Robust Predictions
in
Games with Incomplete Information

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- Game Theoretic Predictions are very sensitive to "higher order beliefs" or (equivalently) information structure
- Higher order beliefs are rarely observed
- What predictions can we make and analysis can we do if we do not observe higher order beliefs?

Robust Predictions Agenda

- Fix "payoff relevant environment"
  - $=$ action sets, payoff-relevant variables ("states"), payoff functions, distribution over states
  - $=$ incomplete information game without higher order beliefs about states
- Assume payoff relevant environment is observed by the econometrician
- Analyze what could happen for all possible higher order beliefs (maintaining common prior assumption and equilibrium assumptions)
- Make set valued predictions about joint distribution of actions and states

Partial Identification

Having identified mapping from "payoff relevant environment" to action-state distributions, we can analyze its inverse:

- Given knowledge of the action-state distribution distribution, or some moments of it, what can be deduced about the payoff relevant environment?

Robust Comparative Statics / Policy Analysis

- What can say about the impact of changes in parameters of the payoff relevant environment (for example, policy choices) for the set of possible outcomes?

Partial Information about Information Structure

- Perhaps you don't observe all higher order beliefs but you are sure of some aspects of the information structure:
  - You are sure that bidders know their private values of an object in an auction, but you have no idea what their beliefs and higher order beliefs about others' private values are...
  - You are sure that oligopolists know their own costs, but you have no idea what beliefs and higher order beliefs about demand and others' private values.
- What can you say then?

This Talk

1. General Approach
2. Illustration with Continuum Player, Symmetric, Linear Best Response, Normal Distribution Games

Results Preview

1. General Approach
   - Set valued prediction is set of "Bayes Correlated Equilibria"
   - Partial information monotonically reduces the set of "Bayes Correlated Equilibria"
2. Illustration with Continuum Player, Symmetric, Linear Best Response, Normal Distribution Games
   - These sets are tractable and intuitive
   - Cannot distinguish strategic substitutes and complements

- players $i = 1, ..., I$
- (payoff relevant) states Θ

Payoff Relevant Environment

- actions $(A_i)_{i=1}^{I}$
- utility functions $(u_i)_{i=1}^{I}$, each $u_i : A \times \Theta \to \mathbb{R}$
- state distribution $\psi \in \Delta(\Theta)$
- $G = \left((A_i, u_i)_{i=1}^{I}, \psi\right)$
- ("basic game", "pre-game")

- signals (types) $(T_i)_{i=1}^{I}$
- signal distribution $\pi : \Theta \rightarrow \Delta(T_1 \times T_2 \times ... \times T_I)$
- $S = \left((T_i)_{i=1}^{I}, \pi\right)$
- ("higher order beliefs", "type space," "signal space")

- The pair (G, S) is a standard game of incomplete information
- A (behavioral) strategy for player i is a mapping $b_i : T_i \to \Delta(A_i)$

DEFINITION. A strategy profile b is a Bayes Nash Equilibrium (BNE) of (G, S) if, for all i, t_i and a_i with $b_i(a_i|t_i) > 0$,

$$\sum_{a_{-i}\in A_{-i}, t_{-i}\in T_{-i}, \theta\in\Theta} \left(\prod_{j\neq i} b_j(a_j|t_j)\right) u_i((a_i, a_{-i}), \theta)\, \psi(\theta)\, \pi(t|\theta)$$

$$\geq \sum_{a_{-i}\in A_{-i}, t_{-i}\in T_{-i}, \theta\in\Theta} \left(\prod_{j\neq i} b_j(a_j|t_j)\right) u_i\left(\left(a_i', a_{-i}\right), \theta\right) \psi(\theta)\, \pi(t|\theta)$$

for all $a_i' \in A_i$.

