14.461: Problem Set 1

Due: September 18, 2014.

1. Exercise 13.13 from *Introduction to Modern Economic Growth.*
2. Exercise 14.14 from *Introduction to Modern Economic Growth.*
3. Optimal State Dependent IPR in a Partial Equilibrium Framework:

   Suppose that there are only two firms ($i \in \{1, 2\}$) in each industry, each maximizing its expected present discounted value of net profits. Moreover we assume:

   - maximum number of gaps between two firms is two ($n \in \{-2, -1, 0, 1, 2\}$)
   - the profit of each firm (without taking into account expenditure on R&D) is only a function of the number of the gaps in the quality of the firm and its rival, and let us denote these by $\pi^i = \pi_n \in \{\pi_{-2}, \pi_{-1}, \pi_0, \pi_1, \pi_2\}$
   - R&D leads to to step by step innovation and the cost of R&D is linear in the arrival rate of innovation. $\Phi(x^i) = \phi x^i$
   - interest rate is constant. $r = r_0$

   In this framework, intellectual property right policy is modeled as the rate at which a technology becomes available for the rival firm for free and it is only a function of number of gaps between the leader and follower.

$$\eta = \begin{cases} \eta_1 & n = 1 \\ \eta_2 & n = 2 \\ \to \infty & n > 2 \end{cases}$$

   Throughout this question we are focusing on the Markov Perfect Equilibrium in which strategies of firms are only function of the number of gaps between the firm and its rival. $x^i = x_n \in \{x_{-2}, x_{-1}, x_0, x_1, x_2\}$.

   (a) Write down the value function for a firm that is n step ahead (behind) its rival.

   (b) Solve the system of the equations for the optimal R&D decisions, $\{x_{-2}, x_{-1}, x_0, x_1, x_2\}$.

(c) Let assume we are restricted to the policy $\eta_1 = \eta_2 = \eta$. Can $\eta > 0$ increase R&D ? Interpret this result.

(d) If we relax the above restriction, show that IPR policy can lead to greater R&D investment than the case $\eta_1 = \eta_2 = 0$? Provide an intuition for why $\eta_1 > 0$ can increase R&D.