Labor Economics, 14.661, Second Part, Problem Set 5

This problem set is for practice. You are strongly encouraged to work out these problems even though they are not to be handed in.

Exercise 1 Consider the McCall search model with a mass 1 of risk neutral individuals with discount factor equal to β and an exogenously given stationary distribution of wages $F(w)$. Assume that there is no unemployment benefit, so unemployed workers receive zero wage. Once a worker finds and accepts a job, he will be employed in this job until the job is destroyed exogenously, which happens with independent probability equal to s in every period. Once the job is destroyed, the individual returns to the unemployment pool. Suppose that at $t = 0$ all workers start out as unemployed.

1. Show that, provided that the worker never quits, the value of a worker who accepts a job at the wage w is given by
$$v^a(w) = w + \beta\left[(1-s)\, v^a(w) + sv\right],$$
where v is the value of an unemployed (searching) worker. Explain the intuition for this equation. Will the worker ever quit a job (unless there is an exogenous separation)?

2. Write down the dynamic programming recursion that characterizes the optimal behavior of an unemployed worker. Be specific about he assumptions you are making in writing this recursion (and justify these assumptions). Derive an expression for the value of an unemployed worker, v.

3. Find the reservation wage of the individual. Explain intuitively why this is constant over time. (Hint: use the fact that at the reservation wage R, the worker is indifferent between accepting the job and continuing to search, and combine this with the expression for v obtained in 2).

4. Find the the law of motion of unemployment. Why is unemployment not necessarily constant? Where does it converge to? Provide an interpretation of the limiting value unemployment in terms of separations and job creation.

5. What happens to reservation wages and the unemployment process when s increases?

6. Define the notion of "second-order stochastic dominance". What happens when $F(w)$ shifts to a new distribution $\tilde{F}(w)$ that has the same expected wage but second-order stochastically dominates F? Provide an intuition for this result.

Exercise 2 Consider a standard search model in continuous time where all workers have the same level of productivity, y. Workers and firms get together via a constant returns to scale matching function $M(U, V)$ where U is the number of unemployed workers and V is the number of vacancies. The flow cost of holding an open vacancy is γ, and unemployed workers get utility of leisure equal to z. Potential or existing firms open vacancies until a marginal vacancy makes zero-profits. Worker-firm matches come to an end at the flow rate s and all agents are risk-neutral and discount the future at the rate r. Wages are determined by Nash Bargaining where the bargaining power of the worker is β.

1. Write the Bellman equations, define an equilibrium and characterize it.

2. Suppose a utilitarian Social Planner (that means, the planner's objective is a simple average of the utility of all agents) can choose job creation and acceptance decisions. Characterize her choice, i.e., "the second-best allocation" (in deriving this result you can set $r = 0$).

3. Now, suppose the planner can only choose β of the wage determination rule. Show that there is a β^* such that if $\beta = \beta^*$, then the equilibrium achieves the best allocation from the planner's point of view.

4. Now suppose that z is unemployment benefit financed by lump-sum taxation. Suppose that the planner cannot directly choose job creation and acceptance decisions, and has to take β is given. She can only control z, and has to take the equilibrium as given conditional on a value of z. What is the value of z that the planner would like to choose (call this z^*)?

5. Now, suppose the planner chooses β and z. Determine the value of z, $\hat{z}$, that she would like to choose. Explain why $\hat{z}$ is different than z^*. Explain why the results regarding the choice of $\hat{z}$ are special, and discuss how you would modify this model to reach more realistic normative conclusions.

6. Show that as $\beta \to 1$, the unemployment rate, u, also tends to 1. Now consider the following critique of the model

   "The case of $\beta \to 1$ emphasizes that this is not a good model. β captures the division of output after the match. If β is too high, then the worker must be able to make an upfront payment, b, and get employed. By ruling out such payments, this model is ruling out the price mechanism."

   Discuss this claim. You might first want to show that in the logic of the model such payments are not possible, and then discuss how one could introduce such payments in this model, and whether or not they should be there for a realistic analysis of labor markets. Add any other angle that you see appropriate.

Exercise 3 Consider the following search model. Time is continuous, and all agents are risk-neutral and discount the future at the rate r. There is a mass 1 of workers. Workers and firms come together according to constant returns to scale matching technology, $M(U, V)$ where U is unemployment, and V is the mass of unfilled vacancies. A free entry condition determines V: a large mass of firms can open a new vacancy at the flow cost k. Once together, pairs separate at the flow rate s. Output of a pair is equal to $f(h)$ where h is the skill level of the worker, and assume $f(0) > 0$. Wages are determined by Nash Bargaining where the worker's bargaining power is β.

1. Find the steady state equilibrium assuming that $h = 0$ for all workers.

2. Now let us allow investments in human capital. Start the economy in steady state at time $t = 0$, and assume that at this point, firms can invest in h (training). Assume that this is the only time firms can invest in training (i.e., there are no further training opportunities at any $t > 0$) and also that workers cannot pay firms for this investment, nor can they commit to a wage cut. A higher h for a worker increases his productivity not only with this firm, but with all future employers because h is general human capital (and h is perfectly observed by all firms). Suppose that the cost of investing in human capital for the firm is $c(h)$ such that $c(0) = 0$, and c is strictly convex. Suppose also that there is a unique steady state. Characterize this steady state. In particular, (i) find the wage $w(h)$ as a function of human capital framework are matched at time $t = 0$; (ii) characterize the level of human capital h^* that firms at time $t = 0$ will invest in; and (iii) write down the free entry condition, recognizing that workers that were not initially employed will have human capital $h = 0$.

3. Now take the limit as $M(U, V) \to \infty$ (that is, if the probability of a match for a worker is p and that for a firm is q, then we have $p, q \to \infty$). Show that in this limit, firms do not invest in h.

4. Relate this finding to the general results on role of labor market imperfections and wage compression in general training investments.