Market Risk with Interdependent Choice*

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Abstract

Risks faced by traders from price movements are sometimes magnified by the actions of other traders. Risk management systems which neglect this feature may give a seriously misleading picture of the true risks. The hazards arising from this potential blindspot are at their most dangerous when the prevailing conventional wisdom lulls traders into a false sense of security on the attractivenss of a trading position. The efforts of one trader to reverse his trade makes more acute the

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need to follow suit on the part of others. For markets dominated by traders with short time horizons, such interdependence leads to exaggerated price movements. Estimates of 'value at risk' which recognize such interdependence of actions can diverge substantially from those given by conventional techniques.

1. Introduction

The summer and autumn of 1998 were exceptionally turbulent times for financial markets and the risk management systems of financial institutions engaged in proprietary trading went through a searching examination. Although the financial system pulled back from the brink and the feared financial meltdown did not materialize, many institutions suffered significant losses on their trading activities.

One theme which has emerged in the subsequent debate on the performance of the risk management systems has been the criticism that many financial entities entered the period of turbulence with very similar trading positions[1]. In one respect, this was entirely natural. If the prevailing conventional wisdom deems certain trades as being the most profitable, and the commonly available data butress this conventional wisdom, then it is understandable that institutions end up with similar trading positions. However, the consequence of this was that, when many of the institutions attempted to unwind their trading positions, they encountered similar attempts by others, leading to exaggerated price movements and the drying up of liquidity even in the most widely traded instruments. The collapse of the dollar against the yen on October 7th and 8th 1998 illustrated how even the most liquid of markets were vulnerable to concerted selling pressure. Thus, although we can explain why institutions entered the crisis with similar positions, this cannot be an excuse for any failures of risk management systems in place at the time. Why did so many sophisticated financial institutions with highly developed risk management tools get caught out? What was the blindspot?

Conventional risk management techniques rest on the assumption that risk management is a single-person decision problem - in the jargon, a "game against nature". That is, uncertainty governing price movements is assumed to be exogenous, and assumed not to depend on the actions of other decision makers. The analogy is with a gambler facing a spin of a roulette wheel, where the bets placed by other gamblers do not affect the outcome of the spin. The roulette wheel may have an unknown number of outcomes with differing probabilities, but as long as the outcome is unaffected by the actions of other gamblers, it is simply a matter of applying standard statistical techniques to past outcomes to enumerate what these outcomes are, and to estimate their respective probabilities. Much of the sophisticated techniques in the current state of the art can be seen as alternative ways of refining such estimation procedures, as well as tracking the non-linear payoff structures arising from derivative securities such as options[2].

[1] See, for instance, Economist, November 14th 1998, pp. 140 - 145.

[2] The technical documents provided by the RiskMetrics Group (1999) set out perhaps the most common techniques, based on the covariance structure of asset returns. Other approaches include simulations

In normal market conditions, when trading is orderly and markets function well, there is little harm in treating uncertainty as being exogenous in this way. However, during a crisis, such a world view is likely to throw up nasty surprises. When short run changes in prices depend on the actions of other traders, the "roulette wheel" view of uncertainty is no longer adequate. Since short run price changes depend on what others do, my decision depends on what others do. In other words, the uncertainty is *strategic*, in the sense used in game theory. When the outcomes of trading decisions depend on what others do, the uncertainty facing a trader has elements of poker, as well as roulette.

The neglect of strategic effects in risk management is all the more puzzling when set against the lessons drawn after the October 1987 crash of the stock market, barely a dozen years ago. The Brady Commission's report (1988) attributed the magnitude and swiftness of the price decline to practices such as portfolio insurance and dynamic hedging techniques. Such trading techniques have the property that they dictate selling an asset when its price falls and buying it when the price rises. Best estimates at the time suggested that around $100 billion in funds were following formal portfolio insurance programs, representing around 3 percent of the pre-crash market value. However, this is almost certainly an underestimate of total selling pressure arising from informal hedging techniques such as stop-loss orders (see the survey evidence presented in Shiller (1987)).

There are similarities between the 1987 crash and the events of 1998. Perhaps more than any other market, the fall of the dollar agains the yen in October share many of the same themes - dynamic hedging strategies, stop-loss orders and price magnification effects of selling into a falling market. There is also an irony here. Some of the institutions which suffered the largest losses due to the fall in the dollar were precisely those which had exploited the price-feedback effect of a market stampede of selling into a falling market during the Asian crisis of 1997. Thus, to understand the failure of risk management due to strategic uncertainty, it is instructive to examine first one of the trading "successes" of such institutions.

2. Currency Attacks

Defending a currency peg in adverse circumstances entails large costs for the government or monetary authorities. The costs bear many depressingly familiar symptoms -

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based on historical returns, and on Monte Carlo experiments. See Jorion (1997) or Goodhart et al. (1998, ch. 5) for an introduction.

collapsing asset values, rising bankruptcies, the loss of foreign exchange reserves, high interest rates and the resulting reduction in demand leading to increases in unemployment and slower growth. Whatever the perceived benefits of maintaining a currency peg, and whatever their official pronouncements, all monetary authorities have a pain threshold at which the costs of defending the peg outweighs the benefits of doing so. Understanding the source and the severity of this pain is a key to understanding the onset of currency attacks.

Facing the monetary authority is an array of diverse private sector actors, both domestic and foreign, whose interests are affected by the actions of the other members of this group, and by the actions of the monetary authority. The main actors are domestic corporations, domestic banks and their depositors, foreign creditor banks, and outright speculators - whether in the form of hedge funds or the proprietary trading desks of investment banks. Two features deserve emphasis.

- Each actor faces a choice between actions which exacerbate the pain of maintaining the peg and actions which are more benign.
- The more prevalent are the actions which increase the pain of holding the peg, the greater is the incentive for an individual actor to adopt the action which increases the pain. In other words, the actions which tend to undermine the currency peg are mutually reinforcing.

For domestic corporations with unhedged dollar liabilities, they can either attempt to hedge their positions or not. The action to hedge their exposure - of selling Baht to buy dollars in forward contracts, for example, is identical in its mechanics (if not in its intention) to the action of a hedge fund which takes a net short position in Baht. For domestic banks and finance houses which have facilitated such dollar loans to local firms, they can either attempt to hedge their dollar exposure on their balance sheets or not. Again, the former action is identical in its consequence to a hedge fund short-selling Baht. As a greater proportion of these actors adopt the action of selling the domestic currency, the greater is the pain to the monetary authorities, and hence the greater is the likelihood of abandonment of the peg. This increases the attractiveness of selling Baht. In this sense, the actions which undermine the currency peg are mutually reinforcing. They are "strategic complements", in the sense used in game theory.

