Recitation 1: Review of Endogenous Technological Change Models
(Based on Daron's lecture 1 slides)

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Introduction

- R&D and technology adoption are purposeful activities resulting from endogenous innovation
- This lecture reviews the two textbook models of technological change:
  - Expanding variety of machines used in production, by Romer (1990)
  - "Schumpeterian models" with quality improvements and creative destruction as in Aghion and Howitt (1992) and Grossman and Helpman (1991)

Romer (1990) - Overview

- Innovation is modelled as generating new blueprints or *ideas* for production
- Model features:
  - Increasing returns to scale: constant returns to scale to capital, labor, material etc, but increasing returns to scale when ideas and blueprints are also produced
  - Costs of R&D paid as fixed costs upfront
  - Monopolistic competition: firms that successfully innovate become monopolists and make profits
    - Dixit-Stiglitz CES demand structure for simplicity
    - For a model of innovation with perfect competition, see Bodrin and Levin (2008)
  - Major shortcoming: all firms are identical, hence no easy way to map to data

Romer (1990) - Preferences and Technology

- Infinite horizon, continuous time
- Representative household with preferences:

$$\int_0^\infty \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt$$

- Constant population of workers L with inelastic labor supply
- Representative household owns a balanced portfolio of all the firms in the economy

Romer (1990) - Preferences and Technology

- Unique consumption good produced competitively with aggregate production function:

$$Y(t) = \frac{1}{1-\beta}\left[\int_0^{N(t)} x(\nu,t)^{1-\beta}\,d\nu\right] L^{\beta}$$

  where

  - $N(t)$ is the number of varieties of inputs (can also be interpreted as machines, intermediate goods, or capital) at time t
  - $x(\nu,t)$ is the amount of input type ν used at time t. They fully depreciate after use, hence not state variables

- For given $N(t)$, which final good producers take as given, the aggregate production function exhibits CRTS.

Romer (1990) - Preferences and Technology

- The resource constraint of the economy at time t is

$$C(t) + X(t) + Z(t) \leq Y(t) \tag{1}$$

  where $X(t)$ is the resource spent on inputs and $Z(t)$ is expenditure on R&D
- Once the blueprint of a particular input is invented, the research firm can create one unit of that machine at marginal cost $\psi > 0$ units of the final good

Romer (1990) - Innovation Possibility Frontier and Patents

- Innovation Possibility Frontier:

$$\dot{N}(t) = \eta Z(t)$$

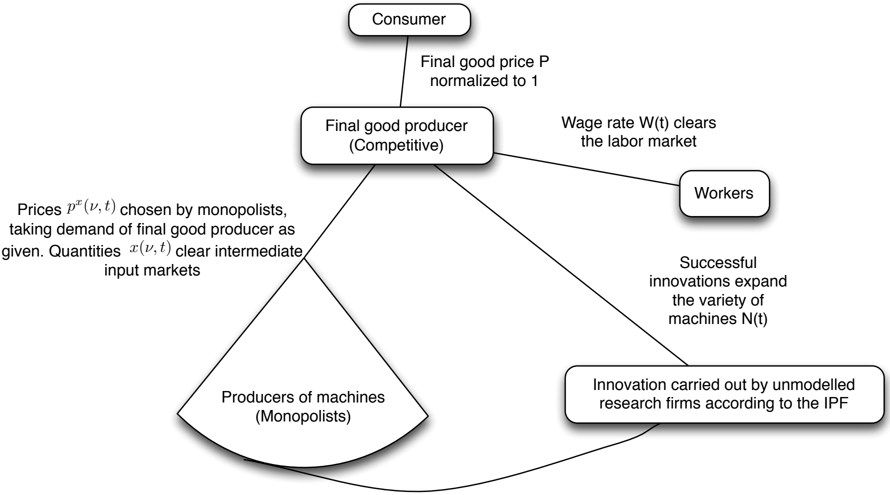
where $\eta > 0$ and the economy starts with some $N(0) > 0$

