Advanced Economic Growth, Problem Set 2

This problem set is due on or before the recitation on Friday, October 12

Please answer the following questions:

Exercise 1   *1. What is the effect of competition on the rate of growth of the economy in a standard product variety model of endogenous growth? What about the quality-ladder model? Explain the intuition.*

2. Now consider the following one-period model. There are two Bertrand duopolists, producing a homogeneous good. At the beginning of each period, duopolist 1's marginal cost of production is determined as a draw from the uniform distribution $[0, \bar{c}_1]$ and the marginal cost of the second duopolist is determined as an independent draw from $[0, \bar{c}_2]$. Both cost realizations are observed and then prices are set. Demand is given by $Q = A - P$, , with $A > 2 \max \{\bar{c}_1, \bar{c}_2\}$.

(a) Characterize the equilibrium pricing strategies and calculate expected ex ante profits of the two duopolists.

(b) Now imagine that both duopolists start with a cost distribution $[0, \bar{c}]$, and can undertake R&D at cost μ. If they do, with probability η, their cost distribution shifts to $[0, \ \bar{c} - \alpha]$ where $\alpha < \bar{c}$. Find the conditions under which one of the duopolists will invest in R&D and the conditions under which both will.

(c) What happens when $\bar{c}$ declines? Interpreting the decline in $\bar{c}$ as increased competition, discuss the effect of increased competition on innovation incentives. Why is the answer different from that implied by the baseline endogenous technological change models of expanding varieties or Schumpeterian growth?

Exercise 2 *Consider a version of the baseline directed technological change model introduced in the lectures with the only difference that technological change is driven by quality improvements rather than expanding machine varieties. In particular, let us suppose that the intermediate goods are produced with the production functions:*

$$Y_L(t) = \frac{1}{1-\beta} \left[\int_0^1 q_L(\nu, t) x_L(\nu, t \mid q)^{1-\beta} d\nu \right] L^\beta, \text{ and}$$

$$Y_H(t) = \frac{1}{1-\beta} \left[\int_0^1 q_H(\nu, t) x_H(\nu, t \mid q)^{1-\beta} d\nu \right] H^\beta.$$

Producing a machine of quality q costs ψq, where we again normalize $\psi \equiv 1 - \beta$. R&D of amount $Z_f(\nu, t)$ directed at a particular machine of quality $q_f(\nu, t)$ leads to an innovation at the flow rate $\eta_f Z_f(\nu, t)/q_f(\nu, t)$ and leads to an improved machine of quality $\lambda q_f(\nu, t)$, where $f = L$ or H, and $\lambda \bmod el (1-\beta)^{-(1-\beta)/\beta}$, so that firms that undertake an innovation can charge the unconstrained monopoly price.

1. *Define an equilibrium in this economy.*

2. *Specify the free entry conditions for each machine variety.*

3. *Characterize the BGP equilibrium, show that it is uniquely defined and determine conditions such that the growth rate is positive and the transversality condition is satisfied.*

4. *Derive the BGP equilibrium relationship between relative technologies.*

5. *Show that the equivalents of weak and strong bias results hold in this environment.*

6. *Characterize the transitional dynamics in this economy.*

7. *Characterize the Pareto optimal allocation and compare it to the decentralized equilibrium.*

8. *What are the pros and cons of this model relative to the baseline model we studied in the lectures.*

Exercise 3 *Consider the directed technological change model discussed in the lectures and recall that in the baseline model the supplies of the two factors were exogenous and we focused on the impact of relative supplies on factor prices. In this exercise, we look at the joint determination of relative supplies and technologies.*

Let us focus on a model with the two factors corresponding to skilled and unskilled labor. Suppose a continuum v of unskilled agents are born every period, and each faces a flow rate of death equal to v, so that population is constant at 1. Each agent chooses upon birth whether to acquire education to become a skilled worker. For agent x it takes K_x periods to become skilled, and during this time, he earns no labor income. The distribution of K_x is given by the distribution function $\Gamma(K)$ which is the only source of heterogeneity in this economy. The rest of the setup is the same as in the text. Suppose that $\Gamma(K)$ has no mass points. Define a BGP as a situation in which H/L and the skill premium remain constant.