Indeed, the strategic effects run deeper. As domestic firms with dollar liabilities experience difficulties in servicing their debt, the banks which have facilitated such dollar loans attempt to cover their foreign currency losses and improve their balance sheet by a contraction of credit. This in turn is accompanied by a rise in interest rates, fall in profit and a further increase in corporate distress. For foreign creditor banks with short-term exposure, this is normally a cue to cut off credit lines, or to refuse to roll over short term debt. Even for firms with no dollar exposure, the general contraction of credit increases corporate distress. Such deterioration in the domestic economic environment exacerbates the pain of maintaining the peg, thereby serving to reinforce the actions which tend to undermine it. To make matters worse still, the belated hedging activity by banks is usually accompanied by a run on their deposits, as depositors scramble to withdraw their money.

The following table contains a (somewhat simplistic) taxonomy of the various actors and their actions which undermine the peg. The feature to be emphasized is the increased pain of maintaining the peg in the face of widespread adoption of such actions, and hence the *mutually reinforcing* nature of the action which undermines the peg. The greater is the prevalence of such actions, the more attractive such actions become to the individual actor.

Actor	Action(s) undermining peg
Speculators	Short sell Baht
Domestic firms	Sell Baht for hedging purposes
Domestic banks	{ Sell Baht for hedging purposes Reduce credit to domestic firms
Foreign banks	Refuse to roll over debt
Depositors	Withdraw deposits

To be sure, the actual *motives* behind these actions are as diverse as the actors themselves. A currency speculator rubbing his hands and looking on in glee as his target country descends into economic chaos has very different motives from a desperate owner of a firm in that country trying frantically to salvage what he can, or a depositor queuing to salvage her meagre life savings. However, whatever the motives underlying these actions, they are similar in their consequences. They all lead to greater pains of holding to the peg, and hence hasten its demise.

2.1. Dollar/yen in October 1998

The mutually reinforcing effect of individual traders' actions in the context of large unhedged trading positions may be a useful way to understand the behaviour of the dollar agains the yen over two memorable days (October 7 - 8, 1998) when the dollar fell from 131 yen to 112 yen by lunchtime in London on Thursday the 8th, bouncing back sharply to end New York trading at 119 yen. October 7th and 8th were perhaps two of the most turbulent days of trading in financial markets in recent memory, which also saw sharp falls in longer dated government bonds and the virtual seizing up of markets for corporate debt, and for less liquid government debt instruments.

The fall in the dollar was especially dramatic given its strength throughout the spring and summer of 1998, reaching its high of 147.26 yen on August 11th. Many commentators were predicting that dollar/yen would reach 150 or perhaps 200 by the end of the year, especially in the light of the apparent failure (in June) of the joint intervention by the U.S. and Japan to support the yen more than temporarily. The conventional wisdom among academics, commentators and traders alike was that the yen was bound to fall, and that it was a matter of the speed and the magnitude of its fall rather than the directon. Indeed, by the summer of 98, this conventional wisdom had almost acquired the status of an immutable truth. Although such arrogance seems misplaced with the benefit of hindsight, it is easy to see how such a confident view of the world arose. Since the spring of 1995, the dollar had continued to appreciate against the yen (with a brief respite in mid-1997), and the contrasting macroeconomic fortunes of the U.S. and Japan, with strong growth in the former and weakness in the latter seemed to presage more of the same in the months ahead.

The combination of an appreciating dollar and the large interest rate differential between Japan and the U.S. gave rise to the singularly profitable trading opportunity of borrowing yen, buying dollar assets, and gaining both on the appreciation of the dollar and the interest rate differential. This "yen carry" trade was widespread among hedge funds, the proprietary trading desks of investment and commercial banks, and even some corporations. Funds were raised in the interbank market through term repo agreements, or by issuing money market paper. Then these funds would be swapped for foreign currency or exchanged in the spot market to fund purchases of higher-yielding assets, including U.S. Treasuries, corporate bonds, mortgage-backed securities and also even riskier instruments such as Russian GKOs. Japanese banks also resorted to the yen-carry trade by accumulating foreign assets. In the first three quarters of 1998, the net holdings of assets denominated in foreign currencies increased by about $44 billion, while the holdings of yen-denominated assets abroad declined by $103 billion (IMF (1998, p.126)). Thus, the conventional wisdom concerning the relentless rise in dollar/yen was also apparently shared by the Japanese instutitions.

The tide began to turn after the Russian default in August, but the initial weakening of the dollar was relatively orderly, falling by less than 10 percent against both the yen and the deutschmark between mid-August and early October. However, in the week beginning October 5th, the decline of the dollar against the yen accelerated sharply - closing down roughly 15 percent over the week. Significantly, the fall in the dollar against the deutschmark was much less pronounced, falling less than 2 percent during the week. It was also noteworthy how this fall in dollar/yen coincided with an unprecendented steepening of the yield curve for mature debt markets outside Japan, as bond yields bounced back from their historical lows. During the same week, the yield gap between three month rates and 10 year rates widened by 85 basis points in the U. S., 60 basis points in the U. K., and 50 basis points in Germany. The coincidence of (i) the rapid fall in dollar/yen (ii) less precipitous fall in dollar/deutschmark and (iii) rapid steepening of the yield curve in markets outside Japan is consistent with the unwinding of the yen carry trades.

The logic of the mutually reinforcing effects of selling into a falling market dictates that a *moderate* fall in asset value is highly unlikely. Either the asset does not fall in value at all, or the value falls by a large amount. Our main task in this paper will be to model this phenomenon, and to capture the true value at risk in such contexts. The fall in dollar/yen is also likely to have been exaggerated by stop-loss orders, and by the cancellation of barrier options and the unwinding of associated hedging positions by dealers. One estimate of the volume of outstanding yen foreign currency contracts at the end of June was in excess of $3.3 trillion (Bank of Japan (1998)). Just as in the stock market crash of 1987, the effect of such trading techniques is to exaggerate price movements, by selling into a falling market. In retrospect, sharp price movements are exactly what one should expect in a market which is marked by such high levels of leverage, undertaken by so many diverse institutions. The unwinding of yen carry trades proceeded at such a pace that press reports referred to market rumours of imminent collapse of one or more hedge funds. The Bank of Japan reported large buying of yen by at least one large hedge fund (*Financial Times*, October 9th, p.19).

