

## Labor Economics, 14.661, Second Part, Problem Set 2

This problem set is due on or before November 28.

Please answer the following questions:

**Exercise 1** Consider a society  $S$  consisting of  $M \times N$  students, and  $M$  schools, each of size  $N$ . Each student has a pre-schooling human capital  $e_i$ . The post-schooling human capital of a student  $i$  with pre-schooling human capital  $e_i$  attending school  $j$  is

$$h_i = G(e_i, e_{-i,j})$$

where  $e_{-i,j}$  is a vector of pre-schooling human capital of students other than  $i$  attending school  $j$ .

1. Show that if  $G$  is supermodular in its arguments, then to maximize the sum of post-schooling human capitals in society,  $\sum_{i \in S} h_i$ , the  $N$  students with the highest pre-schooling human capital should be allocated to one school, then the next  $N$  students with the highest pre-schooling human capital should be allocated to another school and so on.
2. Show that if parents maximize  $wh - p$ , where  $p$  is their cost of schooling, then the allocation that maximizes the sum of post-schooling human capitals can be decentralized by each school choosing some cost of attendance (tuition)  $p_j$  and accepting all students who want to attend at this price. What could go wrong with this decentralization scheme?
3. How is the allocation that maximizes the sum of post-schooling human capitals affected when  $G$  is submodular? Can this allocation be decentralized in the same manner as in part 2? If not, why not?
4. A researcher tries to determine whether  $G$  is supermodular or not by running a regression of GPAs of college students on SATs of classmates. What might go wrong with this empirical approach? (Hint: you may want to discuss separately the issues of identification, the difficulty of determining who the peers are, and the difficulty of mapping GPAs into “human capital”).

**Exercise 2** A worker has utility  $U = \log(s) - e$  where  $s$  is reward and  $e$  is effort which can be equal to either 0 or 1. The effort choice of the worker is not observed by the risk-neutral principal who only cares about net profits. The worker on the other hand needs to be given an ex ante expected utility level equal to  $u^*$ . The probability density for revenue,  $x$ , defined only for  $x \geq 1$ , is  $f(x) = 3x^{-4}$  if  $e = 0$  and  $f(x) = 2x^{-3}$  if  $e = 1$ .

1. Suppose low effort is preferred, determine the optimal contract.
2. Suppose high effort is preferred, determine the optimal contract. (Hint: the optimal contract will have the form  $s(x) = A - B/x$ ).
3. Write down the condition for high effort to be preferred to low effort.

**Exercise 3** A worker can choose  $e = 0$  or  $e = 1$ . The worker's utility is

$$u(w) - Ke,$$

where  $w$  is the wage,  $K > 0$  and  $u(w)$  is concave. The worker cannot be paid less than zero, so  $w \geq 0$ , and has a reservation utility equal to 0. If  $e = 0$ , the project has a probability of success equal to  $p$ , in which case it produces a revenue of  $y$ . If it's unsuccessful, it produces a revenue of zero. If  $e = 1$  the project has a probability of success equal to  $q > p$ .

1. Write the optimization problem of the principal assuming that she wants to implement  $e = 1$ .

2. Determine the wage contract and indicate the conditions under which the worker needs to be paid an “efficiency wage”.
3. Determine whether the principal prefers  $e = 1$  to  $e = 0$ .

**Exercise 4** Consider the following career concerns model. The world lasts two periods. All firms and workers are risk neutral and there is no discounting. Workers are high or low ability, with ability denoted by  $\eta \in \{\eta^H, \eta^L\}$ , with  $\eta^H > \eta^L$ . The fraction of high ability workers in the population is  $p \in (0, 1)$ . Worker ability is observed neither by the worker nor by the firms in the market. Each worker chooses an effort  $a \in R_+$  at each date, and with probability  $q(\eta, a) \in (0, 1)$ , he generates high output  $Y^h > 0$  and with the complementary probability, he generates low output  $Y^l$  which we normalize to  $Y^l = 0$ . Assume that  $q(\eta, a)$  is continuous, increasing and differentiable in  $a$  and increasing in  $\eta$ . The output level of each worker is publicly observed, but his effort level is not observed by potential employers. After the first period output level  $Y_1$  is realized, a large number of firms compete a la Bertrand to hire the workers. Finally, assume that workers have a continuous, differentiable, and convex cost of effort,  $c(a)$ .