- There is free entry into research: any individual or firm can spend one unit of the final good at time t in order to generate a flow rate η of the blueprints of new machines
- The firm that invents a blueprint receives a fully-enforced perpetual patent on this variety of machine
- There is no aggregate uncertainty in the innovation process:
  - There is uncertainty at the level of the individual firm, but with a continuum of research labs undertaking such R&D, the IPF holds deterministically at the aggregate level

Romer (1990) - Innovation Possibility Frontier and Patents

- A firm that invents a new machine variety ν is the sole supplier of that type of machine, and sets a profit-maximizing price of $p^x(\nu, t)$ at time t to maximize profits
- Since machines fully depreciate after use, $p^x(\nu, t)$ can also be interpreted as a "rental price" or the user cost of that machine

Romer (1990) - Solving the Model



Innovation is done in pursuit of future stream of monopolist profits.
Free-entry condition ensures PDV profits from innovators equals costs

Romer (1990) - The Final Good Sector

- Maximization by final the good producer:

$$\max_{[x(\nu,t)]_{\nu\in[0,N(t)]}} \frac{1}{1-\beta}\left[\int_0^{N(t)} x(\nu,t)^{1-\beta}\, d\nu\right] L^{\beta} - \int_0^{N(t)} p^x(\nu,t)\, x(\nu,t)\, d\nu - w(t)\, L$$

- Demand for machines:

$$x(\nu,t) = p^x(\nu,t)^{-1/\beta} L$$

- Isoelastic demand that does not depend on equilibrium interest rate, wage rate, or the total measure of available machines

Romer (1990) - Technology Monopolists

- A monopolist owning the blueprint of a machine of type ν at time t maximizes the PDV of profits:

$$V(\nu, t) = \int_t^{\infty} \exp\left[-\int_t^s r(s')\, ds'\right] \pi(\nu, s)\, ds$$

where

$$\pi(\nu, t) \equiv \max_{p(\nu,t)} \left[p(\nu, t) - \psi\right] x(\nu, t)$$

- Value function in the alternative HJB form:

$$r(t)\, V(\nu, t) - \dot{V}(\nu, t) = \pi(\nu, t)$$

Romer (1990) - Technology Monopolists

- Since demand for intermediate machines is isoelastic, all monopolists set the same price in every period:

$$p^x(\nu, t) = \frac{\psi}{1-\beta} \text{ for all } \nu \text{and } t \tag{2}$$

- As usual, normalize $\psi \equiv (1-\beta)$ so that $p^x(\nu, t) = 1$ for all ν and t
- The quantity of machines that clear the markets is also the same across varieties and time:

$$x(\nu, t) = L \text{ for all } \nu \text{and } t \tag{3}$$

- A monopolist's flow profit is

$$\pi(\nu, t) = \beta L \text{ for all } \nu \text{and } t$$

Romer (1990) - Equilibrium

- Substitute quantity of machines into final good production function, we get the level of output:

$$Y(t) = \frac{1}{1-\beta} N(t) L$$

- Note
    - CRTS from the viewpoint of final good firms, but IRTS for the entire economy
    - Similarity to AK models: $Y(t) = AK(t)$
- Total expenditures on machines:

$$X(t) = N(t) L \tag{4}$$

Romer (1990) - Equilibrium

- Equilibrium wages:

$$w(t) = \frac{\beta}{1-\beta} N(t) \tag{5}$$

- Free entry

$$\begin{aligned} \eta V(\nu, t) \le 1, &\qquad Z(\nu, t) \ge 0 \text{ and} \\ (\eta V(\nu, t) - 1) Z(\nu, t) = 0, &\qquad \text{for all } \nu \text{and } t \end{aligned} \tag{6}$$

