14.773 Political Economy of Institutions and Development.
Lectures 4 and 5: Introduction to Dynamic Voting and Constitutions

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Introduction

- Markov Perfect Equilibria different from myopic rules because they take into account the effect of current votes on future political decisions.
- These issues are more salient and important when current political decisions affect the *distribution of political power* in the future.
- The set of issues that arise here are very similar to those that will be central when we think about endogenous institutions.
- Thus useful to start considering more general dynamic voting models.

Why Worry about the Dynamics of Political Power?

- Why does political equilibrium lead to “inefficiency”?
  - Why doesn't even the most corrupt and kleptocratic dictator just choose economically efficient actions and then redistribute things towards himself?
- In static models, as we have seen, political equilibria are often Pareto efficient (though often inefficient in other ways).
- In dynamic models, there is a new reason why inefficiency will arise: *the political losers effect*.
  - If you do the right thing, this may reduce your political power and your future rents.
- To study these issues, we need dynamic models with endogenous distribution of political power.

Dynamic Voting in Clubs

- Let us start with a model due to Robert's (1999).
- Voting directly over club size (utilities directly from club size).
- Relatively parsimonious model, but it gets quickly complicated.
- Nevertheless, some important insights can be obtained.
- We will see both later in the lecture and when we study endogenous institutions later in the class how similar insights arise in different settings.
- Key issue: what type of structure we should impose on dynamic models so that they are tractable, while capturing real-world relevant phenomena?

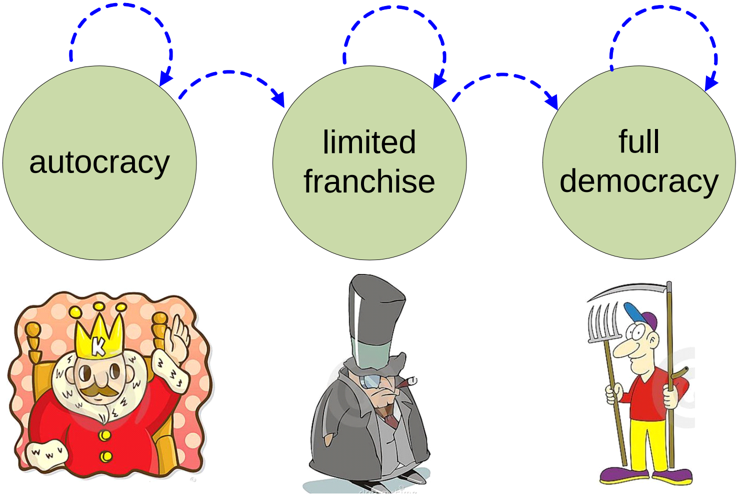
Specifics of Roberts's Model

- An economy consisting of a finite group $\mathcal{N} = \{1, 2..., n\}$.
- There is a seniority system so that if the voting population is of size x, it includes individuals $\{1, 2, ..., x\}$, i.e., lower index individuals are always included before higher index individuals.
- Size of voting club at time t is x_t and instantaneous utility of individual ξ when the size of the (voting) club is x is $w_\xi(x)$.
- In terms of more micro models, this instantaneous utility function incorporates what the utility of individual ξ will be when tax policies are determined by a club of x individuals.
- Key assumption: **(Strict) Increasing Differences:** For all $x > x'$, $\xi > \xi'$, we have

$$w_\xi(x) - w_\xi(x') > w_{\xi'}(x) - w_{\xi'}(x').$$

- Slight variant on single crossing: Higher ranked individuals included later in the club than lower-ranked individuals, but also have preference towards larger groups.

Constitutional Choice



Constitutional Choice—Simple Example

- Three states: absolutism a, constitutional monarchy c, full democracy d
- Two agents: elite E, middle class M

$$\begin{aligned} w_E(d) &< w_E(a) < w_E(c) \\ w_M(a) &< w_M(c) < w_M(d) \end{aligned}$$

- E rules in a, M rules in c and d.
- Myopic elite: starting from a, move to c
- Farsighted elite (high discount factor): stay in a—as moving to c will lead to M moving to d
- But very different insights when there are stochastic elements and intermediate discount factors.

States and Utilities

- More formally, "society" starts period in "state" (e.g., size of club, constitution, policy) s_{t-1} and decides on (feasible) s_t
- Individual i in period t gets instantaneous utility

$$w_i(s_t)$$

- **Strict increasing differences**: For any agents $i, j \in \mathcal{N}$ such that $i > j$,

$$w_i(s) - w_j(s)$$

  is increasing in s

  - This could be weakened to weak increasing differences for some results.

- In addition, possibly transition cost $c_i(s_t, s_{t-1})$.

Transition Mapping

- Let us consider Markov transition rules for analyzing how the "state" changes over time.
- A Markov transition rule is denoted by ϕ such that

$$\phi : \mathcal{S} \to \mathcal{S}.$$

- A transition rule is useful because it defines the path of the state s recursively such that for all t, i.e.,

$$s_{t+1} = \phi(s_t).$$

- Why Markov?
- If there is an s_∞ such that $s_\infty = \phi(s_\infty)$, then s_∞ is a *steady state* of the system.
- We will consider both deterministic and stochastic transition rules $\phi(\cdot)$. But for now, useful to think of it as non-stochastic.

Recursive Representation

- Value function (conditioned on transition mapping ϕ):

$$V_i^{\phi}(s) = w_i(s) + \sum_{k=1}^{\infty} \beta^k \left[w_i\left(\phi^k(s)\right) - c_i\left(\phi^{k-1}(s), \phi^k(s)\right) \right].$$

- Recursively

$$\begin{aligned} V_i^{\phi}(s) &= w_i(s) + \beta \left[V_i^{\phi}(\phi(s)) - c_i(s, \phi(s)) \right], \text{ or} \\ V_i^{\phi}(s) &= w_i(s) - \beta c_i(s, \phi(s)) + \beta V_i^{\phi}(\phi(s)) \end{aligned}$$

- Also define continuation value inclusive of transition costs:

$$V_i^{\phi}(s \mid x) = V_i^{\phi}(s) - c_i(x, s)$$

Recursive Representation (continued)

- In the stochastic case:

$$V^{\phi}_{E,i}(s) = w_{E,i}(s) + \beta_E \sum_{E'} q(E, E') \left[V^{\phi}_{E',i}(\phi_{E'}(s)) - c_{E',i}(s, \phi_{E'}(s)) \right]$$

  where E denotes different "environments" with different payoffs, transition costs or political processes, and $q(E, E')$ denotes transition probabilities.

- Also:

$$V^{\phi}_{E,i}(s \mid x) = V^{\phi}_{E,i}(s)) - c_{E,i}(x, s).$$

- **Key observation:** If w satisfies (strict) increasing differences, then so does V.

Markov Voting Equilibrium

- Let $\mathcal{W}_{E,x}$ denote the set of "winning coalitions"—i.e., the set of agents politically powerful enough to change the state—starting in state x when the environment is E. The structure of these sets will be explained in detail below.
- $\phi = \{\phi_E : S \to S\}$ is a *Markov Voting Equilibrium* if for any $x, y \in S$,

$$\left\{i \in \mathcal{N} : V^{\phi}_{E,i}(y \mid x) > V^{\phi}_{E,i}(\phi_E(x) \mid x)\right\} \notin \mathcal{W}_{E,x}$$
$$\left\{i \in \mathcal{N} : V^{\phi}_{E,i}(\phi_E(x) \mid x) \geq V^{\phi}_{E,i}(x)\right\} \in \mathcal{W}_{E,x}$$

- The first is ensures that there isn't another state transition to which would gather sufficient support.
  - Analogy to "core".
- The second one ensures that there is a winning coalition supporting the transition.

Roadmap

- We now study some special cases, then returning to the general framework so far outlined.
  - A (finite) game of political eliminations.
  - Characterization for the general model without stochastic elements and with β close to 1.
  - Applications.
  - Characterization for the general model with stochastic elements and arbitrary discount factor β.
  - Applications.