The poignant irony could not have been lost on observers of the Asian financial crisis. Just a year earlier, the hedge funds and assorted proprietary trading desks of banks had profited handsomely from the stampede by Asian borrowers with unhedged dollar liabilities to cover their positions in a desperate attempt to keep afloat. In October 1998, these same "sharks" had become their own bait. It was now they who were scrambling to cover their positions. The logic of mutually reinforcing sales meant that the harder they tried to swim away, the more they provoked the feeding frenzy. The sense of fear was palpable during the turbulent trading of October 8th. With sentiment already fragile after the forced rescue of Long Term Capital Management, rumours of the immient collapse of a major hedge fund further reinforced the disengagement from risk. Can the events of October be seen as a "currency attack" on the dollar? Although it may seem incongruous even to entertain such a question, there is an uncanny resemblance to the Asian financial crisis - large unhedged foreign currency positions, and the scramble to unwind these positions exacerbated by the price feedback effect of selling into a falling market. There is, of course, one important difference between the collapse of the dollar and the Asian financial crisis. Unlike its Asian precursors, the U. S. Federal Reserve *cut* U. S. interest rates in response to the crisis, injecting liquidity and curtailing the vicious circle of selling. This was very different from the policy response to the Asian crisis. The medicine prescribed for these countries was for steeply *higher* interest rates, exacerbating financial distress and fuelling the vicious circle of selling[3].

3. Modelling Market Risk

Although the mutually reinforcing effects of certain actions have been well understood in the academic debate, one of the difficulties in developing this theme for risk management is that a formal analysis of this problem yields multiple equilibria, and hence do not yield a definitive prediction of the outcome. This indeterminancy is largely due to the self-fulfilling nature of the belief in an imminent sell off. If speculators and exposed borrowers believe that a currency will come under attack, their actions in anticipation of this precipitate the crisis itself, while if they believe that a currency is not in danger of imminent attack, their inaction spares the currency from attack, thereby vindicating their initial beliefs.

In the context of 2 person binary action games, Carlsson and van Damme (1993) showed how a unique equilibrium may be obtained when the payoffs cease to be comment knowledge. Morris and Shin (1998, 1999)) have extended this method to examine coordination problems in a number of applications in finance. The theory rests on two features.

- The actions of market participants can be mutually reinforcing.
- Market participants have access to a large mass of information concerning market fundamentals, and hence are often well informed of the underlying state. However, perhaps because of the sheer volume of information, there are small disparities in the information at the disposal of each market participant.

The first of these features has already been discussed. The innovation comes with the second feature. When there are small disparities in the information of the market

[3] This is not the place to pursue this issue further, but it is a useful exercise to think of the appropriate dimensions in which the two cases differ. The issue of whether an attack on one's currency should be met by a tight or loose monetary policy deserves more systematic treatment.

participants, the indeterminacy of beliefs inherent in the multiple equilibrium story is largely removed. Instead, it is possible to track the shifts in beliefs as we track the shifts in the economic fundamentals. This is so, since uncertainty about others' beliefs now takes on a critical role, and such uncertainty often dictates a particular course of action as being the uniquely optimal one. Even vanishingly small differences in information suffice to generate such uncertainty about others' beliefs. When we consider the sheer quantity of information available to market participants - the news wire services, in-house research, leaks from official sources, as well as the press and broadcasters, exact uniformity of information is the last thing we can expect.

Indeed, the fragmentation of the media in modern times has generated the paradoxical situation in which ever greater quantities of information is generated and disseminated, but comes at the expense of the shared knowledge of its recipients. Apart from totalitarian regimes in which there is a singe source of information (or perhaps in the heyday of the BBC Home Service), the receipt of information is rarely accompanied by the knowledge that everyone else is also receiving precisely this information at that time. Even among financial markets, the foreign exchange market is especially fragmented. Its market microstructure is characterized by the decentralized nature of the trade necessitated by round-the-clock trading, and the geographical spread which goes with it. At its most basic, a speculative attack is a resolution of a coordination problem among the diverse interested parties - both foreign and domestic. Small disparities of information determine the outcome of such coordination problems.

When these two ingredients are brought together, the apparent mulitplicity of equilibria induced by mutually reinforcing actions makes way for a unique equilibrium in which market participants employ a "switching strategy". That is, market participants base their actions on their best estimate of the underlying fundamentals, bearing in mind that all other market participants are engaged in the same exercise. A switching strategy is a rule of action in which the action chosen is determined by whether the best estimate of the underlying fundamentals is above or below some pre-determined benchmark level. This equilibrium also happens to be a symmetric equilibrium, in the sense that the same benchmark switching point is used by all the market participants. In what follows, we develop a simple model of market risk along these lines.

3.1. Trading Game

There is a continuum of traders, each of whom have identical holdings of a risky asset. The traders have the log utility function:

$$u(c) = \log c. \tag{3.1}$$

There are three dates - *initial, interim* and *final*. The value of the asset at the final date is determined in part by a log-normal random variable R. The logarithm of R is denoted by r, so that r is a normal random variable. We suppose that r has mean $\overline{r}$ and precision α (i.e., variance $\frac{1}{\alpha}$). This random variable should be thought of as determining the "fundamental" value of the asset. The actual liquidation value of the asset will depend on the incidence of selling pressure, as we will describe below.

At the interim date, the traders have access to information about r. Trader i observes the realization of the signal

$$x_i = r + \varepsilon_i \tag{3.2}$$

where ε_i is normally distributed with mean 0 and precision β. The noise terms are i.i.d. across traders. Based on his own information, each trader decides whether to hold on to the asset, or to sell it. We will assume a very simple liquidation process whereby any trader can sell his asset for 1 unit of the consumption good. We denote by

$$\ell \tag{3.3}$$

the proportion of traders who sell their asset at the interim date, and suppose that the liquidation value of the asset at the final date is given by

$$Re^{-\ell} \tag{3.4}$$

Thus, the greater is the incidence of selling at the interim date, the lower is the value of the asset at the final date.

