14.773 Political Economy of Institutions and Development.
Lectures 4 and 5: Introduction to Dynamic Voting and Constitutions

Daron Acemoglu

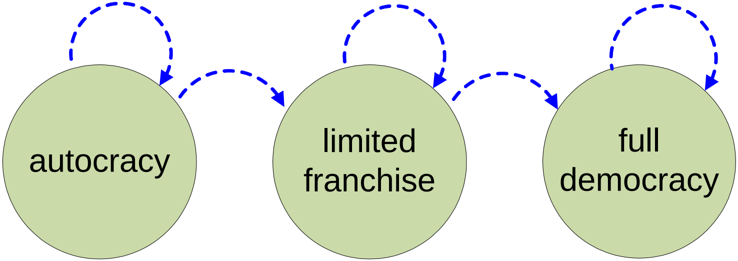
MIT

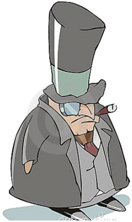
February 2024

Introduction

- **Political losers**: Anticipation of future political changes creates incentives for distorting policies/institutions/actions now.
- More generally related to: How does the anticipation of changes in political power effects political equilibria and economic efficiency?
- We now investigate this question focusing on Markov Perfect Equilibria of dynamic political games.
- These issues are more salient and important when current political decisions affect the *distribution of political power* in the future.
- The set of issues that arise here are very similar to those that will be central when we think about endogenous institutions.
- Thus useful to start considering more general dynamic voting models.

Constitutional Choice Example





Constitutional Choice (continued)

- Three states: absolutism a, limited franchise l, full democracy d
- Two agents: elite E, middle class M

$$\begin{aligned} w_E(d) &< w_E(a) < w_E(l) \\ w_M(a) &< w_M(l) < w_M(d) \end{aligned}$$

- E rules in a, M rules in l and d.
- Myopic elite: starting from a, move to l
- Farsighted elite (high discount factor): stay in a—as moving to l will lead to M moving to d
- But very different insights when there are stochastic elements and intermediate discount factors.

States and Utilities

- "Society" starts period in "state" (e.g., size of club, constitution, policy) s_{t-1} and decides on (feasible) s_t
- A finite set of individuals/players, and a finite set of states, $\mathcal{S}$
- All players maximize discounted utility, with discount factor $\beta < 1$
- Player i in period t gets instantaneous utility (in general, this is derived from the "within-state" game)

$$w_i\left(s_t\right)$$

- **Strict increasing differences**: For any agents $i, j \in \mathcal{N}$ such that $i > j$,

$$w_i\left(s\right) - w_j\left(s\right)$$

  is increasing in s

  - This could be weakened to weak increasing differences for some results.

Transition Mapping

- Let us consider Markov transition rules for analyzing how the "state" changes over time.
- A Markov transition rule is denoted by ϕ such that

$$\phi : \mathcal{S} \to \mathcal{S}.$$

- A transition rule is useful because it defines the path of the state s recursively such that for all t, i.e.,

$$s_{t+1} = \phi(s_t).$$

- Why Markov?
- If there is an s_∞ such that $s_\infty = \phi(s_\infty)$, then s_∞ is a *steady state* of the system (and we also use $\phi^\infty(s)$ to denote limiting value starting with s).
- We will consider both deterministic and stochastic transition rules $\phi(\cdot)$. But for now, useful to think of it as non-stochastic.

Recursive Representation

- Value function (conditioned on transition mapping ϕ):

$$V_i^{\phi}(s) = w_i(s) + \sum_{k=1}^{\infty} \beta^k w_i\left(\phi^k(s)\right).$$

- Recursively

$$V_i^{\phi}(s) = w_i(s) + \beta V_i^{\phi}(\phi(s)).$$

- Can be generalized with stochastic realization of states and powers.
- How do we go from here?

Roadmap

- We now study some special cases, then returning to the general framework so far outlined.
  - A (finite) game of political eliminations.
  - Characterization for the general model without stochastic elements and with β close to 1.
  - Applications.
  - (Omitted) Characterization and applications of the general model with stochastic elements and arbitrary discount factor β.

