

The Power of Confidence and Ignorance in Coordination Problems

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Coordination

- ▶ Coordination Problems:
 - ▶ Players would like to coordinate their actions with others' actions and a fundamental state
- ▶ Economic Examples:
 - ▶ Forecasting: I want my forecast to be accurate but not too far from others' forecast
 - ▶ Finance: my willingness to pay for an asset depends on my expectation of fundamentals and my expectation of others' willingness to pay
 - ▶ Team Problem: my private return to effort depends on the effort of others (complementary inputs) and a common state
 - ▶ Macro: my investment will depend on my expectation of productivity and others' investment decisions
- ▶ We will study linear best response game, where players set their actions equal to a weighted combination of their expectations of the state and the average action.

Beliefs

- ▶ Players differ in the signals they observe and their prior beliefs about the state
- ▶ Formally, players have heterogeneous prior beliefs and asymmetric information about a payoff relevant state (the **combination** is studied too little)
- ▶ Thus players differ in
 1. their prior means;
 2. their confidence: a player is more **confident** if his prior beliefs are more concentrated around his prior mean
 3. how informed they are: a player is more **informed** if his signal is more concentrated around the true state

Main Result: The Power of Confidence and Ignorance

- ▶ We identify a **focal mean**, which will play the role of the common/objective prior mean in common prior analysis.
- ▶ In particular,
 - ▶ If the coordination motive dominates, all players will choose the focal mean
 - ▶ More generally, the equilibrium average action will be a convex combination of the true state and the focal mean
- ▶ The focal mean
 - ▶ is a weighted sum of prior means
 - ▶ **over-weights** the prior means of more **confident** and more **ignorant** players

Economic Implications

1. (Private Benefit of Private Ignorance) A player has a private incentive to be less informed in order to have the focal mean and thus equilibrium actions reflect her prior mean
2. (Accurate Information Leads to "Bias"). When all players have no information, the focal mean is a simple average of prior means. When all players have very accurate information, the weights in focal mean are based on relative confidence.
3. (Common Benefit of Ignorant Optimists) If all players want all other players to choose higher actions (there are externalities), all players will want the most "optimistic" player to be uninformed and all other players to be informed.

Assumptions

Some assumptions we make:

1. Linear Best Response Game
2. Normality
3. Uniform interaction (i.e., not networks)
 - ▶ Key assumption is (1)
 - ▶ Our old working paper on "Expectations, Networks and Conventions" relaxes (2) and (3) and derives analogous - but more complicated to state - results

Model

- ▶ Group i has mass s_i
- ▶ Player $z \in [0, 1]$ in group i believes that state $\theta \sim N\left(y_i, \frac{1}{v_i}\right)$ and observes a conditionally independent signal $x_i \sim N\left(\theta, \frac{1}{\pi_i}\right)$
- ▶ Let (i) a_{iz} be the action of player z in group i ; (ii) a_i be the average group i action; (iii) $\bar{a} = \sum_j s_j \bar{a}_j$ be the average population action.
- ▶ Best response: Player z in population i wants to set

$$a_{iz} = r \mathbb{E}_i(\bar{a} | x_{iz}) + (1 - r) \mathbb{E}_i(\theta | x_{iz})$$

- ▶ "Coordination motive" is $r \in [0, 1]$.
- ▶ Benchmark **coordination payoff** function:

$$u_{iz}^C(a_{iz}, \bar{a}, \theta) = - \left[r (a_{iz} - \bar{a})^2 + (1 - r) (a_{iz} - \theta)^2 \right]$$

Resoluteness

- ▶ Recall that

$$a_{iz} = r\mathbb{E}_i(\bar{a} | x_{iz}) + (1 - r)\mathbb{E}_i(\theta | x_{iz})$$

- ▶ First order beliefs about state

$$\mathbb{E}_i(\theta | x_{iz}) = \frac{\nu_i y_i + \pi_i x_{iz}}{\nu_i + \pi_i}$$

- ▶ Group i 's **resoluteness** is the weight they put on their prior mean:

$$\zeta_i = \frac{\nu_i}{\nu_i + \pi_i}$$

- ▶ "Resoluteness" measures "insensitivity to signal"
- ▶ Resoluteness of leaders is discussed both informally and formally in an organizational literature on leadership
- ▶ We will see that resoluteness is a sufficient statistic for "first order average beliefs", focal mean and thus equilibrium behavior.