3.2. Equilibrium

We now solve for the equilibrium of this trading game. The focus is on the traders' decisions at the interim date. Trader i observes his signal x_i, and forms the updated belief concerning the return r, and the possible signals obtained by other traders. Based on this information, trader i decides whether to sell or hold. A *strategy* for a trader is a rule of action which prescribes an action for each realization of the signal. A profile of strategies (one for each trader) is an equilibrium if, conditional on the information available to trader i and given the strategies followed by other traders, the action prescribed by i's strategy maxmizes his conditional expected utility. Treating each realization of i's signal as a possible "type" of this trader, we are solving for the Bayes Nash equilibria of the imperfect information game. To economize on the statement of the results, we assume that if selling yields the same expected utility as holding, then the trader prefers to hold. This assumption plays no substantial role in what follows.

Since both r and x are normally distributed, a trader's updated belief of r upon observing signal x is

$$\rho = \frac{\alpha \bar{r} + \beta x}{\alpha + \beta} \tag{3.5}$$

The introduction of uncertainty yields a unique equilibrium if private signals are sufficiently accurate. The result depends on the prior and posterior precision of r. Specifically, let

$$\gamma \equiv \frac{\alpha^2 (\alpha + \beta)}{\beta (\alpha + 2\beta)}, \tag{3.6}$$

and write $\Phi(.)$ for the standard normal distribution function. Our main result states that there is a unique equilibrium in this context, provided that γ is small enough.

Theorem. *Provided that $\gamma \leq 2\pi$, there is a unique equilibrium. In this equilibrium, every trader sells if and only if $\rho < \rho^*$ where ρ^* is the unique solution to*

$$\rho^* = \Phi\left(\sqrt{\gamma}\left(\rho^* - \bar{r}\right)\right).$$

In the limit as γ tends to zero, ρ^* tends to $\frac{1}{2}$.

Provided that the traders' signals are precise enough (β is high relative to α), every trader follows the switching strategy around the critical value ρ^*. This critical value is obtained as the intersection of a cumulative normal distribution function with the 45° line, as depicted in figure 3.1. In the limiting case when the noise becomes negligible, the curve flattens out and the critical value ρ^* tends to 0.5.

Let us sketch the argument behind this result. For ρ^* to be an equilibrium switching point, a trader whose updated belief is exactly ρ^* ought to be indifferent between holding the asset and selling it. The utility of selling the asset is

$$\log 1 = 0 \tag{3.7}$$

while the utility from holding the asset is

$$\log\left(Re^{-\ell}\right) = r - \ell \tag{3.8}$$

At the switching point ρ^*, the expectation of $r - \ell$ conditional on ρ^* must therefore be zero. The expectation of r conditional on ρ^* is simply ρ^* itself. Thus, consider the

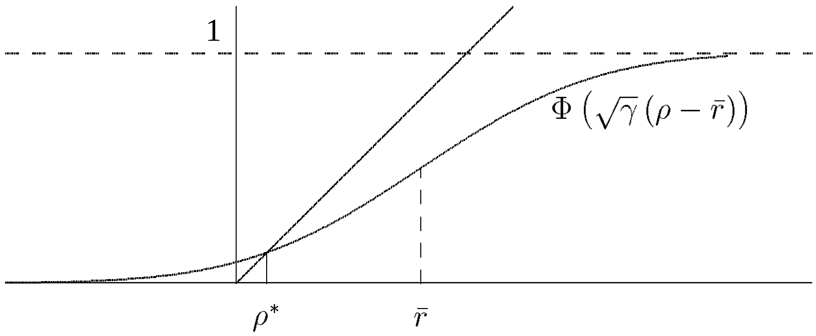

Figure 3.1: Switching point ρ^*

expectation of ℓ conditional on ρ^*. Since noise is independent of the true return r, the expected proportion of traders who sell is equal to the probability that any particular trader sells. And since the hypothesis is that every trader follows the switching strategy around ρ^*, the probability that any particular trader sells is given by the probability that this trader's updated belief falls below ρ^*.

When trader i has posterior belief ρ_i, what is the probability that i attaches to some other trader j having posterior belief lower than himself? Figure 3.2 illustrates the reasoning.

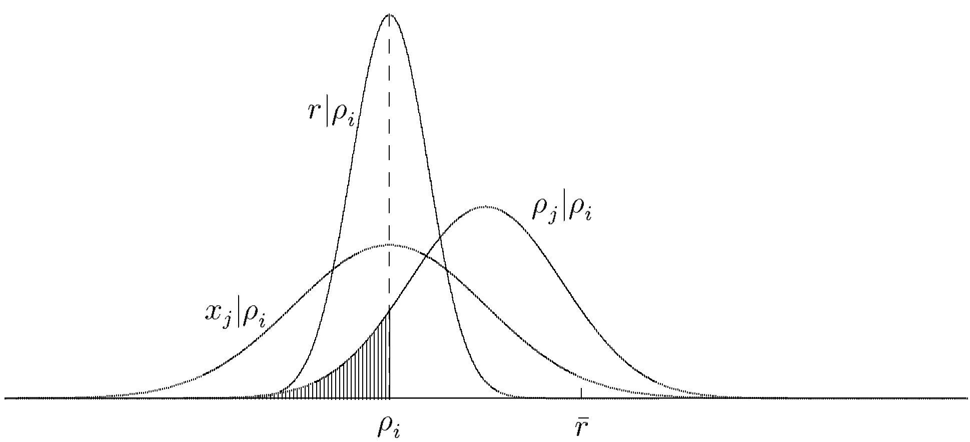

Figure 3.2: Beliefs conditional on ρ_i

Conditional on ρ_i, return r is normal with mean ρ_i and precision $\alpha+\beta$. Since $x_j = r+\varepsilon_j$, the distribution of x_j conditional on ρ_i is normal with mean ρ_i and precision:

$$\frac{1}{\frac{1}{\alpha+\beta}+\frac{1}{\beta}} = \frac{\beta\left(\alpha+\beta\right)}{\alpha+2\beta}. \tag{3.9}$$

But $\rho_j = \left(\alpha\bar{r}+\beta x_j\right)/\left(\alpha+\beta\right)$, so that the distribution of $\rho_j|\rho_i$ is as depicted in figure 3.2, and the probability that ρ_j is less than ρ_i conditional on ρ_i is given by the shaded area. Moreover,

$$\rho_j < \rho_i \Leftrightarrow \frac{\alpha\bar{r}+\beta x_j}{\alpha+\beta} < \rho_i \Leftrightarrow x_j < \rho_i + \frac{\alpha}{\beta}\left(\rho_i-\bar{r}\right) \tag{3.10}$$

so the question of whether ρ_j is smaller than ρ_i can be reduced to the question of whether x_j is smaller than $\rho_i + \frac{\alpha}{\beta}\left(\rho_i-\bar{r}\right)$. Hence,

$$\begin{aligned}
\text{Prob}\left(\rho_j < \rho_i|\rho_i\right) &= \text{Prob}\left(x_j < \rho_i + \frac{\alpha}{\beta}\left(\rho_i-\bar{r}\right)\middle|\,\rho_i\right) \\
&= \Phi\left(\sqrt{\tfrac{\beta(\alpha+\beta)}{\alpha+\beta}}\left(\rho_i + \frac{\alpha}{\beta}\left(\rho_i-\bar{r}\right)-\rho_i\right)\right) \\
&= \Phi\left(\sqrt{\gamma}\left(\rho_i-\bar{r}\right)\right). \qquad (3.11)
\end{aligned}$$

So, the shaded area in figure 3.2 can be represented in terms of the area under a normal density which is centered on the ex ante mean $\bar{r}$. Figure 3.3 illustrates.

