

# 14.461: Technological Change, Lectures 12 and 13

## Propagation of Shocks over Economic Networks

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# Introduction

- A myriad of economic interactions reflect the propagation (and sometimes amplification) of shocks or small impulses across different units in the economy — from one industry, firm, bank, region, innovator, to another.
- Often, it is both convenient and conceptually useful to think of the interaction structure of the economy as a **network**, so that we are looking at the propagation of shocks over networks.
- The last two lectures of this course will focus on some canonical models of such propagation in the context of production networks (input-output economies) and financial networks.
  - Propagation of supply and demand shocks across industries in the context of an **input-output economy**.
  - Estimating such propagation (and problems of causal inference).
  - **Sources of aggregate fluctuations**—from micro shocks.
  - Financial networks and **systemic risk**.
  - The innovation network and the propagation of ideas.

# Plan

- Shocks and interactions across industries in production networks.
- Empirical challenges of causal inference in networks.
- Estimating the propagation of shocks over input-output networks.
- Do microeconomic shocks wash out in the aggregate? Some theoretical insights and suggestive evidence.
- Systemic risk in financial networks.
- The innovation network and the propagation of ideas.
- Conclusion.

# Production Networks

- Let us consider a simple model of input-output linkages.
- Based on Long and Plosser (*JPE*, 1993), Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (*Econometrica*, 2012), Acemoglu, Ozdaglar and Tahbaz-Salehi, (2015), and Acemoglu, Akcigit and Kerr (*NBER Macroeconomics Annual*, 2016).
- The output of each sector is used by a subset of all sectors as input (intermediate goods) for production.
- A static economy (without capital) consisting of  $n$  sectors—generalization to dynamics, including capital accumulation relatively straightforward and omitted for simplicity.

# Model

- Static, perfectly competitive economy with  $n$  industries.
- Production function for each industry:

$$y_i = e^{z_i} l_i^{\alpha_i} \prod_{j=1}^n x_{ij}^{a_{ij}}, \quad i \in \{1, \dots, n\}$$

$x_{ij}$  : quantity of goods produced by industry  $j$  and used by industry  $i$ ,

$l_i$  : labor,

$z_i$  : Hicks-neutral productivity shock

Cobb-Douglas:  $\alpha_i + \sum_{j=1}^n a_{ij} = 1$ .

- Market clearing:

$$y_i = c_i + \sum_{j=1}^n x_{ji} + G_i,$$

$c_i$  : final consumption of output  $i$ ,

$G_i$  : government purchases of good  $i$ , (assumed to be wasted).

# Preferences

- Representative household's utility:

$$u(c_1, c_2, \dots, c_n) = \prod_{i=1}^n c_i^{1/n},$$

- Lump-sum tax,  $T$ , to finance  $G_j$ .
- Budget constraint of the household:

$$\sum_{i=1}^n p_i c_i = wL - T.$$

# Supply-side Shocks

## Proposition (Part A)

The full impact of sectoral supply-side (productivity) shocks  $\mathbf{dz}$  on sector  $i$  is

$$d \ln y_i = dz_i + \sum_j a_{ij} \times d \ln y_j.$$

which can be solved as:

$$d \ln y_i = \underbrace{dz_i}_{\text{own effect}} + \underbrace{\sum_j (h_{ij} - \mathbf{1}_{j=i}) \times dz_j}_{\text{network effect}}$$

In matrix form:

$$\mathbf{d} \ln \mathbf{y} = \mathbf{H} \mathbf{dz} \equiv \underbrace{(\mathbf{I} - \mathbf{A})^{-1}}_{\text{Leontief inverse}} \mathbf{dz}$$

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*Implication 1:*

No upstream effects, and only downstream effects

(i.e., no effect on suppliers, only on customers of affected industries).



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*Implication 2:*

Own effect and network effect have the same level of impact.

# Demand-side Shocks

## Proposition (Part B)

The full impact of demand-side ( $\mathbf{dG}$ ) shocks is

$$d \ln y_i = \underbrace{\frac{dG_i}{y_i}}_{\text{own effect}} + \underbrace{\sum_{j=1}^n a_{ji} \frac{dp_j y_j}{p_i y_i}}_{\text{network effect}}.$$

In equilibrium: 
$$\mathbf{d} \ln \mathbf{y} = \left( \mathbf{I} - \hat{\mathbf{A}}^T \right)^{-1} \mathbf{D} \left( \mathbf{I} - \frac{\mathbf{1}'}{n} \right) \mathbf{d}\tilde{\mathbf{G}}$$

where  $\mathbf{d}\tilde{\mathbf{G}}$  is the vector of  $p_i G_i$ ,  $\hat{a}_{ij} = \frac{x_{ij}}{y_j}$ , and  $\mathbf{D} = \text{diag}\left(\frac{1}{p_i y_i}\right)$ .

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*Implication 3:*

No downstream effects, and only upstream effects

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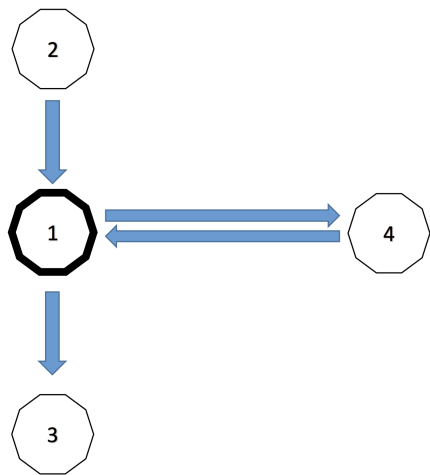
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*Remark:*

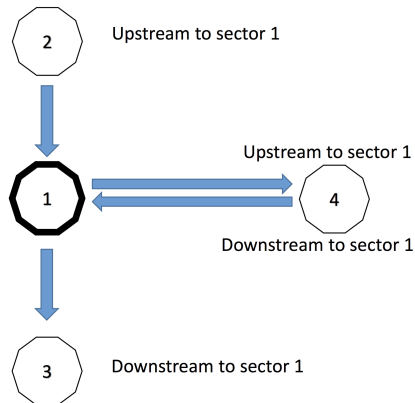
$\mathbf{d}\tilde{\mathbf{G}}$  has an additional impact on ALL sectors due to increased taxes. This effect is different than what we define as “downstream effect”.

# Intuition for Supply-side Shocks

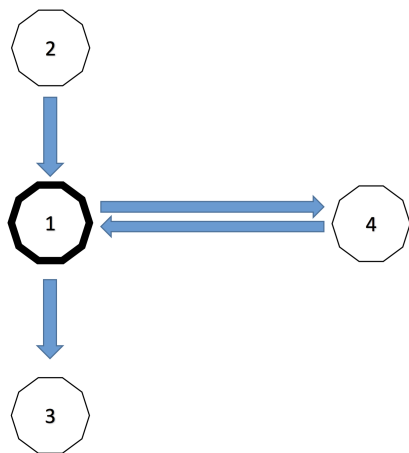


Focal sector 1 is connected to sectors 2, 3 and 4.

# Intuition for Supply-side Shocks

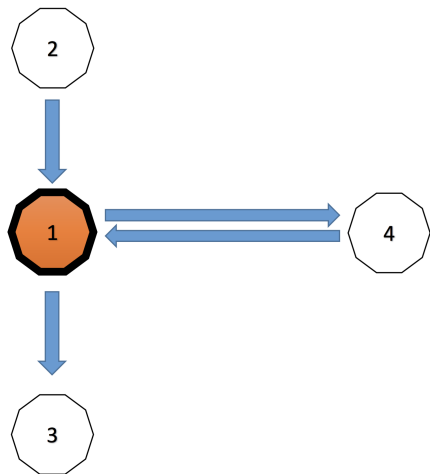


# Intuition for Supply-side Shocks



Sector 1 gets hit by a negative productivity shock  $dz_1 < 0$ .

