Abstract

Essays on Evolution, Reputation and Rationality

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In the first chapter of my dissertation, I analyze a model in which different agents have different non-rational expectations about the future price and cash flows of a risky asset. The beliefs in the society evolve according to a very general class of evolution functions that are monotone; that is, if one type has increased its share then all types with higher profit should also increase their shares. I show that the price of the risky asset converges to the risk-neutral fundamental price even though all agents in the economy are risk-averse. Thus effectively the asset is going to be overvalued and will pay zero risk premium. If the evolutionary process depends on the realization of random dividend shocks, the equilibrium price is absorbed with probability one by a symmetric interval around the risk-neutral price. The paper thus provides a behavioral explanation of the asset overvaluation and the recently observed decline in the equity premium.

In the second chapter, I analyze a reputational dynamic in the credit market for sovereign debt. The infinitely repeated game is considered where investors are competitive, and the country's discount factor is private information. I show that there is a unique efficient and a continuum of inefficient equilibria. In the efficient equilibrium, the investment risk is asymptotically eliminated. In the continuum of inefficient equilibria two cases are possible: either the country's reputation increases at first but then starts to decline, or the reputation declines from the beginning of the game. In either case, all types will eventually default. The result helps to explain why developing countries often fail to create good reputation in order to attract potential investors.

In the third chapter, I experimentally analyze the extent to which recent behavioral theories can explain subjects' overcontribution in public good games. I divide possible explanations into three groups: *non-monetary considerations* (fairness, altruism, etc.), *strategic considerations* (reciprocation, reputation, etc.) and *confusion*. I suggest several treatments that make one or two of these groups inapplicable, but do not change the rest of the game. Most importantly, they do not change the strategic uncertainty that subjects face. The main result is that non-monetary and strategic considerations explain less than half of the observed overcontribution. In addition to that, I perform an econometric analysis of the data to demonstrate that the suggested treatments *did* remove these considerations from the actual subjects' behavior.

Essays on Evolution, Reputation and Rationality

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To my parents, my wife, my friends and ...nobody else.

Chapter 1

Equity Premium Decline and Evolution of Heterogeneous Beliefs

1.1 Introduction

The nature of asset market behavior in the presence of heterogenous agents has been debated in the literature since as early as the nineteen fifties. In 1953, Milton Friedman argued that economists should restrict their attention to models where all agents have rational expectations. His argument was that even if there were irrational (in terms of beliefs) traders on the market, they would be making consistently less money than rational traders and thus eventually would be driven out of the market, and their price impact would become negligible.

Despite the intuitive appeal of this argument, over the past years researchers have accumulated a large body of evidence that shows that heterogeneity of beliefs can be persistent. In particular, it has been shown that under some reasonable conditions such as incomplete markets (Blume and Easley (2001)) the market selection hypothesis can fail. Indeed, it is possible that traders with rational expectations will be driven out of the market and their consumption path will converge to zero. DeLong, Shleifer, Summers, Waldmann (1990) (DSSW) argue that the noisy optimistic traders can make higher profit than the agents with rational expectations. The reason is that optimists have higher demand for risky assets, and thus as long as the assets have positive excess return, optimists will be making more money.

The model in this paper is built upon the intuition in DSSW. I investigate asset market behavior under two assumptions. First, I assume that different types of agents have different beliefs, and second, I assume that the types who earn higher profit increase their market weight. Unlike DSSW, I consider the model where all agents have non-rational expectations. This enables me to completely characterize the equilibrium dynamic, whereas in the DSSW only a single limiting case was considered.

The setting without rational expectations is used by many authors. See, for example, Brock and Hommes (1997), (1998), Brock et al. (2005), Chiarella and He (2001), (2002), Levy and Levy (1996) Lux and Marchesi (2000), Hommes et al. (2005), etc. My paper is different from this literature in several crucial aspects that will be discussed later. However, the basic framework is quite standard. I assume that there are two assets: a riskless bond available in perfectly elastic supply, and a risky asset that pays random dividends and has a fixed supply. Agents are different in their beliefs about future price and dividend distributions of the risky asset. Since different agents have different beliefs, they will make different portfolio choices and will earn different profits. The weights of different types evolve in such a way that agents with higher profit gain higher market share. We can think of this as a market selection mechanism, similar to that suggested by Friedman.

The analysis in this paper consists of two parts. In the first part, it is assumed that the evolution of market weights depends on the conditional expectations of individual profits. In the second part this assumption is substituted by a more realistic one — that the evolution function depends on actually realized profits.

The main result of the first part is as follows: under very general conditions on the class of evolution functions, the asset price will converge to the *risk-neutral* fundamental price even though all market participants are *risk-averse*. This means that first, in the long-run, the asset is going to be overvalued from the standard point of view. Second, its return will be equal to the riskless return and the equity premium will be zero. I also show that even in the long-run heterogeneity of beliefs will persist. That is, agents with different beliefs (including incorrect beliefs) will co-exist. When evolution depends on the actually realized profits, I show that with probability one the equilibrium price

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will be absorbed by the symmetric interval around the risk-neutral price. Thus, it is still the risk-neutral price that drives the price dynamic and not the (correct) risk-adjusted price.

Even though my model predicts overvaluation, the result does not rely on built-in assumptions which are asymmetric, such as an optimistic bias or short-sale constraints. Starting from Miller (1977), it is well-known that short-sale constraints and heterogenous beliefs can generate asset overvaluation. The idea is simple: more optimistic people can push the price up by having high demand for assets. In this situation, pessimists would short, and that would bring the price down. However, when there are short-sale constraints, pessimists have to stay out of the market, and thus the asset remains overpriced.

In my model, there are both pessimists and optimists, and no short-sale constraints. Nonetheless, the eventual price of the asset is higher than the correct price. The basic intuition behind this result is similar to DSSW. Optimists are going to hold more units of the risky asset, and so if the asset pays a positive excess return, they will make more profit. While the optimists do take more risk than is optimal given their risk attitudes, the market selection mechanism (that is, which types grow and which types decline in their market weight) is "risk neutral". It does not care about the risk taken except insofar as it affects the wealth made. Since on average the optimists will make more money, their share will increase and it will push prices up.

There is a catch, however. As it was shown by Samuelson (1971), in the long-run a portfolio with high mean return and high variance can be dominated with probability one by a less risky portfolio with smaller return[1]. In my paper, when the evolution depends only on conditional expected profit, this issue does not appear since the dynamic is determined by the expected, and not realized, wealth. On the other hand, in the second part of the paper where the evolution depends on realized wealth, taking more risk for higher returns can be dangerous. In fact, as I will show in numerical simulations, the most extreme types who hold the riskiest portfolios will die out.

The reason why riskier choices do not prevent optimists from pushing the price up is

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[1]In other words, with probability one the wealth generated by a riskier portfolio will be a negligible part of the wealth generated by safer investments.

as follows. The Samuelson result applies when the mean and the variance of portfolios are constant. In my setting, this is not the case. The portfolio choice, and thus its return and variance, are endogenously determined by prices. In particular, when the price increases, optimists will hold fewer units of the risky asset whereas pessimists might be willing to short it. Thus, as the price goes up, optimists are taking less risk, while pessimists are taking more risk. On the other hand, since the optimists' portfolios still have higher expected return, their share on average will grow and the price will go up. This upward pressure decreases as the price moves closer to the risk-neutral level, since the expected excess return becomes closer to zero and the optimists' advantage declines.

My model is different from the literature, specifically papers by Brock and Hommes (BH), in that the evolution is modeled at the level of the aggregate distribution of beliefs and not at the level of individual forecasting rules. When the evolution is modeled at the individual level, as is commonly done in the literature, one has to specify the exact procedures that agents use to update their beliefs. The problem is first, that it is not clear what combination of update rules should be used to model the stock market. Second, it is not clear how robust the results are to any particular choice. On the other hand, when modeling the evolution at the aggregate level, I avoid placing specific restrictions on the individual update rules. Furthermore, my results are robust over a wide class of aggregate evolutionary processes. Effectively individuals can use any update technique as long as the change in aggregate distribution of beliefs satisfies natural properties such as monotonicity.

The exact properties that I impose on the evolution process are monotonicity, slow-speed and non-triviality. Monotonicity is a requirement that makes the evolution functions reflect Friedman's idea. It says that if some type has increased its market weight, then all types with higher profit should increase their market weights as well. Slow-speed assumption requires that the market weights of agents change sufficiently slowly. Non-triviality states that if agents make different profits, then their weights should change.

The main predictions of the model are asset overvaluation and zero equity premium. The predictions are consistent with the recently observed decline of equity premium in

the United States market, see Fama and French (2002), Jagannathan, McGrattan and Scherbina (2000). In particular, the last paper shows "*the roughly equal returns that investments in stocks and console bonds of the same duration would have earned between 1982 and 1999, years when the equity premium is estimated to have been zero.*" Even after the market decline in 2000, the stock prices still remain well-above historical norms (see Lattau et al. (2004)).

Recently in the literature, many explanations have been offered using both rational and behavioral models. Rational theories explain the decline by increasing opportunities for portfolio diversification (see Merton (1987) and Heaton and Lucas (2000)) or by the decline in macroeconomic risk in the US economy (see Lattau et al. (2004)). More behavioral explanations follow Miller's idea, which I discussed above. They assume that due to overconfidence, investors have different opinions, which together with short-sale constraints make assets overpriced (see, for example, Scheinkman and Xiong (2003), Nagel (2005)). Unarguably, the short-sale constraints are real, and even though the evidence from empirical literature is mixed and somewhat weak, more recent findings claim that Miller's hypothesis is confirmed by the data (see Boehme et al. (2005), also an extensive review of previous work can be found there). In my paper, I provide another story of overvaluation that is based on market selection. I show that if beliefs which earn a higher return get higher market shares, then under some general conditions the asset price is going to be higher than the fundamental price.

The paper is structured in the following way: in Section 1.2 describes the model. Section 1.3 solves the model for the case when the evolution depends on the conditional expectation of individual profits. And Section 1.4 generalizes this result to the case when the evolution depends on the realized wealth.

1.2　The model

1.2.1　The static version of the model.

I consider an economy with one riskless and one risky asset. The riskless asset is available in a perfectly elastic supply at a price of 1, and its return is equal to R every period. The

risky asset pays dividends y_t that are iid with mean y and finite variance. The supply of the risky asset is equal to $S \geq 0$.

There is a continuum of agents in the economy that is normalized to one. All agents live for two periods. They are born with the initial endowment W of the riskless asset, and they consume only when they are old. To transfer the wealth between periods agents use the stock market. Thus every period, the trade happens between old and young generations. The old generation sells its portfolio to spend the money on consumption, whereas the young generation forms its portfolio to transfer wealth into the next period.

Different types of agents are different in their beliefs about the distribution of future price and dividends of the risky assets. Specifically, agents of type h believe that the distribution of $p_{t+1} + y_{t+1}$ has the mean ρ_h and the variance σ^2. I assume that ρ_h does not change over time, and I will discuss this assumption later in the text. Finally, following Brock and Hommes I assume that beliefs about the variance are equal and constant for all types, and the types are different only in their beliefs about the mean.

There is a finite number of types, and agents of all types have the same utility function

$$U = E_{ht}W_{t+1} - (a/2)V(W_{t+1}),$$

where E_{ht} and V represent beliefs of agents of type h about their wealth next period. Recall that variance is assumed to be the same across agents and time periods.

If an agent of type h holds z_{ht} units of the risky asset, then the wealth next period is equal to

$$W_{ht+1} = R(W - p_t z_{ht}) + (p_{t+1} + y_{t+1})z_{ht} = RW + (p_{t+1} + y_{t+1} - Rp_t)z_{ht}, \quad (1.1)$$

where W is the initial endowment and the current price of the risky asset is p_t.

We have that $V(W_{t+1}) = z_{ht}^2 \cdot V(p_{t+1} + y_{t+1} - Rp_t) = z_{ht}^2 \sigma^2$ for any type h. And so the maximization problem for type h is

$$\max_z E_{ht}(p_{t+1} + y_{t+1} - Rp_t)z - (a/2)\sigma^2 z^2, \quad (1.2)$$

where a is the risk-aversion coefficient. Without loss of generality, we normalize it to 2,

and then we have that the demand function of type h is equal to

$$z_h(p_t) = \frac{1}{\sigma^2}(\rho_h - Rp_t),\qquad(1.3)$$

where ρ_h is $E_{ht}(p_{t+1} + y_{t+1})$.

Assume that the share of young individuals of type h at period t is equal to n_{ht}, then in equilibrium

$$S = \sum_h z_{ht}(p_t) \cdot n_{ht} = \frac{1}{\sigma^2}\left(\sum_h n_{ht}\rho_h - Rp_t\right).$$

Thus we have that the equilibrium price for asset i is equal to

$$p_t = \frac{1}{R}(\sum_h n_{ht}\rho_h - \sigma^2 S).\qquad(1.4)$$

Notice that if there is only one type in the society, then the rational equilibrium price would be equal to

$$p_r = \frac{y_i - \sigma^2 S}{R - 1},$$

and in particular if all agents are risk-neutral, then the market price will be equal to $p_n = y/(R - 1)$.

1.2.2 Evolutionary dynamics.

So far the model formulation was relatively static and nothing has been said about the evolution of type shares. In this section we are going to define a very general class of evolution functions that will govern the equilibrium dynamic.

Before that, I need to introduce some notation. Denote the excess return on the risky asset $p_t + y_t - Rp_{t-1}$ as R_t. The profit made by an agent of type h at time t is $W_{ht} = R_t z_{ht-1} + WR$. Since all individuals of type h make the same decisions and have the same initial endowment, W_{ht} is the same within type h. The total wealth of all individuals of type h is then equal to $n_{ht}W_{ht}$. We denote the total number of types as H. Vector W_t is the per capita wealth vector $(W_{1t}, \ldots, W_{Ht})$, and the vector of shares in period t is denoted as n_t. In the text I am going to use words share, weight and market

weight interchangeably.

Finally from now on, I am going to assume that the types are ordered with respect to their optimism. That is the most pessimistic type is indexed with 1, and the most optimistic type is indexed with H.

1.1 Definition. *The evolutionary dynamic* is determined by a function f that satisfies the following assumptions:

(A1) The vector share in period t is a deterministic C^1-function of per capita wealth vector, W_t, shares vector, n_{t-1}, and some other variables, $\xi_t \in \Xi$, where Ξ is a compact set. That is

$$n_t = f(n_{t-1}, W_t, \xi_t),$$

where $f : \Delta^{H-1} \times \mathbb{R}^H \times \Xi \to \Delta^{H-1}$ is a C^1-function.

(A2) (*Weak monotonicity*) For given W_t and n_{t-1} if $n_{ht} = f(n_{t-1}, W_t, \xi_t) > n_{ht-1}$ for some type h, then $n_{kt} > n_{kt-1}$ for any type k such that $W_{kt} \geq W_{ht}$.

(A3) *(Non-triviality)* If the vector of shares does not change then either all types with positive shares earned the same profit, or one type has a share equal to one.

(A4) *(Evolution speed)* The evolution process is sufficiently slow.

The first axiom, while a seemingly benign technical assumption, has two important and non-trivial features. Formally, it specifies the variables that determine the evolution dynamics. It says that given shares in period $t-1$, the per capita vector wealth that each type made in period t, and possibly some other parameters ξ_t, function f will define the vector of shares in period t. The important features of this axiom are that the vector of weight today n_t depends on W_t and not W_{t-1}, and that the wealth vector is not risk-adjusted. These (implicit) assumptions are very crucial, and they will be carefully discussed in the next section.

The weak monotonicity axiom is an extremely important assumption that implements the Friedman selection mechanism. Its requirements are quite weak. What it says is

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that if some type has increased its share then all types that were more successful and earned higher per capita wealth should increase their shares as well.

The non-triviality axiom is needed to remove trivial evolutionary rules that do not change the type shares even though the types earned different profit. Obviously, if the evolution does not change the shares of the types, then the market clearing can stay constant at any arbitrary level.

The last axiom ensures that the evolution speed is slow. Later in the text it will be formalized as a condition that the first partial derivatives of f with respect to wealth vector components are sufficiently small. The requirement of low evolution speed is typical in evolutionary economics. The reason is that if it is not satisfied then it is possible that many types would just die out very fast because of few negative shocks, which will lead the system to some strange outcomes.

1.2 Example. A simple example of the evolution rule that would satisfy the definition above is the case when the share of a type is equal to the share of its profit in the total wealth

$$n_{ht} = \frac{n_{ht-1} W_{ht}}{\sum_k n_{kt-1} W_{kt}}.$$

1.3 Definition. Given (n_{t-1}, p_{t-1}), the equilibrium in period t is a pair (n_t, p_t) that is defined as follows:

i) given shares vector n_t, the equilibrium price p_t clears the market of the risky asset as specified in (1.4);

ii) given the price p_t, the vector of shares n_t is determined by the evolution function $n_t = f(n_{t-1}, W_t, \xi_t)$.

The relationship between n_t and p_t in the last part of the definition might seem a little bit obscure, but in fact it is quite straightforward. In period $t-1$ young agents of each type formed a portfolio based on their beliefs. After that in period t, the market price and dividends are realized, and we can calculate the profit that old individuals of each type earned in period t. It is equal to $W_{ht}(p_t) = (p_t + y_t - Rp_{t-1})z_{ht-1} + WR$. By plugging it into the evolution function we get that n_t is a function of p_t, y_t and other parameters determined in period $t-1$. On the other hand, p_t is a function of the

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weights vector n_t and it it is determined from the condition that the demand of young individuals should be equal to the supply of the risky asset. Thus n_t and p_t are defined simultaneously, and so the equilibrium dynamic is described by the following system:

$$\begin{cases} p_t = \dfrac{1}{R}(\sum_h n_{ht}\rho_h - \sigma^2 S) \\ n_t = f(n_{t-1}, W_t, \xi_t) \end{cases} \qquad (1.5)$$

1.2.3 Discussion

There are two interpretations that justify the existence of a market selection mechanism. The first interpretation is that the wealth of successful agents is increasing, and thus the market weights are increasing. The second interpretation is that agents adopt the beliefs of more successful types.

In the first case, two timings are possible. It can be either the current wealth or the last period wealth that determines the current market weight. Since, the first timing is more natural (if an agent is rich today, his market weight should be also high today), I use it in Definition 1.1. It has been also used in the literature, see e.g. Guerdjikova (2003), Chiarella and He (2001). Assuming the second timing would not change the results of my paper, and I show it in the appendix. However, if n_t and p_t were determined simultaneously, the dynamic becomes sequential: wealth vector W_{t-1} determines n_t which determines p_t and W_t, which determines n_{t+1} and so on.

The second interpretation, where agents adopt more successful beliefs is used, for example, in BH and DSSW papers. The natural timing for the second interpretation is also sequential. When agents in time t decide what type to join, they base their decision, on W_{t-1}, since W_t is not known yet. Not all reasonable adoption rules could be incorporated into the dynamic suggested in the paper. For example, the rule suggested by DSSW is a partial case of Definition 1.1, but the probabilistic choice rule used in BH is not[2].

To sum up, depending on the context, n_t might be considered as a function of W_t or W_{t-1}. In the main part of the paper, I assume the former, however, as it is shown in

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[2]In my paper if all types make the same profit, then their shares do not change. On the other hand, according to the probabilistic choice rule, if all types make the same profit their shares become equal.

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the appendix, the results do not depend on a particular timing assumption.

Another implicit assumption that is made in the first axiom is that the evolution depends on the raw wealth and not on the risk-adjusted wealth. If we think that market shares change because agents are wealthier, the wealth should not be risk-adjusted, and this is the right assumption to make. If, however, we think that agents switch to better strategies, then it is more reasonable to expect that they take into account the risk.

The financial literature has very strong evidence that suggests that switching to more successful strategies exists. In particular, it is documented by many researchers that there is a strong relationship between the inflow of new investments into a mutual fund and the fund's past performance (see Ippolito (1992), Gruber (1996), Chevallier and Ellison (1997), Sirri and Tuffano (1998) and many others). The common way in the literature to measure funds' performance is to use risk-adjusted returns. However, when the raw returns are used to estimate the relationship, the results remain the same. It is not surprising since raw and risk-adjusted returns are highly correlated and so one would serve as a proxy for another. Nonetheless, as some researchers independently noticed (Grubber (1996) and Sirri and Tuffano (1998)), the raw performance had an impact on fund flows which was separate from risk-adjusted measures. Moreover, if the risk measure is not correlated with the raw returns then its explanatory power seems to be very low. For example, in Sirri and Tuffano (1998) the standard deviation of past returns is used as alternative risk measure. Despite the fact that it was the only risk measure publicly reported to the agents in the sample, it turned out to be only "marginally significant" with p-value 0.105.

The evidence of individual investor behavior is even more supportive for the raw wealth assumption. In Barber, Odean and Zhu (2003) and Barber and Odean (2005) it is shown that the individual investors tend to buy stocks that have "grabbed" their attention, in particular, those stocks that had good long-term past performance, or the stocks with extreme one-day returns. In both cases, the criteria do not take risk into account, since in the first case the risk is averaged out, and in the second case, the decision is based on the one day performance.

Finally, I am going to discuss the assumption that types do not update their beliefs.

At first, this assumption appears to be very restrictive and unrealistic. However, notice that this assumption is about types of beliefs, and *not* about the agents' behavior. Individuals are allowed to switch between types. Furthermore, the way they switch is governed not directly by explicit constraints on individual behavior, but rather indirectly by placing some restrictions on the evolution of aggregated beliefs. Effectively, agents can be trend-extrapolators, contrarians, fundamentalists, or anyone else, as long as the change in the aggregate distribution of beliefs is consistent with properties of the evolution function f in Definition 1.1.

A more standard approach adopted in the literature is to model the evolution at the level of individual learning/forecasting rules. For example, in Hommes et al. (2005) and Chiarela and He (2001) there are two types of agents. One type forms its expectations based on the extrapolation of the trend, and the other type believes that the price will go back to fundamentals. While the choice of these particular rules is supported by psychological evidence, the problem is that there are many other different rules that are also supported by psychological findings. A priori it is not clear what choice of rules would be the best to model the stock market, and how robust the results are to different choice of rules. With my approach I avoid this problem, by putting restrictions not on the individual but on the aggregate learning.

1.3 The deterministic evolution

The evolution process defined in the previous section depends on the realization of random dividend payoffs, and so the whole system evolves according to some stochastic process. Following Brock and Hommes (1998) and Brock et al. (2005), in this section I will provide the analysis of what is called the "deterministic skeleton" of the process. It means that I will assume that the evolution of shares depends on the conditional expectations of per capita profits and not on their actual realizations.

To see the difference, recall that the profit of type h is $W_{ht} = (p_t + y_t - Rp_{t-1})z_{ht-1} + WR$, whereas the conditional expectation of type h profit is $E_{t-1}(W_{ht}) = (E_{t-1}(p_t) + y - Rp_{t-1})z_{ht-1} + WR$. In this section I am going to assume that the evolution depends on the conditional expectation of the profit, and thus $n_t = f(n_{t-1}, E_{t-1}(W_t), \xi_t)$. In particular,

it implies that p_t is not random and so $E_{t-1}(W_{ht}) = (p_t + y - Rp_{t-1})z_{ht-1} + WR$. For the sake of notation simplicity, everywhere in this section I am going to denote $E_{t-1}(W_{ht})$ as simply W_{ht}.

Given this assumption, we can completely describe the behavior of the economy in the long-run. We start with a simple statement that shows that if all types make the same profit then their shares do not change.