Aside: Operational Meaning and Observability of Our Model

- ▶ Prior means are theoretical constructs, and not clear what the empirical counterpart is?
- ▶ But I can imagine observing the precision of the signal π , the parameters a and resoluteness $b < 1$ of the regression:

$$\mathbb{E}_i(\theta | x_{iz}) = a + bx_{iz}$$

- ▶ Question for audience: are these parameters observed and measured in practise?
- ▶ The observable variables partially identify our parameters y_i and v_i :

$$v_i = \frac{1-b}{b} \pi$$
$$y_i = \frac{a}{1-b}$$

Resoluteness Determines First Order Beliefs

- ▶ So player first order beliefs are

$$\mathbb{E}_i(\theta | x_{iz}) = \zeta_i y_i + (1 - \zeta_i) x_{iz}$$

by standard normal Bayes updating, where $\zeta_i = \frac{v_i}{v_i + \pi_i}$

- ▶ Group average first order beliefs:

$$\bar{\mathbb{E}}_i(\theta) = \zeta_i y_i + (1 - \zeta_i) \theta$$

- ▶ Population average first order beliefs:

$$\bar{\mathbb{E}}(\theta) = \bar{\zeta} \bar{y} + (1 - \bar{\zeta}) \theta$$

where population (average) resoluteness and population (resolution weighted) prior mean are

$$\bar{\zeta} = \sum_i s_i \zeta_i \text{ and } \bar{y} = \bar{\zeta}^{-1} \sum_i s_i \zeta_i y_i$$

and we will call \bar{y} the **focal mean**.

Individual to Population Best Responses

- ▶ Recall best response function:

$$a_{iz} = r\mathbb{E}_i(\bar{a} | x_{iz}) + (1 - r)\mathbb{E}_i(\theta | x_{iz})$$

- ▶ Group average action best response is:

$$a_i = r\bar{\mathbb{E}}_i(\bar{a}) + (1 - r)\bar{\mathbb{E}}_i(\theta)$$

- ▶ we could replace group i with a single player and get (almost) the same game
- ▶ Population best response is:

$$\bar{a} = r\bar{\mathbb{E}}(\bar{a}) + (1 - r)\bar{\mathbb{E}}(\theta)$$

- ▶ Morris and Shin (2002) analyzed this game with a homogenous population interpretation

Population Equilibrium

Population equilibrium has:

$$\bar{a}^* = \bar{\alpha}(r) \cdot \bar{y} + (1 - \bar{\alpha}(r)) \cdot \theta$$

where

$$\bar{\alpha}(r) = \frac{\bar{\zeta}}{\bar{\zeta} + (1 - r)(1 - \bar{\zeta})}$$

- ▶ could give iterated average expectations derivation

Individual Equilibrium Actions

The equilibrium action of player z in group i is a weighted sum of \bar{y} , y_i and x_{iz} :

$$\begin{aligned} a_{iz}^* &= r \mathbb{E}_i(\bar{a} | x_{iz}) + (1-r) \mathbb{E}_i(\theta | x_{iz}) \\ &= r(\bar{\alpha}(r) \cdot \bar{y} + (1 - \bar{\alpha}(r)) \cdot x_{iz}) + (1-r)(\xi_i y_i + (1 - \xi_i) x_{iz}) \\ &= r\bar{\alpha}(r) \cdot \bar{y} + (1-r)\xi_i y_i + \left(\frac{r(1 - \bar{\alpha}(r))}{+(1-r)(1 - \xi_i)} \right) x_{iz} \end{aligned}$$

Equilibrium and Focal Mean Summary

- ▶ In the $r \rightarrow 1$ limit, the individual equilibrium action.....
 1. ...is deterministic
 2. ...with all players choosing the focal mean for sure.
- ▶ For $r < 1$, the equilibrium average action (\bar{a}) is a weighted average of the focal mean (\bar{y}) and the state (θ)

Common Prior Means

Suppose that $y_i = y$ for all i .

- ▶ The focal mean is y (the "public signal")
- ▶ For all r , the ex ante expected action is y
 - ▶ Dispersed information has no impact on first moments in linear best response games (an implication of the law of iterated expectations). All the welfare action is in second moments.
- ▶ "Common prior assumption" would require in addition that $v_i = v$ for all i
 - ▶ this extra restriction doesn't matter for our results....
 - ▶ with this restriction, the result is a (very) special case of Samet (1998)

- ▶ Recall focal mean:

$$\bar{y} = \bar{\zeta}^{-1} \sum_i s_i \zeta_i y_i$$

- ▶ As a group becomes less informed ($\pi_i \rightarrow 0$), they become more resolute ($\zeta_i = \frac{v_i}{v_i + \pi_i} \uparrow 1$) and the focal mean becomes closer to their prior mean.
- ▶ Conjecture based on focal mean: a group has an incentive to **not** acquire information in order to bring the focal mean closer to her prior mean
- ▶ However, there is a cost to being less informed: actions are not well-coordinated with the state
- ▶ If coordination motive (r) is high, first moment benefit will exceed second moment cost
- ▶ For some $\bar{r} < 1$, player i 's equilibrium utility is decreasing in π_i

Application 1: Forecasting

- ▶ Suppose that
 1. an investment firm is optimistic about the market (their prior mean is high)
 2. they would like their market forecasts to be accurate (to keep their clients happy or for reputational reasons more generally)
 3. but they would also like their forecasts to be close to others' forecasts (other things equal, this may also be important for reputation and also they would like the market to bet the same way as them)
- ▶ Then they have an incentive not to acquire market research.