DEFINITION. An action state distribution $\mu \in \Delta(A \times \Theta)$ is a BNE action state distribution of (G, S) if there exists a BNE strategy profile b such that

$$\mu(a, \theta) = \sum_{t \in T} \psi(\theta) \pi(t|\theta) \left(\prod_{i=1}^{I} b_i(a_i|t_i) \right).$$

Bayes Correlated Equilibrium (with Null Information)

DEFINITION. An action state distribution $\mu \in \Delta(A \times \Theta)$ is a Bayes Correlated Equilibrium (BCE) of G if is *obedient*, i.e., for each i, a_i and a_i',

$$\sum_{\theta \in \Theta} u_i\left(\left(a_i, a_{-i}\right), \theta\right) \mu\left(\left(a_i, a_{-i}\right), \theta\right) \geq \sum_{\theta \in \Theta} u_i\left(\left(a_i', a_{-i}\right), \theta\right) \mu\left(\left(a_i, a_{-i}\right), \theta\right)$$

and *consistent*, i.e., for each θ

$$\sum_{a \in A} \mu(a, \theta) = \psi(\theta).$$

PROPOSITION 1. Action state distribution μ is a BNE action state distribution of (G, S) for some S if and only if it is a BCE of G.

c.f. Aumann 1987, Forges 1993

- We know players observe S but we dont know what additional information they observe.
- Augmented information system $\widetilde{S} = \left((Z_i)_{i=1}^{I}, \phi \right)$, where $\phi : \Theta \times T \to \Delta(Z)$
- Augmented information information game $\left(G, S, \widetilde{S} \right)$
- Player i's behavioral strategy $\tau_i : T_i \times Z_i \to \Delta(A_i)$

DEFINITION. A strategy profile τ is a Bayes Nash Equilibrium (BNE) of (G, S, S') if, for all i, t_i, z_i and a_i with $b_i(a_i|t_i, z_i) > 0$,

$$\sum_{a_{-i}, t_{-i}, z_{-i}, \theta} \left(\prod_{j \neq i} b_j(a_j|t_j, z_j) \right) u_i((a_i, a_{-i}), \theta) \psi(\theta) \pi(t|\theta) \phi(z|$$

$$\geq \sum_{a_{-i}, t_{-i}, z_{-i}, \theta} \left(\prod_{j \neq i} b_j(a_j|t_j, z_j) \right) u_i\left((a_i', a_{-i}), \theta\right) \psi(\theta) \pi(t|\theta) \phi(z$$

for all $a_i' \in A_i$.

BNE Action Type State Distributions

DEFINITION. An action type state distribution $\nu \in \Delta(A \times T \times \Theta)$ is a BNE action type state distribution of (G, S, S') if there exists a BNE strategy profile τ such that

$$\mu(a, t, \theta) = \psi(\theta)\, \pi(t|\theta) \sum_{z \in Z} \left(\prod_{i=1}^{I} b_i(a_i|t_i, z_i) \right) \phi(z|t, \theta).$$

DEFINITION. An action type state distribution $\nu \in \Delta(A \times T \times \Theta)$ is a Bayes Correlated Equilibrium (BCE) of (G, S) it is *obedient*, i.e., for each i, t_i, a_i and a_i',

$$\sum_{a_{-i} \in A_{-i}, t_{-i} \in T_{-i}, \theta \in \Theta} u_i\left(\left(a_i, a_{-i}\right), \theta\right) \nu\left(\left(a_i, a_{-i}\right), \left(t_i, t_{-i}\right), \theta\right)$$

$$\geq \sum_{a_{-i} \in A_{-i}, t_{-i} \in T_{-i}, \theta \in \Theta} u_i\left(\left(a_i', a_{-i}\right), \theta\right) \nu\left(\left(a_i, a_{-i}\right), \left(t_i, t_{-i}\right), \theta\right)$$

and *consistent*, i.e.,

$$\sum_{a \in A} \nu\left(a, t, \theta\right) = \psi\left(\theta\right) \pi\left(t | \theta\right).$$

- If S is null information system, reduces to earlier definition.

PROPOSITION 2. Action type state distribution ν is a BNE action type state distribution of (G, S, S') for some S' if and only if it is a BCE of (G, S).

c.f. Forges 1993

- If S is null information system, reduces to Proposition 1.

Legitimate Definitions

- Forges (1993): "Five Legitimate Definitions of Correlated Equilibrium in Games with Incomplete Information"; Forges (2006) gives $\#6$
- This definition is "illegitimate" because it fails "join feasibility"

DEFINITION. Action type state distribution ν is join feasible for (G, S) if there exists $f : T \rightarrow \Delta(A)$ such that

$$\nu(a, t, \theta) = \psi(\theta) \pi(t|\theta) f(a|t)$$

for each a, t, θ.

