

## Labor Economics, 14.661, Second Part, Problem Set 3

This problem set is for practice. It's not to be handed in.

**Exercise 1** Consider an economy consisting of a large number of workers and firms. Each worker is infinitely lived in discrete time and maximizes the expected discounted value of income, with a discount factor  $\beta < 1$ . There is no ex ante heterogeneity among the workers, but the quality of the match between a worker and its employer is random, and is not directly observed by either. Suppose that the worker is a good match to its employer with probability  $\mu_0 \in (0, 1)$  (and this does not change over time for a given worker-firm match). A worker who is a good match to its employer produces a stochastic output  $y$  drawn from a distribution  $F_h$ , while a worker who is not a good match produces a stochastic output  $y$  drawn from a distribution  $F_l$ . Suppose that workers are paid a constant fraction of their expected marginal product (with expectation taken with respect to all information available up to then). At any point in time, the worker can decide to quit. If he does so, he becomes unemployed. Unemployed workers receive an income of  $b < \int y dF_l(y)$  and find a new match with probability  $q < 1$ .

1. What conditions do we need to impose on  $F_h$  and  $F_l$  such that workers who produce more are paid more?
2. Suppose that we have imposed this restriction on  $F_h$  and  $F_l$ . Is it also the case that workers who produce more are more likely to have a longer tenure with their firm?
3. Provide the explanation/intuition for why, conditional on staying with the firm, a worker has, on average, an upward sloping wage profile. Is this profile likely to be linear? Convex? Can you say about the variability of the wages of the worker who has longer tenure?
4. Provide conditions on  $F_h$  and  $F_l$  such that workers that are a good match (and only workers that are good match) can have infinite tenure (with some positive probability). What happens to the wages of workers that are in this “infinite tenure” range? What happens if these conditions are not satisfied?
5. Show that the wage that a worker receives just before quitting the firm is necessarily lower than the wage he will receive just after getting a new job. Is this also true when  $q = 1$ ?
6. What are some additional facts that the model of this sort can account for, and what are some facts that it will have difficulty explaining?

**Exercise 2** Consider the McCall search model with a mass 1 of risk neutral individuals with discount factor equal to  $\beta$  and an exogenously given stationary distribution of wages  $F(w)$ . Assume that there is no unemployment benefit, so unemployed workers receive zero wage. Once a worker finds and accepts a job, he will be employed in this job until the job is destroyed exogenously, which happens with independent probability equal to  $s$  in every period. Once the job is destroyed, the individual returns to the unemployment pool. Suppose that at  $t = 0$  all workers start out as unemployed.

1. Show that, provided that the worker never quits, the value of a worker who accepts a job at the wage  $w$  is given by

$$v^a(w) = w + \beta[(1-s)v^a(w) + sv],$$

where  $v$  is the value of an unemployed (searching) worker. Explain the intuition for this equation. Will the worker ever quit a job (unless there is an exogenous separation)?

2. Write down the dynamic programming recursion that characterizes the optimal behavior of an unemployed worker. Be specific about the assumptions you are making in writing this recursion (and justify these assumptions). Derive an expression for the value of an unemployed worker,  $v$ .

3. Find the reservation wage of the individual. Explain intuitively why this is constant over time. (Hint: use the fact that at the reservation wage  $R$ , the worker is indifferent between accepting the job and continuing to search, and combine this with the expression for  $v$  obtained in 2).
4. Find the the law of motion of unemployment. Why is unemployment not necessarily constant? Where does it converge to? Provide an interpretation of the limiting value unemployment in terms of separations and job creation.
5. What happens to reservation wages and the unemployment process when  $s$  increases?
6. Define the notion of “second-order stochastic dominance”. What happens when  $F(w)$  shifts to a new distribution  $\tilde{F}(w)$  that has the same expected wage but second-order stochastically dominates  $F$ ? Provide an intuition for this result.

