Political Economy of Institutions and Development: 14.773
Problem Set 2
Due Date: Thursday, March 15, 2019.

Please answer Questions 1, 2 and 3.

Question 1

Consider an infinite-horizon dynamic game between two groups, an elite and a group of citizens. The elite is currently in power in a "non-democratic" regime, N. In the non-democratic regime, the elite receives income y^N, while citizens receive income w^N. At the beginning of each period t, the elite (acting as a single agent in this game) chooses how much to spend to defend the regime, denoted by $x_t \geq 0$. If

$$x_t \geq v_t,$$

the elite are able to maintain the non-democratic regime. Here v_t is a random term representing the stochastic power of the citizens to contest the regime. Assume that v_t is distributed according to continuous, smooth density f, which is in addition single peaked and symmetric around zero.

If the citizens are able to change the regime, then a new democratic regime, D, is instituted, and in this regime, the elite receives income $y^D < y^N$, while each citizen receives $w^D > w^N$ (so that the elite prefers non-democracy and citizens prefer democracy). In democracy, the timing of events is similar: at the beginning of each period t, the elite decide how much to invest in activities to subvert democracy back to non–democracy, again denoted by x_t. The only difference is that because in democracy the citizens are more powerful, the elite will succeed only if

$$x_t \geq v_t + \gamma,$$

where $\gamma > 0$, and v_t have the same distribution, f.

1. Write down the dynamic maximization problem of the elite in non-democracy and separately in democracy as a dynamic programming problem (with maximization over x_t).

2. Assume that the maximization problem of the elite has an interior solution in both non-democracy and democracy, and derive the first-order condition for both problems. Interpret the conditions.

3. Show, again under the assumption of interior solution, that the equilibrium probability that next period there will be non-democracy is the same starting from democracy and nondemocracy. Interpret this result.

4. What happens if γ increases (still assuming an interior solution)? Interpret this result.

5. What are the implications of these results for welfare (Hint: compute the expected discounted net present value of both the elite and citizens starting from the two regimes and compare them, and then determine the implications of a greater γ for welfare).

6. Explain why the result obtained in this model is special and provide three different modifications of the model (without doing the math) that will change this result. Do you think there will be an overlap in terms of the qualitative effect of institutions between these modified models and the one in this question?

Question 2

Ideas dating back to de Tocqueville suggest that greater social mobility makes democracy work better or become more stable. Consider a simple two-period model with three groups, poor, middle-class and rich (individuals within each group are completely homogeneous). Preferences satisfy single crossing and single peakedness, and the ideal policies of the three groups are distinct. Suppose there are also three political institutions: democracy in which the middle class is the median voter, a left dictatorship in which the poor make all the decisions, and an elite dictatorship in which the rich make all the decisions. In the second period, the relevant decision-maker just decides policy. In the first period, the relevant decision-maker decides both policy and what tomorrow's institutions should be. Suppose that we start in democracy, so that the first-period decisions will be made by the middle class.

Suppose also that there are 200 rich individuals, 150 middle-class individuals and 300 poor individuals, so that in democracy a member of the middle class is the median voter. Suppose that there is the following type of social mobility: $K \leq 150$ middle-class individuals become poor in the second period, while K poor individuals become middle-class (leaving the overall distribution of society across the three groups unchanged, with no mobility for the rich). All individuals know their new social group before the voting stage in the second period. Show that if $K < 75$, democracy is stable (in the sense that the society remains democratic in the second period), while if $K > 75$, democracy is unstable. Explain the intuition for this result and discuss de Tocqueville's hypothesis in this light.

Question 3

Consider an economy populated by λ rich agents who initially hold power, and $1-\lambda$ poor agents who are excluded from power, with $\lambda < 1/2$. All agents are infinitely lived and discount the future at the rate $\beta \in (0,1)$. Each rich agent has income θ/λ while each poor agent has income $(1-\theta)/(1-\lambda)$ where $\theta > \lambda$. The political system determines a linear tax rate, τ, the proceeds of which are redistributed lump-sum. Each agent can hide their money in an alternative non-taxable production technology, and in the process they lose a fraction ϕ of their income. There are no other costs of taxation. The poor can undertake a revolution, and if they do so, in all future periods, they obtain a fraction $\mu(t)$ of the total income of the society (i.e., an income of $\mu(t)/(1-\lambda)$ per poor agent). The poor cannot revolt against democracy. The rich lose everything and receive zero payoff after a revolution. At the beginning of every period, the rich can also decide to extend the franchise to the poor, and this is irreversible. If the franchise is extended, the poor decide the tax rate in all future periods.

1. Define MPE in this game.

2. First suppose that $\mu(t) = \mu^l$ at all times. Also assume that $0 < \mu^l < 1-\theta$. Show that in the MPE, there will be no taxation when the rich are in power, and the tax rate will be $\tau = \phi$ when the poor are in power. Show that in the MPE, there is no extension of the franchise and no taxation.