Voting Over Coalitions

- Another obvious example of dynamic voting with changing constituencies.
- Model based on Acemoglu, Egorov and Sonin (2008).
- A coalition, which will determine the distribution of a pie (more generally payoffs), both over its own membership.
- Possibility of future votes shaping the stability of current clubs illustrated more clearly.
- *Motivation:*
  1. the three-player divide the dollar game.
  2. eliminations in the Soviet Politburo.

Political Game

- Let $\mathcal{I}$ denote the collection of all individuals, which is assumed to be finite.
- The non-empty subsets of $\mathcal{I}$ are *coalitions* and the set of coalitions is denoted by $\mathcal{C}$.
- For any $X \subset \mathcal{I}$, $\mathcal{C}_X$ denotes the set of coalitions that are subsets of X and $|X|$ is the number of members in X.
- In each period there is a designated *ruling coalition*, which can change over time.
- The game starts with ruling coalition N, and eventually the *ultimate ruling coalition* (URC) forms.
- When the URC is X, then player i obtains *baseline* utility $w_i(X) \in \mathbb{R}$.
- $w(\cdot) \equiv \{w_i(\cdot)\}_{i \in \mathcal{I}}$.
- Important assumption: game of "non-transferable utility". Why?

Political Power

- So far, our focus has been on "democratic" situations. One person one vote.
- Now allow differential powers across individuals.
- *Power* mapping to:

$$\gamma : \mathcal{I} \rightarrow \mathbb{R}_{++},$$

- $\gamma_i \equiv \gamma(i)$: political *power* of individual $i \in \mathcal{I}$ and $\gamma_X \equiv \sum_{i \in X} \gamma_i$ political power of coalition X.

Winning Coalitions

- Coalition $Y \subset X$ is *winning* within coalition X if and only if

$$\gamma_Y > \alpha\gamma_X,$$

  where $\alpha \in [1/2, 1)$ is a (weighted) supermajority rule ($\alpha = 1/2$ corresponds to majority rule).
- Let us write: $Y \in \mathcal{W}_X$ for $Y \subset X$ winning within X.
- Since $\alpha \geq 1/2$, if $Y, Z \in \mathcal{W}_X$, then $Y \cap Z \neq \varnothing$.

Payoffs

Assumption: Let $i \in I$ and $X, Y \in C$. Then:
(1) If $i \in X$ and $i \notin Y$, then $w_i(X) > w_i(Y)$ [i.e., each player prefers to be part of the URC].
(2) For $i \in X$ and $i \in Y$, $w_i(X) > w_i(Y) \iff \gamma_i/\gamma_X > \gamma_i/\gamma_Y$ ($\iff \gamma_X < \gamma_Y$) [i.e., for any two URCs that he is part of, each player prefers the one where his relative power is greater].
(3) If $i \notin X$ and $i \notin Y$, then $w_i(X) = w_i(Y) \equiv w_i^-$ [i.e., a player is indifferent between URCs he is not part of].

- Interpretation.
- Example:

$$w_i(X) = \frac{\gamma_{X \cap \{i\}}}{\gamma_X} = \begin{cases} \gamma_i/\gamma_X & \text{if } i \in X \\ 0 & \text{if } i \notin X \end{cases}. \tag{1}$$

Extensive-Form Game

- Choose $\varepsilon > 0$ arbitrarily small. Then, the extensive form of the game $\Gamma = (N, \gamma|_N, w(\cdot), \alpha)$ is as follows. Each *stage j* of the game starts with some ruling coalition N_j (at the beginning of the game $N_0 = N$). Then:

1. Nature randomly picks agenda setter $a_{j,q} \in N_j$ for $q = 1$.
2. [Agenda-setting step] Agenda setter $a_{j,q}$ makes proposal $P_{j,q} \in \mathcal{C}_{N_j}$, which is a subcoalition of N_j such that $a_{j,q} \in P_{j,q}$ (for simplicity, we assume that a player cannot propose to eliminate himself).
3. [Voting step] Players in $P_{j,q}$ vote sequentially over the proposal. More specifically, Nature randomly chooses the first voter, $v_{j,q,1}$, who then casts his vote vote $\tilde{v}(v_{j,q,1}) \in \{\tilde{y}, \tilde{n}\}$ (Yes or No), then Nature chooses the second voter $v_{j,q,2} \neq v_{j,q,1}$ etc. After all $|P_{j,q}|$ players have voted, the game proceeds to step 4 if players who supported the proposal form a winning coalition within N_j (i.e., if $\{i \in P_{j,q} : \tilde{v}(i) = \tilde{y}\} \in \mathcal{W}_{N_j}$), and otherwise it proceeds to step 5.

Extensive-Form Game (continued)

4. If $P_{j,q} = N_j$, then the game proceeds to step 6. Otherwise, players from $N_j \setminus P_{j,q}$ are eliminated and the game proceeds to step 1 with $N_{j+1} = P_{j,q}$ (and j increases by 1 as a new transition has taken place).
5. If $q < |N_j|$, then next agenda setter $a_{j,q+1} \in N_j$ is randomly picked by Nature among members of N_j who have not yet proposed at this stage (so $a_{j,q+1} \neq a_{j,r}$ for $1 \leq r \leq q$), and the game proceeds to step 2 (with q increased by 1). If $q = |N_j|$, the game proceeds to step 6.
6. N_j becomes the ultimate ruling coalition. Each player $i \in N$ receives total payoff

$$U_i = w_i(N_j) - \varepsilon \sum_{1 \leq k \leq j} \mathbf{I}_{\{i \in N_k\}}, \tag{2}$$

where $\mathbf{I}_{\{\cdot\}}$ is the indicator function taking the value of 0 or 1.

Discussion

- Natural game of sequential choice of coalitions.
- ε introduced for technical reasons (otherwise, indifferences lead to uninteresting transitions).
- Important assumption: players eliminated have no say in the future.
- Stark representation of changing constituencies, but not a good approximation to democratic decision-making.
- More reminiscent to "dealmaking in autocracies"—or coalition formation in nondemocracies.

Axiomatic Analysis

- Games of coalition formation have noncooperative and cooperative features.
- Ideally, the two perspectives give congruent results.
- Key idea in the extensive-form game: players will not support a coalition that will later eliminate themselves.
  $\longrightarrow$ *stability*
- Let us first capture this notion using an axiomatic approach.

Axioms

- Consider the set of games $(N, \gamma|_N, w(\cdot), \alpha)$.
- Holding γ, w and α fixed, consider the correspondence $\phi^\infty : \mathcal{C} \rightrightarrows \mathcal{C}$ such that $\phi^\infty(X)$ gives the ultimate ruling coalition starting with coalition $X \in \mathcal{C}$.To simplify notation, let us denote this by ϕ.
- Axioms on ϕ.

Axiom 1 (Inclusion) For any $X \in \mathcal{C}$, $\phi(X) \neq \varnothing$ and if $Y \in \phi(X)$, then $Y \subset X$.

Axiom 2 (Power) For any $X \in \mathcal{C}$, $Y \in \phi(X)$ only if $Y \in \mathcal{W}_X$.

Axiom 3 (Self-Enforcement) For any $X \in \mathcal{C}$, $Y \in \phi(X)$ only if $Y \in \phi(Y)$.

Axiom 4 (Rationality) For any $X \in \mathcal{C}$, for any $Y \in \phi(X)$ and for any $Z \subset X$ such that $Z \in \mathcal{W}_X$ and $Z \in \phi(Z)$, we have that $Z \notin \phi(X) \iff \gamma_Y < \gamma_Z$.

- Interpretation.

Self-Enforcing Coalitions

- Motivated by the self-enforcement axiom:

Definition

Coalition $X \in P(\mathcal{I})$ *is* self-enforcing *if* $X \in \phi(X)$.

Assumption: The power mapping γ is *generic* in the sense that if for any $X, Y \in \mathcal{C}$, $\gamma_X = \gamma_Y$ implies $X = Y$.

- We also say that coalition N is generic or that numbers $\{\gamma_i\}_{i \in N}$ are generic if mapping $\gamma|_N$ is generic.