- BCE fails join feasibility, Forges' weakest definition (Bayesian solution) is BCE satisfying join feasibility

Trivial One Player Example

- $I = 1$
- $\Theta = \{\theta, \theta'\}$
- $\psi(\theta) = \psi(\theta') = \frac{1}{2}$
- Payoffs u_1

	θ	θ'
a_1	2	-1
a_1'	0	0

- unique Bayesian solution: $\mu(a_1, \theta) = \mu(a_1, \theta') = \frac{1}{2}$
- a BCE: $\mu(a_1, \theta) = \mu(a_1', \theta') = \frac{1}{2}$

PROPOSITION. (Informal Statement). If information system S' is less informed than S, then S' has a larger set of BCE action state distributions.

Compare

- Gossner 00, Lehrer, Rosenberg and Schmaya 06, 10;
- in Gossner, "more" information lead to more equilibria

Payoff Environment: Quadratic Payoffs

- utility of each agent i is given by quadratic payoff function:
- determined by individual action $a_i \in \mathbb{R}$, state of the world $\theta \in \mathbb{R}$, and average action $A \in \mathbb{R}$:

$$A = \int_0^1 a_i di$$

and thus:

$$u_i(a_i, A, \theta) = (a_i, A, \theta) \begin{pmatrix} \gamma_{aa} & \gamma_{aA} & \gamma_{a\theta} \\ \gamma_{aA} & \gamma_{AA} & \gamma_{A\theta} \\ \gamma_{a\theta} & \gamma_{A\theta} & \gamma_{\theta\theta} \end{pmatrix} (a_i, A, \theta)^T$$

- game is completely described by interaction matrix $\Gamma = \{\gamma_{ij}\}$

- the state of the world θ is normally distributed

$$\theta \sim N\left(\mu_\theta, \sigma_\theta^2\right)$$

  with mean $\mu_\theta \in \mathbb{R}$ and variance $\sigma_\theta^2 \in \mathbb{R}_+$
- the distribution of the state of the world is commonly known common prior

Interaction Matrix

- given the interaction matrix Γ, complete information game is a potential game (Monderer and Shapley (1996)):

$$\Gamma = \begin{pmatrix} \gamma_a & \gamma_{aA} & \gamma_{a\theta} \\ \gamma_{aA} & \gamma_A & \gamma_{A\theta} \\ \gamma_{a\theta} & \gamma_{A\theta} & \gamma_\theta \end{pmatrix}$$

- diagonal entries: $\gamma_a, \gamma_A, \gamma_\theta$ describe "own effects"
- off-diagonal entries: $\gamma_{a\theta}, \gamma_{A\theta}, \gamma_{aA}$ "interaction effects"
- fundamentals matter, "return shocks":

$$\gamma_{a\theta} \neq 0;$$

- strategic complements and strategic substitutes:

$$\gamma_{aA} > 0 \quad \text{vs.} \quad \gamma_{aA} < 0$$

- concavity at the individual level (well-defined best response):

$$\gamma_a < 0$$

- concavity at the aggregate level (existence of an interior equilibrium)

$$\gamma_a + \gamma_{aA} < 0$$

- concave payoffs imply that the complete information game has unique Nash **and** unique correlated equilibrium (Neyman (1997))

- continuum of agents: $i \in [0, 1]$
- action (= message): $a \in \mathbb{R}$
- state of the world: $\theta \in \mathbb{R}$
- payoff function

$$u_i = -(1-r)(a_i - \theta)^2 - r(a_i - A)^2$$

with $r \in (0, 1)$

- see Morris and Shin (2002), Angeletos and Pavan (2007)

Example 2: Competitive Market

- action (= quantity): $a_i \in \mathbb{R}$
- cost of production $c(a_i) = \frac{1}{2}\gamma_a (a_i)^2$
- state of the world (= demand intercept): $\theta \in \mathbb{R}$
- inverse demand (= price):

$$p(A) = \gamma_{a\theta}\theta - \gamma_{aA}A$$

where A is average supply:

$$A = \int_0^1 a_i di$$

- see Guesnerie (1992) and Vives (2008)