1. *Show first that in BGP, there is a single-crossing property: if an individual with cost of education K_x chooses schooling, another with $K_{x'} < K_x$ must also acquire skills. Conclude from this that there exists a cutoff level of talent, $\bar{K}$, such that all $K_x > \bar{K}$ do not get education.*

2. *Show that, along BGP relative supplies take the form:*

$$\frac{H}{L} = \frac{\Gamma(\bar{K})}{1 - \Gamma(\bar{K})}.$$

   Explain why such a simple expression would not hold away from the BGP.

3. *How would you determine $\bar{K}$? [Hint: agent with talent $\bar{K}$ has to be indifferent between acquiring skills and not].*

   Show that the relative supply of skills as a function of the skill premium must satisfy

$$\frac{H}{L} = \frac{\Gamma\left(\ln\omega/\left(r^* + v - g^*\right)\right)}{1 - \Gamma\left(\ln\omega/\left(r^* + v - g^*\right)\right)},$$

   where r^* and g^* refer to the BGP interest-rate and growth rate.

4. *Determine the BGP skill premium by combining this equation with BGP relationship between relative technologies. Can there be multiple equilibria? Explain the intuition.*

Exercise 4 *Consider the Jones model presented in the lectures, where ideas have a Pareto distribution and the economy functions at the "most productive" idea.*

1. *Show that if capital and labor are allocated in competitive markets, in general more than one technique will be used in equilibrium. [Hint: construct an example in which there are three ideas $i = 1, 2$ and 3, such that when only one can be used, it will be $i = 1$, but output can be increased by allocating some of labor and capital to ideas 2 and 3].*

2. *Show that in this case the exact aggregation result used in this model does not apply.*

Exercise 5 *Consider the modified Schumpeterian model with innovation by the incumbents. Set up the social planner's problem (of maximizing the utility of the representative household).*

1. *Show that this maximization problem corresponds to a concave current-value Hamiltonian and derive the unique solution to this problem. Show that this solution involves the consumption of the representative household growing at a constant rate at all points.*

2. *Show that the social planner will tend to increase growth because she avoids the monopoly markup over machines.*

3. *Show that the social planner will tend to choose lower entry because of the negative externality in the research process.*

4. *Give numerical examples in which the growth rate in the Pareto optimal allocation is greater than or less than the decentralized growth rate.*

Exercise 6 *Consider the the modified Schumpeterian model with innovation by the incumbentsand suppose that the R&D technology of the incumbents for innovation is such that if an incumbent with a machine of quality q spends an amount zq for incremental innovations, then the flow rate of innovation is $\phi(z)$ (and this innovation again increases the quality of the incumbent's machine to λq). Assume that $\phi(z)$ is strictly increasing, strictly concave, continuously differentiable, and satisfies $\lim_{z\to 0}\phi'(z) = \infty$ and $\lim_{z\to\infty}\phi'(z) = 0$.*

1. *Focus on steady-state equilibria and conjecture that $V(q) = vq$. Using this conjecture, show that incumbents will choose R&D intensity z^* such that $(\lambda - 1)v = \phi'(z^*)$. Combining this equation with the free entry condition for entrants and the accounting equation for growth rate, $g^* = (\lambda - 1)\phi z^* + (\kappa - 1)\hat{z}^*\eta(\hat{z}^*)$, show that there exists a unique BGP equilibrium (under the conjecture that $V(q)$ is linear).*

2. *Show that this equilibrium involves positive R&D both by incumbents and entrants.*

3. *Now introduce taxes on R&D by incumbents and entrants at the rates τ_i and τ_e. Show that the effects of both taxes on growth are ambiguous. What happens if $\eta(z) =$constant?*