Main Axiomatic Result

Theorem

Fix $\mathcal{I}$, γ, $w(\cdot)$ and $\alpha \in [1/2, 1)$. Then:
1. There exists a unique mapping ϕ that satisfies Axioms 1–4. Moreover, when γ is generic, ϕ is single-valued.
2. This mapping ϕ may be obtained by the following inductive procedure. For any $k \in \mathbb{N}$, let $\mathcal{C}^k = \{X \in \mathcal{C} : |X| = k\}$. Clearly, $\mathcal{C} = \cup_{k \in \mathbb{N}} \mathcal{C}^k$. If $X \in \mathcal{C}^1$, then let $\phi(X) = \{X\}$. If $\phi(Z)$ has been defined for all $Z \in \mathcal{C}^n$ for all $n < k$, then define $\phi(X)$ for $X \in \mathcal{C}^k$ as

$$\phi(X) = \operatorname*{argmin}_{A \in \mathcal{M}(X) \cup \{X\}} \gamma_A, \text{ and} \tag{3}$$

$$\mathcal{M}(X) = \{Z \in \mathcal{C}_X \setminus \{X\} : Z \in \mathcal{W}_X \text{ and } Z \in \phi(Z)\}. \tag{4}$$

Proceeding inductively $\phi(X)$ is defined for all $X \in \mathcal{C}$.

Intuition

- For each X, (4) defines $\mathcal{M}(X)$ as the set of proper subcoalitions which are both winning and self-enforcing. Equation (3) then picks the coalitions in $\mathcal{M}(X)$ that have the least power.
- When there are no proper winning and self-enforcing subcoalitions, $\mathcal{M}(X)$ is empty and X becomes the URC), which is captured by (3).
- What does this mean?

Implication

Corollary

Coalition N is self-enforcing, that is, $N \in \phi(N)$, if and only if there exists no coalition $X \subset N$, $X \neq N$, that is winning within N and self-enforcing. Moreover, if N is self-enforcing, then $\phi(N) = \{N\}$.

- Main implication: a coalition that includes a winning and self-enforcing subcoalition cannot be self-enforcing. This captures the notion that the stability of smaller coalitions undermines stability of larger ones.
- Application: coalition formation among three players with approximately equal powers.

Noncooperative Game

Theorem

Suppose that $\phi(N)$ satisfies Axioms 1-4 (cfr. (3) in the axiomatic analysis). Then, for any $K \in \phi(N)$, there exists a pure strategy profile σ_K that is an SPE and leads to URC K in at most one transition. In this equilibrium player $i \in N$ receives payoff

$$U_i = w_i(K) - \varepsilon \mathbf{I}_{\{i \in K\}} \mathbf{I}_{\{N \neq K\}}. \tag{5}$$

This equilibrium payoff does not depend on the random moves by Nature.

- Thus equivalence between cooperative and noncooperative approaches.

Intuition

- Suppose each player anticipates members of a self-enforcing ruling coalition to play a strategy profile such that they will turn down any offers other than K and they will accept K;
- then, it is in the interest of all the players in K to play such a strategy for any history.
- This follows immediately because by the definition of the set $\phi(N)$, because for any deviation to be profitable, the URC that emerges after such deviation must be either not self-enforcing or not winning.
- But the the first option is ruled out by induction while a deviation to a non-winning URC will be blocked by sufficiently many players.
- The payoff in (5) is also intuitive.
- Each player receives his baseline payoff $w_i(K)$ resulting from URC K and then incurs the cost ε if he is part of K and if the initial coalition N is not equal to K (because in this latter case, there will be one transition).

Stronger Results Under Genericity

Theorem

Suppose the genericity Assumption holds and suppose $\phi(N) = K$. Then any (pure or mixed strategy) SPE results in K as the URC. The payoff player $i \in N$ receives in this equilibrium is given by (5).

- Intuition.

Characterization

- Equilibrium characterize simply by a set of recursive equations.
- What are the implications of equilibrium coalition formation
- Let us impose one more assumption

Assumption: *For no $X, Y \in \mathcal{C}$ such that $X \subset Y$ the equality $\gamma_Y = \alpha\gamma_X$ is satisfied.*

Continuity of Ruling Coalitions

Proposition Consider $\Gamma = (N, \gamma, w(\cdot), \alpha)$ with $\alpha \in [1/2, 1)$. Then:
1. There exists $\delta > 0$ such that if $\gamma' : N \to \mathbb{R}_{++}$ lies within δ-neighborhood of γ, then $\Phi(N, \gamma, w, \alpha) = \Phi(N, \gamma', w, \alpha)$.
2. There exists $\delta' > 0$ such that if $\alpha' \in [1/2, 1)$ satisfies $|\alpha' - \alpha| < \delta'$, then $\Phi(N, \gamma, w, \alpha) = \Phi(N, \gamma, w, \alpha')$.
3. Let $N = N_1 \cup N_2$ with N_1 and N_2 disjoint. Then, there exists $\delta > 0$ such that for all N_2 such that $\gamma_{N_2} < \delta$, $\phi(N_1) = \phi(N_1 \cup N_2)$.

Fragility of Self-Enforcing Coalitions

Proposition Suppose $\alpha = 1/2$ and fix a power mapping $\gamma : \mathcal{I} \to \mathbb{R}_{++}$. Then:
1. If coalitions X and Y such that $X \cap Y = \varnothing$ are both self-enforcing, then coalition $X \cup Y$ is not self-enforcing.
2. If X is a self-enforcing coalition, then $X \cup \{i\}$ for $i \notin X$ and $X \setminus \{i\}$ for $i \in X$ are not self-enforcing.

- Implication: under majority rule $\alpha = 1/2$, the addition or the elimination of a single agent from a self-enforcing coalitions makes this coalition no longer self-enforcing. Why?

Size of Ruling Coalitions I

Proposition Consider $\Gamma = (N, \gamma, w(\cdot), \alpha)$ with $\alpha \in [1/2, 1)$. Suppose that there exists $\delta > 0$ such that $\max_{i,j \in N} \{\gamma_i / \gamma_j\} < 1 + \delta$. Then:
1. When $\alpha = 1/2$, any ruling coalition must have size $k_m = 2^m - 1$ for some $m \in \mathbb{Z}$, and moreover, $\phi(N) = N$ if and only if $|N| = k_m$ for $k_m = 2^m - 1$.
2. When $\alpha \in [1/2, 1)$, $\phi(N) = N$ if and only if $|N| = k_{m,\alpha}$ where $k_{1,\alpha} = 1$ and $k_{m,\alpha} = \lfloor k_{m-1,\alpha} / \alpha \rfloor + 1$ for $m > 1$, where $\lfloor z \rfloor$ denotes the integer part of z.

- When powers are approximately equal, the size of the URC is determined tightly.

Rules and Coalitions

- Should an increase in α raise the size of the URC? Should an individual always gain from an increase in his power?
- Intuitive, but the answers are no and no.

Proposition

1. An increase in α may reduce the size of the ruling coalition. That is, there exists a society N, a power mapping γ and $\alpha, \alpha' \in [1/2, 1)$, such that $\alpha' > \alpha$ but for all $X \in \Phi(N, \gamma, w, \alpha)$ and $X' \in \Phi(N, \gamma, w, \alpha')$, $|X| > |X'|$ and $\gamma_X > \gamma_{X'}$.
2. There exist a society N, $\alpha \in [1/2, 1)$, two mappings $\gamma, \gamma' : N \to \mathbb{R}_{++}$ satisfying $\gamma_i = \gamma'_i$ for all $i \neq j$, $\gamma_j < \gamma'_j$ such that $j \in \Phi(N, \gamma, w, \alpha)$, but $j \notin \Phi(N, \gamma', w, \alpha)$. Moreover, this result applies even when j is the most powerful player in both cases, i.e. $\gamma'_i = \gamma_i < \gamma_j < \gamma'_j$ for all $i \neq j$.

- Why?

Conclusions

- Once dynamic voting also affects the distribution of political power, richer set of issues arise.
- Endogeneity of constituencies is both practically relevant and related to endogenous institutions.
- Ensuring equilibria in situations of dynamic voting harder, but often we can put economically interesting structure to ensure equilibria (once we know what we are trying to model).

Introduction

- Why do constitutions matter?
- What is written in constitutions seems to matter, but constitutions can be disobeyed and rewritten.
- How do we think about the role of constitutions?
- Different approaches.

