Regulation and Design of Financial Markets

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Introduction

Financial Intermediation in Endogenous Incomplete Markets:

- They play the role of creating assets for agents to invest in
- Non-convexities in aggregate production set, such as trading fix costs (Pesendorfer (95), Bisin (98))
- In general, financial intermediaries have a coordination purpose: obtain resources from consumers and invest them in the productive sector.
- If there are economies of scale in aggregate risk sharing, intermediaries help coordinate investment.
- In this paper, we study a (very simple) model of constrained efficient incomplete markets, (from min scale constraints)
- We also study optimal financial regulation = optimal arrangements for intermediation (contracts, institutions and rules of competition)

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- The paper is divided in two parts

1. An optimal mechanism design problem: Abstracting away from markets, choose optimal
    1. span of assets (or technologies)
    2. investment portfolio
    3. "deposit" insurance for consumers
2. Implementation: Decentralize the optimal allocation with familiar institutions
    1. Broker - Dealers (acting as commercial banks) with free entry
    2. Firms (with free entry)

    - However, this institutions must be regulated in order to implement the optimal mechanism.
    - In our simple example, these are quite stark: shut down consumer $\leftrightarrow$ firms channel.

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Setup

Households

- Diamond-Dygvig model
- Continuum set of households I, with measure 1.
- One consumption good, which is perishable. Households live for 3 periods $t = 0, 1, 2$.
- At $t = 0$, all households are identical, and receive an endowment of $\omega > 0$ units of $t = 0$ consumption
- Consumers have no endowment in $t = 1, 2$.
- Only derive utility from c_1, c_2
- Private Information:
  - Ex-ante identical households.
  - At $t = 1$ a taste shock $\theta \in \Theta$ is drawn from a distribution $F(\theta)$ (compact, Banach space)
- At $t = 1$ there is also a publicly observed shock $s \sim \text{Uniform}[0, 1]$

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Technology

Two types of securities:

- Storage (short): Safe, that pays only 1 unit of next period consumption.
- Long assets, or productive technology, that pay off in period 2 only.
- For each $\hat{s} \in [0,1]$ there is an asset $A_{\hat{s}}$ that pays a rate of return $r_{\hat{s}}(s)$

$$r_{\hat{s}}(s) = \begin{cases} 0 & \text{if } s \neq \hat{s} \\ R > 1 & \text{if } s = \hat{s} \end{cases}$$

- If invest y in all long technologies equally, then gets Ry w.p.1
- This would map directly to classical Diamond-Dygvig model

Setup

Minimum Scale

- **Constraint:** Minimum scale requirement for investment $y(s)$ in asset s:

$$r_s(\hat{s} = s) = R \iff y(s) \geq M(s)$$

- This constraint is binding in the aggregate; i.e. there is not enough endowment to invest in all technologies

$$\int_0^1 M(s) > \omega$$

- Acemoglu and Zilibotti (1997): model of endogenous incomplete markets.
- Extra assumptions:
  - $M(s)$ is weakly increasing (w.l.o.g)
  - Continuous
  - $M(s) = 0$ for all $s \in [0, \delta]$ for some $\delta > 0$

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Timeline

- $t = 0$:
  - agents ex-ante identical, have $\omega > 0$ for investing
  - investments made in storage (x) and long technologies $(y(s))$
- $t = 1$:
  - Aggregate shocks s (publicly observed) and θ_i (private) are realized
  - Agents report types $\hat{\theta}_i$ and receive consumption $c_1\left(\hat{\theta}_i, s\right)$
  - Invest remainder $= x - \int c_1(\theta, s)\, dF(\theta)$ to storage technology again
- $t = 2$
  - Agents consume $c_2\left(\hat{\theta}_i, s\right)$

