

Political Economy of Institutions and Development: 14.773

Problem Set 1

Due Date February Thursday 21st

Question 1

- (1) Consider the example of a three-person three-policy society with preferences

$$\begin{array}{l} 1 \quad a \succ b \succ c \\ 2 \quad b \succ c \succ a \\ 3 \quad c \succ b \succ a \end{array}$$

Voting is dynamic: first, there is a vote between a and b . Then, the winner goes against c , and the winner of this contest is the social choice. Find the subgame perfect Nash equilibrium voting strategy profiles in this two-stage game in “weakly undominated” (recall that each player’s strategy has to specify how they will vote in the first round, and how they will vote in the second round as a function of the outcome the first round). [Hint: for “weakly undominated” strategies, first eliminate weakly dominated strategies in the last round, and then eliminate whatever is weakly dominated in the previous round, etc.].

- (2) Suppose a generalization whereby there are finite number of policies, $Q = \{q_1, q_2, \dots, q_N\}$ and M agents (which you can take to be an odd number for simplicity). Voting takes $N - 1$ stages. In the first stage, there is a vote between q_1 and q_2 . In the second stage, there is a vote between the winner of the first stage and q_3 , until we have a final vote against q_N . The winner of the final vote is the policy choice of the society. Prove that, if preferences of all agents are single peaked (with a unique bliss point for each), then the unique subgame perfect Nash equilibrium in “weakly undominated” implements the bliss point of the median voter.
- (3) Why is “weakly undominated” in quotation marks? [Hint: can you construct other equilibria in parts 1 and 2, when you simply focus on weakly undominated strategies (that is, without doing the sequential elimination described in the Hint of part 1)?].

Question 2

Consider party competition in a society consisting of a continuum 1 of agents. The policy space is the $[0, 1]$ interval and assume that preferences of all agents are single peaked and political bliss points are uniformly distributed over this interval.

- (1) To start with, suppose that there are two parties, A and B. They both would like to maximize the probability of coming to power. The game involves both parties simultaneously announcing $q_A \in [0, 1]$ and $q_B \in [0, 1]$, and then voters voting for one of the two parties. The platform of the party with most votes gets implemented. Determine the equilibrium of this game. How would the result be different if the parties maximized their vote share rather than the probability of coming to power?
- (2) Now assume that there are three parties, simultaneously announcing their policies $q_A \in [0, 1]$, $q_B \in [0, 1]$, and $q_C \in [0, 1]$, and the platform of the party with most votes is implemented. Assume that parties maximize the probability of coming to power. Characterize all pure strategy equilibria.
- (3) Now assume that the three parties maximize their vote shares. Prove that there exists no pure strategy equilibrium. Characterize the mixed strategy equilibrium (Hint: assume the same symmetric probability distribution for two parties, and make sure that given these distributions, the third party is indifferent over all policies in the support of the distribution).

Question 3

Consider the following one-period economy populated by a mass 1 of agents. A fraction λ of these agents are capitalists, each owning capital k . The remainder have only human capital, with human capital distribution $F(h)$. Output is produced in competitive markets, with aggregate production function

$$Y = K^{1-\alpha} H^\alpha,$$

where uppercase letters denote total supplies. Assume that factor markets are competitive and denote the market clearing rental price of capital by r and that of human capital by w .

- (1) Suppose that agents vote over a linear income tax, τ . Because of tax distortions, total tax revenue is

$$Tax = (\tau - v(\tau)) \left(\lambda r k + (1 - \lambda) w \int h dF(h) \right)$$

where $v(\tau)$ is strictly increasing and convex, with $v(0) = v'(0) = 0$ and $v'(1) = \infty$ (why are these conditions useful?). Tax revenues are redistributed lump sum. Find

the ideal tax rate for each agent. Find conditions under which preferences are single peaked, and determine the equilibrium tax rate. How does the equilibrium tax rate change when k increases? How does it change when λ increases? Explain.

- (2) Suppose now that agents vote over capital and labor income taxes, τ_k and τ_h , with corresponding costs $v(\tau_k)$ and $v(\tau_h)$, so that tax revenues are

$$Tax = (\tau_k - v(\tau_k)) \lambda r k + (\tau_h - v(\tau_h)) (1 - \lambda) w \int h dF(h)$$

Determine ideal tax rates for each agent. Suppose that $\lambda < 1/2$. Does a voting equilibrium exist? Explain. How does it change when λ increases? Explain why this would be different from the case with only one tax instrument?

- (3) In this model with two taxes, now suppose that agents first vote over the capital income tax, and then taking the capital income tax as given, they vote on the labor income tax. Does a voting equilibrium exist? Explain. If an equilibrium exists, how does the equilibrium tax rate change when k increases? How does it change when λ increases?

Question 4

A society is a two party democracy with population normalized to 1, with political parties R and D competing to maximize their vote share. Parties compete by proposing a tax rate τ with proceeds distributed lump sum to each member of society. Taxing income introduces distortions, so the tax revenue, distributed lump-sum, is $(\tau - v(\tau)) \bar{y}$, where \bar{y} is average income in society and $v(0) = v'(0) = 0$ and $v'(1) = \infty$, and the government budget constraint is

$$T \leq (\tau - v(\tau)) \bar{y}$$

The society is stratified into n groups. The size of each group varies, but members of the same group have the same income, denoted by y_j , but differing political ideology. Let the political leaning towards party R of individual i in group j be σ_j^i and the size of group j be α_j , with $\sum_{j=1}^n \alpha_j = 1$ and naturally $\sum_{j=1}^n \alpha_j y_j = \bar{y}$. Assume that σ_j^i has a symmetric distribution $\phi_j^{-1} F(x)$ with $\phi_j > 0$.

Assume that individuals of the society all share a common utility function

$$U_j^i(c_i, \sigma_j^i) = c_i + [\sigma_j^i + \delta] I_R$$

where I_R is an indicator for party R coming to power, and δ is a random popularity measure for party R, with distribution $G(\cdot)$.

- (1) First ignore the ideological leanings of each group and the relative popularity measure (i.e., $\sigma_j^i = \delta = 0$). Find the equilibrium in the party competition game and the tax rate announced by the two parties. Does a pure strategy equilibrium always exist?

- (2) Now characterize the equilibrium with the ideological leanings. Does a pure strategy equilibrium always exist?
- (3) Now assume the parties can offer group-specific transfers (instead of the lump sum redistribution) denoted by $\omega_j \geq 0$, so the government budget constraint thus becomes

$$\sum_{j=1}^n \alpha_j \omega_j \leq (\tau - v(\tau)) \bar{y}$$

Show, using an example, that there may exist no pure strategy equilibrium for the game when δ is known in advance to be 0. Determine conditions for an equilibrium to exist when δ is random with distribution G , and characterize such an equilibrium. Explain why a pure strategy equilibrium is more likely to exist when δ is random? Will the two parties necessarily offer the same policy platform?

- (4) Now fully characterize the equilibrium in this probabilistic voting model assuming that σ_j^i is uniform over $[-\phi_j^{-1}, \phi_j^{-1}]$ for all j and δ is uniform over $[-\psi^{-1}, \psi^{-1}]$.