Philosophical

- What is written on paper should not matter:
  - because whatever is written down could have been expected even when it was not written down
  - a constitution is as good as the force behind it
- But this perspective may not be too useful in studying how constitutions are written in practice, why they persist and why and when they matter.

Dynamics and Stability: a More General Approach

- A more general approach towards stability and change in social arrangements (political regimes, constitutions, coalitions, clubs, firms)—without giving up existence.
- Essential ingredients:
  - **Payoffs:** different arrangements imply different payoffs
  - **Power:** different arrangements reallocate political or decision-making power
- In this light, we need to study:
  - *Change*: which arrangements will be changed by force or reform
  - *Stability*: which arrangements will resist change
- Are there any general insights?
- *Strategy:* Formulate a general dynamic framework to investigate the interplay of these two factors in a relatively "detail-free" manner.
  - Details useful to go beyond general insights.

Simple Example: Recap

- Consider a simple extension of franchise story
- Three states: absolutism a, constitutional monarchy c, full democracy d
- Two agents: elite E, middle class M

$$
\begin{aligned}
w_E(d) &< w_E(a) < w_E(c) \\
w_M(a) &< w_M(c) < w_M(d)
\end{aligned}
$$

- E rules in a, M rules in c and d.
- Myopic elite: starting from a, move to c
- Farsighted elite: stay in a: move to c will lead to M moving to d.
- Same example to illustrate resistance against socially beneficial reform.

Naïve and Dynamic Insights

- *Naïve insight:* a social arrangement will emerge and persist if a "sufficiently powerful group" prefers it to alternatives.
- Simple example illustrates: power to change towards a more preferred outcome is not enough to implement change
  - because of further dynamics
- Social arrangements might be stable even if there are powerful groups that prefer change in the short run.
- **Key:** social arrangements change the distribution of political power (decision-making capacity).
- **Dynamic decision-making:** future changes also matter (especially if discounting is limited)

Main Results of General Framework

- An axiomatic characterization of "outcome mappings" corresponding to dynamic game (based on a simple *stability* axiom incorporating the notion of forward-looking decisions).
- Equivalence between the MPE of the dynamic game (with high discount factor) and the axiomatic characterization
- Full characterization: *recursive* and *simple*
- Under slightly stronger conditions, the stable outcome (dynamically stable state) is unique given the initial state
  - but depends on the initial state
- Model general enough to nest specific examples in the literature.
- In particular, main theorems directly applicable to situations in which states can be ordered and static payoffs satisfy single crossing or single peakedness.

Simple Implications

- A particular social arrangement is made stable by the instability of alternative arrangements that are preferred by sufficiently many members of the society.
  - stability of a constitution does not require absence of powerful groups opposing it, but the absence of an alternative stable constitution favored by powerful groups.
- Efficiency-enhancing changes are often resisted because of further social changes that they will engender.
  - Pareto inefficient social arrangements often emerge as stable outcomes.

Model: Basics

- Finite set of individuals $\mathcal{I}$ ($|\mathcal{I}|$ total)
  - Set of coalitions $\mathcal{C}$ (non-empty subsets $X \subset \mathcal{I}$)
- Each individual maximizes discounted sum of playoffs with discount factor $\beta \in [0, 1)$.
- Finite set of states $\mathcal{S}$ ($|\mathcal{S}|$ total)
- Discrete time $t \geq 1$
- State s_t is determined in period t; s_0 is given
- Each state $s \in \mathcal{S}$ is characterized by
  - Payoff $w_i(s)$ of individual $i \in \mathcal{I}$ (normalize $w_i(s) > 0$)
  - Set of winning coalitions $\mathcal{W}_s \subset \mathcal{C}$ capable of implementing a change
  - Protocol $\pi_s(k)$, $1 \leq k \leq K_s$: sequence of agenda-setters or proposals ($\pi_s(k) \in \mathcal{I} \cup \mathcal{S}$)

Winning Coalitions

Assumption (Winning Coalitions) For any state $s \in \mathcal{S}$, $\mathcal{W}_s \subset \mathcal{C}$ satisfies two properties:
(a) If $X, Y \in \mathcal{C}$, $X \subset Y$, and $X \in \mathcal{W}_s$ then $Y \in \mathcal{W}_s$.
(b) If $X, Y \in \mathcal{W}_s$, then $X \cap Y \neq \varnothing$.

- (a) says that a superset of a winning coalition is winning in each state
- (b) says that there are no two disjoint winning coalitions in any state
- $\mathcal{W}_s = \varnothing$ is allowed (exogenously stable state)
- Example:
  - Three players $1, 2, 3$
  - $\mathcal{W}_s = \{\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$ is valid (1 is dictator)
  - $\mathcal{W}_s = \{\{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$ is valid (majority voting)
  - $\mathcal{W}_s = \{\{1\}, \{2,3\}\}$ is not valid (both properties are violated)

Dynamic Game

1. Period t begins with state s_{t-1} from the previous period.
2. For $k = 1, \ldots, K_{s_{t-1}}$, the kth proposal $P_{k,t}$ is determined as follows. If $\pi_{s_{t-1}}(k) \in \mathcal{S}$, then $P_{k,t} = \pi_{s_{t-1}}(k)$. If $\pi_{s_{t-1}}(k) \in \mathcal{I}$, then player $\pi_{s_{t-1}}(k)$ chooses $P_{k,t} \in \mathcal{S}$.
3. If $P_{k,t} \neq s_{t-1}$, each player votes (sequentially) *yes* (for $P_{k,t}$) or *no* (for s_{t-1}). Let $Y_{k,t}$ denote the set of players who voted *yes*. If $Y_{k,t} \in \mathcal{W}_{t-1}$, then $P_{k,t}$ is accepted, otherwise it is rejected.
4. If $P_{k,t}$ is accepted, then $s_t = P_{k,t}$. If $P_{k,t}$ is rejected, then the game moves to step 2 with $k \mapsto k+1$ if $k < K_{s_{t-1}}$. If $k = K_{s_{t-1}}$, $s_t = s_{t-1}$.
5. At the end of each period (once s_t is determined), each player receives instantaneous utility $u_i(t)$:

$$u_i(t) = \begin{cases} w_i(s) & \text{if } s_t = s_{t-1} = s \\ 0 & \text{if } s_t \neq s_{t-1} \end{cases}$$

Key Notation and Concepts

- Define binary relations:
  - states x and y are payoff-equivalent

$$x \sim y \iff \forall i \in \mathcal{I} : w_x(i) = w_y(i)$$

  - y is weakly preferred to x in z

$$y \succeq_z x \iff \{i \in \mathcal{I} : w_y(i) \geq w_x(i)\} \in \mathcal{W}_z$$

  - y is strictly preferred to x in z

$$y \succ_z x \iff \{i \in \mathcal{I} : w_y(i) > w_x(i)\} \in \mathcal{W}_z$$

- Notice that these binary relations are **not** simply preference relations
  - *they encode information about preferences and political power.*

Preferences and Acyclicity

Assumption (Payoffs) Payoff functions $\{w_i(\cdot)\}_{i \in \mathcal{I}}$ satisfy:
(a) For any sequence of states $s_1, \ldots, s_k$ in $\mathcal{S}$,

$$s_{j+1} \succ_{s_j} s_j \text{ for all } 1 \leq j \leq k-1 \Longrightarrow s_1 \not\succ_{s_k} s_k.$$

(b) For any sequence of states $s, s_1, \ldots, s_k$ in $\mathcal{S}$ such that $s_j \not\sim s_l$ (for any $1 \leq j < l \leq k$) and $s_j \succ_s s$ (for any $1 \leq j \leq k$)

$$s_{j+1} \succsim_s s_j \text{ for all } 1 \leq j < k-1 \Longrightarrow s_1 \not\succsim_s s_k.$$

Moreover, if for x, y, s in $\mathcal{S}$, we have $x \succ_s s$ and $y \not\succ_s s$, then $y \not\succ_s x$.

- (a) rules out cycles of the form $y \succ_z z$, $x \succ_y y$, $z \succ_x x$
- (b) rules out cycles for $\succeq$ and also an additional condition that is weaker than transitivity within states, i.e., $x \succ_s s$, $y \not\succ_s s$, then $y \not\succ_s x$, whereas transitivity would require $x \succ_s s$, $s \succ_s y$, then $x \succ_s y$, which implies our condition, but is much stronger.
- Alternative (with equivalent results): voting *yes* has a small cost.
- These assumptions rule out Condorcet-type cycles emerge.