Planners Problem

- **Planner Problem:** Choose optimal "consumption allocation" and "portfolio plan" to maximize consumers ex-ante expected utility
  - Consumption Allocation: Functions $c_1(\theta, s), c_2(\theta, s)$
  - Portfolio Allocation: Investment in short technology $x \geq 0$ and in long technologies $(y(s))_{s \in [0,1]}$

Planners Problem

- Feasibility:

$$\int c_1(\theta,s)\,dF(\theta) \le x \text{ for all } s \in [0,1] \tag{1}$$

and

$$\int c_2(\theta,s)\,dF(\theta) \le Ry(s) + \left(x - \int c_1(\theta,s)\,dF(\theta)\right) \text{ for all } s \in [0,1]$$

- Inada Condition: if $\exists \hat{\Theta} \subseteq \Theta$ with $\Pr\left(\hat{\Theta}\right) > 0$ such that

$$\frac{\partial U}{\partial c_1}(c_1, 0, \theta) = \infty \text{ for all } \theta \in \hat{\Theta}, c_1 \ge 0$$

then (1) is not binding, and

$$\int [c_1(\theta,s) + c_2(\theta,s)]\,dF(\theta) \le x + Ry(s) \text{ for all } s \in [0,1]$$

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$$W^* = \max_{c_1(\theta,s), c_2(\theta,s), x, y(s)} \int_0^1 ds \int U\left(c_1(\theta,s), c_2(\theta,s)\right) dF(\theta)$$

1. Inter-temporal RC: for all $s \in [0,1]$:

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2. IC constraints: for all $s \in [0,1]$ and all $\theta, \hat{\theta} \in \Theta$:

$$U(c_1(\theta,s), c_2(\theta,s), \theta) \geq U(c_1(\hat{\theta},s), c_2(\hat{\theta},s), \theta) \tag{3}$$

3. Minimum scale constraints:

$$x \geq 0 \text{ and } y(s) \geq M(s) \text{ whenever } y(s) > 0 \tag{4}$$

4. Portfolio Budget:

$$x + \int_0^1 y(s)\, ds \leq \omega \tag{5}$$

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Separate into two Programs

1. **Incentive program:** Given output $Y \geq 0$:

$$V(Y) := \max_{c_1(\theta), c_2(\theta)} \int U(c_1(\theta), c_2(\theta), \theta)\, dF(\theta)$$

$$\int [c_1(\theta) + c_2(\theta)]\, dF(\theta) \leq Y \qquad (6)$$

$$U(c_1(\theta, s), c_2(\theta, s), \theta) \geq U(c_1(\hat{\theta}, s), c_2(\hat{\theta}, s), \theta) \text{ for all } \theta, \hat{\theta} \in \Theta \qquad (7)$$

2. **Investment program:** Given $V(\cdot)$, choose investments:

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$$V(Y) := \max_{c_1(\theta), c_2(\theta) \in \mathscr{C}} \int U(c_1(\theta), c_2(\theta), \theta)\, dF(\theta)$$

subject to

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- What is in $\mathscr{C}$?

1. Hidden trades (Farhi, Golosov and Tsyvinski (2009))
2. Hidden Savings (Allen and Gale 2004)
3. Not in $\mathscr{C}$: incomplete contracts (i.e. not contingent on $Y \sim s$)

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- Goal: characterize the optimal investment profile.
- **Steps:**

1. Solve Investment Program (8) for general $V(\cdot)$, assuming $V(Y)$ to be strictly concave
2. Find conditions on $U(c_1, c_2, \theta)$ and $\mathscr{C}$ such that $V(Y)$ is in fact, strictly concave.