Voting Over Coalitions

- Model based on Acemoglu, Egorov and Sonin (2008).
- A coalition, which will determine the distribution of a pie (more generally payoffs), both over its own membership.
- Possibility of future votes shaping the stability of current clubs illustrated more clearly.
- *Motivation:*
  1. the three-player divide the dollar game.
  2. eliminations in the Soviet Politburo.

Political Game

- Let $\mathcal{I}$ denote the collection of all individuals, which is assumed to be finite.
- The non-empty subsets of $\mathcal{I}$ are *coalitions* and the set of coalitions is denoted by $\mathcal{C}$.
- For any $X \subset \mathcal{I}$, $\mathcal{C}_X$ denotes the set of coalitions that are subsets of X and $|X|$ is the number of members in X.
- In each period there is a designated *ruling coalition*, which can change over time.
- The game starts with ruling coalition N, and eventually the *ultimate ruling coalition* (URC) forms.
- When the URC is X, then player i obtains *baseline* utility $w_i(X) \in \mathbb{R}$.
- $w(\cdot) \equiv \{w_i(\cdot)\}_{i \in \mathcal{I}}$.
- Important assumption: game of "non-transferable utility". Why?

Political Power and Winning Coalitions

- Allow differential powers across individuals.
- *Power* mapping to:

$$\gamma : \mathcal{I} \rightarrow \mathbb{R}_{++},$$

- $\gamma_i \equiv \gamma(i)$: political *power* of individual $i \in \mathcal{I}$ and $\gamma_X \equiv \sum_{i \in X} \gamma_i$ political power of coalition X.
- Coalition $Y \subset X$ is **winning** within coalition X if and only if

$$\gamma_Y > \alpha \gamma_X,$$

  where $\alpha \in [1/2, 1)$ is a (weighted) supermajority rule ($\alpha = 1/2$ corresponds to majority rule).
- Let us write: $Y \in \mathcal{W}_X$ for $Y \subset X$ winning within X.
- Since $\alpha \geq 1/2$, if $Y, Z \in \mathcal{W}_X$, then $Y \cap Z \neq \varnothing$.

Payoffs

Assumption: Let $i \in I$ and $X, Y \in C$. Then:
(1) If $i \in X$ and $i \notin Y$, then $w_i(X) > w_i(Y)$ [i.e., each player prefers to be part of the URC].
(2) For $i \in X$ and $i \in Y$, $w_i(X) > w_i(Y) \iff \gamma_i/\gamma_X > \gamma_i/\gamma_Y$ ($\iff \gamma_X < \gamma_Y$) [i.e., for any two URCs that he is part of, each player prefers the one where his relative power is greater].
(3) If $i \notin X$ and $i \notin Y$, then $w_i(X) = w_i(Y) \equiv w_i^-$ [i.e., a player is indifferent between URCs he is not part of].

- Interpretation.
- Example:

$$w_i(X) = \frac{\gamma_{X \cap \{i\}}}{\gamma_X} = \begin{cases} \gamma_i/\gamma_X & \text{if } i \in X \\ 0 & \text{if } i \notin X \end{cases}. \tag{1}$$

Extensive-Form Game

- Choose $\varepsilon > 0$ arbitrarily small. Then, the extensive form of the game $\Gamma = (N, \gamma|_N, w(\cdot), \alpha)$ is as follows. Each *stage j* of the game starts with some ruling coalition N_j (at the beginning of the game $N_0 = N$). Then:

1. Nature randomly picks agenda setter $a_{j,q} \in N_j$ for $q = 1$.
2. [Agenda-setting step] Agenda setter $a_{j,q}$ makes proposal $P_{j,q} \in \mathcal{C}_{N_j}$, which is a subcoalition of N_j such that $a_{j,q} \in P_{j,q}$ (for simplicity, we assume that a player cannot propose to eliminate himself).
3. [Voting step] Players in $P_{j,q}$ vote sequentially over the proposal. More specifically, Nature randomly chooses the first voter, $v_{j,q,1}$, who then casts his vote vote $\tilde{v}(v_{j,q,1}) \in \{\tilde{y}, \tilde{n}\}$ (Yes or No), then Nature chooses the second voter $v_{j,q,2} \neq v_{j,q,1}$ etc. After all $|P_{j,q}|$ players have voted, the game proceeds to step 4 if players who supported the proposal form a winning coalition within N_j (i.e., if $\{i \in P_{j,q} : \tilde{v}(i) = \tilde{y}\} \in \mathcal{W}_{N_j}$), and otherwise it proceeds to step 5.