# Intuition for Supply-side Shocks

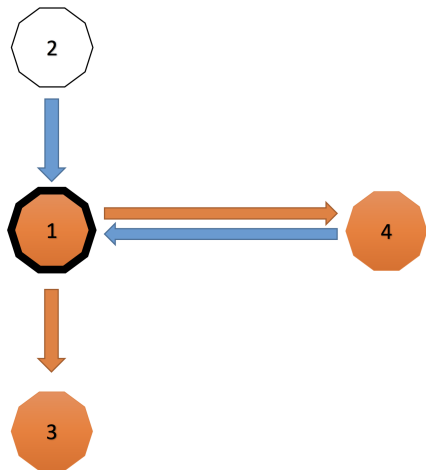


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$$dz_1 < 0 \implies p_1 \uparrow$$



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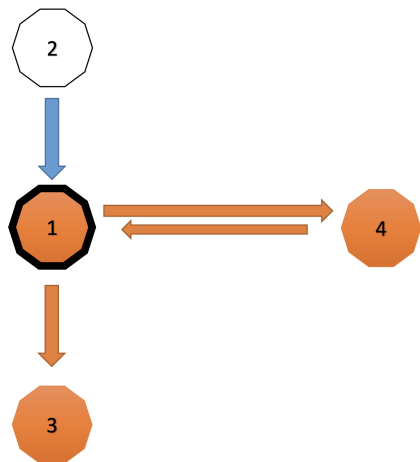


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Downstream effect:  
customers are adversely affected,  
 $p_1 \uparrow \implies x_{31}, x_{41} \downarrow$  &  $p_3, p_4 \uparrow$

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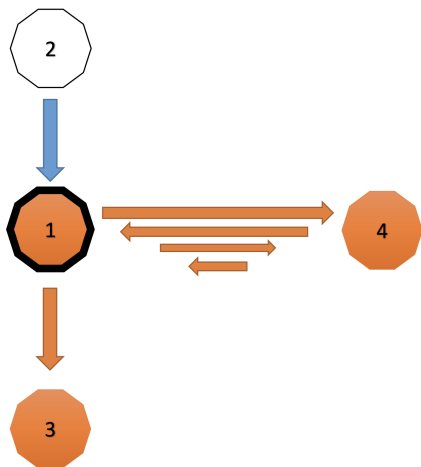
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$\implies$  second round effect:  
 $p_4 \uparrow \implies y_1 \downarrow$  &  $p_1 \uparrow$

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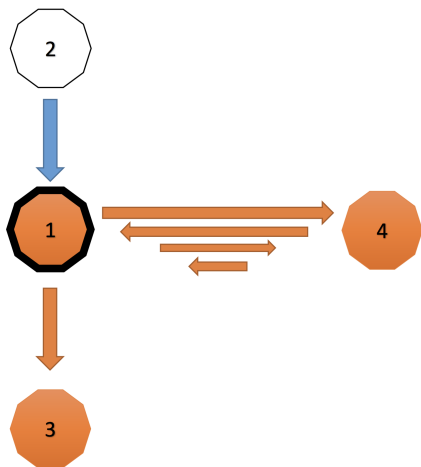
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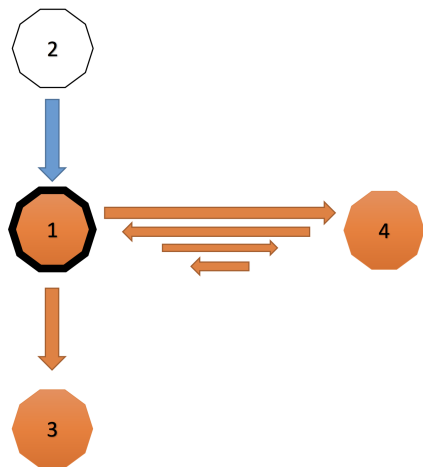
$\implies$  so on...

# Intuition for Supply-side Shocks



Why is there no upstream effect?

# Intuition for Supply-side Shocks

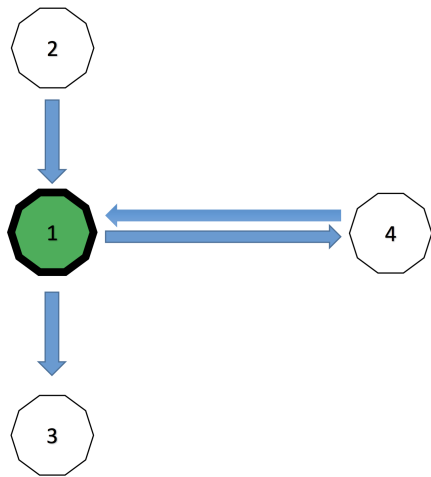


Why is there no upstream effect?

Because the price and quantity effects cancel out!

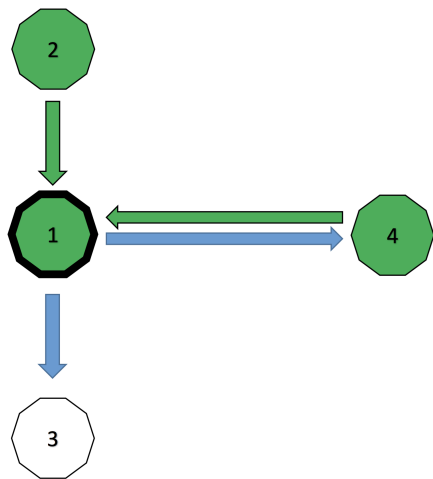
$$p_2 x_{12} = a_{12} p_1 y_1$$

# Intuition for Demand-side Shocks



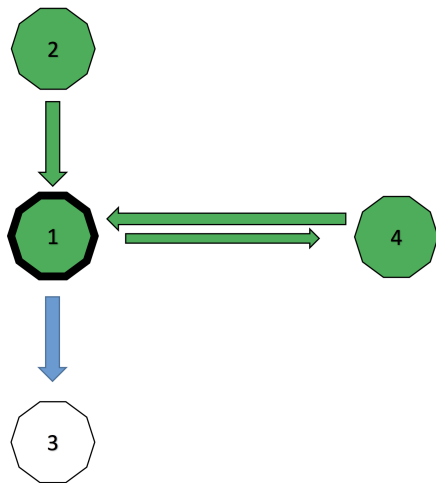
Sector 1 gets hit by a negative demand shock  $dG_1 < 0$ .

# Intuition for Demand-side Shocks



$$dG_1 < 0 \implies y_1 \downarrow$$

# Intuition for Demand-side Shocks



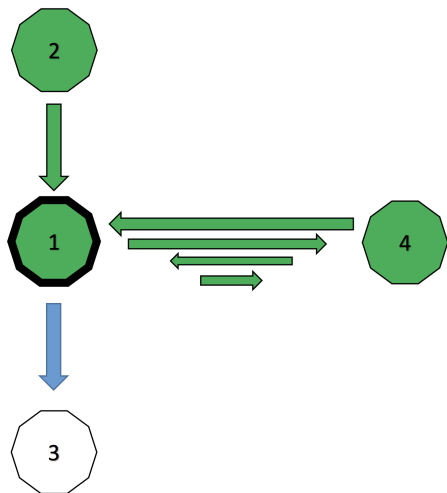
$$dG_1 < 0 \implies y_1 \downarrow$$

Upstream effect:  
suppliers are adversely affected,

$$y_1 \downarrow \implies x_{12}, x_{14} \downarrow$$



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$$dG_1 < 0 \implies y_1 \downarrow$$

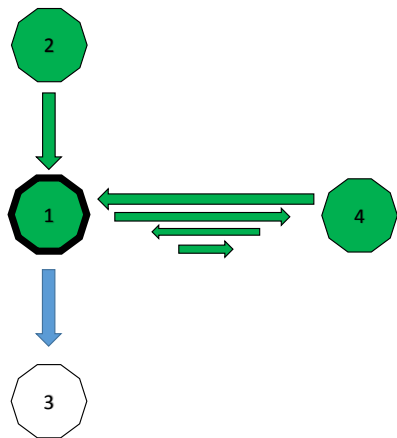
Upstream effect:  
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$\implies$  second round effect:

$$x_{14} \downarrow \implies x_{41} \downarrow$$

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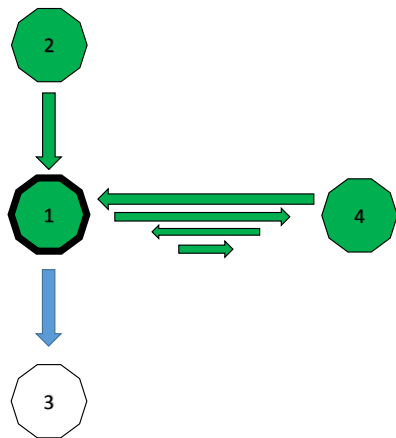
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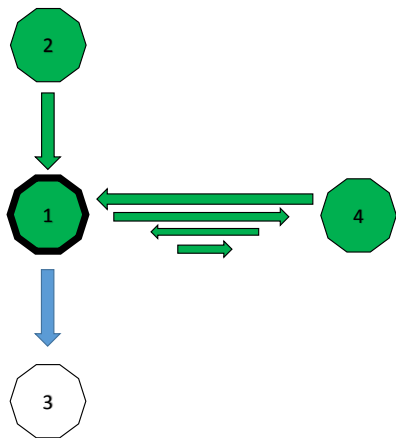
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# Intuition for Demand-side Shocks



Why is there no downstream effect?