If ρ^* is an equilibrium switching point, the expectation of $r-\ell$ conditional on ρ^* must be zero. Since

$$\text{E}\left(r-\ell|\rho^*\right) = \rho^* - \Phi\left(\sqrt{\gamma}\left(\rho^*-\bar{r}\right)\right), \tag{3.12}$$

ρ^* must be the point at which $\Phi\left(\sqrt{\gamma}\left(\rho-\bar{r}\right)\right)$ intersects the 45° line, exactly as depicted in figure 3.1. Provided that γ is small enough, the slope of $\Phi\left(\sqrt{\gamma}\left(\rho-\bar{r}\right)\right)$ is less than one, so that there can be at most one point of intersection. Since the slope of the cumulative normal is given by the corresponding density function (which has the maximum value of $\sqrt{\gamma/2\pi}$), we can guarantee that there is a unique intersection point provided that γ is less than 2π. All that remains is to show that if there is a unique symmetric equilibrium in switching strategies, there can be no other equilibrium. The appendix completes the argument.

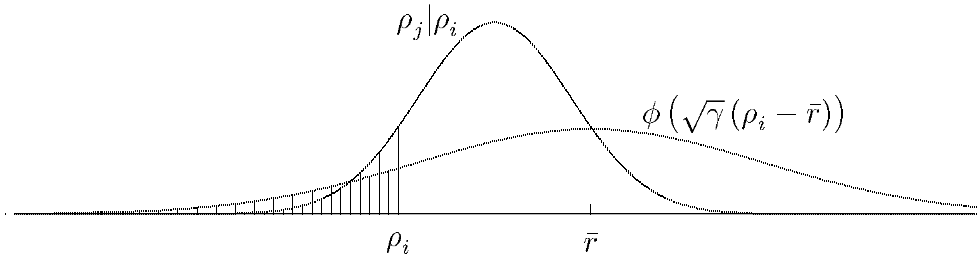

Figure 3.3: Density $\phi\left(\sqrt{\gamma}\left(\rho_i - \bar{r}\right)\right)$

3.3. Value at Risk

Consider the incidence of selling as given by the equilibrium value of ℓ. This incidence is a random variable that depends on the realized return r. A trader sells whenever his posterior belief falls below the critical value ρ^*, which happens whenever

$$\frac{\alpha\bar{r} + \beta x_i}{\alpha + \beta} < \rho^*.$$

In other words, a trader sells whenever the realization of his signal x_i falls below the critical value

$$x^*\left(\rho^*, \bar{r}\right) \equiv \frac{\alpha + \beta}{\alpha}\rho^* - \frac{\alpha}{\beta}\bar{r} \tag{3.13}$$

Since $x_i = r + \varepsilon_i$, the incidence of selling is a function of the realized return r, and is given by

$$\ell\left(r\right) = \Phi\left(\sqrt{\beta}\left(x^*\left(\rho^*, \bar{r}\right) - r\right)\right). \tag{3.14}$$

Figure 3.4 illustrates.

From this, we can examine the distribution of log returns. Since the incidence of selling is a function of r, we can write the liquidation value also as a function of r. Let

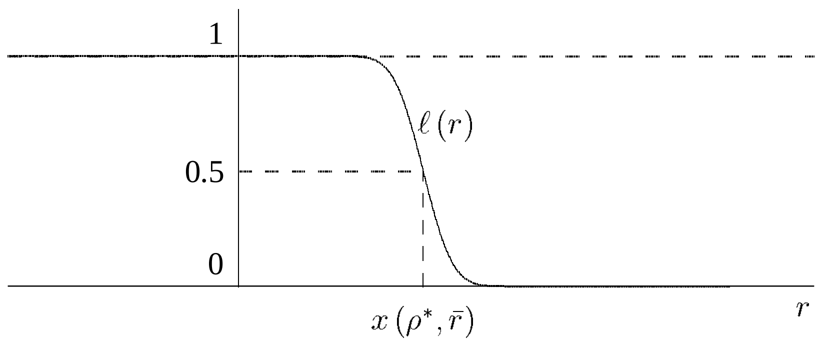

Figure 3.4: Incidence $\ell(r)$ of selling

$Q(r)$ be this liquidation value. Then,

$$\begin{aligned} q(r) &\equiv \log Q(r) \\ &= r - \ell(r) \\ &= r - \Phi\left(\sqrt{\beta}\left(x^*(\rho^*, \bar{r}) - r\right)\right). \end{aligned}$$

Since we know the distribution of r (it is normally distributed), we can now examine the distribution of $q(r)$. For any number y,

$$\begin{aligned} \text{Prob}\left(q(r) \le y\right) &= \text{Prob}\left(r \le q^{-1}(y)\right) \\ &= \Phi\left(\sqrt{\alpha}\left(q^{-1}(y) - \bar{r}\right)\right). \end{aligned} \tag{3.15}$$

where $\bar{r}$ is the ex ante mean of r. Denoting by

$$h(y) \equiv q^{-1}(y),$$

we can derive the density function for $q(r)$ by differentiating (3.15) with respect to y. It is given by

$$\sqrt{\alpha} \cdot h'(y) \cdot \phi\left(\sqrt{\alpha}\left(h(y) - \bar{r}\right)\right) \tag{3.16}$$

The shape of the density is determined by the slope of the $h(\cdot)$ function. Figure 3.5 illustrates the h function.