Extensive-Form Game (continued)

4. If $P_{j,q} = N_j$, then the game proceeds to step 6. Otherwise, players from $N_j \setminus P_{j,q}$ are eliminated and the game proceeds to step 1 with $N_{j+1} = P_{j,q}$ (and j increases by 1 as a new transition has taken place).
5. If $q < |N_j|$, then next agenda setter $a_{j,q+1} \in N_j$ is randomly picked by Nature among members of N_j who have not yet proposed at this stage (so $a_{j,q+1} \neq a_{j,r}$ for $1 \leq r \leq q$), and the game proceeds to step 2 (with q increased by 1). If $q = |N_j|$, the game proceeds to step 6.
6. N_j becomes the ultimate ruling coalition. Each player $i \in N$ receives total payoff

$$U_i = w_i\left(N_j\right) - \varepsilon \sum\nolimits_{1 \leq k \leq j} \mathbf{I}_{\{i \in N_k\}}, \tag{2}$$

where $\mathbf{I}_{\{\cdot\}}$ is the indicator function taking the value of 0 or 1.

Discussion

- Natural game of sequential choice of coalitions.
- ε introduced for technical reasons (otherwise, indifferences lead to uninteresting transitions).
- Important assumption: players eliminated have no say in the future.
- Stark representation of changing constituencies, but not a good approximation to democratic decision-making.
- More reminiscent to "dealmaking in autocracies"—or coalition formation in nondemocracies.

Main Result

Theorem

Fix $\mathcal{I}$, γ, $w(\cdot)$ and $\alpha \in [1/2, 1)$. Then there exists a set $\phi^{\infty}(N)$ such that:
1. For any $K \in \phi^{\infty}(N)$, there exists a pure strategy profile σ_K that is an SPE and leads to URC K in at most one transition. In this equilibrium player $i \in N$ receives payoff

$$U_i = w_i(K) - \varepsilon \mathbf{I}_{\{i \in K\}} \mathbf{I}_{\{N \neq K\}}.$$

This equilibrium payoff does not depend on the random moves by Nature.
2. Suppose that γ is generic (in the sense that no two coalitions have the same power), then $\phi^{\infty}(N)$ is a singleton (and $\phi^{\infty}(\cdot)$ is single-valued).

Main Result (continued)

Theorem

(continued)
3. This mapping ϕ^∞ may be obtained by the following inductive procedure. For any $k \in \mathbb{N}$, let $\mathcal{C}^k = \{X \in \mathcal{C} : |X| = k\}$. Clearly, $\mathcal{C} = \cup_{k \in \mathbb{N}} \mathcal{C}^k$. If $X \in \mathcal{C}^1$, then let $\phi^\infty(X) = \{X\}$. If $\phi^\infty(Z)$ has been defined for all $Z \in \mathcal{C}^n$ for all $n < k$, then define $\phi^\infty(X)$ for $X \in \mathcal{C}^k$ as

$$\phi^\infty(X) = \underset{A \in \mathcal{M}(X) \cup \{X\}}{\operatorname{argmin}} \gamma_A, \textit{ and}$$

$$\mathcal{M}(X) = \{Z \in \mathcal{C}_X \setminus \{X\} : Z \in \mathcal{W}_X \textit{ and } Z \in \phi^\infty(Z)\}.$$

Proceeding inductively $\phi^\infty(X)$ is defined for all $X \in \mathcal{C}$.