1.4 Statement. *If all agents make the same profit in period t then $n_{ht} = n_{ht-1}$ for any h. In particular, this is going to be the case if $R_t = 0$.*

◄ Assume that type h has increased its share, that is $n_{ht} > n_{ht-1}$. Then from weak monotonicity it follows that $\sum_h n_{ht} > \sum_h n_{ht-1} = 1$ which is impossible. If there is a type that decreased its share, there should be a type that increased its share, and we can apply the reasoning above. Thus it has to be the case that the shares of all types did not change.

If $R_t = 0$ then the wealth of each type is equal to $W_{ht} = R_t z_{ht-1} + WR = WR$, and thus all types have the same wealth.►

The statement above highlights the key difference between the evolutionary dynamics defined here, and the dynamic described by Brock and Hommes (1998, 2005) and Hommes et al (2004). In my paper, if individuals of all types make the same profit then the type shares do not change. In Brock and Hommes, if individuals of all types make the same profit then the shares become equal to each other[3]. This difference is crucial in explaining the difference in the results.

To ensure the existence of the market clearing price in each period we need to make one more technical assumption.

1.5 Definition. The risky asset is *scarce* if when everyone in the society has the lowest possible belief, its price is still positive.

1.6 Theorem. *For any share vector n_{t-1} and any equilibrium price in period $t - 1$, there exists p_t that defines the equilibrium (market-clearing) price in period t.*

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[3]Hommes et al. (2004) consider a little bit more general dynamics, however, it is still based on the discrete choice probabilities.

◄ The proof of this theorem is just a simple continuity argument. The market clearing price p_t is determined from the equation $TD(p) = \sum_h n_{ht} \cdot z_{ht}(p) = S$. Then we have that

$$\sum_h n_{ht} \cdot z_{ht}(0) \geq z_{1t}(0) \geq S.$$

Here the first inequality follows from the convention that types are ordered with respect to their optimism, and that type 1 is the most pessimistic and thus have the lowest demand. The second inequality is true because the asset is scarce.

On the other hand we have that when p is very large, the demands of all types will be negative, and thus the total demand will be less than S. Note, that given (A1), we have that the total demand is a continuous function of p, and thus there exists a market-clearing price p_t. ►

1.7 Theorem. *If the evolution function satisfies all axioms in Definition 1.1 the sequence of prices converges.*

◄ To prove this theorem we are going to use the following very important lemma.

1.8 Lemma. *If (A1), weak monotonicity and non-triviality axioms are satisfied and all types have shares smaller than one in period t, then in the equilibrium $R_t > 0 \Leftrightarrow \Delta p_t > 0$.*

◄ Since types are ordered with respect to their optimism, for any price p it is the case that $z_{1t}(p) < z_{2t}(p) < \cdots < z_{Ht}(p)$. From $W_{ht} = R_t z_{ht} + WR$, we have that if $R_t > 0$ then $W_{1t} < W_{2t} < \cdots < W_{Ht}$, and if $R_t < 0$ then $W_{1t} > W_{2t} > \cdots > W_{Ht}$.

Now let us take equilibria equations in period t and $t - 1$ and deduct one from each other. We get the following equation

$$R\Delta p_t = \sum_h \rho_h(n_{ht} - n_{ht-1}). \tag{1.6}$$

We already know that if $R_t = 0$ then the RHS of (1.6) is equal to zero, and thus $p_t = p_{t-1}$.

Consider the case when $R_t > 0$. Then individuals of different types will have different wealths, and so by the non-triviality axiom we have that the share vector will change[4].

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[4] If one type has share one, then two cases are possible, either the share vector changes, or it does not. If it does change, then the following reasoning is fully applicable

Moreover, there is a type k such that $1 < k \le H$ and $n_{it} \le n_{it-1}$ for any $i < k$, and $n_{it} > n_{it-1}$ for any $i \ge k$. Indeed, whenever there is a type with positive change in shares, by weak monotonicity it follows that more optimistic types should also get an increase in shares. However, it cannot be the most pessimistic type because then it would mean that all types have strictly increased their shares which is impossible and so that proves that $k > 1$.

Now we have that

$$R\Delta p_t = \sum_{h=1}^{H}(n_{ht} - n_{ht-1})\rho_h = \sum_{h=1}^{k-1}(n_{ht} - n_{ht-1})\rho_h + \sum_{h=k}^{H}(n_{ht} - n_{ht-1})\rho_h >$$

$$> \rho_k\sum_{h=1}^{k-1}(n_{ht} - n_{ht-1}) + \rho_k\sum_{h=k}^{H}(n_{ht} - n_{ht-1}) = 0.$$

Notice that since there are at least two types that change shares, the last inequality is strict.

Now consider the case when $R_t < 0$. Then we know that there is a type k such that $1 \le k < I$ and $n_{it} \ge n_{it-1}$ for any $i \le k$, and $n_{it} < n_{it-1}$ for any $i > k$. As before we have that

$$R\Delta p_t = \sum_{h=1}^{H}(n_{ht} - n_{ht-1})\rho_h = \sum_{h=1}^{k}(n_{ht} - n_{ht-1})\rho_h + \sum_{h=k+1}^{H}(n_{ht} - n_{ht-1})\rho_h <$$

$$< \rho_k\sum_{h=1}^{k}(n_{ht} - n_{ht-1}) + \rho_k\sum_{h=k+1}^{H}(n_{ht} - n_{ht-1}) = 0,$$

which completes the proof.▶

Next we are going to use the continuity argument to show that there is such value W^0 that $f(W^0, n_{ht-1}, W_{-ht}, n_{-ht-1}, \xi_t) = n_{ht-1}$. Take W_{min} equal to the lowest wealth among $(W_{1t}, \dots, W_{Ht})$. Then $f(W_{min}, n_{ht-1}, W_{-ht}, n_{-ht-1}, \xi_t) \le n_{ht-1}$ since otherwise by weak monotonicity we would have that all types have increased shares, which is impossible. Similarly, if W_{max} is the highest wealth among $(W_{1t}, \dots, W_{Ht})$ we have that $f(W_{max}, n_{ht-1}, W_{-ht}, n_{-ht-1}, \xi_t) \ge n_{ht-1}$. Indeed, if the share strictly decreases, it

means that there is a type k whose share strictly increased, but then since type h has (weakly) higher wealth, its share should also increase.

Thus, from continuity of f it follows that there is W^0 such that $W_{min} \leq W^0 \leq W_{max}$ and $f(W^0, n_{ht-1}, W_{-ht}, n_{-ht-1}, \xi_t) = n_{ht-1}$.

Now we can write the following:

$$n_{ht} - n_{ht-1} = f(W_{ht}, n_{ht-1}, W_{-ht}, n_{-ht-1}, \xi_t) - f(W^0, n_{ht-1}, W_{-ht}, n_{-ht-1}, \xi_t) =$$

$$= \int_{W^0}^{W_{ht}} f_1'(x, n_{ht-1}, W_{-ht}, n_{-ht-1}, \xi_t) dx,$$

where $f_1'(\cdot)$ is the partial derivative of f with respect to the first argument[5]. We know that $W_{ht} = R_t z_{ht-1} + WR$, and since W^0 is between W_{min} and W_{max} we also know that $W^0 = R_t z_{t-1}^0 + WR$, where $z_{min} \leq z_{t-1}^0 \leq z_{max}$. Thus we have that

$$\int_{W^0}^{W_{ht}} f_1'(x, \dots) dx = R_t(z_{ht-1} - z_{t-1}^0) f_1'(x_0, \dots), \qquad (1.7)$$

where x_0 is some intermediate value, and other arguments of f are not shown for brevity of notations.

Now we are ready to prove convergence. From (1.6) and (1.7) it follows that

$$R\Delta p_t = \sum_h \rho_h(n_{ht} - n_{ht-1}) = R_t \sum_h \rho_h(z_{ht-1} - z_{t-1}^0) f_1'(x_0, \dots). \qquad (1.8)$$

Denote $p_t + y - Rp_t$ as b_t. Notice that $b_t = 0$ if and only if p_t is equal to the risk-neutral fundamental price, and $b_t > 0 \Leftrightarrow p_t < p_n$. Another thing to notice is that $R_t - b_t = R\Delta p_t$, which is nothing more than algebraic identity. Using our notations we can re-write (1.8) as:

$$R_t \left(1 - \sum_h \rho_h(z_{ht-1} - z_{t-1}^0) f_h'(x_0, \dots) \right) = b_t. \qquad (1.9)$$

---

[5]The requirement that f is C^1 might be too strict because for example it can eliminate rules of the form $n_{ht} = max\{g(n, W, \xi), 0\}$, where we have a kink when g hits zero. However, the reasoning will still go through in a little bit more complicated manner. Specifically, even though now x_0 in (1.8) might not exist, we nonetheless can re-write it as $R\Delta p_t = \sum_h \rho_h(n_{ht} - n_{ht-1}) = R_t \sum_h \rho_h(z_{ht-1} - z_{t-1}^0)F$, where $F < \sup_x f_1'(x, n_{ht-1}, W_{-ht}, n_{-ht-1}, \xi_t)$.

Here comes the time for the evolution speed assumption. The evolution is slow when the change of the wealth of one type does not change its share *too much*. In other words the partial derivative of the evolution function with respect to W_{ht} should be sufficiently small for all h.

Our next step is to prove that the expression in the parenthesis is strictly positive. By the evolution speed assumption we can assume that f'_h is sufficiently small[6], so the only remaining thing to show is that other terms are bounded. It follows immediately from the fact that belief support is bounded. Therefore, the set of possible equilibrium prices is bounded and thus the set of possible demand values is bounded as well.

Since the expression in parenthesis is positive, we can conclude that $R_t > 0 \Leftrightarrow b_t > 0$, which is equivalent to $R_t > 0 \Leftrightarrow p_t < p_n$. Finally we have that

$$R_t > 0 \Rightarrow (\Delta p_t > 0, b_t > 0) \Rightarrow b_{t-1} > 0,$$

$$R_t < 0 \Rightarrow (\Delta p_t < 0, b_t < 0) \Rightarrow b_{t-1} < 0.$$

Now it is easy to prove the monotonicity. If $p_{t-1} < p_n$ then $b_{t-1} > 0$, then $R_t > 0$ then $p_t > p_{t-1}$ and $b_t > 0$, so $p_t < p_n$, which means that $R_{t+1} > 0$, and so on. Similar, it can be shown that if $p_m > p_n$ then the price sequence will be decreasing.

Thus we have the following behavior of the dynamics: if the initial price p_1 is less than p_n, then the price sequence will be increasing, and if p_1 is greater than p_n then the price sequence will be decreasing. ▶

The next theorem will specify the exact limit of price sequence for any initial distribution of beliefs.

1.9 Definition. We say that all beliefs in the economy are *too low (high)* if the highest (lowest) possible equilibrium price at time t is less (more) than p_n, that is $1/R \cdot (p_{hmax} + y - \sigma^2 S) < p_n$ (or $1/R \cdot (p_{hmin} + y - \sigma^2 S) > p_n$). If beliefs are neither too low nor too high, we say that the beliefs in the economy are *diverse*.

The purpose of this definition is to separate the following three cases. The first two

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[6]We can be more precise about how small f'_h should be. First, notice $\sigma^2 (z_{ht-1} - z^0_{t-1}) = R(p_0 - p_{t-1}) \leq R(p_{max} - p_{min}) = \rho_{max} - \rho_{min}$. Then the sum in (1.9) is less or equal to $(1/\sigma^2)\rho_{max}(\rho_{max} - \rho_{min}) \sum f'_h$. For this expression to be less than one it is sufficient that $|f'_h| < \sigma^2/(H\rho_{max}(\rho_{max} - \rho_{min}))$.

cases is when everyone in the society is too pessimistic or too optimistic so that the risk-neutral price is not reachable, that is it cannot be a market-clearing price. The third case, is when the risk-neutral price is reachable that is there is a distribution of beliefs such that the risk-neutral price would clear the market. The next theorem shows that whenever the risk-neutral price is reachable, the system will converge to it. If, however, the risk-neutral price cannot be reached given the set of beliefs, the long-run price will be as close to the risk-neutral price as possible.

1.10 Theorem. *If beliefs are diverse then the limit price will be equal to p_n and agents with different beliefs will co-exist. If beliefs are too high (too low) then only the highest (lowest) type will survive in the long-run.*

◀ We start the proof with a couple of useful lemmas.

1.11 Lemma. *Assume that at least two types with positive weights have different wealths. In that case, the type with the lowest wealth will decrease its share and the type with the highest wealth will increase its share.*

◀ We know from non-triviality that if at least two types with positive weights have different wealths then the vector of shares change. Given that, there is at least one type whose share will strictly increase and thus from the weak monotonicity it follows that the type with the highest wealth will strictly increase its share as well.

As for the type with the lowest wealth, its share cannot strictly increase since then by weak monotonicity we would have that the shares of all other types also strictly increase which is impossible. Thus, the share of type with the lowest wealth should decrease, at least weakly. ▶

1.12 Lemma. *If $R_\infty \neq 0$ then only one type survives in the end.*

◀ Step 1. Define S as a subset of the following set:

$$S \subset \{n_1, \ldots, n_H, W_1, \ldots, W_H, \xi | \exists h, i : n_h > 0, n_i > 0 \ \& \ W_h < W_i\},$$

and assume that S is a compact set. For any vector (n, W, ξ) we can calculate the smallest positive increase in shares:

$$I(n, W, \xi) = \min_{\{i : f_i(n, W, \xi) > n_i\}} [f_i(n, W, \xi) - n_i],$$

where min of empty set is defined to be zero.

In words we compare the corresponding components of the vector of the previous shares n with the vector of new shares f and the function $I(n, W, \xi)$ is equal to the lowest positive increase in the shares, or zero if there is none.

By definition $I(n, W, \xi) \geq 0$, but if $(n, W, \xi) \in S$ then by the non-triviality axiom we have that $I(n, W, \xi) > 0$. Now since S is compact it means that $\inf_S I(n, W, \xi) > 0$, because the infimum is reached at some point in S, and at this point $I(n, W, \xi) > 0$.

Step 2: Assume that $R_t \to R_\infty > 0$. Then we know that the type with the highest belief will always have the highest wealth. Thus, the sequence of its shares will be increasing and, thus, it converges to some positive limit n_H. If $n_H = 1$ then the shares of other types will converge to zero and we are done. Now assume that $n_H < 1$.

Given that $R_\infty > 0$, after some point as well as in the limit, the wealth of all types will be different from each other. Thus,

$$\exists T \ \forall t > T \ \forall h \ \exists H_1, H_2 : W_{ht} \in [H_1; H_2] \ \& \ \forall k \neq h \ W_{kt} \notin [H_1; H_2].$$

Now we are going to define

$$S' = \left\{ (n, W, \xi) \middle| \ 0 \leq n_i; a \leq n_H \leq b < 1; \sum n_i = 1; H_1 \leq W_{ht} \leq H_2; \xi \in \Xi \right\}.$$

First of all, S' is a compact set. Second, for any point in S' there are at least two types with positive shares and different wealths (in fact, all types have different wealths). Hence, we can apply Step 1 to S'.

By definition of S' after some moment T, the equilibrium sequence (n_t, W_t, ξ_t) will belong to S', and thus the smallest positive increase in shares is bounded away from zero. But then it is impossible that n_{Ht} converges.

The main conclusion of step 2 is that if $R_\infty > 0$ then the sequence of shares converges and in the limit only the type with the highest belief survives. In a similar way, we can

show that if $R_\infty < 0$ then the sequence converges and only the type with the lowest belief survives. ▶

If beliefs are too high then $p_t < p_\infty < p_n$, and from the proof of Theorem 1.7 it follows that $R_\infty > 0$. Thus only the type with the highest beliefs will survive. Similarly, if beliefs are too low then $R_\infty < 0$ which means that only the type with the lowest beliefs will survive.

Now let's consider the case when beliefs are diverse and $p_1 < p_n$ (the other case is similar). We know from Theorem 1.7 that then $p_t < p_n$ for all t, $R_t > 0$ for all t and the price sequence is increasing. Since the type with the highest belief will make the highest profit every period, its share is monotone and thus has a positive limit. Notice, however, that it cannot be the case that in the end only one type survives, since then it would have to be the highest type which would mean that at some point $p_t > p_n$, which is impossible. Thus several types will survive, and thus from Lemma 1.12 it follows that $R_\infty = 0$, and thus $p_\infty = p_n$. ▶

1.13 Corollary. *If beliefs are too high or too low the long-run price will be as close to the risk-neutral price as possible given the set of beliefs.*

To sum up, the dynamic of the system is fairly simple. If the risk-neutral price can be achieved given the set of initial beliefs, then the price monotonically converges to it. Otherwise, the price still monotonically moves towards the risk-neutral price, and in the limit it reaches the closest point to p_n given the set of beliefs. If the beliefs are diverse then the return on the risky asset will be equal to zero, and different types will co-exist in the market. If beliefs are too high (too low), then only the most pessimistic (optimistic) type will survive.

The driving force here is similar to DSSW (1990). If the initial distribution of beliefs is such that p_1 is less than p_n, then the excess return on the risky asset is positive, and so since more optimistic agents will hold more of the risky asset, they will make higher profit. Therefore, their share will grow, and it will push the price up, until it reaches the risk-neutral level, where the portfolio choice does not matter.

From symmetry, another scenario, arguably less realistic, is possible when the initial

Figure 1.1: Deterministic Evolution
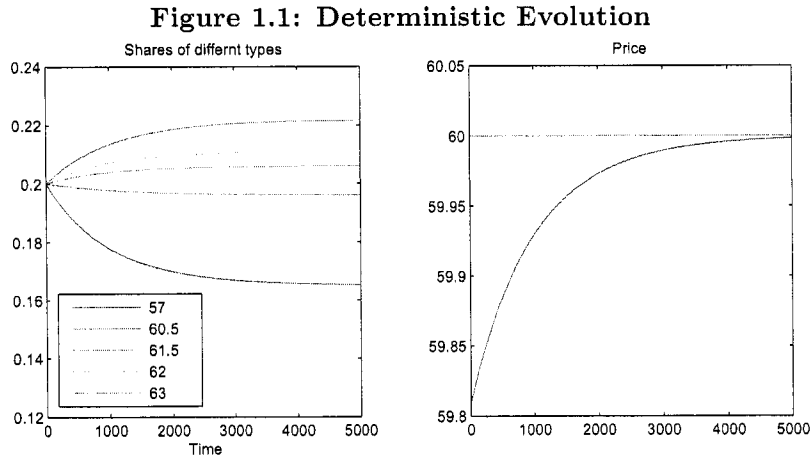
Figure 1.1: The horizontal line is the risk-neutral fundamental price 60. The fundamental price adjusted to risk is 40. Type beliefs are (57,60.5,61.5,62,63). The most optimistic type ends up having the highest share.

price is so high that the excess return is negative and then by the same logic the pessimists will grow and bring the price down to the risk-neutral level.

However, there is one caveat. All the results above are shown for the deterministic evolution, that is when the next period shares are functions of the expected wealth vector, and not of the actually realized wealth. Specifically, I assumed that the next period shares depend only on $(p_t + y - Rp_{t-1})z_{ht-1} + WR$, which is the expected wealth of type h, whereas the realized wealth of type h is equal to $(p_t + y + \varepsilon_t - Rp_{t-1}) \cdot z_{ht-1} + WR$, and it depends on the random shock ε_t.

The main shortcoming of the deterministic evolution, is that by ignoring the random shock, the evolution loses its ability to punish for the excessive risk. Specifically, the types who have crazily optimistic or crazily pessimistic beliefs will never be punished for their completely irrational portfolio choices. Instead, depending on initial conditions, either extreme optimists or extreme pessimists will be making the highest profit, and their shares will grow. As it can be seen from Figures 1.1 and 1.2, this is not the case if the evolution depends on random shocks. In the deterministic case (Figure 1.1) the most optimistic beliefs strictly increase their shares. However, when the evolution is affected by random shocks (Figure 1.2), agents that are too optimistic and too pessimistic die out for the exact reasons that were discussed in the Samuelson (1971) result.

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Figure 1.2 Evolution with Random Shocks
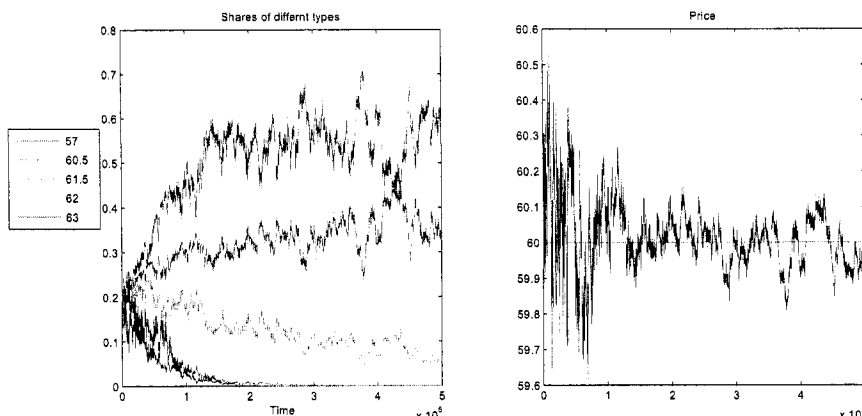
Figure 1.2: The horizontal line is the risk-neutral fundamental price 60. The dividend shock $\varepsilon_t \sim U[-1,1]$. The fundamental price adjusted to risk is 40. Type beliefs are (57,60.5,61.5,62,63). The most optimistic and pessimistic types die out.

1.4 Evolution with random shocks

The next theorem shows that the results from the previous section are robust in the following sense: with probability one the equilibrium price will be absorbed by some interval around p_n, and the size of the interval is proportional to the support of the random error. Thus, in particular, if the error is small, the equilibrium price will be very close to the risk-neutral price level. This result is distribution-free with the only requirement that the error is iid.

1.14 Theorem. *Assume that beliefs are diverse and that ε_t is iid with support $[-M; M]$. Then either with probability one there will be a moment $T < \infty$ such that for any $t > T$*

$$p_n - \frac{M}{R-1} \le p_t \le p_n + \frac{M}{R-1},$$

or $\lim p_t$ *will exist and belong to this range.*

◄ To begin with, we are going to adjust the theorems stated above to the fact that now R_t includes a random error ε_t. The first thing to notice is that the proof of Lemma 1.8 does not change, except that now $R_t = p_t + y_t - Rp_{t-1}$ includes a random shock.

Similarly, the derivation of formula (1.8) goes through unchanged except that R_t now

includes the random shock.

Recall equation (1.8):

$$R\Delta p_t = \sum_h \rho_h(n_{ht} - n_{ht-1}) = R_t \sum_h \rho_h(z_{ht-1} - z^0_{t-1})f'_1(x_0, \dots). \qquad (1.8)$$

Using it and the fact that $R_t > 0 \Leftrightarrow \Delta p_t > 0$ we have that

$$R\Delta p_t = R_t A_t, \qquad (1.10)$$

where A_t denotes the sum on the RHS of (1.8), and is positive for any t. From the evolution speed assumption we also know that $A_t < 1$.

Now we are going to use some algebra. First of all we notice that $R_t = b_{t-1} + \Delta p_t + \varepsilon_t$, and then we can re-write (1.10) as

$$(R - A_t)p_t = (R - A_t)p_{t-1} + A_t b_{t-1} + \varepsilon_t A_t.$$

By adding $(R - A_t)y$ to both sides of the equation and by deducting the same inequality multiplied by R we have that

$$(R - A_t)b_t = R(1 - A_t)b_{t-1} - (R - 1)A_t \varepsilon_t.$$

Finally from here we can get:

$$b_t = \frac{R(1 - A_t)}{R - A_t}b_{t-1} - \frac{(R - 1)A_t}{R - A_t}\varepsilon_t, \qquad (1.11)$$

and

$$\Delta b_t = -\frac{(R - 1)A_t}{R - A_t}(b_{t-1} + \varepsilon_t). \qquad (1.12)$$

Step 1: If $|b_{t-1}| \leq M$ then $|b_t| \leq M$ with probability 1, and thus $|b_t| \leq M$ is the absorbing range.