Fix an information system

1. every agent i observes a public signal y about θ :

$$y \sim N\left(\theta, \sigma_y^2\right)$$

2. every agent i observes a private signal x_i about θ :

$$x_i \sim N\left(\theta, \sigma_x^2\right)$$

- the best response of each agent is:

$$a = -\frac{1}{\gamma_a}\left(\gamma_{a\theta}\mathbb{E}\left[\theta\left|x,y\right.\right] + \gamma_{Aa}\mathbb{E}\left[A\left|x,y\right.\right]\right)$$

- suppose the equilibrium strategy is given by a linear function:

$$a\left(x,y\right) = \alpha_0 + \alpha_x x + \alpha_y y,$$

- denote the sum of the precisions: $\sigma^{-2} = \sigma_\theta^{-2} + \sigma_x^{-2} + \sigma_y^{-2}$

Theorem

The unique Bayesian Nash equilibrium (given the bivariate information structure) is a linear equilibrium, $\alpha_0^* + \alpha_x^* x + \alpha_y^* y$, with

$$\alpha_x^* = -\frac{\gamma_{a\theta}\sigma_x^{-2}}{\gamma_{Aa}\sigma_x^{-2} + \gamma_a \sigma^{-2}},$$

and

$$\alpha_y^* = -\frac{\gamma_a}{\gamma_a + \gamma_{aA}} \frac{\gamma_{a\theta}\sigma_y^{-2}}{\gamma_{Aa}\sigma_x^{-2} + \gamma_a \sigma^{-2}}.$$

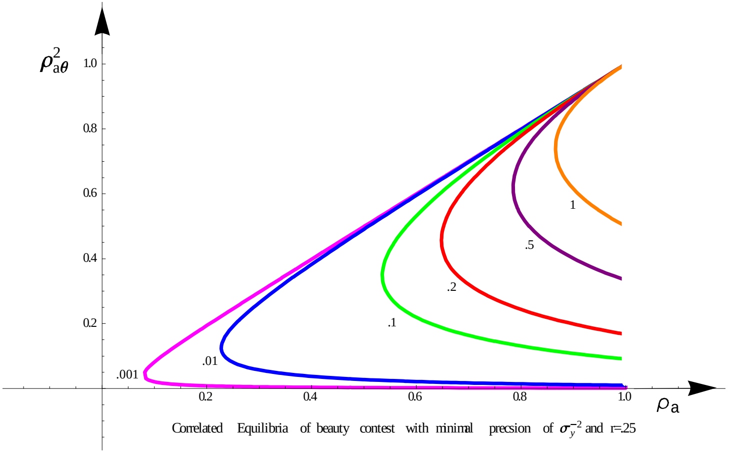
There is an implied joint distribution of (a, A, θ)

Standard Approach

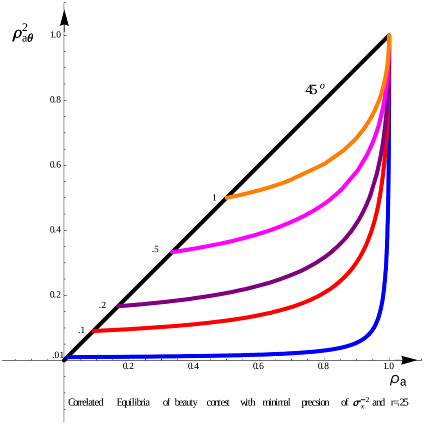
- There is an implied joint distribution of (a, A, θ)

$$\begin{pmatrix} a_i \\ A \\ \theta \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_a \\ \mu_A \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \rho_{aA}\sigma_a\sigma_A & \rho_{a\theta}\sigma_a\sigma_\theta \\ \rho_{aA}\sigma_a\sigma_A & \sigma_A^2 & \rho_{A\theta}\sigma_A\sigma_\theta \\ \rho_{a\theta}\sigma_a\sigma_\theta & \rho_{A\theta}\sigma_A\sigma_\theta & \sigma_\theta^2 \end{pmatrix} \right)$$