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Investment Program

Choose

- $x =$ storage from $t = 0$ to $t = 1$
- $y(s) =$ investment in technology A_s

to solve

$$W^* = \max_{x,(y(s))_{s\in[0,1]}} \int_0^1 V(x + Ry(s))\,ds$$

subject to

$$y(s) \geq M(s) \text{ for all } s : y(s) > 0 \quad (11)$$

$$x + \int_0^1 y(s)\,ds = \omega \quad (12)$$

Investment Program

- **Problem:** Non-convex feasible set
- Investment has two margins:
  - Extensive: which technologies to fund
  - Intensive: how much
- We prove a series of conditions on the shape of the optimal investment profile

Result 1: If $y^*(s) > M(s)$ and $y^*(s') > M(s') \Longrightarrow y^*(s) = y^*(s')$

Result 2: $\exists s^* \in (0,1)$ such that $y^*(s) = y^* := M(s^*)$ for all $s \leq s^*$

Result 3: $\exists \hat{s} \in (s^*,1)$ such that $y^*(s) = M(s)$ for all $s \in [s^*, \hat{s}]$

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Result 4: $y^*(s) = 0$ for all $s > \hat{s}$

Investment Program

- **Problem:** Non-convex feasible set
- Investment has two margins:
  - Extensive: which technologies to fund
  - Intensive: how much
- We prove a series of conditions on the shape of the optimal investment profile

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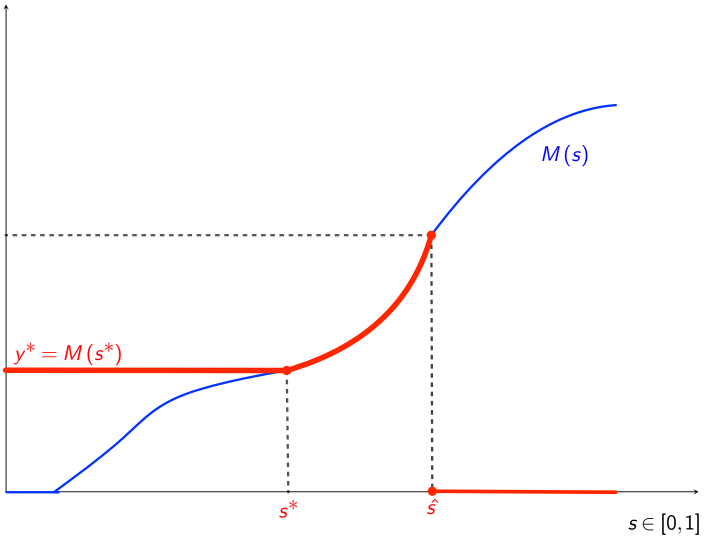

Figure: Optimal $y^*(s)$ schedule

Investment Program

- Find optimal cutoffs $s^*, \hat{s}$
- Storage investment is $x^* = \omega - s^* y^* - \int_{s^*}^{\hat{s}} M(s)\,ds$

Intensive/Extensive margin tradeoff:

- Intensive margin: Given a set of available assets $\Longrightarrow$ optimal to invest the same amount in all of themselves
- Extensive margin: Lower investment in low MS assets to increase asset span

Pecuniary externality:

- A single agent cannot cover the minimum scale of any given asset.
- In a decentralized economy with no trading constraints, each agent takes asset span given
- Since she does not affect the set of available assets $\Longrightarrow$ invests the same in all
- In the aggregate, equivalent to a portfolio

$$y(s) = \begin{cases} \tilde{y} & \text{for all } s \leq \tilde{s} \\ 0 & \text{for all } s > \tilde{s} \end{cases}$$

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Investment Program

- It generates 3 type of states for output:

1. Normal states $(s < s^*)$: there, intertemporal output is constant and equal to $Y = x^* + Ry^*$
2. "Crisis" states $(s > \hat{s})$: no long technology gives return, so $Y = x^*$
3. Boom states $(s^* < s < \hat{s})$: output is variable, $Y(s) = x^* + RM(s)$

- When compared to previous case, output is more volatile
- But strictly welfare improving.

