Screening with Persuasion

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Introduction

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- the seller can choose....

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  2. the selling mechanism
- classic screening problem [Mussa-Rosen (1982)] combined with Bayesian persuasion / information design

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2. in fact, the seller will choose a single-item menu (with or without exclusion) under weak conditions

intuition: efficiency versus information rents

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  - seller supresses socially valuable information to reduce information rents at the cost of efficiency

digital economy motivation

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- implementation of information structure by (explicit) recommendation systems or (implicit) by presentation of options

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    - relate to "one majorization constraint" related literature, e.g., Loertscher and Muir (JPE22), Myerson (MOR81), Bergemann et al. (AERi22); also Kolotolin and Wolitsky (2020wp) and Akbarpour, Dworczak and Kominers (2022wp)

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  - preview importance of interaction of screening and persuasion

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- buyers' values v have cdf F on $[\underline{v}, \overline{v}]$
- buyers have quasi-linear utility; willingness to pay for quality q of buyer with "value" v is

$$v \cdot q$$

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- now $G^{-1} : [0,1] \to [\underline{v}, \overline{v}]$ and $G^{-1}(t)$ is the expected value of the tth quantile buyer
- useful fact: $F^{-1} \succ G^{-1}$ if and only if $G \succ F$ (Shaked and Shanthikumar (2007))

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  3. *feasibility*: the expected qualities sold must be consistent with available supply Q

3. revenue as a function of allocation

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- payoff equivalence fails with discrete support, but formula still follows from optimality

4. feasibility

key change of variables

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- Kleiner et al (2021) say that Q^{-1} weakly majorizes R^{-1} (or $Q^{-1} \succ_w R^{-1}$)

maximization with two majorization constraints

- re-writing revenue with this change of variables (and integration by parts), we have

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- we think this representation of the problem is pretty cool.....

integration by parts and change of variable algebra

$$\int_{\underline{v}}^{\overline{v}} \left(\overbrace{wq(w)}^{\text{surplus}} - \overbrace{\int_{\underline{v}}^{w} q(t)\, dt}^{\text{information rent}} \right) dG(w)$$

$$= \int_{\underline{v}}^{\overline{v}} \left(w - \frac{1 - G(w)}{g(w)} \right) dG(w)\text{, by IP}$$

$$= \int_{0}^{1} \left(G^{-1}(t) - (1-t) \frac{dG^{-1}(t)}{dt} \right) R^{-1}(t)\text{, by CV } t = G(w)$$

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- how to sell a fixed distribution of qualities optimally...
- ironing solution (in continuum case): under irregular distribution, alternating pooled intervals and full separation regions

fixed information: many player re-interpretation

ex ante symmetric buyers of a single (fixed quality) good

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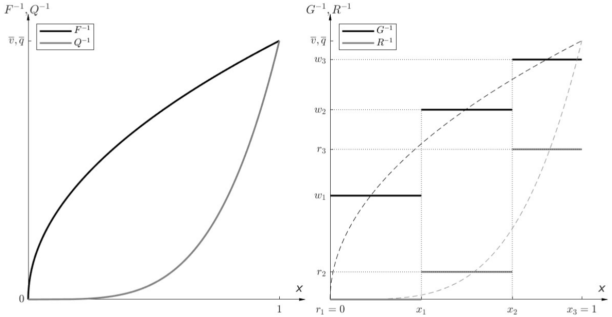
  - this is closest paper to us (similarities and differences outlined in paper)

Results

example

- value distribution $F(v) = t^2$ and $Q(q) = q^{\frac{1}{4}}$



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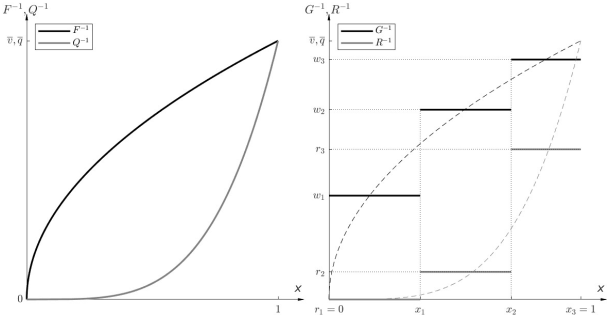
- quantile distributions $F^{-1}(t) = t^{1/2}$ and $Q^{-1}(t) = t^4$ (left panel)



