Political Economy of Institutions and Development: 14.773
Problem Set 1
Due Date: Friday, February 21, 2020.

Please answer questions 1 and 2. Question 3 is for practice and does not need to be handed in.

Question 1

Consider a variant of the model of politics under elite domination studied in the lecture, whereby elite and middle class individuals produce intermediate goods that are imperfectly substitutable. In particular, assume that the economy is closed and the total supply of the unique final good at time t is

$$Y_t = (Y_t^e)^{\mu} (Y_t^m)^{1-\mu},$$

where Y_t^e is the total supply of the intermediate good produced by the elite and Y_t^m is the total supply of the intermediate good produced by the middle class. As in the lecture notes, assume that all individuals have utility given by $U_0^j = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t^j$ with $\beta < 1$. There are θ_e elite agents, θ_m middle-class agents, and a mass of workers normalized to 1. Elite and middle-class producers have access to the production function

$$y_t^j = \frac{1}{1-\alpha}(A^j)^{\alpha}(k_t^j)^{1-\alpha}(l_t^j)^{\alpha}, \tag{1}$$

where each elite agent has productivity A^e in each period, and each middle class agent's productivity is A^m. Capital here is denoted in units of the final good and depreciates fully after use (i.e. the cost is normalized to 1 and there is no stock), and $l_t^j \leq \lambda$ (i.e. there are capacity constraints which impose a maximum firm size).

Political power is again in the hands of the elite, and the fiscal instruments are tax rates on elite and middle-class producers, $\tau^e \geq 0$ and $\tau^m \geq 0$, and nonnegative lump-sum transfers targeted towards each group, $T^w \geq 0$, $T^m \geq 0$ and $T^e \geq 0$. The government budget constraint is now

$$T_t^w + \theta^m T_t^m + \theta^e T_t^e \leq \phi \int_{j \in S^e \cup S^m} \tau_t^j p_t^j y_t^j \, dj, \tag{2}$$

where p_t^j is the price of the intermediate good produced by entrepreneur j (elite or middle-class) in terms of the final good, and $S^e \cup S^m$ is the total set entrepreneurs.

1. Define and characterize the economic equilibrium taking the sequence of policies as given (Hint: you first have to characterize prices p_t^e and p_t^m).

2. Now suppose that there is excess labor supply, i.e., $\theta^e + \theta^m \leq 1/\lambda$, and find the MPE policy sequence that maximizes the elite's utility. Show that this is also the unique SPE.

3. Now imagine that this economy is opened to trade, so that it can sell and buy the two intermediate goods at constant world prices given by $\bar{p}^e$ and $\bar{p}^m$. Again under the assumption that $\theta^e + \theta^m \leq 1/\lambda$, find the MPE policy sequence maximizing the elite's utility. Explain carefully the intuition for why taxes on middle class producers are now higher. What does this model imply about the political economy implications of trade opening?

Question 2

Suppose that the economy consists of two groups, the elite and the producers. Suppose that both groups are of equal size (each consisting of a continuum of agents). Both groups have instantaneous utility $u(c) = \log c$ and discount factor β. Producers have access to the production technology $f(k) = Ak^\alpha$, where k is capital. The elite impose a linear tax rate of τ_t on production at time t and consume the proceeds. The capital stock for time t must be chosen at time t after the tax rate τ_t has been announced. There is full depreciation of capital after use; capital here functions as a stock, so unconsumed units of output become capital in the next period.

1. Given a tax sequence, set up the dynamic optimization problem of entrepreneurs and show that the evolution of the capital stock is given by

$$k_{t+1} = \alpha\beta(1 - \tau_t) Ak_t^\alpha. \tag{3}$$

   Explain why the capital stock for time $t+1$ does not depend on the current tax rate but only on the past tax rate. [Hint: to derive this, set up the maximization problem of the entrepreneur is a dynamic program and conjecture a decision rule of the form $k_{i,t+1} = \kappa(1 - \tau)y_{i,t}$ for entrepreneur i, where $y_{i,t}$ is his output at time t].

