Information and Market Power

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Introduction: Price Impact

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- Such effects overwhelm large number of trader effects

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Introduction: Methodology

- We will characterize what can happen for any information structure
- We have been thinking about this in a variety of settings
- Application of tools developed elsewhere to an environment with **linear best responses, normal information and maintaining symmetry**
- But can now compare all outcomes that can arise in the same environment for different mechanisms (e.g., Cournot, Kyle)

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- Kyle model relaxes both constraints

Talk

1. Environment
2. Noise Free Information and Demand Function Competition
3. General Information Structures
4. General Mechanisms

Demand

- $i = 1, ..., N$ agents (buyers)
- agent i's net utility from a_i units of an asset (good) purchased at price p is

$$u_i(\theta_i, a_i) = \theta_i a_i - \frac{1}{2}a_i^2 - pa_i$$

- agent i's "valuation" (marginal value of first unit) is θ_i
- valuations are normally and symmetrically distributed:

$$\begin{pmatrix} \theta_i \\ \theta_j \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \rho_{\theta\theta}\sigma_\theta^2 \\ \rho_{\theta\theta}\sigma_\theta^2 & \sigma_\theta^2 \end{pmatrix}\right)$$

with mean $\mu_\theta > 0$, standard deviation $\sigma_\theta > 0$ and correlation coefficient $\rho_{\theta\theta} \in (0, 1)$

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  with mean $\mu_\theta > 0$, standard deviation $\sigma_\theta > 0$ and correlation coefficient $\rho_{\theta\theta} \in (0, 1)$
- interdependent valuations: idiosyncratic and common payoff shocks
  - as $\rho_{\theta\theta} \to 0$: pure private values
  - as $\rho_{\theta\theta} \to 1$: pure common values

- (inverse) aggregate supply function:

$$p = c_0 + cA, \quad c_0, c \in \mathbb{R}_+$$

- could be derived from quadratic cost function

- individual values are normally distributed:0

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- useful alternative representation by orthogonal elements: common payoff shock

$$\overline{\theta} \triangleq \frac{1}{N} \sum_i \theta_i$$

and idiosyncratic payoff shock

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- resulting distribution of payoff uncertainty:

$$\begin{pmatrix} \Delta\theta_i \\ \overline{\theta} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} (1-\rho_{\theta\theta})\,\sigma_\theta^2 & 0 \\ 0 & \rho_{\theta\theta}\sigma_\theta^2 \end{pmatrix} \right)$$

- agent i has private but imperfect information about the payoff shocks
- signals $s_i \in \mathbb{R}^K$ are normally and symmetrically distributed:

$$\begin{pmatrix} \theta_i \\ \theta_j \\ s_i \\ s_j \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ \mu_\theta \\ \mu_s \\ \mu_s \end{pmatrix}, \begin{pmatrix} \Sigma_{\theta\theta} & \Sigma_{\theta s} \\ \Sigma_{\theta s} & \Sigma_{ss} \end{pmatrix}\right)$$

- signal $s_i \in \mathbb{R}^K$ of each agent can be multi-dimensional
- large class of possible information structures

Trading Mechanisms: Demand Function Competition

- Each agent submits a demand function (schedule):

$$x_i : \mathbb{R}^K \times \mathbb{R} \to \mathbb{R}$$

expressing a price contingent demand:

$$x_i (s_i, p) \in \mathbb{R}$$

- aggregate demand:

$$\sum_i x_i (s_i, p)$$

- market clearing:

$$p^* = c_0 + c \sum_i x_i (s_i, p^*)$$

- Each agent observes

$$s_i = \Delta\theta_i + \lambda \cdot \overline{\theta} + \varepsilon_i,$$

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- Price Impact

$$m = \frac{\partial p}{\partial x}$$

will also be an equilibrium parameter because agent i will want to set

$$x_i = \frac{\mathbb{E}[\theta_i | s_i, p] - p}{1 + m}$$

- Solve for $(\beta_0, \beta_s, \beta_p, m)$
- We will focus on price impact (m) and price sensitivity (β_p)