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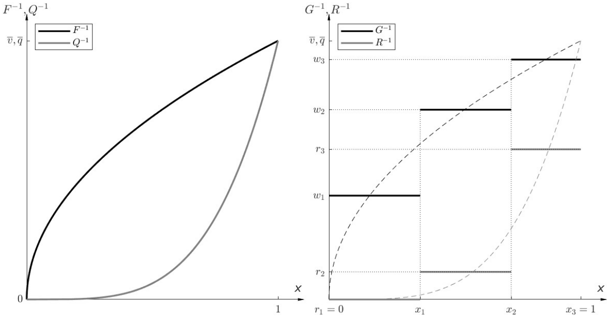
- optimal quantile distributions G^{-1} and R^{-1}.....



main result: the structure of the optimal mechanism

Theorem

The optimal G and R are finite monotone partitional with common support.

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- monotone partitional G is *finite* if the partition is a finite collection of sets (only intervals, always pooling)
- *common support*: same partition of quantiles

proof in three steps

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3. optimal G and R consist of finite intervals only

proof in three steps

1. optimal G and R are monotone partitional with common support (but possibly with full revelation over some intervals and thus infinite items)
    - like Myerson and other "one majorization constraint" arguments
2. optimal G and R consist of intervals only (no full separation)
    - key novelty
3. optimal G and R consist of finite intervals only
    - boring

step 1: "standard"

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- similarly, fixing G^{-1}....

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- idea of proof:
  - pooling allocation over a small interval leads to a third-order decrease in revenue
  - pooling information over that small interval leads to a second-order increase in revenue (via a decrease in information rents)

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- the decrease in total surplus is of order

$$\overbrace{(v_2 - v_1)}^{\text{change in value}} \times \overbrace{(q^*(v_2) - q^*(v_1))}^{\text{change in quality}} \times \overbrace{(F(v_2) - F(v_1))}^{\text{probability of } v \in [v_1, v_2]}$$

or

$$\Delta^3$$

2b: pooling information, decrease in information rents / increase in revenue

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$$\overbrace{(\mu_v - v_1)}^{\text{increase in payment}} \times \overbrace{\left(\mu_q - q^*(v_1)\right)}^{\text{quality increase}} \times \overbrace{(1 - F(v_1))}^{\text{probability } v \geq v_1}$$

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- i.e., of order Δ^2

step 3: boring

optimal G and R are finite monotone partitional

- preliminary result: quality increments are non-decreasing, i.e., if we let q_k be the quality level

$$\Delta q_{k+1} = q_{k+1} - q_k \geq q_k - q_{k-1} = \Delta q_k$$

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  - fails if $\Delta q_k > \Delta q_{k+1}$, i.e., it is optimal to lower threshold

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- preliminary result: quality increments are non-decreasing
- no accumulation points

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- pooling result breaks if "less than"...

how many items?

Theorem (the optimality of single item)

If Q is convex, the optimal menu has a single item.

- i.e., uniform lottery over qualities is sold at posted price to included agents, full surplus extraction
- Q convex $=$ increasing density of qualities
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  - number of items is less than $\frac{\bar{q}}{\underline{q}}$
  - if we drop the upper bound on values, (countably) infinite partition

endogenizing qualities

- exogenous distribution of qualities Q
  - as in (published) model of Loertscher and Muir (2002)
- endogenous distribution of qualities
  - convex cost $c(q)$ of producing quality q, where $c(\cdot)$ is convex
  - as in model of Mussa and Rosen (1978)
- earlier version of paper analyzed latter problem, current version gives it as an extension
  - exogenous case cleaner theoretically
  - endogenous case more canonical

digital economy motivation

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2. little personalized pricing, perhaps because....
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    - they can search under friends' or artificial digital identities
3. but buyers receive information in the form of (implicit or explicit) recommendations....

recommender system implementation

- the seller chooses
  - a finite menu
  - a recommendation rule mapping buyers' values to items
- the menu is public
- the recommendation rule satisfies an interim obedience constraint

conclusion

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- methodological takeaway: two majorization constraints
- theory takeaway: classic conflict between efficiency and minimizing information rent translates into simple menus (i.e., finite or single item)
- a digital market takeaway: recommender systems are more likely to be observed for horizontally differentiated goods than vertically differentiated goods

signal properties: monotone partitional

- a signal G is *monotone partitional* if it partitions values into convex informations sets (i.e., singletons or intervals)