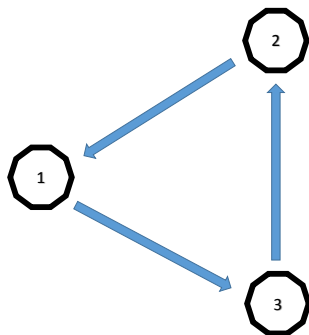
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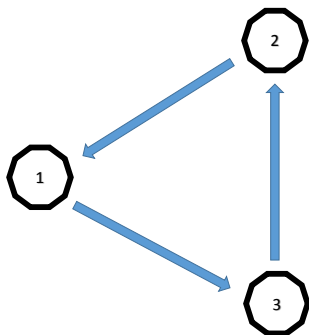
Why is there no downstream effect?

Because the relative prices remain unaffected!

# Ex: Downward propagation of productivity shocks

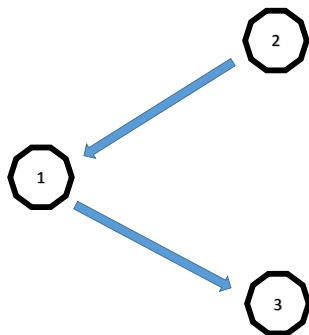


# Ex: Downward propagation of productivity shocks

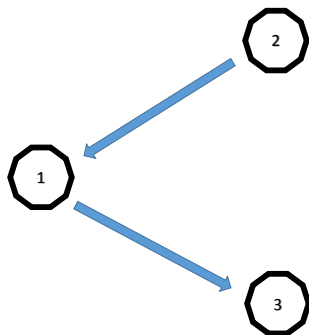


$$d \ln y_1 = \frac{dz_1 + a_{12}dz_2 + a_{12}a_{23}dz_3}{1 - a_{12}a_{23}a_{31}} + \text{cons.}$$

# Ex: Downward propagation of productivity shocks



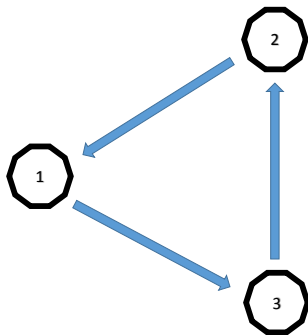
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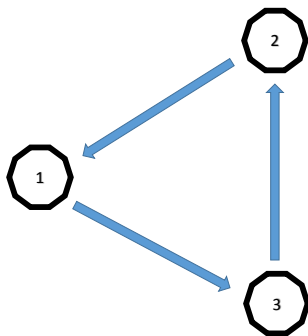
$$d \ln y_1 = dz_1 + a_{12} dz_2 + \text{cons.}$$



# Ex: Upstream propagation of demand-side shocks

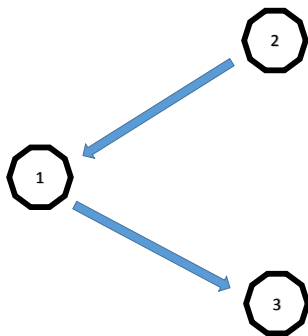


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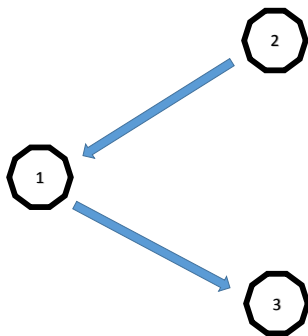


$$d\tilde{y}_1 = \frac{1}{1 - a_{12}a_{23}a_{31}} \left\{ \begin{array}{l} d\tilde{G}_1 + a_{31}a_{23}d\tilde{G}_2 + a_{31}d\tilde{G}_3 \\ -\frac{(1+a_{31}+a_{31}a_{23})}{3} [d\tilde{G}_1 + d\tilde{G}_2 + d\tilde{G}_3] \end{array} \right\}$$

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# Data

- Industry-level data: NBER-CES Manufacturing Industry Database  
⇒ 1991-2009.
- In the first four years: 392 four-digit industries; thereafter, 384 industries for 6560 total observations.
- Industry linkages: Bureau of Economic Analysis' 1992 Input-Output Matrix.
- We compute the Leontief inverse as in theory.

# Identification Challenges

- The difficulty of identifying are well recognized since Deaton (1990) and Manski (1993).
- Throughout, “identification” refers not to lack of identification of the regression coefficient (of say  $x_i$  or of  $y_i$  on the  $x$  choices of some neighbors/connected units), but to lack of information from an estimation approach on the “structural” or “causal” parameters.

## Challenge I: Mechanical Biases

- A regression of the form

$$y_i = b_{\text{own}}x_i + b_{\text{spillover}}\bar{x}_i + \text{controls} + u_i^x, \quad (1)$$

where  $\bar{x}_i$  is the average of  $i$ 's neighbors, will not always lead to interpretable estimates of  $b_{\text{spillover}}$ .

- Acemoglu and Angrist (2002): in the case where neighborhoods are “partitioned”—i.e., neighbors are other units in the same area—even if there are no spillovers,  $b_{\text{spillover}}$  will be estimated to be positive *provided that* OLS estimates of own effects differ from IV estimates using group dummies (e.g., because of different local averages or because of measurement error).
- The problem is even worse when the regression takes the form

$$y_i = b_{\text{own}}x_i + b_{\text{spillover}}\bar{y}_i + \text{controls} + u_i^y \quad (2)$$

## Challenge II: Correlated Effects

- Unobserved errors are likely to be correlated between “neighbors” .
- In terms of (1) or (2),  $u_i^x$  and  $u_i^y$  are likely to be correlated across  $i$ .
- This is for two distinct but related reasons:
  - 1 Suppose friendships are exogenously given. Two friends are still likely to be influenced by similar taste shocks, information and influences (because they are spending time together or because roommates are affected by the same disturbances, a problem even for papers such as Sacerdote (*QJE*, 2001) attempting to estimate endogenous effects based on random assignment).
  - 2 Suppose friendships are endogenously given. Then people choosing to be friends are likely to share similar observed and unobserved characteristics.
- Empirical approaches outlined below will attempt to deal with both mechanical biases and correlated effects.



# Empirical Approaches

- Two approaches:
  - ① Exploit network structure.
  - ② “Network instruments”.
- Both approaches assume that the network structure is known and measured without error.

# Exploiting Network Structure

- The most well-known example of exploiting network structure is the creative paper by Bramouille, Djebbari, and Fortin (*Journal of Econometrics*, 2009).
- Consider three agents  $i, j$  and  $k$ , and let us use the notation  $k \in N(j)$  to denote that  $k$  is linked to (is a neighbor of)  $j$ .
- Suppose that  $k \in N(j)$ ,  $j \in N(i)$ , and  $k \notin N(i)$ —i.e.,  $k$  is  $j$ 's friend/neighbor and  $j$  is  $i$ 's friend/neighbor, but  $k$  is not links to  $i$ .
- Then, in terms of estimating (1) for of the impact of  $x_j$  on  $y_i$ , we can use covariates of  $k$ ,  $\mathbf{z}_k$ , as instruments.

## Exploiting Network Structure (continued)

- But this identification strategy works only if error terms (1) and (2) are orthogonal across non-neighbor agents.
  - Bramoulle et al. show how one might deal with some instances of a *priori known* correlated effects.
- If  $k$  and  $i$  have correlated error terms that are also correlated with their characteristics (their  $x$ 's), then  $k$ 's covariates cannot be an instrument for estimating  $j$ 's endogenous effect on  $i$ .
- But such correlation is likely to be endemic:
  - Geographic or social proximity between  $k$  and  $i$  likely to be high because they share friends.
  - Unlikely that  $k$  and  $j$  are correlated,  $j$  and  $i$  are correlated, but  $k$  and  $i$  are uncorrelated.
- Additional problem: if the network is measured with error, then neighbors  $k$  and  $i$  may appear not to be neighbors, creating a violation of the exclusion restrictions.