It can be immediately seen from (1.11). The highest possible value of b_t is when b_{t-1} is equal to M and ε_t is equal to $-M$. Then b_t is equal M. The lowest possible value of b_t is reached when b_{t-1} is equal to $-M$, and ε_t is equal to M, and then b_t is equal to

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$-M$, which proves step 1.

Step 2: If $b_{t-1} > M$ then with probability one $b_{t-1} > b_t$ and $b_t > -M$. If $b_{t-1} < -M$ then with probability one $b_{t-1} < b_t$, and $b_t < M$.

The second step says that if b_{t-1} is outside of the absorbing range, say below it, then b_t will increase with probability one, but will not jump above the absorbing range. To see this recall that from the evolution speed assumption and Lemma 1.8 we know that $0 \leq A_t < 1$. Thus from (1.12) we immediately have that if $b_{t-1} < -M$ then Δb_t is non-negative, which means that $b_t > b_{t-1}$. The fact that $b_t < M$ follows immediately from (1.11).

From Steps 1 and 2 we have that there are two possibilities: either at some point b_t gets in the absorbing range $|b_t| \leq M$ and stays there forever, or it never gets into it, and then it either always stays positive or always stays negative.

Step 3: b_t will reach the absorbing range with probability 1.

Assume that there is some realization of shocks such that b_t never reaches the absorbing state. In the latter case from Step 2 we know that the b-sequence is monotone with probability one, and thus p-sequence is monotone as well. Both sequences are bounded, and so they converge to some limit.

Denote the limit of b_t as b, and assume that $|b| > M$. If we look at the algebraic identity $R_t = b_t + R\Delta p_t + \varepsilon_t$ we see that as t goes to infinity $b_t \to b$ and $R\Delta p_t \to 0$. Thus if $b > M$ we have[7] that $\inf_t R_t > 0$ for all t, and if $b < -M$ we have that $\sup_t R_t < 0$. Given that we can use Lemma 1.12 which proof remains unchanged, and thus we know that only one type should survive. That should be the type that is either the most optimistic one if $p_t < p_n$ or the most pessimistic one otherwise. But then we have a contradiction since if it is the most optimistic type that survived, it would mean that p_t at some point will be above p_n which is impossible. If $|b| = M$ the statement of the theorem is still satisfied, since we reach, at least asymptotically, the absorbing interval.

Now from the fact that with probability 1 we end up in the price range such that $|b_t| \leq M$ we can calculate boundaries on the asset price as stated in the Theorem. ▶

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[7]Here R_t incluses the random error, and keep in mind that I fixed a particular realization of errors.

The theorem above shows that the results obtained for deterministic evolution are indeed robust to the introduction of small random errors. Due to the very general form of the evolution function it is difficult to say something more specific about the long-run distribution of the equilibrium price. So we are going to look at some very simple evolution rule to show some numerical simulations and also to demonstrate that at least for this particular rule, it is still the case that if beliefs are too high or too low then only one type survives with probability one, and if the beliefs are diverse then the equilibrium price will cross p_n infinitely often.

1.15 Example. Assume that the evolution function has the following functional form:

$$n_{ht} = n_{ht-1} + R_t \tau (z_{ht-1} - S) n_{ht-1}, \tag{1.13}$$

where $\tau > 0$ is a parameter that governs the speed of the evolution.

The meaning of this rule is that the increase in shares is proportional to the difference between the type's wealth and the average wealth. To see this notice that the wealth of an old individual of type h in period t is equal to $R_t z_{ht-1} + WR$. The average wealth of old individuals is equal to $\sum_h (R_t z_{ht-1} + WR) n_{ht-1} = R_t S + WR$. The difference between the wealth of type h and the average wealth is thus $R_t(z_{ht-1} - S)$ and so the change of the type share is proportional to the difference between type's wealth and the average wealth.

It is easy to check that this function satisfies all axioms of definition 1.1. To make sure that vector of new shares belongs to the simplex, τ has to be small enough so that n_{ht} is always positive[8]. Also, small τ implies slow evolution speed, which guarantees the last axiom.

1.16 Statement. *Assume that ε_t is iid with symmetric distribution around zero and with support $[-M; M]$. If all beliefs are too low (high) then with probability one only the type with the highest (lowest) belief will survive.*

This statement is the exact analogue of Theorem 1.10 that was proved for the deterministic evolution.

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[8]Recall that the possible values of R_t and z_{ht-1} are bounded, and so it is possible to pick τ to guarantee the positivity of n_{ht} for any values of R_t, n_{ht-1}, etc.

◄ In the proof I am going to consider the case when all beliefs are too low. The other case is identical.

From (1.13) we have that

$$n_{ht} - n_{ht-1} = R_t\tau(z_{ht-1} - S)n_{ht-1}.$$

Let us look at the most pessimistic type indexed as h. I am going to prove that its share is supermartingale and thus it converges to some limit with probability one.

The demand of this type is always going to be strictly less than S, and so $z_{ht-1} - S < 0$. Thus the relation between $E_{t-1}n_{ht}$ and n_{ht-1} depends on the sign of $E_{t-1}R_t$, and if $E_{t-1}R_t > 0$ for any t it would immediately mean that $E_{t-1}n_{ht} - n_{ht-1} < 0$.

The first step is to find a more explicit expression for A_t in (1.10). It can be done easily:

$$Rp_t = \sum_h n_{ht}\rho_h - \sigma^2 S = \sum_h n_{ht-1}\rho_h - \sigma^2 S +$$

$$+\tau R_t\sum(z_{ht-1} - S)n_{ht-1}\rho_h = Rp_{t-1} + \tau R_t A_{t-1},$$

where A_{t-1} depends only on variables determined in period $t-1$. Thus

$$R\Delta p_t = R_t\tau A_{t-1}.$$

We can simplify A_{t-1} even further, and show that it is positive.

$$A_{t-1} = \sum_h z_{ht-1}\rho_h n_{ht-1} - S\sum_h \rho_h n_{ht-1} =$$

$$= \frac{1}{\sigma^2}\sum_h \rho_h^2 n_{ht-1} + \frac{y - Rp_{t-1} - \sigma^2 S}{\sigma^2}\sum_h \rho_h n_{ht-1} =$$

$$= \frac{1}{\sigma^2}\left(\sum_h \rho_h^2 n_{ht-1} - \left(\sum_h \rho_h n_{ht-1}\right)^2\right) \geq 0,$$

where the last inequality follows from the Jensen inequality.

Now we can show that $E_{t-1}R_t$ is positive. Since beliefs are too low it means that $b_{t-1} > 0$ with probability 1. From (1.10), (1.11) and the fact that $b_t = R_t - R\Delta p_t - \varepsilon_t$

it follows that

$$R_t = \frac{R}{R - \tau A_{t-1}} b_{t-1} + \frac{R}{R - \tau A_{t-1}} \varepsilon_t, \qquad (1.14)$$

and so

$$E_{t-1} R_t = \frac{R}{R - \tau A_{t-1}} b_{t-1} + \frac{R}{R - \tau A_{t-1}} E_{t-1} \varepsilon_t = \frac{R}{R - \tau A_{t-1}} b_{t-1} > 0.$$

Thus, the shares of the most pessimistic type form a (non-negative) supermartingale, and hence it converges to some random variable. Moreover, we know that the supermartingale sequence can oscillate infinitely with probability zero, and thus for any shock realization, the share of the most pessimistic type will converge with probability one.

Assume now that with some positive probability, ν, the share sequence converges, but not to zero. Then, since $n_{ht} - n_{ht-1} = R_t \tau (z_{ht-1} - S) n_{ht-1}$ we still have that $R_t \tau (z_{ht-1} - S) n_{ht-1} \to 0$, and so with the same probability, ν, it has to be the case that R_t converges to zero. Now let us show that this is impossible.

Take a sequence of shocks such that R_t converges to zero. Then it means that $R_t = p_t + y + \varepsilon_t - R p_{t-1} = b_t + R \Delta p_t + \varepsilon_t \to 0$. From (1.10) it follows that $R_t \to 0 \Rightarrow R \Delta p_t \to 0$, and so $b_t + \varepsilon_t \to 0$. If we look at (1.12) we have

$$\Delta b_t = -\frac{(R-1)\tau A_{t-1}}{R - \tau A_{t-1}} (b_{t-1} + \varepsilon_t) = -\frac{(R-1)\tau A_{t-1}}{R - \tau A_{t-1}} (b_t + \varepsilon_t - b_t + b_{t-1}). \qquad (1.15)$$

Since $b_t + \varepsilon_t$ converges to zero we have that $\dfrac{(R-1)\tau A_{t-1}}{R - \tau A_{t-1}}$ should be either close to one, which is impossible since τ is small, or converge to zero. The latter is also impossible. To see this notice that the most optimistic type forms a submartingale that is bounded from above, and thus it also converges with probability one to some positive number. Given that in the limit there are at least two types with positive weights, $A_{t-1} = \sum_h (\rho_h - \sum_h \rho_h n_{ht-1})^2 n_{ht-1}$ will be also positive in the limit.

Thus it proves that the share of the most pessimistic type converges to zero with probability one. Therefore, eventually, with probability one the next pessimistic type will have that $z_{ht} - S < 0$ and so by the similar reasoning this type will die out as well,

and so on.▶

1.17 Corollary. *If beliefs are diverse then the equilibrium price will cross p_n infinitely often.*

◄ Assume that this is not true, then after some point b_t will always have the same sign. Then we can apply the statement above to show that only one type with survive with probability one. If this b_t is always positive it would mean that only the most optimistic type survives, which then would mean that p_t will be greater than p_n which is contradiction to the positivity of b_t. ▶

Finally, numerical simulations for this specific evolution function show that $ER_t = 0$, and thus the mean equity premium is going to be equal to zero, as in the deterministic case.

1.5 Conclusions

In the paper I provide a behavioral model that explains the asset overvaluation recently observed in the stock markets. The contribution of this paper is that the proposed model is intrinsically symmetric, in a sense there is no optimistic bias in agents' decision making, and there are no short-sale constraints that would decrease the opportunities for the bearish agents to bring the price down. Another contribution of the paper is that the analysis does not depend on a particular form of the evolution function or individual learning, which helps to make the analysis more general.

The main result is that in an economy with the Friedman selection mechanism, the price of the risky asset converges to the risk-neutral price level, even though all agents are risk-averse. The primary intuition is based on the DSSW (1990) intuition, which is that it is true that the optimists choose suboptimal portfolios given their preferences. However, the sub-optimality here means more risk than the agents would optimally prefer. The expected return of the optimistic portfolio is higher than the return of more cautious people and that increases the share of optimists and pushes the price up towards the risk-neutral level.

In the paper, I develop this intuition even further by addressing Samuelson (1971)

results, which indicate that portfolios with higher risk and higher expected return can be dominated with probability one by less risky and less profitable alternatives. I show that despite bearing more risk, the optimists can still push the asset price above the correct level. The intuition now is more subtle: Samuelson's results are applicable to the case when the mean and the variance of returns are fixed, whereas in my paper they are endogenously determined by the prices. In particular, as the price goes up, the optimists demand less of the risky asset, while more cautious agents might think that the asset is overvalued and short it. Thus the riskiness of the optimists' portfolios decreases while the pessimists returns become more risky. That reinforces the upward price pressure.

The analysis in the paper demonstrates and limits the optimists' power. It shows that optimists can significantly affect the asset prices and push them up, but only to the risk-neutral fundamental price.

1.6 Appendix. Lagged timing

In this paper, I assumed that n_t was determined by the vector of the current wealth. As I show in the appendix the results of the paper do not change, if I instead consider the lagged timing, where n_t is determined by W_{t-1}. To do that I am going to go through the reasoning of Section 1.3 and adjust it where necessary. While in most of the cases, the adjustment will be trivial, one complication will arise. As I will show, under the lagged timing I have to introduce an additional state variable, which will make the proof of convergence more complicated.

1.18 Definition. *The evolutionary dynamic* is determined by a function f that satisfies the following assumptions:

(A1) The share of type h in period t is a deterministic C^1-function of per capita wealth vector, W_{t-1}, shares vector, n_{t-1} and some other variables, $\xi_{t-1} \in \Xi$, where Ξ is a compact set. That is,

$$n_t = f(n_{t-1}, W_{t-1}, \xi_{t-1}),$$

where $f : \Delta^{H-1} \times \mathbb{R}^H \times \Xi \to \Delta^{H-1}$ is C^1-function.

(A2) (*Weak monotonicity*) For given W_{t-1} and n_{t-1} if $n_{ht} = f(n_{t-1}, W_{t-1}, \xi_{t-1}) > n_{ht-1}$ for some type h, then $n_{kt} > n_{kt-1}$ for any type k such that $W_{kt} \geq W_{ht}$.

(A3) (*Non-triviality*) If the vector of shares does not change, then either all types with positive shares earned the same profit in period $t-1$, or one type has a share equal to one.

(A4) (*Evolution speed*) The evolution process is sufficiently slow.

Given this definition, the equilibrium dynamics is defined in the following way:

1.19 Definition. Given $(n_{t-1}, W_{t-1}, p_{t-1})$, the equilibrium in period t is a triple (n_t, W_t, p_t) such that $n_t = f(n_{t-1}, W_{t-1}, \xi_{t-1})$, the price in period t clears the market. That is, $p_t = \frac{1}{R}(\sum_h n_{ht}\rho_h - \sigma^2 S)$ and $W_{ht} = (p_t + y - Rp_{t-1})z_{ht-1} + WR$.

Now we are going to prove that the analogue of Theorem 1.10 with the new timing holds.

1.20 Theorem. *Consider the equilibrium dynamics as defined in Definition 1.19 with the evolution function as in Definition 1.18. Then the conclusion of Theorem 1.10 will hold.*

◄ We are going to re-write the argument that was used in Section 2.2 for the new evolution function. Some of the proof will work just by changing index t of the wealth vector on index $t-1$. However, there will be two non-trivial changes in the proof caused by the fact that now the evolution of the system is determined by three state variables and not two as before.

Lemma 1.8 adapted to the new dynamics states that $\Delta p_t > 0 \Leftrightarrow R_{t-1} > 0$. Its proof can be trivially re-written just using the change of indices. The next step is to show that the price sequence converges. The previous proof of this section cannot be applied to the new dynamics, and in the next lemma a different argument for the convergence is developed.

1.21 Lemma. *If the evolution function satisfies all axioms in Definition 1.18 then the price sequence converges.*

◄ Similarly to Theorem 1.7, we can get that

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$$R\Delta p_t = R_{t-1}A_t, \tag{1.16}$$

where A_t is positive by Lemma 1.8. Denote x_t as $p_t - y/(R-1)$. Then we can re-write (1.16) as

$$(x_t - x_{t-1}) = (x_{t-1} - Rx_{t-2})A_t,$$

or

$$x_t = (1 + \frac{A_t}{R})x_{t-1} - A_t x_{t-2}. \tag{1.17}$$

If A_t were constant then the behavior of the difference equation (1.17) would be determined by its characteristic roots. Specifically, if $A < 1$ then the zero solution is globally stable (see Agarwal (2000)). We cannot apply this result directly to our case, since A_t is not a constant and we have little information about its behavior. Nonetheless, the characteristic roots of this equation will still be useful. For each A_t the characteristic roots are the solutions of the quadratic equation:

$$\lambda_t^2 - \frac{A_t + R}{R}\lambda_t + A_t = 0,$$

and thus

$$\lambda_{t,12}(A_t) = \frac{1}{2} + \frac{A_t}{2R} \pm \sqrt{\frac{A_t^2}{4R^2} - (1 - \frac{1}{2R})A_t + \frac{1}{4}}.$$

I am going to index the smallest root with one, that is $\lambda_{t1}(A_t) < \lambda_{t2}(A_t)$

Step 1: There is such $\bar{A} > 0$ that for any non-negative $A < \bar{A}$, the characteristic equation has two real roots that are non-negative and smaller than one. Moreover, for any $A_t, A_t' < \bar{A}$ it is the case that $\lambda_1(A_t) < \lambda_2(A_t')$.

The easiest way to see this is to notice that the minimum of the characteristic function is reached at point $(A_t + R)/2R$. Thus as long as $A_t < R$ it is reached at point between 0 and 1. By plugging 0 and 1 instead of λ_t into the characteristic function we have that it is non-negative as long as A_t is non-negative. Thus, if there are real roots, they should both be between 0 and 1. By looking at the discriminant we see that it is positive when $A_t = 0$, and so there is some number $\bar{A}_1$ such that when $0 \leq A_t < \bar{A}_1$ the discriminant is also positive and thus there are two real characteristic roots. As for the second part,

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when $A_t = 0$ there are two roots 0 and 1. As we increase A_t by continuity it is still going to be the case that all smallest roots are less than all largest roots. Assume that this property is satisfied for any $A_t, A'_t < \bar{A}_2$. Then the required $\bar{A} = \min\{\bar{A}_1, \bar{A}_2\}$.

Now we are going to use the assumption on the evolution speed, specifically we are going to assume that all possible values of A_t are less than $\bar{A}$. Denote $\lambda_{2min} = \inf\{\lambda_2(A_t)|A_t < \bar{A}\}$.

Divide both sides of (1.17) on x_{t-1} and denote x_t/x_{t-1} as u_t. Then we have the following equation

$$u_t = 1 + \frac{A_{t-1}}{R} - \frac{A_{t-1}}{u_{t-1}}. \tag{1.18}$$

Step 2: If $u_{t-1} > \lambda_{2min}$ then $u_t > \lambda_{2min}$.

Indeed,

$$u_t = 1 + \frac{A_{t-1}}{R} - \frac{A_{t-1}}{u_{t-1}} > 1 + \frac{A_{t-1}}{R} - \frac{A_{t-1}}{\lambda_{2min}} > \lambda_{2min},$$

where the last inequality follows from the fact that for any A_t the smallest characteristic root is less than λ_{2min} and the largest characteristic root is greater than λ_{2min}.

Step 3: The sequence $\{u_t\}$ is eventually positive.

Assume that $u_{t-1} < 0$ for some $t-1$, then $u_t > 1 + A/R > \lambda_{2min}$ and for Step 2 we know that it will remain positive. Assume that $u_{t-1} = 0$. It means that $x_{t-1} = 0$, which in turn means that $u_{t+1} = x_{t+1}/x_t = (A+R)/R > \lambda_{2min}$.

Step 4: The sequence x_t converges.

From Step 3 it follows that x_t will eventually have the same sign. Assume that it is positive. Then either $x_t > x_{t-1}$ for any t, which means that it converges since the prices are bounded. Alternatively, $x_t < x_{t-1}$ for at least some t. In that case, if we re-write (1.17) as

$$x_t - x_{t-1} = A_{t-1}\left(\frac{x_{t-1}}{R} - x_{t-2}\right),$$

we see that $x_t < x_{t-1}$ is equivalent to $x_{t-1}/R - x_{t-2} < 0$. Also from $x_t - x_{t-1} < 0$ and $x_t > 0$ it follows that $x_t/R - x_{t-1} < 0$ and thus $x_{t+1} < x_t$ and so on. Thus, the sequence

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is decreasing and bounded by zero, so it converges. The case of negative x_t is similar. ▶

Lemma 1.12 that claims that if $R_\infty \neq 0$ then only one type survives in the end is still valid, and its proof can be easily re-written by changing the wealth index from t to $t-1$.

Now we are ready to finish the proof of the theorem. Assume that the price sequence converges to some limit p. If $p < p_n$ then $R_\infty = p + y - Rp > 0$, and if $p > p_n$ then $R_\infty < 0$. Since when beliefs are too low (high) the limit price has to be less (greater) than risk-neutral, from Lemma 1.12 we know that only the most optimistic (pessimistic) type survives.

Now assume that beliefs are diverse and $R_\infty > 0$. Then it has to be the case that only the most optimistic type survives. But then it would mean that the limit price is higher than the risk-neutral price, and thus at some point the excess return would become negative, which is a contradiction.

The case when R_∞ is negative is similar. And hence it proves that when beliefs are diverse, R_t should converge to zero and the equilibrium price should converge to the risk-neutral level. ▶

Chapter 2

Reputation Dynamic in Credit Markets

2.1 Introduction

A lack of investments is one of the most serious problems for developing and transitional economies. Many countries with high economic potential cannot realize it without the necessary capital inflow. A low credit ranking of the country makes otherwise lucrative projects too risky for investors. But, the recipe in this situation seems to be simple. One would think that all the country has to do is to pay back the investors, which should gradually improve its reputation, decrease the risk of investments and increase the inflow of capital. Nonetheless, in the last 20 years there were more than 500 defaults and debt restructuring. Countries fail to establish reputation and the question is why.

To address this question, I model interactions between a country and investors as an infinitely repeated game. The country is a long-lived player, and its discount factor is private information. Given the flow of future investments, the country maximizes its net present value of wealth by choosing the best time, if ever, to default. Investors are short-lived players who are competitive and risk-neutral. They have a common prior that is updated based on the history of the game. In equilibrium, investors correctly estimate the probability of default, and given that, investments flow into the country until their expected marginal return is equal to the market riskless rate. If at some period the country defaults, the investors abandon the country.

It turns out that there is a continuum of equilibria with all equilibria but one being inefficient. Which one is realized depends on investors' initial confidence, i.e. initial

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beliefs about the probability of default. In the unique efficient equilibrium, the types who choose the honest strategy under complete information also choose the honest strategy under incomplete information. That being so, the investment risk is asymptotically eliminated — *permanent reputation improvement*. The inefficient equilibria can be divided into two groups. There is a continuum of equilibria when initially the confidence and investment levels grow, but after some period of time confidence starts to decline, and all types choose to default — *temporary reputation improvement*. That happens for intermediate values of initial beliefs. Finally there is a continuum of equilibria when the confidence in the country declines from the start, even if the country pays its loans back. In this case, again, sooner or later all types will default — *no reputation improvement*. This happens for low values of initial beliefs.

The inefficiency of the equilibria is a very striking and surprising result. One would think that as the game goes on and the country does not default, the investors should become more confident that the country is sufficiently patient. Thus one would expect that the risk of investing would decrease while the level of investment increase. However, the result is exactly the opposite: in all but the efficient equilibrium, the level of investments converges to zero, while the risk of default converges to one.

There are two reasons why the predictions of my model are different from our intuitive expectations. First, starting from Kreps and Wilson (1982) and Milgrom and Roberts (1982) the reputation effect has been modeled by adding a positive probability of a commitment type who for some exogenous reasons always plays the same strategy. I, however, assume that all types are rational, which leads to a completely different outcome. Indeed, if with small probability the country is the honest type which always pays back, then as the game proceeds without defaults, the updated probability of dealing with the honest country increases. In turn, this makes the investments less risky, and inefficient equilibria become impossible. When all types are rational, the only information that is revealed over time is that the country is more and more patient. However, this information *per se* does not decrease the risk of investment because even though the country is patient it still might prefer to default depending on the path of future payoffs.

Second, investors are short-sighted, and so it does not matter for them how patient

the types in the game are. The only relevant parameter is the probability of confiscation in a particular period of investment. Furthermore, if the investment level declines, it is irrational for the current investor to "use" information about the country's high patience and to invest more, because the country is a long-run player and its decision is based on the whole path of payments. The high payoff today and low payoffs tomorrow only increase the country's incentive to cheat.