Incentive Program

- Given output $Y \geq 0$:

$$V(Y) := \max_{c_1(\theta), c_2(\theta)} \int U(c_1(\theta), c_2(\theta), \theta) \, dF(\theta)$$

subject to

$$\int [c_1(\theta) + c_2(\theta)] \, dF(\theta) \leq Y$$

$$U(c_1(\theta, s), c_2(\theta, s), \theta) \geq U(c_1(\hat{\theta}, s), c_2(\hat{\theta}, s), \theta) \text{ for all } \theta, \hat{\theta} \in \Theta$$

- Gives optimum $(c_1(\theta, Y), c_2(\theta, Y))$. Optimal contract is

$$c^* = (c_1^*(\theta, s), c_2^*(\theta, s)) = (c_1(\theta, x + Ry(s)), c_2(\theta, x + Ry^*(s)))$$

Convexity of IC Contracts

$$IC_Y = \left\{ c = (c_1, c_2) \in \mathscr{C} : \begin{cases} U(c_1(\theta, s), c_2(\theta, s), \theta) \geq U(c_1(\hat{\theta}, s), c_2(\hat{\theta}, s), \theta) \\ \mathbb{E}_\theta \left[c_1(\theta) + c_2(\theta) \right] \leq Y \end{cases} \right.$$

where $\mathscr{C}$ is a potential set of extra constraints

- **Example:** access to secondary lending market (Golosov, Farhi and Tvinsky 2009) with price $q =$ of c_2/c_1
- Agents report type $\hat{\theta}$ to maximize

$$V\left(\hat{\theta}, \theta\right) = \max_{c_1, c_2} U\left(c_1, c_2, \theta\right)$$

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Convexity of IC Contracts

- **Result:** If IC_Y is convex and $U(\cdot, \theta)$ is strictly concave $\Longrightarrow V(Y)$ is strictly concave.
- **Problem:** In general IC_Y set is not convex, particularly if extra constraints added.

Theorem

If $U(c_1, c_2, \theta) = g_1(\theta)\, u_1(c_1) + g_2(\theta)\, u_2(c_2)$ and $\mathscr{C}$ has no extra constraints $\Longrightarrow IC_Y$ is convex

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Convexity of IC Contracts with Re-trading Constraints

- With re-trading constraints, we need to add more constraints
- An equivalent formulation is to directly choose the price q and Income I
- Let $v(q, I, \theta)$ be the indirect utility function for type θ, with demand functions $c_1(q, I, \theta), c_2(q, I, \theta)$
- Incentive problem is

$$V(Y) = \max_{q,I} \int v(q, I, \theta)\, dF(\theta)$$

s.t

$$\int [c_1(q, I, \theta) + c_2(q, I, \theta)]\, dF(\theta) \leq Y$$

$$c_1(q, I, \theta) + qc_2(q, I, \theta) = I \text{ for all } \theta$$

Convexity of IC Contracts with Re-trading Constraints

- The preferences are Gorman* if

$$v(q, I, \theta) = g(a(q) + b(q, \theta) I)$$

for some function $g(\cdot)$ strictly increasing and concave

- Preferences that satisfy this condition:
  - $U = \theta \ln(c_1) + (1-\theta)\ln(c_2)$
  - $U = \left(\alpha(\theta) c_1^{\theta} + \beta(\theta) c_2^{\theta}\right)^{\frac{1}{\theta}}$
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If $v(q, I, \theta) = g(a(q) + b(q, \theta) I)$ then $V(Y)$ is strictly concave as well.

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Decentralization

- We propose a decentralization with three distinct sectors:

1. Consumers: Buy (lotteries) over deposit contracts
2. Firms: They manage short and long productive technologies (free entry)
3. Broker-dealers (or financial intermediaries): They sell contracts, and invest directly in firms

- There is free entry in all sectors (anyone can run a firm or be a financial intermediary)
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  - Study first equilibria for a given set of contracts and financial intermediaries
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1) Consumers

