Labor Economics, 14.661, Second Part, Problem Set 4

Please answer questions 2, 3 and 4. This problem set is due Friday, December 1, before recitation.

Exercise 1 Consider the following economy. At $t = 0$, the firm decides how much to invest in its employee's general skills. The cost of an investment τ is $c(\tau)$, which is incurred by the firm. A worker with general skills τ produces $1+\tau$ output in period $t = 1$. At this point, he can also move to a different firm where his wage will be $1 + \tau - \theta$ where θ is the cost of moving to a different firm. θ is a random variable, drawn from a uniform distribution $[0, 1]$, and is the private information of the worker (i.e., the firm does not observe it). The exact sequence of events is as follows: at $t = 0$, the firm chooses τ and makes a wage offer (w) to the worker; next, the worker, knowing her own θ, decides whether to quit or to stay.

1. Characterize the firm's wage offer as a function of τ. In particular, is $w'(\tau)$ positive, negative, zero, or ambiguously determined? Why?

2. Solve for the firm's level of training and wage offer that maximize expected profit. Explain why the firm is not investing in τ?

3. Suppose now that the worker can finance his own training investment. Solve for the worker's choice of training and the firm's wage offer.

4. Suppose again that the worker cannot finance her training, but that her wage, if she quits the firm, is given by $1+\tau(1-\theta)$. Explain why the mobility cost might take this form. Solve for τ and w. Why is the firm investing in training in this case? Contrast these results with those obtained in part 2.

5. Contrast these results with the Becker view of training (in particular, contrast how the costs of training are shared between firms and workers in the two different views).

Exercise 2 Suppose that a worker's productivity is $f(h, s)$, where h is her general human capital, s is her firm-specific human capital, and $f(\cdot, \cdot)$ is strictly increasing in both of its arguments, continuous, differentiable and concave. If the worker quits her current employer, she receives an outside wage of $v(h) = f(h, 0)$. The wage with the current employer is determined by Nash bargaining where the worker's bargaining power is β.

1. Suppose that the worker's firm-specific human capital is fixed at $s = 0$, and the firm, and only the firm, can invest in the worker's general human capital h at the cost $c(h)$, where $c(\cdot)$ is continuous, differentiable and convex, and satisfies $c'(0) = 0$. Determine the equilibrium level of general human capital.

2. Suppose now that there is firm-specific human capital, $s > 0$, and also that $f(h, s) = f_1(h)+f_2(s)$. Determine the equilibrium level of general human capital in this case.

3. Suppose next that $s > 0$, and $\partial^2 f(h, s)/\partial h \partial s > 0$. Determine the equilibrium level of general human capital in this case and compare it to those in parts 1 and 2. Carefully explain why the result is different in this case.

4. What is the effect of β on the level of general human capital investment in the previous part?

5. Suppose now that the worker invests in her own firm-specific human capital with cost function $\gamma(\cdot)$, which is assumed to be continuous, differentiable and convex, and satisfies $\gamma'(0) = 0$. Investments in the two types of human capital are undertaken simultaneously. Determine the equilibrium level of general human capital investment in this case.

6. Show that in part 5, a higher level of β can increase the equilibrium level of general human capital investment. Provide an intuition and contrast this result to part 4.

Exercise 3 Consider a firm with two ex ante identical employees, $i = 1, 2$. At time $t = 0$ each employee decides whether to invest in his firm-specific skills at private cost c. If worker i makes this investment, we denote it by $s_i = 1$ and otherwise by $s_i = 0$. At the beginning of time $t = 1$, the firm decides the allocation of the two workers to tasks. There are two tasks, production and management. If both workers are assigned to the production task, then total output of the firm is

$$y^P(s_1) + y^P(s_2),$$

where $y^P(1) > y^P(0)$. If worker 1 is assigned to management and worker 2 to production, then the total output of the firm is

$$y^M(s_1) + y^P(s_2),$$

where

$$y^M(1) \geq y^P(1)\, y^P(0) \geq y^M(0).$$

Both workers cannot be assigned to management. Suppose that firm-specific skills and investments are observable (by the firm) but not contractible (i.e., neither task assignments nor wages can be conditioned on firm-specific skills), but the firm can commit to different wages for different tasks (w^M for workers employed in management and w^P for workers employed in production; thus can commit to a "wage structure" $\left(w^P, w^M\right)$). A worker can quit at any point and receive an outside option normalized to 0.