Implication 2: Accurate Information Leads to "Bias"

- ▶ Suppose that the all players observe signals of the same precision π
- ▶ When $\pi = 0$, resolution is 1 for all groups, $\bar{y} = \sum_j s_j y_j$.
- ▶ If the designer picks common high precision, the focal mean will put more weight on more confident groups:

$$\bar{y} = \frac{\sum_j s_j \tilde{\zeta}_j y_j}{\sum_j s_j \tilde{\zeta}_j} = \frac{\sum_j s_j \frac{v_j}{v_j + \pi} y_j}{\sum_j s_j \frac{v_j}{v_j + \pi}} \rightarrow \frac{\sum_j s_j v_j y_j}{\sum_j s_j v_j} \text{ as } \pi \rightarrow \infty$$

- ▶ "bias" from simple average

Implication of Bias from Accurate Information

- ▶ Suppose that:
 1. there is heterogeneity in group confidence (v_i).
 2. an information designer thinks that (i) over-confident (high v_i) players are also over-pessimistic (low y_i); but (ii) less confident / more open minded (lower v_i) players also have correct prior means (i.e., y_i close to his)
 3. the information designer can control only the common signal precision π of all groups
 4. the information designer wants the average action to be close to the true state

- ▶ What common precision should the information designer choose?

Implication of Bias from Accurate Information

- ▶ Conjecture based on focal mean: He would keep everyone uninformed (in order to bring the focal mean closer to (his expectation of) the state).
- ▶ Again, there is a cost to players being uninformed which is that their actions do not match state well away from the $r \rightarrow 1$ limit.
- ▶ We can verify conjecture taking into account cost of mis-coordination: for any fixed π , there exist $\bar{r} < 1$ such that the designer prefers lower π whenever $r > \bar{r}$, even if we take into account the loss from mis-coordination.
 - ▶ this is consistent with the important observation that $\pi = \infty$ is always optimal for the designer for any fixed r

Application 2: Fed Communication

- ▶ Suppose that the Fed is deciding about their communication policy and would like to coordinate expectations on the state
- ▶ Interpretation in this paper:
 - ▶ The Fed cannot send public signals (as in, e.g., Morris and Shin (2002)) because there will be inevitable heterogeneity in how their statements are interpreted
 - ▶ Instead, "more communication" is assumed to translate into more accurate noisy private signals
- ▶ Conventional wisdom? Communication is good because sensible market participants have sensible prior means and respond sensibly to information from the Fed; and this coordinates expectations on the state.
- ▶ But suppose that there are also clueless market participants (i) have "incorrect" prior means, (ii) also matter to market outcomes and (iii) are less responsive to information. If coordination motive is high, communication may coordinate expectations **away** from the true state.

Implication 3: The Common Benefit of Ignorant Optimists

- ▶ Consider a game with the same best response functions, but externalities, so all players would like all other players to choose high actions
- ▶ More specifically, consider a team problem with strategic complementarities and uncertainty about the exogenous component of the return to effort (θ)....

$$u_{iz}^T(a_{iz}, \bar{a}, \theta) = (r\bar{a} + (1-r)\theta) a_{iz} - \frac{1}{2} a_{iz}^2$$

- ▶ note that this does not change best responses/positive predictions, but will change welfare calculations
- ▶ The most optimistic player is the one with the highest expectation of the exogenous component of the return to effort (θ).....
- ▶ Conjecture: All players (including the optimist) would like the optimist to be uninformed and everyone else to be very well informed.
- ▶ Again, we verify this welfare implication away from the $r \rightarrow 1$ limit

Application 3: Leadership

1. The value of resolute leadership:
 - ▶ Discussed extensively (informally) in the organization literature
 - ▶ A formalization in Bolton, Brunnermeier and Veldcamp (2013): "a resolute leader ... attaches an exaggerated value to his initial information"
 - ▶ In a dynamic model, BBV show a resolute leader will coordinate behavior well, because he can commit to not make (rational) adjustments in response to new information ex post, after others have looked into their actions.
2. The endogeneity and value of optimistic leaders (Van den Steen (2004))
 - ▶ With heterogeneous prior beliefs, leaders tend to be optimistic by selection
 - ▶ Complementarity between optimism and effort, which would help with a free rider problem in teams
 - ▶ Strategic complementarity reinforces benefit of having optimistic leaders/team members
3. Our contribution: there is a complementarity in social value between resoluteness and optimism