- Intuitively, $\mathcal{M}(X)$ is the set of proper subcoalitions which are both **winning** and **self-enforcing**. When there are no proper winning and self-enforcing subcoalitions, $\mathcal{M}(X)$ is empty and $\phi^\infty(X) = X$.

Discussion and Implication

- Implication: Essential uniqueness when there are no "ties".
- Application: coalition formation among three players with approximately equal powers.
- What happens among k players with approximately equal powers?

Corollary

Coalition N is self-enforcing, that is, $N \in \phi^{\infty}(N)$, if and only if there exists no coalition $X \subset N$, $X \neq N$, that is winning within N and self-enforcing. Moreover, if N is self-enforcing, then $\phi^{\infty}(N) = \{N\}$.

- Main implication: a coalition that includes a winning and self-enforcing subcoalition cannot be self-enforcing. This captures the notion that the stability of smaller coalitions undermines stability of larger ones.

Dynamics and Stability: A More General Approach

- A more general approach towards stability and change in social arrangements (political regimes, constitutions, coalitions, clubs, firms) based on Acemoglu, Egorov and Sonin (2012). Main trade-off between current economic payoffs and future political power.
- **Recap:** Consider the same simple extension of franchise story
- Three states: absolutism a, constitutional monarchy l, full democracy d
- Two agents: elite E, middle class M

$$
\begin{aligned}
w_E(d) &< w_E(a) < w_E(l) \\
w_M(a) &< w_M(l) < w_M(d)
\end{aligned}
$$

- E rules in a, M rules in l and d.
- Myopic elite: starting from a, move to l
- Farsighted elite: stay in a: move to l will lead to M moving to d.
- Same example to illustrate resistance against socially beneficial reform.

Naïve and Dynamic Insights

- *Naïve insight:* a social arrangement will emerge and persist if a "sufficiently powerful group" prefers it to alternatives.
- Simple example illustrates: power to change towards a more preferred outcome is not enough to implement change
  - because of further dynamics
- Social arrangements might be stable even if there are powerful groups that prefer change in the short run.
- **Key:** social arrangements change the distribution of political power (decision-making capacity).
- **Dynamic decision-making:** future changes also matter (especially if discounting is limited)

Simple Implications

- A particular social arrangement is made stable by the instability of alternative arrangements that are preferred by sufficiently many members of the society.
    - stability of a constitution does not require absence of powerful groups opposing it, but the absence of an alternative stable constitution favored by powerful groups.
- Efficiency-enhancing changes are often resisted because of further social changes that they will engender.
    - Pareto inefficient social arrangements often emerge as stable outcomes.

Model: Basics

- Finite set of individuals $\mathcal{I}$ ($|\mathcal{I}|$ total)
  - Set of coalitions $\mathcal{C}$ (non-empty subsets $X \subset \mathcal{I}$)
- Each individual maximizes discounted sum of playoffs with discount factor $\beta \in [0, 1)$.
- Finite set of states $\mathcal{S}$ ($|\mathcal{S}|$ total)
- Discrete time $t \geq 1$
- State s_t is determined in period t; s_0 is given
- Each state $s \in \mathcal{S}$ is characterized by
  - Payoff $w_i(s)$ of individual $i \in \mathcal{I}$ (normalize $w_i(s) > 0$)
  - Set of winning coalitions $\mathcal{W}_s \subset \mathcal{C}$ capable of implementing a change
  - Protocol $\pi_s(k)$, $1 \leq k \leq K_s$: sequence of agenda-setters or proposals ($\pi_s(k) \in \mathcal{I} \cup \mathcal{S}$)

Winning Coalitions

Assumption (Winning Coalitions) For any state $s \in \mathcal{S}$, $\mathcal{W}_s \subset \mathcal{C}$ satisfies two properties:
(a) If $X, Y \in \mathcal{C}$, $X \subset Y$, and $X \in \mathcal{W}_s$ then $Y \in \mathcal{W}_s$.
(b) If $X, Y \in \mathcal{W}_s$, then $X \cap Y \neq \varnothing$.