- For relevant parameter values with positive entry and economic growth:

$$\eta V(\nu, t) = 1$$

Romer (1990) - Equilibrium

- Finally, the Euler equation as usual:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho) \tag{7}$$

with transversality condition

$$\lim_{t\to\infty}\left[\exp\left(-\int_0^t r(s)\,ds\right)N(t)\,V(t)\right] = 0 \tag{8}$$

- An equilibrium is given by time paths
  - $[C(t), X(t), Z(t), N(t)]_{t=0}^{\infty}$ such that (1), (4), (6), (7), (8) are satisfied;
  - $[p^x(\nu, t), x(\nu, t)]_{\nu\in N(t), t=0}^{\infty}$ that satisfy (2) and (3);
  - $[w(t), r(t)]_{t=0}^{\infty}$ such that (5) and (7) hold.

Romer (1990) - Balanced Growth Path

- A balanced growth path (BGP) is an equilibrium path where $C(t), X(t), Z(t)$, and $N(t)$ grow at a constant rate. Such an equilibrium can also be referred to as a "steady state", since it is a steady state in transformed variables
- A BGP requires constant growth rate g_c for consumption. From the Euler equation, this is only possible if

$$r(t) = r^* \text{ for all } t$$

- Since profits and interest rate are both constant, $\dot{V}(t) = 0$ and from the HJB equation we have

$$V^* = \frac{\beta L}{r^*}$$

Romer (1990) - Balanced Growth Path

- Suppose that the free entry condition holds as an equality, in which case we also have

$$\frac{\eta\beta L}{r^*} = 1$$

This equation pins down the steady-state interest rate, r^*, as:

$$r^* = \eta\beta L$$

- The consumer Euler equation then implies that the rate of growth of consumption must be given by

$$g_C^* = \frac{1}{\theta}(r^* - \rho)$$

Note the current-value Hamiltonian for the consumer's maximization problem is concave, thus this condition, together with the transversality condition, characterizes the optimal consumption plans of the consumer

Romer (1990) - Balanced Growth Path

- In BGP, consumption grows at the same rate as total output

$$g^* = g_C^*$$

Therefore, given r^*, the long-run growth rate of the economy is:

$$g^* = \frac{1}{\theta}(\eta\beta L - \rho) \tag{9}$$

- Finally, suppose that

$$\eta\beta L > \rho \text{ and } (1-\theta)\eta\beta L < \rho, \tag{10}$$

which ensures $g^* > 0$ and the transversality condition is satisfied

Romer (1990) - Transitional Dynamics

- Note that $V(\nu, t)$ is independent of ν. If there is positive growth at some t, i.e., $\eta V(t) = 1$ for any t, then $\eta V(t) = 1$ for all t
- This implies that $\dot{V}(t) = 0$ and interest rate is constant
- Hence there are no transitional dynamics in this model

Proposition Suppose that condition (10) holds. Then, in this model there exists a unique balanced growth path in which technology, output and consumption all grow at the same rate, g^*, given by (9). Moreover, there are no transitional dynamics. That is, starting with initial technology stock $N(0) > 0$, there is a unique equilibrium path in which technology, output and consumption always grow at the rate g^*.

Romer (1990) - Pareto Optimal Allocations

- The competitive equilibrium is Pareto inefficient. Two sources of inefficiencies:
  - Monopoly markup
  - Number of inputs produced at any time may not be optimal
- The second source of inefficiency emerges from the fact that the set of traded (Arrow-Debreu) commodities is endogenously determined
- The socially-planned economy *always has a higher growth rate* than the decentralized economy
- The social planner values innovation more because she is able to use the machines more intensively after innovation, and this encourages more innovation
- For derivations and characterization, see Daron's lecture slides