2. To determine the Markov Perfect Equilibrium tax rates, write the value to a representative elite agent at time $t+1$ as a function of the tax rate $\tau = \tau_{t+1}$, taking into account that the capital stock of entrepreneurs at date $t+1$, $k = k_{t+1}$ is given from (3). Show that this value function takes the form

$$W(k) = \max_{\tau \in [0,1]} \{\log[\tau Ak^\alpha] + \beta W(\alpha\beta(1-\tau)Ak^\alpha)\}. \tag{4}$$

   Use standard dynamic programming arguments show that W is strictly concave and differentiable for $k > 0$ (and denote the derivative by W'). Show that the Euler equation for the elite is

$$\frac{1}{\tau} = \beta^2 \alpha A k^{\alpha} W'\left(k'\right) = \beta \frac{k' W'\left(k'\right)}{1-\tau}.$$

3. Now conjecturing that $W(k) = \eta + \gamma \log k$ and using the Envelope condition, show that $\gamma = \alpha / (1 - \alpha\beta)$ and derive the law of motion of the capital stock of each entrepreneur (and the aggregate capital stock). Explain the role of logarithmic preferences in this result.

Question 3

Consider the following infinite horizon economy populated by two groups, denoted 1 and 2, of equal size. All agents in both groups maximize the expected present discounted value of income, with discount factor β. In any period one of the groups is in power while the other group is out of power. When either group is in power, it loses power with probability $q < 1/2$ in every period.

Income is generated in the following way: group j has an asset stock of $A_{j,t}$ at time t. Using these assets, it can produce income $A_{j,t} f(i_{j,t})$ if it invests $i_{j,t}$ which costs $i_{j,t}$ in terms of utility. Investments are made before taxation decisions (detailed timing below). Initially, the asset stock of both groups is the same $A_{1,0} = A_{2,0} = A$.

Assume that income can be hidden in a non-taxable sector that generates a net return of $(1-\tau) A_{j,t} f(i_{j,t})$, as long as the assets are not expropriated.

So the net return to the group is

$$(1 - z_{j,t})(1 - e_{j,t})(1 - T_{j,t}) A_{j,t} f(i_{j,t}) + z_{j,t}(1 - e_{j,t})(1 - \tau) A_{j,t} f(i_{j,t}) + G_{j,t} - i_{j,t} \quad (5)$$

where $z_{j,t} \in [0, 1]$ denotes the fraction of assets placed in the non-taxable sector, $T_{j,t}$ is a tax rate faced by this group, $e_{j,t} \in [0, 1]$ denotes the proportion of group j's assets that are expropriated in period t, and $G_{j,t}$ is a transfer to group j in period t..

The law of motion of assets, as a function of expropriation of assets, is given by:

$$\begin{aligned} A_{1,t} &= A_{1,t-1} - e_{1,t} A_{1,t-1} + e_{2,t} A_{2,t-1} \\ A_{2,t} &= A_{2,t-1} - e_{2,t} A_{2,t-1} + e_{1,t} A_{1,t-1} \end{aligned} \quad (6)$$

The timing of events is as follows:

- The in- and out-of-power groups are selected
- Each group chooses investment decisions
- The group in power sets tax and expropriation rates (where applicable)

- Each group chooses what fraction of assets to hide
- Net income is realized

1. First suppose that asset expropriation is not allowed, so $e_{j,t} = 0$, and the only decision each group takes is the tax rate it sets when in power. Characterize the pure strategy Markov Perfect Equilibria (MPE) of this repeated game. Show that the output level is less than first-best, and is constant over time.

2. Next suppose that the group in power can expropriate the assets of the other group (so the two decisions now are taxes and expropriation). Characterize the MPE, and show that output can actually be higher in this economy than the economy without asset expropriation. Explain why. Show also that now output is no longer constant, but fluctuates over time.

3. Next consider a model endogenizing q. In particular, imagine that the group out of power can choose to take power in any period but to do so must pay a non-pecuniary cost c. This cost c is drawn each period from the distribution $G(c)$. First consider the case without asset expropriation. Show that there will exist a level of c^* such that when $c \leq c^*$, the group out of power will take power (Hint: write the Bellman equations in terms of c^*—or the probability of regime change in the future—and obtain a fixed-point recursion for c^*).

4. Next consider the case with asset expropriation (where the group that comes to power can cost is the expropriate all the assets of the other group). Show that there will exist a level of c^{**} such that when $c \leq c^{**}$, the group out of power will take power, and show that $c^{**} > c^*$. Show also that this economy with endogenous power switches has higher volatility than the economy in the previous problem part.

5. Discuss whether the two theoretical channels, highlighted by the model, linking security of property rights to economic instability are plausible. Feel free to give real world examples.