- Let B be the (finite) set of Broker-Dealers (BD) active.
- A **contract** is $c = (c_1(\theta, s), c_2(\theta, s))_{(\theta,s)\in\Theta\times S}$ that is incentive-compatible
- For $b \in B$, let C_b be the (finite) set of contracts offered by b
- Contracts are ex-post exclusive: each consumer can only use one contract ex-post.
- **Competition:** BDs sell lotteries over contracts, at a price $P(b, c)$ per lottery unit
- Budget constraint for consumers is

$$\sum_{b\in B, \mathbf{c}\in C_b} P(b,c) \times \underbrace{q^d(b,c)}_{=\text{lot. units bought}} \leq \omega$$

where $q^d \in \Delta :=$ simplex over $\mathbb{R}^{\#\{(b,c):b\in B, c\in C_b\}}$ (we omit BD profits, which are 0 in eqm).

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1) Consumers

- The value of a contract for each consumer ex-ante is

$$V(b,c) := \mathbb{E}_{\theta,s}\{U(c_1(\theta,s), c_2(\theta,s), \theta)\}$$

- Consumer problem is then

$$\max_{q^d \in \Delta} \sum_{b \in B, c \in C_B} q^d(b,c) V(b,c) \text{ s.t. } \sum_{b \in B, \mathbf{c} \in C_b} P(b,c) q^d(b,c) \leq \omega$$

Implicit trading constraints needed for consumers:

(a): Contract exclusivity (only trade ex-post with one BD)

(b): Cannot trade ex-post with other consumers

(c): Cannot trade directly with firms

2) Productive Sector

- Technologies: All productive assets $Y = \{A_s\}_{s \in [0,1]} \cup S$, where $S =$ storage technology.
- Firm f has access to technology $Y_f = A_{\hat{s}}$ for some $\hat{s} \in [0,1]$ or $Y_f = S$
- Firms need to get financing to manage the asset.
- It offers to a potential financiers, a menu of payoffs $\rho(y,s)$ such that

1. $\rho(y,s) = 0$ for all s if $y < M(\hat{s})$
2. $0 \leq \rho(y,s) \leq r_{\hat{s}}(s)\, y$

Trading constraint:

(d): Firms cannot have more than one source of financing

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2) Productive Sector

- Because agents are atomistic, this implies that they can only ask one BD
- Firm profits are then $= r_{\hat{s}}(s)\,y - \rho(y,s)$
- Assumption: Free entry in productive sector
- This pushes profits to zero, so in equilibrium

$$\rho(y,s) = r_{\hat{s}}(s)\,y \times \mathbf{1}\{y \geq M(\hat{s})\}$$

- Question: Why not just consider them as Arrow-Debreu securities, and study classical GE?

3) Broker Dealers

- Each BD has an (exogenous, for now) set of available contracts C_b
- BD chooses:

1. Its supply of each contract lotteries, $q^s(b,c)$
2. Its investments in firms to fund the contracts expenditures.

- In period 1 it has to pay out

$$e_1(s) = \sum_{c \in C_b} q^s(b,c) \mathbb{E}_\theta [c_1(\theta, s)]$$

which can only be financed by an investment in a firm running short tech.

