Political Economy of Institutions and Development: 14.773
Problem Set 3
Not to be handed in. For practice.

Question 1

Consider a polity run by a triumvirate (three rulers). Time is discrete and runs to infinity. There is a status quo institution which generates a certain payoff of s for each of the three rulers at each date. Alternatively, they could implement a reform. Payoffs following reform are uncertain. There is a probability q that each ruler will get a high payoff of H from the reform with probability $1-q$ that each will get a low payoff which is normalized to 0. The reform can be reversed in any period. Suppose that the event that the reform generates the high payoff for a ruler is independent from the same event for the other two rulers. Suppose also that whether the payoff is high or low for a ruler is revealed immediately after the first period of reform. Suppose that all three rulers have a common discount factor of β. Finally suppose that

$$qH < s < (1-\beta)qH + \beta[qH + (1-q)s]. \qquad (*)$$

1. Show that if instead of a triumvirate, there was a single decision-maker, she would experiment with the reform.

2. Suppose now that decisions are made by the triumvirate by majoritarian voting. Show that (*) is no longer sufficient to ensure experimentation with the reform. Derive a sufficient condition for the reform to be tried. Interpret this result.

3. Suppose now that decisions are made by the triumvirate by unanimity. Show again that (*) is not sufficient to ensure experimentation with the reform, and derive a sufficient condition for reform. Compare this to the one you derived in the second part and discuss why either majoritarian voting or unanimity might be more conducive to experimenting with the reform.

Question 2

Consider a country consisting of two ethnic groups, A and B. All agents are infinitely lived (in discrete time) and maximize the net present discounted value of their income with discount factor $\beta \in (0,1)$. Suppose that both groups are of equal size, and have exogenous income levels y_j, $j \in \{A, B\}$ in each period. At the beginning of the period t, there are two possible political regimes S_{t-1} inherited from yesterday. A-dominance ($S_{t-1} = A$) and B-dominance ($S_{t-1} = B$). A secession shock $x_t = \{0, A, B\}$ takes place with probabilities $\{1 - q_A - q_B, q_A, q_B\}$ where $q_A, q_B < 1/2$, where x_t denotes the identity of the group which will have the opportunity to secede at the end of the period. Whoever has political power as determined by S_{t-1} chooses S_t which determines the group which can set policies today (and institutions S_{t+1} tomorrow), where policies consist of a policy vector $\left(\tau_t^A, \tau_t^B\right)$, where τ_t^j is a lump-sum tax imposed on group j, satisfying $\tau_t^j \leq y_j$. Negative values of τ's are allowed (as transfers). Since both groups are of the same size, the government budget constraint is

$$\tau_t^A + \tau_t^B \leq 0.$$

Following the setting of policies, if $x_t = A$, group A has an opportunity to secede from this country, in which case each of its members receive an income of δy_A from t onward, where $\delta \in (0,1)$ and the members of group B receive zero from t onward. If $x_t = B$, an analogous situation occurs where if group B secedes, all of its members receive an income of δy_B from t onward and the members of group A receive zero from t onward. Let $s_t^j = \{0, 1\}$ represent the secession decision which can be taken by group j if has the opportunity where 1 denotes secession. Note that only one group can receive a secession shock at a time.

The timing for the game is as follows:

- The group that was in power in the previous period starts out in power, and the secession shock x_t is realized.
- The group in power determines taxes and whether to transfer power to the other group.
- A secession decision is made if a group has the ability to secede.

1. Define the payoff-relevant state vector, the strategies and a Markov Perfect Equilibrium (MPE) in this game.
2. Show that if $\delta = 0$, there exists a unique MPE such that if $S_t = A$ then $\left(\tau_t^A = -y_B, \tau_t^B = y_B\right)$ and if $S_t = B$ then $\left(\tau_t^A = y_A, \tau_t^B = -y_A\right)$.
3. Explain why the above strategy profile may not be an equilibrium when $\delta > 0$.
4. Construct an equilibrium in which for $\delta \in \left(\underline{\delta}, \overline{\delta}\right)$, there exists an MPE in which whenever group j has the opportunity to secede, the political regime switches to group j-dominance (unless it is already under group j-dominance). Explain why this equilibrium needs both parameter conditions $\delta > \underline{\delta}$ and $\delta < \overline{\delta}$.

5. How does this theory relate to and differs from other models of equilibrium institutional change?

6. How would to enrich this model to make it applicable more broadly to situations of within-country ethnic conflict?

Question 3

Suppose that society consists of two parties 0 and 1. We start with party 0 as the incumbent, which is in power at date $t = 0$. Party 1 is the opposition. There are two possible political states at each date, *Normal* and *Crisis*. In each period, the probability that there is a crisis is s (independently of all other events). In the Crisis state, the opposition can start an armed uprising. In the Normal state no uprising is possible. There is an election in every period in which there is no armed uprising, and the incumbent wins this election with probability $q > 1/2$. There is a unique policy decision at time t, $\hat{p}_t \in [0, 1]$, set by the incumbent. If there is no armed uprising, this decision is implemented, and thus $p_t = \hat{p}_t$. If there is an armed uprising at date t, then the implemented policy $p_t \in [0, 1]$ is set by whoever wins the armed uprising. Armed uprising leads to the victory of either the current incumbent or the opposition with equal probabilities, and whoever wins sets the current policy and becomes the incumbent next period.

The stage utility of party $i = 0, 1$ is

$$u_i(p_t) = -\alpha(p_t - b_i)^2 - C \times 1(\text{uprising at } t),$$

where $\alpha > 0$, b_i is the bliss point of party i, and we take it such that $b_i = i$, and C is the cost of uprising, taking to be the same for both parties for simplicity. Both players discount the future with discount factor $\beta < 1$.

1. Compute the net present discounted values of the current incumbent and opposition parties, V^I and V^O under the assumption that there will never be any uprising. [Hint: you will find that these values are independent of the identity of which party is in power, and just depend on whether a given party is the incumbent or the opposition].

2. Derive a *no armed uprising constraint* such that if this constraint binds and there is never any policy concession by the incumbent, the opposition will start an armed uprising. [Hint: this condition should be derived from assuming that there will be no armed uprising in the future and then computed from the continuation utility of the opposition undertaking an armed uprising. It should take the form of $V^{\text{armed uprising}} \leq V^O$].

3. Suppose the no armed uprising constraint binds, i.e., $V^{\text{armed uprising}} > V^O$. Focus first on Markov Perfect Equilibria (MPE) where the incumbent's policy choice depends only on whether we are in the Normal or the Crisis state. Derive the maximum

value that the incumbent can give to the opposition in any MPE, $\hat{V}^O$. [Hint: to derive this, recall that the incumbent chooses policy before the armed uprising decision, so can make a policy concessions to give higher utility to the opposition].

4. Show that for certain combinations of the parameters α, s, q, C and β, it's impossible to prevent an armed uprising in any MPE. Provide an intuition for this. Does an increase in α make this more or less likely? Does an increase in β? Interpret these comparative statics.

5. Show that it is possible to construct Subgame Perfect Equilibria (SPE) that can prevent an armed uprising even when it could not be prevented with any MPE. Provide an intuition for this. What are the effects of an increase in β on SPE? Do you find MPE or SPE more reasonable?

6. Show that for certain parameters, it's impossible to prevent an armed uprising in any SPE. [Hint: to do this, focus on the worst punishment following a deviation from a candidate equilibrium, which is to start armed uprisings in all future dates whenever an opportunity arises].

7. What are applications of the logic highlighted by this model? How would you enrich this logic?