- (a) says that a superset of a winning coalition is winning in each state
- (b) says that there are no two disjoint winning coalitions in any state
- $\mathcal{W}_s = \varnothing$ is allowed (exogenously stable state)
- Example:
    - Three players $1, 2, 3$
    - $\mathcal{W}_s = \{\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$ is valid (1 is dictator)
    - $\mathcal{W}_s = \{\{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$ is valid (majority voting)
    - $\mathcal{W}_s = \{\{1\}, \{2,3\}\}$ is not valid (both properties are violated)

Dynamic Game

1. Period t begins with state s_{t-1} from the previous period.
2. For $k = 1, \ldots, K_{s_{t-1}}$, the kth proposal $P_{k,t}$ is determined as follows. If $\pi_{s_{t-1}}(k) \in \mathcal{S}$, then $P_{k,t} = \pi_{s_{t-1}}(k)$. If $\pi_{s_{t-1}}(k) \in \mathcal{I}$, then player $\pi_{s_{t-1}}(k)$ chooses $P_{k,t} \in \mathcal{S}$.
3. If $P_{k,t} \neq s_{t-1}$, each player votes (sequentially) *yes* (for $P_{k,t}$) or *no* (for s_{t-1}). Let $Y_{k,t}$ denote the set of players who voted *yes*. If $Y_{k,t} \in \mathcal{W}_{t-1}$, then $P_{k,t}$ is accepted, otherwise it is rejected.
4. If $P_{k,t}$ is accepted, then $s_t = P_{k,t}$. If $P_{k,t}$ is rejected, then the game moves to step 2 with $k \mapsto k+1$ if $k < K_{s_{t-1}}$. If $k = K_{s_{t-1}}$, $s_t = s_{t-1}$.
5. At the end of each period (once s_t is determined), each player receives instantaneous utility $u_i(t)$:

$$u_i(t) = \begin{cases} w_i(s) & \text{if } s_t = s_{t-1} = s \\ 0 & \text{if } s_t \neq s_{t-1} \end{cases}$$

Recursive Representation

- Take a transition mapping ϕ
- Value function conditioned on transition mapping ϕ:

$$V_i^{\phi}(s) = w_i(s) + \sum_{k=1}^{\infty} \beta^k w_i\left(\phi^k(s)\right).$$

- Recursively

$$V_i^{\phi}(s) = w_i(s) + \beta V_i^{\phi}(\phi(s)).$$

- **Key observation:** If w satisfies (strict) increasing differences, then so does V.

Markov Voting Equilibrium

- Let $\mathcal{W}_x$ denote the set of "winning coalitions"—i.e., the set of agents politically powerful enough to change the state—starting in state x. The structure of these sets will be explained in detail below.
- $\phi : S \to S$ is a *Markov Voting Equilibrium* (MVE) if for any $x, y \in S$,

$$\begin{aligned} \left\{ i \in \mathcal{N} : V_i^{\phi}(y) > V_{E,i}^{\phi}(\phi(x)) \right\} &\notin \mathcal{W}_x \\ \left\{ i \in \mathcal{N} : V_i^{\phi}(\phi(x)) \geq V_i^{\phi}(x) \right\} &\in \mathcal{W}_x \end{aligned}$$

- The first is ensures that there isn't another state transition to which would gather sufficient support.
  - Analogy to "core".
- The second one ensures that there is a winning coalition supporting the transition relative to the "status quo".

Single Crossing and Single Peakedness

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, set of states $\mathcal{S} \subset \mathbb{R}$, and payoff functions $w_{\cdot}(\cdot)$. Then, single crossing condition holds if whenever for any $i, j \in \mathcal{I}$ and $x, y \in \mathcal{S}$ such that $i < j$ and $x < y$, $w_i(y) > w_i(x)$ implies $w_j(y) > w_j(x)$ and $w_j(y) < w_j(x)$ implies $w_i(y) < w_i(x)$.

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, set of states $\mathcal{S} \subset \mathbb{R}$, and payoff functions $w_{\cdot}(\cdot)$. Then, single-peaked preferences assumption holds if for any $i \in \mathcal{I}$ there exists state x such that for any $y, z \in \mathcal{S}$, if $y < z \leq x$ or $x \geq z > y$, then $w_i(y) \leq w_i(z)$.