# Network Instruments

- Suppose that there is a variable—unrelated to the network— $c_i$  orthogonal to  $u_i^x$  and  $u_i^y$  that can be used as an instrument for  $x_i$  absent any externalities, peer effects or network interactions.
- Then this is a candidate to be a variable that is orthogonal to  $u_k^x$  and  $u_k^y$  for all  $k \neq i$ .
- In other words, if we have

$$\text{cov}(c_i, u_i^x) = \text{cov}(c_i, u_i^y) = 0,$$

then it is also plausible that (for any integer  $p$ )

$$\text{cov}(\mathbf{G}'_i \mathbf{c}, u_i^x) = \text{cov}((\mathbf{G}_i^p)' \mathbf{c}, u_i^x) = \text{cov}(\mathbf{G}'_i \mathbf{c}, u_i^y) = \text{cov}((\mathbf{G}_i^p)' \mathbf{c}, u_i^y) = 0.$$

## Network Instruments (continued)

- But  $c_i$  should ideally satisfy an additional condition: *lack of correlation over the network*, i.e.,

$$\text{cov}(\mathbf{c}, (\mathbf{G}_i^p)'\mathbf{c}) \approx 0.$$

- Why?
- Because, otherwise, the correlated unobserved effects  $u_i^x$  and  $u_i^y$  could project onto  $\mathbf{c}$ .

# Estimating the Network

- Does it matter if the network is not known?
- Yes and no.
- If there is no information on the network, then instead of a single parameter  $\phi$  or a well-defined local average of  $\gamma_i$ 's, we would need to estimate  $n(n - 1)$  parameters, which is not feasible.
- But if the network is known up to some parameter  $\delta$ , that parameter (or parameter vector) can also be consistently estimated.

# Empirical Approach

- We focus on four shocks:
  - ① import shocks from China;
  - ② federal spending changes;
  - ③ TFP growth;
  - ④ foreign patenting growth.
  
- Our main regression equation:

$$\begin{aligned}\Delta \ln Y_{i,t} &= \psi \Delta \ln Y_{i,t-1} \\ &\quad + \beta^{\text{own}} \textit{Shock}_{i,t-1} \\ &\quad + \beta^{\text{upstream}} \textit{Upstream}_{i,t-1} \\ &\quad + \beta^{\text{downstream}} \textit{Downstream}_{i,t-1} \\ &\quad + \eta_t + \varepsilon_{i,t}.\end{aligned}$$

## Demand-side Shock I

## China Trade Shocks

	<u><math>\Delta</math> Log real value added</u>		<u><math>\Delta</math> Log employment</u>	
$\Delta$ Dependent variable t-1	0.019 (0.025)	0.020 (0.025)	0.149*** (0.020)	0.132*** (0.019)
$\Delta$ Dependent variable t-2		0.047** (0.024)		0.109*** (0.020)
$\Delta$ Dependent variable t-3		0.033 (0.021)		0.089*** (0.016)
Downstream effects t-1	-0.140 (0.086)	-0.124 (0.081)	-0.056 (0.040)	-0.044 (0.037)
Upstream effects t-1	0.076*** (0.024)	0.076*** (0.023)	0.049*** (0.016)	0.039*** (0.015)
Own effects t-1	0.034*** (0.009)	0.031*** (0.009)	0.023*** (0.005)	0.018*** (0.004)
Observations	6560	5776	6560	5776
p-value: Upstream=Own	0.078	0.058	0.108	0.161



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	<u><math>\Delta</math> Log real value added</u>		<u><math>\Delta</math> Log employment</u>	
$\Delta$ Dependent variable t-1	0.019 (0.025)	0.020 (0.025)	0.149*** (0.020)	0.132*** (0.019)
$\Delta$ Dependent variable t-2		0.047** (0.024)		0.109*** (0.020)
$\Delta$ Dependent variable t-3		0.033 (0.021)		0.089*** (0.016)
Downstream effects t-1	-0.140 (0.086)	-0.124 (0.081)	-0.056 (0.040)	-0.044 (0.037)
Upstream effects t-1	0.076*** (0.024)	0.076*** (0.023)	0.049*** (0.016)	0.039*** (0.015)
Own effects t-1	0.034*** (0.009)	0.031*** (0.009)	0.023*** (0.005)	0.018*** (0.004)
Observations	6560	5776	6560	5776
p-value: Upstream=Own	0.078	0.058	0.108	0.161

## Demand-side Shock I

## China Trade Shocks

	<u><math>\Delta</math> Log real value added</u>		<u><math>\Delta</math> Log employment</u>	
$\Delta$ Dependent variable t-1	0.019 (0.025)	0.020 (0.025)	0.149*** (0.020)	0.132*** (0.019)
$\Delta$ Dependent variable t-2		0.047** (0.024)		0.109*** (0.020)
$\Delta$ Dependent variable t-3		0.033 (0.021)		0.089*** (0.016)
Downstream effects t-1	-0.140 (0.086)	-0.124 (0.081)	-0.056 (0.040)	-0.044 (0.037)
Upstream effects t-1	0.076*** (0.024)	0.076*** (0.023)	0.049*** (0.016)	0.039*** (0.015)
Own effects t-1	0.034*** (0.009)	0.031*** (0.009)	0.023*** (0.005)	0.018*** (0.004)
Observations	6560	5776	6560	5776
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$\Delta$ Dependent variable t-2		0.047** (0.024)		0.109*** (0.020)
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## Demand-side Shock II

## Federal Spending Shocks

	<i>Δ Log real value added</i>		<i>Δ Log employment</i>	
Δ Dependent variable t-1	0.019 (0.025)	0.018 (0.024)	0.158*** (0.021)	0.135*** (0.019)
Δ Dependent variable t-2		0.051** (0.023)		0.116*** (0.019)
Δ Dependent variable t-3		0.038* (0.021)		0.102*** (0.016)
Downstream effects t-1	0.017 (0.021)	0.023 (0.021)	0.007 (0.015)	0.013 (0.012)
Upstream effects t-1	0.022** (0.009)	0.020** (0.008)	0.010* (0.006)	0.011** (0.005)
Own effects t-1	0.004 (0.003)	0.008** (0.004)	0.003 (0.003)	0.006*** (0.002)
Observations	6560	5776	6560	5776
p-value: Upstream=Own	0.090	0.197	0.338	0.401

## Demand-side Shock II

## Federal Spending Shocks

	<i>Δ Log real value added</i>		<i>Δ Log employment</i>	
Δ Dependent variable t-1	0.019 (0.025)	0.018 (0.024)	0.158*** (0.021)	0.135*** (0.019)
Δ Dependent variable t-2		0.051** (0.023)		0.116*** (0.019)
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Own effects t-1	0.004 (0.003)	0.008** (0.004)	0.003 (0.003)	0.006*** (0.002)
Observations	6560	5776	6560	5776
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## Demand-side Shock II

## Federal Spending Shocks

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## Federal Spending Shocks

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p-value: Upstream=Own	0.090	0.197	0.338	0.401



# Supply-side Shock I

## TFP Shocks

	<u><math>\Delta</math> Log real value added</u>		<u><math>\Delta</math> Log employment</u>	
$\Delta$ Dependent variable t-1	-0.024 (0.040)	-0.031 (0.041)	0.141*** (0.021)	0.118*** (0.020)
$\Delta$ Dependent variable t-2		0.049** (0.023)		0.118*** (0.019)
$\Delta$ Dependent variable t-3		0.037* (0.020)		0.102*** (0.016)
Downstream effects t-1	0.060*** (0.020)	0.047** (0.020)	0.016* (0.009)	0.011 (0.009)
Upstream effects t-1	0.024** (0.011)	0.020* (0.012)	0.009 (0.006)	0.008 (0.006)
Own effects t-1	0.004 (0.007)	0.007 (0.006)	0.006*** (0.002)	0.007*** (0.002)
Observations	6560	5776	6560	5776
p-value: Downstream=Own	0.005	0.039	0.299	0.654