Price Impact depends on Price Sensitivity

- if agent i demanded x units of the good at price p, then market clearing would imply that

$$p = c_0 + c\left(x + \sum_{j \neq i}\left(\beta_0 + \beta_s s_j + \beta_p p\right)\right)$$

and so

$$m = \frac{\partial p}{\partial x} = \frac{c}{1 - (N-1)\, c\beta_p} \tag{2}$$

- by symmetry and linearity, these two equilibrium variables (m, β_p) are numbers and, in particular, so not depend on the agent, signals (of him and others) and the price

Price Sensitivity depends on Price Impact

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- In this case, will set

$$\beta_p = -\frac{1}{1+m}$$

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        - the agent's signal is mostly about the idiosyncratic component
        - his expectation of the common component will go up
        - his valuation of the good will increase

- Overall (including both effects):

$$\beta_p = -\frac{1}{1+m} + (1-\lambda)\left(\frac{1}{Nc} + \frac{1}{1+m}\right)$$

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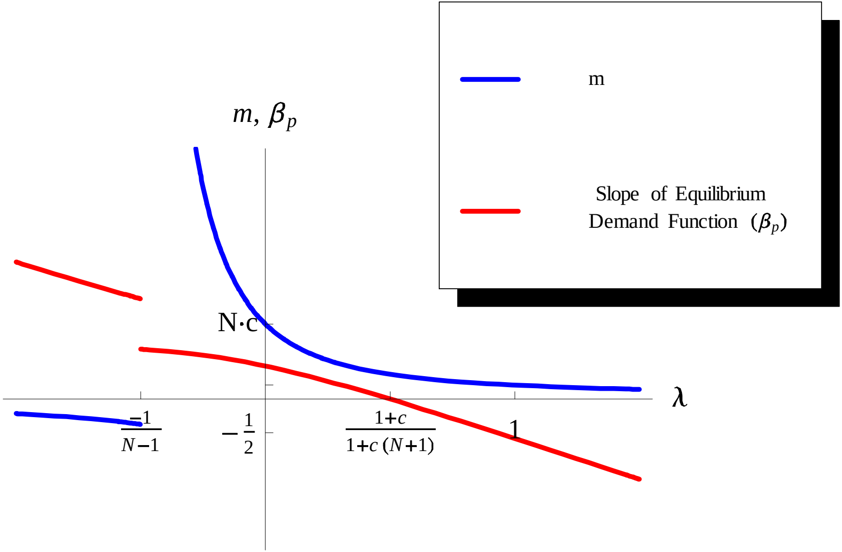
- Price sensitivity switches from negative to positive for some λ between 0 and 1

Two Equations in Two Unknowns

$$m = \frac{c}{1 - (N-1)\, c\beta_p}$$

$$\beta_p = -\frac{1}{1+m} + (1-\lambda)\left(\frac{1}{Nc} + \frac{1}{1+m}\right)$$

Price Impact and Price Sensitivity



A trick

- Solving for each information structure at once is hard work
- Without loss of generality, we can restrict attention to information structures where all an agent knows is his action in equilibrium (i.e., demand function):
  - "Bayes correlated equilibrium"
  - Bergemann and Morris (2012, 2015), Bergemann, Heumann and Morris (2015)

- write $\Delta a_i = a_i - \overline{a}$
- symmetry implies statistically equivalent description over 4 variables

$$\begin{pmatrix} \Delta a_i \\ \bar{a} \\ \Delta\theta_i \\ \bar{\theta} \end{pmatrix}$$

with mean

$$\begin{pmatrix} 0 \\ \mu_a \\ 0 \\ \mu_\theta \end{pmatrix}$$

and variance-covariance matrix....

$$\begin{pmatrix} \frac{N-1}{N}\left(1-\rho_{aa}\right)\sigma_a^2 & 0 & \rho_{\Delta\Delta}\sigma_{\Delta a}\sigma_{\Delta\theta} & 0 \\ 0 & \frac{(1+(N-1)\rho_{aa})\sigma_a^2}{N} & 0 & \rho_{\bar{a}\bar{\theta}}\sigma_{\bar{\theta}}\sigma_{\bar{a}} \\ \rho_{\Delta\Delta}\sigma_{\Delta a}\sigma_{\Delta\theta} & 0 & \frac{N-1}{N}\left(1-\rho_{\theta\theta}\right)\sigma_\theta^2 & 0 \\ 0 & \rho_{\bar{a}\bar{\theta}}\sigma_{\bar{\theta}}\sigma_{\bar{a}} & 0 & \frac{(1+(N-1)\rho_{\theta\theta})\sigma_\theta^2}{N} \end{pmatrix},$$