I also consider several extensions of the basic model. In particular, I show that the results do not depend on the competitiveness assumption. I prove that as long as investors are short-sighted and the inflow of investments is positively correlated with the country's reputation, the results remain the same, even if investors have some market power. It suggests that the myopia of investors stands behind the inefficiency of the equilibria. To investigate this conjecture, I introduce a competitive long-sighted investor who can offer the infinite sequence of loans. As I show in the paper, the existence of such an investor removes all inefficient equilibria. The result has a clear policy implication — that it is vital for the government of a developing country to try to attract long-run investments. This is especially important since the easiest way to obtain foreign financing is via short-term debt[1], which I will show is more prone to default.

While, it is beneficial to attract long-sighted investors, clearly it can be quite a problem for a country with a developing or transitional economy. To allow for this I consider the following extension: the country still has access to short-term investments, but in addition it can send a costly signal that credibly reveals that it is sufficiently patient. Signalling would attract the long-sighted investors who rationally invest the efficient amount of capital into the country. Introducing signaling into the model reveals that the higher reputation a country has, the smaller is the size of the signal required to credibly reveal its patience. Given that developing countries typically do not have extra money to "burn", they can use the following strategy to reduce the burden of the signal: if the initial confidence is not too low for the country to temporarily improve its reputation, then it can start out by taking small credits, paying them back, and

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[1]For example, in Russia in the mid-nineties, the major way for the government to get money was through issuing short-term obligations (GKO) with extremely high rate of return. For instance, annual inflation in Russia in 1996 was estimated to be approximately 20-25%, while the 6-month GKO issued in June provided a return of 250% annually. Eventually it led to the default in August, 1998.

gradually improving its reputation. As confidence in the country increases, the size of the required signal decreases, and so signalling becomes more affordable.

My paper is different from the literature in several aspects. First of all, as I discussed above, all types in my model are rational, whereas the standard assumption in the reputation literature is that with a small probability there is a commitment type. As I will show in the paper, this difference in assumptions leads to completely different outcomes.

Second, in game-theoretical papers analyzing a lender-borrower relationship with strategic default, the market of lenders is not a part of the model, see Sobel (1985), Watson (1999) and (2002). For example, in the last two papers, there are two players playing a partnership game in continuous time with two-sided incomplete information. It is shown that no matter how pessimistic players are about their partners, there is an eventually cooperative equilibrium as long as the partners start small, i.e. when the partnership level is low. The reason why the results in Watson's papers are different from mine is because the partnership level (which is an analogue of the investment's level in my paper) is considered to be exogenous. Watson assumes that before the game starts both parties decide about the partnership dynamics given some reasonable criteria (e.g. renegotiation proofness), and then as the game goes their only decision is at what time to betray the other player. In my paper, the partnership level is determined by the market, and starting small is suicidal. In the case of competitive market, starting small would mean that all investors are very pessimistic about the country and want to charge very high interest rate as a compensation for the risk. In the equilibrium their pessimism is correct since the low investment level does not give the country incentives to pay back, and so it eventually defaults.

There are many papers, that explicitly model the credit market (see Boot and Thakor (1994), Berger and Udell (1995), Greenbaum (1990), Sharpe (1990), etc.). However, the focus and strategic setting of those papers are different. In my model a borrower's strategy is to decide whether to pay back or not. Other papers that analyze the credit market look at adverse selection and moral hazard types of problems, where either the probability of borrower's success is exogenously determined by its type, or, as in

Diamond (1989), the borrower's strategy is a choice between different projects with exogenous returns.

Some authors (Petersen and Rajan (1995), Cetorelli (1997), Caminal and Matutes (1997)) look at the concentrated credit market and argue that despite standard monopolistic distortions, it might be more efficient than a competitive market. A possible explanation is that a lender with market power can first charge smaller interest rate, but then as the uncertainty about the borrower decreases, the lender will be able to extract higher surplus. This possibility makes the lender more willing to give credit in the situations when there is large uncertainty about the borrower's type. In my paper, the monopoly is strictly worse than competitive lenders. It charges higher interest rate, and provides less capital, while all qualitative properties of equilibria remain the same. The reason why there is no benefit from monopoly is that lenders are myopic maximizers. Effectively, for them the relationship will terminate by the end of the current period, whereas, according to the logic above, the long-term lender-borrower relationship is essential.

The non-monotone reputational dynamic described in the paper, appears also in Mailath and Samuelson (2001) and Cripps et al. (2004), where it is shown that under imperfect monitoring a rational long-run player can only temporarily but not permanently maintain a reputation for a strategy that does not play an equilibrium of the complete information game. While the result has a similar spirit to mine, it is based on a different mechanism. In the setting with a commitment type and imperfect monitoring, a long-lived player is willing to create good reputation at first, but then as the prior gets high enough, the benefit of improving it becomes smaller than the benefit of "cheating" once in a while. Eventually the long-lived player will eat up all the reputation he created.

In my model the country is not trying to create a reputation of being someone else, simply because there is no one else, i.e. no commitment type. While on one hand, it might be good for the country if investors believe that it is patient, on the other hand, the country's patience does not necessarily mean that it is going to be honest. Thus, if investors think that the country is likely to default they are going to provide a low level

of capital even if they know that the country is patient. And consistent with that if the flow of investments is low and does not provide the incentives for honest behavior, the country will default.

The chapter has the following structure: in Section 2.2, I describe the model's assumptions, the timing of the game, and the payoff structure. In section 2.3, I derive the system of equations that defines the equilibria, classify the different types of the equilibria and interpret them. In section 2.4, the model extensions are considered. Section 5 is the conclusion, and all the proofs are given in the appendix.

2.2   The model

In this paper, I consider an infinitely repeated game between investors and the country. The country is a long-lived player that plays through the whole game, and investors are short-lived players, playing for one period.

The economy has discount factor δ unknown to investors. It is common knowledge that δ is distributed in $0 \leq [\delta_{min}; \delta_{max}] \leq 1$ with cdf $\Phi(\delta)$. We assume that $\Phi(\delta)$ is differentiable, and has a strictly positive density on $[\delta_{min}; \delta_{max}]$. The economy can produce output from capital but it does not have any, so it needs external investments. Economy's production function is $y = F(K)$ (known to the investor), and we assume that it satisfies Inada conditions.

The time is discrete, and at each period the following stage game is played: the investor sets the price of capital R_t, which depends on his beliefs on the probability of default. Given that, the investments flow into the country until their marginal return is exactly equal to R_t. The country produces output and decides whether to honor the investor or not. In the former case it pays to the investor the return on capital $R_t K_t$, and has $F(K_t) - R_t K_t (= F(K_t)(1 - \varepsilon(K_t)))$ left, while in the latter case it pays nothing and has $F(K_t)$, but as the penalty the investors abandon the country. Here $\varepsilon(K_t)$ is the elasticity of the production function. In the end of each period the full depreciation of capital occurs.

Given, the sequence of invested capital $\{K_t\}$, the country wants to find the best

moment, if ever, to confiscate the investor's wealth. So its problem is

$$\max_T \left\{ \sum_{t=0}^{T} (1 - \varepsilon(K_t)) F(K_t) \delta^t + F(K_{T+1}) \delta^{T+1} \right\}. \tag{2.1}$$

Note that T might be equal to infinity, which means that the honest behavior is the most preferable.

Investors know the history of the game which is that the country has not defaulted till the current moment. Each investor plays for one period only, and in each period there is only one investor. I assume that the investors are risk-neutral, competitive, short-sighted. The investor in period t believes that with probability q_t the country will pay back. Given that, he sets R_t in such a way that his expected return on capital should be exactly rK, where r is the market riskless interest rate. So it is easy to see that $R_t = \frac{r}{q_t}$. In the equilibrium the investors' beliefs are rational.

Denote the level of capital that would be invested in the absence of risk (that is under interest rate r) as $\bar{K}$.

ASSUMPTION 1. $K \cdot F'(K)$ *is an increasing function of capital.*

Under this assumption, the country facing an increasing sequence of investments will have a non-trivial trade-off between cheating now and cheating later when more capital is invested.

ASSUMPTION 2. *In order to have the efficient equilibrium we need to assume that there are patient types in the game. More precisely, we assume that $\delta_{max} \geq \varepsilon(K)$ for any $K \leq \bar{K}$, where ε is the elasticity of production function.*

If Assumption 2 is violated, the efficient equilibrium (when some types choose the honest strategy) is impossible, since the share of the profit that the country gets is not sufficient to prevent cheating.

2.3   The Equilibria

In the equilibrium, given the sequence of invested capital, the country chooses the best moment to confiscate the investor's wealth (that is it solves maximization problem (2.1)).

At each period the current investor has belief on the probability of confiscation, and given it he sets the price of the capital R_t and the investments flow into the country until their marginal return is equal to it. Since the investors are competitive and risk-neutral, the interest rate R_t is set in such a way that $E_t(R_t K_t) = r K_t$. In the equilibrium the investors' beliefs should be rational.

In order to solve the country's problem (2.1) we introduce the sequence of auxiliary variables. Let δ_t be a discount factor such that the economy with it is indifferent between cheating at moment t and moment $t + 1$. It can be found from the following equation:

$$F(K_t) = F(K_t)(1 - \varepsilon(K_t)) + \delta_t F(K_{t+1}), \tag{2.2}$$

which is equivalent to

$$\delta_t = \varepsilon(K_t) \frac{F(K_t)}{F(K_{t+1})}. \tag{2.3}$$

All economies with $\delta < \delta_t$ will prefer cheating at moment t to cheating at moment $t + 1$, and on the other hand all economies with $\delta > \delta_t$ will prefer cheating at moment $t + 1$ to cheating at moment t.

The next two statements are crucial in solving the country's problem.

2.1 Statement. *If $\{\delta_t\}$ is a non-decreasing sequence, the economy with $\delta_t < \delta < \delta_{t+1}$ will default at moment $t + 1$ (or will not default at all).*

◀ Assume that $\delta_t < \delta < \delta_{t+1}$. Let π_t denote the net present value (from zero period point of view) of the economy's profit if it defaults at moment t. It is easy to see that

$$\pi_0 < \pi_1 \cdots < \pi_t < \pi_{t+1} > \pi_{t+2} > \ldots. \tag{2.4}$$

For example, let us prove that $\pi_0 < \pi_1$ — indeed, since $\delta > \delta_0$ the economy prefers cheating at moment 1 to cheating at moment 0. Using the fact that for any $s \le t$ $\delta_s < \delta$, and for any $s \ge t + 1$, $\delta < \delta_s$, we can prove all the inequalities in (2.4) in a similar way. ▶

2.2 Statement. *In the equilibrium $\{\delta_t\}$ is a non-decreasing sequence.*

◀ Assume that it is not the case and that there is an equilibrium such that $\delta_t > \delta_{t+1}$.

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Then I claim that no one will cheat at time $t + 1$. Indeed, let's look at the country with discount factor δ. If $\delta < \delta_t$ then by definition of δ_t, $\pi_t > \pi_{t+1}$. On the other hand, if $\delta > \delta_t$ then it is also the case that $\delta > \delta_{t+1}$ and so it has to be that $\pi_{t+2} > \pi_{t+1}$. Thus there is no type that would like to cheat at moment $t + 1$.

Since investors in the equilibrium have rational expectations it has to be the case that $q_{t+1} = 1$ and the amount of invested capital in period $t + 1$ is equal to $\bar{K}$. Now let us prove that it is impossible. By definition of δ_t and δ_{t+1} we have that

$$\delta_t = \varepsilon(K_t)\frac{F(K_t)}{F(\bar{K})} = \frac{F'(K_t)K_t}{F(\bar{K})}$$

$$\delta_{t+1} = \varepsilon(\bar{K})\frac{F(\bar{K})}{F(K_{t+2})} = \frac{F'(\bar{K})\bar{K}}{F(K_{t+2})}.$$

Since $K_t \leq \bar{K}$ and $K_{t+2} \leq \bar{K}$ we have that $F'(K_t)K_t \leq F'(\bar{K})\bar{K}$ and $F(\bar{K}) \geq F(K_{t+2})$, but then δ_t should be less than δ_{t+1} which is contradiction. ▶

These two extremely simple statements give us a key to the model, because using it we can conclude that the country with discount factor δ either cheats at such t that $\delta_t < \delta \leq \delta_{t+1}$ or does not cheat at all. In Theorem 2.16 in the appendix we prove that the country with $\delta_t < \delta \leq \delta_{t+1}$ will prefer to cheat.

Given $\{\delta_t\}$, the conditional probability q_t that the country will not default in period t is $q_t = Prob\{\delta \geq \delta_t | \delta \geq \delta_{t-1}\}$. Indeed it is known that the economy has $\delta \geq \delta_{t-1}$ and that the economies with $\delta \geq \delta_t$ will not default now. Thus the following system characterizes the equilibrium dynamic:

$$\begin{cases} \delta_t = \varepsilon(K_t)\dfrac{F(K_t)}{F(K_{t+1})} \\ q_{t+1} = \dfrac{1 - \Phi(\delta_{t+1})}{1 - \Phi(\delta_t)} \\ q_0 = 1 - \Phi(\delta_0) \end{cases} \tag{2.5}$$

Some explanations are required: first of all in (2.5) there are three sets of equations. The first set describes the behavior of the country, given the investment's path (i.e it determines $\{\delta_t\}$ given $\{q_t\}$), the second set describes the behavior of the investors given the behavior of the country (i.e it determines $\{q_t\}$ given $\{\delta_t\}$), and the last equation

provides that the initial confidence is rational. Second, the only unknown variables in (2.5) are $\{q_t\}$ and $\{\delta_t\}$. The sequence $\{K_t\}$ is uniquely determined by $\{q_t\}$, since for any t: $F'(K_t) = R_t = r/q_t$ and F' is a strictly decreasing function. Third, the solution of (2.5) is completely determined by q_0. Indeed, given q_0 from the third equation one determines δ_0. Given δ_0 and q_0, from the first equation one determines q_1 and so on. Fourth, at any moment t the future behavior of the system is uniquely determined by a pair (q_t, δ_t).

2.3.1 Complete information case.

In the complete information case δ is common knowledge, and then it is relatively easy to find the pure-strategy equilibrium for the case of complete information. The behavior of the country is still determined by the first equation of (2.5). The probability of confiscation is either 1 or 0, and so in equilibrium there are two cases: either q_t is always 1, or q_t is always zero[2]. Plugging $q_t = 1$ into the first equation of (2.5), we have that $\delta_t = \varepsilon(\bar{K})$.

Thus, if $\delta < \varepsilon(\bar{K})$ then the only equilibrium is $q_t \equiv 0$ and the country confiscates everything invested. If $\delta \geq \varepsilon(\bar{K})$ then there are two equilibria: $q_t \equiv 1$, the country behaves honestly; $q_t \equiv 0$, the country confiscates everything invested.

I am going to call types with $\delta \geq \varepsilon(\bar{K})$ as patient or safe, because in the complete information environment, there is an equilibrium where the country does not default. On the other hand, countries with $\delta < \varepsilon(\bar{K})$ are impatient and will cheat regardless of the investment path.

2.3.2 Incomplete information case without commitment type.

In the case of incomplete information without commitment type, the equilibrium dynamic is described by (2.5). As it is shown in Lemma 2.6.11 there are two steady states: $(1, \varepsilon(\bar{K}))$ and $(0, \delta_{max})$. Later I will show that any equilibrium trajectory will converge to one of these steady states. Therefore there are two asymptotic outcomes: *the effi-*

---

[2]It is impossible, if $q_t = 0$ and $q_{t+1} = 1$, since if $\delta \geq \varepsilon(\bar{K})$ then the investor could invest some positive amount in period t and the country would not cheat. If $\delta < \varepsilon(\bar{K})$ then the country prefers cheating to the sequence of riskless amount of capital, and thus, it will cheat at moment $t+1$ regardless of the future path.

cient outcome when the risk of investing is eventually eliminated, and only the patient types with $\delta \geq \varepsilon(\bar{K})$ are left in the game; and the inefficient outcome when there are no investments at all, because even the most patient types preferred to default at some moment.

It is very tempting to anticipate that the efficient outcome has a higher chance to happen. Indeed, as time goes and the game continues, it is revealed to the investors that the country is more and more likely to have the patient type, which means that the risk of investment declines with the time. Since by Assumption 2 there are sufficiently patient types in the game, it is quite natural to expect that eventually we should converge to the riskless outcome. However, as the next theorem shows, the intuition is not correct, and in fact almost all equilibria are inefficient.

Theorem 1. *Given the assumptions above on the production function F and cdf $\Phi(\delta)$ there are only four possible kinds of solutions to (2.5) and all but the last one are equilibria:*

a) (No reputation improvement) The confidence in the country and the corresponding flow of investments decrease from the beginning of the game, and all types will eventually default[3].

Figure 2.1: No reputation improvement case. The q-line characterizes investors' confidence to the country in period t. The δ-line shows what types have not defaulted yet: those with δ higher than δ_t.

b) (Temporary reputation improvement) Initially, the confidence in the country grows, however ultimately it also falls to zero, and all types will eventually default.

---

[3]These pictures are drawn for $F(K) = \sqrt{K}$ and uniform distribution on $[0,1]$, however it is shown in Appendix that the result is the same for any production and distribution functions satisfying the assumptions of the model.
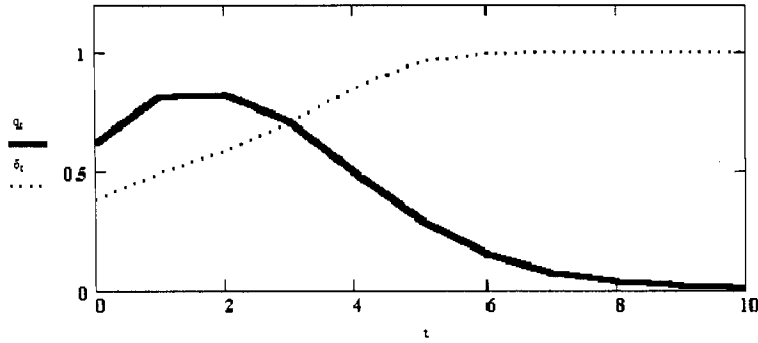
Figure 2.2: Temporary reputation improvement case. The q-line characterizes investors' confidence to the country in period t. The δ-line shows what types have not defaulted yet: those with δ higher than δ_t.

c) *(Permanent reputation improvement) There exists the unique efficient equilibrium*

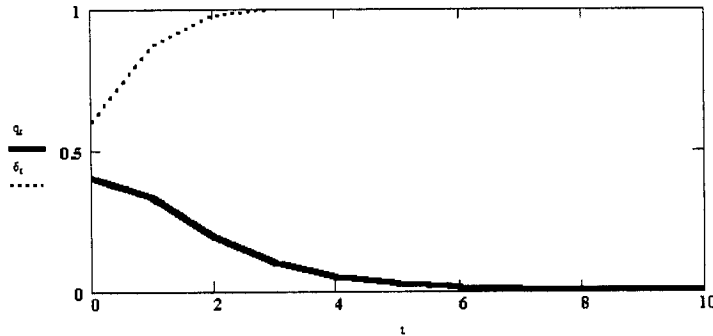*such that* $q_t \to 1$, $\delta_t \to \varepsilon(\bar{K})$.
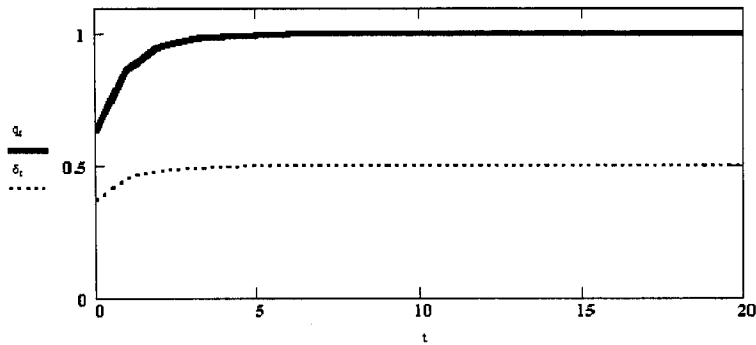
Figure 2.3: Permanent reputation improvement case. The q-line characterizes investors' confidence to the country in period t. The δ-line shows what types have not defaulted yet: those with δ higher than δ_t.

d) *A very high initial confidence cannot be rational, and thus these trajectories will*

not be an equilibrium.

The pictures should be understood as follows. The q-line shows the investors' confidence in the country, and also it represents the dynamic of investments, since the higher is q_t, the higher is invested K_t. The δ-line shows when different types are going to default. For example, by period $t + 1$ types in $[\delta_{min}; \delta_t]$ have already defaulted, types in $[\delta_t; \delta_{t+1}]$ are going to default in period $t+1$, and types in $[\delta_{t+1}; \delta_{max}]$ are going to default later, or are not going to default at all.

From now on I will refer to the particular type of equilibria as a)-case, b)-case or

c)-case.

Clearly, there is a multiplicity of equilibria. Moreover, if we denote the q_0 that generates the efficient outcome c) as q_0^c then it can be shown that any $q_0 \leq q_0^c$ generates the equilibrium trajectory (it follows from the Monotonicity property, see Lemma 2.6.7). In Lemma 2.6.10 it is proved that the feasible solutions of (2.5) should look like in a), or b) or c)-case, and Theorem 2.6.16 shows that they are equilibria. Here by feasible solution we mean any sequence $\{q_t, \delta_t\}$ that $0 \leq \delta_t, q_t \leq 1$ for all t. Theorems 2.6.12 and 2.6.13 show that the efficient equilibrium (when $q_t \to 1$) exists and is unique.

Now we can interpret the results and compare them with our expectations. First of all, the initial confidence plays the crucial role in the trajectory behavior. Specifically, the higher is q_0, the more investments the country receives (see Monotonicity property in the Appendix). Second, there is an efficient equilibrium, where the impact of incomplete information is asymptotically eliminated. The patient countries with $\delta \geq \varepsilon(\bar{K})$ choose an honest strategy, as well as they did it under complete information, and the flow of investment asymptotically converges to the efficient level.

The bad news is that there is only one efficient equilibrium, which is highly surprising given that in a) and b)-cases, there is such a moment T that after $t = T$ it is known that $\delta > \varepsilon(\bar{K})$, and so the country would honor the riskless flow of investments. Yet, the confidence in the country keeps declining and eventually converges to zero. There are two reasons behind this result. First, the safe types are rational and so the fact that the only safe types are left does not necessarily mean that there is no more risk of investing. Indeed, if the flow of investments dramatically decreases, as it happens in both a) and b)-cases, then even the safe types prefer cheating.

The second reason is that the investors are short-sighted, and so they do not care how patient the types in the game are. The only important parameter for them is the rate of default in the current period, or in other words, what is the probability of the confiscation in the period of the investment. And although it is known that the types are safe, the rate of default can be very high.

This result seems to be extremely pessimistic, and as I show in the next section, the possibility of attracting a long-sighted investor remedies the problem and leaves the

efficient equilibrium only.

To complete the description of the equilibrium I would like to consider one special case when the production function has a Cobb-Douglas form. In this case the following statement holds.

2.3 Proposition. *There is such a value of the initial confidence denoted as q_0^a that*

a) $0 \leq q_0^a < q_0^c$;

b) if $q_0 < q_0^a$ we have the a)-case, while if $q_0^a < q_0 < q_0^c$ we have the b)-case[4].

The proof is the simple corollary of Lemma 2.6.10, where it is shown that the a)-case occurs iff $\delta_0 > \varepsilon(K_0)$, and the fact that the elasticity of the Cobb-Douglas function is constant [5].

This statement implies the following: when the initial reputation is low, it is impossible to improve it /a)-case/, whereas a higher initial reputation enables the country to (at least temporarily) improve it, and moreover, the higher is q_0, the stronger (and for the longer time) is reputation improvement /b)-case/; and finally we have the efficient c)-outcome, when it is possible to permanently improve the reputation.