1. Define a Perfect Bayesian Equilibrium for this game.
2. Show that in period 2, a worker will be paid

$$w_2(Y_1) = \pi(Y_1) q(\eta^H, 0) Y^h + (1 - \pi(Y_1)) q(\eta^L, 0) Y^h,$$

where  $\pi(Y_1)$  is the probability that the market assigns to the worker being high ability after observing his output level  $Y_1 \in \{Y^h, Y^l = 0\}$  in the first period.

3. Suppose that all workers choose effort  $a$  in the first period and derive  $\pi(Y_1)$  from Bayes’s rule.
4. Given  $w_2(Y_1)$  and  $\pi(Y_1)$ , derive the best response first period effort  $a$  of workers. Show that in equilibrium this effort must satisfy  $a = \bar{a}$ .
5. Provide conditions such that a symmetric pure strategy equilibrium exists. Can there be multiple equilibria? Provide an economic intuition.
6. Suppose that a unique symmetric pure strategy equilibrium exists. What is the impact of an increase in  $Y^h$  on equilibrium effort level? How does this effort depend on the form of the function  $q(\eta, a)$ ? Can you relate this to any real-world labor market facts?
7. Define the “first-best” effort level. Can the equilibrium level of effort be greater than the first-best effort?
8. What is the difference of this model from the Holmstrom’s baseline career concerns model? What are the advantages and disadvantages of this model?

**Exercise 5** Consider the Shapiro-Stiglitz model where workers and firms are infinitely lived and risk-neutral, both with discount rate  $r$ . Effort costs  $e$ , and without effort there is no output produced. There are  $N$  firms each with production function  $AF(L)$  which is increasing and strictly concave where  $L$  is the number of workers employed by the firm who exert effort. There is an exogenous separation rate equal to  $b$ , and unemployed workers get disutility of leisure (and benefits) equal to  $z$ . Unemployed workers are randomly allocated to new job openings (which are due to separations). Firms decide what wage to offer to their workers. Workers who shirk (do not exert effort) are caught with probability  $q$ . The difference from the standard model is that  $q$  is chosen by the firm. It costs  $C(q)$  per worker (thus a total of  $C(q)L$ ) to choose a level of monitoring equal to  $q$ .

1. First write the Bellman equations for given  $q$  and derive the incentive compatibility condition (or the no shirking constraint).

2. Now find a first-order condition to determine the optimal level of  $q$  for a firm (Hint: be careful here; a common mistake is not to distinguish between the “ $q$ ” of the firm in question, say  $q_i$ , and the “ $q$ ” of all other firms which enters through  $V^U$  and which is obviously not controlled by the firm).
3. Show that an increase in  $A$ , which reduces unemployment, leaves  $q$  unchanged. Explain this result. Is it counter-intuitive?
4. How would you modify the model so that changes in  $A$  have an impact on  $q$ ? Outline, if you can, possible ways of and generating the prediction that  $dq/dA > 0$  and that  $dq/dA < 0$ .
5. Informally discuss whether  $C(q)L$  as the cost of monitoring is plausible. In particular, would  $C(q)$  be better? What would change in the model if instead of “ $q$ ” we had workers supervising other workers (Hint: think of wages as costs)?

**Exercise 6** (*optional*) The efficiency wage models we analyzed in the lecture were of the moral hazard variety (or effort-elicitation models). Another strand of the efficiency wage literature relies on adverse selection (type-elicitation).

Suppose that there are  $N$  workers.  $\phi N$  of the workers are low type and have 1 efficiency unit of labor.  $(1 - \phi)N$  of the workers are high type and they have  $\alpha > 1$  efficiency units of labor. The type of the worker is his private information and never observed by any other agent. High type workers have a reservation return  $u_h$  and low type workers have a reservation return  $u_l < u_h$ . There are  $M$  firms each with a decreasing returns to scale production function  $F(H)$  where  $H$  is the efficiency units of labor.

1. Draw the supply and demand curves for labor.
2. Assume that these two curves intersect at  $w < u_h$ . Show mathematically that it may be profitable for a firm to offer a wage  $w = u_h$ . Explain the intuition. Characterize diagrammatically the equilibrium in which all firms offer  $w = u_h$ . Find the unemployment rate of this economy. Is all of this “involuntary”? Why don’t the employers cut wages?
3. The implicit assumption that you have used so far is that workers can apply to as many firms as they like. Now assume that each worker can only apply to one firm and choose which firm to apply after seeing the whole distribution of wage offers by firms. Show that starting from the allocation characterized in part (2) where  $w = u_h$  for all firms, there is a profitable deviation for a firm.
4. Can you guess the form of the equilibrium in this case where each worker can only apply to one firm?