# Supply-side Shock I

## TFP Shocks

	<u><math>\Delta</math> Log real value added</u>		<u><math>\Delta</math> Log employment</u>	
$\Delta$ Dependent variable t-1	-0.024 (0.040)	-0.031 (0.041)	0.141*** (0.021)	0.118*** (0.020)
$\Delta$ Dependent variable t-2		0.049** (0.023)		0.118*** (0.019)
$\Delta$ Dependent variable t-3		0.037* (0.020)		0.102*** (0.016)
Downstream effects t-1	0.060*** (0.020)	0.047** (0.020)	0.016* (0.009)	0.011 (0.009)
Upstream effects t-1	0.024** (0.011)	0.020* (0.012)	0.009 (0.006)	0.008 (0.006)
Own effects t-1	0.004 (0.007)	0.007 (0.006)	0.006*** (0.002)	0.007*** (0.002)
Observations	6560	5776	6560	5776
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# Supply-side Shock I

## TFP Shocks

	<u><math>\Delta</math> Log real value added</u>		<u><math>\Delta</math> Log employment</u>	
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Own effects t-1	0.004 (0.007)	0.007 (0.006)	0.006*** (0.002)	0.007*** (0.002)
Observations	6560	5776	6560	5776
p-value: Downstream=Own	0.005	0.039	0.299	0.654

# Supply-side Shock II

## Foreign Patent Shocks

	<i><math>\Delta</math> Log real value added</i>		<i><math>\Delta</math> Log employment</i>	
$\Delta$ Dependent variable t-1	0.020 (0.025)	0.020 (0.025)	0.159*** (0.021)	0.138*** (0.020)
$\Delta$ Dependent variable t-2		0.051** (0.023)		0.117*** (0.020)
$\Delta$ Dependent variable t-3		0.037* (0.021)		0.100*** (0.016)
Downstream effects t-1	0.043*** (0.011)	0.044*** (0.011)	0.018*** (0.006)	0.018*** (0.006)
Upstream effects t-1	-0.000 (0.005)	0.000 (0.005)	-0.001 (0.003)	-0.000 (0.003)
Own effects t-1	-0.006 (0.004)	-0.007* (0.004)	-0.008*** (0.003)	-0.006** (0.003)
Observations	6543	5761	6543	5761
p-value: Downstream=Own	0.000	0.000	0.000	0.001

# Supply-side Shock II

## Foreign Patent Shocks

	<i><math>\Delta</math> Log real value added</i>		<i><math>\Delta</math> Log employment</i>	
$\Delta$ Dependent variable t-1	0.020 (0.025)	0.020 (0.025)	0.159*** (0.021)	0.138*** (0.020)
$\Delta$ Dependent variable t-2		0.051** (0.023)		0.117*** (0.020)
$\Delta$ Dependent variable t-3		0.037* (0.021)		0.100*** (0.016)
Downstream effects t-1	0.043*** (0.011)	0.044*** (0.011)	0.018*** (0.006)	0.018*** (0.006)
Upstream effects t-1	-0.000 (0.005)	0.000 (0.005)	-0.001 (0.003)	-0.000 (0.003)
Own effects t-1	-0.006 (0.004)	-0.007* (0.004)	-0.008*** (0.003)	-0.006** (0.003)
Observations	6543	5761	6543	5761
p-value: Downstream=Own	0.000	0.000	0.000	0.001

## Supply-side Shock II

## Foreign Patent Shocks

	<i><math>\Delta</math> Log real value added</i>		<i><math>\Delta</math> Log employment</i>	
$\Delta$ Dependent variable t-1	0.020 (0.025)	0.020 (0.025)	0.159*** (0.021)	0.138*** (0.020)
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Observations	6543	5761	6543	5761
p-value: Downstream=Own	0.000	0.000	0.000	0.001

## Aggregate Implications

- Let us now return to the model introduced above, and first arrive its aggregate implications.
- The most important observation is that log GDP or real value added is given as a *convex combination* of sectoral shocks:

$$y \equiv \log(GDP) = \mathbf{v}'\mathbf{z},$$

where  $\mathbf{z} \equiv [z_1, \dots, z_n]'$  is the vector of sectoral shocks, and  $\mathbf{v}$  the **influence vector** or the vector of **Domar weights** or the vector of **Bonacich centrality** indices defined as

$$v_i \equiv \sum_{j=1}^n \beta_j h_{ji},$$

where  $h_{ij}$  denotes entries of the Leontief inverse.

# Aggregate Implications

- As originally observed by Hulten (*Review of Economic Studies*, 1978) and Gabaix (*Econometrica*, 2011),  $\mathbf{v}$  is also the “sales vector” of the economy, with its elements given by

$$v_i = \frac{p_i x_i}{\sum_{j=1}^n p_j x_j}.$$

- In fact representation with Domar weights is true with any constant returns to scale production function (but not the representation in terms of Leontief inverses).



# Aggregate Volatility

- Let us go back to the general framework presented above and consider **aggregate volatility**—meaning the volatility of log GDP—measured as.

$$\sigma_{agg} \equiv \sqrt{\text{var } y}.$$

- Recall that

$$y \equiv \log(\text{GDP}) = \mathbf{v}'\mathbf{z},$$

- Hence, assuming that sectoral shocks are independent (thus approximating idiosyncratic shocks):

$$\sigma_{agg} = \sqrt{\sum_{i=1}^n \sigma_i^2 v_i^2}.$$

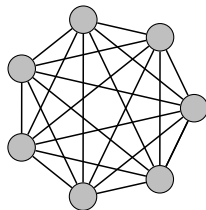
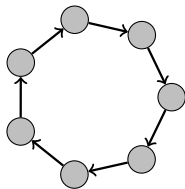
- From this expression, the “conventional wisdom”—e.g., as articulated by Lucas (*Theories of Business Cycles*, 1984)—can be understood:
  - suppose  $v_i \approx \frac{1}{n}$  and  $n$  is large (the economy is “well diversified”), then  $\sigma_{agg}$  is trivial—*no aggregate fluctuations without aggregate shocks*.

## Some Theoretical Results

- We first start with some simple theoretical observations questioning the above “diversification argument” and then link the structure of the input-output network to aggregate volatility.
- We next turn to a structural empirical strategy to shed more light on the relationship between aggregate volatility and sectoral shocks.
- Finally, we provide sharper results by studying “large” (highly diversified) economies—i.e., those with  $n$  large.

# Macroeconomic Irrelevance of Micro Shocks

- We say that the network is **regular** if  $d_i = d$  for each  $i$ , where  $d_i = \sum_{j=1}^n a_{ji}$ .
  - That is, each sector has a similar degree of importance as a supplier to other sectors.
- Examples of regular networks:
  - **rings:** the most “*sparse*” input-output matrix, where each sector grows all of its inputs from a single other sector.
  - **complete graphs:** where each sector equally draws inputs from all other sectors.



## Irrelevance of Micro Shocks (continued)

- Suppose also that

$$\sigma_i = \sigma \text{ for each } i.$$

- Then we have that for all regular networks:

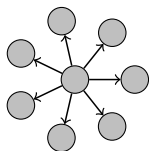
$$\sigma_{agg} = \frac{\sigma}{\sqrt{n}}$$

(see also Dupor, *Journal of Monetary Economics*, 1999).

- Intuition: with the (log) linearity implied by the Cobb-Douglas technologies, shocks average out exactly *provided that all sectors have the same degree*.
- This result is particularly interesting because rings are often conjecture to be unstable or prone to “domino effects” (or other types of contagion).

# Asymmetric Networks Are Fragile

- However, this irrelevance is not generally correct.
- In particular, Lucas's argument is incorrect when  $v_i$ 's are far from  $1/n$ , which happens when the network is highly asymmetric—in terms of degrees.
- The extreme example is the **star network**, with degrees summing to  $1 - \alpha$ :



## Asymmetric Networks Are Fragile (continued)

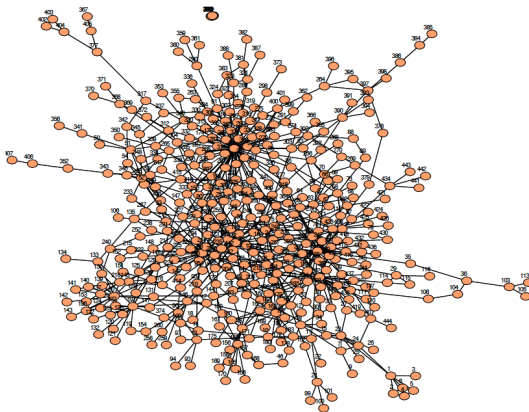
- In fact, it can be shown that the highest level of aggregate volatility is generated by the **star network** and is equal to

$$\sigma_{agg} = \frac{\sigma}{\sqrt{1 - \left(\frac{n-1}{n}\right) \alpha (1 - \alpha)}},$$

which is much greater than  $\sigma/\sqrt{n}$  when  $n$  is large.