Preferences and Acyclicity (continued)

- We will also strengthen our results under:

Assumption (Comparability) For $x, y, z \in \mathcal{S}$ such that $x \succ_z z$, $y \succ_z z$, and $x \nsim y$, either $y \succ_z x$ or $x \succ_z y$.

- This condition sufficient (and "necessary") for uniqueness.

Approach and Motivation

- **Key economic insight:** *with sufficiently forward-looking behavior, an individual should not wish to transition to a state that will ultimately lead to another lower utility state.*
- Characterize the set of allocations that are consistent with this insight—without specifying the details of the dynamic game.
  - Introduce three simple and intuitive axioms.
  - Characterize set of mappings Φ such that for any $\phi \in \Phi$, $\phi : \mathcal{S} \rightarrow \mathcal{S}$ satisfies these axioms and assigns an **axiomatically stable state** $s^{\infty} \in \mathcal{S}$ to each initial state $s_0 \in \mathcal{S}$ (i.e., $\phi(s) = s^{\infty} \in \mathcal{S}$ loosely corresponding to $s_t = s^{\infty}$ for all $t \geq T$ for some T).
- Interesting in its own right, but the main utility of this axiomatic approach is as an input into the characterization of the (two-strategy) MPE of the dynamic game.

Axiom 1

(Desirability) If $x, y \in \mathcal{S}$ are such that $y = \phi(x)$, then either $y = x$ or $y \succ_x x$.

- A winning coalition can always stay in x (even a blocking coalition can)
- A winning coalition can move to y
- If there is a transition to y, a winning coalition must have voted for that

Axiom 2

(Stability) If $x, y \in \mathcal{S}$ are such that $y = \phi(x)$, then $y = \phi(y)$.

- Holds "by definition" of $\phi(\cdot)$: $\exists T : s_t = \phi(s)$ for all $t \geq T$; when $\phi(s)$ is reached, there are no more transitions
- If y were unstable ($y \neq \phi(y)$), then why not move to $\phi(y)$ instead of y

Axiom 3

(Rationality) If $x, y, z \in \mathcal{S}$ are such that $z \succ_x x$, $z = \phi(z)$, and $z \succ_x y$, then $y \neq \phi(x)$.

- A winning coalition can move to y and to z
- A winning coalition can stay in x
- When will a transition to y be blocked?
  - If there is another z preferred by some winning coalition
  - If this z is also preferred to x by some winning coalition (so blocking y will lead to z, not to x)
  - If transition to z is credible in the sense that this will not lead to some other state in perpetuity

Stable States

- State $s \in \mathcal{S}$ is ϕ-stable if $\phi(s) = s$ for $\phi \in \Phi$
- Set of ϕ-stable states: $\mathcal{D}_\phi = \{s \in \mathcal{S} \colon \phi(s) = s \text{ for } \phi \in \Phi\}$
- We will show that if ϕ_1 and ϕ_2 satisfy the Axioms, then $\mathcal{D}_{\phi_1} = \mathcal{D}_{\phi_2} = \mathcal{D}$
  - Even if ϕ is non-unique, notion of stable state is well-defined
  - But $\phi_1(s)$ and $\phi_2(s)$ may be different elements of $\mathcal{D}$

Axiomatic Characterization of Stable States

Theorem

Suppose Assumptions on Winning Coalitions and Payoffs hold. Then:

1. *There exists mapping* ϕ *satisfying Axioms* 1–3.
2. *This mapping* ϕ *may be obtained through a recursive procedure (next slide)*
3. *For any two mappings* ϕ_1 *and* ϕ_2 *that satisfy Axioms* 1–3 *the the sets of stable states of these mappings coincide (i.e.,* $\mathcal{D}_{\phi_1} = \mathcal{D}_{\phi_2} = \mathcal{D}$*).*
4. *If, in addition, the Comparability Assumption holds, then the mapping that satisfies Axioms* 1–3 *is "payoff-unique" in the sense that for any two mappings* ϕ_1 *and* ϕ_2 *that satisfy Axioms* 1–3 *and for any* $s \in \mathcal{S}$, $\phi_1(s) \sim \phi_2(s)$.

Recursive Procedure

Theorem (continued)

Any ϕ *that satisfies Axioms 1–3 can be recursively computed as follows. Construct the sequence of states* $\left\{\mu_1, ..., \mu_{|\mathcal{S}|}\right\}$ *with the property that if for any* $l \in (j, |\mathcal{S}|]$, $\mu_l \not\succ_{\mu_j} \mu_j$. *Let* $\mu_1 \in \mathcal{S}$ *be such that* $\phi(\mu_1) = \mu_1$. *For* $k = 2, ..., |\mathcal{S}|$, *let*

$$\mathcal{M}_k = \left\{s \in \{\mu_1, \ldots, \mu_{k-1}\} : s \succ_{\mu_k} \mu_k \text{ and } \phi(s) = s\right\}.$$

Define, for $k = 2, ..., |\mathcal{S}|$,

$$\phi(\mu_k) = \begin{cases} \mu_k & \text{if } \mathcal{M}_k = \varnothing \\ z \in \mathcal{M}_k \text{: } \nexists x \in \mathcal{M}_k \text{ with } x \succ_{\mu_k} z & \text{if } \mathcal{M}_k \neq \varnothing \end{cases}.$$

(If there exist more than one $s \in \mathcal{M}_k$*:* $\nexists z \in \mathcal{M}_k$ *with* $z \succ_{\mu_k} s$*, we pick any of these; this corresponds to multiple* ϕ *functions).*

Extension of Franchise Example

- Get back to the simple extension of franchise story
- Three states: absolutism a, constitutional monarchy c, full democracy d
- Two agents: elite E, middle class M

$$
\begin{aligned}
w_E(d) &< w_E(a) < w_E(c) \\
w_M(a) &< w_M(c) < w_M(d)
\end{aligned}
$$

- $\mathcal{W}_a = \{\{E\}, \{E, M\}\}$, $\mathcal{W}_c = \{\{M\}, \{E, M\}\}$, $\mathcal{W}_d = \{\{M\}, \{E, M\}\}$
- Then: $\phi(d) = d$, $\phi(c) = d$, therefore, $\phi(a) = a$
  - Indeed, c is unstable, and among a and d player E, who is part of any winning coalition, prefers a
  - Intuitively, if limited franchise immediately leads to full democracy, elite will not undertake it

Example (continued)

- Assume $\mathcal{W}_c = \{\{E, M\}\}$ instead of $\mathcal{W}_c = \{\{M\}, \{E, M\}\}$
- Then: $\phi(d) = d$, $\phi(c) = c$, and, $\phi(a) = c$
- a became unstable because c became stable

- Now assume $\mathcal{W}_a = \mathcal{W}_c = \mathcal{W}_d = \{\{E, M\}\}$ and

$$\begin{aligned} w_E(a) &< w_E(d) < w_E(c) \\ w_M(a) &< w_M(c) < w_M(d) \end{aligned}$$

  - a is disliked by everyone, but otherwise preferences differ
- Then: $\phi(d) = d$, $\phi(c) = c$, and $\phi(a)$ may be c or d
- In any case, $\mathcal{D} = \{c, d\}$ is the same

Back to Dynamic Game

Assumption (Agenda-Setting and Proposals) For every state $s \in \mathcal{S}$, one (or both) of the following two conditions is satisfied:
(a) For any state $q \in \mathcal{S} \setminus \{s\}$, there is an element $k : 1 \leq k \leq K_s$ of sequence π_s such that $\pi_s(k) = q$.
(b) For any player $i \in \mathcal{I}$ there is an element $k : 1 \leq k \leq K_s$ of sequence π_s such that $\pi_s(k) = i$.

- Exogenous agenda, sequence of agenda-setters, or mixture.
- This assumption ensures that all proposals will be considered (or all agenda-setters will have a chance to propose)

Definition

(Dynamically Stable States) State $s^\infty \in \mathcal{S}$ is a **dynamically stable state** if there exist a protocol $\{\pi_s\}_{s \in \mathcal{S}}$, a MPE strategy profile σ (for a game starting with initial state s_0) and $T < \infty$, such that in MPE $s_t = s^\infty$ for all $t \geq T$.