- In period 2, it has to pay out

$$e_2(s) = \sum_{c \in C_b} q^s(b,c) \mathbb{E}_\theta [c_2(\theta, s)]$$

which is financed with (a) storage and (b) long technologies

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3) Broker Dealers

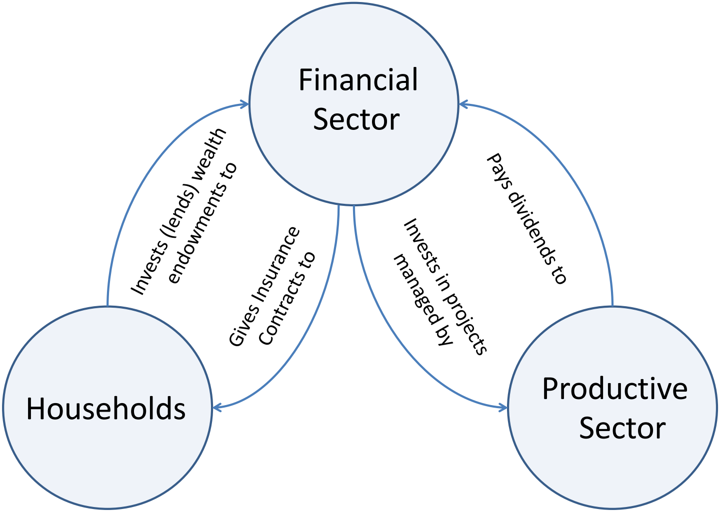
- Each BD has an (exogenous, for now) set of available contracts C_b

$$\pi_b = \max_{q^s, x_b, y_b(s)} \sum_{c \in C_b} P(b,c)\, q^s(b,c) - \left(\underbrace{x_b + \int_0^1 y_b(s)\, ds}_{\text{investments}} \right) \tag{13}$$

subject to

$$\begin{cases} e_1(s) \leq x & \text{for all } s \in [0,1] \\ e_2(s) \leq x - e_1(s) + \rho(y(s), s) & \text{for all } s \in [0,1] \end{cases}$$

- Free-entry into BS industry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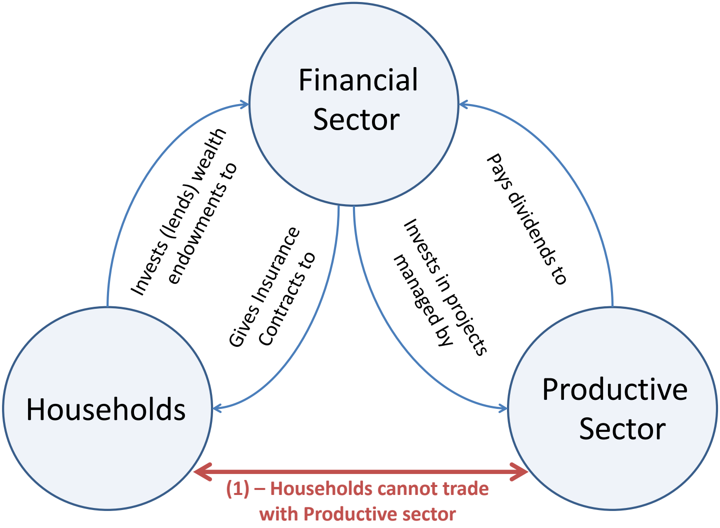
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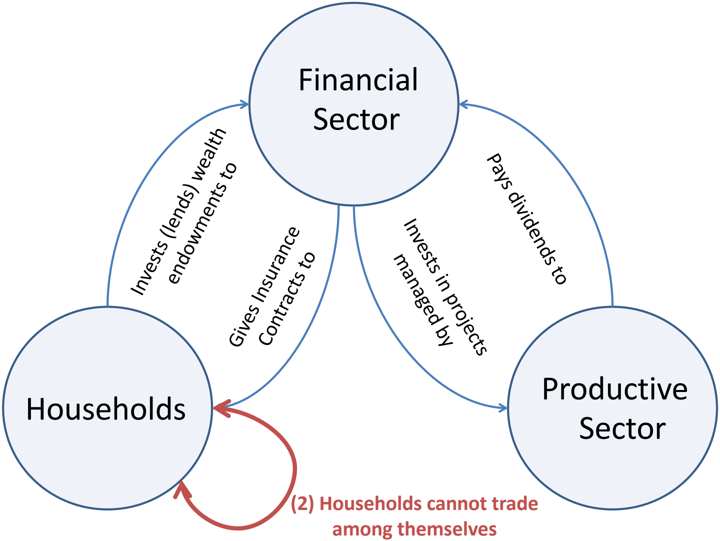
Financial Sector
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Productive Sector
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Pays dividends to
(2) Households cannot trade among themselves