- In fact, this is not just high volatility, but **systemic volatility** ( $\approx$  “system-wide” volatility: shocks to the central sector spread to the rest, creating system-wide co-movement—we return to systemic volatility below).
- Intuition: the shock to the central sector of the star does not “wash out”.
- More general result: **unequal degrees**—or asymmetric networks—create additional volatility.

# What Does the US Input-Output Network Look Like?



- Intersectoral network corresponding to the US input-output matrix in 1997. For every input transaction above 5% of the total input purchases of the destination sector, a link between two vertices is drawn.

## Asymptotic Results

- To obtain sharper theoretical results, consider a sequence of economies with input-output matrix  $A_n$  and  $n \rightarrow \infty$ .
- So we will be looking at “*law of large numbers*”-type results.
- Suppose that  $\sigma_i \in (\underline{\sigma}, \bar{\sigma})$ .
- Then the greatest degree of “*stability*” or “*robustness*” (least systemic risk) corresponds to

$$\sigma_{agg} \sim 1/\sqrt{n}$$

(as in standard law of large numbers for independent variables).

- Define the **coefficient of variation of degrees** (of an economy with  $n$  sectors) as

$$CV_n \equiv \frac{1}{d_{avg}} \left[ \frac{1}{n-1} \sum_{i=1}^n (d_i - d_{avg})^2 \right]^{1/2},$$

where  $d_{avg} = \frac{1}{n} \sum_i d_i$  is the average degree.



# First-Order Results

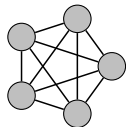
- Just considering the first-order downstream impacts,

$$\sigma_{agg} = \Omega \left( \frac{1}{\sqrt{n}} + \frac{CV_n}{\sqrt{n}} \right).$$

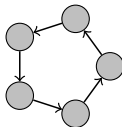
The  $\Omega$  notation implies  $\sigma_{agg} \rightarrow 0$  as  $n \rightarrow 0$  no faster than  $\frac{1+CV_n}{\sqrt{n}}$ .

- For regular networks,  $CV_n = 0$ , so  $\sigma_{agg} \rightarrow 0$  could (should) go to zero at the rate  $\frac{1}{\sqrt{n}}$ .
- For the star network,  $CV_n \not\rightarrow 0$  as  $n \rightarrow 0$ , so  $\sigma_{agg} \not\rightarrow 0$  and the law of large numbers fails.

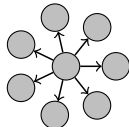
$$c_n = \Omega(b_n) \iff \liminf_{n \rightarrow \infty} c_n / b_n > 0$$



$$CV_n = 0$$



$$CV_n = 0$$



$$CV_n \sim \sqrt{n}$$

## First-Order Results (continued)

- We can also make these results easier to apply.
- We say that the degree distribution for a sequence of economies has **power law tail** if, there exists  $\beta > 1$  such that for each  $n$  and for large  $k$ ,

$$P_n(k) \propto k^{-\beta},$$

where  $P_n(k)$  is the counter-cumulative distribution of degrees and  $\beta$  is the shape parameter.

- It can be shown that if a sequence of economies has power law tail with shape parameter  $\beta \in (1, 2)$ , then

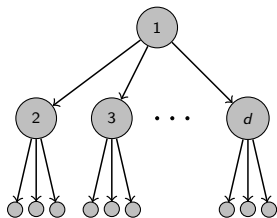
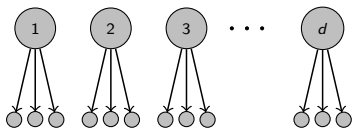
$$\sigma_{agg} = \Omega \left( n^{-\frac{\beta-1}{\beta}-\varepsilon} \right)$$

where  $\varepsilon > 0$  is arbitrary.

- A smaller  $\beta$  corresponds to a “thicker” tail and thus higher coefficient of variation, and greater fragility.

## Higher-Order Results

- In the same way that first-order downstream effects do not capture the full implications of negative shocks to a sector, the degree distribution does not capture the full extent of asymmetry/inequality of “connections”.
- Two economies with the same degree distribution can have very different structures of connections and very different nature of volatility:

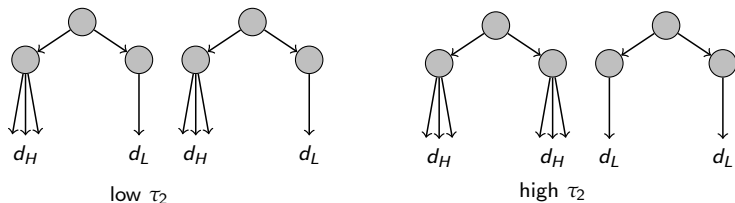


## Higher-Order Results (continued)

- We define the **second-order interconnectivity coefficient** as

$$\tau_2(A_n) \equiv \sum_{i=1}^n \sum_{j \neq i} \sum_{k \neq i,j} a_{ji} a_{ki} d_j d_k.$$

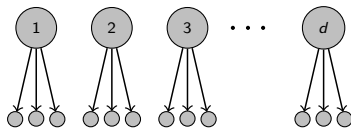
- This will be higher when high degree sectors share “upstream parents”:



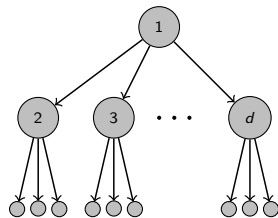
# Higher-Order Results (continued)

- It can be shown that

$$\sigma_{agg} = \Omega \left( \frac{1}{\sqrt{n}} + \frac{CV_n}{\sqrt{n}} + \frac{\sqrt{\tau_2(A_n)}}{n} \right).$$



$$\tau_2 = 0$$



$$\tau_2 \sim n^2$$

## Higher-Order Results (continued)

- Define **second-order degree** as

$$q_i \equiv \sum_{j=1}^n d_j a_{ji}.$$

- For a sequence of economies with a power law tail for the second-order degree with shape parameter  $\zeta \in (1, 2)$ , we have

$$\sigma_{agg} = \Omega \left( n^{-\frac{\zeta-1}{\zeta}-\varepsilon} \right),$$

for any  $\varepsilon > 0$ .

- If both first and second-order degrees have power laws, then

$$\sigma_{agg} = \Omega \left( n^{-\frac{\zeta-1}{\zeta}-\varepsilon} + n^{-\frac{\beta-1}{\beta}} \right),$$

i.e., dominant term:  $\min \{ \beta, \zeta \}$ .

# When Network Structure Does Not Matter

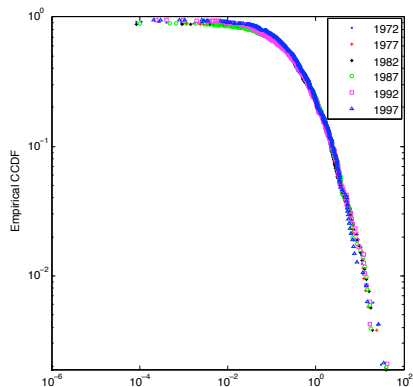
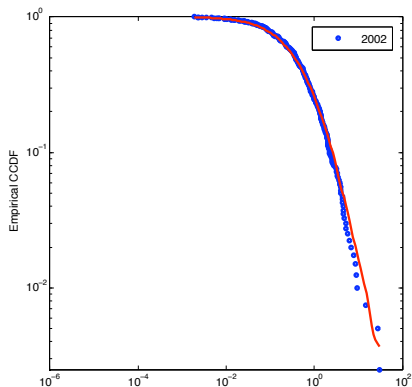
- We say that a sequence of economies is **balanced** if  $\max_i d_i < c$  for some  $c$ .
- This is clearly much weaker than regularity.
- It can be shown that, for any sequence of balanced economies,

$$\sigma_{agg} \sim \frac{1}{\sqrt{n}}.$$

- Once again rings and complete networks are equally stable (emphasizing that sparseness of the input-output matrix has little to do with aggregate volatility).