Noncooperative Characterization

Theorem

(Noncooperative Characterization) *Suppose Assumptions on Winning Coalitions and Payoffs hold. Then there exists* $\beta_0 \in [0, 1)$ *such that for all* $\beta \geq \beta_0$, *the following results hold.*

1. *For any mapping* ϕ *satisfying Axioms* 1–3 *there is a protocol* $\{\pi_s\}_{s \in \mathcal{S}}$ *and a MPE* σ *of the game such that* $s_t = \phi(s_0)$ *for any* $t \geq 1$; *that is, the game reaches* $\phi(s_0)$ *after one period and stays in this state thereafter. Therefore,* $s = \phi(s_0)$ *is a dynamically stable state.*

Noncooperative Characterization (continued)

Theorem

... *Moreover:*

2. *For any protocol* $\{\pi_s\}_{s\in\mathcal{S}}$ *there exists a MPE in pure strategies. Any such MPE* σ *has the property that for any initial state* $s_0 \in \mathcal{S}$*, it reaches some state,* s^∞ *by* $t = 1$ *and thus for* $t \geq 1$, $s_t = s^\infty$*. Moreover, there exists mapping* $\phi : \mathcal{S} \to \mathcal{S}$ *that satisfies Axioms* 1–3 *such that* $s^\infty = \phi(s_0)$*. Therefore, all dynamically stable states are axiomatically stable.*
3. *If, in addition, Assumption (Comparability) holds, then the MPE is essentially unique in the sense that for any protocol* $\{\pi_s\}_{s\in\mathcal{S}}$*, any MPE strategy profile in pure strategies* σ *induces* $s_t \sim \phi(s_0)$ *for all* $t \geq 1$*, where* ϕ *satisfies Axioms* 1–3*.*

Dynamic vs. Myopic Stability

Definition

State $s^m \in \mathcal{S}$ is *myopically stable* if there does not exist $s \in \mathcal{S}$ with $s \succ_{s^m} s^m$.

Corollary

1.
2. *State $s^\infty \in S$ is a (dynamically and axiomatically) stable state only if for any $s' \in \mathcal{S}$ with $s' \succ_{s^\infty} s^\infty$, and any ϕ satisfying Axioms 1–3, $s' \neq \phi(s')$.*
3. *A myopically stable state s^m is a stable state.*
4. *A stable state s^∞ is not necessarily myopically stable.*

- E.g., state a in extension of franchise story

Inefficiency

Definition

(Infficiency) State $s \in S$ is *(strictly) Pareto inefficient* if there exists $s' \in S$ such that $w_i(s') > w_i(s)$ for all $i \in \mathcal{I}$.
State $s \in S$ is *(strictly) winning coalition inefficient* if there exists a winning coalition $\mathcal{W}_s \subset \mathcal{I}$ in s and $s' \in S$ such that $w_i(s') > w_i(s)$ for all $i \in \mathcal{W}_s$.

- Clearly, if a state s is Pareto inefficient, it is winning coalition inefficient, but not vice versa.

Corollary

1. *A stable state $s^\infty \in S$ can be (strictly) winning coalition inefficient and Pareto inefficient.*
2. *Whenever s^∞ is not myopically stable, it is winning coalition inefficient.*

Applying the Theorems in Ordered Spaces

- The characterization theorems provided so far are easily applicable in a wide variety of settings.
- In particular, if the set of states is ordered and static preferences satisfy single crossing or single peakedness, all the results provided so far can be applied directly.
- Here, for simplicity, suppose that $\mathcal{I} \subset \mathbb{R}$ and $\mathcal{S} \subset \mathbb{R}$ (more generally, other orders on the set of individuals and the set of states would work as well)

Single Crossing and Single Peakedness

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, set of states $\mathcal{S} \subset \mathbb{R}$, and payoff functions $w_{\cdot}(\cdot)$. Then, single crossing condition holds if whenever for any $i, j \in \mathcal{I}$ and $x, y \in \mathcal{S}$ such that $i < j$ and $x < y$, $w_i(y) > w_i(x)$ implies $w_j(y) > w_j(x)$ and $w_j(y) < w_j(x)$ implies $w_i(y) < w_i(x)$.

Definition

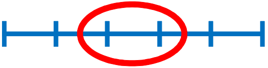
Take set of individuals $\mathcal{I} \subset \mathbb{R}$, set of states $\mathcal{S} \subset \mathbb{R}$, and payoff functions $w_{\cdot}(\cdot)$. Then, single-peaked preferences assumption holds if for any $i \in \mathcal{I}$ there exists state x such that for any $y, z \in \mathcal{S}$, if $y < z \leq x$ or $x \geq z > y$, then $w_i(y) \leq w_i(z)$.

Generalizations of Majority Rule and Median Voter

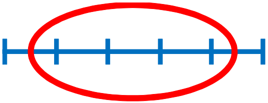
Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, state $s \in \mathcal{S}$, and set of winning coalitions $\mathcal{W}_s$ that satisfies Assumption on Winning Coalitions. Player $i \in \mathcal{I}$ is called quasi-median voter (in state s) if $i \in X$ for any $X \in \mathcal{W}_s$ such that $X = \{j \in \mathcal{I} : a \leq j \leq b\}$ for some $a, b \in \mathbb{R}$.

- That is, quasi-median voter is a player who belongs to any "connected" winning coalition.
- Quasi-median voters:



simple majority



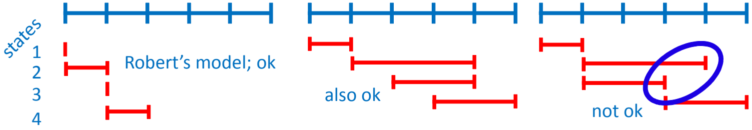
5/6 supermajority

Generalizations of Majority Rule and Median Voter (continued)

- Denote the set of quasi-median voters in state s by M_s (it will be nonempty)

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, set of states $\mathcal{S} \subset \mathbb{R}$. The sets of winning coalitions $\{\mathcal{W}_s\}_{s \in \mathcal{S}}$ has monotonic quasi-median voter property if for each $x, y \in \mathcal{S}$ satisfying $x < y$ there exist $i \in M_x$, $j \in M_y$ such that $i \leq j$.



A Weak Genericity Assumption

- Let us say that preferences $w_{\cdot}(\cdot)$, given the set of winning coalitions $\{\mathcal{W}_s\}_{s\in\mathcal{S}}$, are *generic* if for all $x, y, z \in \mathcal{S}$, $x \succeq_z y$ implies $x \succ_z y$ or $x \sim y$.
- This is (much) weaker than the comparability assumption used for uniqueness above.
  - In particular, it holds generically.

Theorem on Single Crossing and Single Peakedness

Theorem

Suppose the Assumption on Winning Coalitions holds.

1. *If preferences are generic and satisfy single crossing and the monotonic quasi-median voter property holds, then Assumptions on Payoffs above are satisfied and Theorems 1 and 2 apply.*
2. *If preferences are generic and single peaked and all winning coalitions intersect (i.e., $X \in \mathcal{W}_x$ and $Y \in \mathcal{W}_y$ imply $X \cap Y \neq \varnothing$), then Assumptions on Payoffs are satisfied and Theorems 1 and 2 apply.*

- Note monotonic median voter property is weaker than the assumption that $X \in \mathcal{W}_x \wedge Y \in \mathcal{W}_y \Longrightarrow X \cap Y \neq \varnothing$.

Voting in Clubs

- N individuals, $\mathcal{I} = \{1, \ldots, N\}$
- N states (clubs), $s_k = \{1, \ldots, k\}$
- Assume single-crossing condition

$$\text{for all } l > k \text{ and } j > i,\ w_j(s_l) - w_j(s_k) > w_i(s_l) - w_i(s_k)$$

- Assume "genericity":

$$\text{for all } l > k,\ w_j(s_l) \neq w_j(s_k)$$

- Then, the theorem for ordered spaces applies and shows existence of MPE in pure strategies for any majority or supermajority rule.
- It also provides a full characterization of these equilibria.