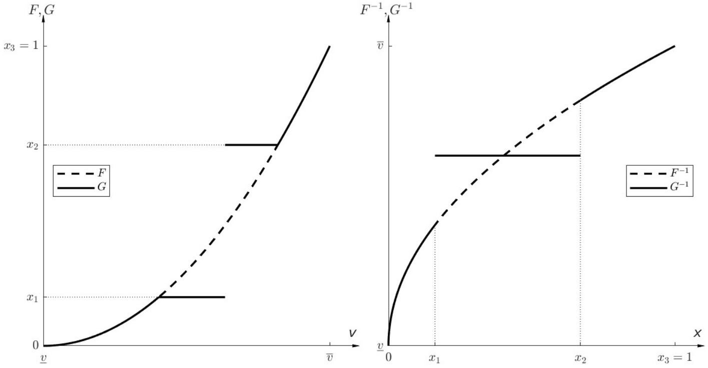

Figure: A monotone partitional distribution G which majorizes $F(v) = v^2$. The distribution G has intervals of complete disclosure and of pooled disclosure. The distributions F, G are on the left, the quantile distributions F^{-1}, G^{-1} on the right.

signal properties 2: pooling

- a monotone partitional signal G is *pooling* if every set in the partition is an interval (i.e., no singletons)
- a monotone partitional signal G is *finite* if it consists of a finite collection of sets

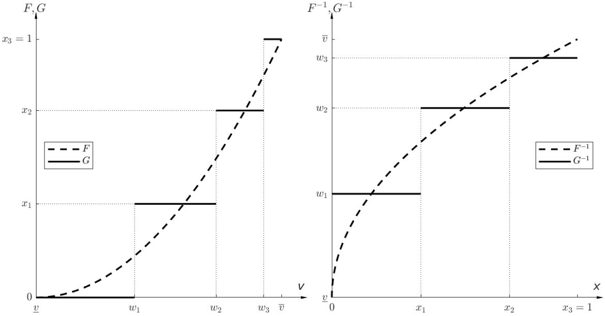

Figure: A finite and pooling monotone partitional distribution G which majorizes $F(v) = v^2$ and has only intervals of pooled disclosure. The specific distribution G is the optimal distribution for a quality distribution $Q(q) = q^{1/4}$.

example

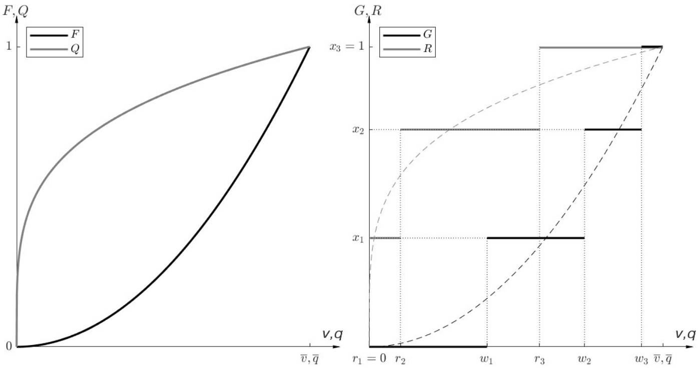

Figure: The given value and quality distributions $F(v) = v^2$ and $Q(q) = q^{1/4}$ are depicted on the left. The associated optimal monotone pooling distributions G and R are depicted on the right.

pooling argument I

- we will argue that **if** there was any small interval $[v_1, v_2]$ with full separation, then profits would be improved by pooling a small neighborhood of values....
- the optimal allocation $q^*(v)$ is strictly increasing on $[v_1, v_2]$
- suppose we pooled values in this interval (and assigned the average quality of the optimal allocation) but kept information unchanged
- the decrease in revenue is of order

$$\overbrace{(v_2 - v_1)}^{\text{change in value}} \times \overbrace{(q^*(v_2) - q^*(v_1))}^{\text{change in quality}} \times \overbrace{(F(v_2) - F(v_1))}^{\text{probability of } v \in [v_1, v_2]}$$

or (if $\Delta = v_1 - v_2$)

$$\Delta^3$$

pooling argument II

- so decrease in profit is of order Δ^3

pooling argument III

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- so increase in payments and thus profit is at least

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- this is of order Δ^2