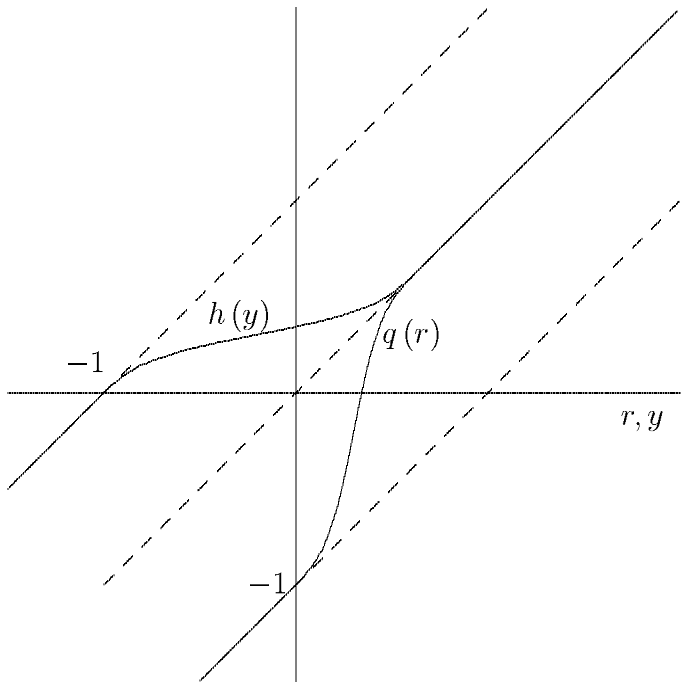

Figure 3.5: The function $h\left(y\right)$

For the range over which h has a shallow slope, the corresponding density is low. As noise becomes smaller so that the incidence of sales $\ell\left(r\right)$ becomes a step function, the function h becomes flatter. The consequence is that the density of log return becomes bi-modal. Figure 3.6 illustrates.

[Figure 3.6 here]

Estimates of value at risk based on normally distributed returns will severely underestimate the true value at risk in this context. Nor is it clear that realized historical returns capture the appropriate effect if the trading context in the past differ in material ways from current market conditions. Correct estimates must rest on the extent of the strategic interaction across traders. This in turn depends on the degree to which selling by one trader will force selling by others. Uniformity of trading positions and the degree of leverage will be prime determinants of this spillover effect. The lesson to be drawn from the formal analysis is that a proper estimate of value at risk must pay heed to the strategies followed by *other* market participants. Seasoned traders will see this point as being rather obvious. The onus is on those who formulate overall risk management policies to catch up, and to quantify this effect.

4. Systemic Risk and Regulation

The adoption of explicit risk management techniques has been accompanied by a growing acceptance by regulators of self-policing by the financial institutions themselves using their own internal risk management models. This growing acceptance has raised the stakes in the search for adequate risk management systems. The initial proposals by the Basel Committee on Banking Supervision (1993) to deal with market risk generated by proprietary trading was much more cautious, and placed relatively little weight on the internal risk management models. Indeed, it had more in common with the "building blocks" approach of the original 1988 Basel Accord. However, during the consultation process which followed, the banking industry mounted a successful campaign to establish the use of internal models. In two BIS documents two year later (1995a, 1995b), the principle was conceded by the Basel Committee, and this concession was enshrined in the amendment to the Basel Accord the following year (BIS 1996). Thus, from January 1st 1998, the provisions of the amendment came into effect, requiring internationally active banks in the G-10 countries to maintain regulatory capital to cover market risk.

It was unfortunate that 1998, the first year of the new regimen, saw such unprecedented market turbulence. Much of this turbulence had its roots in trading decisions taken much earlier on, but it should give food for thought for both regulators and market participants. What's at issue is whether such bouts of turbulence will subside as more sophisticated versions of current risk management techniques become more widely adopted, or whether the more widespread adoption of such techniques merely serve to increase the fragility of the system. If the argument in this paper has any force, then the latter possibility cannot be ruled out. As long as the world view underlying the risk management models discounts the feedback effect from actions to outcomes, the building blocks underlying such models remain suspect. If the externalities generated by one trader's actions on the payoff distribution of another is not taken into account, then assumptions supporting the model are undermined.

The term "externality" is used advisedly. The usual context in which this notion appears is in welfare economics, such as when applied to environmental issues, in which the absence of markets generates inefficient outcomes among market participants. Thus, when I take my car out on to the congested roads, I am contributing to the congestion, but this added inconvenience to others is not priced by the market, as there is no market for unencumbered use of the road. There is an analogy with the trading decisions of market participants. When one hedge fund decides to engage in the yen carry trade, the decision is based on the profitability for that trader alone. However, by short-selling the yen, this trader generates an externality for all other market participants who are engaged in the same trade. This is so, since when the yen begins to rise, its rise will be that much more accentuated by the attempt to cover the short yen position by this trader. Thus, just as a driver discounts the inconvenience caused by his own driving on the welfare of other drivers, the hedge fund discounts the possible losses inflicted on other market participants by his own trades.

Indeed, the externalities inflicted by traders on other traders will be worse than this analogy suggests. For a driver taking his car out on to the road, he at least will anticipate the selfish actions of other drivers - daily experience of congestion will have reinforced this. However, the hedge fund engaging in the yen carry trade will underestimate the risks if the trading positions of other traders are ignored. The hedge fund may hold *incorrect* beliefs if his risk management model is based on a "roulette-wheel" view of the world in which there is no feedback effect from the actions of other traders on the market outcome. During normal, tranquil market conditions, the daily signs from the market do not serve to warn the hedge fund of impending danger. As seen from figure 3.6, the price distribution is only distorted for one of the tails of the distribution. As long as the underlying fundamentals move within a small interval of the median, the outcomes are indistinguishable from that generated by the symmetric normal distribution. It is only when the underlying fundamentals wander off to the left that the hedge fund will realize that something is seriously wrong. But by then, it is too late.

Externalities justify a role for the regulator, whether it be in reducing congestion on the roads, or in reducing the damaging effects of market turbulence. This role can be justified even though the individual decision makers are perfectly rational, and are able to take informed decisions themselves. The incentives for individuals, whether they be individual drivers or traders, do not always take into account the effect of their decisions on others' welfare. The Basel Committee (BIS 1995b) has provided for a "buffer" in the captial requirements set against market risk, in which the value at risk obtained from the internal risk-management models of the banks is multiplied by a factor of three to reach the capital requirement. Indeed, this factor is raised by a colour-coded "plus factor" if the internal models of the banks perform inadequately in actual trading. The green zone attracts no plus factor, while the yellow zone attracts a plus factor of 0.4 to 0.85, rising in the red zone to a full point. Thus, banks in the red zone must set aside four times the value at risk obtained from their internal model. Such a buffer may serve to extinguish some of the dangers arising from the externalities generated by traders' trading positions on others' but it is likely to be a subject of some controversy in the days to come, since such provisions undermine the profitability of banks.