Romer (1990) - Effects of Competition

- Recall that the monopoly price is:

$$p^x = \frac{\psi}{1-\beta}$$

- Imagine, instead, that a fringe of competitive firms can copy the innovation of any monopolist and produce at marginal cost of $\gamma\psi$ with $1/(1-\beta) > \gamma > 1$
- The fringe forces the monopolist to set a "*limit price*",

$$p^x = \gamma\psi$$

- Profits under the limit price:

$$\text{profits per unit } = (\gamma - 1)\psi = (\gamma - 1)(1-\beta) < \beta$$

- Since innovation is driven by monopoly profits, growth is slower under competition:

$$\hat{g} = \frac{1}{\theta}\left(\eta\gamma^{-1/\beta}(\gamma-1)(1-\beta)^{-(1-\beta)/\beta} L - \rho\right) < g^*$$

Romer (1990) - Growth with Knowledge Spillovers

- In the lab equipment model, growth resulted from the use of final output for R&D. The accumulation equation is linear in accumulable factors, stock of knowledge in this model, AN form instead of Rebelo (1991)'s AK form
- An alternative is to have "scarce factors" used in R&D: we have scientists as the key creators of R&D
- With this alternative, there cannot be endogenous growth unless there are knowledge spillovers from past R&D, making the scarce factors used in R&D more and more productive over time
- Innovation possibilities frontier in this case:

$$\dot{N}(t) = \eta N(t) L_R(t)$$

- See Daron's lecture slides for details

Schumpetarian Growth - Overview

- Most process innovations either increase the quality of an existing product or reduce the costs of production.
- *Competitive* aspect of innovations: a newly-invented superior computer often *replaces* existing vintages.
- Realm of Schumpeterian *creative destruction*.
- Schumpeterian growth raises important issues:
  1. Direct price competition between producers with different vintages of quality or different costs of producing
  2. Competition between incumbents and entrants: *business stealing effect.*

Schumpetarian - Preferences and Technology

- Preferences and resource constraints same as in the expanding variety model
- Normalize the measure of inputs to 1, and denote each machine line by $\nu \in [0, 1]$
- Engine of economic growth: *quality improvement*.
- $q(\nu, t)$ =quality of machine line ν at time t.
- "Quality ladder" for each machine type:

$$q(\nu, t) = \lambda^{n(\nu,t)} q(\nu, 0) \text{ for all } \nu \text{ and } t, \tag{11}$$

where:

- $\lambda > 1$
- $n(\nu, t)$ =innovations on this machine line between 0 and t.

Schumpetarian - Technology

- Production function of the final good:

$$Y(t) = \frac{1}{1-\beta}\left[\int_0^1 q(\nu,t)x(\nu,t \mid q)^{1-\beta}d\nu\right]L^{\beta}$$

  where $x(\nu, t \mid q)$ is the quantity of machine of type ν quality q
- Implicit assumption: at any point in time only one quality of any machine is used
- *Creative destruction*: when a higher-quality machine is invented it will replace ("destroy") the previous vintage of machines

Schumpetarian - Innovation Possibility Frontier

- Cumulative R&D process and free entry into research
- $Z(\nu, t)$ units of the final good for research on machine line ν, quality $q(\nu, t)$ generate a flow rate

$$\eta Z(\nu, t) / q(\nu, t)$$

  of innovation
- Note one unit of R&D spending is proportionately less effective when applied to a more advanced machine
- The firm that makes an innovation has a perpetual patent, but other firms can undertake research based on the product invented by this firm

Schumpeterian - Innovation Possibility Frontier

- Once a machine of quality $q(\nu, t)$ has been invented, any quantity can be produced at the marginal cost $\psi q(\nu, t)$.
- New entrants undertake the R&D and innovation:
  - The incumbent has weaker incentives to innovate, since it would be replacing its own machine, and thus destroying the profits that it is already making (*Arrow's replacement effect*).