Generalizations of Majority Rule and Median Voter

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, state $s \in \mathcal{S}$. Player $i \in \mathcal{I}$ is a **quasi-median voter** (in state s) if $i \in X$ for any $X \in \mathcal{W}_s$ such that $X = \{j \in \mathcal{I} : a \leq j \leq b\}$ for some $a, b \in \mathbb{R}$.

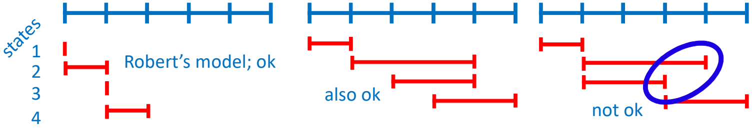
- That is, quasi-median voter is a player who belongs to any "connected" winning coalition.
- **Quasi-median voters:**

Generalizations of Majority Rule and Median Voter (continued)

- Denote the set of quasi-median voters in state s by M_s (it will be nonempty)

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, set of states $\mathcal{S} \subset \mathbb{R}$. The sets of winning coalitions $\{\mathcal{W}_s\}_{s\in\mathcal{S}}$ has **monotonic quasi-median voter property** if for each $x, y \in \mathcal{S}$ satisfying $x < y$ there exist $i \in M_x$, $j \in M_y$ such that $i \leq j$.



Some More Notation

- Define binary relations:
  - states x and y are payoff-equivalent

$$x \sim y \iff \forall i \in \mathcal{I} : w_i(x) = w_i(y)$$

  - y is weakly preferred to x in z

$$y \succeq_z x \iff \{i \in \mathcal{I} : w_i(y) \geq w_i(x)\} \in \mathcal{W}_z$$

  - y is strictly preferred to x in z

$$y \succ_z x \iff \{i \in \mathcal{I} : w_i(y) > w_i(x)\} \in \mathcal{W}_z$$

- Notice that these binary relations are **not** simply preference relations
  - *they encode information about preferences and political power.*

Theorem on Single Crossing and Single Peakedness

Theorem

If preferences are generic (extending our previous definition) and satisfy **single crossing** *and* **the monotonic quasi-median voter property** *holds, or if preferences are generic and* **single peaked** *and* **all winning coalitions intersect** *(i.e., $X \in \mathcal{W}_x$ and $Y \in \mathcal{W}_y$ imply $X \cap Y \neq \varnothing$), then $\succ_{s_j}$ is* **acyclic***. That is:*

1. For any sequence of states $s_1, \ldots, s_k$ in $\mathcal{S}$,

$$s_{j+1} \succ_{s_j} s_j \text{ for all } 1 \leq j \leq k-1 \Longrightarrow s_1 \not\succ_{s_k} s_k \text{, and}$$

2. For any sequence of states $s, s_1, \ldots, s_k$ in $\mathcal{S}$ such that $s_j \nsim s_l$ and $s_j \succ_s s$,

$$s_{j+1} \succsim_s s_j \text{ for all } 1 \leq j < k-1 \Longrightarrow s_1 \not\succsim_s s_k.$$

Noncooperative Characterization

Theorem

There exists $\beta_0 \in [0,1)$ such that for all $\beta \geq \beta_0$, the following results hold.
1. Any MVE can be characterized by a mapping ϕ^∞ constructed as follows: reorder states as $\left\{\mu_1, ..., \mu_{|\mathcal{S}|}\right\}$ such that if for any $l \in (j, |\mathcal{S}|]$, $\mu_l \not\succ_{\mu_j} \mu_j$. Let $\mu_1 \in \mathcal{S}$ be such that $\phi^\infty(\mu_1) = \mu_1$. For $k = 2, ..., |\mathcal{S}|$, let

$$\mathcal{M}_k = \left\{s \in \left\{\mu_1, \ldots, \mu_{k-1}\right\} : s \succ_{\mu_k} \mu_k \text{ and } \phi^\infty(s) = s\right\}.$$

Define, for $k = 2, ..., |\mathcal{S}|$,

$$\phi^\infty(\mu_k) = \begin{cases} \mu_k & \text{if } \mathcal{M}_k = \varnothing \\ z \in \mathcal{M}_k \text{: } \nexists x \in \mathcal{M}_k \text{ with } x \succ_{\mu_k} z & \text{if } \mathcal{M}_k \neq \varnothing \end{cases}.$$

(If there exist more than one $s \in \mathcal{M}_k$: $\nexists z \in \mathcal{M}_k$ with $z \succ_{\mu_k} s$, pick any of these).