# Another Look at the US Input-Output Network

- Empirical counter-cumulative distribution of first-order and second-order degrees
- Linear tail in the log-log scale  $\rightarrow$  power law tail





## Higher-Order Results (continued)

- Average (across years) estimates:  $\hat{\beta} = 1.38$  ,  $\hat{\zeta} = 1.18$ .
- $\hat{\zeta} < \hat{\beta}$ : second-order effects dominate first-order effects.
- Average (annual) standard deviation of total factor productivity across 459 four-digit (SIC) manufacturing industries between 1958 and 2005 is 0.058.
- Since manufacturing is about 20% of the economy, for the entire economy this corresponds to  $5 \times 459 = 2295$  sectors at a comparable level of disaggregation.
- Had the structure been balanced:  $\sigma_{\text{agg}} = 0.058 / \sqrt{2295} \simeq 0.001$ .
- But from the lower bound from the second-order degree distribution:

$$\sigma_{\text{agg}} \sim \sigma / \sqrt{n} \approx 0.018.$$

# Financial Contagion

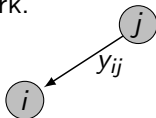
- An at-first surprising implication of the analysis so far is the result that aggregate volatility is the *same* in complete and ring networks.
- Is this a general result?
- *The answer is no*, and underscores that the implications of different network structures crucially depend on what types of interactions are taking place over the network.
- In particular, the linearity (log-linearity) is responsible for this result—positive and negative shocks cancel out when all units have similar “influence”.
- But linearity may be a good approximation for input-output that works, but not for finance—where, in the presence of debt-like contracts, **default** (and bankruptcy) creates a major nonlinearity.

## A Simple Model of Counterparty Relations

- Based on Acemoglu, Ozdaglar and Tahbaz-Salehi (mimeo, 2013). See also Allen and Gale (*JPE*, 2000) and Elliott, Golub and Jackson (mimeo, 2013) on a non-linear financial model due to cross-firm shareholdings and bankruptcy.
- Consider a network of banks (financial institutions) potentially borrowing and lending to each other (as well as from outside creditors and senior creditors).
- All borrowing and lending is through short-term, uncollateralized debt contracts.
- Suppose that all contracts are signed at date  $t = 0$ .
- Banks have long-term assets that will pay out at date  $t = 2$ , but are illiquid, and cannot be liquidated at date  $t = 1$ .
- Banks are hit by liquidity shocks at date  $t = 1$  and also receive and make payments on their interbank contracts.

## A Simple Model of Counterparty Relations (continued)

- More specifically, banks lend to one another at  $t = 0$  through **standard debt contracts** to be repaid at  $t = 1$ .
- Face values of debt of bank  $j$  to bank  $i$ :  $y_{ij}$ .
- $\{y_{ij}\}$  defines a financial network.



- Related problem: chains of trade credit—Kiyotaki and Moore (mimeo, 1997) for theory and Jacobson and von Schedvin (mimeo, 2013) for evidence.

## A Simple Model of Counterparty Relations (continued)

- Bank  $i$  invests in a project with returns at  $t = 1, 2$ .
- Random return of  $z_i$  at  $t = 1$ .
- Deterministic return of  $A$  at  $t = 2$  if the entire project is held to maturity.
- In addition, bank  $i$  has **senior** obligations in the amount  $v > 0$ .
- If the bank cannot meet its obligations, it will be in bankruptcy and has to liquidate its project with  $\zeta A$ .
- If it still has insufficient funds, the bank will have to **default** on its creditors, which will be paid on pro rata basis.
- Simplify the discussion here by assuming that  $\zeta \approx 0$ , so that liquidation of long-term assets is never sufficient to stave off default.

## Payment Equilibrium

- From the above description, we have that bank  $j$ 's actual payments are:

$$x_{ij} = \begin{cases} y_{ij} & \text{if } z_j + \sum_s x_{js} \geq v + \sum_s y_{sj} \\ \frac{y_{ij}}{\sum_s y_{sj}} (z_j - v + \sum_s x_{js}) & \text{if } v \leq z_j + \sum_s x_{js} < v + \sum_s y_{sj} \\ 0 & \text{if } z_j + \sum_s x_{js} < v. \end{cases}$$

- The first branch is when the bank is not in default.
- The second is when the bank is in default but senior creditors are not hurt.
- The third is when senior creditors are not paid in full (and the rest are not paid at all).

## Payment Equilibrium (continued)

- A **payment equilibrium** is a fixed point  $\{x_{ij}\}$  of the above set of equations (one for each bank  $j$ ).
- *A payment equilibrium exists and is generically unique.*
- This generalizes Eisenberg and Noe (*Mathematics of Operations Research*, 2001).

# Volatility in the Financial Network

- To discuss volatility in this financial network, let us focus on the case in which:
  - The financial network is **regular**, i.e.,  $\sum_s y_{sj} = y$  for all  $j$ . (We know from our analysis of input-output networks that asymmetries in this quantity create one source of systemic volatility, so we are abstracting from this).
  - $z_j = a$  or  $z_j = a - \varepsilon$ , so that banks are potentially hit by a negative liquidity shock at time  $t = 1$ .
  - Suppose also that only one bank in the network is hit by the negative liquidity shock,  $-\varepsilon$ .
  - Throughout, focus on the network of size  $n$  (i.e., no asymptotic results).



## Volatility in the Financial Network (continued)

- How to quantify volatility?
- The following observation gives us a simple way:

$$\text{Social surplus} = na - \varepsilon + (\text{number of defaults}) - A.$$

- Thus social surplus clearly related to how systemic the shock that hits one bank becomes, suggesting a natural measure of volatility and **stability** in this financial network.
- We say that a network is **less stable** than another if it has greater number of expected defaults.

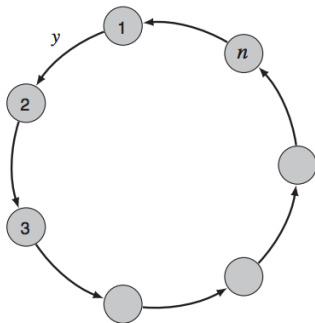
## Small Shock vs. Large Shock Regimes

- It will turn out that the size of the negative shock (or more generally the size and the number of shocks) will matter greatly for what types of networks are stable.
- For this, let us call a regime in which  $\varepsilon < \varepsilon^*$  the **small shock regime**, and the regime in which  $\varepsilon > \varepsilon^*$  the **large shock regime**.

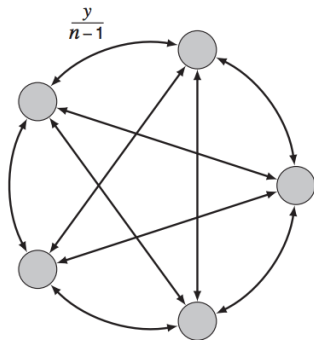
## Stability in the Small Shock Regime

- Suppose that  $\varepsilon < \varepsilon^*$  and  $y > y^*$  (so that the liabilities of banks are not too small). Then:
  - *The complete financial network is the most stable network.*
  - *The ring financial network is the least stable network.*

Panel A. The ring financial network



Panel B. The complete financial network

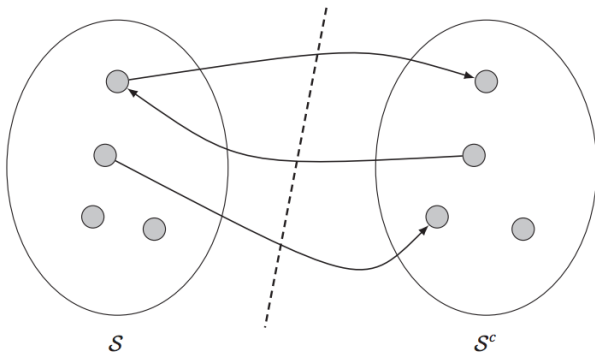


## Stability in the Small Shock Regime (continued)

- In addition, it can be shown that if we take a  $\gamma$  convex combination of the complete and the ring networks (so that  $y_{ij} = (1 - \gamma)y_{ij}^{\text{ring}} + \gamma y_{ij}^{\text{complete}}$ ), then *as  $\gamma$  increases, the network becomes more stable.*
- Intuition: more links out from a bank implies that liabilities of that bank are held in a more *diversified* manner, and losses of that bank can be better absorbed by the financial system.
- The ring is the least diversified network structure, leading to the greatest amount of systemic volatility/instability.
- In the linear/log-linear case, positive shocks and negative shocks in different parts of the regular network canceled out. This no longer happens because of **default**.
- Rather, default creates **domino effects**.
- If a bank is negatively hit, then it is unable to make payments on its debt, and this puts its creditors (that are highly exposed to it) in potential default, and so on.