The term “transparency” has been a touchstone of the policy response following the Asian crisis of 1997/8, and the issue has taken on added significance following market turmoil of the summer and autumn of 1998. It has figured prominently in numerous official publications (IMF (1998b), BIS (1999)). The debate on transparency has many themes, but one presumption running throughout the debate has been that it was a lack of information about the underlying fundamentals which exacerbated the crises, both during the Asian crisis, and the subsequent turbulence in the mature financial markets. There is a sense in which this presumption is well-founded, and another sense in which it is not.

In one respect, lack of information was a key. If a hedge fund uses an incorrect risk management model by, say, disregarding the trading positions of other traders, then the dangers of the situation can be impressed upon the hedge fund manager by showing him the correct model, and educating him on the true risks involved.

However, there is another sense in which “transparency” is a red herring. Suppose that all the traders now begin to use the correct risk management model which takes into account the trading positions of others. The externality problem is not solved by information alone. There is still a mismatch between the incentives of an individual trader and overall welfare, just as the driver taking his car out on to the congested roads will not factor in the environmental harm done by his driving. Now, what is the effect of better information in this instance? What is the effect of the provision of more accurate and timely information to market participants? In terms of the formal theory described above, more accurate information corresponds to a smaller degree of noise in the signals of the market participants, and the switching strategies used by these traders can now rely on better information concerning the fundamentals. In the limit, as the noise becomes negligible, there will then be an exact correspondence between the true state of fundamentals and the perception of these fundamentals by the market participtants, and hence their switching strategies will dictate a much sharper break than before. In other words, the violence of sudden market movements may be exacerbated by better information. More formally, greater precision of the traders’ signals corresponds to an increse in β. This makes the $q(\cdot)$ function steeper, and hence makes its inverse *shallower* (see figure 3.5). Thus, the “double hump” of the return distribution becomes more pronounced. In the limit, as β tends to infinity, the left tail of the return density becomes detached from the main body of the density.

There are some important lessons for the conduct of public policy in dissemination of information. When calling for improved transparency, it is important to be clear as to *how* the improved information will improve the outcome. The mere provision of information may not be enough to preclude market turbulence. With the benefit of theoretical hindsight, it is perhaps not surprising that the provision of more information to market participants does not mitigate the coordination problem. After all, we should draw a distinction between a single-person decision problem and a strategic situation. In a single-person decision problem, more information is always more valuable. When I debate whether to carry an umbrella into work, an accurate weather forecast will minimize both the inconvience of carrying a bulky umbrella on a sunny day, and also the opposite inconvenience of getting caught in a shower without shelter. In such instances, "transparency" works.

However, it is far from clear whether better information will mitigate a coordination problem. There is little guidance from economic theory that better information about payoffs to players of a coordination game leads to greater incidence of successful coordination. Indeed, the intuition conveyed by exisiting theory is of a much more prosaic sort - typified by the debate on the Coase Theorem - in which all the emphasis is placed on the impediments to efficient bargaining. When the interested parties are diffuse and face uncertainty both about the fundamentals and the information of others, it would be overly optimistic to expect ex post efficient bargains to be struck. In the case of credit risk, where the issue is the coordination of diverse creditors facing a distressed borrower, it is possible to contemplate institutions which may, in principle, serve to achieve successful coordination - especially when led by a forceful facilitator. Such institutions could be seen as the "Coasian" solution to the externality problem, relying on the self-interested bargaining of interested parties.

For market risk, however, it is difficult to see how any institutional setup can implement the Coasian solution. Markets, by their nature, rely on the decentralized decision makers making their decisions in isolation from others. The text book alternative to Coasian bargaining is the introduction of taxes and subsidies to align individual incentives towards collectively efficient outcomes. But even here, it is only marginally more plausible than Coasian bargaining itself. The monitoring and enforcement powers available to the regulators will make any fine-tuning all but impossible, while crude measures may do more harm than good. To a large degree, the externalities associated with market risk will be impossible to remove.

5. Concluding Remarks

Episodes of market turbulence such as that experienced last year are a rarity, and the desire to prevent a repeat of such episodes must be tempered by the need to allow financial entities to pursue their legitimate commercial interests. Striking the proper balance rests on the proper recognition of the sources of fragility of the market and the targeting of these weaknesses, guarding against crude, ham-fisted measures borne out of a knee-jerk reaction to market volatility. This essay has emphasized two issues in particular.

- The "roulette wheel" view of market uncertainty is inadequte as a basis for modelling market risk. For markets whose outcomes depend on the actions of market participants, game theoretic issues must be addressed explicitly. The greater the leveraged positions of the traders in those markets, and the greater the uniformity of these trading positions, the more important it is to recognize the feedback effect from outcomes to actions back to outcomes. One role for "transparency" in the market is in aiding the education of market participants and in bringing the potential whiplash effects of such markets to their attention.

- However, transparency is not a panacea. Even if every market participant transcends the roulette wheel world view to recognize the interdependent choices in these markets, this does not fully align the incentives of the market participants towards the collective interest. Just as a driver does not price in the congestion externality when taking his car on the road, a trader does not take into account the externality generated by mimicking the trading position of another trader. In this respect, regulation still has a place.

The foreign exchange market is perhaps the best case where these lessons may usefully be kept in mind. One of the enduring puzzles in financial economics is why "uncovered interest parity" does not hold in practice. That is, why it is that differences in interest rates do not perform well as a predictor of the future movement of exchange rates[4]. On average, it has been profitable to borrow a currency with a low interest rate and buy assets denominated in the currency with a higher interest rate. This being so, there is always a bias towards trading positions which bet against uncovered interest parity. Given the size of the foreign exchange market the collective trading positions can take on enormous magnitudes. The yen carry trade of the late nineties was just an extreme case of this. At the moment of writing, the Euro has fallen to its lowest level to the dollar - just under 0.89 dollars as compared to 1.17 at its launch. So far, it has been another instance in which it has proved profitable to bet against uncovered interest parity (Euro interest rates being lower than U.S. rates). Only time will tell how long this period of Euro weakness will last, but both traders and regulators would do well to keep the lessons of October 1998 in mind.