Voting in Clubs

- If in addition only odd-sized clubs are allowed, unique dynamically stable state.
- Equilibria can easily be Pareto inefficient.
- If "genericity" is relaxed, so that $w_j(s_l) = w_j(s_k)$, then the theorem for ordered spaces no longer applies, but both the axiomatic characterization and the noncooperative theorems can still be applied from first principles.
- Also can be extended to more general pickle structures (e.g., weighted voting or supermajority) and general structure of clubs (e.g., clubs on the form $\{k - n, ..., k, ..., k + n\} \cap \mathcal{I}$ for a fixed n and different values of k).

An Example of Elite Clubs

- Specific example: suppose that preferences are such that

$$w_j(s_n) > w_j(s_{n'}) > w_j(s_{k'}) = w_j(s_{k''})$$

  for all $n' > n \geq j$ and $k', k'' < j$
  - individuals always prefer to be part of the club
  - individuals always prefer smaller clubs.
- Winning coalitions need to have a strict majority (e.g., two out of three, three out of four etc.).
- Then,
  - $\{1\}$ is a stable club (no wish to expand)
  - $\{1, 2\}$ is a stable club (no wish to expand and no majority to contract)
  - $\{1, 2, 3\}$ is not a stable club (3 can be eliminated)
  - $\{1, 2, 3, 4\}$ is a stable club
- More generally, clubs of size 2^k for $k = 0, 1, \ldots$ are stable.
- Starting with the club of size n, the equilibrium involves the largest club of size $2^k \leq n$.

Stable Constitutions

- N individuals, $\mathcal{I} = \{1, \ldots, N\}$
- In period 2, they decide whether to implement a reform (a votes are needed)
- a is determined in period 1
- Two cases:
  - Voting rule a: stable if in period 1 no other rule is supported by a voters
  - Constitution (a, b): stable if in period 1 no other constitution is supported by b voters
- Preferences over reforms translate into preferences over a
  - Barbera and Jackson assume a structure where these preferences are single-crossing and single-peaked
  - Motivated by this, let us assume that they are strictly single-crossing
- Stable voting rules correspond to myopically (and dynamically) stable states
- Stable constitutions correspond to dynamically stable states

Political Eliminations

- The characterization results apply even when states do not form an ordered set.
- Set of states $\mathcal{S}$ coincides with set of coalitions $\mathcal{C}$
- Each agent $i \in \mathcal{I}$ is endowed with political influence γ_i
- Payoffs are given by proportional rule

$$w_i(X) = \begin{cases} \gamma_i / \gamma_X & \text{if } i \in X \\ 0 & \text{if } i \notin X \end{cases} \quad \text{where } \gamma_X = \sum_{j \in X} \gamma_j$$

and X is the "*ruling coalition*".

- this payoff function can be generalized to any function where payoffs are increasing in relative power of the individual in the ruling coalition

Political Eliminations (continued)

- Winning coalitions are determined by weighted (super)majority rule $\alpha \in [1/2, 1)$

$$\mathcal{W}_X = \left\{ Y : \sum_{j \in Y \cap X} \gamma_j > \alpha \sum_{j \in X} \gamma_j \right\}$$

- Genericity: $\gamma_X = \gamma_Y$ only if $X = Y$
- Assumption on Payoffs is satisfied and the axiomatic characterization applies exactly.
- If players who are not part of the ruling coalition have a slight preference for larger ruling coalitions, then Stronger Acyclicity Assumption is also satisfied.

Other Examples

- Inefficient inertia
- The role of the middle class in democratization
- Coalition formation in democratic systems
- Commitment, (civil or international) conflict and peace

Dynamic Game

- We now return to the general model with stochastic elements and discount factor < 1
- Focus on *Markov Voting Equilibrium.*
- Comparison with Markov Perfect Equilibria again similar (and discussed below).

Dynamic Game

1. Period t begins with state s_{t-1} and environment E_{t-1} inherited from the previous period (where s_0 is exogenously given).
2. Shocks are realized.
3. Players become agenda-setters, one at a time, according to the protocol $\pi^{s_{t-1}}$. Agenda-setter i proposes an alternative state $a_{t,i}$
4. Players vote sequentially over the proposal $a_{t,i}$. If the set of players that support the transition is a winning coalition, then $s_t = a_{t,i}$. Otherwise, the next person makes the proposal, and if the last agent in the protocol has already done so, then $s_t = a_{t,i}$.
5. Each player i gets instantaneous utility

$$w_{E_t,i}\left(s_t\right) - c_{E_t,i}\left(s_{t-1}, s_t\right).$$

Approach

- Recall that any MPE in pure strategies can be represented by a set of transition mappings $\{\phi_E\}$ such that
  - if $s_{t-1} = s$, and $E_t = E$, then $s_t = \phi_E(s)$ along the equilibrium path, where recall that

$$\phi : S \to S.$$

- Transition mapping $\phi = \{\phi_E : S \to S\}$ is *monotone* if for any $s_1, s_2 \in S$ with $s_1 \le s_2$, $\phi_E(s_1) \le \phi_E(s_2)$.
  - natural, given monotonic median voter property
- Key steps in analysis
  - fix E
  - characterize ϕ_E and expected payoffs when there is no stochasticity
  - then backward induction and dynamic programming.

General Result

Theorem

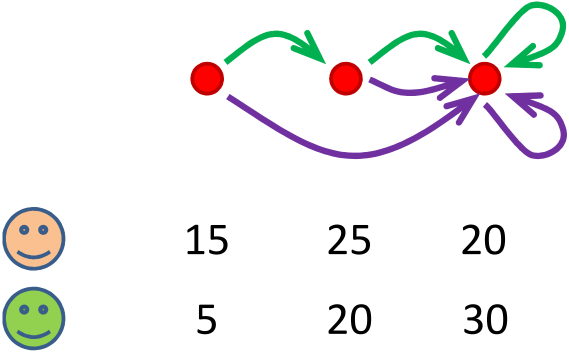
(existence) *There exists a Markov voting equilibrium with monotone transition mapping ϕ.*

Theorem

(uniqueness) *"Generically" there exists no other Markov voting equilibrium with monotone transition mapping if either every set of quasi-median voters is a singleton or preferences are single-peaked (plus additional conditions on transition costs; e.g., only one step transitions).*

- Thus monotone transition mappings arise naturally.
  - though equilibria without such monotonicity may exist.

Non-uniqueness



- Political rule: unanimity in state 1 and the green player dictator in states 2 and 3.

Limiting states and efficiency

- Back to the general model:

Theorem

(limit behavior) *In any Markov voting equilibrium, there is convergence to a limiting state with probability 1.*
The limiting state depends on the timing of shocks.

Theorem

(efficiency) *If each* β_E *is sufficiently small, then the limiting state is Pareto efficient. Otherwise the limiting state may be Pareto inefficient.*

- Example of Pareto inefficiency: elite E, middle class M

$$\begin{aligned} w_E(d) &< w_E(a) < w_E(c) \\ w_M(a) &< w_M(c) < w_M(d) \end{aligned}$$

- E rules in a, M rules in c and d.

Comparative statics

Theorem

("monotone" comparative statics) *Suppose that environments* E^1 *and* E^2 *coincide on* $S' = [1, s] \subset S$ *and* $\beta_{E^1} = \beta_{E^2}$, ϕ_1 *and* ϕ_2 *are MVE in these environments. Suppose* $x \in S'$ *is such that* $\phi_1(x) = x$. *Then* $\phi_2(x) \geq x$.

- Implication, suppose that $\phi_1(x) = x$ is reached before there is a switch to E_2. Then for all subsequent t, $s_t \geq x$.
- Intuition: if some part of the state space is unaffected by shocks, it is either reached without shocks or not reached at all.

MPE vs. Markov voting equilibria

Theorem

(MPE ≈MVE) *For any MVE ϕ (monotone or not) there exists a set of protocols such that there exists a Markov Perfect equilibrium of the game above which implements ϕ.*
Conversely, if for some set of protocols and some MPE σ, the corresponding transition mapping $\phi = \{\phi_E\}_{E \in \mathcal{E}}$ is monotone, then it is MVE.
In addition, if the set of quasi-median voters in two different states have either none or one individual in common, and only one-step transitions are possible, every MPE corresponds to a monotone MVE (under any protocol).