3. Suppose that $\mu^l \in (1-\theta, (1-\phi)(1-\theta) + \phi(1-\lambda))$. Characterize the MPE in this case. Why is the restriction $\mu^l < (1-\phi)(1-\theta) + \phi(1-\lambda)$ necessary?

4. Now consider the SPE of this game when $\mu^l > 1-\theta$. Construct an equilibrium where there is extension of the franchise along the equilibrium path. [Hint: first, to simplify, take $\beta \to 1$, and then consider a strategy profile where the rich are always expected to set $\tau = 0$ in the future; show that in this case the poor would undertake a revolution; also explain why the continuation strategy of $\tau = 0$ by the rich in all future periods could be part of a SPE]. Why is there extension of the franchise now? Can you construct a similar non-Markovian equilibrium when $\mu^l < 1-\theta$?

5. Explain why the MPE led to different predictions than the non-Markovian equilibria. Which one is more satisfactory?

6. Now suppose that $\mu(t) = \mu^l$ with probability $1-q$, and $\mu(t) = \mu^h$ with probability q, where $\mu^h > 1-\theta > \mu^l$. Construct a MPE where the rich extend the franchise, and from there on, a poor agent sets that tax rate. Determine the parameter values that are necessary for such an equilibrium to exist. Explain why extension of the franchise is useful for rich agents?

7. Now consider non-Markovian equilibria again. Suppose that the unique MPE has franchise extension. Can you construct a SPE equilibrium, as $\beta \to 1$, where there is no franchise extension?

8. Contrast the role of restricting strategies to be Markovian in the two cases above [Hint: why is this restriction ruling out franchise extension in the first case, while ensuring that franchise extension is the unique equilibrium in the second?].

Question 4

Consider the following infinite horizon economy. Time is discrete and indexed by t. There is a set of citizens, with mass normalized to 1 and a ruler. Citizens discount the future with the discount factor β, and have the utility function

$$u_t = \sum_{j=t}^{\infty} \beta^j \left[\frac{c_{t+j}^{1-\theta}}{1-\theta} - e_{t+j} \right],$$

where $\theta > 0$, c_{t+j} is consumption and e_{t+j} is effort (the ruler exerts no effort).

Each citizen i has access to the following Cobb-Douglas production technology:

$$y_t^i = A_t^{\alpha} \left(e_t^i\right)^{1-\alpha},$$

where A_t denotes the state of technology and infrastructure at time t, which will be determined by the ruler. In addition, the ruler sets a tax rate τ_t on income. Also, each citizen can decide to hide a fraction z_t^i of his output, which is not taxable, but hiding output is costly, so a fraction δ of it is lost in the process. So consumption of agent i is:

$$c_t^i \leq \left[(1-\tau_t)\left(1-z_t^i\right) + (1-\delta)\, z_t^i\right] y_t^i,$$

and tax revenues are

$$T_t = \tau_t \int \left(1 - z_t^i\right) y_t^i di.$$

The ruler at time t decides how much to spend on A_{t+1} with technology:

$$A_{t+1} = G_t$$

where G_t denotes government spending on infrastructure, and $\phi < 1$. This implies that the consumption of the ruler is

$$c_t^R = T_t - G_t.$$

The ruler is assumed to maximize the net present discounted value of his or her consumption.

The timing of events within every period is as follows:

- The economy inherits A_t from government spending at time $t-1$.
- Citizens choose investment, $\{e_t^i\}$.
- The ruler sets tax rate τ_t.
- Citizens decide how much of their output to hide, $\{z_t^i\}$.
- The ruler decides how much spent on next period's infrastructure G_t.

1. Find the first-best allocation in this economy.
2. Define a Markov Perfect Equilibrium (MPE) where strategies at time t depend on the payoff-relevant state of the game at time t. Be specific about what the strategies are and what the state is.
3. Show that in a MPE $\tau_t = \delta$ for all t. Interpret what δ corresponds to.
4. Given this, find the equilibrium level of effort by citizens and tax revenues as functions of A_t, $T(A_t)$.
5. Explain why we could write the value function of the ruler as

$$V(A_t) = \max_{A_{t+1}} \{T(A_t) - A_{t+1} + \beta V(A_{t+1})\},$$

   and using this, find the equilibrium spending on infrastructure, G_t.
6. What is the effect of δ on spending on infrastructure and equilibrium investments. Does a decline in δ always imply greater effort by citizens? If not, why not? Calculate the equilibrium level output and find the output maximizing level of δ. Discuss what this means, and how you could map this to reality.
7. How would you generalize/modify this model so that some countries tax a lot and use most of the proceeds for infrastructure and public good investments?