For more general production functions, it still follows from Lemma 2.6.10 that low values of q_0 will generate the a)-case, and high values of q_0 will generate the b)-case. However, now it might be possible that as q_0 increases, there will be several intervals of a) and b)-cases in between.

2.3.3   Incomplete information with commitment type.

Now that we have the complete description of the equilibrium dynamics in the incomplete information case, we can compare it with a more standard case when there is a small measure ν of the honest type (so that the initial probability of the honest type is $\frac{\nu}{1+\nu}$). In that case (2.5) can be re-written as

---

[4]If $q_0 = q_0^a$ and the production function is Cobb-Douglas then $q_0 = q_1$ and then we have the b)-case

[5]The less restrictive sufficient condition for this statement is that there is only one value of q_0 such that $\varepsilon(K_0) = \delta_0$ (note, that both the RHS and the LHS are the functions of q_0), and again this follows from Lemma 2.6.10.

$$\begin{cases} \delta_t = \varepsilon(K_t) \dfrac{F(K_t)}{F(K_{t+1})} \\ q_{t+1} = \dfrac{1 - \Phi(\delta_{t+1}) + \nu}{1 - \Phi(\delta_t) + \nu} \\ q_0 = \dfrac{1 - \Phi(\delta_0) + \nu}{1 + \nu} \end{cases} \qquad (2.6)$$

The main change is in the second equation that follows from Bayesian rule. Notice that in period $t + 1$ conditional probabilities of dealing with the honest and rational types are

$$Prob(Honest) = \frac{\nu}{1 - \Phi(\delta_t) + \nu}, \quad Prob(Rat) = \frac{1 - \Phi(\delta_t)}{1 - \Phi(\delta_t) + \nu}.$$

and so then it is easy to see that

$$q_{t+1} = \frac{1 - \Phi(\delta_{t+1})}{1 - \Phi(\delta_t)} \cdot \frac{1 - \Phi(\delta_t)}{1 - \Phi(\delta_t) + \nu} + 1 \cdot \frac{\nu}{1 - \Phi(\delta_t) + \nu} = \frac{1 - \Phi(\delta_{t+1}) + \nu}{1 - \Phi(\delta_t) + \nu}.$$

As it was already mentioned above in this case we cannot have inefficient equilibria since as the game goes without default, investors become more certain that the country has the honest type, and so the risk of investments should fall down. It can be seen immediately that (2.6) has only one steady state $(1, \varepsilon(\bar{K}))$. More generally, in the appendix it is shown that

Theorem 2.6.18 *If there is a positive measure ν of the honest type, then the equilibrium trajectories exist and they all converge to the efficient outcome $(1, \varepsilon(\bar{K}))$.*

2.4 Extension of the model

2.4.1 Myopic monopolists

The results above were obtained given the competitiveness assumption. We assumed that the country is dealing with a competitive market of investors, and thus the interest rate will be set in such a way that the expected return is equal to r. The question is whether the results obtained in the previous section are robust to different competition levels of the credit market.

There is a very extensive discussion in the literature about the effect of the monop-

olization of credit markets. There are some natural inefficiencies of the concentrated credit markets such as smaller supply and higher interest rate for credits. However, there are also positive effects caused by the fact that the monopoly can vary the interest rate across time and in the beginning it can "subsidize" the firm by charging a rate that is smaller than the competitive, and setting a higher rate later. Given that, it is interesting to investigate if the concentrated market would result in more efficient outcomes. Unfortunately, in my model, the effect of the monopoly is unambiguously negative. All the qualitative features of the equilibria remain unchanged, and specifically, there is still only one efficient equilibrium. On the other hand, the investment level is smaller and the interest rate is higher than in the competitive case.

The reason why in my model, the positive effects of monopoly do not appear is because the monopolist is myopic and thus the long-run relationship with a borrower used in the literature is impossible. Effectively, the situation that I consider is when there are one or several big players that are willing to make short-term investments into the country.

To begin the analysis first, I assume that there is only one lender. The country maximization problem is $F(K) - RK \to$ max, and so the inverse demand function is given by $F'(K) = R$.

The problem of the monopolist is $\max_K E(R(K) - r)K$ given the response of the country and the probability of non-default q. Thus, we can re-write it as $\max_K(q \cdot F'(K) - r)K$. Recall that there is an international capital market, where the monopolist can get riskless rate r, and so rK is the opportunity cost. The first-order condition is

$$F'(K) + K \cdot F''(K) = \frac{r}{q}, \tag{2.7}$$

and this expression determines K as a function of q. I will need two additional assumptions.

Assumption 3. $F'(K) + K \cdot F''(K)$ is a decreasing function of K.

This assumption, while very technical, is used to guarantee a very intuitive condition that as the confidence in the country increases, the amount of capital invested is also

increasing. By applying the implicit function theorem to (2.7), we can immediately see that Assumption 3 is equivalent to the statement that $\partial K / \partial q$ is positive. It will be convenient to use this re-formulation.

Assumption 3′. The higher the confidence to the country is, the higher is the level of invested capital.

Assumption 4. As K converges to zero, the limit of $F'(K) + K \cdot F''(K)$ is $+\infty$.

This assumption guarantees that for any positive level of confidence the monopolist will provide a positive level of credit. Notice that we do not need any additional conditions for the other end of the interval. If K goes to infinity, then from Inada conditions we have that $F'(K)$ goes to zero and the second term is negative. Thus, given Assumption 4, for any positive q there is $K(q)$ such that the first-order condition (2.7) is satisfied. Again, less technical re-formulation of this assumption will be useful.

Assumption 4′. For any positive level of confidence, the monopolist will provide positive level of investments.

Given these two assumptions, the results established in section 2.3.2 immediately go through without the slightest change. In fact, the market structure does not even have to be monopolistic. There might be several potential creditors, and as long as they are myopic and assumptions 3′ and 4′ are satisfied all the statements proved in 2.3.2 are still true, without any change in the proof. The simplest way to see that is by noticing that the equilibrium dynamics are given by the same system of equations as before:

$$
\begin{cases}
\delta_t = \varepsilon(K_t) \dfrac{F(K_t)}{F(K_{t+1})} \\[2mm]
q_{t+1} = \dfrac{1 - \Phi(\delta_{t+1})}{1 - \Phi(\delta_t)} \\[2mm]
q_0 = 1 - \Phi(\delta_0)
\end{cases}
$$

The only difference is that now K_t is a different function of q_t than before. For example, in the case of the monopolist it is determined by (2.7). However, the assumptions above ensure that the reasoning used in Section 2.3.2 will go through unchanged.

The equilibrium trajectories are described in terms of q and δ, and in this sense asymptotic behavior does not change. In particular, as before there is one trajectory that

converges to $(1, \varepsilon(\bar{K}))$ and a continuum of trajectories converging to $(0, \delta_{max})$. However, the actual level of invested capital and the interest rate are going to be different from the competitive case. Even in the best outcome, the level of capital provided by monopolist under no risk will be less than the efficient competitive level.

2.4.2 The long-sighted investor

The main prediction of the model seems to be too pessimistic — indeed, there is only one efficient equilibrium, and a continuum of inefficient equilibria. Thus if we assume that each level of initial reputation is equally likely, it means that the country is almost doomed to the no-investment trap. Furthermore, along the inefficient equilibria, after some moment T, investors know with probability 1 that the country is safe, and yet the flow of investments keeps declining. In this section I am going to show that the reason for such inefficiency is the short-sightedness of investors, and that the existence of a long-sighted investor eliminates all inefficient equilibria.

It is easy to see that if the investors are short-sighted and competitive, then even if it is known that the country has safe type, it is still not rational to invest more then the equilibrium prescribes. The reason is that if the investor deviates from the equilibrium path, and invests more than equilibrium level, his expected return is less than rK. Indeed, when the investor increases q_t, it follows from the first equation of (2.5) that δ_t increases as well (given that $q_{t+1}, q_{t+2}, \ldots$ do not change). Therefore, the probability of default at moment t increases, and so the expected return on capital is less than rK. That is why in spite of the investor's having reliable information that the economy is patient, the system has to follow the bad equilibrium path.

Now, assume that there exists a *long-sighted* investor that can propose to the country such an infinite sequence of payments that will guarantee the economy better profit (and thus will be accepted), and still satisfies the competitiveness and rationality conditions. The former means that the investor is the capital market price-taker, and the latter means that the investor's expectations about the probability of confiscation are rational.

Given that, the equilibria should be "intruder proof", that is it should be impossible for the long-sighted investor to introduce a better schedule of payments.

Then, all inefficient trajectories are not equilibria anymore. Indeed, at the moment when $\delta_t > \varepsilon(\bar{K})$ the long-sighted investor would propose the constant riskless flow of capital and this proposal would be

a) accepted by the country;

b) honored by the country, and thus be rational.

On the other hand, the efficient trajectory is still an equilibrium. Indeed, assume that at some moment t the strategic investor can propose a better rational trajectory. Since this trajectory is rational it should satisfy

$$q_t = \frac{1 - \Phi(\delta_t)}{1 - \Phi(\delta_{t-1})},^6 \tag{2.8}$$

and since it is accepted by the economy, it should converge to $(1; \varepsilon(\bar{K}))$, and so we can define the function $q_t = q(\delta_t)$, that associates with δ such (unique) q that if the trajectory starts from $(q(\delta), \delta)$ then it converges to $(1; \varepsilon(\bar{K}))$.

It is shown in Statement 2.6.14 and Lemma 2.6.15, that $q(\delta)$ is the function indeed, and that it is an increasing function. And thus equation (2.8) and $q_t = q(\delta_t)$ have only one solution because the first condition determines a decreasing relationship between q_t and δ_t, and the second one determines an increasing relationship. And we already know this unique solution: it is the original efficient trajectory, since it satisfies both conditions.

2.4.3 Signaling

From the results of the previous section, it follows that it is highly desirable to attract the long-run investments. However, it might be quite a difficult task for developing countries. Thus the question is: how the country can attract long-run investments, which are really vital for it?

In this section we suggest that the economy can send a signal, in order to attract the long-run investments. In the literature there are known many opportunities for the government to signal on the macrolevel (see e.g in (Drazen, 2000)). Usually the government

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[6]Note that δ_{t-1} is given from the game history.

has to bear some cost or perform suboptimal actions to reveal the information about its type.

To introduce signaling into our model we consider a slight modification of it. We assume that the economy sends a credible signal to the (long-sighted) investor, and when the investor receives this signal he agrees to invest the riskless amount of capital to the economy till the end of time. We assume that the economy determines the best moment to send the signal, and we also assume that *even if the economy hasn't sent a signal, it does not add any information to the investor.*

The last assumption

a) enables us to stay in the same set of equilibria as we had before, of course, with the exception that some economies now can leave the game not only by cheating the investor, but also by sending a signal.

b) is not that unrealistic, because there may be plenty of reasons why the country cannot send the signal in a particular moment. Usually the developing countries have a very tight budget, and so it might be not easy to find money to "burn".

The timing of signaling is as follows: if at moment t the economy sends the signal, then from moment $t + 1$ it gets access to the infinite sequence of riskless level of investments.

It is easy to show, that if the country at moment $T - 1$ sends the signal

$$\left(F(\bar{K}) - F(K_T)\right)\varepsilon(\bar{K}) \tag{2.9}$$

it will credibly reveal that the economy is patient enough, that is that $\delta \geq \varepsilon(\bar{K})$, and thus it will honor the riskless flow of investments[7].

Since K_T is an increasing function of q_T, formula (2.9) shows that the higher reputation (i.e. q_T) the economy has, the easier it can send a signal to the investor. Now let

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[7]The signal should be unprofitable for economies with $\delta < \varepsilon(\bar{K})$. So let's look at the impatient economy: suppose that without signaling the best moment for cheating is T. Then at moment $t - 1$ the economy's gain of signaling and following cheating (in comparison with non-signaling) is

$$[(F(\bar{K}) - (1 - \varepsilon(K_t))F(K_t) - (1 - \varepsilon(K_{t+1}))F(K_{t+1})\delta - \ldots$$
$$-F(K_T)\delta^{T-t-1})]\delta. \tag{2.10}$$

It has the highest value at $T - 1$: $(F(\bar{K}) - F(K_T))\delta$, which is less than $(F(\bar{K}) - F(K_T))\varepsilon(\bar{K})$

us examine all types of equilibria that we described in the previous section .

In a)-case the best moment when the country can send a signal is at the zero moment, and if this signal is unaffordable then woe to the country! Indeed, sending the signal later will be even more expensive and thus still unaffordable, so the country is doomed to move along the bad path.

In b)-case, q_t increases at first and then decreases. So the best moment for the country to send the signal is around the peak, since when q_t is high the signal itself is less costly and the country has more money to pay for it. It means that *in the beginning the country can take very expensive credits, in order build some reputation, and only then can it afford to send a signal to the investor.*

In this case, signaling and reputation work as complements. Indeed, without building the reputation the country cannot send the signal to the investor, whereas without signaling, the reputation is not a strong incentive for honest behavior. Hence, without any of these two ingredients the patient countries will not be able to get the efficient level of investment.

2.5   Conclusions

In this paper I consider an infinite game between investors and a country, when the patience of the country is unknown to the investor, and the level of investments is determined endogenously by the market.

Depending on the level of the initial confidence there are three cases: first, there is the unique efficient equilibrium, when the risk of investing is asymptotically eliminated, and safe types choose the honest strategy. Second, there is a continuum of inefficient equilibria when it is impossible to improve the reputation by paying back. Eventually all types default and the investment flow converges to zero. Third, there is a continuum of equilibria that is inefficient as well, but now it is possible to improve the reputation. However, the reputation improvement is only temporary, and again asymptotically the flow of investment converges to zero and all types prefer confiscation.

The existence of inefficient equilibria depends on the assumption that investors are short-sighted and invest for one period only. It is shown that the existence of a strategic

investor who can offer the infinite payments schedule eliminates all inefficient equilibria. This observation enables us to claim that it might be dangerous for transition or developing economies to attract short-sighted investors, whereas attracting strategic investors is beneficial.

We extend the model by assuming that the country can send a signal by "burning money" in order to attract the strategic investor. The higher the reputation of the country is, the less is the size of the signal. This result is especially important if we apply it to the *temporary reputation improvement* case, which are the equilibria where at first, the confidence in the country grows, but eventually falls to zero. It is shown that along these trajectories the reputation and signaling complement each other. That is, first, the country can gain some reputation by taking the small credits and paying them back. Than the improved reputation enables it to send the signal.

2.6   Appendix. Proofs

In this section we are going to prove the above-claimed properties of the equilibrium trajectories. This section is extremely convoluted and filled with technical details of all kind. The most important statements have their own name (e.g. Monotonicity property).

And also, whenever we mention the first/the second/the last equation, it means the corresponding equation of (2.5).

2.4 Lemma. *If $\{q_t\}$, $\{\delta_t\}$ are the solutions of (2.5), then $\frac{\partial q_{t+1}}{\partial q_t} > 0$, $\frac{\partial q_{t+1}}{\partial \delta_t} < 0$*

◀ The proof is just a simple exercise on the Implicit Function Theorem.

First of all,
$$\frac{\partial q_{t+1}}{\partial q_t} = \frac{\partial q_{t+1}}{\partial K_{t+1}} \frac{\partial K_{t+1}}{\partial K_t} \frac{\partial K_t}{\partial q_t}$$

Since we know that K_t is determined as the solution of $\max F(K) - r/q_t K$, it is easy to see that
$$\frac{\partial K_t}{\partial q_t} = -\frac{F'(K_t)}{q_t F''(K_t)}.$$

And if we re-write the first equaiton of (2.5) as $\delta_t F(K_{t+1}) - F'(K_t)K_t = 0$ and use

the implicit function theorem it is easy to get the expression for $\frac{\partial K_{t+1}}{\partial K_t}$. And finally we have that

$$\frac{\partial q_{t+1}}{\partial q_t} = \frac{q_{t+1}F''(K_{t+1})}{F'(K_{t+1})} \frac{F'(K_t) + K_t F''(K_t)}{\delta_t F'(K_{t+1})} \frac{F'(K_t)}{q_t F''(K_t)} > 0. \qquad (2.11)$$

The numerator of the second fraction is positive, by our assumption that $K \cdot F'(K)$ is increasing.

Absolutely, in a similar way we find the sign of $\frac{\partial q_{t+1}}{\partial \delta_t}$:

$$\frac{\partial q_{t+1}}{\partial \delta_t} = \frac{\partial q_{t+1}}{\partial K_{t+1}} \frac{\partial K_{t+1}}{\partial \delta_t} = \frac{q_{t+1}F''(K_{t+1})}{F'(K_{t+1})} \frac{F(K_{t+1})}{\delta_t F'(K_{t+1})} < 0$$

It is very important to notice that we proved that q_{t+1} is a *strictly* increasing function of q_t and a *strictly* decreasing function of δ_t. ▶

Now, let's set up few definitions. First of all, we name (2.5) as Full System (FS), and (2.5) without the last equation as Reduced System (RS) [8]. Note that in RS there should be two initial data, namely q_0 and δ_0 to determine the following dynamics (RS-dynamics). When we consider FS(RS)-trajectories usually we restrict ourselves only to *feasible* trajectories.

2.5 Definition. We call a trajectory *feasible* iff for all t: $0 \leq q_t, \delta_t \leq 1$.

2.6 Definition. Consider two trajectories going through (q_t, δ_t) and (Q_t, Δ_t) correspondingly. If both $q_t > (\geq)Q_t$ and $\Delta_t \geq (>)\delta_t$ than we say that the first trajectory is *higher* than the second one at moment t.

2.7 Lemma. *(Monotonicity property) It can be shown that if feasible trajectory A is higher than feasible trajectory B at moment t, then A is higher than B at any $s > t$. And so, if A is higher than B at 0 then it is higher than B for all moments, and we call it A is higher than B.*

In other words if A is higher than B then for all t $q_t^A > q_t^B$, which explains the word higher.

◀ a) It follows from Lemma 2.4 that if we decrease q_t and increase δ_t than q_{t+1} decreases, that is if $Q_t < q_t$, $\Delta_t > \delta_t$ then $Q_{t+1} < q_{t+1}$.

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[8]The reason why we need RS is because, loosely speaking, RS is FS from any $t > 0$

b) Now look at $q_{t+1} = \frac{1-\Phi(\delta_{t+1})}{1-\Phi(\delta_t)}$. If we increase δ_t and decrease q_{t+1} /here we use a)/ than δ_{t+1} increases; that is $\Delta_{t+1} > \delta_{t+1}$, and then we can apply the induction.

Note that the same reasoning is applicable if (as it allows the definition of the *higher trajectories*) there were one inequality and one equality. ▶

2.8 Remark. Monotonicity property holds for infeasible trajectories as well, but only for $t \leq T$ where T is the least number such that $q_T > 1$. Indeed, note that a)-reasoning is still valid even if $q_{t+1} > 1$, whereas b)-reasoning might be incorrect, and so we are unable to use the induction from $T + 1$.

Now we would like to demonstrate that there exist both the feasible and infeasible solutions of (2.5).

2.9 Lemma. *(Existence of feasible and non-feasible solutions)*

Assume that $\delta_{max} \geq \varepsilon(\bar{K}) \geq \delta_{min}$ then there exist

a) feasible,

b) infeasible solutions of (2.5).

Moreover, we can claim that both in a) and b) $q_0 < 1$

◀ a) Since $\delta_{max} \geq \varepsilon(\bar{K})$, $(1, \varepsilon(\bar{K}))$ is a feasible RS-trajectory. Now if we decrease q_0 we have lower (and thus feasible) trajectorY, so we just take such value of q_0 that $q_0 = 1 - \Phi(\varepsilon(\bar{K}))$, which gives us feasible FS-trajectory.

b) If we take $q_0 = 1$, then $\delta_0 = \delta_{min} \leq \varepsilon(\bar{K})$, and so from the first equation $q_1 > q_0 = 1$. However, since $q_1(q_0)$ is a continuous function, if we take q_0 slightly less than 1, we still will have that $q_1 > 1$, and thus the trajectory is infeasible. ▶

Now, we would like to discuss the assumption $\delta_{max} > \varepsilon(\bar{K}) > \delta_{min}$. If $\delta_{min} > \varepsilon(\bar{K})$ part b) is not true, but we do not need it for this case (this Lemma is used only for the proof of Theorem 2.12). As for part a) $(1, \varepsilon(\bar{K}))$ is the explicit example of the feasible FS-trajectory. In the case $\delta_{max} < \varepsilon(\bar{K})$ the efficient equilibrium is impossible, and so we do not consider this case by ASSUMPTION 2.

2.10 Lemma. *(Curves shape)*

i) $q_t > q_{t+1}$ iff $\delta_t > \varepsilon(K_t)$

ii) If $q_t > q_{t+1}$ then for any $s > t$ $q_s > q_{s+1}$

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iii) For feasible trajectories there may be only three shapes of q-curve, that correspond to the cases a)-c) in the classification of equilibria.

◄ i) Given the first equation $q_t > q_{t+1} \Leftrightarrow K_t > K_{t+1} \Leftrightarrow \delta_t > \varepsilon(K_t)$.

ii) if $q_t > q_{t+1}$, $\delta_{t+1} > \delta_t$ (since $q_{t+1} < 1$) and so from Lemma 2.4 it follows that $q_{t+1} > q_{t+2}$, and then by induction.

iii) From ii) it follows that if q_t began to decrease than it decrease for all the time, which means that there are three possible cases:

- q_t decreases from the zero moment till infinity

- In the beginning q_t increases, but then from some moment it starts to decrease

- q_t increases from the zero moment till infinity

Moreover, given the lemma statements, case a) is possible if and only if $\delta_0 > \varepsilon(K_0)$.

►

2.11 Lemma. *(Steady states) There are possible only two limit points for feasible trajectories $(1, \varepsilon(\bar{K}))$ and $(0, \delta_{max})$, which are steady states.*

◄ As we know from Lemma 2.10, the q-sequence is eventually monotone, and since we consider the feasible trajectories it is bounded, thus it converges. From the second equation it follows that if $q_t \leq 1$ then $\delta_{t+1} \geq \delta_t$, so δ-sequence also converges. Assume that $\delta_t \to \delta (\neq \delta_{max})$ then from the second equation it follows that q_t should converge to 1, and then from the first equation that $\delta_t \to \varepsilon(\bar{K})$.

The only option left is $\delta_t \to \delta_{max}$. Then in the second equation we have uncertainty "0 over 0". If q-sequence converges to positive limit than the first equation converges to $\delta_{max} = \varepsilon(K)$ which is impossible by Assumption 2. And so the only option left is q_t converges to zero, and then in the first equation we have uncertainty "0 over 0", which is consistent with $\delta_t \to \delta_{max}$.

Now let us comment on the fact that these two points are steady states. $(1, \varepsilon(\bar{K}))$ is obviously steady state, as for the second point, we have that both equations contain uncertainty "0 over 0" and so there is no contradiction, that occurs if we would take

$(q_0(\neq 0), \delta_{max})$ or $(0, \delta(< \delta_{max}))$. So in this sense we claim that $(0, \delta_{max})$ is steady state as well.▶

2.12 Theorem. *(Existence of the efficient equilibrium) There exists the efficient equilibrium (that is the equilibrium converging to $(1; \varepsilon(\bar{K}))$)*

◀ Consider the set of all trajectories that are solutions of (2.5). Denote as A the set of all initial values $0 < q_0 < 1$ such that trajectories originating from them are feasible. Denote as $\bar{A}$ the set of all initial values $0 < q_0 < 1$ such that trajectories starting from them are infeasible, that is there exists $q_t > 1$, or formally speaking $\bar{A} = \{q_0 | \exists t : q_t > 1\}$. It is very important to notice that if we had defined $\bar{A}$ as $\bar{A} = \{q_0 | \exists t : q_t \geq 1\}$ it would not have changed the set. That is if for some t $q_t = 1$ then this trajectory is infeasible. Indeed, consider the first such t that $q_t = 1$, then $\delta_t = \delta_{t-1}$, on the other hand $q_t > q_{t-1}$, and so in the first equation for moments t and $t + 1$ in comparison with the same equation for t and $t - 1$ we have that LHS does not change, whereas numerator times elasticity in RHS increases, and so it means that denominator should increase as well, which means that $q_{t+1} > 1$. We know that if $\delta_{min} < \varepsilon(\bar{K})$, than $\bar{A}$ is non-empty (Lemma 2.9) (if $\delta_{min} \geq \varepsilon(\bar{K})$ than the trajectory $\{(1; \varepsilon(\bar{K}))\}$ is the explicit example of the efficient trajectory, and there's nothing to prove anymore).