- movements along level curve are variations in σ_x^{-2} given σ_y^{-2}



- movements along level curve are variations in σ_y^{-2} given σ_x^{-2}



- the object of analysis: joint distribution over actions and states:
$$\mu(a, A, \theta)$$
- characterize the set of (normally distributed) BCE:

$$\begin{pmatrix} a_i \\ A \\ \theta \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_a \\ \mu_A \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \rho_{aA}\sigma_a\sigma_A & \rho_{a\theta}\sigma_a\sigma_\theta \\ \rho_{aA}\sigma_a\sigma_A & \sigma_A^2 & \rho_{A\theta}\sigma_A\sigma_\theta \\ \rho_{a\theta}\sigma_a\sigma_\theta & \rho_{A\theta}\sigma_A\sigma_\theta & \sigma_\theta^2 \end{pmatrix}\right)$$

- σ_A^2 is the aggregate volatility (common variation)
- $\sigma_a^2 - \sigma_A^2$ is the cross-section dispersion (idiosyncratic variation)
- statistical representation of equilibrium in terms of first and second order moments

Symmetric Bayes Correlated Equilibria

- with focus on symmetric equilibria:

$$\mu_A = \mu_a, \qquad \sigma_A^2 = \rho_a \sigma_a^2, \qquad \rho_{aA}\sigma_a\sigma_A = \rho_a\sigma_a^2$$

  where ρ_a is the correlation coefficient across individual actions

- the first and second moments of the correlated equilibria are:

$$\begin{pmatrix} a_i \\ A \\ \theta \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_a \\ \mu_a \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \rho_a\sigma_a^2 & \rho_{a\theta}\sigma_a\sigma_\theta \\ \rho_a\sigma_a^2 & \rho_a\sigma_a^2 & \rho_{a\theta}\sigma_a\sigma_\theta \\ \rho_{a\theta}\sigma_a\sigma_\theta & \rho_{a\theta}\sigma_a\sigma_\theta & \sigma_\theta^2 \end{pmatrix} \right)$$

- correlated equilibria are characterized by:

$$\{\mu_a, \ \sigma_a, \ \rho_a, \ \rho_{a\theta}\}$$

- in the complete information game, the best response is:

$$a = -\theta \frac{\gamma_{a\theta}}{\gamma_a} - A \frac{\gamma_{Aa}}{\gamma_a}$$

- best response is weighted linear combination of fundamental θ and average action A relative to the cost of action: $\gamma_{a\theta}/\gamma_a$, γ_{Aa}/γ_a
- in the incomplete information game, θ and A are uncertain:

$$\mathbb{E}[\theta], \quad \mathbb{E}[A]$$

- given the correlated equilibrium distribution $\mu(a, \theta)$ we can use the conditional expectations:

$$\mathbb{E}_\mu[\theta \,|a], \quad \mathbb{E}_\mu[A \,|a]$$

- in the incomplete information game, the best response is:

$$a = -\mathbb{E}_{\mu}\left[\theta \,|a\right] \frac{\gamma_{a\theta}}{\gamma_{a}} - \mathbb{E}_{\mu}\left[A \,|a\right] \frac{\gamma_{Aa}}{\gamma_{a}}$$

- best response property has to hold for all $a \in \operatorname{supp} \mu\left(a, \theta\right)$
- a fortiori, the best response property has to hold in expectations over all a :

$$\mathbb{E}_{\mu}\left[a\right] = \mathbb{E}_{\mu}\left[-\left(\mathbb{E}_{\mu}\left[\theta \,|a\right] \frac{\gamma_{a\theta}}{\gamma_{a}} + \mathbb{E}_{\mu}\left[A \,|a\right] \frac{\gamma_{Aa}}{\gamma_{a}}\right)\right]$$

- by the law of iterated expectation, or law of total expectation:

$$\mathbb{E}_{\mu}\left[\mathbb{E}_{\mu}\left[\theta \,|a\right]\right] = \mu_{\theta}, \quad \mathbb{E}_{\mu}\left[\mathbb{E}_{\mu}\left[A \,|a\right]\right] = \mathbb{E}_{\mu}\left[A\right] = \mathbb{E}_{\mu}\left[a\right],$$