1. Define a subgame perfect equilibrium. [Hint: this should include an assignment function g for the firm that determines as a function of (s_1, s_2) which worker, if any, will be assigned to the management task].
2. Determine the equilibrium assignment of the firm as a function of (s_1, s_2) and the wage structure $\left(w^P, w^M\right)$.
3. Show that if $y^M(1) = y^P(1)$, then there exists no wage structure $\left(w^P, w^M\right)$ that will induce either employee to undertake investments in firm specific skills in any subgame perfect equilibrium. Provide an intuition for this result. [Hint: distinguish it from the "holdup problem" discussed in the lecture].
4. Now suppose that
$$y^P(1) + 2c > y^M(1) > y^P(1) + c.$$
Show that there exists a wage structure $\left(w^P, w^M\right)$ such that given this wage structure, one of the workers invests in firm-specific skills and the other one does not. At $t = 1$, the firm promotes the worker who has invested in firm-specific skills to the managerial position. [Hint: show that both workers do not want to invest in skills]. Provide an intuition for why this wage structure is providing incentives for firm-specific skills investment.
5. Now suppose that
$$y^M(1) > y^P(1) + 2c.$$
Show that the firm can choose a wage structure $\left(w^P, w^M\right)$ that encourages both workers to invest in firm-specific skills (and then promote one of two workers who have invested in firm-specific skills and management if both of them have done so). Determine the wage structure to achieves this.2
6. Do you find the possibility that the firm can manipulate the organizational structure to encourage firm-specific investments plausible? How else could the firm encourage firm-specific investments in this model?

Exercise 4 Consider an economy consisting of a large number of workers and firms. Each worker is infinitely lived in discrete time and maximizes the expected discounted value of income, with a discount factor $\beta < 1$. There is no ex ante heterogeneity among the workers, but the quality of the match between a worker and its employer is random, and is not directly observed by either. Suppose that the worker is a good match to its employer with probability $\mu_0 \in (0, 1)$ (and this does not change over time for a given worker-firm match). A worker who is a good match to its employer produces a stochastic output y drawn from a distribution F_h, while a worker who is not a good match produces a stochastic output y drawn from a distribution F_l. Suppose that workers are paid a constant fraction of their expected marginal product (with expectation taken with respect to all information available up to then). At any point in time, the worker can decide to quit. If he does so, he becomes unemployed. Unemployed workers receive an income of $b < \int y dF_l(y)$ and find a new match with probability $q < 1$.

1. What conditions do we need to impose on F_h and F_l such that workers who produce more are paid more?
2. Suppose that we have imposed this restriction on F_h and F_l. Is it also the case that workers who produce more are more likely to have a longer tenure with their firm?
3. Provide the explanation/intuition for why, conditional on staying with the firm, a worker has, on average, an upward sloping wage profile. Is this profile likely to be linear? Convex? Can you say about the variability of the wages of the worker who has longer tenure?
4. Provide conditions on F_h and F_l such that workers that are a good match (and only workers that are good match) can have infinite tenure (with some positive probability). What happens to the wages of workers that are in this "infinite tenure" range? What happens if these conditions are not satisfied?
5. Show that the wage that a worker receives just before quitting the firm is necessarily lower than the wage he will receive just after getting a new job. Is this also true when $q = 1$?
6. What are some additional facts that the model of this sort can account for, and what are some facts that it will have *d*ifficulty explaining?