Clearly $A \bigcup \bar{A} = (0; 1)$, and $A \bigcap \bar{A} = \oslash$. It is also clear that if some $q \in \bar{A}$ then all $Q > q$ are belong to $\bar{A}$ as well. And on the other hand if $q \in A$ then all $Q < q$ are in A.

It is relatively easy to see that $\bar{A}$ is an open set. Indeed, let's take the first q_T that is strictly bigger than 1. It follows from the (2.5) that we can slightly decrease q_0 so that q_T will be still bigger than 1. It is essential here that we take the first q_T bigger than 1, because it guarantees us that the function from q_0 to q_T is continuous.

Let $\bar{q} = \inf \bar{A}$. Then first of all, $\bar{q} \notin \bar{A}$, since $\bar{A}$ is an open set. And so $\bar{q} \in A$, and moreover for all $q \in A$ $\bar{q} \geq q$. Actually we are almost done — we claim that the trajectory starting from $\bar{q}$ converges to 1. Indeed, it has the limit (follows from the boundedness and monotonicity of feasible curves). Suppose that its limit is not 1. Than for all t $\bar{q}_t < 1 - \omega < 1$ (where ω is a small positive number), and so if we slightly increase $\bar{q}$ the trajectory will be feasible which is contradiction with the way we define $\bar{q}$.▶

So, now we know that the efficient equilibria exist. However, it turns out that there is only one efficient equilibrium.

2.13 Theorem. *(The uniqueness of the efficient equilibrium) There is only one efficient equilibrium.*

◄ First, let us describe the idea of the proof. We know that there exists such an efficient trajectory that all higher trajectories are infeasible. So we take it and going to prove that all lower trajectories cannot converge to 1. We do it in the following way: we take the efficient trajectory (q_t, δ_t) and take some other lower trajectory. We show that $\Delta_{t+1} - \delta_{t+1} > \Delta_t - \delta_t$ as long as δ_{t+1} and Δ_{t+1} are close enough to $\varepsilon(\bar{K})$, and q_t and Q_t are close enough to 1, which means that the second trajectory cannot converge to the same limit as the first one, and so all lower trajectories have to converge to zero.

It can be done in the following way: (2.5) determines the function Ψ from (q_t, δ_t) to (q_{t+1}, δ_{t+1}). Let us take the small deviation of the efficient trajectory, and consider the Taylor decomposition of Ψ:

$$\begin{pmatrix} Q_{t+1} \\ \Delta_{t+1} \end{pmatrix} = \begin{pmatrix} q_t \\ \delta_t \end{pmatrix} + \Psi' \begin{pmatrix} Q_t - q_t \\ \Delta_t - \delta_t \end{pmatrix}, \tag{2.12}$$

or

$$\begin{pmatrix} Q_{t+1} - q_{t+1} \\ \Delta_{t+1} - \delta_{t+1} \end{pmatrix} = \Psi' \begin{pmatrix} Q_t - q_t \\ \Delta_t - \delta_t \end{pmatrix}. \tag{2.13}$$

In the expression above Ψ' is calculated at point (q_t, δ_t). However, instead of working with $\Psi'(q_t, \delta_t)$ we calculate $\Psi'(1, \varepsilon(\bar{K}))$, given that after some T (q_t, δ_t) will be close to $(1, \varepsilon(\bar{K}))$ and so we can use the continuity of Ψ'.

Clearly Ψ' has the form

$$\Psi' = \begin{pmatrix} \dfrac{\partial q_{t+1}}{\partial q_t} & \dfrac{\partial q_{t+1}}{\partial \delta_t} \\ \dfrac{\partial \delta_{t+1}}{\partial q_t} & \dfrac{\partial \delta_{t+1}}{\partial \delta_t} \end{pmatrix}. \tag{2.14}$$

And as we know from Lemma 2.4 $\dfrac{\partial q_{t+1}}{\partial q_t} > 0$ and $\dfrac{\partial q_{t+1}}{\partial \delta_t} < 0$. However, what we really need is the second row of matrix Ψ'. We are now going to show that the first

element of the second row is negative, whereas the second element is bigger than 1. And since $q_t > Q_t$ and $\delta_t < \Delta_t$ it will give us that $\Delta_t - \delta_t < \Delta_{t+1} - \delta_{t+1}$, and this is exactly what we need.

First of all let us prove that the second element is bigger than 1. It is relatively easy: from the second equation of (2.5) we have that

$$\Phi(\delta_{t+1}) - 1 + q_{t+1}(1 - \Phi(\delta_t)) = 0, \qquad (2.15)$$

from which we have that

$$\frac{\partial \delta_{t+1}}{\partial \delta_t} = q_{t+1}\frac{\phi(\delta_t)}{\phi(\delta_{t+1})} + \frac{1}{\phi(\delta_{t+1})}\left[q'_{t+1}\Phi(\delta_t) - q'_{t+1}\right],$$

where q'_{t+1} is the derivatives of q_{t+1} wrt δ_t. At point $(1; \varepsilon(\bar{K}))$ this expression simplifies to

$$\frac{\partial \delta_{t+1}}{\partial \delta_t} = 1 + \frac{1}{\phi(\varepsilon(\bar{K}))}\left[q'_{t+1}\Phi - q'_{t+1}\right], \qquad (2.16)$$

and since q'_{t+1} is less than zero the expression in the parenthesis is positive, and so (2.16) is bigger than 1.

And finally, let us prove that $\dfrac{\partial \delta_{t+1}}{\partial q_t}$ is negative. From (2.15) we have that

$$\frac{\partial \delta_{t+1}}{\partial q_t} = -\frac{\partial q_{t+1}}{\partial q_t(1 - \Phi(\delta_t))}\phi(\delta_{t+1}) < 0. \blacktriangleright$$

The last two theorems have some very important corollaries.

2.14 Corollary. *For any given $\delta_0 \le \varepsilon(\bar{K})$ there exists only one q_0 that a RS-trajectory starting from (q_0, δ_0) converges to $(1; \varepsilon(\bar{K}))$.*

◄ Actually we can apply absolutely the same reasoning as we used in the existence and uniqueness theorems above. The only thing that need to be changed is the proof of the existence of feasible trajectory. It is just technical issue, however...

Fix $\delta_0 \le \varepsilon(\bar{K})$, as we remember q_1 is determined from the expression $\dfrac{\varepsilon(K_0)F(K_0)}{\delta_0}$. By varying q_0 we can have q_1 anything from 0 to 1 (0 is not included). So let's take such q_0 that $q_1(q_0)$ determines δ_1 in such a way that $\delta_1 = \varepsilon(\bar{K})$. Than at moment 1 we have $(q_1, \varepsilon(\bar{K}))$ which is lower than $(1, \varepsilon(\bar{K}))$ and so the first trajectory is feasible.

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We require that $\delta_0 \leq \varepsilon(\bar{K})$, because otherwise the trajectory converges to inefficient equilibrium. ▶

2.15 Corollary. *So we can define the function $q = q(\delta)$ that corresponds to δ such (unique) q that if the trajectory starts from $(q(\delta), \delta)$ then it converges to $(1; \varepsilon(\bar{K}))$, and this function is increasing.*

◀ The fact that we can define such function is actually the previous corollary, and the fact the $q(\delta)$ is increasing follows from the fact that otherwise the two trajectories (generated by any two points where the increase of $q(\delta)$ is violated) would be comparable and so only one of them can converge to $(1; \varepsilon(\bar{K}))$. ▶

2.16 Theorem. *The solutions of (2.5) that correspond to cases a),b),c) are equilibria.*

◀ Actually it follows from the way we constructed it. The only thing left to show is that the economy cheats the investor iff it has $\delta < \delta_\infty$.

First of all we want to prove the following auxiliary statement:

2.17 Lemma. *If for any t: $\delta \equiv \varepsilon(K_t)\frac{F(K_t)}{F(K_{t+1})}$ then the country with such δ is indifferent between cheating and honest behavior.*

◀ In the case of honest behavior the country's discounted profit is

$$\sum_{s=0}^{\infty}(1 - \varepsilon(K_{t+s}))F(K_{t+s})\delta^s, \tag{2.17}$$

and if it cheats at moment t, then its profit is $F(K_t)$. We are going to show that these two expressions are equal. Here is the sketch of its proof: consider the first three terms of (2.17).

$$(1 - \varepsilon(K_t))F(K_t) + \delta[F(K_{t+1})(1 - \varepsilon(K_{t+1})) + \delta F(K_{t+2})(1 - \varepsilon(K_{t+2}))] =$$

$$(1 - \varepsilon(K_t))F(K_t) + \delta[F(K_{t+1})(1 - \varepsilon(K_{t+1})) + \varepsilon(K_{t+1})F(K_{t+1})(1 - \varepsilon(K_{t+2}))] =$$

$$(1 - \varepsilon(K_t))F(K_t) + \delta[F(K_{t+1}) - F(K_{t+1})\varepsilon(K_{t+1})\varepsilon(K_{t+2})] =$$

$$F(K_t) - \varepsilon(K_{t+2})\varepsilon(K_{t+1})\varepsilon(K_t)F(K_t).$$

Now using the same techniques, and the fact that elasticity is less than unity, we have that (2.17) is equal to $F(K_t)$. ▶

Now assume that $\delta < \delta_\infty$. There exists such t that after it $\delta < \varepsilon(K_{t+s}) \frac{F(K_{t+s})}{F(K_{t+s+1})}$ (for any $s > 0$), and so (similar to as we did in the lemma above)

$$\sum_{s=0}^{\infty}(1 - \varepsilon(K_{t+s}))F(K_{t+s})\delta^{t+s} <$$

$$(1 - \varepsilon(K_t))F(K_t) + (1 - \varepsilon(K_{t+1}))F(K_{t+1})\varepsilon(K_t)\frac{F(K_t)}{F(K_{t+1})} + \cdots = F(K_t),$$

where the LHS is the profit from honest behavior, and the RHS profit from cheating at moment t, and since RHS is bigger, the economy with discount factor δ will not choose the honest behavior.

So we proved that the economies with $\delta < \delta_\infty$ prefer to quit. Now let us prove that the economies with $\delta \geq \delta_\infty$ will chose the honest behavior. First of all, let's remind that π_n denoted the profit of the economy if it quits at moment t. And for economies with $\delta \geq \delta_\infty$ $\pi_0 \leq \pi_1 \leq \pi_2 \leq \ldots$. And clearly that the economy cannot prefer to quit at some moment m to honest behavior, because then moment $m+1$ should be better and so on. And even if we have that the economy is indifferent between quitting at any moment after m and honest behavior (which can be only when $\delta = \delta_\infty$), we just assume that the indifferent economy choose honest strategy.

And so it completes the prove that the found trajectories are equilibria. ▶

2.18 Theorem. *If there is a positive measure ν of the honest type, then the equilibrium trajectories exist and they all converge to the efficient outcome $(1, \varepsilon(\bar{K}))$.*

◀ The proof will be based by adjusting the results that were established in the case of incomplete information without commitment type. The most difficult and non-trivial part is to show the existence.

As it was shown in Section 2.3, the new system that describes the equilibrium dynamic is

$$
\begin{cases}
\delta_t = \varepsilon(K_t) \dfrac{F(K_t)}{F(K_{t+1})} \\[2mm]
q_{t+1} = \dfrac{1 - \Phi(\delta_{t+1}) + \nu}{1 - \Phi(\delta_t) + \nu} \\[2mm]
q_0 = \dfrac{1 - \Phi(\delta_0) + \nu}{1 + \nu}
\end{cases} .
$$

It is easy to see that the only steady state of the system is $(1, \varepsilon(\bar{K}))$. The inefficient outcome when δ_t converges to δ_{max} is not a steady state anymore, since in this case q should converge to one which contradicts the first equation.

Another difference from the original setting is that in (2.5) for any level of q_{t+1} between 0 and 1, it was possible to find a corresponding δ_{t+1}. This is not the case anymore when we have the honest type. It follows from the second equation that the probability of paying back cannot be smaller than

$$
q_{t+1} \geq \frac{\nu}{1 - \Phi(\delta_t) + \nu} \geq \frac{\nu}{1 + \nu}.
$$

Thus if from the first equation it would follow that q_{t+1} is small, we will not be able to find the corresponding δ_{t+1} from the second equation.

It means that now feasible trajectories should satisfy the following definition

2.19 Definition. We call a trajectory *feasible* if for all t: $0 \leq q_t, \delta_t \leq 1$ and also $q_{t+1} \geq \dfrac{\nu}{1 - \Phi(\delta_t) + \nu}$.

2.20 Lemma. *There is a feasible trajectory, and all feasible trajectories converge to the efficient outcome* $(1, \varepsilon(\bar{K}))$.

◀ We are going to adjust the proof of Theorem 2.12 by constructing the efficient trajectory and showing that it is feasible.

Consider the set of all trajectories that are solutions of (2.6) that also include infeasible trajectories. Denote as A the set of all initial values $0 < q_0 < 1$ such that q's of the trajectories originating from them never go above 1. Notice that given the new definition they are not necessarily feasible. Denote as $\bar{A}$ the set of all initial values $0 < q_0 < 1$ such that there exists $q_t > 1$, or formally speaking $\bar{A} = \{q_0 | \exists t : q_t > 1\}$. As it was done in Theorem 2.12 we can show if we had defined $\bar{A}$ as $\bar{A} = \{q_0 | \exists t : q_t \geq 1\}$ it would not have changed the set. We know that if $\delta_{min} < \varepsilon(\bar{K})$, than $\bar{A}$ is non-empty (Lemma 2.9)

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(if $\delta_{min} \geq \varepsilon(\bar{K})$ than the trajectory $\{(1; \varepsilon(\bar{K}))\}$ is the explicit example of the efficient feasible trajectory, and there's nothing to prove anymore).

Clearly $A \bigcup \bar{A} = (0; 1)$, and $A \bigcap \bar{A} = \oslash$. It is also clear that if some $q \in \bar{A}$ then all $Q > q$ are belong to $\bar{A}$ as well. And on the other hand if $q \in A$ then all $Q < q$ are in A.

It is relatively easy to see that $\bar{A}$ is an open set. Indeed, let's take the first q_T that is strictly bigger than 1. It follows from the (2.6) that we can slightly decrease q_0 so that q_T will be still bigger than 1. It is essential here that we take the first q_T bigger than 1, because it guarantees us that the function from q_0 to q_T is continuous.

Let $\bar{q} = \inf \bar{A}$. Then first of all, $\bar{q} \notin \bar{A}$, since $\bar{A}$ is an open set. And so $\bar{q} \in A$, and moreover for all $q \in A$ we know that $\bar{q} \geq q$. I claim that the trajectory that starts at $\bar{q}$ is the efficient and feasible trajectory. Denote as $\bar{q}^s$ the supremum of the q-part of the trajectory starting at $\bar{q}$. If it is infeasible then there is a first moment T when $\bar{q}_T$ becomes too low and the trajectory stops. Consequently, there is a $T' \leq T$ such that $\bar{q}_{T'} = \bar{q}^s$. If $\bar{q}^s$ is below 1 then by the continuity there is a trajectory that starts at $q_0 > \bar{q}$ such that it also becomes infeasible at moment $t = T$ and its highest point is less than 1 (since I have finite number of periods, I can move initial value so that the difference between old $\bar{q}_i$ and new q_i is smaller than ε for all $i \leq T$). It contradicts to the definition of $\bar{q}$. Thus $\bar{q}^s$ should be equal to 1. However, we established above that if there is a finite moment t such that $\bar{q}_t = 1$ then the next period $\bar{q}_{t+1}$ will be strictly greater than 1 which is again impossible, since by the definition of $\bar{q}$ the trajectory never crosses 1. Because of the same reason $\bar{q}^s$ cannot be greater than one. It proves that the trajectory that starts at $\bar{q}$ is feasible.

Given that it is feasible, similarly to Theorem 2.12 we can show that this trajectory is efficient. Indeed, as we know from Lemma 2.6.10, feasible trajectories have limit (follows from the boundedness and monotonicity of feasible curves), and since there is only one limit point of (2.6) the feasible curves are efficient. ▶

The statement of the theorem immediately follows from the lemma.▶

Chapter 3

Separating Non-monetary and Strategic Motives in Public Good Games

3.1 Introduction

A well-established finding in experimental economics is that people in Prisoner's Dilemma and Public Good games tend to behave considerably more cooperative than individual payoff-maximization prescribes. Given that both Prisoner's Dilemma (PD) and Public Good games (PG) have a dominant strategy, the difference between payoff-maximizing and observed behavior is especially hard to reconcile. Any positive amount of contribution is inconsistent with the former no matter what (unobserved) beliefs subjects might have about the opponents' choices. Thus an approach based on the assumption that subjects want to maximize their monetary payoffs and know how to do that fails to explain the observed behavior.

Consequently, new theories have been introduced that stress the role of factors other than payoffs in subjects' decision-making. For instance, fairness considerations, reciprocation or confusion could cause people to play non-dominant strategies. While in theory many factors are capable of explaining cooperative behavior, we do not know to what extent they influence the *actual* decision-making and which of them are responsible for the observed over-contribution.

In my paper, I use an experimental approach to address these questions. I divide explanations of over-contribution suggested in the literature into three groups: *utility*

interdependence (UI), which is that subjects care either positively or negatively about payoffs of their opponents (fairness, altruism); *action interdependence* or (AI), which is that subjects want to affect or reciprocate actions of opponents (reputation, reciprocity); and *confusion,* which is that subjects do not know what is the most optimal way to play the game.

Given this classification, I design several treatments of a public good game where some of the aforementioned considerations are made inapplicable, while the rest of the game remains unaltered. The resulting change in subjects' behavior should be solely due to the removed factors, and the larger the change is, the more important these factors are in the decision-making.

The main challenge in understanding the importance of UI and AI considerations is *not,* however, in designing treatments that would make them inapplicable. That could be easily done by matching subjects with computers. The main challenge is to make sure that other aspects of the game remain unaltered. For instance in the treatment with computerized opponents, subjects would have different strategic uncertainty (i.e. expectations about opponents' decisions) than they would have playing against real people. As a result, it would be impossible to determine to what extent the difference in the behavior is caused by the removed UI and AI considerations versus the change in strategic uncertainty.

Four treatments are suggested in the paper: a benchmark treatment which is a standard public good game; a *phantom treatment* where both UI and AI considerations are not applicable; and two *two-type treatments* where only UI considerations are not applicable.

The benchmark treatment is a standard public good experiment with a linear payoff function such that to contribute zero is a dominant strategy. The phantom treatment is a separate experiment that is conducted with a separate group of subjects. It is identical to the benchmark with the only difference that subjects are randomly matched with the decisions that were made in the benchmark treatment and not with decisions of each other. Figuratively speaking, in the phantom treatment subjects are playing not against real people, but against *"phantom players"* from the previous experiment, and this is

common knowledge among them.

As it can easily be seen, this design removes both UI and AI considerations. The former is removed because subjects are not playing against real people and thus their decisions would not affect anyone's utility. The latter is removed because opponents' decisions are pre-determined, and thus it makes no sense to try to affect or reciprocate them. All other aspects of the game are minimally affected, if at all. The rules of the game, payoff functions and information available to subjects are identical across treatments. Furthermore, the strategic uncertainty is also the same since effectively subjects in the phantom treatment are matched against the same decisions as the subjects in the benchmark. Therefore, if there is a difference in behavior between the treatments it should be exactly due to the removed UI and AI considerations.

The two other treatments designed in this paper remove UI considerations, while leaving AI factors applicable. In these treatments, called the *two-type treatments*, all subjects have one of two types — either type M which stands for *monetary affected* type, or type N which stands for *not affected* type. Type M players have a standard payoff function and so they are affected by decisions of their opponents. Type N players have a fixed payoff regardless of the outcome. This makes them unaffected by opponents' choices.

The information structure of the treatments has two important properties. First, subjects do not know their own type. This way, both types of players have similar monetary incentives at the time they make decisions. Second, subjects know the type of their opponents, which gives a key to separating the effects of UI and AI considerations. If a subject knows that (s)he is matched with type N then (s)he should not have UI considerations. On the other hand, AI considerations are equally applicable against both types. For example, when subjects play cooperatively in order to encourage other people to cooperate it does not matter what is the opponent's type. Thus, if contributions against type M are higher than against type N, then it must be due to utility interdependence. If, however, there is no difference in contributions it must imply that it is *not* utility interdependence that leads to over-contribution.

The main result of the paper is that both UI and AI considerations have relatively

modest effect in explaining over-contributions. Specifically, removal of both utility and strategic interdependence decreases the over-contribution only by 40% as compared to the benchmark. Removal of UI considerations alone decreases the over-contribution by 20-25% on average. The result is rather negative than positive in a sense that it is still unclear what explains more than a half of the remaining over-contribution. The most plausible candidate would be a lack of understanding of the optimal strategy, though the test of this goes beyond the scope of the current paper.

The results in this paper contradict the findings of Andreoni (1995b), where it was shown that over-contribution in public good games is caused by altruism rather than by confusion. However, the treatments suggested in Andreoni's paper introduced some additional factors as compared to the benchmark. In particular, a different payoff function was used so that the game became a zero-sum game. Thus, the Rank treatment from Andreoni (1995b) and the Phantom treatment from this paper are quite different, and moreover, the Rank treatment seems to be further from the benchmark, so it is not surprising that the results documented in my paper and Andreoni's are different.

The remainder of the paper is organized as follows: in section 3.2, the review of relevant literature is given and groups of factors that I will analyze in the paper are classified. In section 3.3, the treatments are described, and their properties discussed. In section 3.4 the experimental procedure is explained, and in section 3.5 the results are presented. Finally, in sections 3.6 and 3.7 I give a discussion of the results and the concluding remarks.

3.2 Literature review

3.2.1 Classification of theories explaining over-contribution

In this paper I am trying to understand the importance of different groups of factors in the decision-making process, and so it is instructive to define precisely the boundaries of each group. Below I give the list of the factors that have gotten the most attention in the literature are likely to be the most relevant in explaining over-contribution. To make it more comprehensive, I add the category "Others" to the list, though the analysis of

this category is beyond the scope of this paper.

Non-understanding or Learning. It is difficult for subjects to understand immediately what the optimal way to play a game is, so they need to experiment to gain some experience, which necessarily implies over-contribution. Another reason why learning can create consistent over-contribution is the fact that in public good games NE is Pareto dominated by the cooperative outcome. And so the payoff-based learning might give "false" feedback and lead to a positive level of contribution.

Actions interdependence (AI) or strategic considerations. Subjects can play sub-optimally in order to change actions of the opponents in the future, or to respond to past opponents' actions. In the former case subjects want to create a reputation or to encourage others to behave cooperatively, in the latter case subjects reciprocate the opponents' actions. See Brandts and Schram (2001), Fischbacher, Gächter and Fehr (2001) for experimental evidence and Falk and Fischbacher (1998) for theoretical model of an equilibrium concept with reciprocation.