Noncooperative Characterization (continued)

Theorem

(continued)

2. There is a protocol $\{\pi_s\}_{s \in \mathcal{S}}$ *and a MPE* σ *such that* $s_t = \phi^{\infty}(s_0)$ *for any* $t \geq 1$*; that is, the game reaches and stays in* $\phi^{\infty}(s_0)$ *after one.*

3. For any protocol $\{\pi_s\}_{s \in \mathcal{S}}$ *there exists a MPE in pure strategies. Any such MPE* σ *has the property that for any initial state* $s_0 \in \mathcal{S}$*, it reaches some state,* s^{∞} *by* $t = 1$ *and thus for* $t \geq 1$*,* $s_t = s^{\infty}$*. Moreover, there exists mapping* $\phi^{\infty} : \mathcal{S} \to \mathcal{S}$ *is constructed above such that* $s^{\infty} = \phi^{\infty}(s_0)$*.*

4. If, in addition, the following property holds: For $x, y, z \in \mathcal{S}$ *such that* $x \succ_z z$*,* $y \succ_z z$*, and* $x \nsim y$*, either* $y \succ_z x$ *or* $x \succ_z y$*, then the MVE is unique, and also, the MPE is essentially unique in the sense that for any protocol* $\{\pi_s\}_{s \in \mathcal{S}}$*, any MPE strategy profile in pure strategies* σ *induces* $s_t \sim \phi^{\infty}(s_0)$ *for all* $t \geq 1$*.*

Efficiency

- A state s is "myopically stable" if $s = \phi(s)$ because there does not exist $s' \succ_s s$
- Clearly a myopically stable state is stable, but not vice versa.

Corollary

If a state s is myopically stable, then it is Pareto efficient. If it is stable but not myopically stable, then it can be Pareto inefficient.

- Previously, no issue of Pareto inefficiency, because you are focusing on a game of pure redistribution (like divide the dollar game). This is no longer the case.

Extension of Franchise Example

- Three states: absolutism a, constitutional monarchy l, full democracy d
- Two agents: elite E, middle class M

$$
\begin{aligned}
w_E(d) &< w_E(a) < w_E(l) \\
w_M(a) &< w_M(l) < w_M(d)
\end{aligned}
$$

- $\mathcal{W}_a = \{\{E\}, \{E, M\}\}$, $\mathcal{W}_l = \{\{M\}, \{E, M\}\}$, $\mathcal{W}_d = \{\{M\}, \{E, M\}\}$
- Choose d as μ_1 and thus $\phi(d) = d$ and $\phi^\infty(d) = d$.
- Next choose l as μ_2 and we have $\phi(l) = d$ and $\phi^\infty(l) = d$
- Therefore, $\phi(a) = a$ (and $\phi^\infty(a) = a$).

Voting in Clubs

- N individuals, $\mathcal{I} = \{1, \ldots, N\}$
- N states (clubs), $s_k = \{1, \ldots, k\}$
- Assume single-crossing condition

$$\text{for all } l > k \text{ and } j > i,\ w_j(s_l) - w_j(s_k) > w_i(s_l) - w_i(s_k)$$

- Assume "genericity":

$$\text{for all } l > k,\ w_j(s_l) \neq w_j(s_k)$$

- Then, the theorem for ordered spaces applies and shows existence of MPE in pure strategies for any majority or supermajority rule.
- It also provides a full characterization of these equilibria.