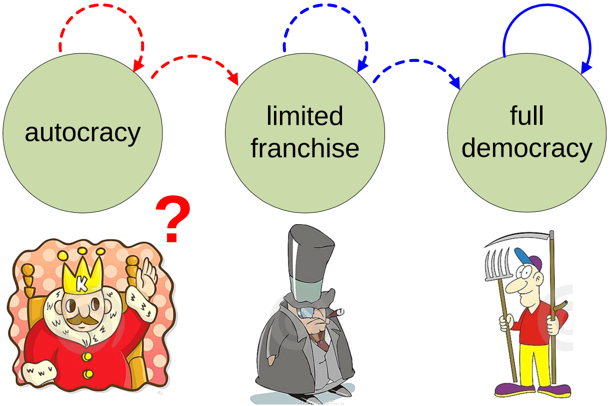
- For each Markov voting equilibrium, there exists a protocol π such that the resulting (pure-strategy) MPE induces transitions that coincide with the Markov voting equilibrium.

Simple example

- Suppose three groups: elite, middle class and workers.
- The elite rule under absolutist monarchy, a.
- Suppose that with limited franchise, c, the middle class rules with probability p and workers rule with probability $1 - p$.
- Workers rule in full democracy, d.
- The middle-class prefer limited franchise, workers prefer full democracy.
- Payoffs

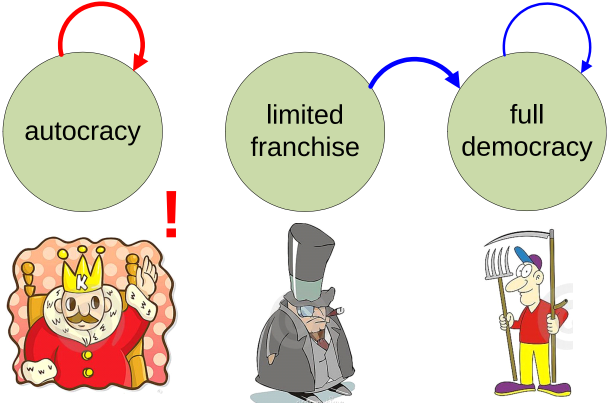
$$\begin{aligned} w_E(d) &< w_E(a) < w_E(c) \\ w_M(a) &< w_M(d) < w_M(c) \\ w_W(a) &< w_W(c) < w_W(d) \end{aligned}$$

Simple example (continued)



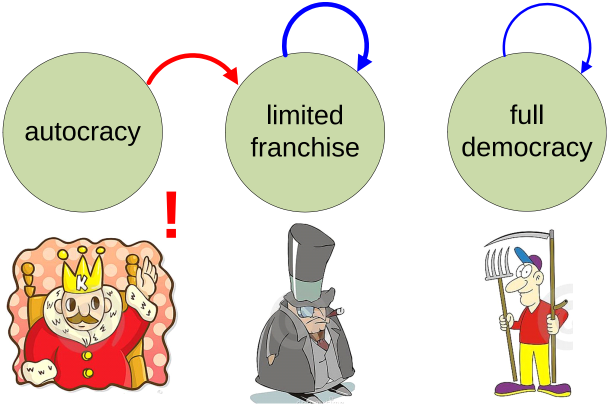
- What happens if β large and p small?

Simple example (continued)



Simple example (continued)

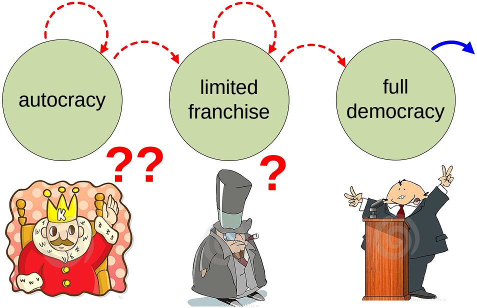
- What happens if $p = 1$ or close to 1?



- What happens if β small or intermediate?

Simple example (continued)

- Now suppose that p takes different values in different environments. We start in E_1 and then stochastically transition to either E_2 or E_3, both of which are absorbing, and $p_{E_2} = 1$ and $p_{E_3} < 1$. Is an early resolution of uncertainty good for transitioning to democracy?



Application: Political Participation

- Suppose that there is a one-dimensional policy space indexed by $\rho \in R = \{\rho^1, \ldots, \rho^r\}$, where higher ρ corresponds to greater tolerance towards religiosity and less tolerance towards non-religious individuals.
- In each period t, the set of individuals who have the right to political participation is Z_t, a *connected* subset of the set of players.
- Suppose that at each date, political decisions are made by α-(super)majorities (i.e., coalitions of at least $\alpha |Z_t|$ members).
- These decisions include the determination of which subset of the society will have the right for political participation in the next period (i.e., the subset Z_{t+1}) and the next period's religiosity policy ρ_{t+1}.
- The *state* can thus be represented by $s = (\rho, Z)$ where $\rho \in R$ and Z is a connected subset of the set of players.

Application (continued)

- Suppose preferences are as follows:

$$w_i(s) = v_i(\rho) + V(Z),$$

where $V(Z)$ is any function, and $v_i(\rho)$ satisfies the strict increasing differences condition:

$$v_i(\rho) - v_j(\rho) \text{ is strictly increasing in } \rho \text{ whenever } i > j.$$

- Since an α-(super)majority in Z chooses the religiousity policy for the next period, ρ.
- Thus this choice will be $\hat{\rho}_{\min M_Z} \leq \rho \leq \hat{\rho}_{\max M_Z}$.

Application (continued)

Proposition

1. *For any degree of (super)majority α, a Markov voting equilibrium exists.*
2. *There exists $\beta_0 < 1$ such that when $\beta > \beta_0$ and $V(Z)$ is (strictly) increasing (i.e., whenever $Z \neq Z'$, $Z \subset Z'$ implies $V(Z) < V(Z')$)) we have: for any initial state s_0, $\phi(s_0) = s = (Z, \rho)$ with Z containing at least one of the extreme players, 1 or n.*

- Intuition: balanced extensions to the extremes are not dangerous.
- But this result no longer true if there are stochastic elements.

Application (continued)

Proposition

Define $A \equiv V(\mathcal{N}) - \max_{i\in\mathcal{N}} V(\mathcal{N}\setminus\{i\})$ *and* $A_i \equiv V(\mathcal{N}) - V(\{i\})$.

1. *Suppose* $v_1(\hat{\rho}(1)) - v_1(\hat{\rho}(\min M_{\mathcal{N}})) < A$ *and* $v_n(\hat{\rho}(n)) - v_n(\hat{\rho}(\max M_{\mathcal{N}})) < A$. *Then for any initial state* s, $Z(\phi(s)) = \mathcal{N}$.
2. *Suppose* $v_1(\hat{\rho}(1)) - v_1(\hat{\rho}(\min M_{\mathcal{N}})) > A_1$ *and* $v_n(\hat{\rho}(n)) - v_n(\hat{\rho}(\max M_{\mathcal{N}})) > A_n$. *There exists* $k \in \mathbb{N}$ *such that if the initial state* s_0 *satisfies* $|Z(s_0)| \leq k$, *then: (i) when* Z_0 *includes the middle player (or at least one of the two middle players if n is even),* $Z(\phi(s_0)) = \mathcal{N}$, *and (ii) when* Z_0 *includes one of the extreme players,* $Z(\phi(s_0)) \neq \mathcal{N}$.
3. *If* $\alpha > \frac{n-1}{n}$, *i.e., the rule is unanimity, then for any initial state* s_0, $Z(\phi(s_0)) = \mathcal{N}$.

Application (continued)

- What happens if there are shocks, for example, increasing the number of votes commanded by extreme groups (which will change the $\mathcal{W}_s$'s)?
- Markov voting equilibria still exist from our general theorems.
- If such shocks to power are sufficiently unlikely, the above characterization still applies, but some shocks will then imply that one of the extremist groups can gain disproportionate power.
- If such shocks to power are likely, say the religious groups may become more powerful in the future, then starting from the middle of political rights may be extended in an asymmetric way or not extended at all.