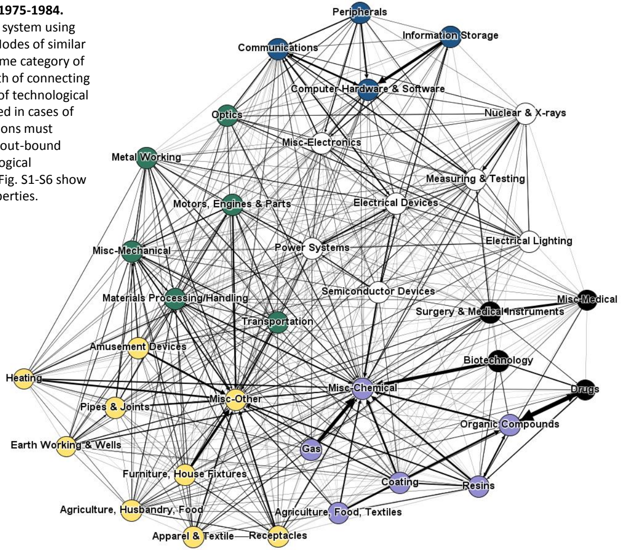
where $\text{Citations}_{j \to k}^{1975-1984}$ is the citation during this period from technology class j to k—thus ideas flowing from k to j.

  - the denominator leaves out "self-cites"—cites from j to j.



The US Innovation Network at the Two-Digit Level

Fig. 2: Innovation network 1975-1984. Network mapping of patent system using technology subcategories. Nodes of similar color are pulled from the same category of the USPTO system. The width of connecting lines indicates the strength of technological flows, with arrows being used in cases of strong asymmetry. Connections must account for at least 0.5% of out-bound citations made by a technological subcategory. Supplemental Fig. S1-S6 show variations and network properties.



Predicting Innovation

- To predict innovation using the innovation network, it is also useful to take account of the citation lags (thus corresponding to a separate G matrix for each citation time gap). For this purpose, construct

$$FlowRate_{j\to j',a}^{1975-1984} = Flow_{j\to j',a}^{1975-1984} / Patent_{j'}^{1975-1984},$$

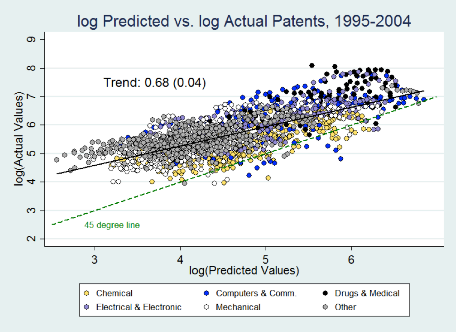
where $Flow_{j\to j',a}^{1975-1984}$ is the total number of cites from technology class j' to j that takes place a years after the patent from j is issued, and $Patent_{j'}^{1975-1984}$ is the number of patents in cited field j'.
- Compute expected patents in sector j at the three-digit technology class level (corresponding to 484 classes):

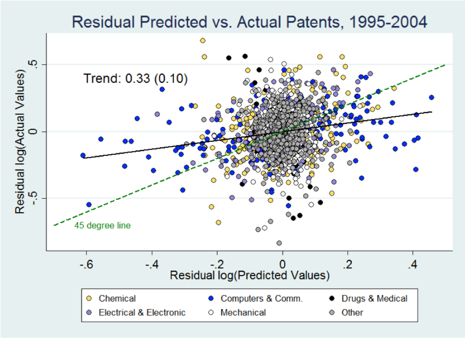
$$ExpectPatents_{j,t}^{1995-2004} = \sum_{j'\neq j} \sum_{a=1,10} FlowRate_{j\to j',a}^{1975-1984} Patents_{j',t=t_0+a}^{1985-1994}.$$

This takes into account a 10-year citation window and sums over all sectors citing j (except $j \to j$), using $FlowRate_{j\to j',a}^{1975-1984}$ as weights.
- Note that the patents on the right-hand side are for 1985-1994, whereas expected patents are for 1995-2004.

Predicting Innovation (continued)

- The relationship between expected patents and actual patents (second panel taking out technology class and year fixed effects).





Interpretation and Current Work

- This descriptive exercise provides fairly strong (albeit reduced-form) evidence that ideas and innovations spread through the citation/innovation network.
- This supports the view that innovation is a cumulative process building on innovation in other fields.
- This evidence would also plausibly suggest that medium-term propagation of "idea shocks" will be through the innovation network.
- One use of this relationship is as a potential source of variation in technology.
- If $ExpectPatents_{j,t}$ is high for some sector relative to others, then we can expect that sector to have a greater number of new innovations and thus a greater improvement in technology.
- Acemoglu, Akcigit and Kerr (2014) use this source of variation to investigate the relationship between technology and employment at the city and industry level.

Conclusion

- Networks playing an increasingly important role in macroeconomic equilibria and also research.
- These three lectures focused on propagation of shocks and distortions across sectors, financial institutions and different types of innovation/technology classes.
- Other important linkages would include geographic areas, Labor markets, firms, and countries.
- This is an area open for new theoretical and empirical work.