Utilities interdependence (UI) or non-monetary considerations. Subjects take into account (positively or negatively) utility of other players, either because of altruism (Andreoni (1990)), or because subjects care about fairness of the outcome (see Bolton (1991), Rabin (1993), Fehr and Schmidt (1999), Bolton and Ockenfels (2000)).

Others. There are other reasons why subjects tend to over-contribute. It is difficult to summarize all of them in only a couple of sentences, so I will just list some of them below.

Emotions. Emotions such as revenge and spitefulness in the ultimatum game or warm-glow in PG games influence the subjects' choices. However, in most cases emotions can be considered as either part of AI or UI groups. For example warm-glow is a UI factor, and spitefulness is an AI factor.

Insufficient motivation. In most of the experimental studies, the payoff range is relatively small, and so the losses from suboptimal playing can be negligible (see Harrison (1989)) and subjects' can be not motivated to look for the optimal strategy. Some studies, however, show that high stakes do not change subjects' behavior significantly

see Camerer and Hogarth(1999) and Hoffman, McCabe and Smith (1996).

Mis-understanding. Subjects do not understand the game they are playing, which is especially likely to happen in more complex games (e.g. in financial markets experiments, see Kagel and Roth (1995) and Hirota and Sunder (2002)). It is a rather technical issue and usually can be addressed by using questionnaires.

3.2.2 Relevant literature

Among the many articles that analyze the role of different factors in subjects' behavior, I would like to mention several papers that are especially relevant to mine. In Blount (1995) the Ultimatum game was played as follows: in addition to a standard treatment, there were conducted two other treatments where the offer was made either by a third person or randomly generated by computer. Clearly the last two treatments eliminate negative reciprocation, or revenge to a low offer, since it is not the proposer who made the offer, and as the result, the rejection rate goes down considerably, especially in the treatment with computer-generated offers. Nonetheless approximately 15-20% of participants said that the smallest acceptable offer should be at least 45% of the pie (see Figure 1, p. 136).

Johnson et al. (2001) analyze three-period sequential bargaining game played with other people in one treatment and with payoff-maximizing robots in another treatment. As a result, the latter treatment produced only a moderate 30% decrease in the average proposer's offer from \$2.11 to \$1.84, while still being above the subgame perfect prediction of \$1.25. It suggests that subjects make generous offers not because they care about fairness, but because they do not understand what the optimal offer is.

In Palfrey and Prisbrey (1996), the authors suggest treatments for linear public goods game where subjects have to make decisions for different returns on the public good. This treatment gives enough variation to estimate econometrically the role of altruism, reputation and noise in subjects' behavior. Their analysis rejects altruism and reputation motives in subjects' behavior. In Fischbacher, Gächter and Fehr (2001) authors analyze the role of conditional cooperation, which can be summarized by the statement "I contribute as long as other people do." They ask subjects how much they

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would contribute as a function of the average contribution of the opponents. They found that approximately one third of all subjects prefer to free-ride, whereas half of the subjects are conditional cooperators.

In Andreoni (1995b) the author suggests the Rank treatment, which helps to separate kindness (altruism) from confusion (non-understanding) by making the former inapplicable. The result is that the level of contribution goes to zero very fast, which suggests that the contribution in the benchmark model is due to altruism and not non-understanding. Although his findings seem to contradict my results, this is probably due to the fact that the treatment he suggested changes the payoff function in such a way that it encourages competition (zero-sum game) and makes payoff-based learning less misleading than in the original public good game.

3.3 Treatment descriptions.

The main goal of this paper is to analyze the relative importance of three groups of factors mentioned above: *non-understanding* (learning), *utility interdependence* or UI and *actions interdependence* or AI. I designed two treatments that help to isolate the effects of different considerations, which I will describe in this section.

3.3.1 Benchmark treatment

All treatments described below are based on and compared to the benchmark treatment, which is a standard public good game with linear utility functions. In the benchmark treatment there are two projects: a private project and a social project (in the instructions, neutral project names are used to avoid framing effects). The private project yields a return of 1.00 to the subject only, and the social project yields a return of 0.75 to ALL members of the group. Thus, investing everything in the private project is the dominant strategy, while the public project is socially efficient.

3.3.2 Phantom treatment

The Phantom treatment (PT) is designed to remove both non-monetary and strategic considerations from subjects' decision-making. Here, it is applied to the public good

game, however as it will be seen from the description the idea is very general and it can be used in many other games. The PT consists of two different sessions A and B with two different groups of subjects. Session A is the benchmark treatment described above, which is a standard public good provision treatment with linear utilities and the strangers[1] matching procedure. Session A is conducted before session B, and its main purpose is to use subjects' decisions from session A during session B.

Session B is different from session A in that B-subjects (those who participate in session B) are matched with A-subjects, and not with other B-subjects. It is realized as follows: assume that in session A, there were N subjects in each group (the actual size of the group in the paper is two). Then in each period, t, for each B-subject, b, the computer randomly chooses $N - 1$ of A-subjects, $a_1, \ldots, a_{N-1}$. The payoff of subject b is determined in the usual way[2] using the decision of subject b made in period t and decisions that were made by the chosen A-subjects $a_1, \ldots, a_{N-1}$ in period t.

It is crucial that subjects in session A are not affected by the outcome of session B, and that matching procedure and relation between sessions A and B are known to B-subjects. It is also important that the game remains simultaneous, that is B-subjects do not know the decisions they are matched with until after they make their decision.

I claim that:

1. *Phantom treatment removes all non-monetary and strategic considerations.*

2. *In **all other aspects** phantom treatment is identical to the benchmark. In particular, B-subjects have the same expectations about the opponents' behavior as in the benchmark.*

The first point is almost obvious. First of all, since A-subjects are not affected by decisions of B-subjects, clearly, the latter will not have any UI considerations. Second, since the decisions that subjects face are pre-determined, it does not make sense for subjects to try to encourage others, to build reputation or to reciprocate.

As for the second point, notice that the decisions which were made by opponents of

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[1]The opponents are determined randomly every period.

[2]Sum of contributions to the public good times return on public good plus the subject's contribution to the private good.

B-subjects were made by *motivated* players who were playing the exact same game. Consequently B-subjects face the same *strategic uncertainty* as if they were playing against other people in the room. This feature makes the treatment more advantageous in comparison with situations when subjects play against computer programs (no strategic uncertainty, some new considerations), or against non-motivated subjects.

In other aspects the phantom treatment is also similar to the benchmark. First of all, in both A and B sessions, subjects have the same information about the game and the same amount of feedback about the outcome of the game. Second, the random matching procedure used in session A is similar to the matching procedure used in session B in a sense that the opponent (real or phantom) is randomly determined every round. Third, B-subjects are matched with the decisions that were made by subjects with the same experience and background as they themselves have. To ensure this, decisions from the same period are matched and subjects are recruited from the same pool of undergraduate students. Fourth, since opponents' decisions are the same as in the benchmark, it does not distort the learning process, which would not be the case if the opponents decisions were on average different. For example, if the opponents' contribution level was close to 0% or 100% of their tokens. Finally, even the instructions are made as similar as possible. To be specific, instructions for B-subjects are the same as instructions for A-subjects plus one section in the end that describes the matching procedure for the B session.

The main problem for the phantom treatment is credibility, namely, it is essential that subjects should believe in the faithful execution of the rules of the treatment. To address this concern in the beginning of the experiment, the printed decision table together with hand-written decision tables that were filled out by A-subjects were demonstrated from a distance, and it was announced that in the end of the experiment, subjects are welcome to look at it, which at least half of the subjects did. This announcement had a two-fold purpose: first, it addressed the credibility concerns, and second, it helped to ensure that subjects understood the specifics of the treatment.

Since *the only* difference between sessions A and B is that in the latter subjects do not have any non-monetary or strategic considerations, by comparing the results of both sessions, we can see the importance of these considerations in decision-making. For

example, if B-subjects over-contribute it should mean that the excessive cooperation in the benchmark model is not caused by non-monetary or strategic considerations, but rather by their inability to solve the game, or some other reason like insufficient motivation, for example. Whereas, if we observe fast convergence to the dominant strategy in experiment B, then we can attribute the excessive contribution in the benchmark model to the removed factors.

3.3.3 Two type treatments.

The two-type treatments are designed to measure the importance of utility interdependence in subjects' decision-making. These treatments enable us to compare the difference in decisions with and without UI considerations. To see the main idea behind these treatments, notice that the phantom treatment is different from the benchmark in two aspects: one, opponents are not affected and, two, their decisions are pre-determined. The former removes UI considerations and the latter removes AI considerations. The two-type treatments are the natural intermediates between the benchmark and the phantom treatment. In the two-type environment some subjects are still not affected by others' decisions, but their decisions are NOT predetermined, and thus the AI factors remain applicable, while UI considerations are removed.

The two-type treatments are designed as follows: at the beginning of each period, half of the players are randomly assigned to type M and another half to type N. Type M players are *monetary affected* players with a usual payoff function. Type N players are *not affected* players and they receive a fixed amount of points regardless of the game outcome. Groups consist of two subjects and each subject is informed about the type of his opponents, and thus subjects matched with type N should not have any UI considerations, whereas for subjects matched with type M non-monetary considerations are applicable to the same extent as in the benchmark.

It is crucial for this treatment to make sure that subjects do not know their *own* type, since otherwise type N subjects, would be unmotivated players. To address this issue, subjects were informed about their type only *after* they made their decisions, and each period types were re-assigned randomly. In addition to that subjects knew that groups

are formed independently of types assignment and thus the opponent's type does not carry any information about the subject's type, and so for example it is quite possible that two players of the same type are matched. Moreover, two practice rounds were programmed in such a way that subjects were assigned against opponents of the same type in the first round and of the different type in the second round to show that both options are equally likely.

There were two two-type treatments designed. The first treatment, called uk2T (2 Types, own type is Unknown, the type of the opponent is Known), has the following timing and information structure: in each round, first, types and groups are assigned, but not disclosed; second, subjects are informed about the type of their opponent and are asked to make a decision; finally, profit is calculated and displayed, and also subjects are informed about the type they had that period.

The main difference in the second two-type treatment, called uk2SM (two types strategy method), is that in each period subjects are not informed about their opponent's type. Instead, they are asked to make two decisions — one, if they are matched with type M, and another if they are matched with type N. This is based on the so-called 'Strategy Method' introduced in Selton (1967). After the two decisions are made, groups are randomly assigned and corresponding decisions are used to calculate profits. For example, if a subject is matched with type M, then the decision he made against M will be used.

Each of the two-type treatments has its own advantages and disadvantages, and this is why both of them were used. The main disadvantage of the uk2T treatment is that it does not control for learning and experience. For example, if subject X decreases his contribution between periods 5 and 6, it can be because in period 6 he is matched with type N, or it can be because he suddenly realized that the private project is better, or because he responds to the low contribution of his opponent in round 5. The uk2SM treatment does not have this problem, since subjects make two decisions given the same experience, and so the difference is only due to the type of the opponent. The problem of the uk2SM treatment is that it might be too suggestive. The very fact that subjects are asked to make two decisions might suggest to them that they are supposed to make

two *different* decisions. To minimize the problem, in the instructions it was said that subjects are allowed to make two decisions just to give them additional flexibility and so it does not mean that they should make the same or different decisions.

The two-type treatments should have the following impact on non-monetary considerations. First of all, clearly, if subjects over-contribute mainly because they care about payoffs of other players, they will contribute less against type N than against type M. Hence, the two-type treatments eliminate UI considerations for those subjects who are matched against type N. However, AI considerations are still applicable. Indeed, if subjects contribute to encourage others, then the type of the opponent does not matter. Similarly, for reciprocative behavior, if subjects reciprocate because they do not know how to play, using the strategies of others as a lead, then M and N strategies are equally useful. If they are generally grateful or outraged by their opponents, and merely want to demonstrate this to someone, then the type of the pair does not matter either. Among reciprocative behaviors, the only exception is the case when subjects reciprocate to specifically hurt or reward the next opponent. But this kind of reciprocation should be attributed to UI factors rather than AI, and is eliminated against type N.

Even though there are many ways how UI considerations can be eliminated from subjects' reasoning, the advantage of this treatments is that all decisions that subjects face are made by *motivated* human beings. The main side effect of the two-type treatments is that since we expect that contribution against type N will be smaller, it might distort the information that subjects will be getting about the behavior of the opponents, and somewhat bias the decisions towards under-contribution. This is a valid concern, and its importance is not clear *a priori*, since N-type subjects after they learn about their type will care less about the low contribution of the opponent.

In summary: two type treatments eliminate non-monetary considerations for subjects matched against type N, while AI considerations are still applicable regardless of the opponent's type. Thus the difference between contributions against type M and type N are solely due to UI factors.

3.4 Experimental procedures

The study consists of four treatments: the benchmark treatment (BT), the phantom treatment (PT) and the two-type treatments: uk2T and uk2SM. Each treatment was run twice to test for robustness of the results. Sessions were conducted at Yale University in July and September of 2004. The subject pool consisted mainly of Yale undergraduate students of non-Econ majors. The experiment was programmed and conducted with the software z-Tree (Fischbacher 1999). On average there were from 10 to 16 subjects at each treatment. Each treatment lasted approximately 45-60 minutes and the average payments were approximately 13-15 USD.

The main structure of all 8 sessions was quite similar. Each session started with subjects reading instructions that were printed out, so that they were readily available throughout the experiment. In addition to that, the summary of instructions was on the screen during the experiment. After reading instructions subjects took a short quiz to make sure that they understand the rules of the treatment. After the quiz, the game was played for 15 rounds and in the end of the session subjects were asked to fill out a post-experimental survey and were privately paid their cash earnings.

In the phantom treatment after instructions were read, the printed table with all of the decisions that were made by participants of the benchmark was demonstrated to subjects. It was announced that in the end of the experiment subjects were welcomed to look at it, as well as at decision sheets that were filled out by participants of the benchmark experiment. As mentioned earlier, the announcement had two purposes. First, it was intended to solve credibility problems, namely, to assure subjects that the experimenter was not lying to them and that the instructions describe the real experimental environment. Second, it made sure that subjects knew that they are not playing with other people in the room, but with the participants of the previous experiment.

In the two-type treatments, after reading instructions, subjects played two practice rounds and only afterwards were asked to take the quiz. Subjects knew that in the practice rounds they were all going to be matched with computer decision 17-3 (17 to private project, 3 to public project). The two practice rounds proved to be very helpful for subjects in understanding the types structure, especially in the 2TSM treatment. In

addition, matching everyone against the same pre-announced computer decisions mini-mized any learning distortion created by the practice rounds.

Each round had the following structure: in the beginning subjects were randomly paired with another person (real or phantom). After that they were given 20 tokens (=10 US dollars) to divide between two projects. Each token invested into Project A (private good) gives a return of 1.00, and each token invested into Project B (public good) gives a return of 0.75 to both members of the group. Thus, it is clearly the dominant strategy to invest everything into project A. After all decisions were made, subjects were informed about the total contribution to project B and their profit. In addition, the history table was available to subjects throughout the experiment that contained the history of their decisions, profits and types.

As it was mentioned above, in total there were 8 different sessions, and special attention was paid to make sure that all the sessions were similar to each other, except for the specifics of the treatment (i.e. phantom players and types). For example, instructions were divided (implicitly) into a general section comprising 60-70% of the instructions (description of the public good game, project returns, examples, cash payments) which was the same across all sessions, and a section that explained the specifics of the treatment.

Nonetheless, there were a few differences between the treatments. One difference was that in the two-type treatments, there were two practice rounds, while in the benchmark and the phantom subjects started to play real rounds from the beginning. This difference has been discussed above, and should have minimal effects on the comparison. Also, there was an important procedural change in the two-type treatments. In the first uk2T treatment subjects read instructions, then took the quiz and then played practice rounds. This procedure turned out to be quite confusing for subjects. In the remaining three two-type treatments (one uk2T and two uk2SM) the following modification was used: first subjects read instructions, then they played two practice rounds, then they took the quiz, and then they played the 15 non-practice rounds. This change made it considerably easier for subjects to understand the rules of the treatments. However, as it will be shown in the next section, it also changed the observed behavior. Specifically, the results observed in the last three treatments are very similar to each other and

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different from the first uk2T treatment. Because of the robustness of results in the last three treatments (versus the first uk2T treatment), and because of budget constraints, another uk2T treatment under the modified procedure was not conducted.

3.5 Results

3.5.1 The phantom treatments

The average contributions in the benchmark and the phantom treatments are given in Figure 3.1. The left picture shows the average contribution in the benchmark - phantom pair, and the right picture shows the average contributions in the second, phantom - phantom pair. One remark is due here: in the second phantom treatment subject 5 contributed 100% of the endowment every period because (as he said in the questionnaire) he "does not want to give up his principles because of some study". Later in the section, this behavior will be discussed and it will be said that this motivation can be quite important for some subjects and is not removed by the phantom design. That said, we are going to exclude subject 5 from the sample for most of the following analysis.

Figure 3.1: Contributions in Benchmark and Phantom Treatments
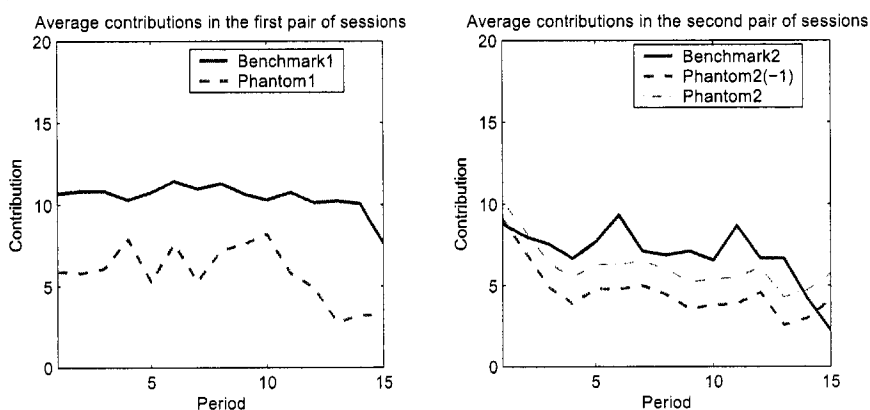
Figure 3.1: The solid lines represent the average contributions in the Benchmark treatments. The dashdotted line in the second picture represents the average contribution of all subjects, the dashed line on the second picture shows the average contribution of all subjects excluding the one who always contributed 20.

As it can be seen from Figure 3.1 the phantom treatments produce lower level of contribution than the benchmark treatments, and using Kruskal-Wallis ranking test it

can be established that the decrease in contributions is significant (see Table 3.1). Thus the first result is:

Result 1: *Phantom treatments produce a lower level of contribution than the corresponding benchmark treatment.*

Table 3.1: Kruskall-Wallis ranking test

Treatments	Kruskal-Wallis ranking test p-value
BT1 - PT1	4.48E-06
BT2 - PT2 (all)	0.0135
BT2 - PT2 (-1)	0.0026

Table 3.1: Comparison of contributions in the benchmark and the phantom treatments. PT2 (all) means the contributions in the phantom treatments including subject 5 (the one who contributed 100% every period), and PT2(-1) excludes this subjects from the sample.

Result 1 confirms the well-known fact that some of the over-contribution in the public good games is caused by non-monetary and strategic considerations. However, as it can be seen from Figure 3.1 the decrease is not as strong as one would expect. We can roughly estimate the difference in the average contributions by comparing the medians. In the first pair of sessions the median contribution goes down from 10.69 to 5.8 (54%), and in the second pair the median contribution goes down from 7.11 to 4.44 (62%).

Result 2: *The non-monetary and strategic considerations on average account for approximately 40% of overcontribution.*

It has to be mentioned here that the number 40% in Result 2 makes sense only as long as we speak on the aggregated level. It is well-known that subjects behavior is heterogenous, and the same phenomenon is observed here. Specifically, on the individual level, the difference between the treatments rises from the fact that subjects who would contribute in the benchmark because of non-monetary or strategic considerations, were contributing zero in the phantom treatments, whereas subjects who were just trying to find the best way to play the game played similar in both treatments. For example, in the phantom treatments, many subjects always contributed zero (7 out of 20 in the phantom versus 3 out of 24 in the benchmark), while the proportion of consistent contributors is

much higher in the benchmark (8 subjects contributed more than half of the endowment in at least 12 rounds, whereas only 2 subjects did so in the phantom).

However, even on the aggregated level, result 2 is somewhat surprising since the drop in the contributions is not particularly high, which means that AI and UI considerations have a relatively modest impact on subjects' behavior, and can explain less than half of the observed over-contributions. Thus the natural question to ask would be *does the phantom treatment work at all?* In other words, does it really eliminate the non-monetary and strategic considerations from subject's reasoning or not? Clearly it should in theory, but it might be the case that some subjects still use the "removed" AI and UI considerations.

To answer this question I ran the Ultimatum game experiment under the phantom treatment rules. This should be a clear test of the phantom treatment performance since the Ultimatum is much simpler than the public good game, and so most of the rejections are more likely to be caused by non-monetary considerations rather than by non-understanding (as it could be in the public good games). As a consequence we should expect considerable drop in the rejection rates under the Phantom treatment rules.

There were 40 subjects who participated in this experiment, and an offer of 1 dollar out of 20 was made to all of them under the phantom rules. The result was that 33 subjects out of 40 accepted the offer and 7 subjects rejected it. The result is consistent with the results published in Blount (1995) where 85% of subjects accepted randomly generated offer of the size $0.50 out of $10 (only 35% in the benchmark). Thus we see that the phantom treatment causes a significant drop in the rejection rates, and so we have the next result:

Result 3: *In the Ultimatum game the phantom treatment decreases the rejection rate of the smallest possible offer to 15%, and thus it removes the non-monetary considerations from subjects' decision-making.*

At least two people of those who rejected the offer said that they understood the treatment and the reason for the rejection was that the offer was "too lame (wrong)". Recall that in the second phantom treatment subject 5 was contributing 20 because

he did not want to give up his principles in the study. Thus it suggests that there is a motivation that the phantom treatment does not remove, which is to follow principles and norms of behavior just for the sake of the principles[3]. Even though at least three subjects demonstrated this motivation in their behavior, the phantom treatment considerably diminishes its impact. The reason is, it is rather easy to realize that it does not make sense to be nice (even from the ethical point of view), if no one is affected, while you can get hurt by doing so. For example, subject 9 in the second phantom treatment said in the questionnaire that she contributed zero every round exactly because it makes no sense to be nice. Even though she still felt irrational guilt about doing so, she nonetheless played optimally.

The hypothesis that the phantom treatment removes non-monetary and strategic considerations from subjects behavior can be (at least partially) tested statistically. Clearly, it is impossible to see from the data whether subjects have any UI considerations, or if subjects contribute to encourage others. It is possible, however, to estimate the subjects' reaction to their opponents' contribution, and thus to see if reciprocation is removed or not.

Table 3.2: Panel-Data Analysis

xtreg, fe	Benchmark		Phantom	
y=$Cont_{t+1}$	Coef.	p-value	Coef.	p-value
$OppCont_t$	0.139404	*0.000*	0.024931	*0.481*
$OppCont_{t-1}$	0.058127	*0.060*	0.056728	*0.107*
Const	6.856729	*0.000*	5.151072	*0.000*

Table 3.2: Panel-data regression with fixed effects of the contribution at period $t+1$ as a function of the previous opponents' contributions.

In Table 3.2 the results of the panel-data regression with fixed effects are demonstrated. The dependent variable is the contribution at round $t+1$, and the regressors are the lags of the opponents' contributions. As it can be seen from the Table, in the benchmark, variable $OppCont_t$ is significant and positive which confirms the well-known fact that subjects tend to reciprocate the behavior of others. However, in the phantom

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[3]This is not as irrational as it sounds. Consider that we ask the following question: you are driving a car and you see an old lady crossing the street. Will you hit her? If you say yes, we'll pay you five dollars, otherwise we'll pay you nothing. Clearly, even though no one gets hit in this scenario many people would prefer to forfeit the money and say no.

treatment opponents' contributions are all insignificant, which suggests that this factor is removed from subjects' reasoning.