- normality implies mean vector μ and variance-covariance matrix Σ is necessary and sufficient for characterization
- outcome variables only, no reference to signals/information
- exogenous variables μ_θ,σ_θ^2, $\rho_{\theta\theta}$
- endogenous variables μ_a,σ_a^2, ρ_{aa}, $\rho_{\bar{a}\bar{\theta}}$, $\rho_{\Delta\Delta}$

Demand Function Competition Best Response Condition

- what happens if you impose best response condition

$$a_i = \frac{1}{1+m}\mathbb{E}[\theta_i - c_0 - cN\bar{a}|a_i, \bar{a}]), \quad \forall i, a_i, \bar{a}$$

  on statistical model?
- where m is a measure of price impact (market power)

Characterization of Demand Function Competition

Theorem

Demand function competition implies:

1. *mean of traded quantity is:*

$$\mu_a = \frac{\mu_\theta - c_0}{1 + Nc + m};$$

2. *second moments of trades are:*

$$\sigma_{\Delta a} = \frac{\rho_{\Delta\Delta}\sigma_{\Delta\theta}}{1 + m}, \sigma_{\bar{a}} = \frac{\rho_{\bar{\theta}\bar{a}}\sigma_{\bar{\theta}}}{1 + c + m};$$

3. *idiosyncratic and average correlation coefficients are:*

$$\rho_{\Delta\Delta}, \rho_{\bar{\theta}\bar{a}} \in (0, 1].$$

4. *market power* $m \in (-1/2, \infty)$

Informational Decentralization

- Bayes correlated equilibrium pins down joint distribution of a_i, $\Delta\theta_i$ and $\overline{\theta}$.
  - with three parameters m, $\rho_{\Delta\Delta}$ and $\rho_{\bar{\theta}\bar{a}}$
- general one dimensional symmetric information structures given by

$$s_i = \Delta\theta_i + \lambda \cdot \overline{\theta} + \varepsilon_i,$$

  - with three parameters λ, $\rho_{\varepsilon\varepsilon}$ and σ_ε
- one to one map in parameter space

- what happens if you impose best response condition

$$a_i = \frac{1}{1+c}\mathbb{E}[\theta_i - c_0 - cN\bar{a}|a_i]$$

on statistical model?

Theorem

(Bergemann, Heumann and Morris (2015)) Demand function competition implies:

1. *mean of traded quantity is:*

$$\mu_a = \frac{\mu_\theta - c_0}{1 + Nc + c};$$

2. *standard deviation of individual actions is:*

$$\sigma_a = \frac{\rho_{a\theta}\sigma_\theta}{1 + Nc\rho_{aa} + c};$$

3. *correlation coefficients satisfy:*

$$\rho_{a\theta} = \rho_{\Delta\Delta}\sqrt{(1-\rho_{aa})(1-\rho_{\theta\theta})} + \rho_{\bar{a}\bar{\theta}}\sqrt{\rho_{aa}\rho_{\theta\theta}}$$

- Can map back into the same three parameter one dimensional signal structure.

Intuition

- First moment:
  - Under Cournot competition, price impact is independent of information structure
  - Under demand function competition
    - price impact varies
    - there is an additional degree of freedom in the first moment
- Second moments
  - Agents are less informed under Cournot competition
    - Arbitrary variance of total output is possible
  - Under demand function competition
    - it is as if agents know the equilibrium price (and thus total quantity)
    - there is an additional restriction in the second moment

More Mechanisms

- Condition on noisy prices: move smoothly from demand function competition to Cournot, continuity in characterizations
- Variation on static Kyle model
  - richer because we have common and idiosyncratic shocks
  - add noise traders
  - market maker plays role of best response function
- outcomes are superset of demand function competition and Cournot
  - do not condition on prices
  - there is variable price impact

- Useful, feasible and insightful to abstract from fine details of the information structure
- Can get new insight into price impact in this framework
- Can compare alternative mechanisms in common outcome space