- the best response property implies that for all $\mu(a,\theta)$:

$$\mathbb{E}_{\mu}[a] = \mathbb{E}_{\mu}\left[-\left(\mathbb{E}_{\mu}[\theta\,|a]\,\frac{\gamma_{a\theta}}{\gamma_a} + \mathbb{E}_{\mu}[A\,|a]\,\frac{\gamma_{Aa}}{\gamma_a}\right)\right]$$

or by the law of iterated expectation:

$$\mu_a = -\mu_\theta \frac{\gamma_{a\theta}}{\gamma_a} - \mu_a \frac{\gamma_{Aa}}{\gamma_a}$$

Theorem (First Moment)

In all Bayes correlated equilibria, the mean action is given by:

$$\mathbb{E}[a] = -\mu_\theta \frac{\gamma_{a\theta}}{\gamma_a + \gamma_{aA}}.$$

- result about "mean action" is independent of symmetry or normal distribution

- in any correlated equilibrium $\mu(a,\theta)$, best response demands

$$a = -\left(\mathbb{E}[\theta \,|a] \frac{\gamma_{a\theta}}{\gamma_a} + \mathbb{E}[A\,|a] \frac{\gamma_{Aa}}{\gamma_a}\right), \qquad \forall a \in \operatorname{supp} \mu(a,\theta)$$

- or varying in a

$$1 = -\left(\frac{\partial \mathbb{E}[\theta \,|a]}{\partial a} \frac{\gamma_{a\theta}}{\gamma_a} + \frac{\partial \mathbb{E}[A\,|a]}{\partial a} \frac{\gamma_{Aa}}{\gamma_a}\right),$$

- the change in the conditional expectation

$$\frac{\partial \mathbb{E}[\theta \,|a]}{\partial a}, \ \frac{\partial \mathbb{E}[A\,|a]}{\partial a}$$

is a statement about the correlation between a, A, θ

- the best response condition **and** the condition that $\Sigma_{a,A,\theta}$ forms a multivariate distribution, meaning that the variance-covariance matrix has to be positive definite
- we need to determine:

$$\{\sigma_a,\ \rho_a,\ \rho_{a\theta}\}$$

Theorem (Second Moment)

The triple $(\sigma_a, \rho_a, \rho_{a\theta})$ forms a Bayes correlated equilibrium iff:

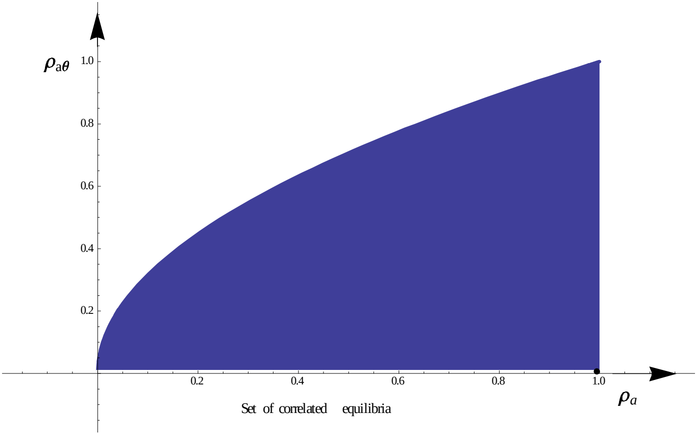
$$\rho_a - \rho_{a\theta}^2 \geq 0,$$

and

$$\sigma_a = -\frac{\sigma_\theta \gamma_{a\theta} \rho_{a\theta}}{\rho_a \gamma_{Aa} + \gamma_a}.$$

Moment Restrictions: Correlation Coefficients

- the equilibrium set is characterized by inequality $\rho_a - \rho_{a\theta}^2 \geq 0$
- ρ_a: correlation of actions across agents; $\rho_{a\theta}$: correlation of actions and fundamental



Equivalence between BCE and BNE

- bivariate information structure which generates volatility (common signal) and dispersion (idiosyncratic signal)

Theorem

There is BCE with $(\rho_a, \rho_{a\theta})$ if and only if there is a BNE with (σ_x^2, σ_y^2).