Voting in Clubs

- If in addition only odd-sized clubs are allowed, unique dynamically stable state.
- Equilibria can easily be Pareto inefficient.
- If "genericity" is relaxed, so that $w_j(s_l) = w_j(s_k)$, then the theorem for ordered spaces no longer applies, but both the axiomatic characterization and the noncooperative theorems can still be applied from first principles.
- Also can be extended to more general pickle structures (e.g., weighted voting or supermajority) and general structure of clubs (e.g., clubs on the form $\{k - n, ..., k, ..., k + n\} \cap \mathcal{I}$ for a fixed n and different values of k).

An Example of Elite Clubs

- Specific example: suppose that preferences are such that

$$w_j(s_n) > w_j(s_{n'}) > w_j(s_{k'}) = w_j(s_{k''})$$

for all $n' > n \geq j$ and $k', k'' < j$

  - individuals always prefer to be part of the club
  - individuals always prefer smaller clubs.
- Winning coalitions need to have a strict majority (e.g., two out of three, three out of four etc.).
- Then,
  - $\{1\}$ is a stable club (no wish to expand)
  - $\{1, 2\}$ is a stable club (no wish to expand and no majority to contract)
  - $\{1, 2, 3\}$ is not a stable club (3 can be eliminated)
  - $\{1, 2, 3, 4\}$ is a stable club
- More generally, clubs of size 2^k for $k = 0, 1, ...$ are stable.
- Starting with the club of size n, the equilibrium involves the largest club of size $2^k \leq n$.

Stable Constitutions

- N individuals, $\mathcal{I} = \{1, \ldots, N\}$
- In period 2, they decide whether to implement a reform (a votes are needed)
- a is determined in period 1
- Two cases:
  - Voting rule a: stable if in period 1 no other rule is supported by a voters
  - Constitution (a, b): stable if in period 1 no other constitution is supported by b voters
- Preferences over reforms translate into preferences over a
  - Barbera and Jackson assume a structure where these preferences are single-crossing and single-peaked
  - Motivated by this, let us assume that they are strictly single-crossing
- Stable voting rules correspond to myopically (and dynamically) stable states
- Stable constitutions correspond to dynamically stable states

Political Eliminations

- The characterization results apply even when states do not form an ordered set.
- Set of states $\mathcal{S}$ coincides with set of coalitions $\mathcal{C}$
- Each agent $i \in \mathcal{I}$ is endowed with political influence γ_i
- Payoffs are given by proportional rule

$$w_i(X) = \begin{cases} \gamma_i / \gamma_X & \text{if } i \in X \\ 0 & \text{if } i \notin X \end{cases} \quad \text{where } \gamma_X = \sum_{j \in X} \gamma_j$$

and X is the "*ruling coalition*".

  - this payoff function can be generalized to any function where payoffs are increasing in relative power of the individual in the ruling coalition

Political Eliminations (continued)

- Winning coalitions are determined by weighted (super)majority rule $\alpha \in [1/2, 1)$

$$\mathcal{W}_X = \left\{ Y : \sum_{j \in Y \cap X} \gamma_j > \alpha \sum_{j \in X} \gamma_j \right\}$$

- Genericity: $\gamma_X = \gamma_Y$ only if $X = Y$
- Assumption on Payoffs is satisfied and the axiomatic characterization applies exactly.
- If players who are not part of the ruling coalition have a slight preference for larger ruling coalitions, then Stronger Acyclicity Assumption is also satisfied.

Other Examples

- Inefficient inertia
- The role of the middle class in democratization
- Coalition formation in democratic systems
- Commitment, (civil or international) conflict and peace

What is Missing?

- Dynamics, stochastic elements, intermediate forward-looking considerations (Discount factor< 1).
- These can be incorporated as well. See Acemoglu, Egorov and Sonin (2015).
- Key challenge: when the game is finite or there is little discounting (and no stochastic shocks), different paths can be evaluated in terms of the utility from the limit state they will lead to.
- This is no longer true in the general model.
- Nevertheless, increasing differences in preferences and the monotonic quasi-median voter property enable us to provide a characterization of MVE.
- New insights: strategic complementarities in good and bad behavior, and anticipation of future political behavior shapes current politics.