## Stability in the Large Shock Regime

- The picture is sharply different in the large shock regime.
- We say that a financial network  $\delta$ -**connected** if there exists a subset  $M$  of banks such that the linkages between this subset and its complement is never greater than  $\delta$ —i.e.,  $y_{ij} \leq \delta$  for any two banks from this subset and its complement.



## Stability in the Large Shock Regime (continued)

- Suppose that  $\varepsilon > \varepsilon^*$  and  $y > y^*$ . Then:
- *The complete and the ring financial networks are the least stable networks.*
- *For  $\delta$  sufficiently small, a  $\delta$ -connected network is more stable than the complete and the ring networks.*

## Stability in the Large Shock Regime (continued)

- This is a type of **phase transition**—meaning that the network properties and comparative statics change sharply at a threshold value.
- *Network Intuition:* When shocks are large, they cannot be contained even with full diversification and spread through the network like an “epidemic”. In that case, insulating parts of the network from others increases stability.
- *Economic Intuition:* weakly connected networks make better use of the liquidity of senior creditors.
- The complete network uses the excess liquidity of non-distressed banks,  $a - v > 0$ , very effectively, but does not use the resources of senior creditors at all. Weakly connected networks do not utilize the liquidity of non-distressed banks much, but do make good use of the resources of senior creditors when needed.

## Further issues in financial networks

- Endogenous networks.
- The response of financial networks to policy (Erol, 2016).
- Network of overlapping assets and fire sales.
- Optimal intervention.
- Overlapping markets (e.g., counterparty risk in interbank relations, overlapping loan networks entry point directions).
- Empirical applications.



# Innovation Networks

- In addition to input-output and financial pathways, shocks the one part of the economy propagate to the rest because of the **innovation network**.
- Ideas in one part of the economy (in one sector, process or technology class) become the basis of innovation or technological improvement in some other part of the economy—“building on the shoulders of giants”.
- Suppose, for example, that we represent innovation relations as a network between  $n$  “technology classes”  $\mathbf{G}$  (again with  $\mathbf{G}_i$  denoting the  $i$ th row of this matrix).
- In the data,  $\mathbf{G}$  corresponds to the matrix given by citation patterns.

## Innovation Networks (continued)

- Then let us posit the following relationship:

$$x_{i,t} = \alpha_i x_{i,t-1} + \phi \mathbf{G}'_i \mathbf{x}_{t-1} + \varepsilon_i,$$

where  $x_{i,t}$  is the innovation rate in technology class  $i$  at time  $t$  and  $\mathbf{x}_t$  denotes the vector of  $x_{i,t}$ 's.

- This implies that successful innovations in sectors that  $i$  cites translate into higher innovations in the future by sector  $i$ .
- In practice, important to estimate  $\mathbf{G}$  from past data (to avoid mechanical biases).

# The US Innovation Network

- Acemoglu, Akcigit and Kerr (mimeo, 2014) perform this task using US citation data for the baseline period, 1975-1984.
- First construct the matrix  $\mathbf{G}$  as

$$g_{jj'} = \sum_{k \neq j} \frac{\text{Citations}_{j \rightarrow j'}^{1975-1984}}{\text{Citations}_{j \rightarrow k}^{1975-1984}}$$

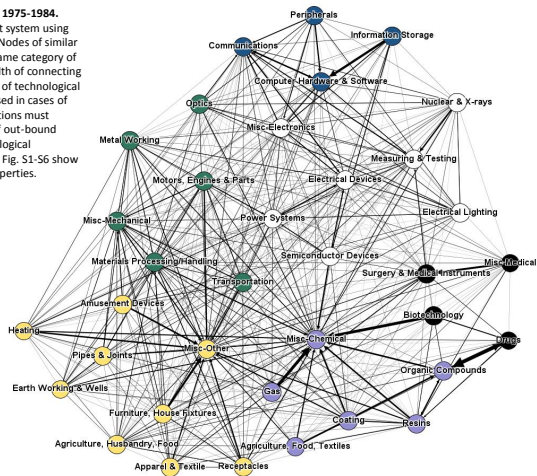
where  $\text{Citations}_{j \rightarrow k}^{1975-1984}$  is the citation during this period from technology class  $j$  to  $k$ —thus ideas flowing from  $k$  to  $j$ .

- the denominator leaves out “self-cites”—cites from  $j$  to  $j$ .

# The US Innovation Network at the Two-Digit Level

**Fig. 2: Innovation network 1975-1984.**

Network mapping of patent system using technology subcategories. Nodes of similar color are pulled from the same category of the USPTO system. The width of connecting lines indicates the strength of technological flows, with arrows being used in cases of strong asymmetry. Connections must account for at least 0.5% of out-bound citations made by a technological subcategory. Supplemental Fig. S1-S6 show variations and network properties.



## Predicting Innovation

- To predict innovation using the innovation network, it is also useful to take account of the citation lags (thus corresponding to a separate  $\mathbf{G}$  matrix for each citation time gap). For this purpose, construct

$$FlowRate_{j \rightarrow j', a}^{1975-1984} = Flow_{j \rightarrow j', a}^{1975-1984} / Patent_{j'}^{1975-1984},$$

where  $Flow_{j \rightarrow j', a}^{1975-1984}$  is the total number of cites from technology class  $j'$  to  $j$  that takes place  $a$  years after the patent from  $j$  is issued, and  $Patent_{j'}^{1975-1984}$  is the number of patents in cited field  $j'$ .

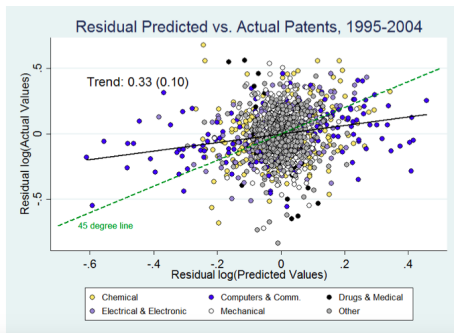
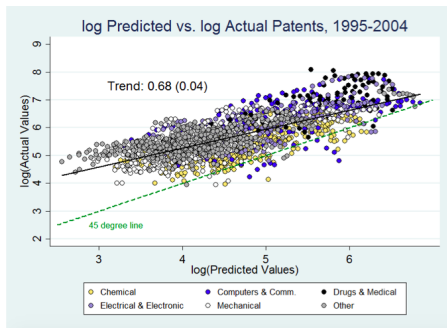
- Compute expected patents in sector  $j$  at the three-digit technology class level (corresponding to 484 classes):

$$ExpectPatents_{j,t}^{1995-2004} = \sum_{j' \neq j} \sum_{a=1,10} FlowRate_{j \rightarrow j', a}^{1975-1984} Patents_{j', t=t_0+a}^{1985-1994}.$$

This only takes into account a 10-year citation window and sums over all sectors citing  $j$ , using  $FlowRate_{j \rightarrow j', a}^{1975-1984}$  as weights (and  $j \rightarrow j$  excluded).

# Predicting Innovation (continued)

- The relationship between expected patents and actual patents (second panel taking out technology class and year fixed effects).



## Interpretation and Current Work

- This descriptive exercise provides fairly strong (albeit reduced-form) evidence that ideas and innovations spread through the citation/innovation network.
- This supports the view that innovation is a cumulative process building on innovation in other fields.
- This evidence would also plausibly suggest that medium-term propagation of “idea shocks” will be through the innovation network.
- One use of this relationship is as a potential source of variation in technology.
- If  $ExpectPatents_{j,t}$  is high for some sector relative to others, then we can expect that sector to have a greater number of new innovations and thus a greater improvement in technology.
- Acemoglu, Akcigit and Kerr (2014) use this source of variation to investigate the relationship between technology and employment at the city and industry level.

# Conclusion

- Networks are also useful vehicle for the study of propagation of shocks at the micro or the microeconomic level across various different units.
- This brief lecture focused on propagation of shocks across sectors, financial institutions and different types of innovations/technology classes.
- Other important linkages would include geographic areas, labor markets, firms, and countries.
- This is another area open for new theoretical and empirical work.