[4] See Obstfeld and Rogoff (1997, pp.621-30) for an introduction to the empirical literature.

Appendix

When there is a unique symmetric equilibrium in switching strategies, there can be no other equilibrium. An argument is sketched here. Denote by $u(\rho, \hat{\rho})$ the expected utility from holding the asset conditional on posterior ρ when all other traders follow a switching strategy around $\hat{\rho}$. Conditional on ρ, the expected proportion of traders who sell is given by the probability that any particular trader receives a signal lower than the critical value $\hat{\rho}$. From the argument in the text, this probability is given by

$$\Phi\left(\sqrt{\tfrac{\beta(\alpha+\beta)}{\alpha+2\beta}}\left(\widehat{\rho}+\frac{\alpha}{\beta}\left(\widehat{\rho}-\overline{r}\right)-\rho\right)\right)=\Phi\left(\sqrt{\gamma}\left(\widehat{\rho}-\bar{r}+\frac{\beta}{\alpha}\left(\widehat{\rho}-\rho\right)\right)\right). \tag{5.1}$$

Hence, $u(\rho, \widehat{\rho})$ is given by

$$u(\rho, \widehat{\rho})=\rho-\Phi\left(\sqrt{\gamma}\left(\widehat{\rho}-\bar{r}+\frac{\beta}{\alpha}\left(\widehat{\rho}-\rho\right)\right)\right). \tag{5.2}$$

If r is negative, the utility to selling is higher than that from holding *irrespective* of what the other traders decide. So, if the posterior belief ρ is sufficiently unfavourable, selling is a dominant action. Let $\underline{\rho}_1$ be the threshold value of the belief for which selling is the dominant action. Any belief $\rho < \underline{\rho}_1$ will then dictate that a trader sells. Both traders realize this, and rule out strategies of the other trader which holds for signals lower than $\underline{\rho}_1$. But then, holding cannot be optimal if one's signal is lower than $\underline{\rho}_2$, where $\underline{\rho}_2$ solves

$$u\left(\underline{\rho}_2, \underline{\rho}_1\right)=0 \tag{5.3}$$

This is so, since the switching strategy around $\underline{\rho}_2$ is the best reply to the switching strategy around $\underline{\rho}_1$, and even the most optimistic trader believes that the incidence of selling is higher than that implied by the switching strategy around $\underline{\rho}_1$. Since the payoff to selling is increasing in the incidence of selling by the other traders, any strategy that holds for signals lower than $\underline{\rho}_2$ is dominated. Thus, after *two* rounds of deletion of dominated strategies, any strategy that holds for signals lower than $\underline{\rho}_2$ is eliminated. Proceeding in this way, one generates the increasing sequence:

$$\underline{\rho}_1 < \underline{\rho}_2 < \cdots < \underline{\rho}_k < \cdots \tag{5.4}$$

where any strategy that holds for signal $\rho < \underline{\rho}_k$ does not survive k rounds of deletion of dominated strategies. The sequence is increasing since $u(\cdot, \cdot)$ is increasing in its first argument, and decreasing in its second. The smallest solution $\underline{\rho}$ to the equation

$u(\rho, \rho) = 0$ is the least upper bound of this sequence, and hence its limit. Any strategy that holds for signal lower than $\underline{\rho}$ does not survive iterated dominance.

Conversely, if ρ is the largest solution to $u(\rho, \rho) = 0$, there is an exactly analogous argument from "above", which demonstrates that a strategy that sells for signals larger than ρ does not survive iterated dominance. But if there is a *unique* solution to $u(\rho, \rho) = 0$, then the smallest solution just *is* the largest solution. There is precisely one strategy remaining after eliminating all iteratively dominated strategies. Needless to say, this also implies that this strategy is the only *equilibrium* strategy.

References

[1] Bank of Japan (1998) *Regular Derivatives Market Statistics in Japan,* Bank of Japan, Tokyo.

[2] Basel Committee on Banking Supervision (1993) *The Supervisory Treatment of Market Risks,* Bank of International Settlements, Basel.

[3] Basel Committee on Banking Supervision (1995a) *Proposal to Issue a Supplement to the Basel Capital Accord*, Bank of International Settlements, Basel.

[4] Basel Committee on Banking Supervision (1995b) *An Internal Model-Based Approach to Market Risk Capital Requirements*, Bank of International Settlements, Basel.

[5] Basel Committee on Banking Supervision (1996) *Amendment to the Capital Accord to Incorporate Market Risks*, Bank of International Settlements, Basel.

[6] Basel Committee on Banking Supervision (1999) *Recommendations for Public Disclosure of Trading and Derivatives Activities of Banks and Securities Firms*, Bank of International Settlements, Basel.

[7] Brady, N. (1988) *Report of the Presidential Task Force on Market Mechanisms,* Government Printing Office, Washington D. C.

[8] Goodhart, C., P. Hartmann, D. Llewellyn, L. Rojas-Suarez and S. Weisbrod (1998) *Financial Regulation*, Routledge, London.

[9] International Monetary Fund (1998a) *World Economic Outlook and International Capital Markets: Interim Assessment*, http://www.imf.org /external/pubs/ft/weo/weo1298/ index.htm

[10] International Monetary Fund (1998b) *Report of the Managing Director to the Interim Committee on Strengthening the Architecture of the Internaltional Monetary System,* http://www.imf.org/external/np/omd/100198.htm

[11] Jorion, P. (1997) *Value at Risk*, McGraw-Hill, New York.

[12] Morris, S. and H. S. Shin (1998a), "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks" *American Economic Review,* 88, 587 - 597.

[13] Morris, S. and H. S. Shin (1998b), "A Theory of the Onset of Currency Attacks", unpublished paper, http://www.nuff.ox.ac.uk /users/Shin/ working.htm

[14] Morris, S. and H. S. Shin (1999), "Coordination Risk and the Price of Debt", unpublished paper, http://www.nuff.ox.ac.uk /users/Shin/ working.htm

[15] Obstfeld, M. and K. Rogoff (1997) *Foundations of International Macroeconomics*, MIT Press, Cambridge.

[16] RiskMetrics Group (1999) *RiskMetrics Technical Manual*, RiskMetrics Group, London, http://www.riskmetrics.com/research/techdocs/index.cgi

[17] Shiller, R. (1987) "Investor Behavior in the October 1987 Stock Market Crash: Survey Evidence" NBER discussion paper 2446, reprinted in *Market Volatility*, MIT Press, 1990.

Figure 3.6