- a public and a private signal are sufficient to generate the entire set of correlated equilibria...
- but a given BCE does not uniquely identify the information environment of a BNE

- for noise terms $\sigma = (\sigma_x, \sigma_y)$, write $C(\sigma_x, \sigma_y) \in [0,1]^2$ for possible values of $(\rho_a, \rho_{a\theta})$

1. For all $\sigma < \sigma'$, $C(\sigma) \subset C(\sigma')$;
2. For all $\sigma \ll \sigma'$:

$$\min_{\rho_a \in C(\sigma)} \rho_a > \min_{\rho_a \in C(\sigma')} \rho_a;$$

Proposition

For all $\sigma \ll \sigma'$:

$$\min_{\rho_{a\theta} \in C(\sigma)} \rho_{a\theta} > \min_{\rho_{a\theta} \in C(\sigma')} \rho_{a\theta}.$$

graph

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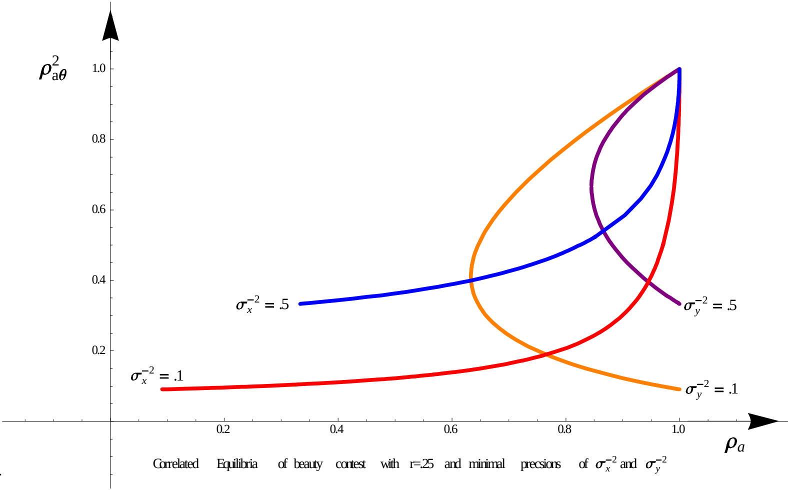

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Figure: Set of BCE with given public and private information

- suppose only the actions are observable:

$$(\mu_a, \sigma_a, \rho_a)$$

but the realization of the state is unobservable, and hence we do not have access to covariate information between a and θ :

- the identification then uses the mean:

$$\mu_\alpha = -\mu_\theta \frac{\gamma_{a\theta}}{\gamma_a + \gamma_{aA}}$$

and variance

$$\sigma_a = -\frac{\sigma_\theta \gamma_{a\theta} \rho_{a\theta}}{\rho_a \gamma_{Aa} + \gamma_a}.$$

Sign Identification

$$\frac{\mu_a}{\mu_\theta} = \frac{\gamma_{a\theta}}{\gamma_a + \gamma_{aA}}$$

$$\frac{\sigma_a}{\sigma_\theta} = -\frac{\gamma_{a\theta}\rho_{a\theta}}{\rho_a\gamma_{Aa} + \gamma_a}$$

- can we identify sign of interaction?
- recall: $\gamma_{a\theta}$ informational externality, γ_{aA} strategic externality

Theorem (Sign Identification)

The Bayes Nash Equilibrium identifies the sign of $\gamma_{a\theta}$ *and* γ_{aA}.

- identification in Bayes Nash equilibrium uses variance-covariance given information structure $\left(\sigma_x^2, \sigma_y^2\right)$

Theorem (Partial Sign Identification)

The Bayes Correlated Equilibrium identifies the sign of $\gamma_{a\theta}$ *but it does not identify the sign of* γ_{aA}.

- failure to identify the strategic nature of the game, strategic complements or strategic substitutes

- Now suppose $\rho_{a\theta}$ is observable
- But now allow a to have unknown intercept μ_a instead of 0
- now we have

$$\mu_\alpha = \mu_0 - \mu_\theta \frac{\gamma_{a\theta}}{\gamma_a + \gamma_{aA}}$$

  and variance

$$\sigma_a = -\frac{\sigma_\theta \gamma_{a\theta} \rho_{a\theta}}{\rho_a \gamma_{Aa} + \gamma_a}.$$

- can identify sign of $\gamma_{a\theta}$ but not sign of γ_{aA}

Classical Supply and Demand Identification

- If the information structure is known, slope of supply and demand can be identified
- uncertainty about information structure gives bounds on slopes