Tables 3.3 and 3.4 show results of fixed effect logit estimations. The dependent variables are *Negch* which is equal to 1 if there was a negative change in contributions, *Negbigch* which is equal to 1 if the decrease in contribution was at least 3, and variables *Posch* and *Posbigch* that are defined similarly for positive and big positive changes. In Table 3.3 the explanatory variable is *Profit* , and in Table 3.4 it is the opponent's contribution last period ($OppCont_t$).

Table 3.3: Fixed-effects Logit Regressions on Profit

| xtlogit, fe | Benchmark | | Phantom | |
	Coef.	p-value	Coef	p-value
negch				
Profit	-0.142	0.000	-0.081	0.013
posch				
Profit	0.136	0.000	0.059	0.077
negbigch				
Profit	-0.185	0.000	-0.132	0.002
posbigch				
Profit	0.127	0.001	0.046	0.242

Table 3.3: Results of fixed-effects logit regressions with Profit as dependent variable. Negch (Posch) is equal to 1 if the change in contribution is negative (positive). Negbigch is equal to 1 if the decrease in contribution is at least 3.

Table 3.4: Fixed-effects Logit Regressions on Opponent's Contributions

| | Benchmark | | Phantom | |
	Coef.	p-value	Coef	p-value
negch				
$OppCont_t$	-0.072	0.003	-0.030	0.219
posch				
$OppCont_t$	0.073	0.003	0.020	0.425
negbigch				
$OppCont_t$	-0.083	0.007	-0.047	0.115
posbigch				
$OppCont_t$	0.062	0.031	0.008	0.790

Table 3.4: Results of the fixed-effects logit regressions where the regressor is the opponent's contribution last period. Negch (Posch) is equal to 1 if the change in contribution is negative (positive). Negbigch is equal to 1 if the decrease in contribution is at least 3.

First of all, the benchmark results are as expected. Both *Profit* and $OppCont_t$ are

significant in all four regressions and their signs are intuitive. In the phantom treatment, the picture is different. First of all, as before, regressions on $OppCont_t$ are all insignificant (see Table 3.4). However, if we regress the binary variables on $Profit$, they are significant in the regressions with negative change variables and insignificant in the regressions with positive change variables.

The latter gives very strong evidence for my hypothesis. Indeed, the reason why subjects in the benchmark respond positively to the increase in profit is that usually this increase is caused by increases in the opponents' contribution and subjects reciprocate. However, this reasoning is not applicable in the phantom treatment, and this is why we have that profit is insignificant in the regressions with positive change variables. Nonetheless, in the regressions with negative change variables, profit is significant, while the opponent contribution is not. The former is due to the fact that subjects in the phantom treatment still use profit to learn about the optimal behavior which creates downward trend, and the latter is again due to the fact that subjects do not reciprocate in the phantom treatment.

Result 4: *Tables 3.2-3.4 give very strong support to the hypothesis that subjects do not reciprocate in the phantom treatment, while they do reciprocate in the benchmark.*

3.5.2 The two-type treatments

In total there were four two-type treatments: two uk2T and two uk2SM treatments. As it was explained in details in the end of Section 3.4, the original procedure that was used in the first uk2T session proved to be difficult for subjects, and so it was slightly modified to make it less confusing.

Average contributions in all four sessions are shown in Figure 3.2. As it can be seen from the picture the results in the last three modified sessions (uk2T-2 and both uk2SM) are different from the results in the uk2t-1 treatment and similar to each other[4]. In the uk2T-1 session contributions have higher variability and often contributions against type N were much higher than contributions against type M which is counterintuitive.

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[4]The best thing to do in this situation would be to throw away the results of uk2T-1 session and run another uk2T session under the modified procedure. However, because the results from the modified sessions are sufficiently robust to make reliable conclusions and and because of the budget constraints, it was not done.

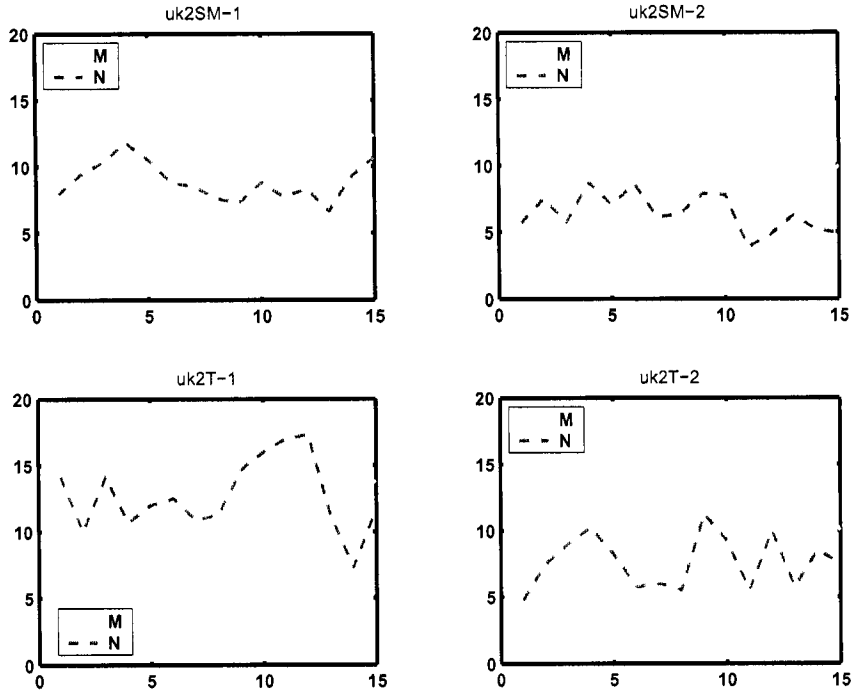## Figure 3.2: Contributions in Two Type Treatments

Figure 3.2: The solid line represents the average contribution against type M, the dashed line against type N. uk2SM is the two-type treatment with the strategy method, and uk2T is a standard two-type treatment.

In uk2T-2 and both uk2SM sessions the average contributions against type N are lower than the average contributions against type M, and Kruskal-Wallis ranking test in Table 3.5 shows that this difference is significant for all treatments but uk2T-1.

Table 3.5: Kruskal-Wallis Ranking Test of the Similarity Between Contributions Against M and N

Treatments	Kruskal-Wallis ranking test p-value
uk2T-1	0.2451
uk2T-2	0.0001
uk2SM-1	0.0007
uk2SM-2	0.0001

Table 3.5: Kruskal-Wallis test shows that in all treatments but uk2T-1 the difference between the contributions against type M and N is statistically significant.

A panel-data model with fixed effects was used to estimate the role of non-monetary considerations in the two-type treatments. The results of the estimation are given in Table 3.6. The dependent variable was subject contribution at period $t + 1$ and the

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regressors were the opponent's type at period $t + 1$ (recall that it was observed before subjects made their decisions) and opponents' contributions in periods t and $t - 1$. The type of the opponent was a binary variable that was equal to 1 if the opponent had type N. As it can be seen from the Table, the coefficient of the opponent's type is significantly negative in all treatments but uk2T-1. Moreover, its value is approximately the same in the three treatments and is somewhere between -2.5 and -2. This result is reasonably robust to using different regressors, in a sense that the coefficients at $OppType_{t+1}$ are always in the range between -3 and -2 and close to each other.

Table 3.6: The Role of Opponent's Type in Contribution Decisions

xtreg, fe	uk2SM-1		uk2SM-2		uk2T-1		uk2T-2	
$y=Cont_{t+1}$	Coef.	p-value	Coef.	p-value	Coef.	p-value	Coef.	p-value
$OppType_{t+1}$	-2.411	0.005	-2.081	0.032	-0.860	0.315	-2.344	0.009
$OppCont_t$	0.108	0.084	0.098	0.142	0.161	0.010	0.214	0.009
$OppCont_{t-1}$	-0.005	0.935	0.086	0.194	0.137	0.029	0.119	0.156
Const	9.809	0.000	7.697	0.000	9.682	0.000	7.534	0.000

Table 3.6: Results of panel-data regressions with fixed effects. The variable OppType is equal to 1 if the opponent's type is N, and 0 otherwise. It is significant in all but uk2T-1 regression. In uk2SM treatments, the variable $Cont_{t+1}$ is equal to the decision that was used for profit calculations.

Thus we have the following two results:

Result 5: *In the two-type treatments the average contributions against type N are significantly lower than the average contributions against type M.*

Result 6: *As it follows from Table 3.6, non-monetary considerations account for an approximately 2-2.5 (out of 20) increase in contributions.*

As it was mentioned before, the numbers presented in Result 6 are meaningful only on the aggregated level. The individual decisions are less uniform and can be seen on Figure 3.3. Notice that even on the individual level, most of the subjects contribute less against N than against M. The precise numbers are as follows: first of all, as it could be expected many subjects made the same decisions against both types. It happened in 216 cases out of 450 (48%), and 12 subjects (out of 30) played the same strategies

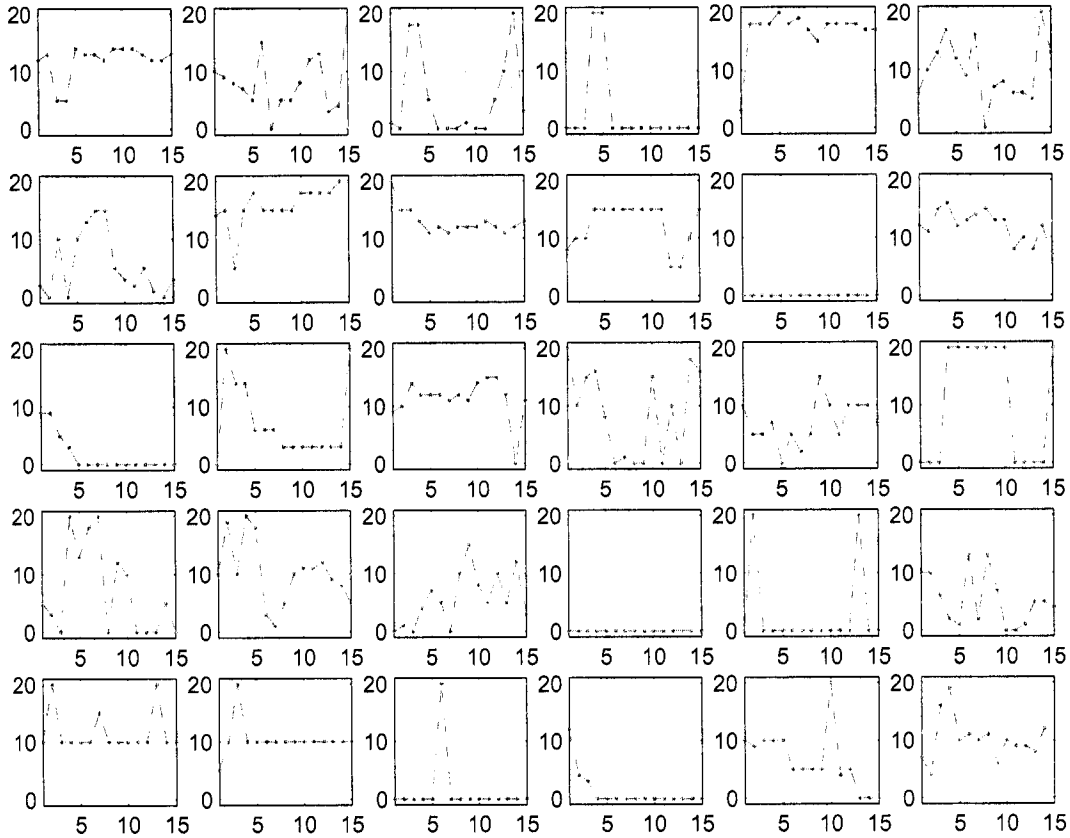**Figure 3.3: Individual Decisions in uk2SM Treatments**

Figure 3.3: The solid line represents contributions against type M. The dashed line with points represents contributions against type N. The horizontal axis represents rounds of the experiment from 1 to 15. The vertical axis represents contributions.

at least for 10 rounds. Clearly, these subjects (and it follows from the questionnaire as well) did not care about non-monetary considerations, and were contributing either because of AI reasonings or because they were trying to maximize their profit, and just did not see any reason to contribute differently. In 150 cases, the difference between M and N-contributions was positive and 7 subjects contributed consistently (at least for 10 rounds) more against M than N (see eg. subjects 4, 7, 11). Another example is subject 17 who was sometimes contributing less against N "since N does not care". These subjects clearly cared about the payoffs of the opponents and fairness. Naturally, some instances of positive difference were not because of the UI considerations, but just due to experimenting by subjects. In 84 decisions out of 450 (19%), the contribution against N was higher. Again as before, many subjects did not care about type differences and were

just experimenting. There were also four subjects who contributed higher against N in at least 8 rounds, and one of them contributed higher against N in 13 rounds. The latter was a typical example of satisficing behavior, namely (s)he said that contributing 14-6 against M, and 6-14 against N gave enough profit. The other three subjects mentioned AI (encouragement) as the main factor that determined their behavior.

In general, it can be seen that the number of subjects who care about the payoffs of the opponents is not very high. In these treatments we have 7 subjects out of 30 (23%) who consistently contributed higher against type M. Most of the other subjects did not use the information about the opponent's type either at all or in any systematic manner. These results are not particularly surprising given the results obtained in the phantom treatment.

3.6   Discussion

The main goal of the paper is to measure the importance of non-monetary and strategic considerations in subjects' decision-making. Such non-classical theories as fairness, reciprocation and others become more and more popular in economics. They seem to be intuitive and psychologically appealing. Moreover, they explain subjects' behavior much better than the classical payoff-maximization approach. Given that, the results obtained in this paper are a little bit shocking, because it follows that these considerations are not as important for subjects as one would think.

To be more specific, applying the phantom treatment to the data from the benchmark caused only 40% decrease in the average contribution. Since in the phantom treatment, subjects do not have any non-monetary or strategic considerations it means that the remaining 60% are due to some other factors, and regardless of whether it is learning or insufficient motivation or something else it still means that subjects do not understand that it is always optimal for them to contribute zero. One must ask, "can it be that the main result of the paper is just that it is possible to construct an experiment that completely confuses subjects?" Not surprisingly, the answer is no, there is obviously more than that, and to show this I would like to mention couple of points.

The first point is that, *if any treatment is misleading, then the benchmark itself should*

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be misleading for subjects, since the phantom treatment does not introduce additional confusion. Indeed, first of all, as it was argued in the paper, the phantom treatment was designed to be as close to the benchmark as possible. In particular, instructions were also made similar to each other, and the only difference was that instructions to the phantom treatment had an additional section that explained the phantom rules. Furthermore, the sentence "you will be matched with decisions made by participants of another experiment" was actually found to be quite natural by subjects and neither the questions during the experiment nor the responses to the questionnaire revealed any sort of confusion. Second, in the paper it was shown that the phantom treatment worked properly, in a sense that it removed all non-monetary and strategic considerations, and in particular, the experimental procedure ensured that all subjects understood that they are playing with phantom players and not with other people in the room (instructions, quiz, showing the benchmark decision tables).

The second point is that *it was a standard public good experiment that was used as the benchmark.* In particular, the story with two projects (public and private) is an absolutely standard story in public good experiments which helps to remove any kind of framing from the instructions. The instructions themselves were based on the instructions in Andreoni and Petrie (2004), and the returns on the public project and group size were used as in Morgan and Sefton (2000). The results obtained in the benchmark are qualitatively similar to standard results in the literature, in particular to those in Morgan and Sefton (2000). Namely, the average contribution level starts at approximately 50% of the endowment, and then fluctuates around a decreasing trend with a sharp drop in the end. It means that the benchmark experiments that were conducted in the paper are no more confusing for participants than the standard public good games in the literature and so we should look at those in order to spot the cause of non-understanding.

To begin with, it is not immediately obvious that the public project is always worse (in terms of profit) than the private project. In addition to that, it is well-known that the strategic uncertainty that subjects face in the game can be a serious obstacle to finding the optimal strategy. The most persuading support to this claim can be found in

Shafir and Tversky (1992), where in the sequential Prisoner' Dilemma, the second-movers were defecting considerably more often than in the benchmark even if the first-mover cooperated. The additional evidence can be found in the post-experimental surveys for this paper. One of the free-response questions was: *Assume you know that your pair's decision is 13-7 (13 to project A). What would you decide?* From 70 subjects who answered this question 33 (47%) said that they would contribute zero to public good, 9 subjects said that they would contribute no more than 5, and quarter of them said that they would play (around) 13-7 as well, which most likely indicated that they just do not know what to do in this situation. However, many subjects who gave selfish response played quite cooperatively during the experiment. Notice also that the question did not ask *What decision would maximize your profit* , thus still leaving non-monetary considerations applicable.

Another possible critique of the phantom treatment is that its results are not robust to changes in instructions. The easiest response would be that nothing is robust to changes in instructions. In most of the standard environments, the subjects behavior can differ in a systematic manner because of some small and seemingly irrelevant changes between treatments. The most recent example is given in Liberman et al. (2004), where the Prisoner's Dilemma was either called "The Wall Street Game" or "The Communal Game", which led to completely different behavior. The difference is of course, due to the framing effect and to the well-known fact that subjects tend to play in such a way that they think the experimenter expects them to play. In my paper, I tried to avoid framing or being suggestive in the experimental procedure, however there is no doubt that if the experimental procedure had been designed in such a way that it specifically encouraged cooperation or alternatively, selfish behavior, it would have changed the impact of the phantom treatment, but it would have changed the benchmark behavior as well. This is why the experiments in this paper were based on the standard public good experiment design, so that the results would be more relevant to what has been done in the literature.

3.7 Conclusions

In this paper two experiments were designed to understand the role of non-monetary and strategic considerations in subjects' reasoning. In the phantom treatment both UI and AI considerations are removed while the rest of the game is unchanged, and thus the positive contributions cannot be explained by the removed factors. In this treatment subjects are playing not with other people in the room, but with decisions made by the participants of the benchmark experiment, *(phantom players)*. Notice that in this environment, the non-monetary considerations are removed because the opponents are not affected and the strategic considerations are removed because their decisions are pre-determined. As it was shown the phantom treatment decreases the over-contribution by approximately 40% in comparison with the benchmark. Given that the standard level of contribution is around 10 (out of 20), the approximate decrease in contributions is 4.

There are two possible intermediate designs between the phantom treatment and the benchmark. The first design is where the opponents' decisions are still pre-determined, but the opponents are affected. This environment is almost the phantom treatment and it has not been realized in the paper because of the credibility concerns. The second intermediate design, is when the decisions of the opponents are not affected, and their decisions are not pre-determined. This was realized by the two-type treatments, where players had one of two types: M — *motivated* and N — *non-motivated or non-affected*. In this setting, subjects who matched with type N did not have non-monetary considerations, and thus the difference between contributions against M and N is exactly due to UI factors. The role of these factors is also quite modest — the removal of UI factors reduces the average contributions by 2-2.5 tokens (out of 20). And thus the overall result is that both non-monetary and strategic considerations explain less than a half of the observed over-contribution in the public good games.

The final remark is that the treatments designed in the paper, specifically the phantom treatment, can be applied to a broader range of questions than just public good games. In literature, the experiments are conducted because of two reasons: either to approximate the theory, or to approximate reality. In the first case, the applicability of the phantom treatment is almost obvious, since most of the theories so far do not use

behavioral assumptions to model subjects' behavior and thus it is not surprising that very often subjects behave differently from the theoretical predictions. Maybe somewhat surprisingly, the phantom treatment can be used to approximate reality as well. It happens when the agents in the real markets are likely to be more rational than subjects in the lab. For instance, the phantom treatment can be used to analyze the performance of auctions. In real life, the participants of the auctions, especially multiple-unit auctions, are firms who are more rational and are less susceptible to such considerations as fairness or altruism. In the laboratory, however, subjects are people and so non-monetary considerations can distort the results as compared to reality[5]. Consequently, by applying the phantom treatment it would be possible to obtain a more precise estimate of auction performance.

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[5]For example in multiple-unit auctions where subjects can prefer to be fair and buy only one good, so that other people can buy some as well.

3.8   Appendix. Phantom Treatment Instructions

Welcome to a decision-making study!

Introduction.

You will participate in a decision making experiment. These instructions describe a game that you will play for 15 rounds. The instructions are simple and include many examples to make sure that you understand the rules of the experiment. If you follow the instructions carefully and make good decisions you will earn a considerable amount of money. Your payoffs in this experiment will depend on the choices made by you and the other players you will be paired with.

If you have any questions while these instructions are being read, please raise your hand. Please do not talk with the other subjects, even to ask questions about the instructions. If we hear you talking at any point in the experiment you will be removed from the room and will not receive any payment.

General Rules. Decisions.

In the experiment you will play the same game for 15 periods. In the beginning of each period you will be randomly paired with another person **according to the rule described in the end of the instructions**. You and the person you are paired with will form a group, and your earnings will depend on what you and your pair decide.

In the beginning of each period you will be given endowment of 20 ECU[6] and your decision will be to choose how to divide it between two investment projects:

- *Project A:* Each ECU you invest in project A will give you a return of 1 ECU.

- *Project B:* Each ECU invested in Project B by either member of the group will yield a return of .75 to EACH member.

Your task is to decide what part of your endowment to invest in project A and what part invest in project B. You can invest some of your endowment in project A and some in project B. Alternatively, you can invest all your money into project A or into project B. **Your experimental profit in each period will be equal to the sum of project returns.**

Example. If both you and the other member of your group invest 10 ECU to project A and the remaining 10 in project B, then your income will be equal to $10 \cdot 1 + (10 + 10) \cdot 0.75 = 25$. The first term, $10 \cdot 1$, is your return from project A. The second, $(10+10) \cdot 0.75$, is your return from the combined (yours and your pair's) investments in project B.

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[6]Experiment Currency Units

Your experimental profit and cash earnings.

You will be paid in private and in cash at the end of the experiment. Your cash earnings are determined in the following way: using a deck of cards we will openly and randomly choose one *payment* period among the periods that you played, and your cash earnings will be equal to your experimental profit in the payment period times 0.5.

For example: assume period 5 was chosen as the payment period, and your profit in period 5 was 100 ECU. Then your cash earnings will be $100 \cdot 0.5 = 50$ USD. (**Number in the example are taken to be higher than the earnings you can get in the experiment. Sorry!**).

How pairs will be formed.

For each decision you will be paired with a randomly chosen participant from another experiment (Experiment A). This experiment was conducted at Yale University on September 14. Experiment A was absolutely identical to the current experiment (Experiment B), except that participants of experiment A were playing with each other, while you will play with participants of the different experiment. Participants of Experiment A were paid based on their performance according to the same rule as you will be paid. All participants of Experiment A were chosen from the same pool as you were, namely students of non-Economics majors at Yale University.

Experiment A was entirely independent of Experiment B. In particular, participants of experiment A were not, and will not be informed about experiment B. Furthermore, they will not receive any actual cash payments as a result of experiment B.

To make sure that you and the random participant of Experiment A, with whom you are paired, are equally experienced in this particular game, you will be matched with decisions made in the same round that you are playing.

Example: assume you are playing round 6. The computer will randomly assign for you Mr. X from Experiment A and your earnings will depend on your decision in round 6 and the decision that Mr. X made in round 6. In round 7 you will be randomly matched with another participant of experiment A (say Ms. Y) and your earnings will depend on your decision in round 7 and the decision Ms. Y made in round 7.

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