Schumpetarian - Equilibrium

- Demand for machines similar to before:

$$x(\nu, t \mid q) = \left(\frac{q(\nu, t)}{p^x(\nu, t \mid q)} \right)^{1/\beta} L \quad \text{for all } \nu \in [0,1] \text{ and all } t, \tag{12}$$

  where $p^x(\nu, t \mid q)$ refers to the price of machine type ν of quality $q(\nu, t)$ at time t.
- Two regimes:
  1. innovation is "drastic" and each firm can charge the unconstrained monopoly price,
  2. limit prices have to be used.
- We focus on drastic innovations regime: λ is sufficiently large

$$\lambda \geq \left(\frac{1}{1-\beta} \right)^{\frac{1-\beta}{\beta}}.$$

- Again normalize $\psi \equiv 1 - \beta$
- See Daron's lecture slides for limit pricing case

Schumpetarian - Equilibrium

- Profit-maximizing monopoly:

$$p^x(\nu, t \mid q) = q(\nu, t).$$

- Combining with (12)

$$x(\nu, t \mid q) = L.$$

- Thus, flow profits of monopolist:

$$\pi(\nu, t \mid q) = \beta q(\nu, t) L.$$

Schumpetarian - Equilibrium

- Substituting the demand for machines into the aggregate production function:

$$Y(t) = \frac{1}{1-\beta} Q(t) L,$$

where

$$Q(t) \equiv \int_0^1 q(\nu, t) d\nu.$$

- Equilibrium wage rate:

$$w(t) = \frac{\beta}{1-\beta} Q(t).$$

Schumpetarian - Equilibrium

- Value function for monopolist of variety ν of quality $q(\nu, t)$ at time t:

$$r(t)\,V(\nu, t \mid q) - \dot{V}(\nu, t \mid q) = \pi(\nu, t \mid q) - z(\nu, t \mid q)V(\nu, t \mid q), \tag{13}$$

  where:
  - $z(\nu, t \mid q)$=rate at which new innovations occur in sector ν at time t,
  - $\pi(\nu, t \mid q)$=flow of profits.
- Last term captures the essence of Schumpeterian growth:
  - when innovation occurs, the monopolist loses its monopoly position and is replaced by the producer of the higher-quality machine.
  - From then on, it receives zero profits, and thus has zero value.
  - Because of Arrow's replacement effect, an entrant undertakes the innovation, thus $z(\nu, t \mid q)$ is the flow rate at which the incumbent will be replaced.

- Free entry:

$$\eta V(\nu, t \mid q) \leq \lambda^{-1} q(\nu, t)$$
$$\text{and } \eta V(\nu, t \mid q) = \lambda^{-1} q(\nu, t) \text{ if } Z(\nu, t \mid q) > 0.$$

- Note: Even though the $q(\nu, t)$'s are stochastic, as long as the $Z(\nu, t \mid q)$'s, are nonstochastic, average quality $Q(t)$, and thus total output, $Y(t)$, and total spending on machines, $X(t)$, will be nonstochastic.
- Consumer maximization implies the Euler equation,

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho),$$

- Transversality condition:

$$\lim_{t \to \infty} \left[\exp\left(-\int_0^t r(s)\, ds \right) \int_0^1 V(\nu, t \mid q)\, d\nu \right] = 0$$

for all q.

Schumpetarian - Equilibrium

- $V(\nu, t \mid q)$, is nonstochastic: either q is not the highest quality in this machine line and $V(\nu, t \mid q)$ is equal to 0, or it is given by (13).
- We have characterized the equilibrium and BGP is defined similarly to before (constant growth of output, constant interest rate).

Schumpeterian - Balanced Growth Path

- In BGP, consumption grows at the constant rate g_C^*, that must be the same rate as output growth, g^*.
- From the Euler equation, $r(t) = r^*$ for all t.
- If there is positive growth in BGP, there must be research at least in some sectors.
- Since profits and R&D costs are proportional to quality, whenever the free entry condition holds as equality for one machine type, it will hold as equality for all of them.
- Thus,

$$V(\nu, t \mid q) = \frac{q(\nu, t)}{\lambda\eta}. \tag{14}$$

- Moreover, if it holds between t and $t + \Delta t$, $\dot{V}(\nu, t \mid q) = 0$, because the right-hand side of equation (14) is constant over time—$q(\nu, t)$ refers to the quality of the machine supplied by the incumbent, which does not change.