**Exercise 3** Consider a standard search model in continuous time where all workers have the same level of productivity,  $y$ . Workers and firms get together via a constant returns to scale matching function  $M(U, V)$  where  $U$  is the number of unemployed workers and  $V$  is the number of vacancies. The flow cost of holding an open vacancy is  $\gamma$ , and unemployed workers get utility of leisure equal to  $z$ . Potential or existing firms open vacancies until a marginal vacancy makes zero-profits. Worker-firm matches come to an end at the flow rate  $s$  and all agents are risk-neutral and discount the future at the rate  $r$ . Wages are determined by Nash Bargaining where the bargaining power of the worker is  $\beta$ .

1. Write the Bellman equations, define an equilibrium and characterize it.
2. Suppose a utilitarian Social Planner (that means, the planner’s objective is a simple average of the utility of all agents) can choose job creation and acceptance decisions. Characterize her choice, i.e., "the second-best allocation" (in deriving this result you can set  $r = 0$ ).
3. Now, suppose the planner can only choose  $\beta$  of the wage determination rule. Show that there is a  $\beta^*$  such that if  $\beta = \beta^*$ , then the equilibrium achieves the best allocation from the planner’s point of view.
4. Now suppose that  $z$  is unemployment benefit financed by lump-sum taxation. Suppose that the planner cannot directly choose job creation and acceptance decisions, and has to take  $\beta$  is given. She can only control  $z$ , and has to take the equilibrium as given conditional on a value of  $z$ . What is the value of  $z$  that the planner would like to choose (call this  $z^*$ )?
5. Now, suppose the planner chooses  $\beta$  and  $z$ . Determine the value of  $z$ ,  $\hat{z}$ , that she would like to choose. Explain why  $\hat{z}$  is different than  $z^*$ . Explain why the results regarding the choice of  $\hat{z}$  are special, and discuss how you would modify this model to reach more realistic normative conclusions.
6. Show that as  $\beta \rightarrow 1$ , the unemployment rate,  $u$ , also tends to 1. Now consider the following critique of the model

“The case of  $\beta \rightarrow 1$  emphasizes that this is not a good model.  $\beta$  captures the division of output after the match. If  $\beta$  is too high, then the worker must be able to make an upfront payment,  $b$ , and get employed. By ruling out such payments, this model is ruling out the price mechanism.”

Discuss this claim. You might first want to show that in the logic of the model such payments are not possible, and then discuss how one could introduce such payments in this model, and whether or not they should be there for a realistic analysis of labor markets. Add any other angle that you see appropriate.

**Exercise 4** Consider the following search model. Time is continuous, and all agents are risk-neutral and discount the future at the rate  $r$ . There is a mass 1 of workers, and a mass  $n$  of firms. Each firm can employ only one worker. Workers and firms come together according to constant returns to scale

matching technology,  $M(U, V)$  where  $U$  is unemployment, and  $V$  is the mass of unfilled vacancies (i.e.,  $1 - U = n - V$ ). Once together, pairs separate at the flow rate  $s$ . Output of a pair is equal to  $f(h)$  where  $h$  is the skill level of the worker. Wages are determined by Nash Bargaining where the worker's bargaining power is  $\beta$ .

1. Find the steady state equilibrium assuming that  $h = 0$  for all workers (but  $f(0) > 0$ ).
2. Now consider an economy in steady state at  $t = 0$ , and assume that for once firms can invest in the human capital,  $h$ , of their workers (training). Assume that workers can not pay firms for this investment, nor can they commit to a wage cut. A higher  $h$  for a worker increases his productivity not only in this relation but in all future relations (thus  $h$  is general human capital). Suppose that the cost of investment is  $c(h)$  such that  $c(0) = c'(0) = 0$ , and  $c$  is strictly convex. Show that as long as  $M(U, V) < \infty$ , firms invest a positive amount in  $h$ .
3. Now take the limit as  $M(U, V) \rightarrow \infty$  (that is, if the probability of a match for a worker is  $p$  and that for a firm is  $q$ , then we have  $p, q \rightarrow \infty$ ). Show that in this limit point, firms do not invest in  $h$ . Discuss and interpret.
4. Informally discuss what would happen if  $n$ , the number of firms, was endogenized via a zero-profit condition (for example, entering costs some amount  $F$ ). Could multiple equilibria arise? Why?