Quasi-equilibrium

- Given B and $C = \{C_b\}_{b \in B}$, a quasi-equilibrium is a tuple $\mathbf{z}$ consisting of
  - Consumer demand q^d
  - BS supply q^s and investment decisions $\left(x_b, (y_b(s))_{s \in [0,1]}\right)$
  - Prices $P(b,s)$ and Payoffs $\rho(y,s)$, such that:

1. q^d solves consumer problem given $P(b,s)$
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4. Resource constraints:

$$\begin{cases} q^d(b,c) = q^s(b,c) & \forall b \in B, c \in C_b \\ \mathbb{E}_\theta[c_1(\theta,s)] \leq \sum_{b \in B} x_b & \forall s \in [0,1] \\ \mathbb{E}_\theta[c_1(\theta,s) + c_2(\theta,s)] \leq \sum_{b \in B} x_b + R \sum_{b \in B} y_b(s) & \forall s \in [0,1] \\ \sum_{b \in B} x_b + \sum_{b \in B} \int_0^1 y_b(s)\, ds = \omega & \end{cases}$$

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1. q^d solves consumer problem given $P(b,s)$
2. q^s and investment decisions $\left(x_b, (y_b(s))_{s\in[0,1]}\right)$ max π_b given prices $P(b,s)$ and $\rho(y,s)$
3. Free-entry: $\rho(y,s) = r_{\hat{s}}(s)\, y \times \mathbf{1}\{y \geq M(\hat{s})\}$
4. Resource constraints:

$$\begin{cases} q^d(b,c) = q^s(b,c) & \forall b \in B, c \in C_b \\ \mathbb{E}_\theta[c_1(\theta,s)] \leq \sum_{b\in B} x_b & \forall s \in [0,1] \\ \mathbb{E}_\theta[c_1(\theta,s) + c_2(\theta,s)] \leq \sum_{b\in B} x_b + R\sum_{b\in B} y_b(s) & \forall s \in [0,1] \\ \sum_{b\in B} x_b + \sum_{b\in B} \int_0^1 y_b(s)\, ds = \omega \end{cases}$$

Quasi-equilibrium

- Given B and $C = \{C_b\}_{b \in B}$, a quasi-equilibrium is a tuple $\mathbf{z}$ consisting of
  - Consumer demand q^d
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Quasi-equilibrium

- We write $\mathbf{z} \in \mathbf{Q}\left(B, \{C_b\}_{b \in B}\right)$ as the set of all quasi-equilibria.

Lemma

There always exist a Quasi-Equilibrium, where only one contract

$$\left(\hat{b}, \hat{c}\right) \in \underset{b \in B, c \in C_b}{argmax} \; V(b, c)$$

is traded in eqm (i.e. $q^d\left(\hat{b}, \hat{c}\right) = q^s\left(\hat{b}, \hat{c}\right) = 1$), and
$P\left(\hat{b}, \hat{c}\right) = x_{\hat{b}} + \int_0^1 y_{\hat{b}}(s)\, ds$

Proof.

Constructive, and straightforward

- So, in any quasi-equilibrium, only one best-ex ante contract is traded
- If two BD b_1, b_2 are selling the same contract $\hat{c}$, then there are two equilibria

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Full Competitive Equilibrium (Makowski 1980)

- So far, we took the technology of each BD as given (i.e. the contracts they propose)
- Equivalent to taking the commodity space as exogenous.
- What contracts should be chosen to be introduced by banks?