Schumpetarian - Balanced Growth Path

- Since R&D for each machine type has the same productivity, constant in BGP:

$$z(\nu, t) = z(t) = z^*$$

- Then (13) implies

$$V(\nu, t \mid q) = \frac{\beta q(\nu, t) L}{r^* + z^*}. \tag{15}$$

- Note the *effective discount rate* is $r^* + z^*$.
- Combining this with (14):

$$r^* + z^* = \lambda \eta \beta L. \tag{16}$$

- From the Euler equation and the fact that $g_C^* = g^*$, $g^* = (r^* - \rho)/\theta$, or

$$r^* = \theta g^* + \rho. \tag{17}$$

Schumpetarian - Balanced Growth Path

- To solve for the BGP equilibrium, we need a final equation relating g^* to z^*
- Note that in an interval of time Δt, $z(t)\Delta t$ sectors experience one innovation, and this will increase their productivity by λ.
- The measure of sectors experiencing more than one innovation within this time interval is $o(\Delta t)$—i.e., it is second-order in Δt, so that

$$\text{as } \Delta t \to 0,\ o(\Delta t)/\Delta t \to 0.$$

- Therefore, we have

$$Q(t+\Delta t) = \lambda Q(t) z(t) \Delta t + (1 - z(t)\Delta t) Q(t) + o(\Delta t).$$

- Now subtracting $Q(t)$ from both sides, dividing by Δt and taking the limit as $\Delta t \to 0$, we obtain

$$\dot{Q}(t) = (\lambda - 1) z(t) Q(t).$$

Schumpetarian - Balanced Growth Path

- Therefore,

$$g^* = \frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{Q}(t)}{Q(t)} = (\lambda - 1) z^*. \tag{18}$$

- Now combining (16)-(18), we obtain:

$$g^* = \frac{\lambda \eta \beta L - \rho}{\theta + (\lambda - 1)^{-1}}. \tag{19}$$

Schumpetarian - Balanced Growth Path

Proposition In the model of Schumpeterian growth, suppose that

$$\lambda\eta\beta L > \rho > (1-\theta)\,\frac{\lambda\eta\beta L - \rho}{\theta + (\lambda - 1)^{-1}}\ . \tag{20}$$

Then, there exists a unique BGP in which average quality of machines, output and consumption grow at rate g^* given by (19). The rate of innovation is $g^*/(\lambda - 1)$. Moreover, starting with any average quality of machines $Q(0) > 0$, there are no transitional dynamics and the equilibrium path always involves constant growth at the rate g^* given by (19).

- Note only the average quality of machines, $Q(t)$, matters for the allocation of resources.
  - In fact, little discipline on firm or micro innovation structure.
- Moreover, the incentives to undertake research are identical for two machine types ν and ν', with different quality levels $q(\nu, t)$ and $q(\nu', t)$.

Schumpetarian - Pareto Optimality

- This equilibrium is typically Pareto suboptimal.
- But now distortions more complex than the expanding varieties model.
  - monopolists are not able to capture the entire social gain created by an innovation.
  - Business stealing effect.
- The equilibrium rate of innovation and growth can be too high or too low.
- See Daron's lecture slides for the characterization of social planner's problem

Schumpetarian - Policy

- Creative destruction implies a natural *conflict of interest*, and certain types of policies may have a constituency.
- Suppose there is a tax τ imposed on R&D spending.
- This has no effect on the flow profits of existing monopolists, and only influences their net present discounted value via replacement.
- Since taxes on R&D will discourage R&D, there will be replacement at a slower rate
- This growth rate is strictly decreasing in τ, but incumbent monopolists would be in favor of increasing τ