Full Competitive Equilibrium (Makowski 1980)

Definition (Profitable Deviation)

Take a set of BD B with contracts C_b, and let $\mathbf{z}$ be a quasi-equilibrium. We say $c' \notin \bigcup_{b \in B} \{C_b\}$ is a profitable deviation if

1. Is incentive compatible
2. $\exists \mathbf{z}' \in Q\left(B, \{C'_b\}\right)$ where $C'_b = C_b$ for all but $\hat{b}$, where $C'_{\hat{b}} = \{c'\} \cup C_{\hat{b}}$, such that

$$\pi_{\hat{b}}\left(\mathbf{z}'\right) > \pi_{\hat{b}}(\mathbf{z})$$

Definition (FCE)

A Full Competitive Equilibrium (**FCE**) is a family of contracts $\{C_b\}_{b \in B}$ and $\mathbf{z}$ such that (1) $\mathbf{z} \in \mathbf{Q}\left(B, \{C_b\}_{b \in B}\right)$ and (2) there are no profitable deviations

Weak FCE

- Problem: in all quasi-equilibria we have $\pi_b(\mathbf{z}) = 0$

Definition (Weak Profitable Deviation)

Take a set of BD B with contracts C_b, and let $\mathbf{z}$ be a quasi-equilibrium. We say $c' \notin \bigcup_{b \in B} \{C_b\}$ is a weak profitable deviation if

1. Is incentive compatible
2. $\exists \mathbf{z}' \in Q\left(B, \{C'_b\}\right)$ where $C'_b = C_b$ for all but $\hat{b}$, where $C'_{\hat{b}} = \{c'\} \cup C_{\hat{b}}$, such that
    1. $\pi_{\hat{b}}(\mathbf{z}') \geq \pi_{\hat{b}}(\mathbf{z})$
    2. $\sum_{c \in C_{\hat{b}}} q^s\left(\hat{b}, c'\right) > \sum_{c \in C'_{\hat{b}}} q^s\left(\hat{b}, c'\right)$

Definition (Weak FCE)

A Weak **FCE** is a family of contracts $\{C_b\}_{b \in B}$ and $\mathbf{z}$ such that (1) $\mathbf{z} \in \mathbf{Q}\left(B, \{C_b\}_{b \in B}\right)$ and (2) there are no weak profitable deviations

Weak FCE Implementation

Lemma

Take $\{C_b\}_{b\in B}$ *and* $\mathbf{z} \in \mathbf{Q}\left(B, \{C_b\}_{b\in B}\right)$. *If* $c^* \notin \bigcup_{b\in B} C_b \Longrightarrow$ $\left(\{C_b\}_{b\in B}, \mathbf{z}\right)$ *is not a weak FCE*

- Proof is trivial: Based on Lemma 3 we know that in any $\mathbf{z}$, the best contract for consumers corners all the market
- c^* is the best feasible contract, so any BD selling it may get all market in an equilibrium.

Theorem (Full Implementation)

In any weak FCE, the first best contract c^* *(with corresponding optimal investment allocation) is implemented.*

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Failures of Implementation

Failure 1: Hidden trades, Hidden savings

- This reduces the set of incentive compatible mechanisms, and therefore changes c^*
- However, if $V(Y)$ is still st. concave $\Longrightarrow$ Shape of optimal portfolio still the same
- Market design still implements the second best allocation

Failures of Implementation

Failure 2: Consumers may directly invest in firms

- Firms, once they cover their minimum scale, can now ask for more external financing
- In this case, consumers would benefit from directly investing in already "open" firms
- ... but they would invest the same amount in all of them.
- No agent would invest in BDs: they would wait for BD to cover MS
- In ex-ante problem, this translates into a reservation utility for each agent
- In constrained optimal mechanism, a BD would give the same utility as direct financing:

$$y^*(s) = \begin{cases} y^* = M(s^*) & \text{for all } s \leq s^* \\ 0 & \text{for all } s > s^* \end{cases}$$

- Same reason behind Acemoglu and Zilibotti (1997)

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Failures of Implementation

Failure 3: Contracts are incomplete

- What if state s is not (perfectly) contractible? (Allen and Gale (2004), etc)
- In this case, separation between incentive and investment programs is not possible.
- Typically, investment in storage would be larger
- However, non-smooth investment in long technologies would still be optimal (conjecture!)

Failure 4: Please tell me us!